Frequency Fitness Assignment: Making Optimization Algorithms Invariant Under Bijective Transformations of the Objective Function Value

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***Abstract*—Under frequency fitness assignment (FFA), the fitness corresponding to an objective value is its encounter frequency in fitness assignment steps and is subject to minimization. FFA renders optimization processes invariant under bijective transformations of the objective function value. On TwoMax, Jump, and Trap functions of dimension s, the classical (1 + 1)-EA with standard mutation at rate 1/s can have expected runtimes exponential in s. In our experiments, a (1 + 1)-FEA, the same algorithm but using FFA, exhibits mean runtimes that seem to scale as s^2 ln s. Since Jump and Trap are bijective transformations of OneMax, it behaves identical on all three. On OneMax, LeadingOnes, and Plateau problems, it seems to be slower than the (1 + 1)-EA by a factor linear in s. The (1 + 1)-FEA performs much better than the (1 + 1)-EA on W-Model and MaxSat instances. We further verify the bijection invariance by applying the Md5 checksum computation as transformation to some of the above problems and yield the same behaviors. Finally, we show that FFA can improve the performance of a memetic algorithm for job shop scheduling.**

ndex Terms—(1 1)-EA, ev tionary algorithm (EA), ency fitness gnment A), j hop sc duling (J), F pr Ju lems, meti on,

which are then the basis for selection. Frequency fitness assignment (FFA) [1], [2] was developed to enable algorithms to escape from local optima. In FFA, the fitness corresponding to an objective value is its encounter frequency so far in fitness assignment steps and is subject to minimization. As we discuss in detail in Section II, FFA turns a static optimization problem into a dynamic one where objective values that are often encountered will receive worse and worse fitness.

In this article, we uncover a so-far unexplored property of FFA: it is invariant under any bijective transformation of the objective function values. This is the strongest invariance known to us and encompasses all order-preserving mappings. Other examples for bijective t ansformations include the negation, permutation, or even en ryption of the objec tive valu s. According to [3], invarianc extends performan observed a single function to an e e associa d inv ance clas hat is, it generalizes from ngle p class p en s. Th , it hopefully es res e pre atic e c