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Statistical Analysis of the Compressive Strength of Concrete in Structures


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The relationship between the in-place compressive strength of concrete in structures and specified strength f_c' is investigated through the use of factors F_1 and F_2. Factor F_1, the ratio of the average strength of standard 28-day-old cylinder specimens to the specified strength, is evaluated using data from 3756 cylinder tests representing 108 concrete mixes produced in Alberta, Canada, between 1988 and 1993. Factor F_2, the ratio of average in-place strength to average cylinder strength, is evaluated using core and cylinder data representing 108 concrete mixes with strengths less than 55 MPa that were investigated by others. A statistical description of the compressive strength of concrete in structures is derived that accounts for the inherent randomness of Factors F_1 and F_2 and also the typical strength variation within a specific structure. The probability of the in-place compressive strength of concrete in a 28-day-old column being less than f_c' is approximately 13 percent. It is likely that a recalibration of the load and resistance factors for the design of new structures in Canada based on these findings would yield greater factored concrete strengths than are currently in use.

Keywords: compressive strength; concrete cores; concretes; cylinders; evaluation; load factors.

INTRODUCTION

Murphy[1] has commented that "the engineer is often able to make little use of any information on the strength of the in-place concrete because this does not figure in any of the normal design processes and is, therefore, unfamiliar to him." The objective of this paper is to address this deficiency, so that in-place strength information can be rationally accounted for in the assessment of the safety of an existing structure. Specified strength f_c' appears throughout the normal design processes and is familiar to engineers. In this study, the relationship between in-place compressive strength and specified strength is sought.

Statistical description of concrete strength in structures may be derived from two separate random quantities, as shown in Fig. 1. Given a deterministic value of f_c', the average in-place strength for a structure $\bar{f}_{c,is}$, as shown in Fig. 1(a), is a random quantity that depends on many factors, including the quality of material supplied, the degree of curing provided, and the age of the structure. As indicated in Fig. 1(b), the in-place strength within a particular structure varies about $\bar{f}_{c,is}$ due to variation of the quality of the material supplied and to systematic variation of strength that occurs due to normal construction practices.

The relationship between mean in-place strength and specified strength may be derived from two independent random quantities, as shown in Fig. 2. The quality of material produced for a particular specified strength is checked using 28-day-old cylinders that are molded, cured, capped, and tested in accordance with the standard provisions.[2,3] Factor F_1 is the ratio of the average strength of the standard cylinders $\bar{f}_{cyl}$ to f_c'

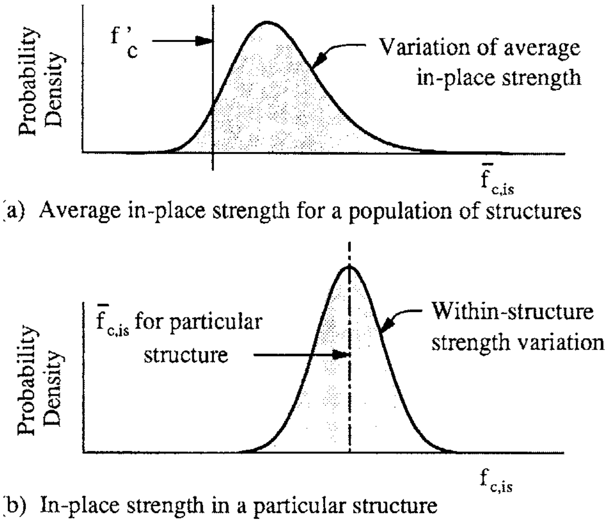

Fig. 1—Statistical description of concrete strength in structures

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$$F_1 = \bar{f}_{cyl}/f_c' \tag{1}$$

Typically, F_1 exceeds 1 because the concrete supplier intentionally overproportions the mix to obtain some assurance that the 28-day strength of the product will be satisfactory. The actual quality of the concrete in the structure may differ from that represented by the cylinders because the age, consolidation, or curing of the in-place material may not be well represented by the standard test specimens. Factor F_2 is the ratio of the average in-place strength of concrete $\bar{f}_{c,is}$ to average cylinder strength

$$F_2 = \bar{f}_{c,is}/\bar{f}_{cyl} \tag{2}$$

The value of F_2 may also depend on the size and type of member.

The relationship between average compressive strength of concrete in structures and specified strength is obtained by elimination of $\bar{f}_{cyl}$ from Eq. (1) and (2)

$$\bar{f}_{c,is} = F_1 F_2 (f_c') \tag{3}$$

In this paper, the expected value and variability of average in-place strength are derived from the expected value and variability of random variables F_1 and F_2, which are estimated empirically. The variability of in-place strength is determined by combining the variability of mean in-place strength with the within-structure strength variation, as suggested in Fig. 1. Finally, the characteristic value of the in-place compressive strength represented by f_c' is obtained.

RESEARCH SIGNIFICANCE

To derive load and resistance factors so that new structures will achieve acceptable levels of safety, the relationship between specified concrete strength and in-place strength must be defined in probabilistic terms. If existing structures are to be evaluated, it is necessary to invert the relationship so that an equivalent specified strength may be obtained from test data that will result in reliability levels that are consistent with those currently accepted for new construction.

From the analysis of a large quantity of data, expressions are obtained for the mean value and coefficient of variation of the ratio of in-place strength to specified strength. These expressions reflect the current quality of concrete produced in Alberta and account for concrete age and type of element. They indicate that the compressive strength of concrete in structures is greater than has been assumed in previous studies.[4,5] It is therefore probable that recalibration based on the statistical description of the in-place compressive strength of concrete presented in this paper would yield larger factored compressive strengths for design than are presently in use.

RATIO F_1 BETWEEN 28-DAY CYLINDER STRENGTH AND SPECIFIED STRENGTH

In Canada, guidelines for determining target strengths to be used for proportioning concrete mixes are given in CAN/CSA-A23.1-M90, "Construction Materials and Methods of Concrete Construction,"[6] which are based on the assumption that cylinder test strengths are normally distributed. To

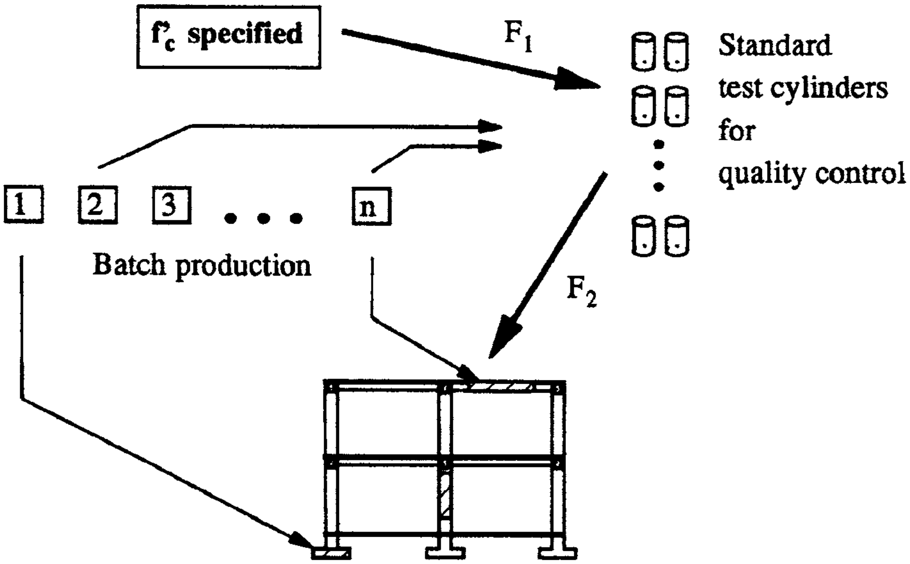

Fig. 2—Nature of relationship between specified and in-place strengths

Table 1—Categorization of cylinder test data

Number of mixes investigated in this study					
General ready-mixed			Bridge construction		
f_c', MPa	Air-entrained	No air	Cast-in-place	Precast	Total
25	18 (10)	18 (7)	15 (3)	0	51
30	4 (3)	5 (1)	13 (4)	0	22
32	1 (1)	1 (1)	0	0	2
35	4 (3)	1	2 (2)	20 (5)	27
40	0	1	0	3	4
45	2	0	0	0	2
Total	29 (17)	26 (9)	30 (9)	23 (5)	108

Note: Numbers in parentheses indicate numbers of mixes with at least one test below f_c'.

achieve a probability of at least 99 percent that the average of three consecutive tests exceeds the specified strength, it is recommended that

$$\bar{f}_{cyl} = f_c' + 1.4s \tag{4}$$

where s is the standard deviation estimated from at least 30 consecutive strength tests of concrete that is similar to that to be produced. To achieve a probability of at least 99 percent that no individual test falls more than 3.5 MPa below the specified strength, it is further recommended that

$$\bar{f}_{cyl} = f_c' + 2.4s - 3.5 \quad \text{(MPa)} \tag{5}$$

If concrete is proportioned to satisfy the criterion given by Eq. (4), which governs if the standard deviation is less than 3.5 MPa, then the average cylinder strength will be 1.4 standard deviations greater than the specified strength, and so the probability of an individual test falling below f_c' will be 8.1 percent. If the standard deviation exceeds 3.5 MPa, Eq. (5) governs and the corresponding probability that an individual test falls below f_c' will be less than 8.1 percent.

The criteria for determining the required average compressive strength used as the basis for selection of mix proportions in ACI 318-89, "Building Code Requirements for Reinforced Concrete,"[7] are similar to those given by Eq. (4) and (5), and yield slightly larger probabilities that individual tests will fall below f_c'. However, a more significant difference is that the criteria given in ACI 318-89[7] are mandatory, whereas those given in CAN/CSA-A23.1-M90[6] are not.

Those who provided strength test data for this study also offered some insight into the current state of practice for proportioning concrete mixes. Durability concerns cause minimum cement content or maximum water-cement ratios to be specified that often require the producer to supply a stronger material than would normally be provided. Air-entrained concretes are usually proportioned to have satisfactory strength at the maximum specified air content, and so are stronger if the air content is less. It is considered prudent by at least one producer to proportion mixes containing fly ash to be just satisfactory if the fly ash does not "kick in" (that is, contribute to the strength). Requirements for strengths at 28 days may be superseded by criteria that are due to the construction schedule, such as early strengths required to meet the floor construction cycle of a high-rise building. Reduced payment schedules for understrength concrete may cause the producer to supply material of a better quality than usual, depending on the financial risk. If the material supplied at the beginning of a job is considerably better than specified, the producer may request permission to reduce the quality, but, in practice, only if the entire project is proceeding smoothly. This anecdotal evidence, although not in a form that can be readily corroborated using analytical statistics, indicates that the process of setting the target strength of a particular mix may be more complex than implied by Eq. (4) and (5).

Standard cylinder test data

Results of 3576 tests of 8662 standard cylinders from 108 concrete mixes were investigated[8] to determine the relationship between the standard cylinder test strength and the specified strength. Each test is an average of two or three cylinder breaks. The data represent four categories of concrete produced in Alberta, Canada, between 1988 and 1993: general ready-mixed concrete with or without entrained air and concretes with entrained air supplied for precast or cast-in-place bridge construction. The breakdown of the data between the various specified strengths and categories is shown in Table 1. The number of strength tests for each mix varied between 14 and 77, and at least 25 tests were reported for 74 percent of the mixes. In cases where a long record of data for a standard product was available, several sets of data have been extracted and are considered different mixes.

Strength test data for ready-mixed concretes were obtained from three local producers who together batch over 300,000 m^3 per year. Most of the mixes had specified strengths of 25 or 30 MPa, and almost all contained a Class C fly ash. All cylinder strength tests were conducted by independent CSA-certified testing laboratories.

Data for concretes used in bridge construction were obtained from the local provincial highway ministry, Alberta Transportation and Utilities. All of the mixes had entrained air and none contained fly ash. The strength tests, each an average of three cylinders, were conducted by independent CSA-certified testing laboratories. Although concretes for cast-in-place and precast construction are produced to meet different specifications, both specifications include a reduced payment schedule for batches that do not achieve the specified strength at 28 days. For cast-in-place concretes, the target strength used to proportion the mix is specified to be 15 percent greater than the specified strength. All precast construction consisted of prestressed girders, for which the concretes were probably proportioned to produce sufficient strength at 18 hr to allow transfer of prestress. Although the precast components received steam curing, the cylinders used to monitor their 28-day strengths did not. Generally, the precast concretes were produced in urban centers, whereas the cast-in-place concretes were supplied by rural producers.

Average cylinder strength given f_c'

Initially, the data were reviewed to see whether Eq. (4) and (5) accurately reflect the current state of practice. The difference in strengths ($\bar{f}_{cyl}$ - f_c') may be isolated on the left hand side of Eq. (4) and (5), and varies with the standard

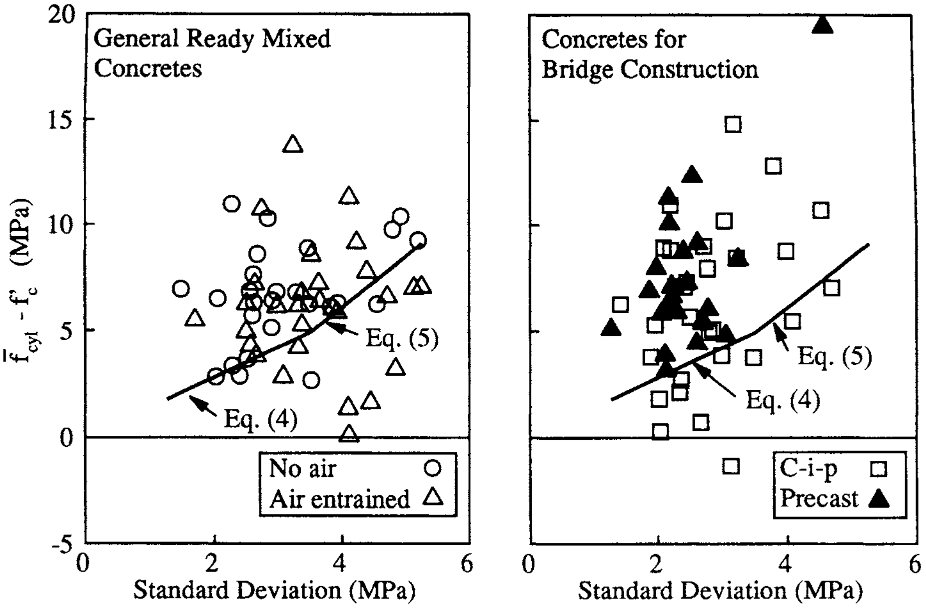

Fig. 3—Relationship between target mix strength and nonmandatory guidelines in CAN/CSA-A23.1-M90[6]

deviation, as shown in Fig. 3. Clearly the actual mean values and standard deviations for bridge and general ready-mixed concretes, also shown in Fig. 3, are not clustered around this relationship. The recommendations in CAN/CSA-A23.1-M90[6] are met by 23 of the 26 general ready-mixed concretes without entrained air, 19 of the 29 general ready-mixed concretes with entrained air, 20 of the 30 mixes for cast-in-place bridge construction, and all of the 23 mixes used for precast bridge construction. The majority of the mixes exceeds the recommended target strengths considerably. Thus, while some producers may base preliminary mix designs on the noncompulsory guidelines given in CAN/CSA-A23.1-M90,[6] the mix proportioning criteria do not accurately represent the degree of overstrength of concretes recently produced in Alberta.

Data for each mix were converted into F_1 values using Eq. (1). The data within each category of concrete were assumed to come from a homogeneous population, because regression analyses indicated that the dependence of F_1 on f_c' is not strong.[8] Cumulative distributions for each data set indicated little difference between the cast-in-place bridge concretes, air-entrained ready-mix concretes, and ready-mix concretes without entrained air. These data were pooled, and a good fit of a normal distribution to the data representing the lower tail of the observed values was obtained. Similarly, a good fit of a normal distribution to the lower tail of the data for precast concrete construction was also obtained. In Table 2, parameters for normal and lognormal distributions fit to the lower tails of the observed F_1 values are shown. The sample sizes are too small to identify which probability distribution is more appropriate.

Batch-to-batch variation

Since each set of strength test cylinders is made from concrete from a single batch, the standard deviation of the test strengths of concrete produced using a particular mix proportioning is a measure of the batch-to-batch variation of the mix. Generally, the standard deviation increases as the average strength increases,[8] which suggests the possibility of expressing the batch-to-batch variability as a constant coefficient of variation. Analysis gave constant coefficients of variation of 10.2 and 8.9 percent, respectively, for ready-mixed concretes with and without entrained air, and 7.9 and 5.6 percent, respectively, for concretes used for cast-in-place and precast bridge construction.

Data from six of the least overdesigned cast-in-place concrete mixes and two of the least overdesigned precast concrete mixes were reviewed to determine the nature of the batch-to-batch variation. The sample cumulative distributions are reasonably straight lines when plotted on normal probability paper,[8] indicating that it is reasonable to assume that the batch-to-batch variation for a given mix follows a normal distribution.

Check of fitted relationships

The parameters describing F_1 shown in Table 2 and the estimated batch-to-batch variation were used to predict the probability that an individual strength test would fall below the specified strength. The distribution of the cylinder test strengths was assumed normal, with the expected value of an individual test equal to the expected value of the average test strength

Table 2—Distribution parameters for F_1

	Variable	
	F_1	ln, F_1
All cast-in-place concrete		
Average	1.250	0.234
Standard deviation	0.131	0.122
Precast concrete		
Average	1.190	0.180
Standard deviation	0.058	0.053

Table 3—Summary of investigated in-place strength data

Reference	Year	N_m	Range of $\overline{f}_{cyl}$, MPa	Test age, days	Element, l x t x h, (m x m x mm)	Element curing details
Mather & Tynes[9]	1961	4	13 to 31	28	1.5 x 1.5 x 500	—
Bloem[26]	1965	1	35	95	? x ? x 200, 100	7 days moist, then in forms until test
		1	29	95	0.7 x 0.2 x 3050	
Bloem[27]	1968	3	25 to 38	28, 91, & 365	2.7 x 1.5 x 150	14 days moist, then in lab air
Meininger[28]	1968	1	21	93	2.0 x 0.4 x 1220 1.1 x 1.1 x 400	Continuous moist
Gaynor[29]	1970	2	26 to 42	91	0.6 x 0.15 x 1220	Continuous moist
Lewis[30]	1976	4	22 to 45	30, 93	3.0 x 3.0 x 350	Initially covered
Meininger, Wagner, & Hall[31]	1977	3	16 to 32	35 to 91	2.1 x 0.3 x 1070	7 days moist, lab air
		3	19 to 45	28, 60 to 102	1.2 x 0.3 x 1830	Continuous moist
Meynink & Samarin[32]	1979	2	26 to 42	28	? x 0.15 x 150	7 days moist, lab air, or continuous moist
Malhotra & Carette[33]	1980	7	18 to 51	28, 91	0.6 x 0.6 x 305	Continuous moist
Keiller[18]	1984	21	16 to 47†	28, 365	1.2 x 0.3 x 600	After 4 days, left exposed outdoors
Yip & Tam[10]	1988	12	16 to 50†	28	0.3 x 0.2 x 300 0.3 x 0.2 x 150	Continuous moist or exposed to lab air
Haque, Day , & Langan[34]	1988	21	13 to 42	182	0.3 x 0.3 x 450	After 1 day, left exposed outdoors
Akers[35]	1990	2	35 to 37	28	"Pads"	—
Szypula & Grossman[36]	1990	1	26	28, 56	9.7 x 2.4 x 240	Continuous moist
Confidential*	1991	5	21 to 29	28	? x ? x 290, 140	Outdoor exposure
Haque, Gopalan, & Ho[12]	1991	8	33 to 52	28, 91, & 365	0.4 x 0.4 x 150	After 1 day, left exposed outdoors
ACI 214[37]	1992	3	34 to 51	56	? x ? x 250	—
Bickley & Read[38]	1992	1	37	28, 91	3.0 x 0.3 x 2000	Outdoor exposure
		1	38		3.0 x 3.0 x 300	
Langley, Carette, and Malhotra[39]	1992	3	22 to 45	42, 365, & 730	3.1 x 3.1 x 3050	2 weeks at 12 C, then outdoors

*Data provided on condition that its source is not revealed.
†Estimated as 0.8 x cube strength.

$$E[f_{cyl}/f_c'] = E[\bar{f}_{cyl}/f_c'] \quad (6)$$

The variance of an individual test is equal to the sum of the variance of the average values and the variance due to batch-to-batch variation s_{bb}^2 within a mix

$$\mathrm{Var}[f_{cyl}/f_c'] = \mathrm{Var}[\bar{f}_{cyl}/f_c'] + \left(\frac{1}{f_c'}\right)^2 s_{bb}^2 \quad (7)$$

Assuming the batch-to-batch variation to have a constant coefficient of variation V_{bb}

$$\left(\frac{1}{f_c'}\right)^2 s_{bb}^2 = \left(\frac{V_{bb}\bar{f}_{cyl}}{f_c'}\right)^2 \approx V_{bb}^2 \left(E[\bar{f}_{cyl}/f_c']\right)^2 \quad (8)$$

so the variance of an individual test result is, by substitution of Eq. (8) into Eq. (7)

$$\mathrm{Var}[f_{cyl}/f_c'] = \mathrm{Var}[\bar{f}_{cyl}/f_c'] + V_{bb}^2 \left(E[\bar{f}_{cyl}/f_c']\right)^2 \quad (9)$$

Probability of a strength test result falling below f_c' was determined using values of $E[\bar{f}_{cyl}/f_c']$ and $\mathrm{Var}[\bar{f}_{cyl}/f_c']$, shown in Table 2 for cast-in-place and precast concretes, and values of coefficients of variation due to batch-to-batch variation, as given in the previous section. For all of the ready-mixed cast-in-place concretes, the estimated probability of a low test result was 7.4 percent, which is slightly larger than the observed probability of 5.8 percent. For the precast concretes, the estimated probability was 1.6 percent, which is, again, slightly larger than the observed value of 0.9 percent. Thus, the parameters shown in Table 2 and the coefficients of variation due to batch-to-batch variation give realistic estimates of the probability of low test results.

RATIO F_2 BETWEEN AVERAGE IN-PLACE STRENGTH AND AVERAGE CYLINDER STRENGTH

A large data set, representing approximately 1080 cores from elements cast from 108 concrete mixes with standard cylinder strengths less than 55 MPa, was abstracted from various published and unpublished investigations and is summarized in Table 3. Complete tabular listings of the data are presented in Reference 8. All cylinders received standard moist-curing[2,3] and were tested at 28 days. For 33 mixes from two sources,[9,10] standard cylinder strength was estimated as 80 percent of the reported strength of moist-cured cubes tested at 28 days. The cores were drilled when the concrete was at least 14 days old, and tested when the concrete was at least 28 days old. The core diameters were typically 100 or 150 mm and always at least 75 mm. The core length-to-diameter ratios varied between 1 and 2. In all cases, the moisture condition of the core at the time of testing was reported.

In-place strengths were calculated from the actual core strengths following procedures derived in Reference 8 that are published elsewhere.[11] The reported core strengths were modified to account for the effects of the core moisture condition and damage due to drilling after first being converted, if necessary, to the equivalent strength of a standard core 100 mm in diameter by 200 mm long. Using these strength correction factors, the in-place strength was 1.155 times the strength of a standard core that had been soaked for at least 40 hr before testing, or 1.018 times the strength of a standard core that had been left to dry in laboratory air for at least 7 days before testing.

Locations from which the cores were drilled within the element are not known for all cases. Cores were obtained from the middle of most columns, walls, and large blocks, where the strength is reasonably uniform.[8] Strengths of cores from the top and bottom of the short column specimens tested by Haque, Gopalan, and Ho[12] were averaged to estimate the strength in the middle. The cores from most of the slabs were drilled through the slab thickness; many were not trimmed before testing.

The quality of curing given to the various elements varied widely and the actual descriptions of the curing provided are often incomplete. Data from a few investigations where the curing conditions were clearly described and carefully controlled were used to assess the effect of curing on in-place strengths.[8] Thin slabs given insufficient moisture during curing had in-place strengths that averaged 77 percent of the strength of identical slabs that received continuous moist- curing. For columns and walls, the in-place strength of poorly cured elements averaged 90 percent of that of well-cured elements. Cores from the elements given good curing in these investigations were also included in the larger data set from the remaining investigations where either the curing conditions were clearly described but not controlled or not clearly described.

Analysis of F_2 data

Data from each element cored were converted to F_2 values using Eq. (2). Preliminary analyses suggested that the F_2 values for columns and walls were consistently larger than those for shallow beams and slabs. This was accounted for using an indicator variable Z_h that was set equal to 1 for elements at least 450-mm-high or zero otherwise. The relative strength of cores from elements less than 450-mm-thick may have been reduced because the cores contained relatively weak concrete from near the top of the element.[8] The difference may also be due to the greater sensitivity of the in-place strength of slabs to poor curing practices, as noted previously. Preliminary analysis also indicated that as the age of the concrete increased, the in-place strength increased relative to the standard 28-day cylinder strength. The strength gain was assumed to increase linearly with the natural logarithm of the age of the concrete.

Regression analyses eventually yielded

$$\hat{F}_2 = 0.948 + 0.084 Z_h + 0.100\, ln\left(\left(\frac{a}{28}\right)\left[1 + 0.090\left(\frac{fa^F}{c}\right)\right]\right) \quad (10)$$

where $\hat{F}_2$ is the predicted F_2 value, a is the concrete age in days, and fa^F and c are the weights of Class F fly ash and cement, respectively, per m^3 of concrete. None of the mixes analyzed contained Class C fly ash, silica fume, or slag. The average in-place strength of 28-day-old concrete is therefore about 95 percent of the standard 28-day cylinder strength for shallow beams and slabs or 103 percent of the average 28-day cylinder strength for higher elements. The average in-place strength is greater at later ages, especially if the mix contains significant quantities of Class F fly ash. It should be noted that Eq. (10) is fit to in-place strength data for concretes between 28 days and 1 year old, and that extrapolation to predict the in-place strengths at greater ages is unwarranted.

The parameter estimates in Eq. (10) differed significantly from zero at the 95 percent level, which indicates that the height of the element, age of concrete, and Class F fly ash content were significantly correlated to in-place strength. The fit had R^2 of only 0.53, and the standard error was 0.15; both suggest that Eq. (10) is not particularly accurate for predicting the in-place concrete strength in a particular element. However, Eq. (10) and its standard error together represent a valid statistical description of the expected value and variability of the average in-place strength in a population of structures. This information is of considerable importance in establishing load and resistance factors.

It has been reported [13,14] that the ratio of in-place strength to standard test specimen strength at the same age decreases as concrete strength increases. The residual errors of Eq. (10) are plotted against the standard cylinder strength in Fig. 4 and do not indicate any such dependence. Thus, for the data considered in this study, which pertain to 108 different mixes produced during 19 different investigations and so simulate the range of concretes encountered in practice, concrete strength level is not an effective indicator of the ratio of in-place strength to cylinder strength. This corroborates the finding by McIntyre and Scanlon[15] that the ratio of core strengths to companion field-cured cylinder strengths does not depend on concrete strength.

It may be convenient for some applications, such as calculation of load and resistance factors, to assume that the distribution of F_2 is lognormal. Nonlinear regression analysis[16] based on this assumption yielded

$$ln\{\hat{F}_2\} = ln\left\{0.936 + 0.085 Z_h + 0.097\, ln\left(\frac{a}{28}\right)\left[1 + 0.88\left(\frac{fa^F}{c}\right)\right]\right\} \quad (10a)$$

with a standard error of 0.139. Analysis of the residuals of Eq. (10a) did not indicate significant correlation with any other variable, including standard cylinder strength. The residuals of Eq. (10a) plot on normal probability paper as a reasonably straight line,[8] indicating that F_2 can indeed be approximated by a lognormal distribution.

There was no clear relationship between the F_2 value or the residual error of the predicted F_2 value and the quality of curing provided for these data, possibly because the "poor curing" conditions were generally not rigorously controlled. In many cases the poorly cured specimens were stored outdoors and hence the curing environment depended on the local seasonal weather conditions. The scatter of F_2 values in these cases tended to be large. Thus, the effects of various curing conditions are represented in the analysis by the mean values from Eq. (10) and (10a) and by associated standard error values that are greater than might be obtained for uniform curing conditions.

Check of predicted F_2 values

Accuracy of the F_2 values predicted using Eq. (10a) was checked using in-place strength data from real structures.[9,17-20] These data were not used in the original analysis because the reported strengths represent cores and cylinders from the same structure but not necessarily the same batch of concrete. It is also possible that the long-term strengths reported in the investigations published before 1970 are appropriate for older cements that were coarsely ground, but may not represent modern cements that are more finely ground.

F_2 values for these data show considerable scatter, as indicated in Fig. 5. The best fit line to the data is

$$\hat{F}_2 = 1.115 + 0.106 ln\left(\frac{a}{28}\right) \quad (11)$$

with both parameters significant at the 95 percent level, $R^2 = 0.2$, and a standard error of 0.20. Values predicted by Eq. (10a) for tall elements are shown by the line superimposed on Fig. 5 and slightly underestimate the average F_2 value predicted using Eq. (11). Eq. (10a) is, therefore, reasonably accurate and slightly conservative for these historical data.

RELATIONSHIP BETWEEN IN-PLACE STRENGTH AND SPECIFIED STRENGTH

Expected value and coefficient of variation of the in-place strength for a given specified strength may now be derived. The relationship between mean in-place strength and specified strength is determined using Eq. (3). The variability of in-place strength consists of the variability of mean in-place strength given f_c', which reflects the random nature of Factors F_1 and F_2 in Eq. (3) and within-structure variability. The probability that the in-place strength is less than the specified strength is then determined from the in-place strength distribution.

Average in-place strength

For concretes used in cast-in-place construction, F_1 can be represented by a lognormal distribution with the parameters shown in Table 2, with a mean value of 1.27 and a coefficient of variation of 12.2 percent. The mean value of F_2 may be approximated by the predicted value $\hat{F}_2$ as obtained from Eq. (10a), with a coefficient of variation of 13.9 percent. It may be assumed that F_1 and F_2 are statistically independent because the quality of the material delivered by the concrete producer is not correlated with the other factors that influence in-place strength such as consolidation and curing practices or age of concrete. Therefore, using Eq. (3), the mean in-place strength value has a lognormal distribution with a mean value of

$$\bar{f}_{c,is} = \left\{1.205 + 0.108 Z_h + 0.125 \ln\left(\frac{a}{28}\right)\right\} f_c' \quad (12)$$

and a coefficient of variation of 18.6 percent. Thus, the average in-place strength of conventional concretes at 28 days is about 1.20 times the specified strength for shallow elements and about 1.31 times the specified strength for tall elements. The coefficient of variation of the average in-place strength is large, reflecting the considerable scatter of the data. For example, the 95 percent confidence limits for the ratio of the average in-place strength at 28 days to the specified strength are 0.82 to 1.70 for shallow elements or 0.90 to 1.85 for tall elements. When the concrete is 1 year old, the ratio of the average in-place strength to the specified strength averages

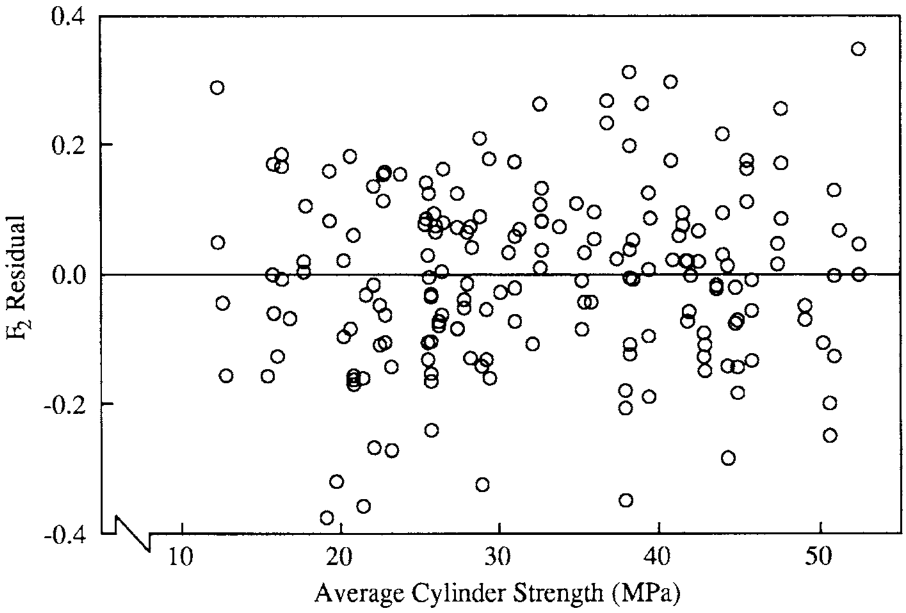

Fig. 4—Residual errors of Eq. (10) versus average cylinder strength

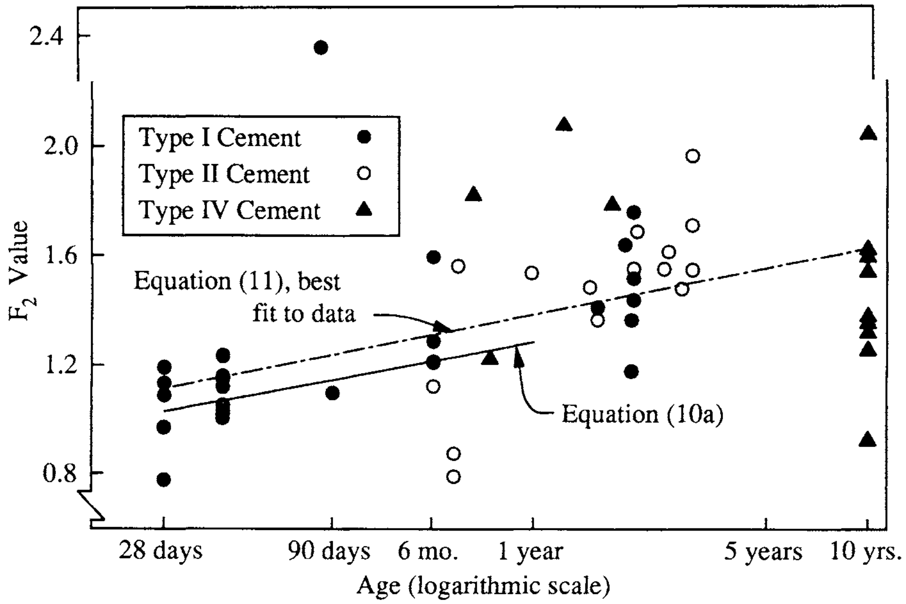

Fig. 5—Check of Eq. (10a) using historical data

1.33 or 1.44 for shallow or tall elements, respectively, with a coefficient of variation of 18.6 percent.

Additional research is necessary to determine the relationship between mean in-place strength and specified strength for precast elements because F_2 values for steam-cured precast elements are not represented by the data in this study. For precast units that are not steam-cured, the ratio of mean in-place strength to specified strength has a lognormal distribution with a mean value roughly 6 percent less than that given by Eq. (12) and a coefficient of variation of 15 percent.

Various relationships between average in-place strength at 28 days and specified strength are shown in Fig. 6. The relationship proposed by MacGregor[4] has been used as the basis for the calibration of the load and resistance factors in CAN-A23.3-M84, "Design of Concrete Structures for Buildings,"[21] and is similar to that reported by Thorenfeld[5] as the basis of provisions in NS 3473, "Design of Concrete Structures."[22] The average in-place strength predicted using Eq. (12) is greater than that obtained using either of the other relationships, which suggests that the associated design provisions may be conservative. Also shown is the in-place strength of columns and walls that were 28 days old in a multistorey car park investigated by Murray and Long[23] using a break-off tester. These data were excluded from the set that was analyzed to obtain Eq. (10) because cores were not tested, but they do corroborate the values predicted using Eq. (12).

Variability of in-place strength

Variability of the in-place strength of concrete can be split into two categories that represent the uncertainty associated with mean in-place strength and variability of strength throughout a structure with a given mean in-place strength, as indicated in Fig. 1. In the previous section, the mean in-place strength is shown to have a lognormal distribution with a mean value given by Eq. (12) and a coefficient of variation of 18.6 percent. This distribution is based on analysis of in-place strength data from single members cast from a single batch of concrete, and does not include batch-to-batch variation or systematic between-member variation. Also, the distribution is not greatly affected by systematic within-member variation or uncertainties attributable to testing factors, since these do not greatly increase the variability of the mean in-place strength.

Table 4—Coefficient of variation due to in-place strength variation throughout structure

Structure composition	One member	Multiple members
One batch of concrete	$\sqrt{V_{wb}^2 + V_M^2}$ $= 0.067$	$\sqrt{V_{wb}^2 + V_M^2 + V_B^2}$ $=0.084$
Multiple batches of concrete	$\sqrt{V_{wb}^2 + V_M^2 + V_{bb}^2}$	$\sqrt{V_{wb}^2 + V_M^2 + V_B^2 + V_{bb}^2}$
Cast-in-place	$= 0.120$	$= 0.130$
Precast	0.090	0.103

Variation of strength throughout the structure for a given mean in-place strength is composed of the inherent within-batch variability of concrete, systematic strength variation within a member, systematic between-member variation if the structure contains more than one member, and batch-to-batch variation if the structure contains more than one concrete batch. The inherent within-batch variability of concrete may be represented by a coefficient of variation V_{wb} of about 3 percent.[8] The coefficient of variation due to systematic within-member strength variation V_M is roughly 5 percent for columns or 6 percent for other elements, including slabs, walls, and large blocks.[8] The coefficient of variation due to systematic between-member variation V_B is not known but probably does not exceed 5 percent if the effects of batch-to-batch variation are accounted for separately. As already noted, the coefficient of variation due to batch-to-batch variation V_{bb} of concretes produced in Alberta from 1988 to 1993 is roughly 10 percent for cast-in-place construction or 6 percent for precast construction. Therefore, the variation of strength throughout the structure for a given mean in-place strength depends on the number of members, number of batches, and type of construction, as shown in Table 4.

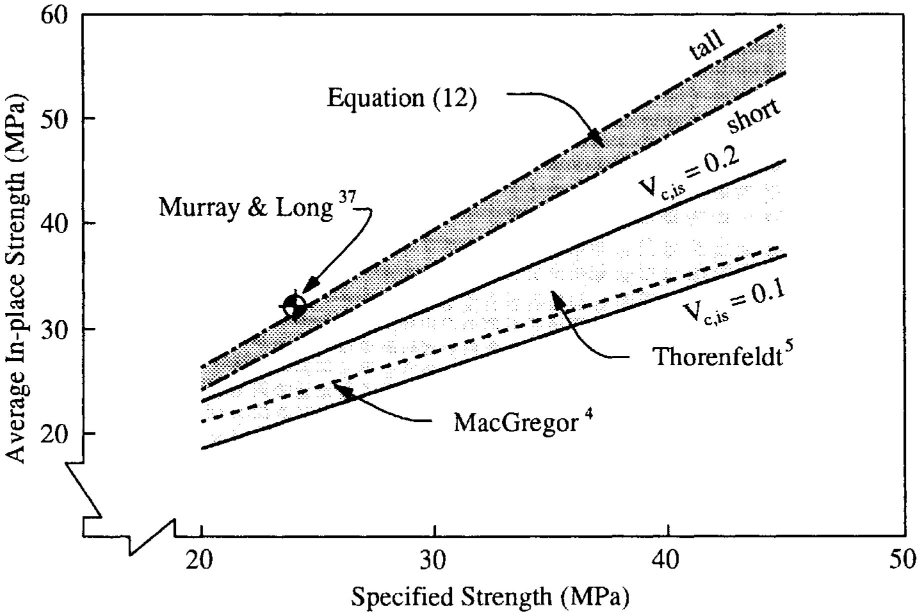

Fig. 6—Average in-place strength at 28 days versus specified strength

Thus, the overall coefficient of variation of the in-place strength of concrete $V_{c,is}$ for a cast-in-place structure composed of many members and cast from many batches is

$$V_{c,\,is} = \sqrt{0.186^2 + 0.130^2} = 0.227 \tag{13}$$

or 22.7 percent. This value pertains to the concrete strength in a structure that has not yet been constructed; that is, before any of the random factors that affect material quality have been realized. It represents considerable uncertainty, since it implies that the approximate 95 percent confidence limits on the unrealized in-place strength in a structure are 55 and 145 percent of the value obtained from Eq. (12).

Characteristic value of in-place strength corresponding to f_c'

The characteristic value of in-place strength of concrete represented by f_c' in conventional resistance equations may now be obtained. It is assumed that the distribution of the estimated in-place strength is lognormal, since the distribution of the estimated average in-place strength given by Eq. (12) is lognormal and the coefficient of variation of the predicted mean in-place strength dominates the overall coefficient of variation in Eq. (13). The desired probability is $\Phi(-\eta/\zeta)$ where $\Phi(\bullet)$ is the standard normal cumulative distribution function and η and ζ are the mean value and standard deviation of the natural logarithms of the in-place strengths. The value of η corresponding to the mean value from Eq. (12) for concrete that is 28 days old is

$$\eta = ln\left(\frac{1.205 + 0.108Z_h}{\sqrt{1 + V_{c,\,is}^2}}\right) \tag{14}$$

where $V_{c,is} = 0.227$ from Eq. (13). The value of ζ corresponding to the standard deviation is

$$\zeta = \sqrt{ln\left(1 + V_{c,\,is}^2\right)} \tag{15}$$

Thus, for tall members $Z_h = 1$, $\eta = 0.247$, and $\zeta = 0.224$, which implies that the probability that the in-place strength is less than the specified strength is $\Phi(-0.247/0.224) = 0.135$. For shallow members, $Z_h = 0$, $\eta = 0.161$, and $\zeta = 0.224$, which infers that the probability that the in-place strength is less than f_c' is $\Phi(-0.72) = 0.235$. The resistance of tall elements including columns and walls is typically more sensitive to the compressive strength of concrete than the resistance of shallow elements, including slabs and beams. Therefore, it can be assumed that the fractile of the in-place strength represented by f_c' is the 13.5 percent characteristic value; that is, 13.5 percent of the in-place strengths at 28 days will be less than f_c'.

It is interesting to note that a much larger characteristic value of in-place strength is associated with the statistical description of concrete reported by Mirza, Hatzinikolas, and MacGregor,[24] which suggests that the fractile of the in-place strength represented by f_c' ranges from the 40 percent characteristic value for 20 MPa concretes produced under poor control to the 98 percent characteristic value for 50 MPa concretes produced under excellent control. Therefore, it is probable that a recalibration of the concrete resistance factor ϕ_c in CAN3-A23.3, "Design of Concrete Structures for Buildings,"[21] based on the statistical description of concrete given by Eq. (12) and (13) would yield a higher concrete resistance factor for design than is presently in use.

SUMMARY

The relationship between the in-place strength of concrete in a structure and the corresponding specified compressive strength may be represented as the product of two independent factors F_1 and F_2. Factor F_1 is the ratio of the average strength of standard cylinder specimens to specified strength f_c'. To have some assurance that the material provided will meet specifications, the concrete producer typically insures that F_1 exceeds 1.0. Factor F_2 is the ratio of the average in-place strength to the average strength of 28-day standard cylinder specimens, and depends on the age and height of the element and the quality of curing provided.

To investigate the degree of overstrength provided by the concrete producer, 3756 strength tests from 108 concrete mixes supplied from 1988 to 1993 in Alberta were obtained. The data represent mixes with and without entrained air used for both cast-in-place and precast construction. Factor F_1 had a mean value of 1.25 and a standard deviation of 0.13 for concretes produced for cast-in-place construction or a mean value of 1.19 and a standard deviation of 0.06 for concretes produced for precast production. In either case, F_1 may be assumed to have a normal or lognormal distribution. Equations that give suggested target strengths for the design of concrete mixes in CAN/CSA-A23.1-M90[6] or ACI 318-89[7] do not accurately reflect the actual margin of overstrength observed.

To investigate the ratio F_2 between the average in-place strength and the strength of standard cylinders tested at 28 days, core and cylinder data from 108 concrete mixes with standard cylinder strengths less than 55 MPa obtained by others were analyzed. At 28 days, the average value of F_2 was 0.95 for elements less than 450-mm-high or 1.03 for elements at least 450-mm-high. The average in-place strength increased by about 25 percent between 28 days and 1 year. The value of F_2 was independent of concrete strength and may be represented by a lognormal distribution with a coefficient of variation of 14 percent.

In-place strength for a given specified strength is uncertain due to the inherent randomness of Factors F_1 and F_2 and also due to the variation of strength within the structure. The coefficient of variation due to within-structure strength variation was roughly 7 percent for a structure consisting of one member cast from a single batch of concrete, or roughly 13 percent for a structure consisting of many members cast from many concrete batches. The coefficient of variation of the in-place compressive strength for a cast-in-place structure that has not yet been constructed was roughly 23 percent.

Based on this statistical description, the probability that the in-place compressive strength of concrete is less than f_c' in columns or walls that are 28 days old is roughly 13 percent. This is a much smaller probability than that associated with the statistical description of concrete used to calibrate the load and resistance factors for the design of new structures in Canada. A recalibration that was based on the findings of this study would probably yield greater factored concrete strengths than are currently used in design.

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CONVERSION FACTORS

25.4 mm = 1 in.
6.895 MPa = 1000 psi

NOTATION

a	=	age of element (days)
c	=	cement content (kg/m^3)
$E[\bullet]$	=	expected value of derived quantity
fa^F	=	Type F fly ash content (kg/m^3)
f_c'	=	specified concrete compressive strength, MPa
$\bar{f}_{c,is}$	=	average in-place concrete strength, MPa
$\bar{f}_{cyl}$	=	average strength of 28-day standard cylinders, MPa
F_1	=	ratio of average standard cylinder strength to specified strength
F_2	=	ratio of average in-place strength to average cylinder strength
N_m	=	number of mixes
R^2	=	coefficient of determination
s_{bb}	=	standard deviation due to batch-to-batch variation of strength
$Var[\bullet]$	=	variance of derived quantity
V_B	=	coefficient of variation due to between-member strength variation
V_{bb}	=	coefficient of variation due to batch-to-batch strength variation
$V_{c,is}$	=	coefficient of variation of in-place strength
V_M	=	coefficient of variation due to within-member strength variation
V_{wb}	=	coefficient of variation due to within-batch strength variation
η	=	standard deviation of transformed lognormal population
ζ	=	mean value of transformed lognormal population
ϕ_c	=	resistance factor for compressive strength of concrete
$\Phi(\bullet)$	=	standard normal cumulative distribution function

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