Dynamic Effects Parameter Uncertainty in Hyperelastic Foam Systems (*Extended Abstract*)

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Abstract Engineering analysts have a need to understand the effect that preload in closed-cell foams has on the dynamic response of a suspended mass. Currently, the effects of preload, or pre-compression, are considered to be negligible; however, any effect to the dynamic response could be detrimental to the involved system. The approximate cubic relationship between force and displacement of the material of interest poses an interesting problem in analyzing how the frequency response of the suspended mass changes with transitions from the cell wall bending to the more linear portion of the stiffness profile. A suite of studies utilizing finite element analysis and numerical simulations was used to perform a parametric study in order to generate a material meta-model that accurately describes the nonlinear system dynamics. Material properties such as porosity, bulk modulus, and other hyper-elastic model parameters were varied in these simulations. A simple two-dimensional plane strain FE model including two sections of closed cell foam and one section of a solid mass was used to perform two tasks – implicitly determine stress states from pre-compression in the foam and explicitly solve for the system's dynamic response when subject to an initial velocity. These results were then verified using numerical simulations of the nonlinear oscillator's second-order equation of motion. A single element, plane strain model of the closed-cell foam was used to generate force-displacement curves that were fitted to create a functional model for the foam's stiffness. Time history signals of the oscillator's response to an initial velocity from both analysis methods were then transformed to the frequency domain where the parameter of interest was the fundamental frequency of the system. The fundamental frequency of the system was extracted from each set of data from the simulation suite. A sensitivity index for material model parameters was then formed based on the effect each parameter had on the stiffness of the model. The meta-model proved to be in fair agreement with the full system model (12.5% error for nominal material values) and was able to bound the frequency response solution of the full system model. The sensitivity analysis revealed that initial porosity was the most significant parameter in the model followed by Neo-Hookean modulus at small displacements and bulk modulus during foam densification.

Keywords: hyperelastic · nonlinear oscillator · sensitivity analysis · finite element analysis · reduced-order model

1. Introduction

Hyperelastic material models are a special class of materials that respond elastically to large compressive or tensile deformations [1]. Rubber-like materials are a common application for hyperelastic constitutive modeling techniques. These materials can exhibit complicated behaviors that extend well beyond linear elastic theory such as large deformations, plastic and viscoelastic properties, and stress softening (Mullins effect) [2]. Starting with a constitutive model form, Ogden or Neo-Hookean for example, experimental data is used to fit constitutive model parameters and thus a hyperelastic material model is born. The downside to constitutive models, however, is that at they rely on a fitting process. A consequence of such a process is the lack of physics involved in using the model outside of its calibration range [3]. Thus, it should be obvious that the typical hyperelastic material models have shortcomings, and this work utilizes a material model that aims to overcome such issues. At the heart of this work is the verification and uncertainty quantification of a physics-based porous foam material model, CHIPFoam. The CHIPFoam material model developed by Lewis [4] was created to improve upon the hyperelastic modeling techniques that are commercially available in finite element codes, such as ABAQUS HYPERFoam. The CHIPFoam model consists of four components – small deformation for capturing buckling behavior in the foam; incompressible considerations for describing stiffening at large deformations; matrix compressibility to allow for consistent response at large compressive strains; and an optional term to model the effects of gas compression in the closed cells of the foam [4]. In this work, it is the inherent complexities that result from parameter uncertainty that are of importance. Additionally, this work aims to produce a reduced order model for predicting the dynamic response of complex systems that utilize components modeled by CHIPFoam.

For use in finite element (FE) models, the CHIPFoam material model is implemented with user-defined subroutines, similar to the UHYPER subroutine templates in ABAQUS [4]. Many systems that are simulated using the CHIPFoam subroutine are extremely computationally expensive; however, the cost of running the material model subroutine is only a small contributor to that computational cost. So, it is beneficial to produce a reduced-order model, or meta-model, of a simple CHIPFoam system for making low-cost predictions about the response of complex models. Two areas that this work extends further than other literature is in regime in which the material model is tested and in that the model is verified with a separate and simpler model. It is much more common for hyperelastic materials to be tested and fit to tensile loading data [3] [5] [6]. This work tests the CHIPFoam model in a purely compression regime with large values of compressive strain up to forty-eight percent. The additional novelty in this work is the combination of an analytical SDOF model and curve fit FE data to generate a meta-model for approximating the response of complex foam and suspended mass systems.

2. Modeling and Methods

At the heart of this work is the data that is extracted from finite element models. Full system models are used for computing the expected response of the suspended mass system via an implicit-explicit hand-off model. Solution verification of this full system model comes in the form of meta-model that is informed by the simulation of a quasit-static compression process in a single element FE model. This meta-model, or model of a model, is generated to provide a simplistic way of predicting the range of responses of the suspended mass system.

The first of the FE models in this work are a series of single element models to generate stiffness data that is representative of the foam. Simple deflection-controlled studies are run on the single element model, and the outputs from each study are a deflection and corresponding reaction force data points. These data points are computed for strains between zero and forty-eight percent (in compression) based on prior knowledge of the material behavior [4]. The force-displacement data that from the displacement-controlled single element analyses will ultimately be used to inform the stiffness of a material meta-model. A functional form of the foam's stiffness is found by curve fitting the F-Δ data using a least-squares regression method to determine polynomial coefficients.

The single element FE model in conjunction with the meta-model exists to provide solution verification for the FE model of the entire suspended mass system. To achieve this, the full system model is first solved implicitly to capture characteristics of precompression in the model and then explicitly to capture the dynamic response of the system. The FE model of the entire suspended mass system consists of two pieces of foam that are used to suspend a solid mass.

A lumped mass model of a single degree-of-freedom oscillator is used to develop the second order equation of motion for the foam and suspended mass system, as shown in Figure 1. Each section of foam is modeled with constant damping β and variable stiffness $k_{1,2}$. The variable stiffness of each section of foam is assumed to be described by a third-degree polynomial. The polynomial-functional form of stiffness for each spring includes the initial precompression in each spring x_0, and the two-dimensional state space formulation of the system's equation of motion can be written to create a series of first order ODEs:

$$\begin{Bmatrix} X \\ \dot{X} \end{Bmatrix} = \begin{Bmatrix} \dot{x} \\ -\frac{1}{m}\left(2\beta\dot{x} + k_1|_{x_0-x}(x) + k_2|_{x+x_0}(x) - F\cos(\omega t)\right) \end{Bmatrix}, \begin{Bmatrix} x \\ \dot{x} \end{Bmatrix}_{t=0} = \begin{Bmatrix} x_i \\ \dot{x}_i \end{Bmatrix}. \tag{1}$$

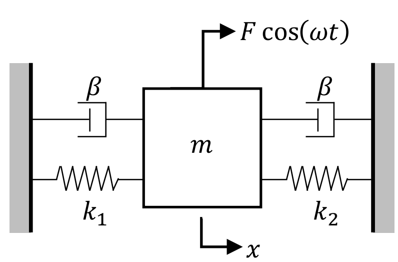

Figure 1. Lumped-mass model of the SDOF suspended mass system, where springs and dampers represent separate pieces of closed-cell foam.

The first-order system of ODEs in the state-space model can be solved with a variety of nonlinear solvers. For this work, the SDOF system is simulated using a fourth-order explicit Runge-Kutta time integration scheme (RK4).

2. The Parametric Study

A Latin Hypercube Sampling process is used to generate suite of simulations for evaluating the full system FE models and the meta-model. The hyperelastic material model contains nine parameters, three of which are varied in this parametric study – initial porosity φ, bulk modulus K, and Neo-Hookean modulus C_{10}. A nominal value of fifteen samples is drawn from a normal distribution of each of the three selected parameters. For each sample of the three model parameters, an additional two Null parameters are sampled. The nominal values and boundaries for the parameter distributions in this study are summarized in Table 1.

Table 1. Nominal values, upper, and lower bounds for parameter selection. Upper and lower bounds on parameters are selected either on physical constraints or numerical stability constraints. Nominal values are calibrated to experimental data. A total of seventy-six runs are generated from the LHS process.

Parameter	Lower Bound	Nominal Value	Upper Bound
Initial Porosity (φ)	0.001	0.633	0.701
Bulk Modulus [MPa] (K)	8.50	10.00	11.50
Neo-Hookean Modulus [MPa] (C_{10})	0.0100	0.0148	0.0400

2.1 Sensitivity Analysis

Each of the seventy-six parameterized single element FE models is solved for displacements up to forty-eight percent strain. Field outputs for reaction force and net displacement of the top surface of the element are differentiated to evaluate stiffness, and stiffness is used as a function of displacement for sensitivity analyses via polynomial chaos expansion (PCE). PCE allows for a global, multi-input sensitivity analysis (SA) that also considers effects from parameter coupling [7] [8] [9]. The PCE SA performed on this data was used to compute Total Effect, Main Effect, and Interaction Effect sensitivity indices, where total effect is simply the combination of main effects and parameter interaction effects. Further derivation and discussion on sensitivity analysis via polynomial chaos expansion can be found in references [7], [8], [9], [10], and [11].

3. Results and Model Comparisons

The entire work to-date presents four main results – computational efficiency of the meta-model, material model sensitivity, meta-model uncertainty, and solution verification of the meta-model via comparison with the full system FE explicit solution. The following sections include results for only computational efficiency, parameter sensitivity, and preliminary solution verification.

3.1 Computational Efficiency

Computational cost of the meta-model and the full system model are evaluated based on the total amount of time it takes to create a set of bounds on the system's frequency response. For the meta-model, all seventy-six single element models need to be evaluated in order to gain information about the bounds on the parameterized force-displacement space; this takes approximately 230 seconds. The RK4 simulation of the meta-model bounds takes approximately thirty-eight seconds for a total meta-model solve time of 268 seconds. In total, the full system parameter study requires 11,332 seconds to compute the bounds on its frequency space. Statistics from the parametric study of the two suites of FE models as well as computational cost summaries are listed in Table 2. All computations for these parametric studies were performed on a single CPU.

Table 2. Statistics from the parametric studies of the two suites of FE models. The meta-model solves over 97% faster than the full system model. Regardless of the complexity of the full system model, the meta-model will always exhibit the same computational cost.

Parameter Study Statistics			
Parameters varied:	Initial Porosity φ, Bulk Modulus K, Neo-Hookean Modulus C_{10}		
Nominal Sample Amount:	15		
Total Simulation Runs:	76 (75 parameterized runs, one nominal run)		
Meta-Model Statistics		**Full System FE Model Statistics**	
Single Element Model:	228 sec.	Implicit Precompression:	76 sec.
RK4 Solver:	38 sec.	Explicit Shock:	11,172 sec.
Curve Fitting	0.2 sec.	Process Acceleration Data:	46 sec.
FFT	2 sec.	FFT:	38 sec.
Total Time:	268 sec. (0.074 hrs.)	Total Time:	11,331 sec. (3.148 hrs.)

3.2 Material Model Sensitivity

Material model sensitivity is evaluated based on the parameterized F-Δ results from the suite of seventy-six single element FE models, as this is the only data that is required to calibrate the meta-model to CHIPFoam's behavior. Parameter sensitivity is computed for three different indices – main effect, interaction effect, and total effect. The most useful sensitivity metric for this parameter study is the total effect sensitivity index. Total sensitivity is the sum of all components of the interaction effect index with the main effect index. The total effect sensitivity index for this parameter study is shown in Figure 2.

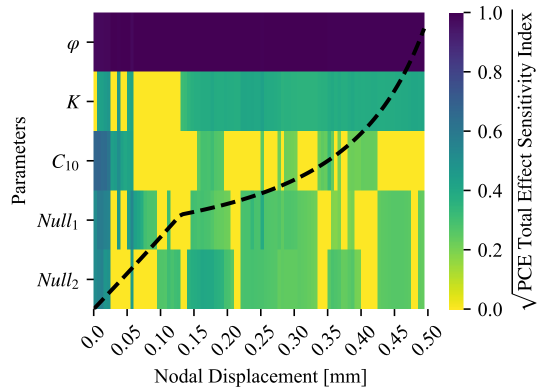

Figure 2. Square root of the total effect sensitivity index from PCE. Higher sensitivity index values indicate a higher effect of the respective parameter on the output of the model. The F-Δ curve for the nominal parameter set is overlaid on the sensitivity index for reference.

Figure 2 shows that initial porosity, φ, has the highest significance of all parameters. Neo-Hookean modulus, C_{10}, has a significant effect at small displacements, and bulk modulus, K, increases in significance as the foam densifies. Another notable observation from Figure 2 is the role of the Null parameters in the parametric study. The noticeable significance of the Null parameters represents physics at play in the model that were not accounted for in the parametric study.

3.3 Comparing the Meta-Model and the Full System FE Model

One of the ultimate goals in generating a reduced-order system model is to come up with an efficient way to compute the approximate solution to the full system model. This goal is verified by comparing the full system explicit model frequency response to the meta-model frequency response using nominal material property values as a case study. Figure 3 shows the FFT of the acceleration signals from both models. The meta-model's fundamental frequency is 4.00 Hz, the full system explicit model's fundamental frequency is 3.50 Hz, and the error between the two solutions is 12.5%.

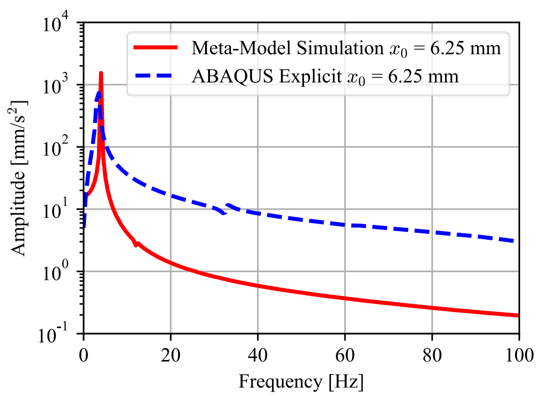

Figure 3. FFT comparison of the meta-model and full system explicit model with nominal material parameter values. The fundamental frequencies of the two models vary by 12.5%.

4. Conclusions

The reduced order model designed in this work was successful in computing the response of the full system model within an acceptable amount of error. The meta-model also generates reasonable bounds on the possible solutions of the full system model when parameter uncertainty is considered. This meta-model is ultimately a tool to be used in future analyses when full system model becomes extremely computationally expensive and needs to be solved as few times as possible. With prior information from the meta-model about the expected response of the system, analysists can be confident in fewer runs of the full system model that the solution is feasible.

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