Number Systems

What is a base?

Base (or radix) is simply the number of digits that a number system has.

Decimal (base 10) has 10 digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

Binary (base 2) has 2 digits: 0, 1

There are other number systems that have a different **base**.

A familiar number system: decimal

Here's how we represent **one hundred and twenty three** in decimal:

123

But how do we know this is **one hundred and twenty three**?

123

As we move from right to left, the number is talking about **increasing powers of the base** (in this case, base 10).

123

We know that the 3 represents three **ones**
(because $10^0 = 1$)

10^0

123

(3 x 1)

the 2 represents two **tens**
(because $10^1 = 10$)

10^1

123

(2 x 10)

and the 1 represents one **hundred**
(because $10^2 = 100$)

10^2

123

(1 x 100)

$$(1 \times 10^2) + (2 \times 10^1) + (3 \times 10^0) =$$

$$(1 \times 100) + (2 \times 10) + (3 \times 1) =$$

123

Let’s look at the **binary** number system

Binary is base 2 (or radix 2), so it’s numbers are: 0, 1

010

How would we convert this to decimal?

010

We work it out the same way we worked out our **base 10** number,

the only difference is we're using **base 2**.

010

2^2 2^1 2^0

0 1 0

2^2 2^1 $2^0 = 1$

0 1 0

0 x 1 = 0

2^2 $2^1 = 2$ 2^0

0 1 0

1 x **2** = 2

$2^2 = 4$ 2^1 2^0

0 1 0

0 x **4** = 0

$$(0 \times 2^2) + (1 \times 2^1) + (0 \times 2^0) =$$

$$(0 \times 4) + (1 \times 2) + (0 \times 1) =$$

2

Let’s whiteboard this binary number together:

1010

Challenge #1: convert this binary number to decimal.

101

Challenge #2: convert this binary number to decimal.

1011101

Challenge #3: work backwards; how would you represent the decimal number **205** in binary?

????????

There's an ~~app~~ method for that.

```
205.to_s(2)
```

```
“11001101”.to_i(2)
```

Let’s look at **Hexadecimal** (base 16)

Base SIXTEEN?! But we’ve only got 10 numbers (0-9)?!

What comes after 9?

Hexadecimal (base 16)

just use letters, bro.

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

So what’s this base 16 number?

B 4

We use the same method.

$16^1 = 16$ $16^0 = 1$

B 4

We use the same method.

$16^1 = 16$ $16^0 = 1$

B 4

B x 16 4 x 1

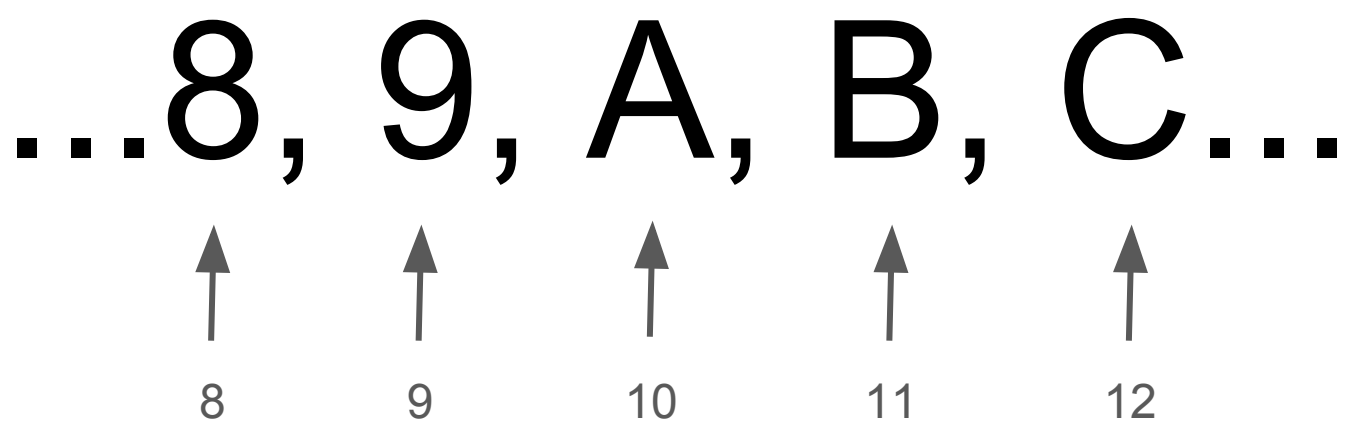
We use the same method.

$16^1 = 16$ $16^0 = 1$

B 4

B x 16 4 x 1

What's B?


...8, 9, A, B, C...
8
9
10
11
12

B is 11

$16^1 = 16$ $16^0 = 1$

B 4

11 x 16 4 x 1

= 176 **= 4**

B 4

176 + 4 = 180

180

Discussion

What’s the largest number can we represent in a 5 digit decimal number?

X X X X X

Discussion

How could we store a larger number in 5 digits?

X X X X X

Discussion

What’s the largest number can we represent in an 8 digit binary number?
(aka one byte)

X X X X X X X X