Damping identification and model updating of boundary conditions for a cantilever beam

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ABSTRACT

Model updating is a crucial tool for dynamically loaded structures as natural frequencies and damping can have a vital impact on normal operation and fatigue life. Finite element models always deviate from experimentally obtained results due to variations in mass and stiffness and the lack of a damping matrix. Thus, it is vital for many large structures, that the dynamic properties be determined experimentally and subsequently apply model updating to "tune" the finite element model.

In this paper, model updating techniques are tested on a small cantilever beam where the fixed boundary condition is assumed to be flexible and the main source of damping. The methods are tested using both simulated data and experimental data for updating the stiffness and damping matrices. The results of the analytical model are compared both before and after updating with its experimental counterpart.

The model updating procedure follows a two-step process. In step one the stiffness matrix is updated using the iterative eigensensitivity approach where selected stiffness related parameters, in this case the boundary conditions, are updated based on the sensitivities to eigenfrequencies and mode shapes. In the second step, the damping of the structure is updated.

Keywords: Model updating, damping identification, modal expansion/reduction, simulated model updating, experimental model updating

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INTRODUCTION

The advent of the finite element method [1] has brought about the ability for more advanced and refined analysis of engineering structures. Stress distributions and modal properties can be analyzed for geometry far more complex than what is possible otherwise. The benefit of this is the ability to create structures optimized to the environmental factors they are subjected to. There is however one caveat, a finite element model is only ever as good as the users ability to capture the physical properties of the system. The stiffness and mass of a system are often fairly well known, however differences often exist due to production tolerances, joints and boundary conditions. These differences can significantly impact modal results and stress distributions. With the advances in experimentally measuring modal parameters [2], methods were in turn developed to update finite element models using the results from experimental analysis. Development started in the 60/70's, however much of the use and further development first came around in the 1990's with the increasing performance of computational hardware. These early methods and the model updating procedure is well described in [3].

The updating of mass and stiffness matrices can generally be split into two categories, direct updating methods and iterative updating methods. The direct methods have the advantage of updating the mass and stiffness matrices in one step and are able to match the experimentally obtained results very accurately. However for this article, only the iterative methods are of interest as the direct methods update the system matrices without consideration of the physical connectivity of the finite element model. Using iterative methods, updating parameters are selected where it is thought that error in the finite element model exists. This could be joints, boundary conditions, youngs modulus, density, etc. Many iterative methods exist, one of the most popular being the inverse eigensensitivty approach [4] which utilizes the eigenvalues and modeshapes of a system for updating. Others have sought to improve the inverse eigensensitivity method, such as Lin [5]. other types use the frequency response functions for updating, such as the normal response function method seen in [6], [7] and [8]. Alternative iterative updating methods exist such as constrained optimization methods [9] or genetic algorithms[10] and neural networks [11] for improving the optimization methods. Another more recent approach, known as the cross-model cross-mode method [12] shows a lot of promise as the method is able to update selected updating parameters in one step. Thus providing the computational efficiency of a direct method but still maintaining the physical connectivity of the system. The method has also been tested on a small experimental structure [13]. For this article, the inverse eigensensitivity approach is used as it is a very proven and robust method of model updating. This is important for the subsequent damping identification as these require accurate stiffness and mass matrices.

Updating of damping is a very complex problem and to a large extent the mechanisms of the force dissipation from a system are unknown. Many damping models have been proposed to capture the energy dissipation from a vibrating system, one of the most common being the viscous Rayleigh/proportional damping model [14], although many other damping models exist. Much like the updating of the mass and stiffness matrices, damping can also be updated either using an iterative or direct approach. Lancaster [15] proposed an iterative method using updated mass and stiffness matrices and complex eigendata. Other methods determine the damping matrix using frequency response functions, such as the work by Lee and Kim [16]. In this article, a modified version of the Lancaster method [17] is used, where the damping matrix is found directly.

One of the issues that arises often when performing model updating, is the incompleteness of measurement data with respect to the finite element model. Finite element models can have millions of DOFs where as with an experimental measurement, you are lucky if you have 100 measurement DOFs. Further, rotational degrees of freedom are challenging to measure. For this reason, it is often necessary to either expand the measured modal data to the DOFs of the finite element model or reduce the finite element model down to the DOFs of the measured modal data. Many methods exist for expansion/reduction, some better then others. One of the best known and earliest methods is the guyan reduction method[18], which is a static reduction technique that closely replicates the modes, however there is some error due to neglecting mass effects. Another common method used is the kidder approach[19], this method is an improvement over the Guyan reduction method at higher frequencies as it includes mass effects. The method used in this article is the SEREP(system equivalent expansion reduction process) method [20].

THEORY

Model reduction and modal expansion

Model reduction and modal expansion are mathematical tools that have been developed to deal with the spatial incompleteness that occurs during experimental modal analysis testing. For the case of model updating, the limited number of measurements during testing presents a problem, as the number of degrees of freedom for a finite element system is often many orders of magnitude higher than that of the experimental setup. Many updating algorithms only work if the the DOF size of the experimental model is equal to the DOF size of the finite element model. Thus a form of mapping is required between the experimental DOFs (master DOFs) and the non-measured analytical DOFs (slave DOFs) [21].

All modal expansion/model reduction techniques are based on the same concept. A transformation matrix is identified that can be used for either expanding an experimental data set from a set of master DOFs to the slave DOFs or to reduce the analytical model down to the slave DOFs. In general, the following relationships for the techniques are thus defined in Equations (1), (2) and (3).

$$\left\{X_n\right\} = \left\{\begin{matrix} X_m \\ X_s \end{matrix}\right\} = \left[T\right]\left\{X_m\right\} \tag{1}$$

Where $\left\{X_n\right\}$ is the expanded mode, $\left\{X_m\right\}$ are the mode shape coefficients for the master modes, $\left\{X_s\right\}$ are the mode shape coefficients for the slave nodes and the transformation matrix between them is given by $\left[T\right]$.

For model reduction, the transformation matrix is multiplied before and after on the mass and stiffness matrices, thus reducing the mass and stiffness matrices down to the master degrees of freedom as seen in Equations (2) and (3).

$$\left[M_m\right] = \left[T\right]^T \left[M_n\right]\left[T\right] \tag{2}$$

$$\left[K_m\right] = \left[T\right]^T \left[K_n\right]\left[T\right] \tag{3}$$

SEREP modal expansion/model reduction:
The SEREP method utilizes the analytical models mode-set in order to interpolate the experimental mode shapes or to reduce the analytical mass and stiffness matrices [20].The transformation matrix is shown in Equation (4).

$$[T_U] = [U_n]([U_m^T][U_m])^{-1}[U_m]^T \tag{4}$$

Where $[T_U]$ is the SEREP transformation matrix, $[U_n]$ is the analytical mode shape matrix and $[U_m]$ is the analytical modeshape matrix containing only master DOFs.

This method of modal expansion and model reduction is used in all subsequent analyses when expansion or reduction is required.

Inverse eigensensitivity approach

The inverse eigensensitivity approach [4] is an iterative model updating technique that uses experimental and analytical eigenvalues and modeshapes for updating selected updating parameters. The method is based on the rate of change of the eigenvalues and modeshapes [22] and utilizes a least-squares updating approach. It can be used for updating both the analytical mass and stiffness matrices. Results obtained from the method are highly dependent upon the choice of updating parameters, this is one of the most crucial steps involved. If incorrect parameters are selected, the updating will not be able to converge on a satisfactory result. If to many updating parameters are selected, the calculation time will increase and the method can potentially become unstable [4]. The parameter sensitivity to eigenvalues $\frac{(\partial\omega_A)_r^2}{\partial p_i}$ is seen in Equation (5) and to modeshapes $\frac{(\partial\phi_A)_r}{\partial p_i}$ in Equation (6).

Sensitivity of eigenvalues:

$$\frac{(\partial\omega_A)_r^2}{\partial p_i} = \{\phi_A\}_r^T \frac{\partial[K_A]}{\partial p_i}\{\phi_X\}_r - (\omega_x)_r^2\{\phi_A\}_r^T \frac{\partial[M_A]}{\partial p_i}\{\phi_X\}_r \tag{5}$$

In equation (5), the parameters eigenvalue sensitivity $\frac{(\partial \omega_A)^2_r}{\partial p_i}$ for mode r is dependent on the analytical modeshape for mode r $\{\phi_A\}_r$, the experimental modeshape for mode r $\{\phi_X\}_r$, the experimental eigenvalue for mode r $(\omega_x)^2_r$ and the partial derivatives of the analytical stiffness $[K_A]$ and mass matrix $[M_A]$ towards the updating parameter p_i.

Sensitivity of modeshapes:

$$\frac{(\partial \phi_A)_r}{\partial p_i} = \sum_{j=1;j\neq r}^{N} \frac{\{\phi_A\}_j\{\phi_A\}_j^T}{(\omega_X)^2_r - (\omega_A)^2_j} \left[\frac{\partial [K_A]}{\partial p_i} - (\omega_X)^2_r \frac{\partial [M_A]}{\partial p_i}\right] \{\phi_X\}_r - \frac{1}{2}\{\phi_A\}_r\{\phi_A\}_r^T \frac{\partial [M_A]}{\partial p_i}\{\phi_X\}_r \quad (6)$$

The parameter sensitivity of modeshapes $\frac{(\partial \phi_A)_r}{\partial p_i}$ in Equation (6) is dependent upon the same input parameters as for the sensitivity to eigenvalues in Equation (5). It is further also dependent on the analytical eigenvalue $(\omega_x)^2_r$.

The parameter updating is then solved using a least squares updating approach as seen in Equation (7). The sensitivities of eigenvalues and modeshapes are added

$$\{\Delta p\} = [S]^+\{\Delta \xi\} \quad (7)$$

Thus the change in updating parameters $\{\Delta p\}$ is found taking the pseudo inverse of the sensitivity matrix $[S]$ (where the $+$ symbol denotes the pseudo inverse) and then multiplying the pseudo inverse sensitivity matrix on the vector $\{\Delta \xi\}$ containing the differences in eigenvalues and modeshapes. The method requires multiple iterations to reach a converged solution.

Non-proportional damping

Proportional damping is defined from a linear combination of the mass and stiffness matrices [2]. The formulation of proportional damping can be seen in Equation (8). It is seen that the damping matrix $[C]$ is a linear combination of $[M]$ and $[K]$, where the coefficients a and b govern the amount of damping contribution from the mass and stiffness matrices.

$$[C] = a[M] + b[K] \quad (8)$$

The damping matrix can be made non-proportional by taking Equation (8) and applying it element-wise, with different values of a and b. Thus each element will have a damping matrix $[C_e]$ formulated by corresponding damping coefficients a_e and b_e and element stiffness and mass matrices $[K_e]$ and $[M_e]$. Thus each element damping matrix can be formulated as in Equation (9). The damping matrix is then assembled based on element connectivity.

$$[C_e] = a_e[M_e] + b_e[K_e] \quad (9)$$

Direct damping updating

The damping updating method used is based on Lancasters iterative method. Pilkey [17] describes how this method can be used as a direct updating method. The approach is based on a viscously damped system as seen in Equation (10). The modeshapes must be scaled to the eigenvalues as in Equation (11), a damping matrix can then be calculated as seen in Equation (12). The mass and stiffness matrix must be accurate in order for this method to produce an adequate damping matrix.

$$[[M]\lambda_i^2 + [C]\lambda_i + [K]]\{\phi_i\} = 0 \quad (10)$$

$$\{\phi_i\}^T([M]\lambda_i^2 - [K])\{\phi_i\} = \lambda_i \quad (11)$$

$$[C] = -[M]([\Phi][\Lambda^2][\Phi]^T + [\bar{\Phi}][\bar{\Lambda}^2][\Phi]^*)[M] \quad (12)$$

For Equations (10),(11) and (12): $[M]$, $[C]$ and $[K]$ are the mass, damping and stiffness matrices. λ_i and $\{\phi_i\}$ are the i'th eigenvalue and modeshape. $[\Phi]$ is the modeshape matrix, $[\Lambda^2]$ the diagonalized eigenvalue matrix and $[\bar{\Phi}]$ and $[\bar{\Lambda}^2]$ their complex conjugate counterparts. $[\Phi]^*$ is the complex conjugate transpose of the modeshape matrix.

SIMULATED BEAM

A simulated "experimental" beam is initially used for testing the inverse eigensensitivity approach and the direct updating method. Using a simulated beam allows for testing the accuracy of the methods, as the true stiffness and damping values are known.

Model setup

The simulated beam is modeled as a fixed-fixed beam as seen in Figure 1. The finite element modal analysis is implemented in Matlab using 39 beam elements. The boundary conditions are made soft, using a rotational and translational spring at each end. Two beams are simulated initially, a simulated analytical beam and a simulated experimental beam. Both beams have identical dimensions and material properties, however the spring stiffnesses are different between the two models. The simulated experimental beam is measured at 8 equidistant points along the beam, simulating the use of accelerometers. Beam properties for both beams can be seen in table 1. For simulating damping, non proportional damping is applied.

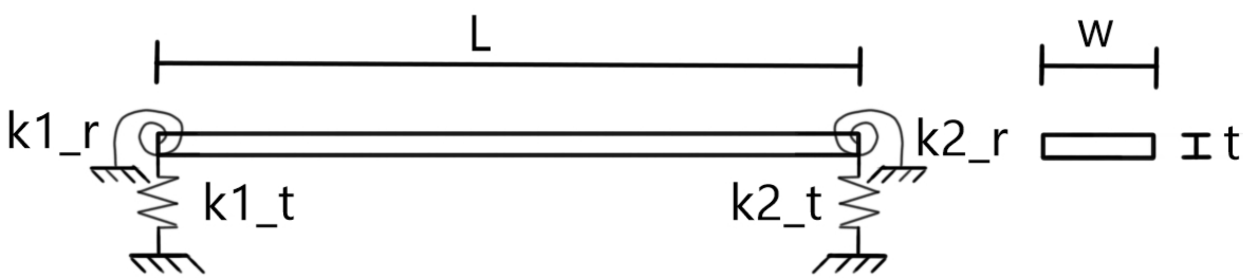


Fig. 1: Simulated beam setup. Rotational and translational springs are used at each end for boundary conditions. The modal results are extracted at 8 equidistant points along the structure.

Tab. 1: Fixed-fixed beam properties.

L $[mm]$	**W** $[mm]$	**t** $[mm]$	**Youngs Modulus** $[GPa]$	**Density** $\left[\frac{kg}{m^3}\right]$
910	50	5	200	7850

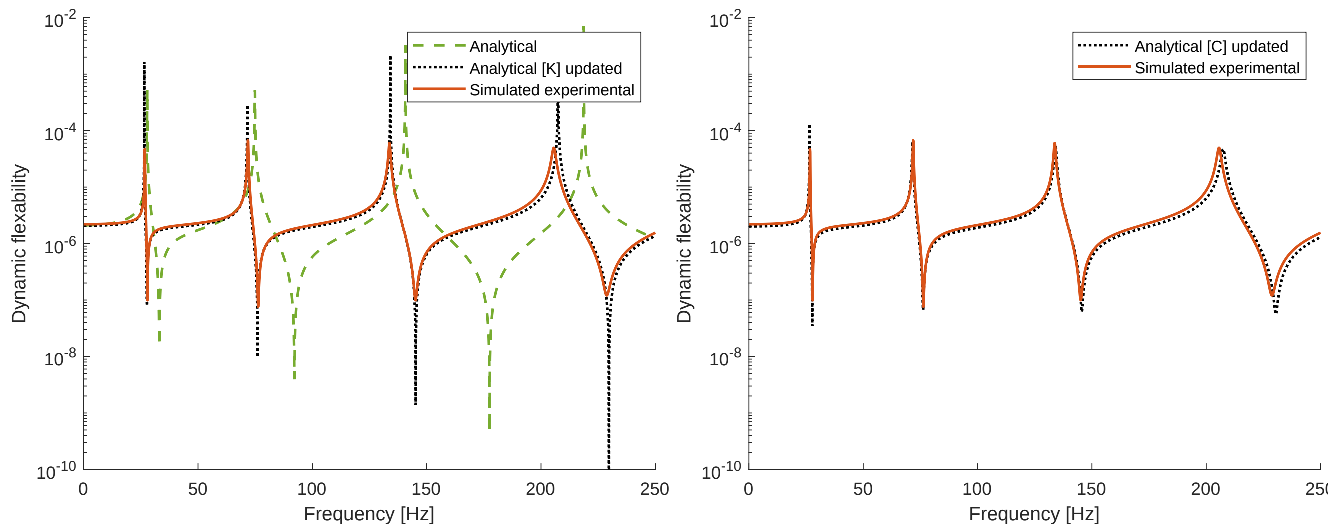
Tab. 2: Fixed-fixed beam spring stiffnesses.

Springs	**Simulated experimental**	**Analytical**
k1_t $\left[\frac{N}{m}\right]$	$5 \cdot 10^5$	$7.5 \cdot 10^5$
k1_r $\left[\frac{N}{m}/^\circ\right]$	$2 \cdot 10^3$	$3 \cdot 10^3$
k2_t $\left[\frac{N}{m}\right]$	$4 \cdot 10^5$	$5.2 \cdot 10^5$
k2_r $\left[\frac{N}{m}/^\circ\right]$	$4 \cdot 10^3$	$5 \cdot 10^3$

Before using the eigensensitivity approach, the simulated experimental modeshapes are expanded up to the same number of DOFs as the analytical modes and when using the Pilkey method for damping updating, the analytical mass and stiffness matrices are reduced down to the number of simulated experimental DOFs.

Results

This section shows the results for the analytical fixed-fixed beam both before and after updating the $[K]$ and $[C]$ matrices. Figure 2 shows the FRF's of the analytical beam compared to the simulated experimental beam both before and after updating. In Table 3, the natural frequencies and MAC values of the simulated experimental, analytical and the analytical after $[K]$ updating are compared and in Table 4, the simulated experimental and analytical model after $[C]$ matrix updating are compared. Finally in Table 5, the spring stiffnesses after updating are compared with the simulated experimental spring stiffnesses.



(a) **(b)**

Fig. 2: (a): FRF's before and after updating the stiffness matrix versus the simulated experimental FRF. **(b)**: Frequency response function after updating the damping matrix versus the simulated experimental FRF.

From Figure 2a it is seen that after updating the stiffness matrix, that the FRFs of the updated model almost coincide, though with a slight shift in frequencies. Likewise in Figure 2b it is seen that after damping identification, that the peeks are nearly identical in amplitude.

Tab. 3: Natural frequency and MAC value comparisons between the simulated experimental beam, the analytical beam and the analytical beam after [K] matrix updating.

Mode [no]	Simulated exp.	Analytical model			[K] updated		
	Freq. [Hz]	Freq. [Hz]	Error [%]	MAC-value	Freq. [Hz]	Error [%]	MAC-value
1	26.75	27.782	3.86	1.00	26.463	-1.07	1.00
2	71.849	74.856	4.19	1.00	71.513	-0.47	1.00
3	133.75	140.69	5.21	1.00	134.08	0.27	1.00
4	205.62	218.69	6.33	0.99	207.57	0.92	1.00

From table 3 it is found that the MAC-values for the analytical model are good, however as expected there is some error of the natural frequencies. After updating the stiffness matrix, the error percentages of the natural frequencies decrease and their is a slight improvement in the MAC values.

Tab. 4: Natural frequency and MAC value comparisons between the simulated experimental beam and the analytical beam after [C] matrix updating.

Mode [nr]	**Simulated exp.**	**[C] updated**		
	Frequency [Hz]	**Frequency [Hz]**	**Error [%]**	**MAC-value**
1	26.75	26.463	-1.07	1.00
2	71.849	71.513	-0.47	1.00
3	133.75	134.08	0.27	1.00
4	205.62	207.57	0.92	1.00

In Table 4 the frequencies and MAC values between the simulated model and the $[C]$ matrix updated model are compared. Comparing the frequency error values for the $[C]$ matrix updated model with the frequency error values of the $[K]$ matrix updated model in Table 3, it is seen that the frequency error does not change. This is logical, as the damping matrix does not affect the natural frequency of the system, only the damped frequency. The MAC values remain the same.

Tab. 5: Fixed-fixed beam spring stiffnesses after updating.

Springs	**Simulated experimental**	**Analytical updated**	**Error** $[\%]$
k1_t $\left[\frac{N}{m}\right]$	$5 \cdot 10^5$	$5.41 \cdot 10^5$	8.2
k1_r $\left[\frac{N}{m}/°\right]$	$2 \cdot 10^3$	$1.79 \cdot 10^3$	-10
k2_t $\left[\frac{N}{m}\right]$	$4 \cdot 10^5$	$4.20 \cdot 10^5$	5
k2_r $\left[\frac{N}{m}/°\right]$	$4 \cdot 10^3$	$3.59 \cdot 10^3$	-10.3

After updating the stiffness matrix, the spring values of the simulated experimental beam and the analytical model after updating are compared in Table 5. Comparing the analytical updated stiffness values to the simulated experimental, it is seen that there still is some error in the spring stiffness's. This explains why in Figure 2a and Table 3 we see discrepancies in the natural frequencies. Comparing the analytical updated values with the analytical model before updating from Table 2, it is seen that the updated model values are still an improvement over those before updating.

EXPERIMENTAL BEAM

After verifying the methods on the simulated setup, the inverse eigensensitivity approach and Pilkey damping updating method are tested on an experimental beam setup. This tests the robustness of the algorithm and method when the ”true” updating parameters are unknown and must be user selected.

Model setup

The experimental beam data used comes from a masters thesis [23] where EMA was preformed on a slender beam structure. The beam was excited using an impact hammer at the tip of the beam and the response was measured at 8 points along the structure using piezoelectric accelerometers. The beam dimensions and accelerometer positions can be seen in Table 6 and Figure 3. As the focus of the article does not cover EMA approaches, for more details on the EMA setup and methods, details are given in the thesis. For finite element analysis of the beam, the beam was modeled in Matlab using 39 beam elements, a rotational spring element and translational spring element.

Tab. 6: Cantilever beam experiment properties.

Length $[mm]$	**Height** $[mm]$	**thickness** $[mm]$	**Youngs Modulus** $[GPa]$	**Density** $\left[\frac{kg}{m^3}\right]$
700	70	2	200	7850

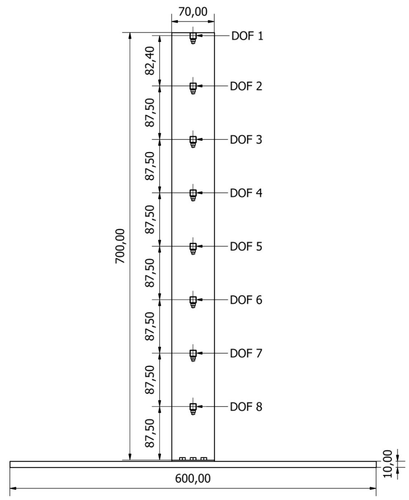


Fig. 3: Experimental beam setup: DOF placements and overall dimensions of the beam.

Before using the eigensensitivity approach, the experimental modeshapes are expanded up to the same number of DOFs as the analytical modes and when using the Pilkey method for damping updating, the analytical mass and stiffness matrices are reduced down to the number of experimental DOFs.

Results

This Section shows the results for the experimental cantilever beam both before and after updating. Figure 4 shows the FRF's of the analytical beam versus the experimental beam both before and after updating the analytical model. In Table 7, the natural frequencies and MAC values of the experimental, analytical and the updated $[K]$ and $[C]$ matrix analytical models are compared. The results shown in Table 8 compare the natural frequencies and MAC values after the $[C]$ matrix is formulated.

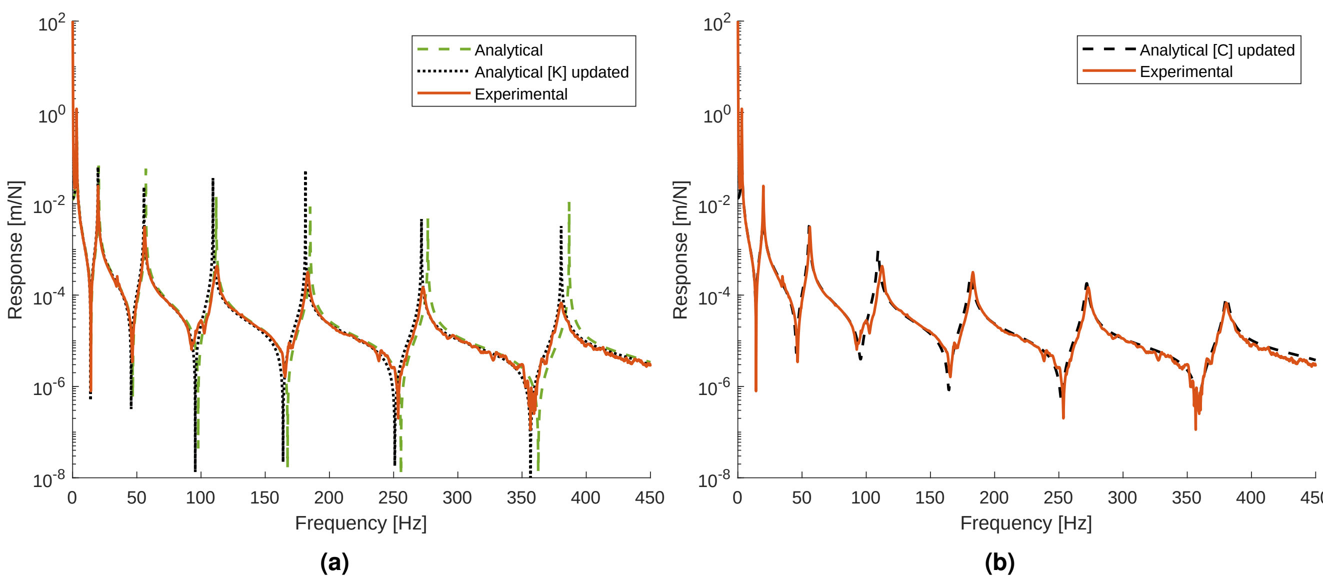


Fig. 4: (a): FRF's before and after updating the stiffness matrix versus the experimental FRF. **(b)**: Frequency response function after updating the damping matrix versus the experimental FRF.

From Figure 4a it is seen that before updating, there is only a slight discrepancy in the natural frequencies in the FRF. After updating the stiffness matrix, the peaks shift slightly and match better with the experimental results. Though still with some error of the peaks lining up. Likewise in Figure 4b it is seen that after damping identification, that the peeks are nearly identical in amplitude.

Tab. 7: Natural frequency and MAC value comparisons between the experimental beam, the analytical beam and the stiffness matrix updated analytical beam.

Mode [no]	**Measured**	**Analytical model**			**[K] updating**		
	Freq. [Hz]	**Freq. [Hz]**	**Error [%]**	**MAC-value**	**Freq. [Hz]**	**Error [%]**	**MAC-value**
1	3.0752	3.2243	5.44	1.00	3.1334	1.89	1.00
2	19.845	20.344	2.51	1.00	19.748	-0.49	1.00
3	56.062	57.017	1.70	1.00	55.545	-0.92	1.00
4	112.19	111.83	-0.30	1.00	109.28	-2.58	0.99
5	182.99	185.03	1.11	1.00	181.29	-0.93	0.99
6	272.78	276.64	1.42	1.00	271.66	-0.41	0.99
7	380.28	386.69	1.69	1.00	380.46	0.05	1.00

From table 7 it is found that the MAC-values for the analytical model are good, however there is some error of the natural frequencies. After updating the stiffness matrix, the error percentages of the natural frequencies decrease but the MAC-values slightly decrease.

Tab. 8: Natural frequency and MAC value comparisons between the experimental beam and the damping matrix updated analytical beam.

Mode [nr]	**Measured**	**[C] Updated model**		
	Frequency [Hz]	**Frequency [Hz]**	**Error [%]**	**MAC-value**
1	3.0752	3.133	1.88	1.00
2	19.845	19.748	-0.49	1.00
3	56.062	55.552	-0.91	1.00
4	112.19	109.28	-2.59	0.99
5	182.99	181.29	-0.93	0.99
6	272.78	271.68	-0.40	0.99
7	380.28	380.47	0.05	1.00

In Table 8 the frequencies and MAC values between the simulated model and the $[C]$ updated model are compared. Comparing the frequency error values for the $[C]$ updated model with the frequency error values of the $[K]$ updated model in Table 7, it is seen that the frequency error does not change. Just as for the simulated experiment, this is as expected, as the damping matrix should not affect the natural frequencies.

Tab. 9: Cantilever beam spring stiffnesses after updating.

Springs	**Analytical**	$[K]$ **updated**	**Change** $[\%]$
k1_t $\left[\frac{N}{m}\right]$	$5 \cdot 10^{10}$	$4.19 \cdot 10^{14}$	-
k1_r $\left[\frac{N}{m}/°\right]$	$1 \cdot 10^{3}$	$4.16 \cdot 10^{2}$	-58.4

In Table 9, the boundary condition spring stiffnesses are compared before and after updating the stiffness matrix. The translational spring sees a huge increase in stiffness, this is probably due to the rigid bolting of the beam and a low stiffness value of the translational spring initially. The rotational spring decreases by 58%, this reduction in stiffness is what accounts for the natural frequency changes in Table 7.

DISCUSSION

Based on the results of the simulated and the experimental beams, it is seen that the inverse-eigensensitivity method and direct damping updating method proprosed by Pilkey [17] provide good results when updating boundary conditions, albeit still with some error in the updated parameters.

For the simulated case, the error in stiffness updated parameters is likely due to the modeshape expansion. According to Jung [4], the inverse eigensensitivity approach can be used on incomplete modal data without modeshape expansion. This is something that must be tested further to see if improvements can be made to the parameter estimation. Further, alternate methods of updating the stiffness matrix could be tested, as the Pilkey damping updating method [17] requires accurate mass and stiffness matrices in order to obtain the best possible results.

For the experimental case, the natural frequencies generally improved well, however the MAC-values slightly decreased. This could indicate that more than just the boundary conditions require updating. The material stiffness can be subjected to slight variations from batch to batch, thus the elastic modulus would be a good candidate for further updating. Also, the beam is fairly slender and quite a few accelerometers are attached. Thus mass loading of the beam could be affecting the natural frequencies and modeshapes. An analytical model with point masses at measurement locations may further improve the results.

CONCLUSION

The inverse-eigensensitivity approach and the direct damping updating method proposed by Pilkey where tested for updating boundary conditions and damping. The methods were both tested on a simulated set of data and corresponding FE model and on an experimentally tested cantilever beam. In both cases, the methods were able to update the stiffness and damping, thus providing more accurate modal results over their non-updated counterparts.

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