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Numerical implementation of temperature and creep in mass concrete

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Abstract

Mass concrete generates internal heat during the hydration period that occurs soon after casting. Added to these internal heat changes are the environmental conditions that the structure is subjected to during its design life. These thermal changes in the material affect the elastic and creep properties of the material, and in turn, the stress fields within the structure. The numerical implementation of these factors is illustrated in a three-dimensional finite element program that simulates the construction process of mass. The mathematical formulation, the numerical implementation and other implementation details are presented herein. The temperature and stress variation of a concrete block was analyzed and results show that temperature plays an important role in concrete structures. © 2001 Elsevier Science B.V. All rights reserved.

Keywords: FEM; Mass concrete; Thermal stresses; Creep; Numerical method

1. Introduction

The design and construction of mass concrete structures involves solving the problem of thermal stresses and temperature control. The material temperature changes due to two factors: (1) the internal hydration of concrete and (2) the environmental boundary conditions. Temperature not only influences the elastic modulus and creep properties of concrete, but it also produces thermal stresses. The temperature increase accelerates the initial elastic modulus of concrete. The creep rate is also increased with higher temperature and the creep strain is enlarged [1,2]. Therefore, the elastic modulus and creep variations at different locations within the mass concrete structures are all a function of temperature and which in turn is a function of time. This requires extending the analysis into another dimension, time. This spatio-temporal problem requires the modification of material properties with time, and if construction is being simulated, changes in gravity dead load

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also need to be combined. This can be accomplished with a modification to a finite element method program that dynamically updates the material properties with time.

This paper introduces a methodology to analyze the unsteady temperature and creep stress fields of mass concrete. Both temperature and stress distributions at different points in time form part of the output of the analysis. The methodology is presented as a numerical implementation using the finite element method, which can simulate the construction process of concrete. A three-dimensional program was developed and a numerical example is used for verification of the methodology and program.

2. Unsteady temperature

A mass concrete structure that generates internal heat due to hydration can be subjected to various boundary conditions, as shown in Fig. 1.

Eq. (1) and the following boundary conditions govern the temperature change in 3D space and time:

$$\frac{\partial}{\partial x}\left(a_x \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(a_y \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(a_z \frac{\partial T}{\partial z}\right) + \frac{\partial \theta}{\partial t} - \frac{\partial T}{\partial t} = 0, \tag{1}$$

where a_x, a_y, and a_z are the material thermal diffusivity.

$$\text{When } t = 0, \quad T = T_0(x, y, z). \tag{2}$$

On boundary C_1, the temperature is known. This is a specified temperature condition, such as a constant source of heat.

$$T = T_\mathrm{b}(t) \quad \text{on boundary } C_1. \tag{3}$$

On boundary C_2, the adiabatic condition is satisfied. This condition applies when the heat flux is equal to zero through that boundary.

$$\frac{\partial T}{\partial n} = 0 \quad \text{on boundary } C_2. \tag{4}$$

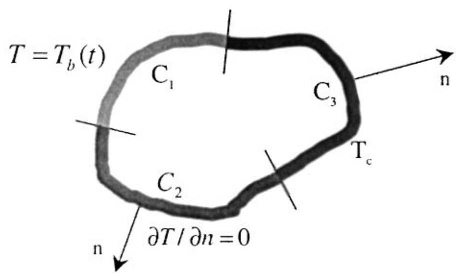

Fig. 1. Boundary conditions on a solid region.

On boundary C_3, convective-type condition is applied. This condition is when there are changes in temperature along that boundary:

$$\lambda_x \frac{\partial T}{\partial x} l_x + \lambda_y \frac{\partial T}{\partial y} l_y + \lambda_z \frac{\partial T}{\partial z} l_z = -\beta(T - T_c), \tag{5}$$

where T is the transient temperature, θ is the adiabatic temperature rise of concrete, T_0 is the initial temperature, n is the outer normal of the boundary, λ_x, λ_y, λ_z are the thermal conductivities for each direction, β is the surface exothermic coefficient, T_c is the temperature of the bounding fluid, and l_x, l_y, l_z are direction cosines of the external normal to the boundary.

The above problem can be solved in 3D using the finite element method, the equations for finite element method are listed as follows:

$$\left[H + \frac{2}{\Delta t} P\right]\{T\}_t + \left[H - \frac{2}{\Delta t} P\right]\{T\}_{t-\Delta t} + \{Q\}_{t-\Delta t} + \{Q\}_t = 0, \tag{6}$$

where

$$H_{ij} = \sum_e h_{ij}^e = \sum_e \iiint_{\Delta R} \left(a_x \frac{\partial N_i}{\partial x} \frac{\partial N_j}{\partial x} + a_y \frac{\partial N_i}{\partial y} \frac{\partial N_j}{\partial y} + a_z \frac{\partial N_i}{\partial z} \frac{\partial N_j}{\partial z} \right) \mathrm{d}x \, \mathrm{d}y \, \mathrm{d}z,$$

$$P_{ij} = \sum_e p_{ij}^e = \sum_e \iiint_{\Delta R} N_i N_j \, \mathrm{d}x \, \mathrm{d}y \, \mathrm{d}z,$$

$$Q_{ij} = \sum_e q_i^e = \sum_e \left(-\iiint_{\Delta R} \frac{\partial \theta}{\partial t} N_i \, \mathrm{d}x \, \mathrm{d}y \, \mathrm{d}z - \iint_{\Delta C} \bar{\beta} T_C N_i \, \mathrm{d}s + \left(\iint_{\Delta C} \bar{\beta} N_i [N_i N_j \cdots] \, \mathrm{d}s \right) \begin{Bmatrix} T_i \\ T_j \\ \vdots \end{Bmatrix} \right).$$

Therefore, if the temperature at time $t - \Delta t$ is known, then the temperature at time t can be calculated, since the initial temperature is known, the temperature at any time can be calculated.

3. Creep stress with temperature effects

In creep analysis, the most widely used models are the CEB-FIP Model [3]; the American Concrete Institute (ACI) Model [4]; the Bazant and Panula's (BP) Model [5] and the Exponential

Model [6]. The first three models have a common requirement for the numerical implementation using FEM, that is, the entire stress history must be stored in a memory array. When a mass concrete structure is analyzed in 3D it demands significant memory resources of the system depending on the detail used in discretizing into elements. If one adds on top of this a change in the material properties with time the task of keeping track of these changes becomes inefficient. The Exponential Model can avoid storing the whole stress history and made the implementation feasible and was selected for use in this methodology.

The Exponential Model of creep compliance can be expressed with the Dirichlet series as [6]

$$J(t,\ \tau) = \sum_{k=1}^{N} \frac{1}{C_k(\tau)}[1 - \exp(y_k(\tau) - y_k(t))], \tag{7}$$

where $J(t,\ \tau)$ is the creep function, τ is the loading age in days, and C_k and y_k are experimental coefficients as functions of time (τ or t).

For mass concrete structures the following specific form is often used [7]:

$$J(t,\ \tau) = \frac{1}{E(\tau)} + C(t,\ \tau),$$

$$C(t,\ \tau) = \sum_{i=1}^{2} (A_i + B_i\tau^{-G_i})[1 - \exp(-S_i(t-\tau))] + D[\exp(-S_3\tau) - \exp(-S_3 t)],$$

$$E(\tau) = E_0(1 - \exp(-\alpha\tau^{\beta})), \tag{8}$$

where $C(t,\ \tau)$ is the creep compliance, $E(\tau)$ and E_0 are transient and ultimate elastic moduli, respectively, and A_i, B_i, G_i, S_i, D, α, β are all experimental fitting parameters.

As mentioned before, the elastic modulus and creep properties of concrete are influenced by temperature. Du and Liu [8] introduced the term equivalent age (τ_e), which represents the hydration period for which the same degree of hydration is reached at a current temperature as that one reached during the actual time (t) at a reference temperature. The concrete age, τ, will be replaced with this equivalent age (τ_e) in the above exponential model. This modified model includes the temperature effects on both the elastic modulus and the creep behavior of concrete.

In the numerical process for creep stress analysis, first the creep strain is calculated, then the corresponding stress can be obtained. Du and Liu [8] derived the mathematical formulation for the creep strain and stress in 3D, the final formula for the increment in strain (Eq. (9)) and stress (Eq. (10)) are presented below.

Let the time interval $[t_0, t]$ be subdivided into N steps, then the creep strain increment $\{\Delta\varepsilon_n^C\}$ at a typical step $[t_{n-1},\ t_n]$ ($n = 1,2,\ldots,N$) can be expressed as

$$\{\Delta\varepsilon_n^C\} = [Q]\sum_{r=1}^{3} [(1 - \exp(-S_r\varphi_{Tn}\Delta\tau_n))\{\omega_{rn}\} + \{\Delta\sigma_n\}\varphi_{rn}^* h_{rn}^*] = [\eta_n] + q_n[Q]\{\Delta\sigma_n\}, \tag{9}$$

where

$$[Q] = \begin{bmatrix} 1-\mu-\mu & 0 & 0 & 0 \\ 1-\mu & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ & 2(1+\mu) & 0 & 0 \\ \text{Sym.} & & 2(1+\mu) & 0 \\ & & & 2(1+\mu) \end{bmatrix}, \quad \mu \text{ is the Poisson's ratio,}$$

$$\{\eta_n\} = \sum_{r=1}^{3} (1 - \exp(-S_r \varphi_{Tn} \Delta\tau_n))\{\omega_{rn}\},$$

$$q_n = \sum_{r=1}^{3} \varphi^*_{rn} h^*_{rn},$$

$$\{\omega_{rn}\} = \{\omega_{rn-1}\}\exp(S_r \varphi_{Tn-1}\Delta\tau_{n-1}) + [Q]\{\Delta\sigma_{n-1}\}\varphi^*_{rn-1} f^*_{rn-1} \exp(-S_r \Delta\tau_{n-1}),$$

$$\varphi^*_{rn} = \varphi_r(\tau_{\mathrm{en}-1/2}),$$

$$h^*_{rn} = 1 - f^*_{rn} \exp\left(-S_r \sum_{i=1}^{n} \varphi_{Ti}\Delta\tau_i\right),$$

$$f^*_{rn} = \frac{1}{\Delta\tau_n}\int_{t_{n-1}}^{t_n} \exp\left(S_r \int_{t_0}^{\tau} \varphi_T \,\mathrm{d}t'\right)\mathrm{d}\tau,$$

$$\Delta\tau_n = t_n - t_{n-1}.$$

After the creep strain increment is calculated, the corresponding stress increment can be obtained:

$$\{\Delta\sigma_n\} = [\bar{D}_n](\{\Delta\varepsilon_n\} - \{\eta_n\} - \{\Delta\varepsilon^{\mathrm{T}}_n\}), \tag{10}$$

where $\{\Delta\varepsilon^{\mathrm{T}}_n\}$ is the thermal strain, $[\bar{D}_n] = [D_n]/(1 + q_n E_n)$, $[D_n]$ is the elastic matrix of the nth time interval, and E_n is the elastic modulus.

The total stress at the moment of the nth step is

$$\{\sigma_n\} = \{\sigma_{n-1}\} + \{\Delta\sigma_n\}. \tag{11}$$

4. Implementation of the methodology in the finite element method

The finite element method is a powerful numerical technique and it is widely used in many fields of engineering. This method discretizes the structure into elements, describes the stress–strain behavior of each element according to its constitutive relationship, then it assembles the elements at "nodes" as if nodes were pins or drops of glue that hold elements together. The general procedure followed in the finite element method for stress–strain behavior is shown in Fig. 2 [9].

Introducing changes in temperature and material properties (elastic modulus and creep) required considering the additional dimension of time in the analysis. Furthermore, if time is now being

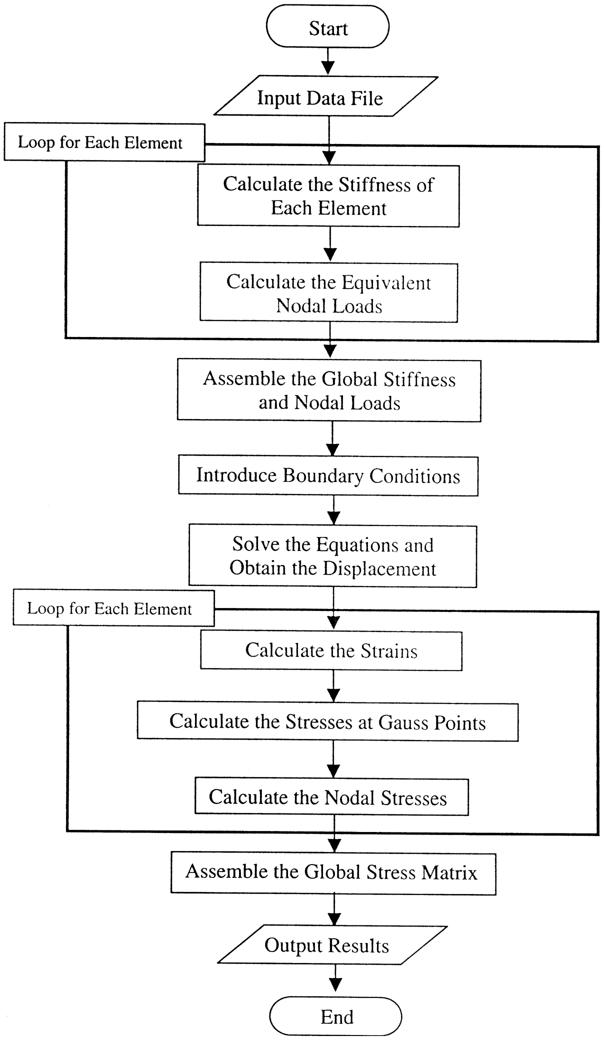

Fig. 2. General approach to the finite element method.

considered then the geometry and gravity loads of mass concrete structures during construction can be updated and included in the analysis. Based on the mathematical formulation described above, a 3D finite element program that can simulate the construction process of a mass concrete structure was developed and a schematic flowchart of the program is shown in Fig. 3. The classic approach to the FEM is modified to include two additional loops for time steps and loading (construction).

The flowchart shown in Fig. 3 represents the architecture and control of the program. The program begins by reading the original data from the input file. The program can simulate the construction process of a mass concrete structure built in a series of thin layers, such as an RCC dam. The outermost loop is for each lift and when a new layer is added, the geometry of the structure, the initial conditions, and the boundary conditions are modified.

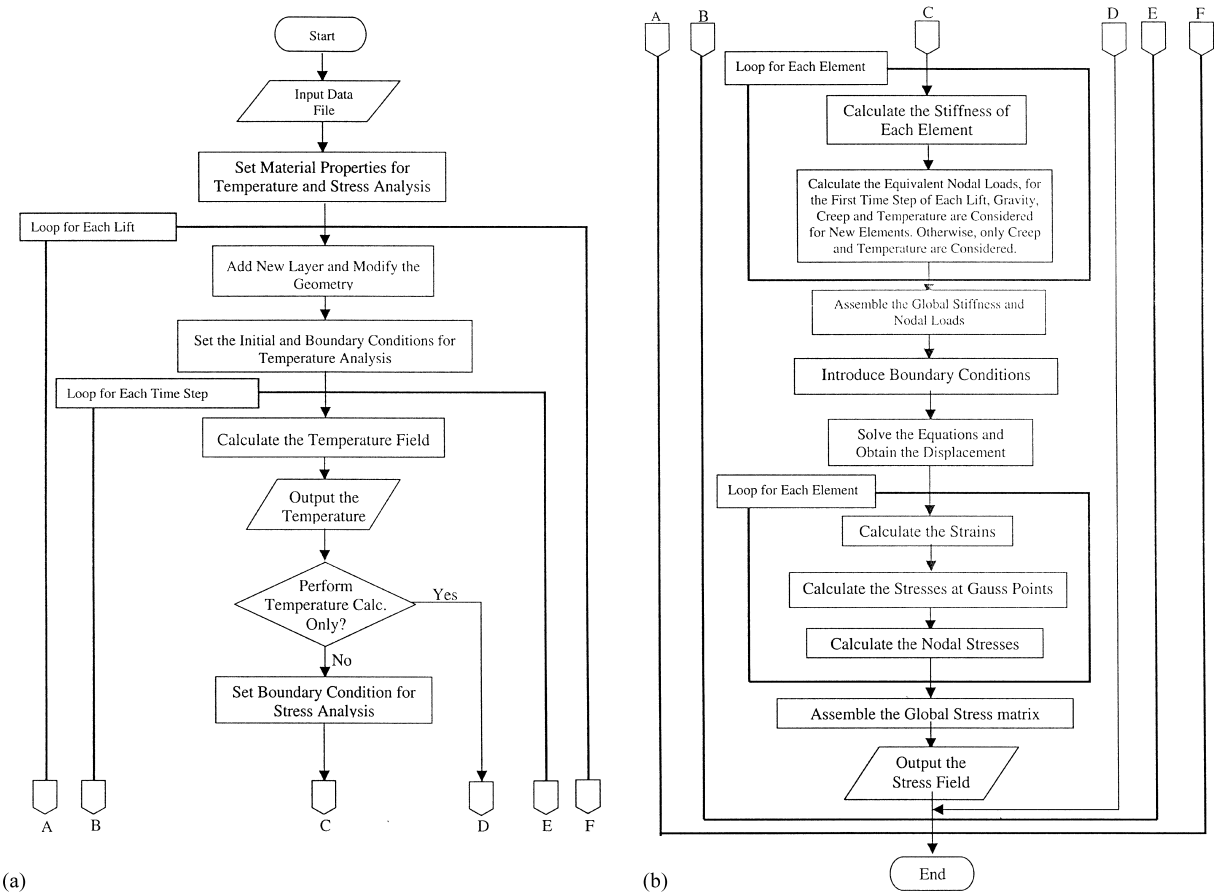

Fig. 3. (a) FEM approach with temperature and creep modifications; and (b) FEM approach with temperature and creep modifications.

After each lift is placed, the control enters a loop for each time step. Using Eq. (6), the temperature field is computed, and the results are written to a file. If only the temperature field analysis is desired, then the program moves to the next time step. If the stress field analysis is also desired, then the boundary conditions for stress analysis are determined and computations for stress analysis are continued following the FEM procedure modified to include the effect of temperature and creep of concrete. Notice that the elastic modulus of concrete is a function of age and temperature, it changes in each time increment, affecting the elastic matrix. The equivalent nodal loads include equivalent load of elastic, temperature, and creep.

After the stiffness and the equivalent nodal loads of all elements are obtained, the global equilibrium equations are obtained. Once this system of equation is solved, the displacement is obtained and then the strain is calculated for each element based on the element type. Finally, the stress field can be computed by assembling a global nodal stress matrix based on Eqs. (10) and (11).

The program was coded in FORTRAN 77 and can be compiled in an IBM RS6000 computer using the UNIX operating system. However, the program can be compiled and run on a Pentium-based personal computer. The main program calls 45 subroutines and the code is about 6500 lines. As mentioned above, the creep compliance relationship was selected based on its computational efficiency. However, the 3D nature of mass concrete analysis, the construction sequence simulated, and the time effect of temperature and creep demand computer time to perform the calculations.

5. Numerical example

In order to demonstrate the methodology and the 3D finite element program, the temperature and stresses of a concrete block were analyzed.

Consider a $30 \times 4 \times 4$ m concrete block supported on a rock base half-space, as shown in Fig. 4. The trivial mesh generating procedure included varying elemnt sizes until satisfactory convergence was achieved at about 0.5 m cube elements. The upper surface of the block is exposed to the air, and the ambient temperature was set to 10°C. All the other sides of the block satisfy the adiabatic condition or $\partial T/\partial n = 0$. The unit weight of concrete was $\gamma = 2400$ kg/m^3. The thermal material properties for concrete were assumed to be as follows: thermal diffusivity, $a = 0.004$ m^2/h, thermal conductivity, $\lambda = 2.16$ cal/(mh°C), and surface exothermic coefficient, $\beta = 20$ cal/(m^{2}h°C). Other values used from previous work are the Poisson's ratio $\mu = \frac{1}{6}$, linear thermal dilatation coefficient, $\alpha = 10^{-5}$/°C, adiabatic temperature rise, $\theta = 30(1 - e^{-0.3\tau})$°C [8]. The elastic modulus and creep compliance models for roller compacted concrete were based on reported values by Zhang [10], as follows:

$$E(\tau) = 2.6 \times 10^9 (1 - e^{-0.14\tau}) \quad \text{in kg/m}^2,$$

$$C(t,\tau) = \left(A_0 + \frac{A_1}{\tau} + \frac{A_2}{\tau^2}\right)[1 - e^{-K_1(t-\tau)}] + \left(B_0 + \frac{B_1}{\tau} + \frac{B_2}{\tau^2}\right) \times [1 - e^{-K_2(t-\tau)}] + D(e^{-K_3\tau} - e^{-K_3 t}),$$

where $A_0 = 0.35494$, $A_1 = 3.7335$, $A_2 = -2.5644$, $B_0 = 0.48368$, $B_1 = -0.186$, $B_2 = 0.13786$, $K_1 = 0.35361$, $K_2 = 0.012486$, $K_3 = 0.032642$, and $D = 0.83509$.

The program can simulate the construction process of mass concrete structures and dynamically update the geometry of the FEM mesh when new lifts are added during construction. However, in

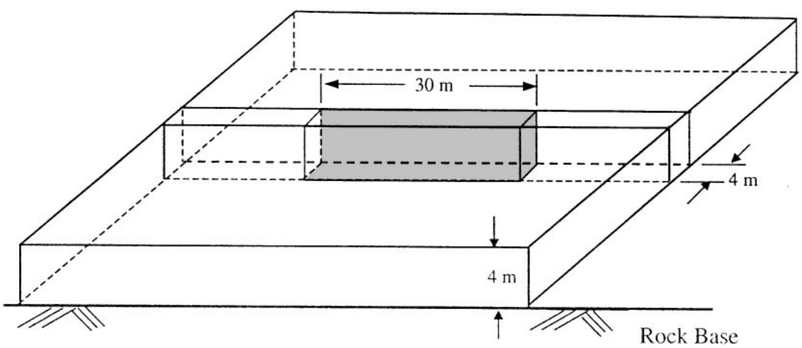

Fig. 4. Concrete block within mass concrete layer.

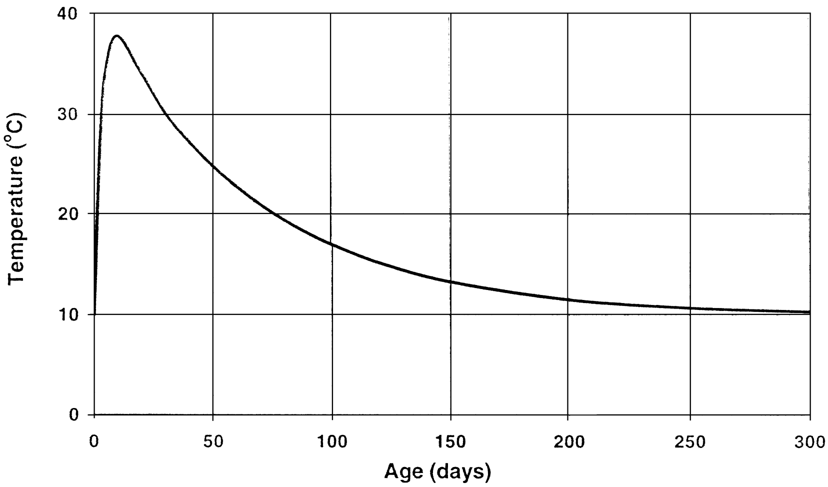

Fig. 5. Temperature curve at the central point of block.

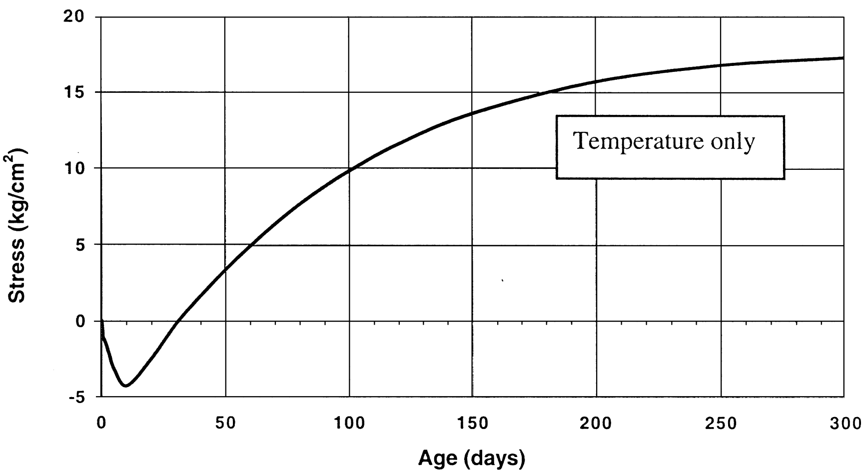

Fig. 6. Stress curve at the central point of block.

order to demonstrate the methodology and the 3D finite element program, the temperature and stresses of a concrete block taken from one lift were analyzed. Fig. 5 shows the temperature at the central point of the concrete block. The temperature increases due to the heat produced by hydration, then reaches a maximum point at about 8 days after casting. Because of the heat transfer between the concrete block and the outside air, the peak temperature is less than the initial temperature plus the adiabatic hydration temperature rise. After 8 days, the block begins to cool down. Only the upper surface is exposed to the air, which requires a long time for the block to reach the boundary environmental temperature. After 300 days the central point of the concrete block is still higher than the boundary environmental temperature. Fig. 6 shows the variation of stress along the same point of the block considering only thermal stresses. At first, a compression stress develops inside the block, this is because the concrete tends to expand due to the temperature increase, while the boundary conditions restrict its expansion. The maximum compression stress also occurs at about 8 days after the block is cast. Then the compression stress decreases which is expected because the block begins to cool down. Finally, the compression stress changes to tension stress and the tension stress becomes larger and larger because the block continues to cool down. This is why concrete often cracks under extreme cold weather conditions.

6. Conclusion

This paper introduced a numerical methodology to simulate the construction process of mass concrete that considers the temperature effects on the elastic modulus and creep behavior of concrete. The implementation of the methodology using the finite element method and the flowchart of a 3D finite element program were presented. A simple numerical example demonstrated the methodology and the capabilities of the proposed approach. This procedure could be used with other materials that are exposed to thermal changes and are sensitive to creep.

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