0.1 Balanced splitters and Catalan numbers

Definition 1. *A (n, r, m)-splitter, S, is a set of functions $s : [n] \to [m]$, such that for any set $R \subseteq [n]$ of size $|R| = r$, there is an $s \in S$ such that*

$$\lfloor r/m \rfloor \leq |s^{-1}(i) \cap R| \leq \lceil r/m \rceil$$

for all $i \in [m]$.

A balanced splitter is a splitter where for all $s \in S$ and $i \in [m]$,

$$\lfloor n/m \rfloor \leq |s^{-1}(i)| \leq \lceil n/m \rceil.$$