Tutorial on Nonlinear Reduced Order Modeling for Nominally Cyclic Symmetric Structures and Rotating Machinery

Mainak Mitra, ANSYS Inc, Canonsburg, PA, USA
Andrea Lupini, University of Michigan, Ann Arbor, MI, USA
Andrew Madden, ANSYS Inc, Ann Arbor, MI, USA
Chiara Gastaldi, Politecnico di Torino, Corso Duca degli Abruzzi 24, 10128 Torino, Italy
Bogdan Epureanu, University of Michigan, Ann Arbor, MI, USA

Abstract

Many important engineering structures such as rotating machinery, including turbine bladed disks, gears, flywheels and satellites are comprised of repeated (nominally identical) substructures arranged circumferentially with cyclic symmetry. Due to this unique arrangement, the system matrices and consequently the dynamics of such structures exhibit specific characteristics [1,2]. Extensive scientific study and analysis has been conducted on this topic in recent decades. Of particular interest is the change in dynamic behavior when there are deviations in substructures from their nominal, even to a small extent. Colloquially termed mistuning, such deviations are practically impossible to avoid. They manifest as material or geometric differences due to causes such as manufacturing tolerances, wear and differential operation conditions [3]. Mistuning can lead to strain energy localization, higher system responses and reduction of the operational life cycle and should therefore be carefully considered in the design and analysis of structures. The current industrial practice is to use Monte Carlo simulations to characterize mistuning effects using randomly generated deviations in substructures of the nominal design [1,3]. Since thousands of dynamic simulations might be required to characterize a single design, full order high fidelity models remain prohibitively expensive. For such tasks, reduced order models (ROMs) are employed instead [3,4]. However, obtaining fast and accurate ROMs for cyclic structures with nonlinearities [1,4,5] remains a challenging task. This tutorial aims at summarizing and highlighting some of the most relevant techniques that have been proposed to date, with a specific focus on nonlinear ROMs including contact nonlinearities.

Keywords

cyclic symmetry, reduced order models, mistuning, nonlinear dynamics, rotordynamics

Cyclic symmetry and constraint equations

The dynamics of a cyclic symmetric structure may be completely captured by establishing displacement constraint equations between degrees of freedom of high (H) and low edges (L) of the structure as shown in Figure 1 (a). Due to symmetry, the displacements at the edges are related by a phase difference, and the familiar dynamic equation corresponding to the full model in Eqn. (1) may be reduced to the order of number of DOFs in a single sector via a transformation to the cyclic domain as shown in Eqn. (2):

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{f};\ \mathbf{q} = [\mathbf{q}_1, \mathbf{q}_2, \dots\ \mathbf{q}_N]^{\mathrm{T}} \tag{1}$$

$$\mathbf{q}^H = Re(\,\mathbf{q}^L\, e^{ip\alpha})\ ;\ \alpha = \frac{2\,\pi}{N}\ ;\ \mathbf{M}_s\ddot{\mathbf{q}}_s + \mathbf{C}_s\dot{\mathbf{q}}_s + \mathbf{K}_s\mathbf{q}_s = \mathbf{f}_s;\ \mathbf{q}_s = [\mathbf{q}^L, \mathbf{q}^I]^{\mathrm{T}} \tag{2}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ represent system mass, damping and stiffness matrices respectively. $\mathbf{q}$ and $\mathbf{f}$ represent the generalized coordinates and excitation force vectors. N is the number of sectors, α is the sector angle. Subscript s represents the sector level constrained cyclic DOFs (L, I). The phase difference imposed by the cyclic constraints arises from a spatial Fourier decomposition of harmonic p. Since the dynamics represented by each spatial harmonic is independent, the full ($N\times$ DOF_s) system reduces to N independent systems of size DOF_s. Moreover, simulating one or two of these independent systems corresponding to the dominant harmonic in the excitation is typically sufficient to obtain predictions when linear effects are predominant in the structure. This is especially true in rotating stages of turbines whose impinging pressure wave has a dominant spatial harmonic imparted by the upstream static stage (stator) comprised of cyclically symmetric vanes. Modal analysis may also be carried out in the cyclic domain. The effect of the Fourier space spanned by the modes of a cyclic symmetric structure are evident in the modes, which may exhibit a certain number of nodal diameters along which displacements are zero. Modes of spatial harmonics (except for p = 0, N/2 when applicable) are commonly paired by identical natural frequencies leading to characteristic Frequency vs. Nodal Diameter plots of several high-density modal regions as shown in Figure 1(b). Rotating structures which tend to be cyclic symmetric are commonly affected by spin softening, Coriolis and rotational prestress effects. Consequently, the natural frequencies are dependent on the rotational velocity, an effect usually analyzed using Campbell diagrams shown in Figure 1(c).

Mistuning and Probabilistic Simulations

The breaking of perfect cyclic symmetry, most commonly due to geometric or structural variations between sectors, is referred to as mistuning. Mistuning is judged to be 'small' when the deviation from the cyclic baseline is significant for natural frequencies but not for the modes. When modes are affected significantly, the mistuning is judged to be 'large'. Even small mistuning can affect the system response significantly as it couples the various spatial harmonics and a linear response is a superposition of multiple modes in a narrow frequency region. The uncertainty in the underlying parametric variations make exact response predictions difficult and Monte Carlo probabilistic simulations are used to characterize the expected response amplification factor (AF) over the nominal cyclic case as shown in Figure 2(a). Further time savings may be realized in probabilistic analysis by fitting available simulation response data to the expected Weibull distribution as shown in Figure 2(b), thereby predicting the statistics with relatively fewer simulations [1].

Nonlinearities

Friction and contacts are the most commonly encountered nonlinearities in cyclic structures such as bladed disks, although their effects vary significantly based on their location and the components affected. For instance, the relative displacements and normal loads may be different by orders of magnitude for a blade and disk joint when compared to contact between a rotor and its casing. Usually, steady state nonlinear amplitude responses are of interest and are modeled by applying harmonic balance (HB) to the time-domain equations to yield a multi-harmonic frequency domain representation as follows:

$$[-h^2\omega^2\mathbf{M} + \mathrm{i}h\mathbf{C} + \mathbf{K}]\bar{\mathbf{q}}^h = \bar{\mathbf{f}}_E^h + \bar{\mathbf{f}}_N^h \tag{3}$$

where $\overline{[\,.\,]}^h$ represents the h^{th} temporal harmonic in the frequency domain and subscripts E and N represent the applied excitation and nonlinear forces respectively. Contact forces which depend on relative displacements and internal slip variables cannot be calculated a-priori or expressed directly in the frequency domain. Consequently, an alternating frequency time (AFT) approach is used. At each simulation iteration, $\bar{\mathbf{f}}_N^h$ is determined by applying an inverse Fourier transform to its time domain periodic solution, which in turn is obtained from time-domain displacements calculated by applying forward Fourier transforms to system displacements. Other nonlinearities such as material nonlinearities (coatings), and multi-physics effects such as contact heat generation and temperature effects are also commonly observed, especially in gas turbine rotors. However, these are beyond the scope of this tutorial whose focus is on contact-related nonlinearities.

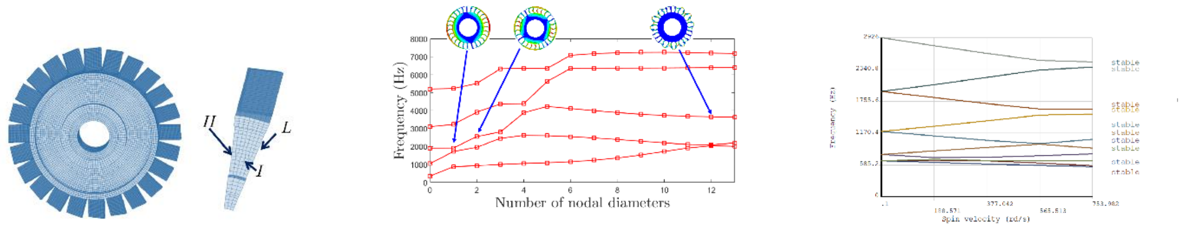
Reduced order modeling

Even when linear effects are dominant, reduction of cyclic systems is challenging due to mistuning. Substructuring using methods such as component mode synthesis (CMS) is a strategy which reduces a system by partitioning it into components and using the interface DOFs between these components to apply appropriate constraints, while reducing internal DOFs of the components, typically via modal truncation. A CMS reduction followed by HB analysis is often used for harmonic analyses of bladed disks with localized nonlinearities such as contact interfaces as illustrated in Figure 3(a). Component mode mistuning (CMM) is a method which adapts CMS to mistuned cyclic structures, treating the baseline tuned structure and the mistuning (often parametrized as deviations in stiffness or mass in blades) as individual components as illustrated in Figure 3(b). For small mistuning, the span of tuned and mistuned modes is assumed to be identical. This allows reduction and projection of the mistuned quantities back onto the Fourier space of the tuned blisk, resulting in computationally efficient simulations.

The nonlinearities representing friction do not have a closed form representation and thus are not conducive to techniques such as polynomial expansion. Other general techniques for generating nonlinear ROMs, such as proper orthogonal modes, nonlinear normal modes prove to be too computationally expensive for practical applications. Instead, nonlinear ROMs for bladed disks commonly focus on approximating nonlinear subspaces representing dynamics specific to the nature of nonlinearity (stick-slip contact, frictionless chatter, amplitude-dependent coatings). Linear reductions are often leveraged in addition to nonlinear reductions, since usually only a portion of the structure such as the blade, shrouds or coatings have significant nonlinearities. Some notable nonlinear ROMs discussed in this tutorial are based on bilinear modes (BLMs), piecewise linear modes (PLMs), adaptive microslip projections (AMP) and equivalent energy single harmonic approximations.

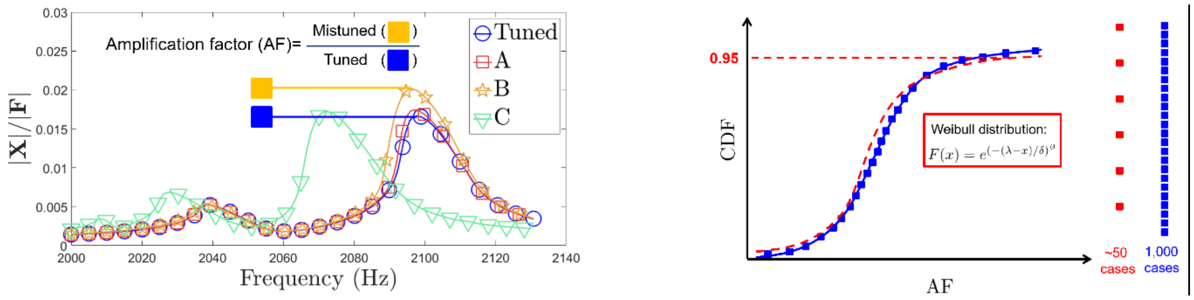
Conclusions

The aim of this tutorial is to present the ideas and techniques commonly employed in dynamic modeling and simulation of cyclic symmetric structures, leading up to a discussion of the state-of-the-art reduced order models used for fast-generation of nonlinear ROMs designed for probabilistic analyses. The tutorial assumes basic knowledge of dynamics, but no prior specific knowledge regarding cyclic symmetric structures. It introduces the participants to concepts unique to the field such as engine order excitation, frequency-nodal diameter plots, block-circulant matrices and amplification factor. It also discusses dynamic reduction techniques for various types of nonlinearities in these structures such as Coulomb friction and amplitude-dependent nonlinearities in the time-domain (harmonic-balance) as well as the spatial domain (projection-based ROMs).



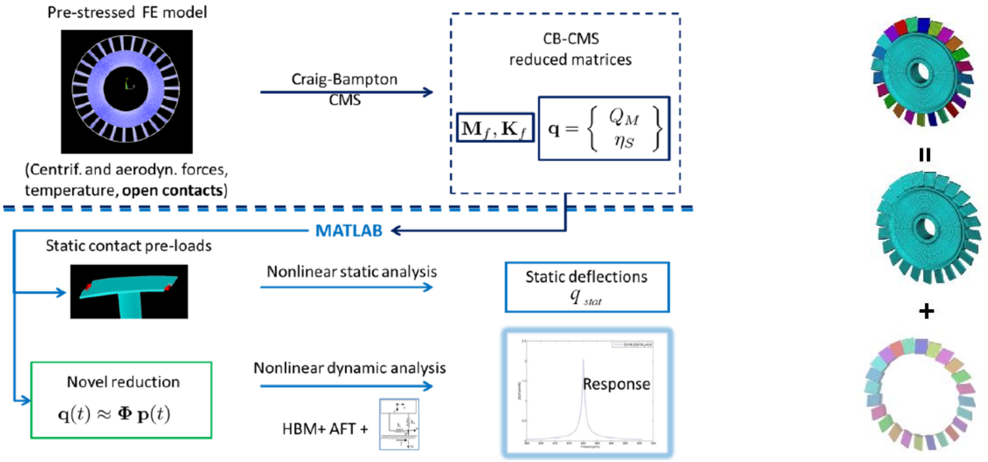
(a) (b) (c)

Figure 1 – (a) Finite element models (b) Frequency vs Nodal Diameter plot for a turbine blisk (c) Campbell diagram (generated using ANSYS®)



(a) (b)

Figure 2 – (a) Nonlinear Response for tuned and mistuned analysis of a shrouded blisk (b) Fitting AFs from 50 mistuned responses to a Weibull distribution (compared to distribution from 1000 cases)



(a) (b)

Figure 3 – (a) CMS followed by HBM for simulating nonlinear dynamics of a shrouded blisk (b) Schematic showing system partitioning for CMM

References

[1] Mitra, M., and Epureanu, B. I. (March 6, 2019). "Dynamic Modeling and Projection-Based Reduction Methods for Bladed Disks with Nonlinear Frictional and Intermittent Contact Interfaces." ASME. *Appl. Mech. Rev*. doi: https://doi.org/10.1115/1.4043083

[2] Olson, B. J., Shaw, S. W., Shi, C., Pierre, C., and Parker, R. G. (June 19, 2014). "Circulant Matrices and Their Application to Vibration Analysis." ASME. *Appl. Mech. Rev*. July 2014; 66(4): 040803. https://doi.org/10.1115/1.4027722

[3] Castanier, M.P., Pierre, C. Modeling and Analysis of Mistuned Bladed Disk Vibration: Current Status and Emerging Directions. Journal of Propulsion and Power,22/2 pp. 384-396, 2006.

[4] Baek, S., and Epureanu, B. (August 2, 2017). "Reduced-Order Modeling of Bladed Disks with Friction Ring Dampers." ASME. *J. Vib. Acoust*. December 2017; 139(6): 061011. https://doi.org/10.1115/1.4036952

[5] Zucca, S., Firrone, C.M., Nonlinear dynamics of mechanical systems with friction contacts: Coupled static and dynamic Multi-Harmonic Balance Method and multiple solutions. Journal of Sound and Vibration 333, 916–926, 2014.