



Optimization Algorithms

1. Introduction

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Outline

1. Transportation Planning Example
2. Optimization Problems
3. Example Framework Scenario: Smart Manufacturing
4. Exact vs. Heuristic Algorithms
5. Summary and Outlook



What is Optimization?

- In this unit, we want to get a rough feeling about what optimization is.

What is Optimization?

- In this unit, we want to get a rough feeling about what optimization is.
- So let us start by looking at some examples for optimization problems.

Transportation Planning Example



Transportation Planning: Task

- Build a system which tells a logistics company what it needs to do to fulfill all transportation orders at minimum costs.³⁻⁷

Transportation Planning: Task

- Build a system which tells a logistics company what it needs to do to fulfill all transportation orders at minimum costs.^{3–7}
- What does this mean?

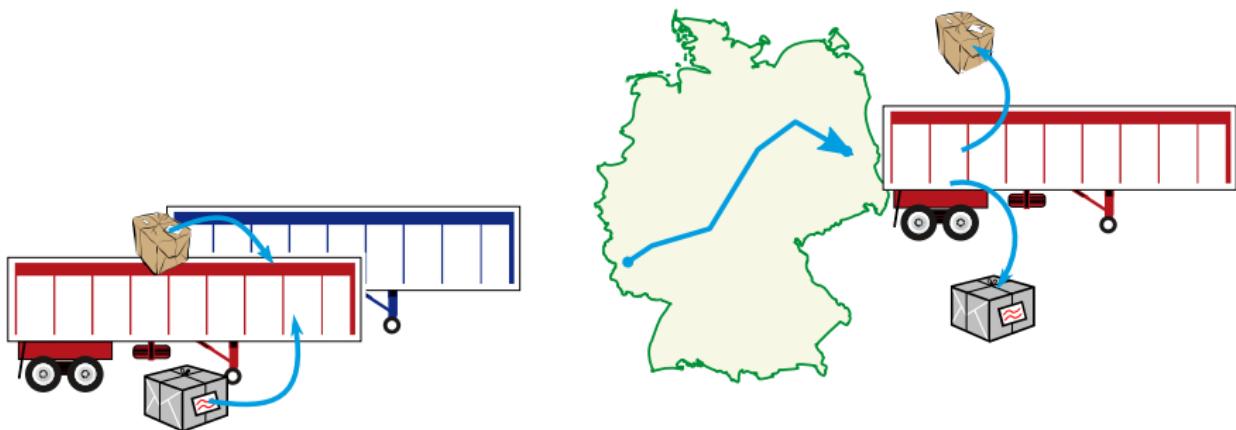
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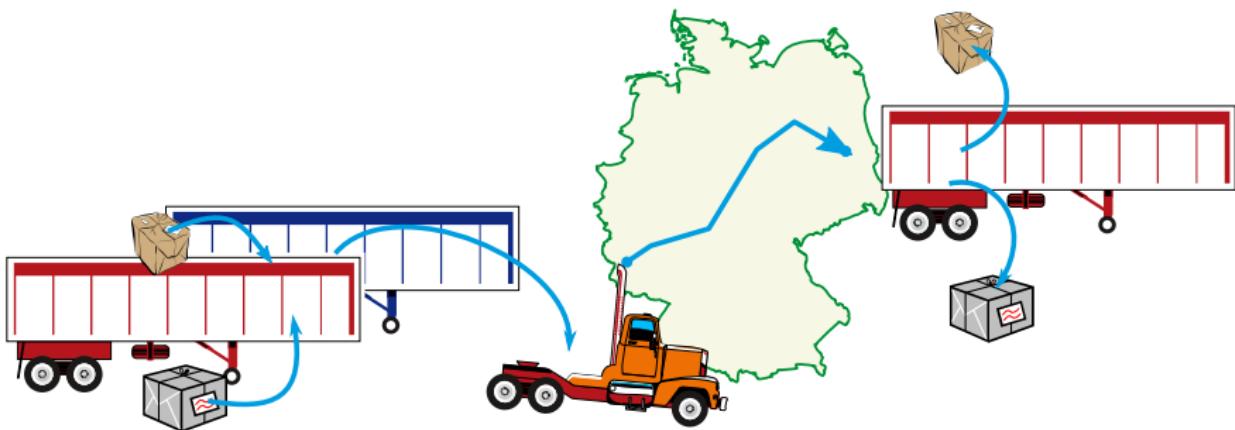
Transportation Planning: Task

- Build a system which tells a logistics company what it needs to do to fulfill all transportation orders at minimum costs.³⁻⁷
 1. Find routes on the map and assignments of orders to containers



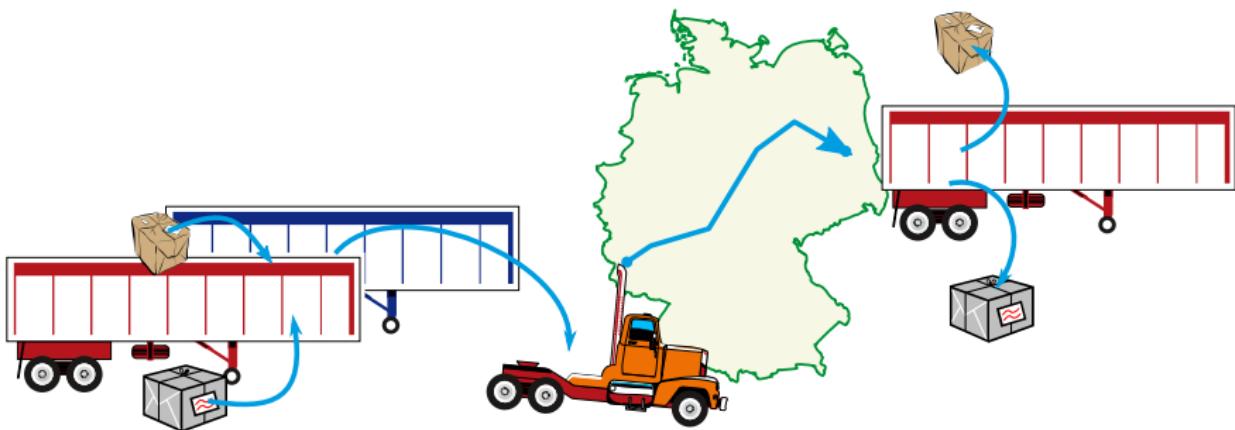
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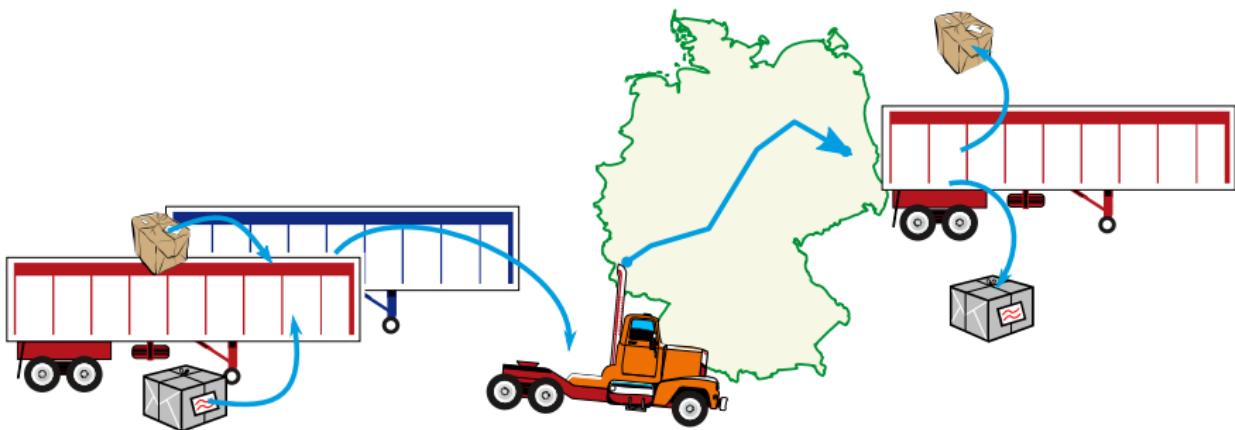
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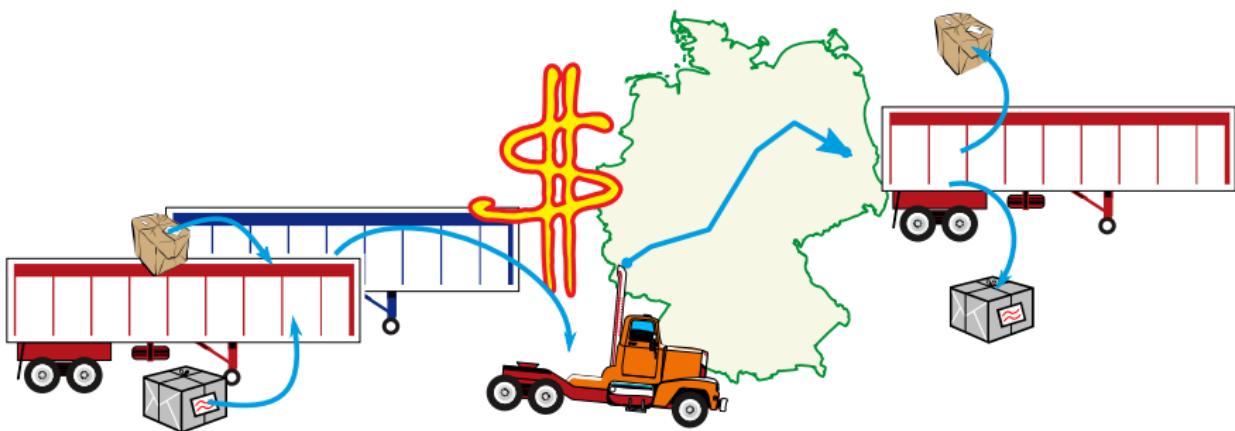
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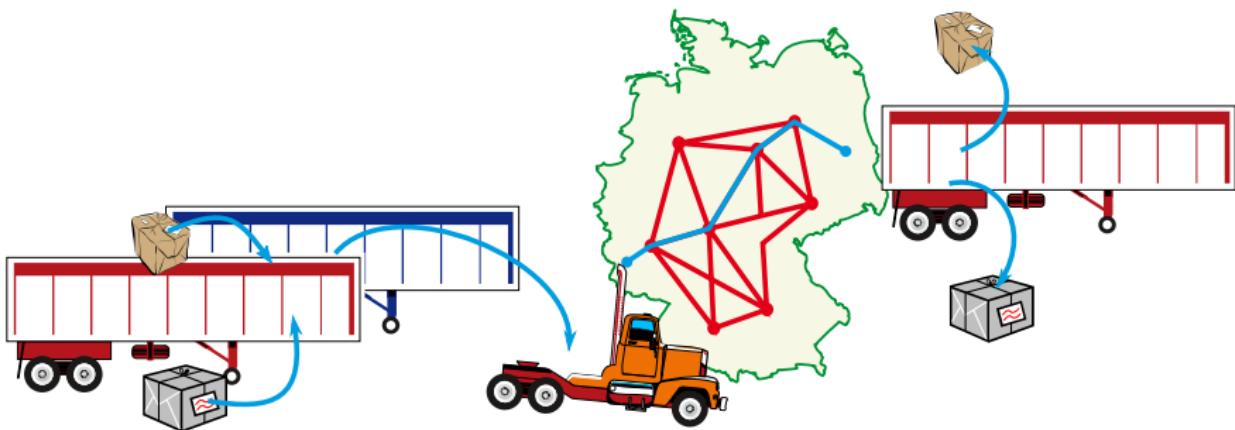
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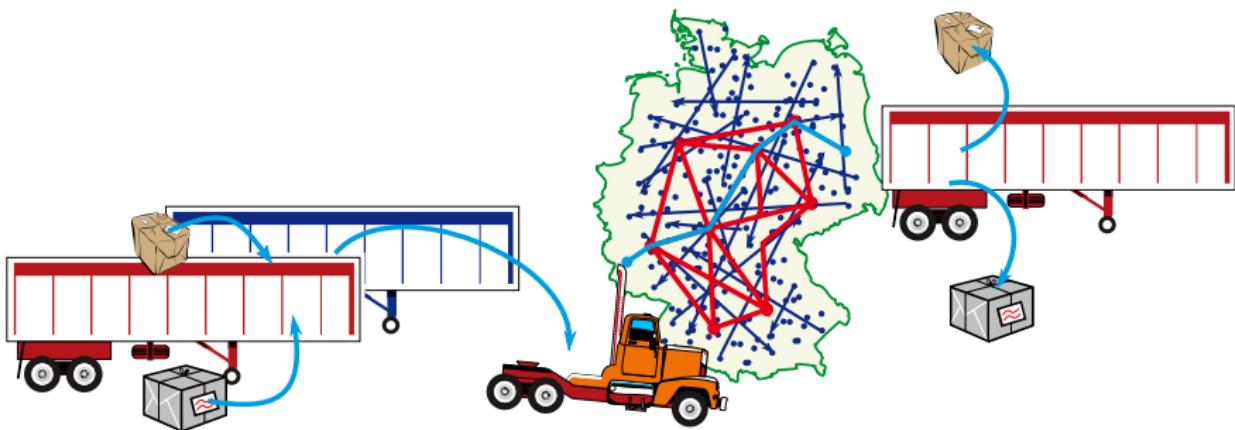
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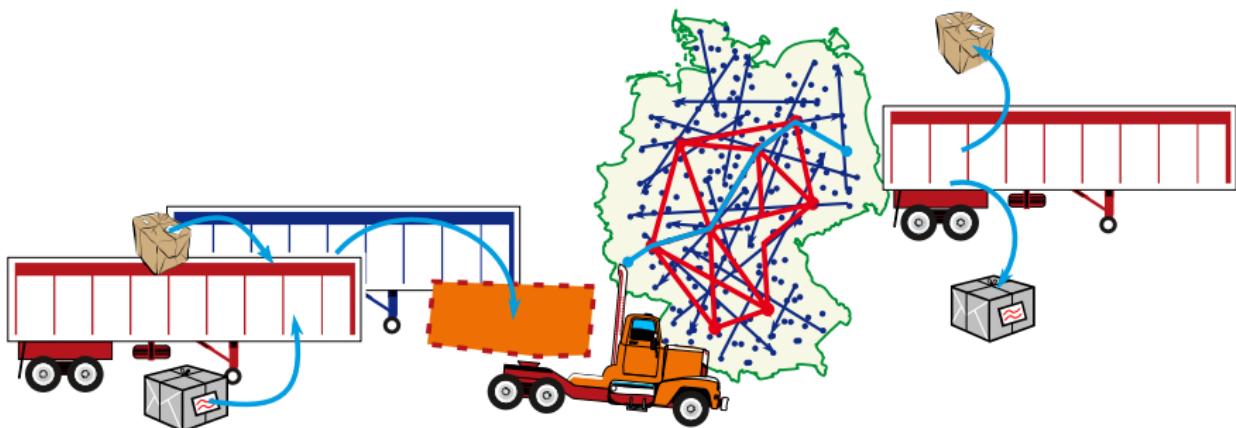
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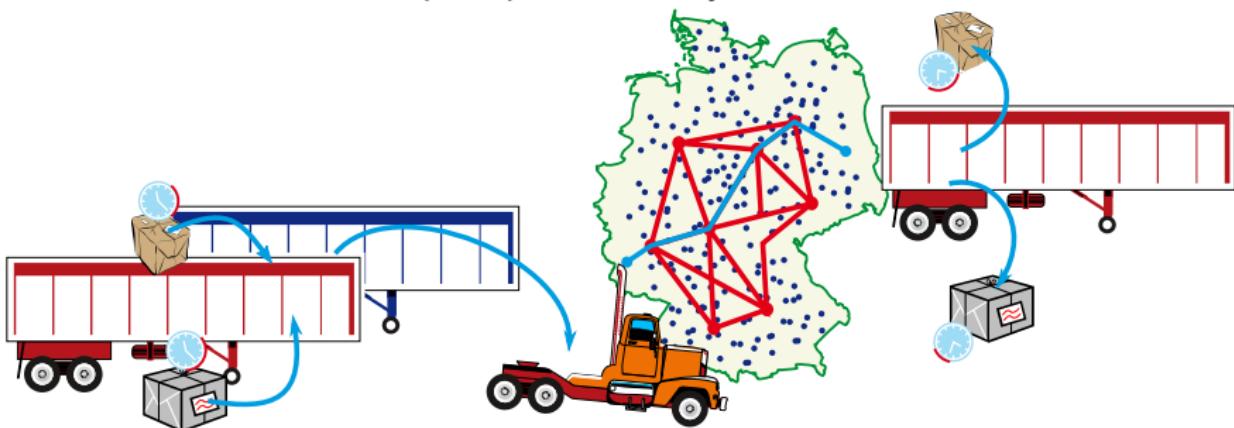
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 5. and constraints and laws.
 6. Time limit for optimization: 1 day

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- Before the problem was solved by hand, by manual planning with Excel sheets...
- With an optimization algorithm, we can get better solutions than that.
- In this course, you will learn how we can do such a thing.

Optimization Problems



What is optimization?

So what actually is optimization?^{1 2 8}

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What is the **cheapest** way to get from Hefei to Beijing?

What is optimization?

So what actually is optimization?^{1 2 8}



What is the **fastest** way for our team to finish all the work?

What is optimization?

So what actually is optimization?^{1 2 8}



How can I package these products using the **fewest** boxes?

What is optimization?

So what actually is optimization?^{1 2 8}



How do I arrange the components on a circuit board so I need the **shortest** electrical cable length?

What is optimization?

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Definition (Optimization Problem: Economical View)

An optimization problem is a situation which requires deciding for one choice from a set of possible alternatives in order to reach a predefined/required benefit at minimal costs.

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Definition (Optimization Problem: Simplified Mathematical View)

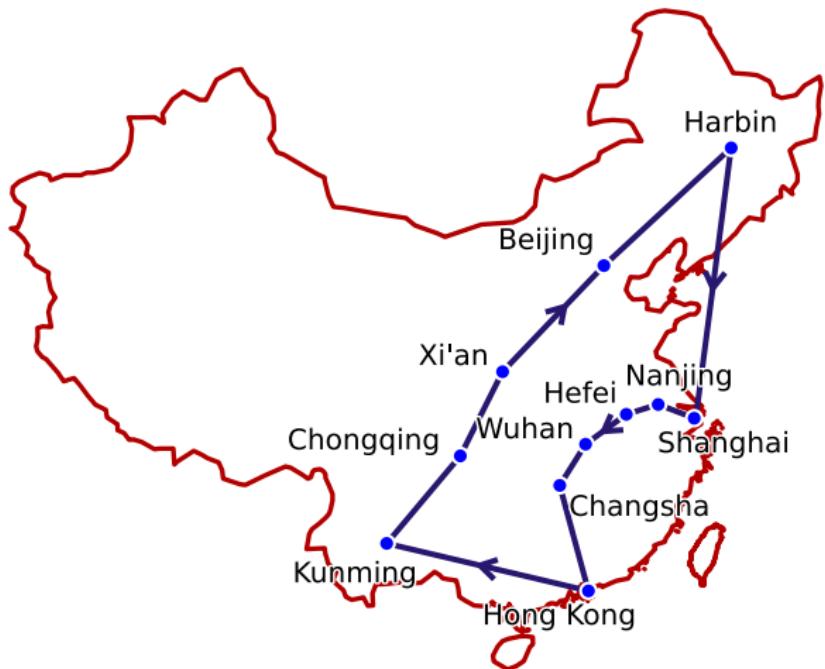
Solving an optimization problem requires finding an input value $y^* \in \mathbb{Y}$ from a set \mathbb{Y} of allowed values for which a mathematical function $f : \mathbb{Y} \mapsto \mathbb{R}$ takes on the smallest possible value.

More Examples

- Many questions in the real world are optimization problems

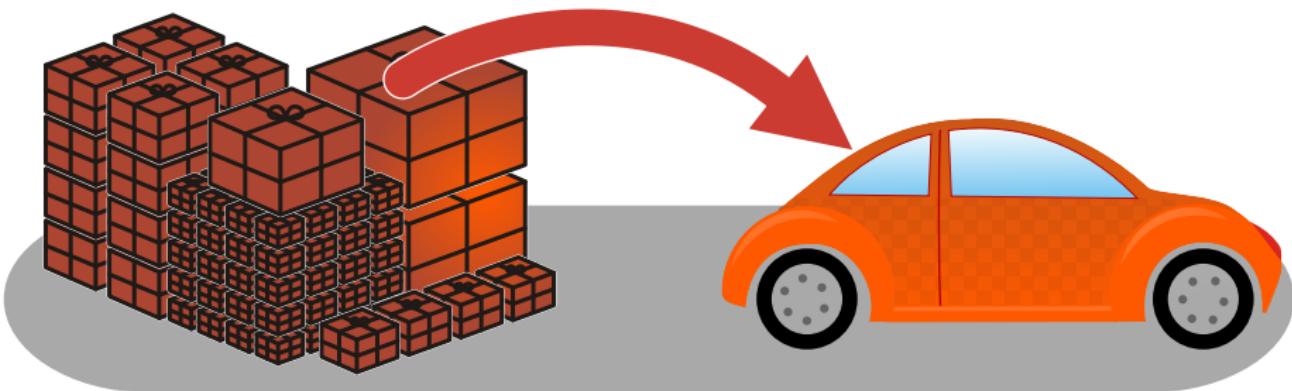
More Examples

- Many questions in the real world are **optimization problems**, e.g.,
 - Find the **shortest** tour for a salesman to visit a certain set of cities in China and return to Hefei!



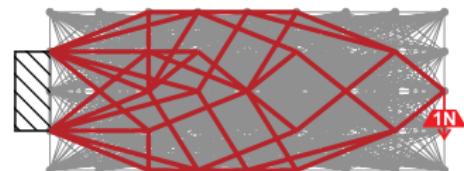
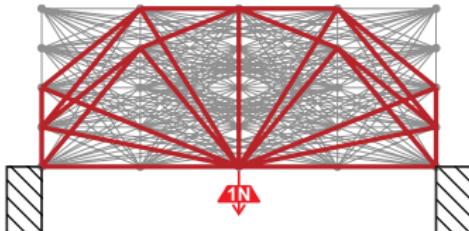
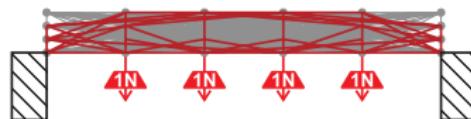
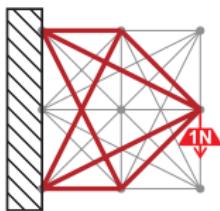
More Examples

- Many questions in the real world are **optimization problems**, e.g.,
 - Find the **shortest** tour for a salesman to visit a certain set of cities
 - I need to transport n items from here to another city but they are too big to transport them all at once. How can I load them best into my car so that I have to travel back and forth the least times?



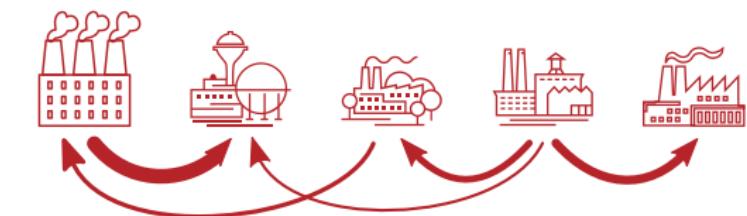
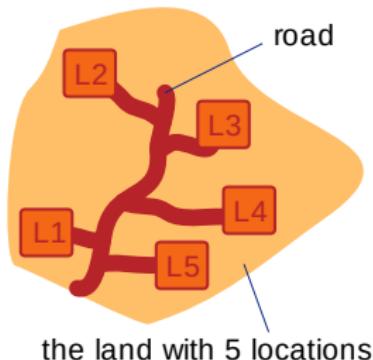
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 - Find the **shortest** tour for a salesman to visit a certain set of cities
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 - How can I construct a truss which can hold a certain weight with at most a certain amount of iron?



More Examples

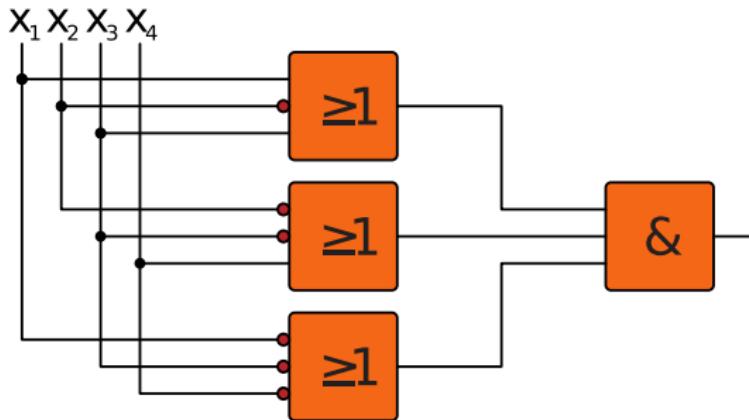
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 - I want to build a large factory with n workshops. I know the flow of material between each two workshops and now need to choose the locations of the workshops such that the overall running cost incurred by material transportation is **minimized**.



the 5 workshops which need to be assigned to the 5 locations and the different material flows between them

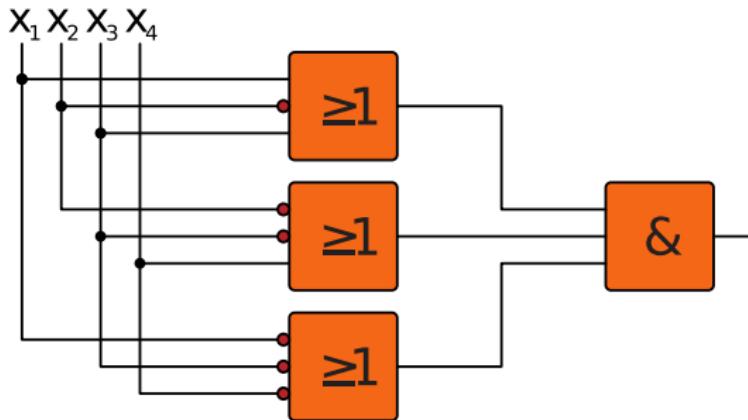
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 - Find the **shortest** tour for a salesman to visit a certain set of cities
 - I need to transport n items from here to another city
 - Construct a truss which can hold a certain weight
 - Assign workshops to locations
 - Which setting of x_1, x_2, x_3 , and x_4 can make $(x_1 \vee \neg x_2 \vee x_3) \wedge (\neg x_2 \vee \neg x_3 \vee x_4) \wedge (\neg x_1 \vee \neg x_3 \vee \neg x_4)$ become true?



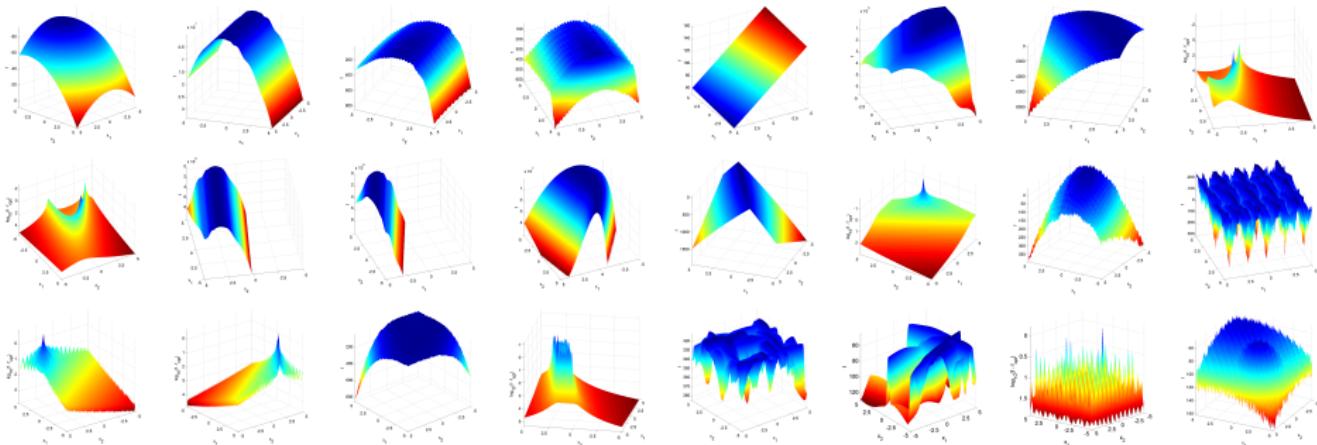
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More Examples

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 - Satisfy Boolean formula
 - Find the minima of complex, multi-dimensional mathematical formulas

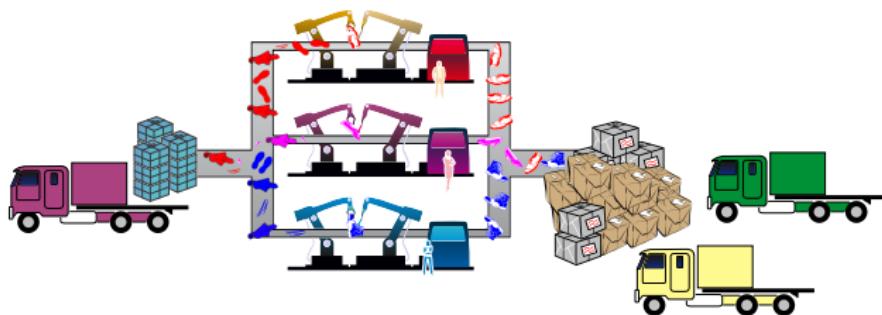


Example Framework Scenario: Smart Manufacturing



What is Smart Manufacturing?

Smart Manufacturing⁹...

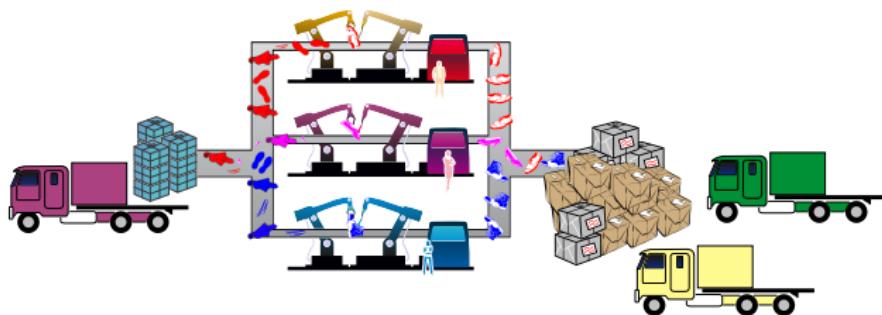


Intelligent
Decisions
by
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Software

What is Smart Manufacturing?

Smart Manufacturing⁹...

- has the goal of optimizing development, production, and logistics.

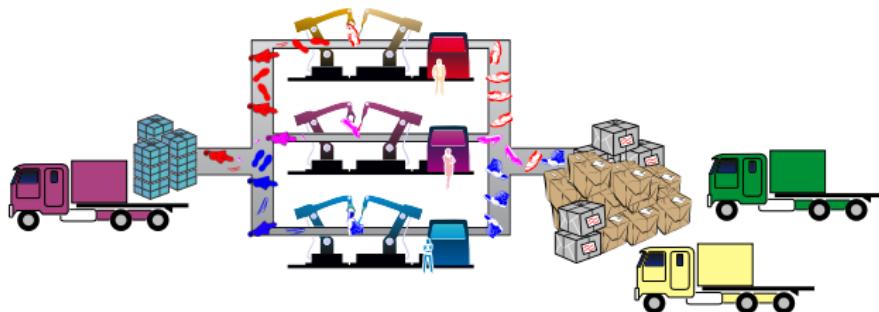


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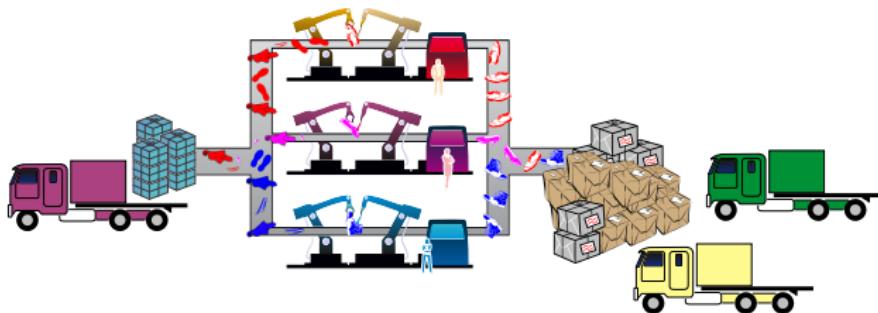


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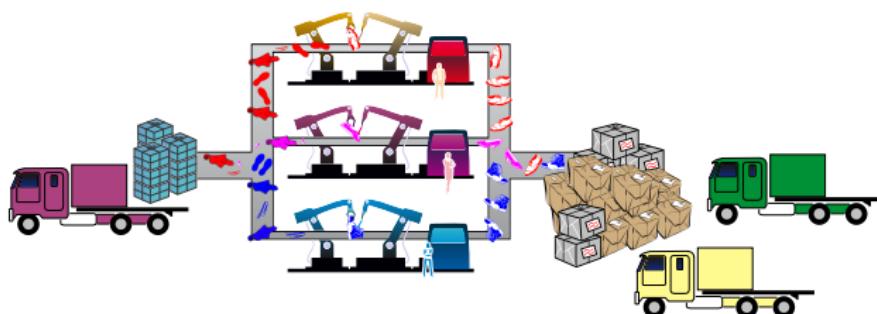


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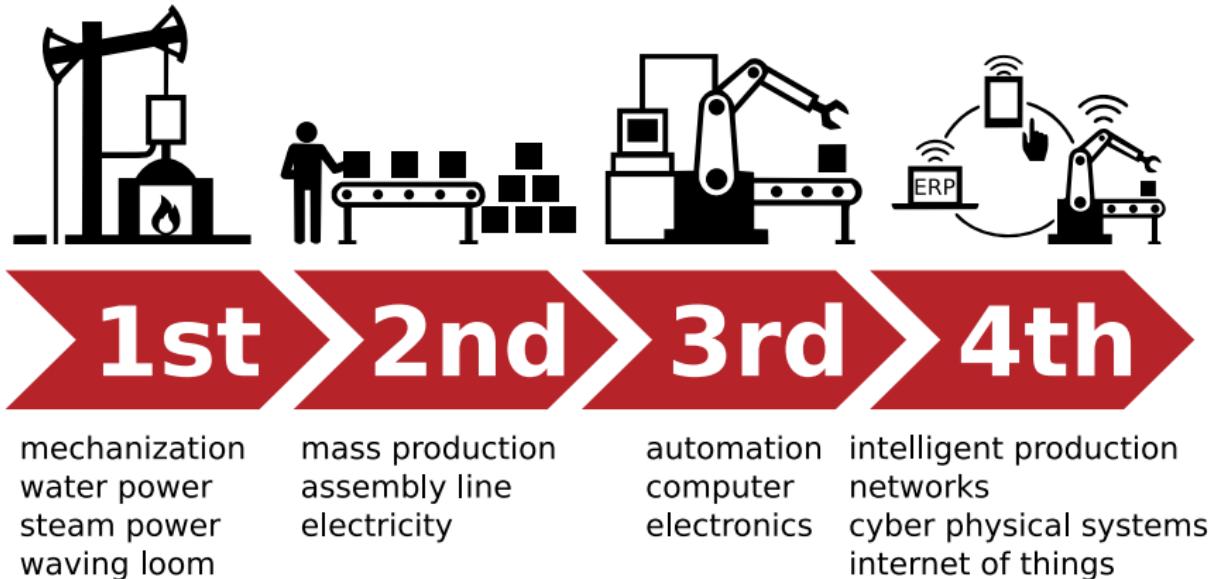
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- employs computer control and high levels of adaptability in the multi-phase process of creating a product from raw material.
- utilizes advanced information and manufacturing technologies to enable flexibility in production processes to address a dynamic market.
- requires increased workforce training for flexibility and use of the technology instead of simple repetitive tasks as in traditional manufacturing.



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Industry 4.0¹⁰



Involved Technologies

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Relationship with Smart Manufacturing

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- An enterprise should want to have **software** that can automatically make good suggestions, which can save costs and resources for the daily operations, the long term planning, and/or even its product/organizational development.
- All kinds of the previously mentioned problems can occur in manufacturing.

Relationship with Smart Manufacturing

- So how is all of this related to smart manufacturing?
- No enterprise can waste money or time or material or energy or any other resource.
- An enterprise must try to make decisions which are optimal from the perspective of cost and resource consumption.
- An enterprise must strive to improve its processes and products.
- An enterprise should want to have **software** that can automatically make good suggestions, which can save costs and resources for the daily operations, the long term planning, and/or even its product/organizational development.
- All kinds of the previously mentioned problems can occur in manufacturing.
- For example, logistics exist inside and outside a company, and even on the factory floor!

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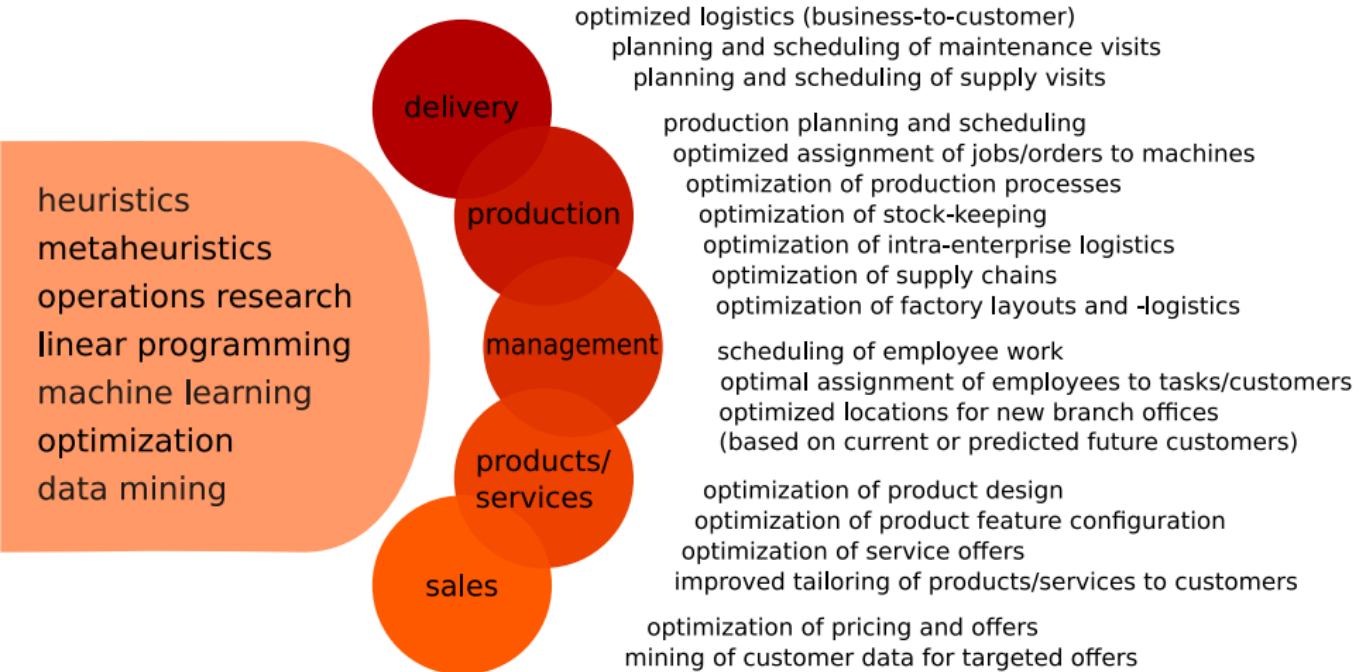
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- We focus only on the first of the two issues: optimization algorithms and their implementation.

Exact vs. Heuristic Algorithms

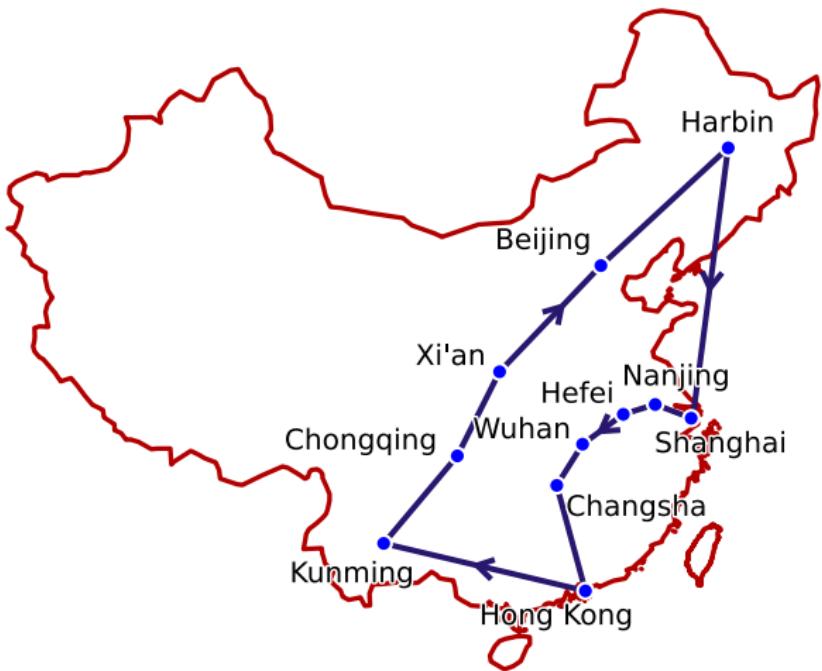


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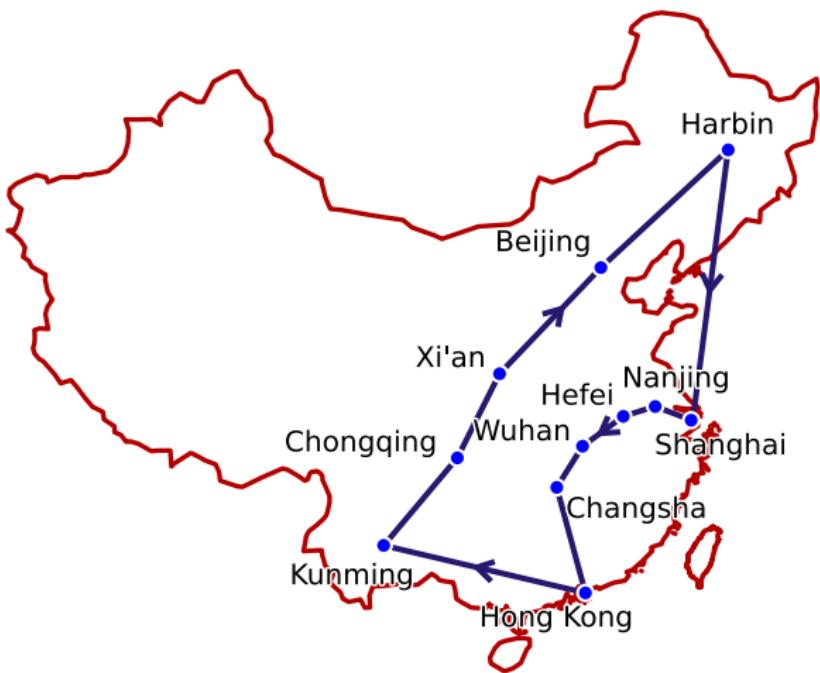
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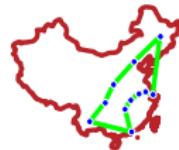
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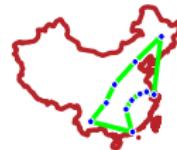
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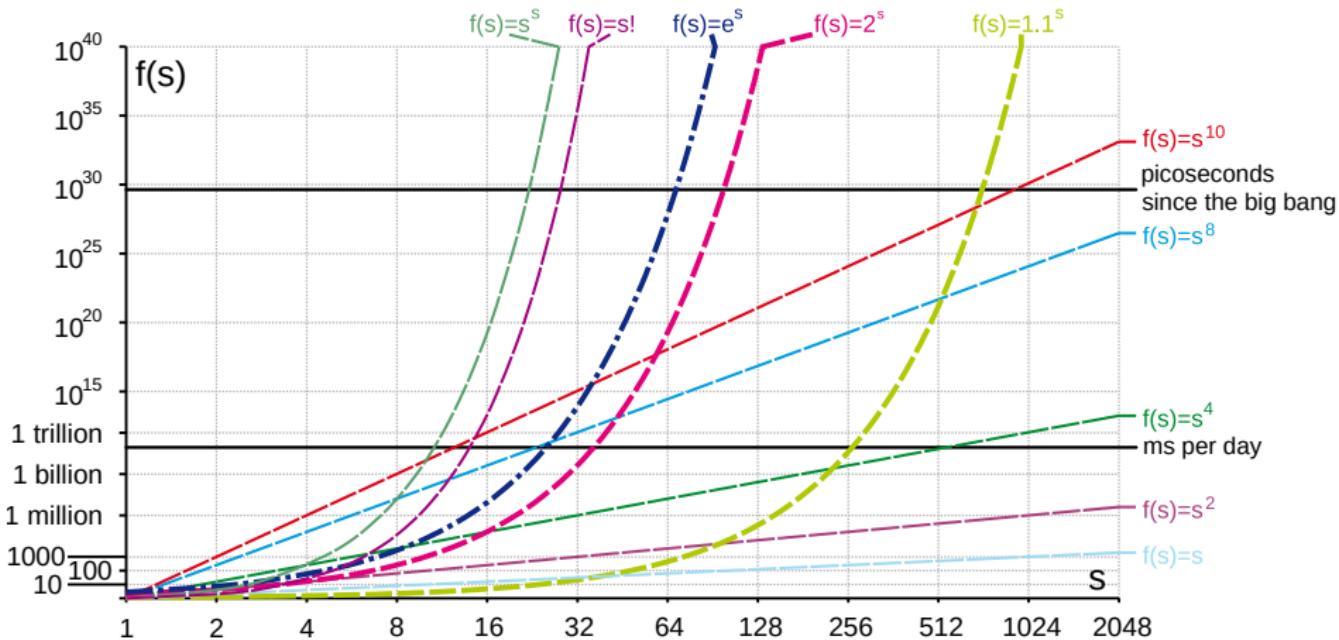
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 - Clearly, there is (at least) one shortest tour.
 - Theory proofs that the time to find this tour may grow exponentially with the number of cities we want to visit in the worst case.¹⁶⁻²⁰

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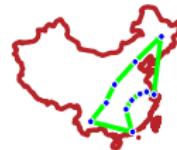
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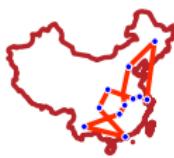
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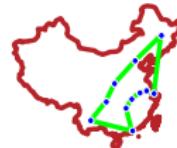
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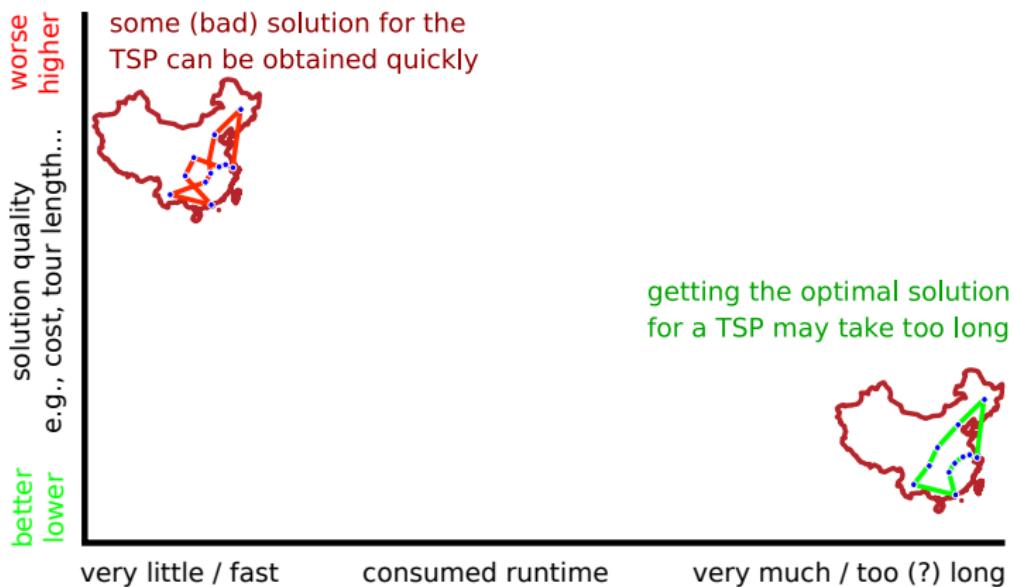
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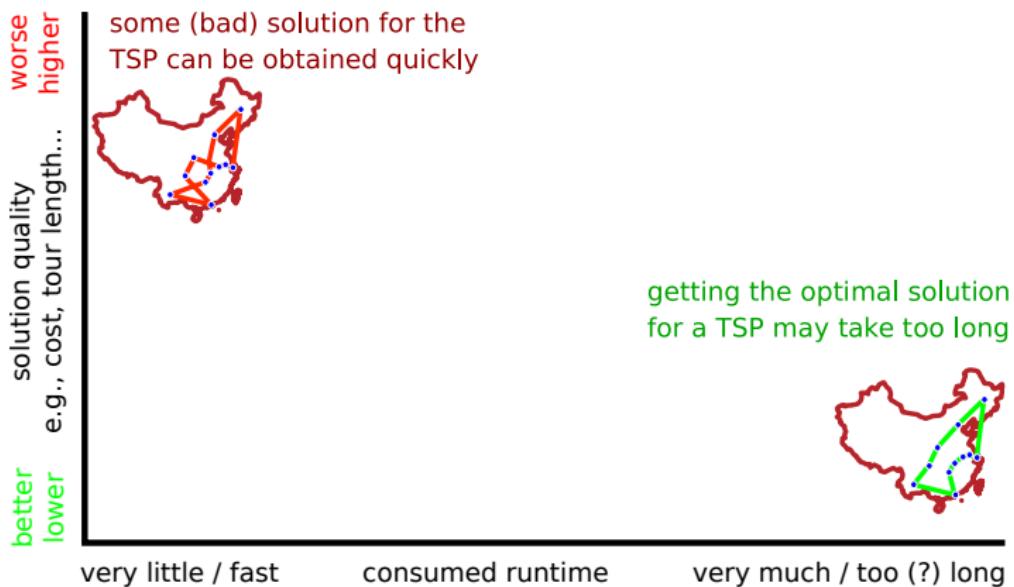
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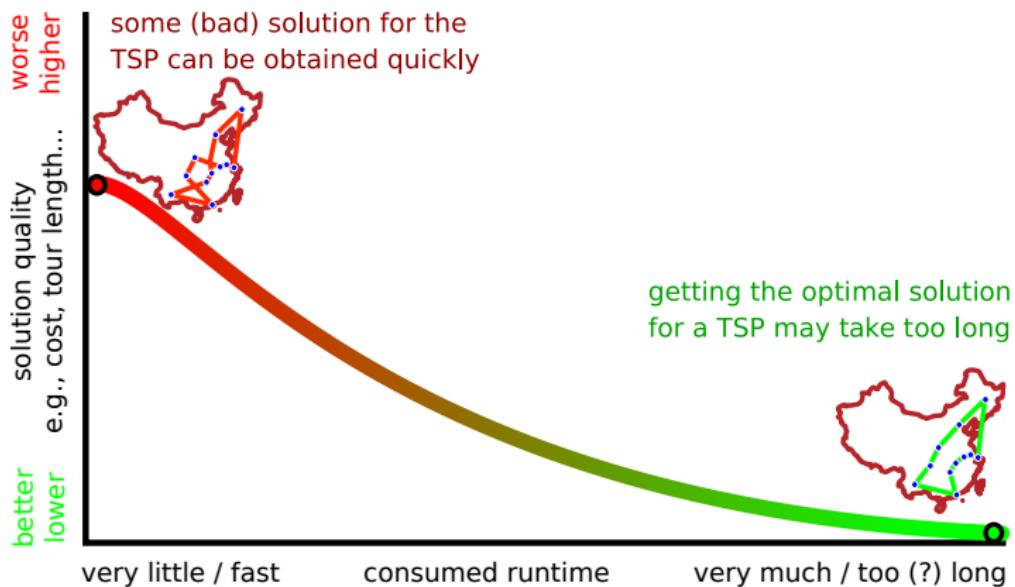
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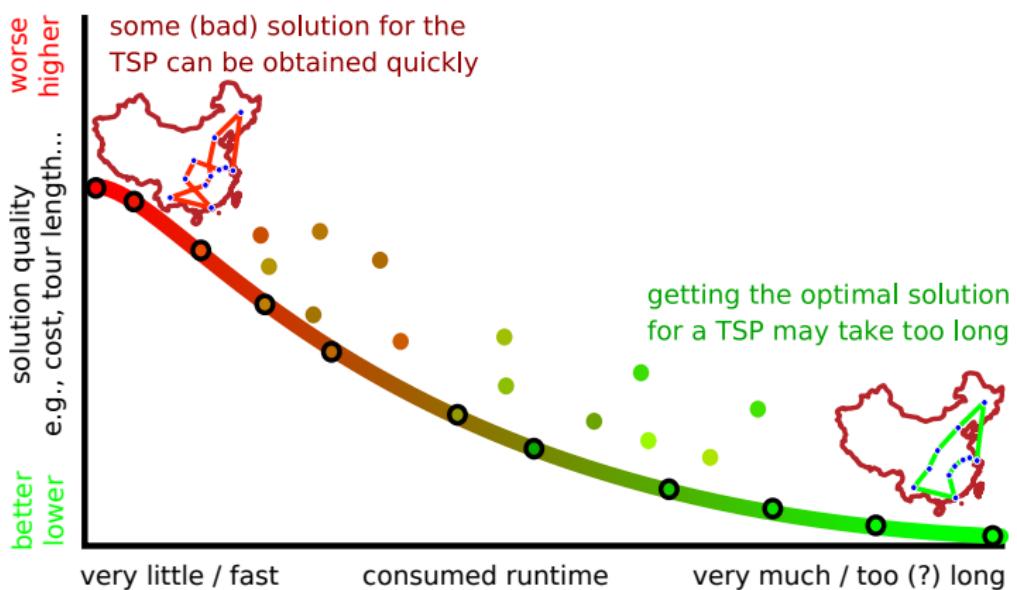
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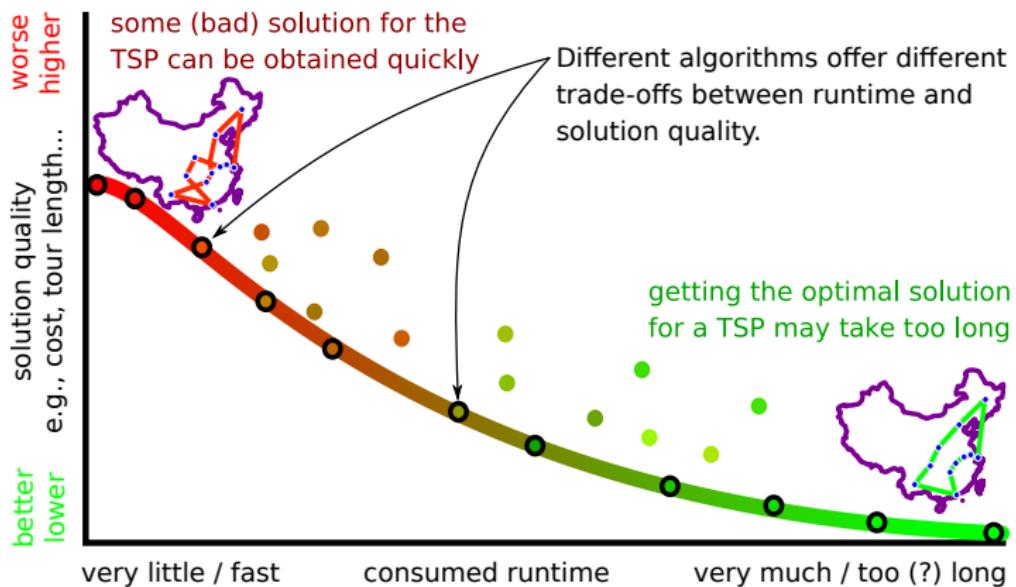
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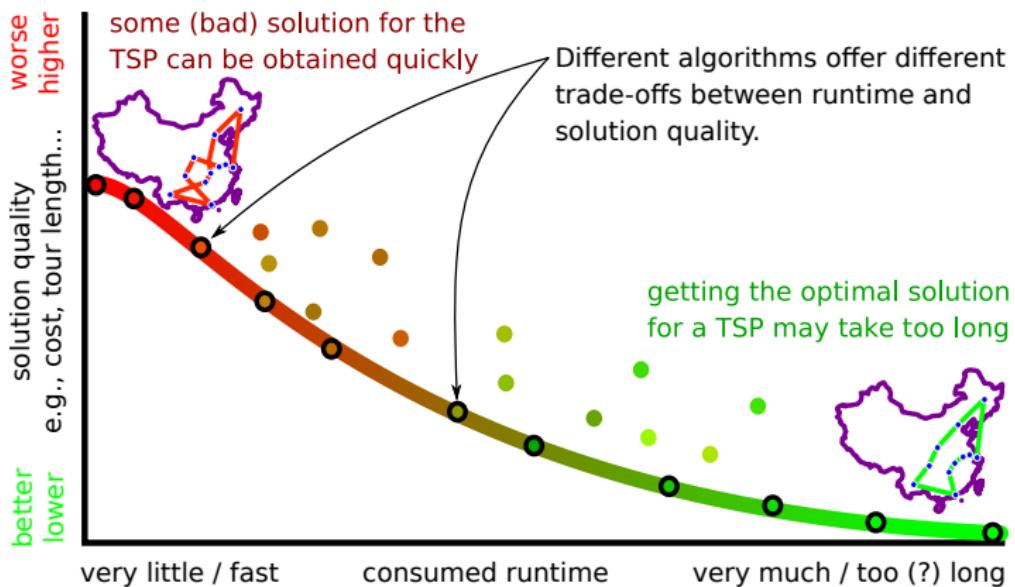
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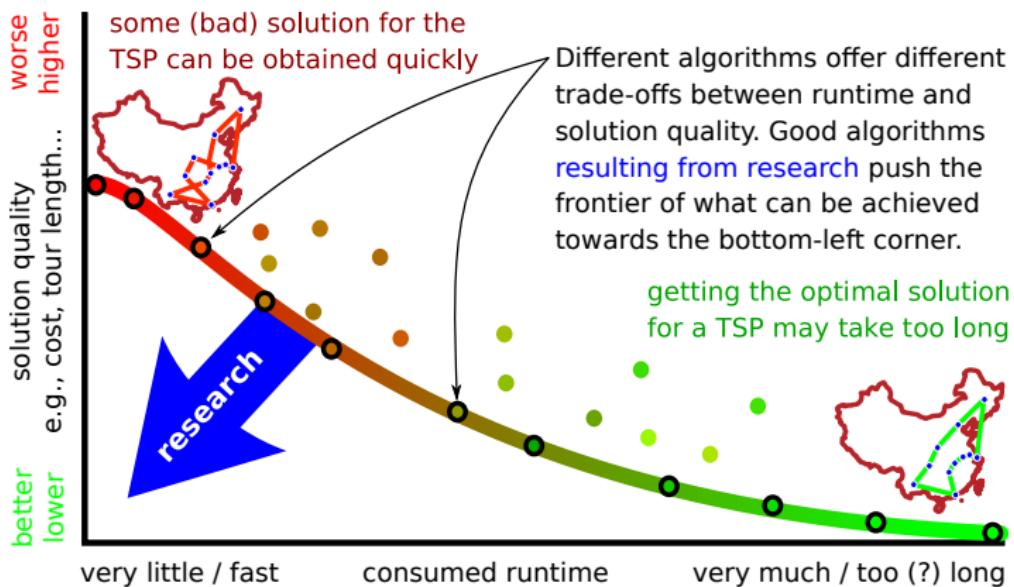
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- Experience is needed: How do I recognize an optimization problem? How can I quickly make a software that can solve it?
- We will try to get a good perspective and understanding of the very basics needed to navigate in the domain of optimization.
- The goal is to be able to recognize and identify optimization problems as they occur in many fields, especially in Intelligent Manufacturing scenarios, and to develop basic algorithms to solve them.

谢谢

Thank you



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