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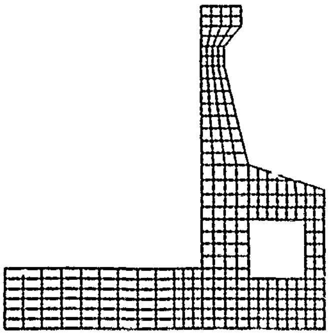
DEVELOPMENT AND IMPLEMENTATION OF TIME-DEPENDENT CRACKING MATERIAL MODEL FOR CONCRETE



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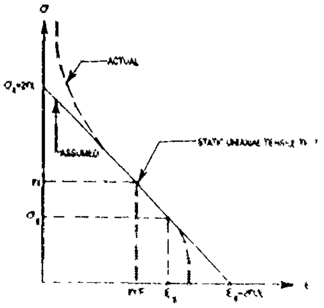

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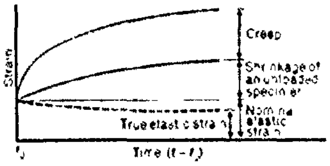

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13. ABSTRACT *(Maximum 200 words)*

Minimizing thermal cracking in mass concrete continues to be a concern for the US Army Corps of Engineers. Thermal cracking is due to the restraint of volume change due to hydration, shrinkage, and creep. The prediction of stresses, strains, and cracking at early times presents special problems because many of the properties of concrete depend on the degree of hydration of the cementitious materials. To better model the time-dependent properties and response of mass concrete, a time-dependent cracking material model was developed for use in a general-purpose heat-transfer and structural analysis finite element code. The model includes the effects of time and temperature on compressive strength, elastic modulus, and creep. An interactive strain-driven, stress-modified cracking criterion based on the smeared-crack approach is included. Examples of calibration and verification of the model are included.

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PREFACE

The research described in this report was conducted for Headquarters, US Army Corps of Engineers (HQUSACE), under the Concrete Research Program, Work Unit 32260, Cracking of Concrete. Dr. Tony Liu, HQUSACE, was the Technical Monitor.

The research was performed by the US Army Engineer Waterways Experiment Station (WES) by personnel of the Structures Laboratory (SL), Concrete Technology Division (CTD), under the general supervision of Messrs. Bryant Mather, Chief, SL; J. T. Ballard, Assistant Chief, SL; and Kenneth L. Saucier, Chief, CTD. Direct supervision was provided by Mr. Steve Ragan, Chief, Engineering Mechanics Branch. Principal Investigators were Dr. C. Dean Norman and Mr. Michael I. Hammons, Applied Mechanics Group (AMG), Engineering Mechanics Branch, CTD. This report was prepared by Mrs. Sharon Garner, AMG, and Mr. Hammons. The authors acknowledge Messrs. Anthony A. Bombich, Donald M. Smith, Dan E. Wilson, Brent Lamb, and Mrs. Linda Mayfield, AMG, for their help during this investigation.

Commander and Director of WES during preparation of the report was COL Larry B. Fulton, EN. Technical Director was Dr. Robert W. Whalin.

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CONVERSION FACTORS, NON-SI TO SI (METRIC) UNITS OF MEASUREMENT

Non-SI units of measurement used in this report can be converted to SI (metric) units as follows:

Multiply	By	To Obtain
Fahrenheit degrees	5/9	Celsius degrees or kelvins*
feet	0.3048	metres
inches	25.4	millimetres
pounds (force) per square inch	0.006894757	megapascals

* To obtain Celsius (C) temperature readings from Fahrenheit (F) readings, use the following formula: C + (5/9)(F - 32). To obtain Kelvin (K) readings, use K = (5/9)(F - 32) + 273.15.

DEVELOPMENT AND IMPLEMENTATION OF TIME-DEPENDENT CRACKING MATERIAL MODEL FOR CONCRETE

PART I: INTRODUCTION

Background

1. Mass concrete structures are built in incremental layers commonly called lifts. This procedure, called incremental construction, is necessary to limit heat rise in the concrete and is further dictated by concrete batch plant capacity and by the cost of formwork. As the cement in the concrete in each lift hydrates, heat is liberated. This heat causes a temperature rise leading to a corresponding increase in volume of the concrete. However, the concrete is usually restrained by boundaries such as the lift of concrete directly beneath and by thermal gradients which exist across the lift. In addition, other mechanisms interact in a complicated fashion to cause additional volume changes. The most important of these are creep (or alternatively, stress relaxation) and shrinkage (both drying and autogenous).

2. The restraint of these volume changes leads to construction-related cracking. Although this cracking has not yet caused a catastrophic failure of a massive structure, it has led to increased maintenance and repair costs over the service life of locks, dams, bridge piers, bridge abutments, and other mass concrete structures. It appears that some investment in measures intended to provide a reduction of construction-related cracking can lead to considerable cost savings over the expected life of the structure by reducing costs associated with remedial repairs to crack-damaged structures. In addition, many of the steps which can be taken to reduce construction-related cracking can lead to substantial savings in the cost of construction. For Locks and Dams 4 & 5 on the Red River Waterway, the use of a high percentage of fly ash in the concrete mixtures resulted in a cost savings of at least $738,000 in the cost of cementitious materials alone. Not only did the use of fly ash in higher percentages than would otherwise have been employed lead to lower temperature rises in the structures, but it also gave mechanical

properties (modulus of elasticity, creep, and shrinkage) which were beneficial in reducing cracking.

3. In 1985, Headquarters, US Army Corps of Engineers (HQUSACE), recognized that a significant research effort was required to modernize the tools available to Corps field offices to analyze and reduce construction-related cracking. As a result, Work Unit Number 32260 entitled "Cracking of Concrète" was established as a part of the Corps' Concrete Civil Wcrks Research Program. This report is a comprehensive review of the development and implementation of the time-dependent cracking model for concrete that was developed under this work unit.

Objective

4. The objective of the research was to develop a computationally efficient, state-of-the-art material model and to implement that model in a general-purpose heat transfer and structural analysis finite element (FE) code. The model was to be capable of predicting the time-dependent changes in material properties which occur during the critical first few days after placement of concrete prior to the time it has developed stable material properties. For the model to be generally applicable and to take fullest advantage of modern supercomputing capabilities, the model was to be generalized to three dimensions.

Scope

5. This report contains a discussion of the theoretical basis of the model as well as the selection of the FE code for implementation in the model. Instructions for the calibration of the model are given. The use of the model is demonstrated in an incremental construction analysis. Examples are included.

PART II: SELECTION OF THE FINITE ELEMENT PROGRAM

Selection Criteria

6. The selection of the FE program for implementation of the time-dependent cracking model was previously discussed by Bombich, Norman, and Jones (1987). The rationale leading to the selection of the program is described in the following paragraphs.

7. Several specific criteria were established in advance for the selection of an FE code for implementation of the model. These criteria were as follows:

a. The FE code must be capable of simulating the incremental construction process. This includes the capability to easily include lifts of concrete and to have flexibility in the selection of solution time-steps.

b. The FE program should have a large element library from which to choose element types (both two-dimensional (2-D) and three-dimensional (3-D) elements).

c. The FE code must allow the implementation of user-defined material models with relative ease.

d. The program should have the capability to model significant numbers of reinforcing bars with relative ease.

e. Because of the computational difficulty of a large, 3-D incremental construction analysis, the program must contain computationally efficient numerical solution procedures to reduce run time on the computer.

f. Finally, the program should be user oriented and receive a high caliber of technical and scientific support from the developer and have a high potential for staying at the state-of-the-art leve'.

Selection of ABAQUS

8. Based upon the criteria set forth in paragraph 7, a review of FE programs was conducted. The review consisted of discussing the experiences of other analysts with various programs, reviewing technical journal articles, and meeting with representatives of both private and governmental entities directly involved with FE applications.

9. After reviewing the available FE software, the program ABAQUS was selected. ABAQUS, developed by Hibbitt, Karlsson and Sorensen, Inc. (1988), is a general-purpose heat transfer and structural analysis FE program that allows either user-selected or automated solution time-step sizing. Input is in free format, has key words, and makes use of set definitions for easy cross reference. A broad element library of both 2-D and 3-D elements is available. User-defined material models can be incorporated through the UMAT subroutine. The incremental construction problem can be simulated through the use of the MODEL CHANGE option in the code. This allows the entire structure to be modeled and then element sets corresponding to lifts to be removed prior to the first solution step. Then the element sets can be added in the appropriate time-step to model the placement of lifts in the field.

10. The current version of ABAQUS is the Version 4.7 Release. For more information on ABAQUS, the reader is referred to the ABAQUS User's Manual Version 4.7 (Hibbitt, Karlsson and Sorensen, Inc. 1988).

PART III: BASIC CONCEPTS

Parameters Affecting Cracking in Mass Concrete

11. All concrete elements and structures are subject to volume change. Cracking in mass concrete is caused by restraint of volume change. These volume changes may be due to heat generation and subsequent cooling, shrinkage, creep/stress relaxation, or other mechanisms. Restraint limits the changes in dimensions and causes corresponding tensile, compressive, or flexural stresses in concrete. Of primary concern in mass concrete structures is restraint which causes tensile stresses, particularly in the first few days after the placement of the concrete when the tensile capacity of the concrete can be quite low.

12. Restraint of volume change may be either external or internal. External restraint is caused by bond or frictional forces between the concrete and the foundation or underlying lifts. The degree of external restraint depends upon the stiffness and strength of the concrete and restraining material and upon the geometry of the section. Internal restraint is caused by temperature gradients within the concrete. The warmer concrete in the interior of the lift provides restraint as the concrete in the periphery of the lift cools due to heat transfer to its surroundings. The degree of internal restraint depends upon the quantity of heat generated, the thermal properties of the concrete, and thermal boundary conditions.

13. A number of parameters may be controlled to limit cracking related to the restraint of volume change. These parameters fall into two categories: material parameters and construction parameters. Among the material parameters are the following:

a. Heat generation of the concrete.

b. Mechanical properties of the concrete including strength, modulus of elasticity, and creep/stress relaxation.

c. Shrinkage of the concrete.

d. Thermal properties of the concrete including coefficient of thermal expansion, specific heat, and thermal conductivity.

The construction parameters are as follows:

- a. Lift height.
- b. Time between placement of lifts.
- c. Placement temperature.
- d. Ambient temperature.
- e. Use of insulation.
- f. Use of cooling coils.
- g. Monolith geometry including section thickness, monolith length, and location and size of inclusions such as galleries, culverts, etc.

14. To be effective, the method used to analyze thermal-related cracking in mass concrete structures must accurately model these complex phenomena. The heat-transfer model must be capable of handling the internal generation of heat and the complex thermal boundary conditions in the incremental construction problem. Similarly, the stress analysis model must be capable of capturing the mechanical properties of the concrete as they change with time. It must also have the ability to predict cracking in a computationally efficient manner.

Definitions

15. Some of the terms used in this report may be unfamiliar to some readers. Therefore, the following definitions have been included.

Adiabatic temperature rise curve

16. The adjective adiabatic refers to a condition in which heat neither leaves nor enters a system. The adiabatic temperature rise curve describes the rise in temperature with time that occurs during hydration of the cement in a specimen in which no heat loss is allowed to occur. This serves as the loading in the heat-transfer analysis.

Creep

17. Creep is defined by American Concrete Institute (ACI) Committee 209 (1990) as "time-dependent increase in strain in hardened concrete subjected to sustained stress" (ACI 1990). Creep strain is obtained in the laboratory by subtracting from the total measured strain in a loaded specimen the sum of: (a) initial instantaneous (usually considered elastic) strain due to the

sustained stress, (b) shrinkage, and (c) and thermal strain in an identical load-free specimen which is subjected to the same history of relative humidity and temperature conditions.

18. The above definicion assumes that strain in a loaded specimen consists of an initial elastic strain, creep strain, shrinkage and thermal expansion or contraction. In a mass concrete structure, however, stresses and moduli are varying with time throughout the structure and construction period, and initial elastic strain has little meaning. Calibration of the material model must be based on time-dependent modulus and creep. The relationship between elastic strain (ϵ^e) and creep strain (ϵ^c) is shown in Figure 1.

Creep compliance

19. Creep compliance is determined from a plot of specific strain (strain per unit stress) versus time from a 3-day creep test and is the difference between the total specific strain and the elastic specific strain. The relationship between total specific strain $J(t)$, creep compliance (C(t)) , and elastic specific strain $(1/E(t))$ is shown in Figure 2.

DFLUX subroutine

20. DFLUX is a user-supplied FORTRAN subroutine used to specify non-uniform distributed fluxes in an ABAQUS heat-transfer analysis. DFLUX is used to define adiabatic curves for one or more concrete mixtures in the heat-transfer analysis.

Incremental construction

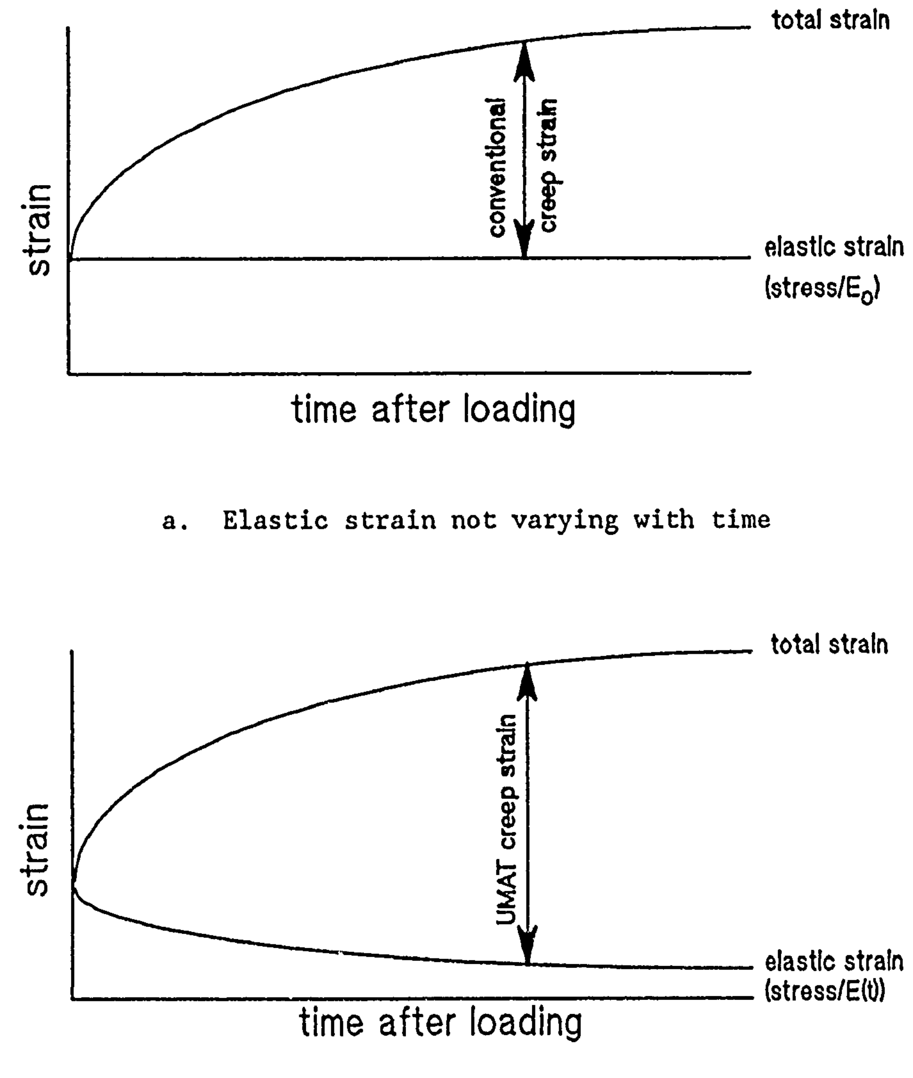
21. Incremental construction is the practice of placing concrete in lifts (or layers). Most mass concrete structures are constructed in lifts (usually 5 to 10 ft in depth) placed at time intervals of several days.

Shrinkage

22. ACI defines shrinkage as "decrease in either length or volume" (ACI 1990). The decrease is due to changes in the moisture content of the concrete and physico-chemical changes which occur without stress attributable to actions external to the concrete. Shrinkage due to moisture loss or drying shrinkage occurs only at the surface of mass concrete structures and is not simulated in the material model. However, additional volumetric changes occur during hydration of the cement that are not directly attributable to changes in temperature. In this report shrinkage refers to these volumetric changes.

UMAT subroutine

23. This refers to a user-supplied material model, usually in the form of a FORTRAN subroutine, which can be linked to ABAQUS. External parameters required by UMAT are input using the USER SUBROUTINE key word in the ABAQUS input file.



a. Elastic strain not varying with time

b. Elastic strain varying with time

Figure 1. Relationship between elastic and creep strains

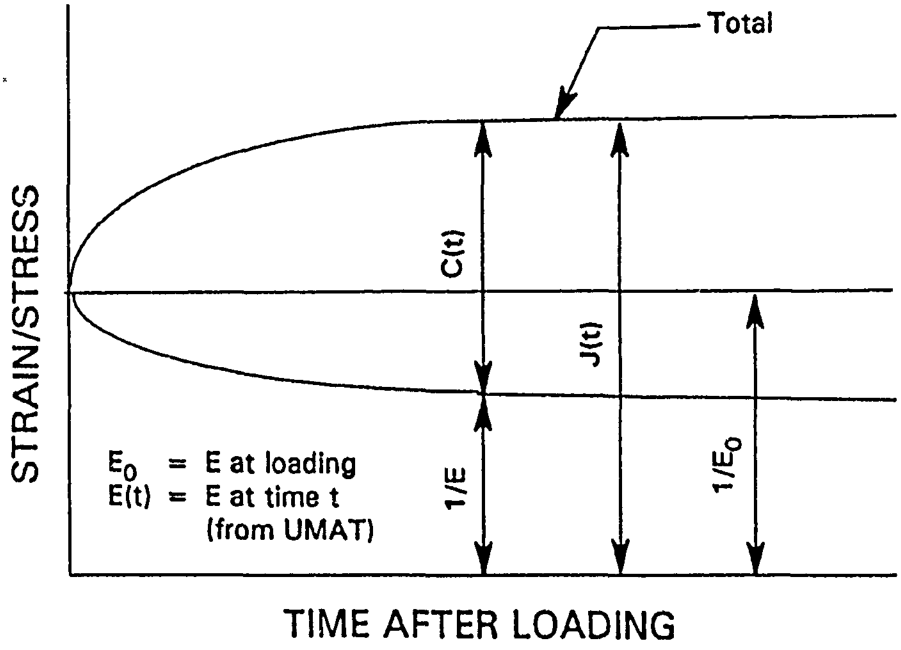


Figure 2. Specific strain relationships

PART IV: MODEL DESCRIPTION

The Prediction of Cracking in Mass Concrete

24. The prediction of stresses, strains, and cracking at early times presents special problems, because many of the properties of concrete depend on the degree of hydration of the cementitious materials. The rate of hydration of the cementitious materials is affected by the type of materials used and by the temperature and moisture history during the period of hydration. At the same time, the internal environment of mass concrete is affected by the hydration of the cementitious materials. Elevated temperatures generated by hydration are maintained for long periods of time in the center of mass concrete structures and affect mechanical properties essential in determining the stress/strain condition of the concrete such as elastic modulus, compressive strength, creep, and volumetric changes associated with hydration.

25. Requirements for the accurate prediction of stresses, strains, and cracking in mass concrete include the following:

a. An FE grid that accurately defines the structure. The grid may consist of 2-D plane strain, plane stress, axisymmetric elements, or 3-D elements. The choice of elements and geometry must be based on an understanding of the problems to be studied.

b. Accurate information about thermal boundary conditions. This includes climatic data such as expected temperatures and wind velocities during the construction period. Also, accurate information about the thermal properties of foundation material is needed to establish heat flow from the structure into the foundation.

c. Accurate thermal and mechanical properties of the concrete.

d. An FE code that incorporates an accurate, reliable heat-transfer capability allows relatively easy incorporation of a concrete constitutive model and is capable of modeling the incremental construction procedures characteristic of mass concrete construction.

e. A material model capable of handling the time- and temperature-dependent properties of concrete and capable of predicting and monitoring cracking in a time- and cost-efficient way.

26. Development of an adequate FE grid is the responsibility of the analyst. It should be undertaken only with a thorough knowledge of the problems to be studied and the tools available for this study.

27. Adequate thermal and mechanical properties data are essential. Often an analyst will try to base an FE study on general concrete properties. However, mass concrete mixtures are usually developed to fulfill specific requirements and may employ diverse chemical admixtures, unusual cements, replacement of cement with fly ash or other pozzolans, and diverse aggregate types. Any one of these can affect the thermal and mechanical properties of the concrete. Also, it is necessary to know how the mechanical properties change with time, beginning immediately after time of final set.

28. Necessary thermal properties include the adiabatic temperature rise thermal conductivity, and specific heat of the concrete. These properties can vary with changes in environmental temperature. Significant variations with temperature should be considered prior to selecting final properties for the analysis.

29. Necessary mechanical properties include time- and temperature-dependent properties such as creep, elastic modulus, compressive strength, and shrinkage. Additional properties commonly required are tensile strain capacity and coefficient of thermal expansion.

30. Many FE programs include a heat-transfer capability and a means for including an adiabatic heat-rise curve as the driving function for a heat-transfer analysis. However, few of these codes are designed to allow easy modeling of incremental construction. Failure to correctly simulate incremental construction in the analysis will result in incorrect predictions.

31. A user-defined, time-dependent material model with cracking capabilities (UMAT) has been developed by the US Army Engineer Waterways Experiment Station (WES) for implementation in ABAQUS through the ABAQUS-UMAT format. The model includes the effects of time and temperature dependency on elastic modulus, compressive strength, and creep. Cracking is included using a smeared-crack approach. Although this approach to cracking does not allow the study of specific cracks, it gives a general indication of when and where cracking is likely to occur without causing the calculations to become too expensive and time consuming. Important features of the model are discussed in the following paragraphs.

UMAT Subroutine

32. The mathematical representation of the time- and temperature-dependent properties of concrete must address three fundamental properties of the material: elastic modulus $E(\tau,T)$, ultimate strength $\sigma_u(\tau,T)$, and creep compliance $C(t,\tau;T)$, where t is current time measured from some reference time, t_o , τ is the time since placement of the concrete, and T is temperature. For an arbitrary stress history $\sigma(\tau)$ and temperature history $T(\imath)$, the stress-strain relationship for an isotropic time-dependent concrete can be written using tensor notation as

$$\underset{\sim}{\Delta\epsilon}(t) = \underset{\approx}{B}\left\{\frac{\underset{\sim}{\Delta\sigma}}{E(t)} - \frac{\underset{\sim}{\sigma}\Delta E(t)}{E^2(t)} + \Delta t\int_{-\infty}^{t} C'[t,\tau;T(\tau)]\,\frac{\partial\underset{\sim}{\sigma}(\tau)}{\partial\tau}\,d\tau\right\} \tag{1}$$

where $\underset{\approx}{B}$ is a material tensor function of Poisson's ratio. Creep compliance is given by

$$C'[t,\tau;T(\tau)] = \frac{\partial C[t,\tau;T(\tau)]}{\partial t} \tag{2}$$

33. The time difference form of Equation 1 is

$$\underset{\sim}{\Delta\epsilon_n} = \underset{\approx}{B}\left\{\frac{\underset{\sim}{\Delta\sigma_n}}{E_n} - \frac{\underset{\sim}{\sigma_n}\Delta E_n}{E_n^{\,2}} + \Delta t\sum_{i=0}^{n-1}\int_{t_i}^{t_{i+1}} C'[t_n,\tau;T(\tau)]\,\frac{\partial\underset{\sim}{\sigma}(\tau)}{\partial\tau}\,d\tau\right\} \tag{3}$$

In this form, the integral at each time-step t_n must be totally reevaluated from t_o to t_n . This is due to the time-dependent nature of creep and results in unnecessarily expensive calculations.

34. For a nontime-dependent material and neglecting temperature, Equation 3 becomes

$$\underset{\sim}{\Delta\epsilon_n} = \underset{\approx}{B}\left[\frac{\underset{\sim}{\Delta\sigma_n}}{E_o} + \Delta t\int_{-\infty}^{t} C'(t-\tau)\,\frac{\partial\underset{\sim}{\sigma}(\tau)}{\partial\tau}\,d\tau\right] \tag{4}$$

where E_o is the elastic modulus at time of loading.

35. Using exponential functions, the nontime-dependent creep compliance can be written in a form that allows separation of the variables t and τ. For

$$C'(t-\tau) = \sum_{i=1}^{m} a_i\left(1 - e^{b_i(t-\tau)}\right) \tag{5}$$

Equation 4 becomes

$$\underset{\sim}{\Delta\epsilon}(t) = \underset{\approx}{B}\left[\frac{\Delta\underset{\sim}{\sigma}}{E_o} + \Delta t\sum_{i=1}^{m} a_i b_i e^{b_i t}\int_{-\infty}^{t} e^{b_i\tau}\,\frac{\partial\underset{\sim}{\sigma}(\tau)}{\partial\tau}\,d\tau\right] \tag{6}$$

The integration of this equation requires only the summation of time-steps.

36. A more generalized form of Equation 5 may be written for a time-dependent material as follows:

$$C'(t-\tau) = \sum_{j=1}^{n}\sum_{i=1}^{m} a_i\left(1 - e^{b_i(t-\tau)}\right) c_j\left(1 - e^{d_j\tau}\right) \tag{7}$$

The time-dependency of creep can then be easily evaluated using the elastic modulus as it is not under the integral sign.

37. Creep properties in the UMAT model are defined by a 3-day creep compliance curve mapped in the time domain by an "aging factor." This aging factor is the ratio of the elastic modulus at the current age to the 3-day elastic modulus. The curve is based on a 70 °F* temperature and modified for current temperature by a temperature factor. The creep equations are given in Equations 8a, 8b, and 8c.

$$C(t,\tau;T) = \sum_{i=1}^{n} A_i(\tau,T)\left(1 - e^{r_i t}\right) + D(\tau,T)\,t \tag{8a}$$

$$A_i(\tau,T) = A_{o_i}\left(\frac{e^{-\frac{Q}{RT}}}{e^{-\frac{Q}{RT_0}}}\right)\left[\frac{E(3)}{E(\tau)}\right]^2 \tag{8b}$$

* A table of factors for converting non-SI units of measurement to SI (metric) units is presented on page 3.

$$D(\tau,T) = D_o\left(\frac{e^{-\frac{Q}{RT}}}{e^{-\frac{Q}{RT_0}}}\right)\left[\frac{E(3)}{E(\tau)}\right]^2 \qquad (8c)$$

where·

C = creep compliance (strain per unit stress)
t = time since loading, days
τ = age of the concrete, days
T = temperature, °K
R = gas constant, 1.98
$E(\tau)$ = modulus of elasticity at age τ
r_i, A_i, D, Q = constants for a given material
Q = the activation energy for creep, 4,345.
T_o = 294 °K (70 °F)

38. The form of the equation for elastic modulus as a function of time is similar to that of Equation 8 and is given in Equation 9.

$$E(\tau) = E(1) + \sum_{i=1}^{2} B_i[1 - e^{-m_i(\tau-1)}] + B_3(\tau-1) \qquad (9)$$

where

τ = total age of the concrete in days
B_i, m_i = constants
$E(1)$ = 1-day modulus, psi
$E(\tau)$ = modulus calculated at 70 °F, psi

The elastic modulus from time of placement to 1 day is assumed to be linear from $E = 0$ at $t = 0$ to $E = E(1)$ at $t = 1$ day. Little data exist to verify this assumption. However, since stresses due to temperature changes during hydration are generally low at very early times, early-time errors in modulus may not produce significant errors in the calculations.

39. The effect of temperature on elastic modulus is accounted for by the temperature factor, $H(T)$ as follows:

$$E(\tau,T) = E(\tau)H(T) \qquad (10a)$$

where

$$H(T) = \frac{E(T, 28days)}{E(70°F, 28days)} \tag{10b}$$

Ultimate strength $\sigma_u(\tau)$ is calculated using a 3-day reference value and the age factor:

$$\sigma_u(\tau) = \sigma_u(3)\left[\frac{E(\tau)}{E(3)}\right] \tag{11}$$

Shrinkage as a function of age of the concrete is given by the following equation.

$$\epsilon^s(\tau) = C_1(1 - e^{s_1\tau}) + C_2(1 - e^{s_2\tau}) \tag{12}$$

where C_1 , C_2 , s_1 , and s_2 are constants. The tensile strain capacity (ϵ_f) , if not defined by the user as a constant, is assumed to be 10 percent of the absolute value of the compressive strain at ultimate strength.

40. Cracking is assumed to occur when a cracking criterion is satisfied. This criterion is strain-driven but is modified by stress. The crack surface normally is in the direction of the principal strain and the cracking criterion is interactive. Figures 3 and 4 have been included to illustrate the cracking criterion. For an isotropic material, such as concrete prior to cracking, the principal strain and stress directions coincide, and the cracking criterion can also be expressed in terms of principal strain.

41. If a cube of concrete is loaded with σ_2 , the cube will split in the direction of the load under the effect of the ϵ_1 strain and σ_1 will be zero. If cracking is based on stress only, the ϵ_1 strain is $-\nu\epsilon_2 = -\nu(\sigma_2/E)$. For $\sigma_2 = f_c'$ and $\nu = 0.2$, the cracking strain is 20 percent of the uniaxial ultimate compressive strain, or twice the value usually assumed. Obviously a strain-dependent criterion is more appropriate.

42. If a small σ_2 is applied and sustained over a long period of time so that creep occurs, cracking could eventually occur under a strain of $\epsilon_1 = -\nu(\sigma_2/E) + \epsilon_1^c$. This indicates a gain in tensile strain capacity under creep. Little creep-cracking data are available, but cracking strain for a specimen undergoing creep appears to be approximately twice that obtained from a uniaxial tensile test (Rashid and Dunham in preparation). A strain-

dependent cracking criterion will not predict this gain, so an interactive c..iterion was adopted to accommodate creep and stress relaxation as well as elastic effects. This criterion is illustrated in Figure 3 by the diagonal line that crosses the stress axis at $2f'_t$ (where f'_t is the uniaxial tensile strength) and the strain axis at $2f'_t/E$. Some common test conditions are indicated on the figure, with a uniaxial tension test located at the midpoint. The actual curve, indicated by the broken line, is asymptotic to the stress axis, since cracking under zero strain is impossible for compressible materials. The criterion is implemented in the model as follows.

a. Calculate maximum principal strain ϵ_1 in UMAT.

b. Enter Figure 3 with ϵ_1 and calculate σ_f .

c. Adjust the failure surface amplitude, using σ_f as the intercept instead of f'_t .

d. Enter Figure 4 using the principal stresses σ_1 and σ_2 calculated in UMAT, and determine whether the (σ_1,σ_2) point penetrates the failure surface. If so, introduce a crack normal to the principal strain direction and formulate the constitutive matrix in the principal coordinate system.

e. Rotate the precracking stresses to the principal coordinate system and adjust these stresses to reflect the new cracking state.

f. Rotate the constitutive matrix and the stresses back to the coordinate system of the structure.

g. The new constitutive matrix and stresses are then used by ABAQUS to calculate the nodal forces and the tangent stiffness matrix in the next step. A smeared crack approach is used to model the cracked regions of the structure. The cracked region is modeled as an anisotropic continuum effectively "smearing" the cracks in a continuous manner throughout the element (Norman and Anderson 1985). When cracking occurs, stress in the tensile direction is allowed to drop to zero while shear transfer due to aggregate interlock is maintained. Cracks are allowed to open or close as conditions in the model vary. Thus, the overall structural response can be modeled adequately without regard to completely realistic crack patterns and local stresses (Chen 1982).

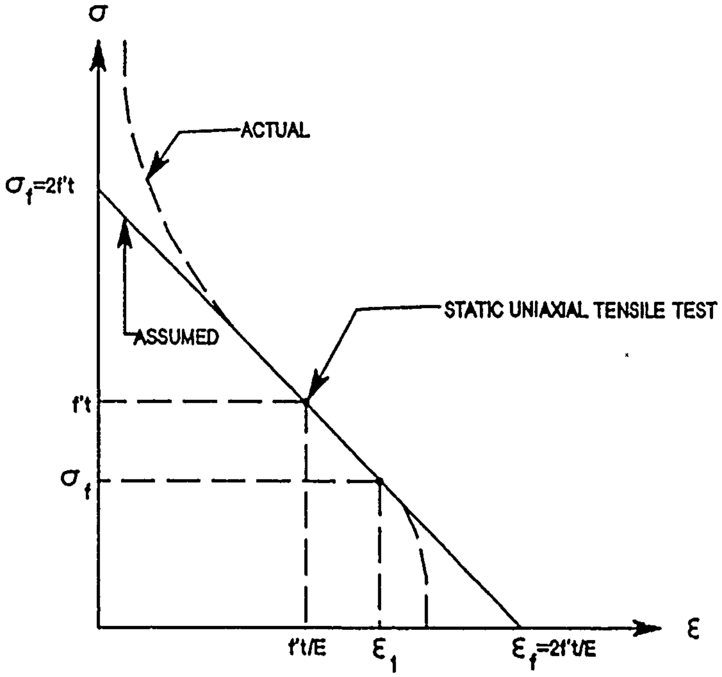


Figure 3. UMAT interactive cracking criterion

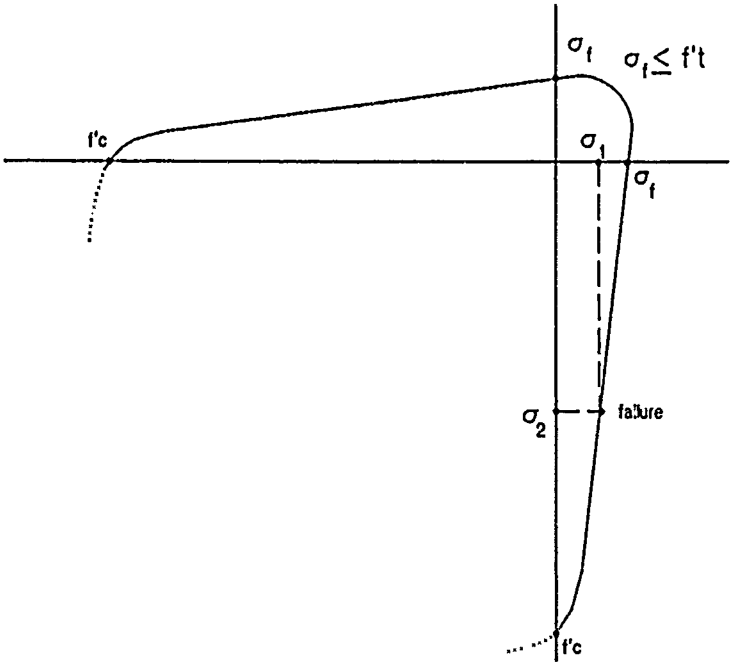


Figure 4. Typical biaxial tensile failure surface for concrete

PART V: CALIBRATION AND VERIFICATION OF THE MATERIAL MODEL

Calibration

43. The UMAT model used in the analysis must be calibrated for each concrete mixture to be simulated in an analysis. Information required for calibration includes 3-day creep compliance, shrinkage, and elastic modulus as a function of time. Each of these are discussed in the following paragraphs.

Creep Compliance

44. Creep compliance is determined from a plot of specific strain (strain per unit stress) versus time from a 3-day creep test and is the difference between the total specific strain and the elastic specific strain (Figure 2). Creep compliance as is given by an equation of the form

$$C(t) = A_1\left(1 - e^{r_1 t}\right) + A_2\left(1 - e^{r_2 t}\right) + A_3\left(1 - e^{r_3 t}\right) + A_4 t \qquad (13)$$

where t is time since loading in days, $C(t)$ is in units of inches per inch per pound per square inch. One or more of the exponential terms in Equation 13 may be eliminated as required to improve the fit to test data. The parameters A_1 , A_2 , A_3 , A_4 , r_1 , r_2 , and r_3 are determined by trial and error fit to test data.

45. An example curve fit is given below. Although an adequate curve fit can usually be obtained using only two or three terms, all four terms have been used in the example. Test specific creep values for days given are listed in Table 1.

Table 1
Example of Test Specific Creep

Time Since Loading days	Specific Creep 10^{-6} in./in/psi
1	0.10
3	0.15
7	0.20
28	0.25
90	0.26

46. Parameters r_1 , r_2 , and r_3 are determined so that the terms $e^{r_i t}$ saturate (i.e. are set equal to a very small number) at three different times. In this example 1, 3, and 28 days were chosen. If the terms $e^{r_i t}$ are set equal to 0.005 at those days, then

$$\begin{aligned} r_1 &= (\ln 0.005)/28. = -0.189226 \\ r_2 &= (\ln 0.005)/3. \;\; = -1.76611 \\ r_3 &= (\ln 0.005)/1. \;\; = -5.29832 \end{aligned}$$

47. Substituting theses values into Equation 13 for ages of 1, 3, 28, and 90 days and setting each equation equal to the specific creep at that age yields four equations with four unknowns. Solving for A_1 , A_2 , A_3 , and D

$$\begin{aligned} C(t) = [&0.17116(1 - e^{-0.18923t}) + 0.029166(1 - e^{-1.766111t}) \\ &+ 0.04640(1 - e^{-5.298321t}) + 0.00015t] \times 10^{-6} \end{aligned} \qquad (14)$$

This equation is plotted against test values in Figure 5. It should be noted that UMAT requires that the number of terms in the creep equation be specified in the subroutine STRN3D. This is done by setting the integer variable JCREEP = N in the subroutine STRN3D, where N is the number of terms in the expression for $C(t)$.

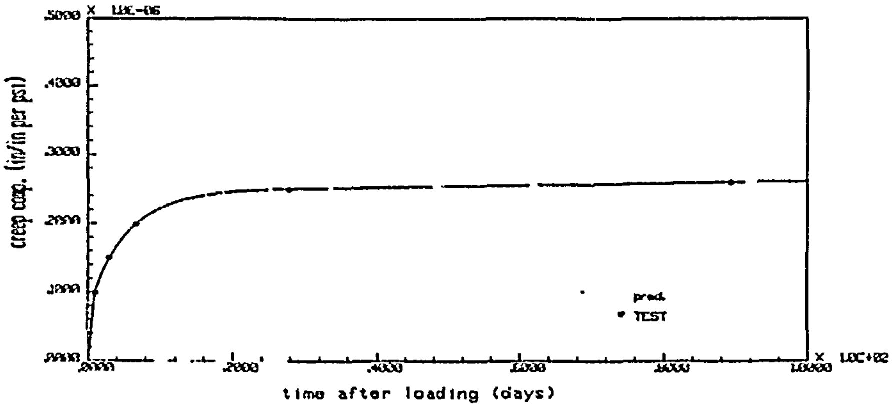


Figure 5. Test specific creep and creep compliance predicted by Equation 14

48. For concretes with extremely low moduli at early times, some modification to the creep aging factor may be required at times when *E(3)/E(τ)* is greater than 1. The effects of varying the creep aging factor on early-time creep predictions can be seen in Figure 6. In this figure the results of UMAT creep predictions are plotted against 1-day test results for a high-fly-ash mass concrete mixture. In Run 1, a creep aging factor of $[E(3)/E(\tau)]^2$ was used throughout, and predicted strains were roughly three times as high as test strains. In Run 2, a creep aging factor of *E(3)/E(τ)* was used prior to $\tau = 3$ days, producing reasonable results.

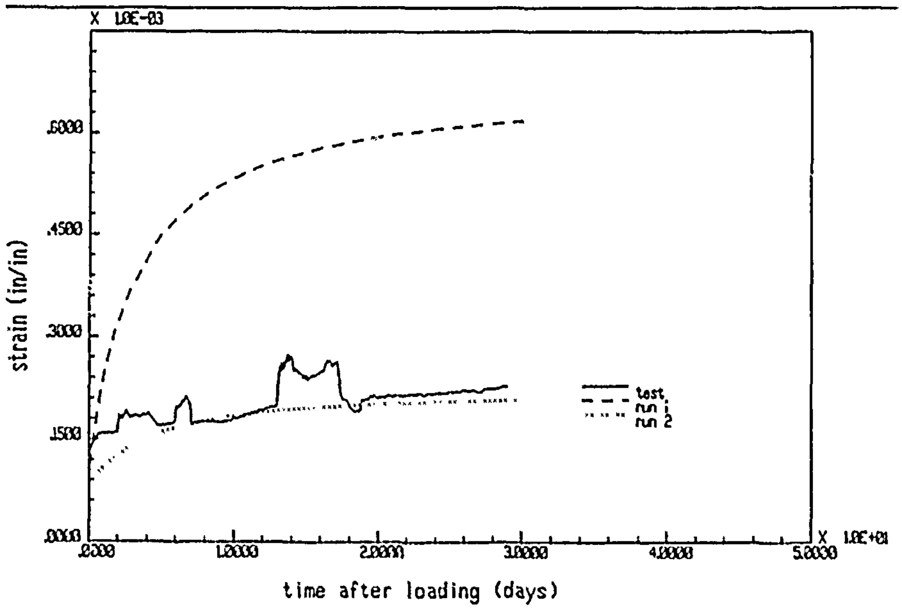


Figure 6. Two different aging factors used in 1-day creep prediction

Elastic Modulus

49. The form of the time-dependent elastic modulus equation is

$$\begin{aligned}&B_1\left[1 - e^{x_1(t-1)}\right] + B_2\left[1 - e^{x_2(t-1)}\right] + B_3\left[1 - x^{r_3(t-1)}\right]\\ &+ B_4(t-1) + E(1) \qquad\qquad \text{for } t \ge 1\end{aligned} \tag{15}$$

The constants B_i and x_i are determined using a procedure similar to that used in Equation 14. Data for calibration of the elastic modulus curve are obtained from unconfined compression tests on specimens stored at 73 °F.

Although the form of the specific creep equation must be maintained, the form of Equation 15 may be changed by the user if desired.

Shrinkage

50. To account for the volumetric changes that occur during the curing of concrete, the UMAT material model includes a shrinkage equation of the form

$$\epsilon^{s} = [204.9(1 - e^{-0.15\tau}) + 145.1(1 - e^{-0.02263\tau})] \times 10^{-6} \qquad (16)$$

where ϵ^{s} has units of inches per inch and τ is time since placement in days. This relationship was developed from test data on silica fume concrete and will predict excessive shrinkage for most mass concrete mixtures.

51. Shrinkage data obtained from sealed specimens for a period of time extending from time of setting until change in strain with time become negligible. These data may be used to develop a new curve or to determine a factor for Equation 16.

52. Additional parameters required by the model are given in the USER MATERIAL statement and are listed in Table 2.

53. Tensile strain capacity, ϵ_{f} , can be entered as a constant in the USER MATERIAL statement or calculated by the program as 10 percent of the absolute value of compressive strain at ultimate strength. A report by Holland, Liu, and Bombich (1982) on the properties of concretes for Lock and Dam No. 2, Red River Waterway, gives some insight into the appropriate choice. Ultimate strain capacity tests using 12- by 12- by 66-in. beams were run for two mixtures, one with a design compressive strength of 3,000 psi at 28 days and the other with a design compressive strength of 3,000 psi at 90 days. Loading rates of 40 psi/min and 25 psi/week were used. For the higher-strength mixture under rapid loading, average tensile strain capacity varied little after 3 days, but average test capacity at 1 day was only 50 millionths as opposed to 80 millionths at 3 days. For the lower-strength mixture, tensile strain capacity under rapid loading varied from an average of 41 millionths at 1 day to an average of 91 millionths at 90 days. Tensile strain capacities for all specimens loaded at the slower rate were well over 100 millionths regardless of age at loading.

Table 2
Other Concrete Parameters Required For Material Model

Parameter	Notes
$E(3)$	3-day elastic modulus in pounds per square inch.
ν	Poisson's ratio. This is assumed to be a constant.
$\sigma_u(3)$	Ultimate strength at 3 days in pounds per square inch.
ϵ_f	Tensile strain capacity.
α	Coefficient of linear thermal expansion.
T_o	Reference temperature in °F. This is the temperature at zero stress. For incremental construction problems, this is the placement temperature.
age	Concrete age at the start of the calculation in days. For incremental construction problems, this is the age at time of setting.
F_s	Shrinkage factor. This used to factor the shrinkage curve in UMAT.
F_c	Creep factor. This used to factor the creep curve in UMAT.
ϵ_0	Initial strain (inches/inch). This is strain existing at the start of the analysis (usually 0).
t_{ref}	The reference time (in days) is used in incremental construction problems and corresponds to the day of placement.
IPRUM	This should be set equal to 2.
IHANOP	This should be set equal to 0.

54. Mass concrete mixtures generally have low strengths at early times and gain stiffness slowly. This means that tensile stresses due to the restraint of strains are induced fairly slowly. Also, cooling, the method by which loads are applied, is a slow process. Because of these factors the constant tensile strain capacity used in the analyses (100 millionths) should be adequate for most mass concrete mixtures. However, some construction procedures and concrete mixtures may result in rapid gains in tensile stress and may require the use of a tensile strain capacity linked to concrete strength and stiffness.

55. The equations necessary to calibrate the model are found in the subroutines listed in Table 3.

Table 3
Location of Equations

Term to be Calculated or Applied	Subroutine Name	Variable Name
$E(\tau)$	COEF	ETATAU, ETA3
$C(t)$	SHIFT1	A,R,D
ϵ^s	USHRNK	SHRNK
$[E(3)/E(\tau)]^2$	CRPROP	-

Verification

56. The UMAT model, incorporating the above algebraic expressions calibrated with test data, is then used with ABAQUS to simulate the entire suite of creep tests for each mixture. Tests normally used for verification of the model are 1-, 7-, 14-, and possibly 28-day creep tests. Each creep cylinder is modeled using a single axisymmetric element supported on rollers at boundaries and uniformly loaded across the top surface (Figure 7). Loads are varied to simulate test loadings. Axial strains from these runs are then plotted against actual test data for comparison. Results of the verification

analyses for a typical concrete mixture are shown in Figure 8. The equations used in UMAT to model the modulus and creep compliance are given in Equations 17 and 18.

$$E(t) = 1.8012 \times 10^{6}(1-e^{-0.031351(t-1)}) + 2.1453 \times 10^{6}(1-e^{-0.407563(t-1)}) - 0.437477(1-e^{-2.649(t-1)}) + 2.25 \times 10^{6} \tag{17}$$

$$C(t) = 0.10576 \times 10^{-6}(1-e^{-0.05887t}) + 0.1589 \times 10^{-6}(1-e^{-0.189226t}) + 0.13887 \times 10^{-6}(1-e^{-1.7661t}) \tag{18}$$

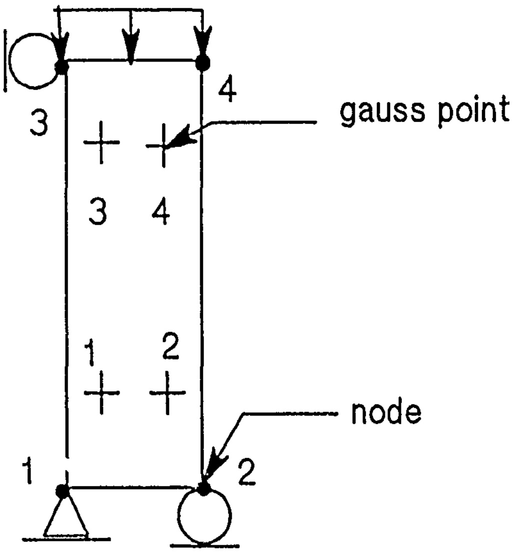
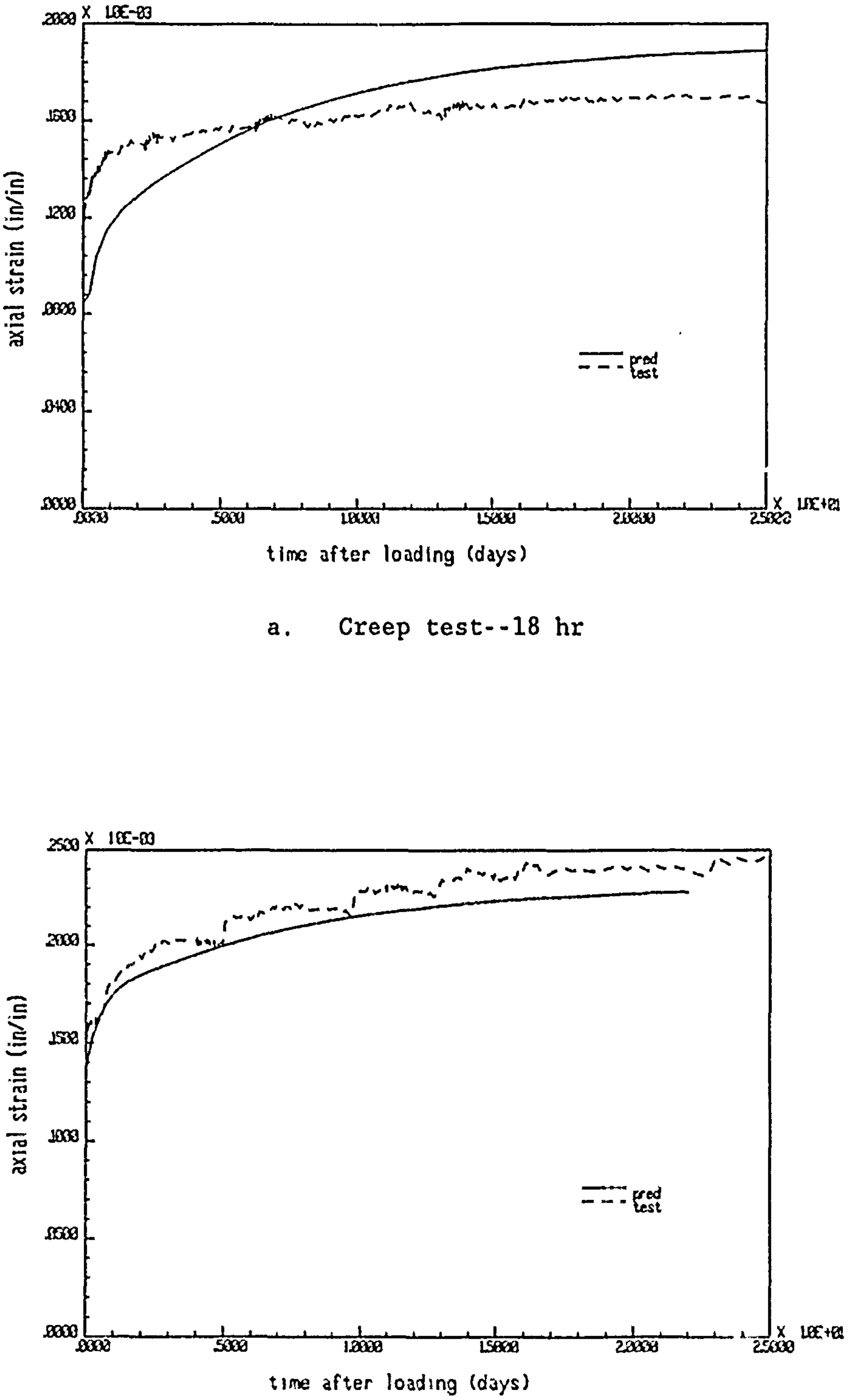


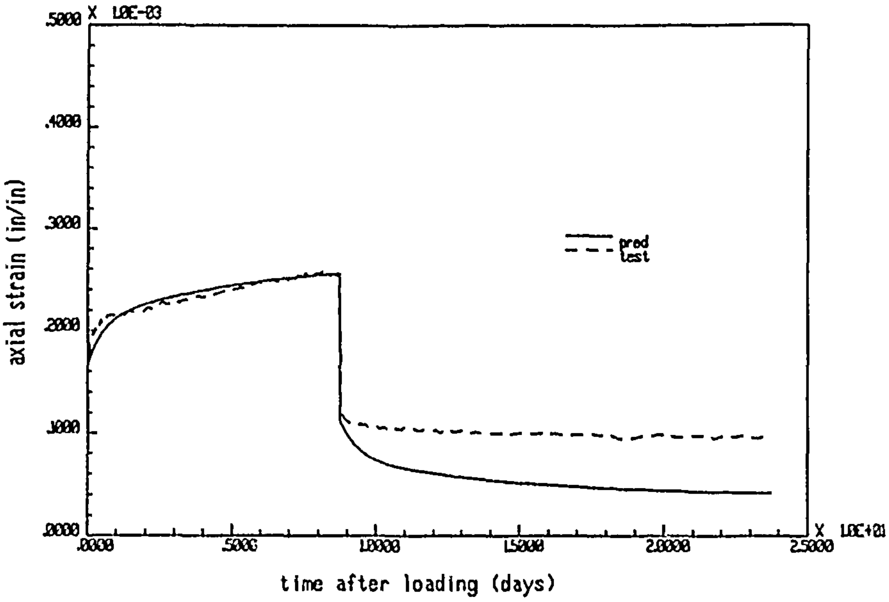
Figure 7. Creep cylinder simulation for FE analysis



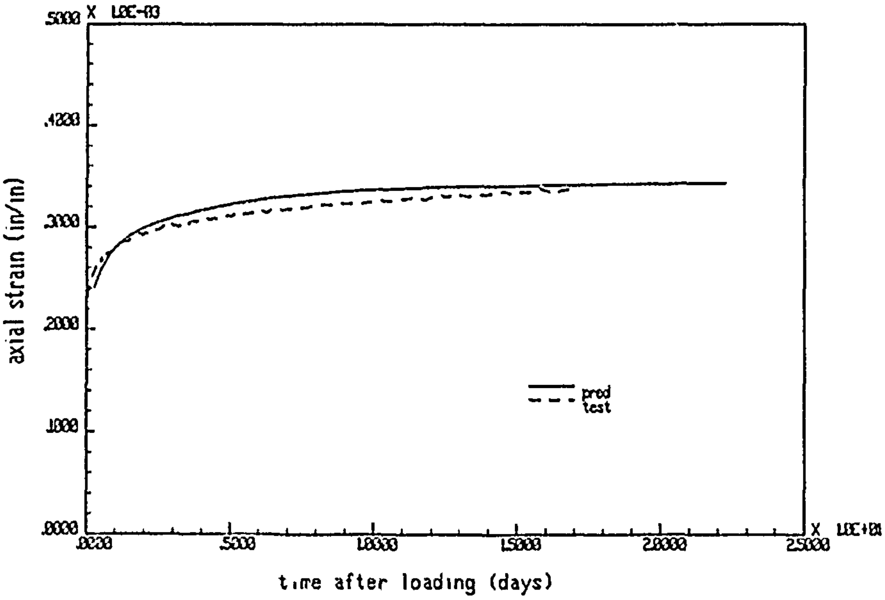
a. Creep test--18 hr

b. Creep test--3 days

Figure 8. Results of verification analyses (Continued)



c. Creep test--7 days



d. Creep test--14 days

Figure 8. (Concluded)

PART VI: USING THE UMAT SUBROUTINE IN AN INCREMENTAL CONSTRUCTION ANALYSIS

Conducting an Incremental Construction Analysis

57. The incremental construction analysis of a mass concrete structure is a two-part procedure. First, a heat-transfer analysis must be performed to determine the temperatures throughout the structure during the construction period. The output from the heat-transfer run in the form of temperatures at nodes is then used as the loading in a stress run. In this section, the mechanics of performing heat transfer and stress analyses pertaining to mass concrete structures are briefly discussed, example input files are developed, factors affecting the accuracy of predictions are discussed, and comparisons are made between 2- and 3-D results.

58. The incremental construction analysis procedure developed at WES uses ABAQUS in conjunction with user-defined, time-dependent routines to define applied heat flux and mechanical behavior of the material. The procedure has been used in several previous projects (Bombich, Norman, and Jones 1987; Norman, Campbell, and Garner 1988; Hammons, Garner, and Smith 1989; Garner, Hammons, and Bombich in preparation). Some of the features of the finite element code are discussed in the following paragraphs.

59. To model the incremental construction, calculations are carried out in time-steps. Using the REMOVE/INCLUDE element options in ABAQUS, new elements are added to the model at regular intervals of time (5, 10, or 15 days) to simulate the placement of additional lifts.

60. The 2- or 3-D transient heat-transfer analysis is performed using heat-transfer elements from the ABAQUS library of elements. The adiabatic temperature rise of the concrete mixture is the loading for the analysis and is supplied by the user in an external subroutine (DFLUX) linked to ABAQUS. Boundary conditions for the heat-transfer analysis are easily varied. External conditions (wind speed, forms, insulation) are modeled using film coefficients applied to external element faces, and ambient and placement temperatures are specified in the input file. The results of the heat-transfer analysis are temperatures at each node for each time increment. The temperature-time history obtained in the heat-transfer analysis is used as the

loading in a stress analysis. This analysis can be conducted using plane stress, plane strain, or 3-D elements from the ABAQUS element library. Time-dependent material characteristics (strength, elastic modulus, creep, and shrinkage), as well ιs cracking, are incorporated into the calculations using +he user-supplied material model, UMAT. The output from the stress analyses includes nodal displacements and stresses and strains at user-selected locations throughout the structure as well as user-selected displacement plots and stress or strain contour plots.

Heat-Transfer Analysis

61. Before an input file for the heat-transfer analysis can be generated, the following information must be obtained.

a. Geometry of the sections to be analyzed.

b. Annual cycle of average ambient temperatures for the area.

c. Depth at which soil temperatures remain constant (usually 10 to 20 ft) and the temperature at that depth.

d. Thermal properties of the soil and concrete (density, specific heat, and conductivity). If any voids are to be included in the analysis, thermal properties must also be determined for air.

e. Adiabatic curve for each concrete mixture to be simulated.

f. Expected lift height and placement schedule.

g. Variables necessary for calculating film coefficients, such as type of formwork and insulation to be used, times for formwork removal, insulation requirements, average wind speed for the area, etc.

62. A sample input file for a 2-D analysis of a lockwall monolith floor and the corresponding DFLUX subroutine are presented in Appendix A. Lift height for this structure wa‿ 4 ft, and three lifts were placed at 10-day intervals. Soil was included for a depth of 10 ft below the base of the structure and 10 ft beyond the outer edge. Further information for setting up an input file can be found in the ABAQUS User's Manual Version 4.7 (Hibbitt, Karlsson and Sorensen, Inc. 1988). The structure is shown in Figure 9, and the FE grid is shown in Figure 10.

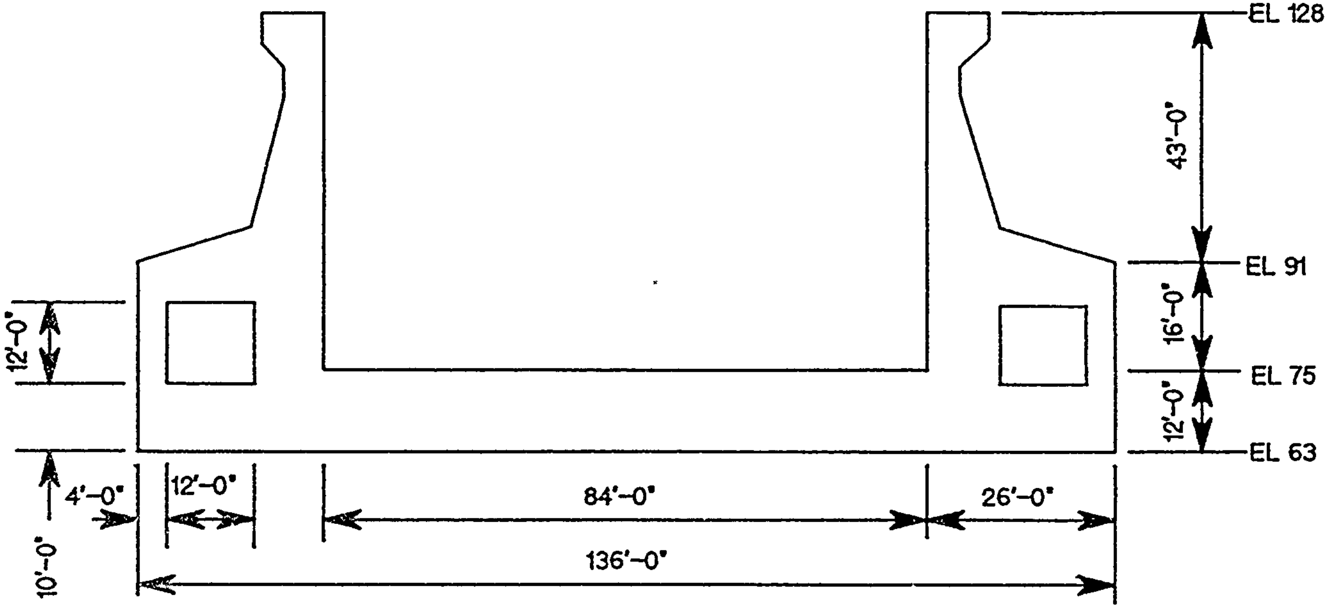
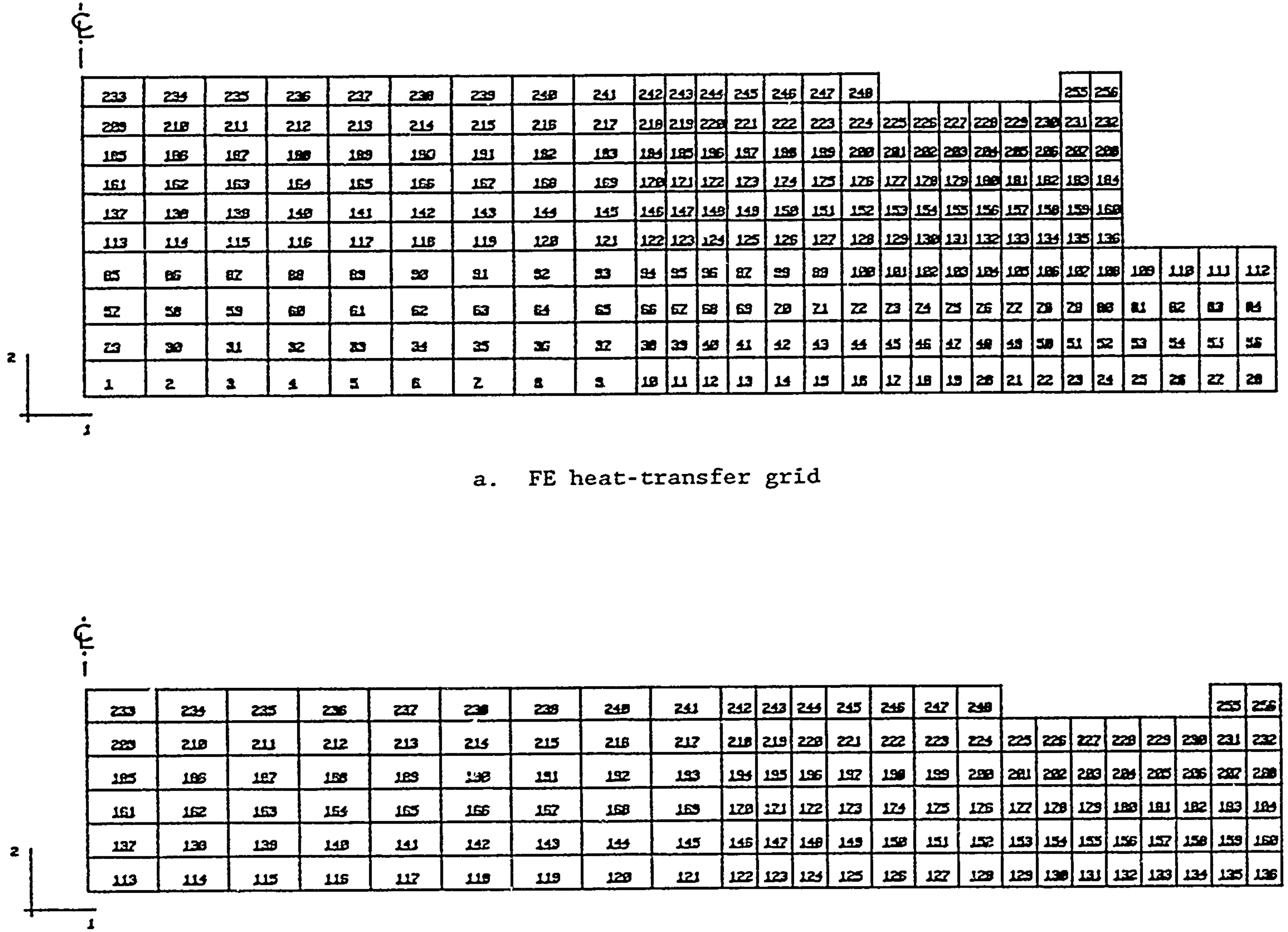


Figure 9. Typical chamber monolith structure



a. FE heat-transfer grid

b. FE stress-analysis grid

Figure 10. FE grids of chamber monolith floor

63. Results from the heat-transfer run were stored in an output file with the extension ".FIL". This is normally a binary file, but can be output in ASCII if requested by the user in the NODE FILE command. This file was then used as the loading for a stress analysis.

Stress Analysis

64. The UMAT subroutine is called by the *USER MATERIAL keyword in the input data file. The material name is given by Mxx where xx is a number from 1 to 99. This allows the material name to be easily converted to an integer in UMAT, which facilitates the modeling of more than one material in the subroutine. The number of material parameters required for the model is 13, and the parameters are listed in the next two lines, with eight parameters per line. A listing of parameters is given in Part III of this report. The number of solution state variables required by the model (specified using the *DEPVAR keyword) is 57. An example call to a user material model is illustrated.

```
*SOLID SECTION,MATERIAL=M1,ELSET=LIFT1
*MATERIAL,NAME=M1
*USER MATERIAL,CONSTANTS=13
3.E6,.15,1000.,100.E-6,5.5E-6,85.,.25,1.
1.,0.,0.,2.,0.
*DEPVAR
57
```

65. For the stress analysis plane-strain elements were used, soil elements were el'minated, and the structure was supported on rollers along the base and axis of symmetry. Prior to removal of the forms (at 2 days), the gravity loading for each new lift was simulated as a pressure on existing concrete. After 2 days the gravity loading was simulated as a body force per unit volume. The input file for the stress analysis is presented in Appendix B. The UMAT subroutine used in this analysis is presented in Appendix C.

66. Various types of output are available in ABAQUS. Output for the stress run in Appendix B was in the form of stress contour and displacement plots at specified time increments and a binary output file containing stresses, strains, and principal stresses at all integration points for each

time increment. A postprocessing routine was then used to convert this binary information into stress-time plots at various locations.

Factors Affecting the Accuracy of the Calculations

67. Several factors can affect the accuracy of calculations using UMAT. The first and most obvious is the accuracy of the information used to develop the creep compliance, modulus, and shrinkage curves in the model. Frequently an analyst will try to cut costs by assuming properties of the concrete. It has been our experience at WES that the properties of mass concrete mixtures are heavily dependent on the amount of cement used, the type and amount of fly ash used, the type and modulus of aggregate, the water-cement ratio of the mixture, and the use of chemical admixtures. Predictions of material behavior are almost impossible to make based on "similar mixtures". For example, two concrete mixtures using the same components and proportions and the same type of aggregate obtained from different sources could still have very different specific creep curves if the moduli of the aggregates were very different.

68. Time-step size also affects accuracy of the results. Small time-steps (0.25 to 0.5 day) must be used after large changes in load to ensure the accuracy of creep predictions. This is true even for loadings applied after the first few days. This can be demonstrated by modeling a creep test using various time-stepping schemes. In the test, a 6- by 12-in. cylinder was loaded to 665 psi at 7 days after placement. The load was removed 9 days later. The test was simulated in four ABAQUS analyses using the time-stepping schemes shown in Table 4. Axial strains predicted for the period prior to unloading are compared with test strains in Figure 11. Predicted strains compared well with test results in the first two analyses (with time-steps less than or equal to 0.5 day for the first 5 days). The third analysis (with 1-day time-steps) overpredicted creep. Predicted and test strains after unloading are compared in Figure 12. Although creep recovery predicted in both Runs 1 and 4 was greater than test creep recovery, Run 1 results closely agreed with strains calculated using superposition, an accepted method of predicting creep recovery (Neville, Dilger, and Brooks 1983). In Run 4, 1-day time-steps were used after unloading, and predicted creep relief was much

Table 4

Time-stepping Schemes for Runs 1 through 4

		Prior to Unloading		After Unloading	
Run	Step	Increment	Step	Increment	Step
No.	No.	days	days	days	days
1	2	0.25	2		
	3	0.50	3		
	4	1.00	4		
	6			0.25	2
	7			0.50	3
	8			1.00	10
2	2	0.50	5		
	3	1.00	4		
	5			0.25	2
	6			0.50	3
	7			1.00	10
3	2	1.00	9		
	4			0.25	2
	5			0.50	3
	6			1.00	10
4	2	0.25	2		
	3	0.50	3		
	4	1.00	4		
	5			1.00	15

greater than in Run 1. In general, the use of time-steps that are too large will result in overpredicting creep strains.

69. The accuracy of predictions may also be affected by the type of elements used in the stress analysis. In a plane-strain analysis, strains in the out-of-plane direction are assumed to be constant along the length of the structure. This type of analysis is considered to be valid for very long structures. In practice, however, the out-of-plane strain is always zero. This condition corresponds to total restraint of out-of-plane strains, a condition which likely does not exist in real mass concrete structures. Because stresses due to this restraint are calculated in the UMAT model, a plane-strain analysis can result in excessive out-of-plane stresses and out-of-plane cracking. In cases where out-of-plane cracking causes convergence problems, plane-stress analyses can be used. In a plane-stress analysis,

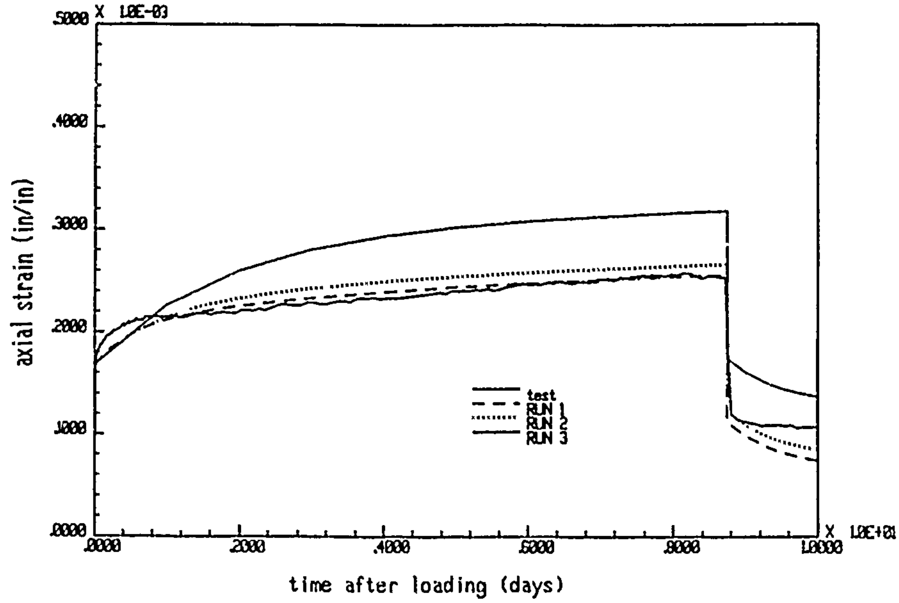


Figure 11. Predicted and test axial strains, 7-day creep test analysis

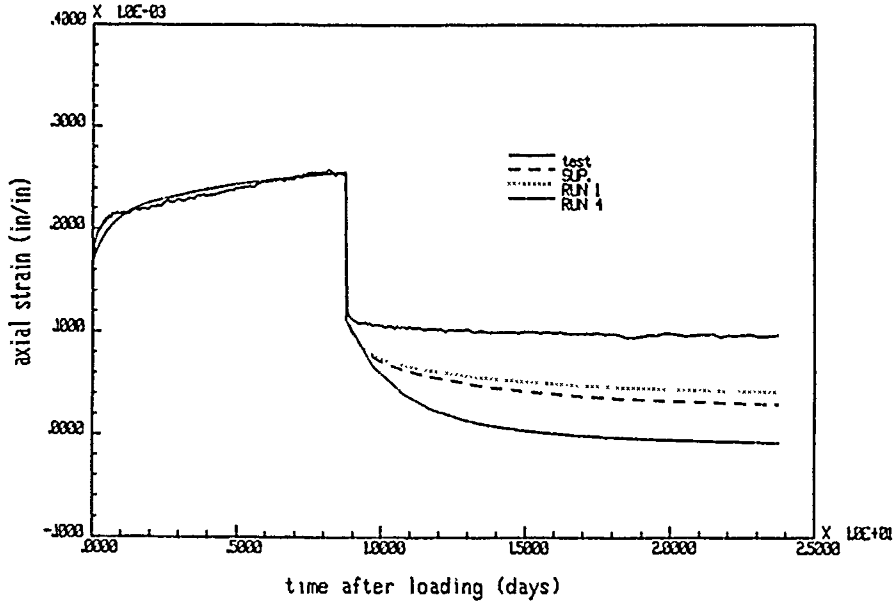


Figure 12. Predicted and test axial strains, 7-day creep test analysis, showing comparison after unloading

out-of-plane stresses are assumed to be constant (or zero) along the length of the structure. This corresponds to no restraint in the out-of-plane direction. Obviously, neither of these analyses gives a complete picture of stresses in most mass concrete structures. However, they can be used to determine the upper and lower limits for in-plane stresses and an upper limit for out-of-plane stresses.

70. To demonstrate the effect of element type on analysis results, three additional analyses were run on the chamber monolith floor shown in Figure 10: (a) plane-stress analysis using 8-node elements with reduced integration; (b) 3-D analysis using 20-node brick elements and a total monolith length of 43 ft; and (c) 3-D analysis using 20-node brick elements and a total monolith length of 86 ft.

71. Grids for the 3-D analyses modeled quarter-symmetric sections of the chamber monolith floor. Full 3 by 3 by 3 integration was used in the heat-transfer analyses and reduced integration in the stress analyses. The grid used in the 43-ft monolith analyses is shown in Figure 13. Integration point locations for the stress elements are shown in Figure 14.

72. Since the highest tensile stresses in the plane-strain analysis occurred at the center of Lift 3, stresses in the x- and z-directions at the center of Lift 3 in the four analyses have been compared in Figures 15 and 16. Stresses in the x-direction in the two 3-D analyses were almost exactly the same. Plane-strain predictions were slightly higher than those for the 3-D analyses, and plane-stress predictions were lower. Stresses in the z-direction increased with monolith length in the 3-D analyses but never reached those predicted by a plane-strain analysis. The sudden drop in stress in Figure 16 for the plane-strain analysis indicates cracking at that integration point.

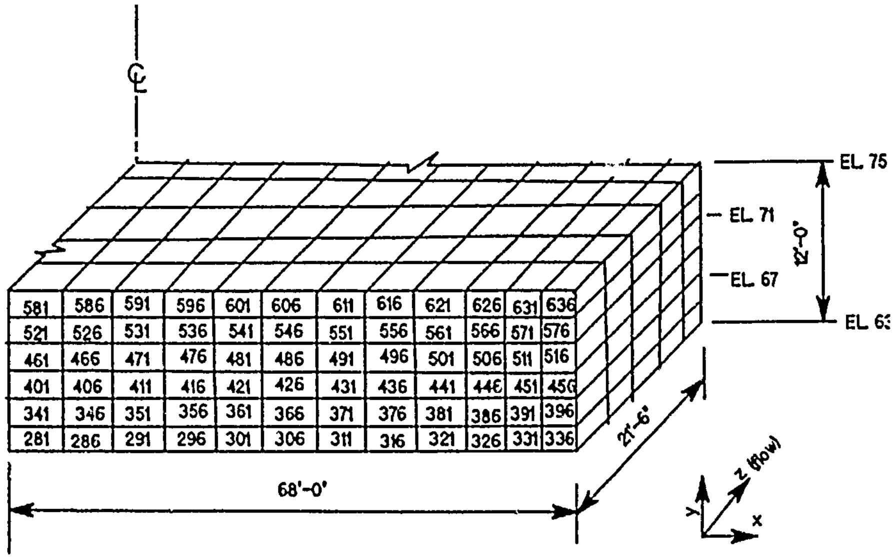


Figure 13. 3-D grid for FΓ stress analysis

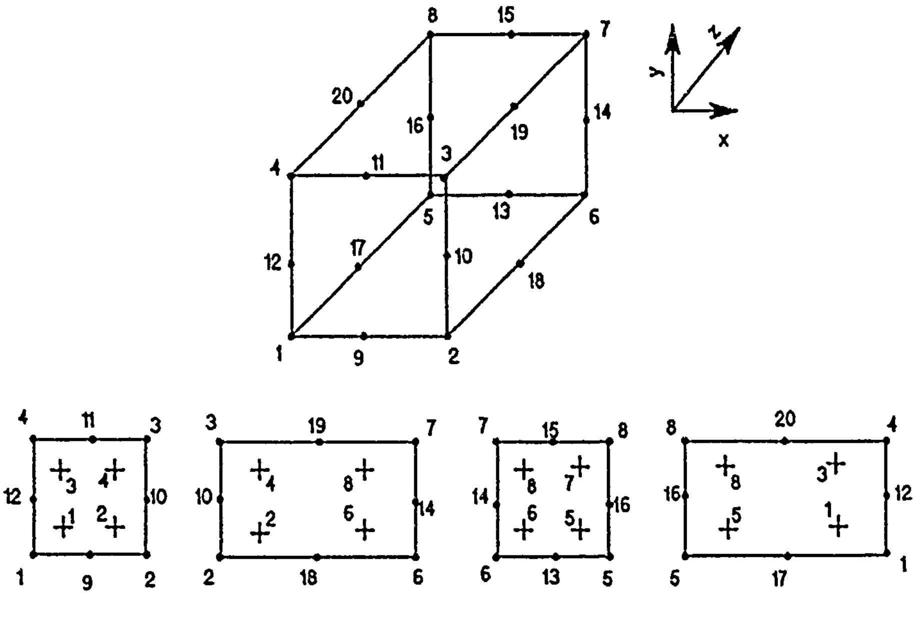


Figure 14. Location of integration points for 20-node element with reduced integration

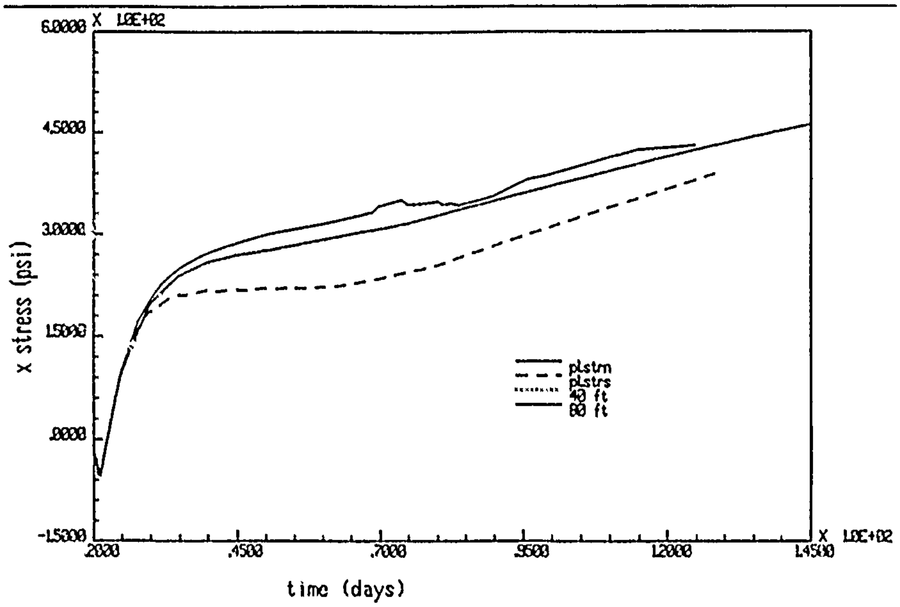


Figure 15. Stresses in the x-direction at the center of Lift 3

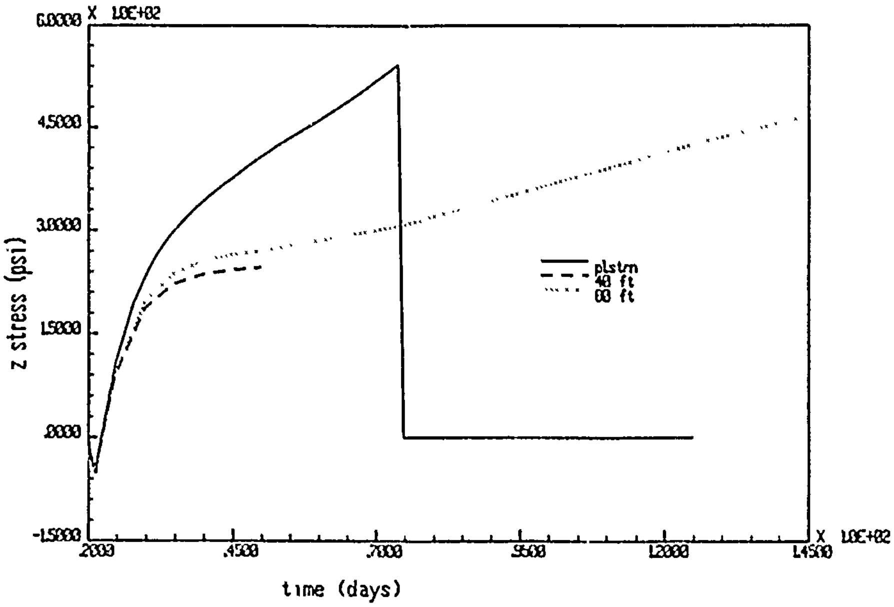


Figure 16. Stresses in the z-direction at the center of Lift 3.

PART VII: CONCLUSIONS AND RECOMMENDATIONS

Conclusions

73. Cracking in mass concrete is due to the restraint of volume change. These volume changes are due to thermal expansion and contraction, shrinkage, and creep/stress relaxation. Restraint of these volume changes is due to external boundary conditions and/or internal thermal gradients.

74. In the construction of mass concrete structures, due consideration should be given to reducing construction-related cracking. Although no Corps of Engineers structures have failed catastrophically due to construction-related cracking, a number of structures have required costly remedial repairs to prevent leakage or to increase the service life of the structure. Therefore, it is prudent to take measures prior to construction of the structure to reduce the potential for cracking.

75. Toward this end, a modern, computationally efficient analysis tool has been developed to predict cracking in concrete. This tool is a constitutive model which keeps track of the time-dependent changes in material response parameters such as elastic modulus, creep, and shrinkage. An interactive cracking criterion is included in the model based upon the smeared-crack approach. Both 2- or 3-D versions of the model are currently available. The model has been developed for use with ABAQUS, a modern, general-purpose heat-transfer and structural analysis FE code. ABAQUS features an option which allows user-defined material models to be easily incorporated as well as user-selected time-stepping for solution of the incremental construction problem. Through the MODEL CHANGE option, ABAQUS also allows the addition of lifts of concrete at user determined intervals of time.

76. The results from the use of this model are sensitive to the input values of the various material parameters. To accurately simulate time-dependent material behavior, the model requires accurate test data for calibration of the user-defined algebraic functions which govern the material properties. These data are critical for obtaining a meaningful representation of material behavior. Because concrete mixtures for mass concrete construction are site- and material-specific, no known data base of test data exists which would allow the model user to confidently estimate changes in

material properties with time. Therefore, until such an extensive data base of material properties data is developed, mechanical tests must be conducted to develop the information needed to calibrate the creep, shrinkage, and elastic modulus formulations in the model. These data should be obtained as soon as possible, beginning no later than 1 day after time of final setting.

77. The results from incremental construction analyses should be used to develop construction specification for projects which will reduce the potential for construction-related cracking. In addition, these analyses will provide information on the characteristics of concrete mixtures which are advantageous for reducing cracking. This information could be used as additional guidance for the proportioning of concrete mixtures for future mass concrete construction.

Recommendations

78. We recommend that accurate early-time material properties data be obtained on a project-by-project basis in the laboratory on project-specific materials and concrete mixtures when possible. If the exact project materials and concrete mixtures are unknown, the analyst should seek assistance from materials experts on the most likely materials to be used to construct the structure. Material properties from these can be developed as an estimate for project materials and then verified at a later date when information on project-specific information is available. The use of material properties from a generic material is not recommended.

79. As more early-time material properties data are gathered from a variety of concrete mixtures and materials, a data base of these properties should be maintained. This data base could be used for reference in the future to possibly establish bounding material properties for use in incremental construction analyses.

80. The analytical formulations presented for creep, elastic modulus, and shrinkage could be further refined as additional data and experience are obtained.

81. To realize the maximum benefit from incremental construction analysis, we recommend that the mixture-proportioning phase of the project be integrated with the incremental construction analysis phase. This will lead

to a more cost-effective, crack-free structure. It has been our experience that too often consideration of cracking has been delayed until after many key decisions have been made about the selection of materials, mixture proportions, and other parameters. Much can be gained from timely consideration of the effects of these factors prior to initiating the incremental construction analysis.

'82. We recommend that the procedures presented in this report be extended to include roller-compacted concrete (RCC) applications. An investigation into the early-time mechanical properties of RCC mixtures along with an analytical study of the construction procedures used to construct RCC structures should be conducted.

83. The material model developed in this investigation incorporates sound theory. However, a disadvantage of the approach used in developing this model is that verification of the predictions of the model under field conditions is quite difficult and expensive and, therefore, has not been accomplished. We recommend that a comprehensive evaluation of the model be conducted on a mass concrete structure in the field. This would require extensive instrumentation of the structure and analysis of the data to verifiy model predictions.

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APPENDIX A

EXAMPLE HEAT-TRANSFER ANALYSIS DECK

```
C RED RIVER CHAMBER MONOLITH FLOOR 2-DIMENSIONAL MODEL
C TEMPERATURE DECK - 112 SOIL ELEMENTS
C                    138 CONCRETE ELEMENTS
C PLACEMENT SCHEDULE:
C CHAMBER FLOOR, LIFTS 1-3 PLACED AT 10-DAY INCR., E.G.
C     PLACED AT 0, 10, 20, & 30 DAYS.
C FORM REMOVAL: FORMS ON VERTICAL SURFACES REMOVED AT 2 DAYS AFTER PLMT
C STEP/INCREMENT SCHEME (SCHEME BEYOND 5 DAYS ONLY AFTER PLACING L1-4,14
C     FIRST 2 DAYS AFTER PLACEMENT- 8 INCREMENTS AT DT=0.25 DAYS EACH.
C     DAYS 3 -  5 AFTER PLACEMENT - 6 INCREMENTS AT DT=0.50 DAYS EACH.
C     DAYS 6 - 10 AFTER PLACEMENT - 5 INCREMENTS AT DT=1.00 DAYS EACH.
C     DAYS 11- 20 AFTER PLACEMENT - 5 INCREMENTS AT DT=2.00 DAYS EACH.
C     DAYS 21- 45 AFTER PLACEMENT - 5 INCREMENTS AT DT=5.00 DAYS EACH.
C     DAYS 46- 95 AFTER PLACEMENT - 5 INCREMENTS AT DT=10.00 DAYS EACH.
C LIFT ELEMENTS - LNN, WHERE NN=LIFT NO. FOR ALL ELEMENTS IN A LIFT
C TOP SURFACE ELEMENTS - LNNT; WHERE NN=LIFT NO.,T=TOP SURFACE ELEMENT
C     UPON WHICH ADDITIONAL CONCRETE IS PLACED, AND
C                      - LNNF; WHERE NN=LIFT NO., F=PERMANENTLY EXPOSED
C     FLOOR SURFACE(SUCH AS CHAMBER FLOOR, CULVERT FLOOR)
C
C COMBINED LIFT ELEMENT SETS:
C     1.  FULL LIFTS:  L1_2 =ALL ELEMENTS IN LIFTS 1 AND 2
C                      L1_3 =ALL ELEMENTS IN LIFTS 1 - 3
C
*HEADING
2-D CHAMBER MONOLITH FLOOR, HEAT TRANSFER DECK #TR1
*NODE
2,0,636
38,432,636
50,504,636
66,624,636
98,816,636
114,936,636
1202,0,756
1238,432,756
1250,504,756
1266,624,756
1298,816,756
1314,936,756
3002,0,900
3038,432,900
3050,504,900
3066,624,900
3098,816,900
*NGEN
2,38,2
38,50,2
50,66,2
66,98,2
98,114,2
1202,1238,2
1238,1250,2
```

```
1250,1266,2
1266,1298,2
1298,1314,2
3002,3038,2
3038,3050,2
3050,3066,2
3066,3098,2
2,1202,150
38,1238,150
50,1250,150
66,1266,150
98,1298,150
114,1314,150
152,188,4
188,200,4
200,216,4
216,248,4
248,264,4
302,338,2
338,350,2
350,366,2
366,398,2
398,414,2
452,488,4
488,500,4
500,516,4
516,548,4
548,564,4
602,638,2
638,650,2
650,666,2
666,698,2
698,714,2
752,788,4
788,800,4
800,816,4
816,848,4
848,864,4
902,938,2
938,950,2
950,966,2
966,998,2
998,1014,2
1052,1088,4
1088,1100,4
1100,1116,4
1116,1148,4
1148,1164,4
1202,3002,150
1238,3038,150
1250,3050,150
1266,3066,150
```

```
1298,3098,150
1352,1388,4
1388,1400,4
1400,1416,4
1416,1448,4
1502,1538,2
1538,1550,2
1550,1566,2
1566,1598,2
1652,1688,4
1688,1700,4
1700,1716,4
1716,1748,4
1802,1838,2
1838,1850,2
1850,1866,2
1866,1898,2
1952,1988,4
1988,2000,4
2000,2016,4
2016,2048,4
2102,2138,2
2138,2150,2
2150,2166,2
2166,2198,2
2252,2288,4
2288,2300,4
2300,2316,4
2316,2348,4
2402,2438,2
2438,2450,2
2450,2466,2
2466,2498,2
2552,2588,4
2588,2600,4
2600,2616,4
2616,2648,4
2852,2888,4
2888,2900,4
2900,2916,4
2916,2948,4
2702,2738,2
2738,2750,2
2750,2766,2
2766,2798,2
*ELEMENT,TYPE=DC2D8
1,2,6,306,302,4,156,304,152
113,1202,1206,1506,1502,1204,1356,1504,1352
255,2790,2794,3094,3090,2792,2944,3092,2940
*ELGEN
1,28,4,1,4,300,28
113,24,4,1,5,300,24
```

```
209,2,300,24
233,16,4,1
255,2,4,1
*ELSET,ELSET=SL,GENERATE
1,112
*ELSET,ELSET=L1,GENERATE
113,160
*ELSET,ELSET=L1T,GENERATE
137,160
*ELSET,ELSET=L1R
136,160
*ELSET,ELSET=L2,GENERATE
161,208
*ELSET,ELSET=L2T,GENERATE
185,208
*ELSET,ELSET=L2R
184,208
*ELSET,ELSET=L1_2R
L1R,L2R
*ELSET,ELSET=L3,GENERATE
209,248
255,256
*ELSET,ELSET=L3T,GENERATE
233,248
255,256
*ELSET,ELSET=L3TA,GENERATE
225,230
*ELSET,ELSET=L3R
232,248,256
*ELSET,ELSET=L3L
255
*ELSET,ELSET=L1_2
L1,L2
*ELSET,ELSET=L1_3
L1_2,L3
*ELSET,ELSET=L1_3R
L1R,L2R,L3R
*ELSET,ELSET=SOILT
109,110,111,112
*ELSET,ELSET=REMV
L2,L3
*ELSET,ELSET=ALL
SOIL,L1_3
*NSET,NSET=SL1,GENERATE
2,114,2
*NSET,NSET=SL2,GENERATE
152,264,4
*NSET,NSET=SL3,GENERATE
302,414,2
*NSET,NSET=SL4,GENERATE
452,564,4
*NSET,NSET=SL5,GENERATE
```

```
602,714,2
*NSET,NSET=SL6,GENERATE
752,864,4
*NSET,NSET=SL7,GENERATE
902,1014,2
*NSET,NSET=SL8,GENERATE
1052,1164,4
*NSET,NSET=SL9,GENERATE
1202,1314,2
*NSET,NSET=NL1,GENERATE
1352,1448,4
1502,1598,2
1652,1748,4
1802,1898,2
*NSET,NSET=NL2,GENERATE
1952,2048,4
2102,2198,2
2252,2348,4
2402,2498,2
*NSET,NSET=NL3,GENERATE
2552,2648,4
2702,2798,2
2852,2948,4
3002,3066,2
3090,3098,2 ]
*SOLID SECTION,MATERIAL=SOIL,ELSET=SL
*MATERIAL,NAME=SOIL
*DENSITY
0.04285
*CONDUCTIVITY
2.2
*SPECIFIC HEAT
0.266
*SOLID SECTION,MATERIAL=CONCR,ELSET=L1_3
*MATERIAL,NAME=CONCR
*DENSITY
0.0865
*CONDUCTIVITY
2.3
*SPECIFIC HEAT
.21
*INITIAL CONDITIONS,TYPE=TEMPERATURE
SL1,70.0
SL2,70.5
SL3,71.0
SL4,72.0
SL5,73.0
SL6,76.0
SL7,79.0
SL8,81.0
SL9,82.0
NL1,85.0
```

```
NL2,85.0
NL3,85.0
*BOUNDARY
SL1,11,,70
*AMPLITUDE,NAME=AMB,TIME=HEAT,VALUE=ABS
     0.0        82.9       8.25       83.6       16.5       83.8       36.5       83.8
    43.1        83.5      47.5        83.0       58.3       82.0       64.0       81.0
    68.5        80.0      78.0        77.7      108.5       67.3      139.0       57.4
   169.5        50.8     178.8        49.0      187.0       48.5      194.5       48.3
   206.3        48.3     215.2        48.7      217.4       49.0      230.0       52.1
   259.5        59.0     290.0        67.4      320.5       74.1      351.0       80.5
*PLOT
CM FLOOR PREPROCESSOR PLOT
200,180,190,160,10,15,10,5
3,,,,1,12,.5
*DETAIL,ELSET=ALL
*DRAW,ELNUM
*WAVEFRONT MINIMIZATION,SUPPRESS
*STEP,INC=8
PLACE LIFT 1, EL63-67, T=0, DAY=1-2, DT=0.25D
*HEAT TRANSFER
0.25,2.0
*MODEL CHANGE,REMOVE
REMV
*FILM,AMPLITUDE=AMB,OP=NEW
SOILT,F3,,0.53867
L1R,F2,,0.16549
L1T,F3,,0.53867
*DFLUX
L1_3,BFNU
*NODE FILE
NT
*PLOT,FREQUENCY=3
CM HEAT TRANSFER RUN, JULY 1 START, L1
200,180,190,160,10,15,10,5
3,,,,1,12,.5
*DETAIL,ELSET=L1
*CONTOUR
TEMP,10,,
*END STEP
*STEP,INC=6
LIFT 1, REMOVE FORMS AT T=2.0 DAYS, RUN DAYS 3-5, DT=0.5
*HEAT TRANSFER
0.5,3.0
*FILM,AMPLITUDE=AMB
L1R,F2,,0.53867
*END STEP
*STEP,INC=5
LIFT 1, CONTINUE CALC AT T=5.0 DAYS, RUN DAYS 6-10, DT=1.0
*HEAT TRANSFER
1.0,5.0
*END STEP
```

```
*STEP,INC=8
PLACE LIFT 2, EL67-71, T=2.00, DAY=3-4, DT=0.5D
*HEAT TRANSFER
0.25,2.0
*MODEL CHANGE,INCLUDE
L2
*FILM,AMPLITUDE=AMB,OP=NEW
SOILT,F3,,0.53867
L1R,F2,,0.53867
L2R,F2,,0.16549
L2T,F3,,0.53867
*FLOT,FREQUENCY=3
CM HEAT TRANSFER RUN, JULY 1 START, L1_2
200,180,190,160,10,15,10,5
3,,,,1,12,.5
*DETAIL,ELSET=L1_2
*CONTOUR
TEMP,10,,
*END STEP
*STEP,INC=6
LIFT 2, REMOVE FORMS AT T=12.0 DAYS, RUN DAYS 13-15, DT=0.5
*HEAT TRANSFER
0.5,3.0
*FILM,AMPLITUDE=AMB
L2R,F2,,0.53867
*END STEP
*STEP,INC=5
LIFT 2, CONTINUE CALC AT T=15.0 DAYS, RUN DAYS 16-20, DT=1.0
*HEAT TRANSFER
1.0,5.0
*END STEP
*STEP,INC=8
PLACE LIFT 3, EL71-75, T=20.00, DAY=21-22, DT=0.25D
*HEAT TRANSFER
0.25,2.0
*MODEL CHANGE,INCLUDE
L3
*FILM,AMPLITUDE=AMB,OP=NEW
SOILT,F3,,0.53867
L1_2R,F2,,0.53867
L3L,F4,,0.16549
L3R,F2,,0.16549
L3TA,F3,,0.16549
L3T,F3,,0.53867
*PLOT,FREQUENCY=3
CM HEAT TRANSFER RUN, JULY 1 START, L1_3
200,180,190,160,10,15,10,5
3,,,,1,12,.5
*DETAIL,ELSET=L1_3
*CONTOUR
TEMP,10,,
*END STEP
```

```
*STEP,INC=6
LIFT 3, REMOVE FORMS AT T=22.0 DAYS, RUN DAYS 23-25, DT=0.5
*HEAT TRANSFER
0.5,3.0
*FILM,AMPLITUDE=AMB
L3L,F4,,0.53867
L3R,F2,,0.53867
L3TA,F3,,0.53867
*END STEP
*STEP,INC=5
LIFT 3, CONTINUE CALC AT T=25.0 DAYS, RUN DAYS 26-30, DT=1.0
*HEAT TRANSFER
1.0,5.0
*END STEP
*STEP,INC=5
LIFT 3, CONTINUE CALC AT T=30. DAYS, RUN DAYS 31-40, DT=2.
*HEAT TRANSFER
2.,10.
*END STEP
*STEP,INC=5
LIFT 3, CONTINUE CALC AT T=40. DAYS, RUN DAYS 41-65, DT=5.
*HEAT TRANSFER
5.,25.
*END STEP
*STEP,INC=5
LIFT 3, CONTINUE CALC AT T=65. DAYS, RUN DAYS 66-106, DT=10.
*HEAT TRANSFER
10.,50.
*END STEP
```

```
      subroutine dflux(flux,temp,kstep,kinc,time,noel,npt,coords,
     $                       jltyp)
c**********************************************************************
c this version interpolates along the adiabatic curve for more than
c one lift of any number of elements
c
c it also permits the use of two adiabatic curves in the same model.
c the concrete represented by the two curves must exist in two distinct
c groups of single or multiple lifts in which the elements in each
c of the two groups of lifts is consecutive  that is, the two concretes
c cannot exist in the same lift or exist in a lower and then a higher
c numbered lift so as to alternate between successive lifts.  in
c other words, there must be a single lift interface between the two
c concretes.  appropriate dimension statements, data statements, and
c coding must be modified to include:
c    q1,t1,q2,t2,prop1,prop2,lmix2
c
c note: although two curves were not required for this analysis, the option
c has been left in to demonstrate its use
c
c  units in the t array are hours.
c  units in the q array are temperature in degrees F and will be
c   converted to btu/(lb-in**3)
c  nq     = no. of points in t & q arrays
c  entime = endtime for dflux (last time in t array + .01)
c  sttime = array of starttimes for each element in hours
c           + one dummy time. the array must be dimensioned
c           number of elements + 1.
c  prop(1)= density (lb/cu.in.)
c  prop(2)= specific heat
c  nmax   = number of integration points per element
c  nlifts = number of lifts
c  nstme  = number of start times (nlifts + 1 dummy time)
c  nelem(nlifts) = array of number of elements in each lift
c  stm(i) = array of starttimes for each lift + 1 dummy time(hrs)
c  nstel  = number of first element using dflux
c  prop1  = density and specific heat of concrete mix 1
c  prop2  = density and specific heat of concrete mix 2
c  lmix2  = lowest lift including concrete mix2
c*********************************************************************
c
c  for double precision versions of ABAQUS 4.7
c
      implicit real*8(a-h,o-z)
c  parameter statement to hold no. lifts and no lifts +1
      parameter (nlifts=3,nstme=4,nq1=9,nq2=26)
      common/eldef/sttime(139)
      dimension coords(3),q(26),t(26),prop(2),nelem(nlifts),
     &          oltime(nlifts),oldq(nlifts),nolincr(nlifts),
     &          nseter(nlifts),stm(nstme),q1(nq1),t1(nq1),
     &          q2(nq2),t2(nq2),prop1(2),prop2(2)
      save nolincr,oltime,oldq nseter,nwhere,nnoel
```

```
c  hansen's al3 curve (hh)
      data q1 /7.33,14.67,22.,25.9,30.5,35.,35.9,36.4,36.4/
      data t1 /5.,10.,15.,25.,40.,70.,80.,140.,672./
c
      data q2 / 11.18, 27.86, 42.11, 49.95, 53.48, 55.35, 56.86,
     &          58.09, 59.15, 59.89, 60.58, 61.17, 62.35, 63.31, 64.15,
     &          64.97, 66.10, 66.99, 67.68, 68.75, 69.57, 70.31, 70.94,
     &          72.13, 72.88, 73.51/
      data t2 /  6.00, 12.00, 18.00, 24.00, 30.00, 36.00, 42.00,
     &          48.00, 54.00, 60.00, 66.00, 72.00, 84.00, 96.00,108.00,
     &         120.00,144.00,168.00,192.00,240.00,288.00,336.00,384.00,
     &         480.00,576.00,672.00/
c
      data prop1/0.0865,0.21/
      data prop2/0.08000,0.20/
      data entime/672.01/
      data lmix2/4/
      data nmax/9/
      data nelem/48,48,42/
      data stm/0.,240.,480.,720./
      data nstel/113/
c
c renumber elements
c
      if(noel.lt.nstel)return
      if(noel.eq.nstel.and.npt.eq.1)nnoel=0
      if(npt.eq.1)nnoel=nnoel+1
      noel=nnoel
c
c fill start-time array
c
      nst=1
      ntot=0
      do 200 i=1,nlifts
        ntot=nelem(i)+ntot
        do 201 j=nst,ntot
          sttime(j)=stm(i)
201     continue
        nst=nst+nelem(i)
200   continue
      sttime(ntot+1)=stm(nstme)
c
c determine lift number (k)
c
      ne=0
      nem=0
      do 202 k=1,nlifts
        ne=ne+nelem(k)
        if(k.eq.1.and.noel.le.ne)go to 24
        if(k.gt.1)nem=nem+nelem(k-1)
        if(noel.gt.nem.and.noel.le.ne)go to 24
202   continue
```

```
 24   if(kstep.eq.1.and.kinc.eq.1)then
        nolincr(k)=1
        oltime(k)=0.
        oldq(k)=0.
      end if
c
c     set up multiple adiabatics & thermal properties
c
      if(k.lt.lmix2)then
c       assign t and q for mix 1
        nq=nq1
        do 203 i=1,nq
          t(i)=t1(i)
          q(i)=q1(i)
203     continue
c       assign props for mix 1
        prop(1)=prop1(1)
        prop(2)=prop2(2)
      else
c       assign t and q for mix 2
        nq=nq2
        do 204 i=1,nq
          t(i)=t2(i)
          q(i)=q2(i)
204     continue
c       assign props for mix 2
        prop(1)=prop2(1)
        prop(2)=prop2(2)
      end if
c     **************
      trel = time - sttime(noel) / 24.
      end = entime / 24.
      flux = 0.0
      if( trel.gt.0.0.and.trel.lt.end ) go to 10
      return
c
  10  continue
      do 20 i=1,nq
      nch=0
      td = t(i) / 24.
      if(i.gt.1)tb=t(i-1)/24.
      dif=abs(trel-td)
      if(trel.lt.td.or.dif.lt.0.01)go to 30
  20  continue
c
      write(6,35) kstep,kinc,time,noel
  35  format(/,"  warning - passed through dflux without assigning",
     &      /,"             flux.  step =",i5,"  inc =",i5,
     &       /,"            time =",f12.2,"  element =",i5)
      return
  30  continue
c
```

```
c calculate flux
c
      if(i.eq.1)then
        tq=q(i)*trel/td
      end if
      if(i.gt.1)tq=(q(i)-q(i-1))*(trel-tb)/(td-tb)+q(i-1)
      flux=(tq-oldq(k))/(trel-oltime(k))*prop(1)*prop(2)
c
c set pointers
c
      if(sttime(noel+1).gt.sttime(noel))then
        if(kinc.eq.nolincr(k).and.npt.eq.nmax.and.nseter(k).eq.4)
     &    go to 100
        if(kinc.eq.nolincr(k).and.npt.eq.nmax)then
          nseter(k)=4
        end if
      end if
      if(kinc.ne.nolincr(k))nolincr(k)=kinc
      go to 999
 100  oltime(k)=trel
      oldq(k)=tq
      nseter(k)=1
 999  continue
      return
      stop
      end
```

APPENDIX B

EXAMPLE STRESS ANALYSIS DECK

```
C RED RIVER CHAMBER MONOLITH FLOOR 2-DIMENSIONAL MODEL
C STRESS DECK - 138 CONCRETE ELEMENTS
C PLACEMENT SCHEDULE:
C CHAMBER FLOOR, LIFTS 1-3 PLACED AT 10-DAY INCR., E.G.
C     PLACED AT 0, 10, 20, & 30 DAYS.
C FORM REMOVAL: FORMS ON VERTICAL SURFACES REMOVED AT 2 DAYS AFTER PLMT
C STEP/INCREMENT SCHEME (SCHEME BEYOND 5 DAYS ONLY AFTER PLACING L1-4,14
C     FIRST 2 DAYS AFTER PLACEMENT- 8 INCREMENTS AT DT=0.25 DAYS EACH.
C     DAYS 3 -  5 AFTER PLACEMENT - 6 INCREMENTS AT DT=0.50 DAYS EACH.
C     DAYS 6 - 10 AFTER PLACEMENT - 5 INCREMENTS AT DT=1.00 DAYS EACH.
C     DAYS 11- 20 AFTER PLACEMENT - 5 INCREMENTS AT DT=2.00 DAYS EACH.
C     DAYS 21- 45 AFTER PLACEMENT - 5 INCREMENTS AT DT=5.00 DAYS EACH.
C     DAYS 46- 95 AFTER PLACEMENT - 5 INCREMENTS AT DT=10.00 DAYS EACH.
C LIFT ELEMENTS - LNN, WHERE NN=LIFT NO. FOR ALL ELEMENTS IN A LIFT
C TOP SURFACE ELEMENTS - LNNT; WHERE NN=LIFT NO.,T=TOP SURFACE ELEMENT
C     UPON WHICH ADDITIONAL CONCRETE IS PLACED, AND
C                       - LNNF; WHERE NN=LIFT NO., F=PERMANENTLY EXPOSED
C     FLOOR SURFACE(SUCH AS CHAMBER FLOOR, CULVERT FLOOR)
C
C COMBINED LIFT ELEMENT SETS:
C     1.  FULL LIFTS:  L1_2 =ALL ELEMENTS IN LIFTS 1 AND 2
C                      L1_3 =ALL ELEMENTS IN LIFTS 1 - 3
C
*HEADING
2-D CHAMBER MONOLITH FLOOR, STRESS DECK #SR1
*NODE
1202,0,756
1238,432,756
1250,504,756
1266,624,756
1298,816,756
3002,0,900
3038,432,900
3050,504,900
3066,624,900
3098,816,900
*NGEN
1202,1238,2
1238,1250,2
1250,1266,2
1266,1298,2
3002,3038,2
3038,3050,2
3050,3066,2
3066,3098,2
1202,3002,150
1238,3038,150
1250,3050,150
1266,3066,150
1298,3098,150
1352,1388,4
1388,1400,4
```

```
1400,1416,4
1416,1448,4
1502,1538,2
1538,1550,2
1550,1566,2
1566,1598,2
1652,1688,4
1688,1700,4
1700,1716,4
1716,1748,4
1802,1838,2
1838,1850,2
1850,1866,2
1866,1898,2
1952,1988,4
1988,2000,4
2000,2016,4
2016,2048,4
2102,2138,2
2138,2150,2
2150,2166,2
2166,2198,2
2252,2288,4
2288,2300,4
2300,2316,4
2316,2348,4
2402,2438,2
2438,2450,2
2450,2466,2
2466,2498,2
2552,2588,4
2588,2600,4
2600,2616,4
2616,2648,4
2852,2888,4
2888,2900,4
2900,2916,4
2916,2948,4
2702,2738,2
2738,2750,2
2750,2766,2
2766,2798,2
*ELEMENT,TYPE=CPE8R
113,1202,1206,1506,1502,1204,1356,1504,1352
255,2790,2794,3094,3090,2792,2944,3092,2940
*ELGEN
113,24,4,1,5,300,24
209,2,300,24
233,16,4,1
255,2,4,1
*ELSET,ELSET=L1,GENERATE
113,160
```

```
*ELSET,ELSET=L1T,GENERATE
137,160
*ELSET,ELSET=L2,GENERATE
161,208
*ELSET,ELSET=L2T,GENERATE
185,208
*ELSET,ELSET=L3,GENERATE
209,248
255,256
*ELSET,ELSET=L1_2
L1,L2
*ELSET,ELSET=L1_3
L1_2,L3
*ELSET,ELSET=REMV
L2,L3
*NSET,NSET=CL,GENERATE
1202,3002,150
*NSET,NSET=BASE,GENERATE
1202,1298,2
*NSET,NSET=SUP
CL,BASE
*NSET,NSET=NLFT1,GENERATE
1352,1448,4
1502,1598,2
1652,1748,4
1802,1898,2
*NSET,NSET=NLFT2,GENERATE
1952,2048,4
2102,2198,2
2252,2348,4
2402,2498,2
*NSET,NSET=NLFT3,GENERATE
2552,2648,4
2702,2798,2
2852,2916,4
2940,2948,4
3002,3066,2
3090,3098,2
*SOLID SECTION,MATERIAL=M1,ELSET=L1
*MATERIAL,NAME=M1
*USER MATERIAL,CONSTANTS=13
1.88E6,.15,600.,100.E-6,5.5E-6,85.,.24,1.,
1.00,0.,0.,2.,0.
*DEPVAR
57
*SOLID SECTION,MATERIAL=M2,ELSET=L2
*MATERIAL,NAME=M2
*USER MATERIAL,CONSTANTS=13
1.88E6,.15,600.,100.E-6,7.E-6,85.,.24,1.,
1.00,0.,10.,2.,0.
*DEPVAR
57
```

```
*SOLID SECTION,MATERIAL=M3,ELSET=L3
*MATERIAL,NAME=M3
*USER MATERIAL,CONSTANTS=13
1.88E6,.15,600.,100.E-6,7.E-6,85.,.24,1.,
1.00,0.,20.,2.,0.
*DEPVAR
57
*INITIAL CONDITIONS,TYPE=TEMPERATURE
NLFT1,85.0
NLFT2,85.0
NLFT3,85.0
*BOUNDARY
BASE,2
CL,1
*WAVEFRONT MINIMIZATION,SUPPRESS
*STEP
PLACE LIFT 1, EL63-67, T=0, DAY=1-2, DT=0.25D
*STATIC,PTOL=10.,DIRECT=NOSTOP
0.01,.01
*MODEL CHANGE,REMOVE
REMV
*TEMPERATURE,FILE=15,BSTEP=1(INC=1),ESTEP=1(INC=1)
*EL FILE,ELSET=L1
S,E
*EL PRINT,ELSET=L1,FREQUENCY=0
*PLOT,FREQUENCY=3
CM RUN 1, JULY 1 START, PL STRN, L1
200,180,190,160,10,15,10,5
3,,140,,1,12,.5
*DETAIL,ELSET=L1
*CONTOUR
S11,6
*CONTOUR
S22,6
*CONTOUR
S33,6
*CONTOUR
PRIN3,6
*DISPLACED
U,
*END STEP
*STEP,INC=7
PLACE LIFT 1, EL63-67, T=0, DAY=1-2, DT=0.25D
*STATIC,PTOL=10.,DIRECT=NOSTOP
.25,1.75
*DLOAD
L1,BY,-.0865
*TEMPERATURE,FILE=15,BSTEP=1(INC=2),ESTEP=1(INC=8)
*END STEP
*STEP,INC=6
LIFT 1, REMOVE FORMS AT T=2.0 DAYS, RUN DAYS 3-5, DT=0.5
*STATIC,PTOL=10.,DIRECT=NOSTOP
```

```
0.5,3.0
*TEMPERATURE,FILE=15,BSTEP=2(INC=1),ESTEP=2(INC=6)
*END STEP
*STEP,INC=5
LIFT 1, CONTINUE CALC AT T=5.0 DAYS, RUN DAYS 6-10, DT=1.0
*STATIC,PTOL=10.,DIRECT=NOSTOP
1.0,5.0
*TEMPERATURE,FILE=15,BSTEP=3(INC=1),ESTEP=3(INC=5)
*END STEP
*STEP
LIFT 1, CONTINUE CALC AT T=11. DAYS, RUN DAY 11, DT=.24
*STATIC,PTOL=10.,DIRECT=NOSTOP
.24,.24
*TEMPERATURE,FILE=15,BSTEP=4(INC=1),ESTEP=4(INC=1)
*END STEP
*STEP
PLACE LIFT 2, EL67-71, T=11.00, DT=0.01D
*STATIC,PTOL=10.,DIRECT=NOSTOP
.01,.01
*DLOAD
L1T,P3,4.152
*TEMPERATURE,FILE=15,BSTEP=4(INC=1),ESTEP=4(INC=1)
*MODEL CHANGE,INCLUDE
L2
*EL FILE,ELSET=L1_2
S,E
*EL PRINT,ELSET=L1_2,FREQUENCY=0
*PLOT,FREQUENCY=3
CM RUN 1, JULY 1 START, PL STRN, L1_2
200,180,190,160,10,15,10,5
3,,140,,1,12,.5
*DETAIL,ELSET=L1_2
*CONTOUR
S11,6
*CONTOUR
S22,6
*CONTOUR
S33,6
*CONTOUR
PRIN3,6
*DISPLACED
U,
*END STEP
*STEP,INC=7
LIFT 2, EL67-71, T=11.00, DAYS=11-12, DT=.25D
*STATIC,PTOL=10.,DIRECT=NOSTOP
0.25,1.75
*TEMPERATURE,FILE=15,BSTEP=4(INC=2),ESTEP=4(INC=8)
*END STEP
*STEP,INC=1
LIFT 2, REMOVE FORMS AT T=12.0 DAYS, RUN DAYS 13, DT=0.5
*STATIC,PTOL=10.,DIRECT=NOSTOP
```

```
0.5,.50
*DLOAD,OP=NEW
L1_2,BY,-.0865
*TEMPERATURE,FILE=15,BSTEP=5(INC=1),ESTEP=5(INC=1)
*END STEP
*STEP,INC=5
LIFT 2, REMOVE FORMS AT T=12.0 DAYS, RUN DAYS 13-15, DT=0.5
*STATIC,PTOL=10.,DIRECT=NOSTOP
0.5,2.5
*TEMPERATURE,FILE=15,BSTEP=5(INC=2),ESTEP=5(INC=6)
*END STEP
*STEP,INC=5
LIFT 2, CONTINUE CALC AT T=16.0 DAYS, RUN DAYS 16-20, DT=1.0
*STATIC,PTOL=10.,DIRECT=NOSTOP
1.0,5.0
*TEMPERATURE,FILE=15,BSTEP=6(INC=1),ESTEP=6(INC=5)
*END STEP
*STEP
LIFT 2, CONTINUE CALC AT T=21.0 DAYS, RUN DAY 21, DT=.24
*STATIC,PTOL=10.,DIRECT=NOSTOP
.24,.24
*TEMPERATURE,FILE=15,BSTEP=7(INC=1),ESTEP=7(INC=1)
*END STEP
*STEP
PLACE LIFT 3, EL71-75, T=21.00, DAY=21, DT=0.01D
*STATIC,PTOL=10.,DIRECT=NOSTOP
.01,.01
*DLOAD
L2T,P3,4.152
*MODEL CHANGE,INCLUDE
L3
*TEMPERATURE,FILE=15,BSTEP=7(INC=1),ESTEP=7(INC=1)
*EL FILE,ELSET=L1_3
S,E
*EL PRINT,ELSET=L1_3,FREQUENCY=0
*PLOT,FREQUENCY=3
CM RUN 1, JULY 1 START, PL STRN, L1_3
200,180,190,160,10,15,10,5
3,,140,,1,12,.5
*DETAIL,ELSET=L1_3
*CONTOUR
S11,6,
*CONTOUR
S22,6,
*CONTOUR
S33,6,
*CONTOUR
PRIN3,6,
*DISPLACED
U,
*END STEP
*STEP,INC=7
```

```
PLACE LIFT 3, EL71-75, T=21.00, DAY=21-22, DT=0.25D
*STATIC,PTOL=10.,DIRECT=NOSTOP
.25,1.75
*TEMPERATURE,FILE=15,BSTEP=7(INC=2),ESTEP=7(INC=8)
*END STEP
*STEP,INC=1
LIFT 3, REMOVE FORMS AT T=22.0 DAYS, RUN DAYS 23, DT=0.5
*STATIC,PTOL=10.,DIRECT=NOSTOP
0.5,.50
*DLOAD,OP=NEW
L1_3,BY,-.0865
*TEMPERATURE,FILE=15,BSTEP=8(INC=1),ESTEP=8(INC=1)
*END STEP
*STEP,INC=5
LIFT 3, REMOVE FORMS AT T=22.0 DAYS, RUN DAYS 23-25, DT=0.5
*STATIC,PTOL=10.,DIRECT=NOSTOP
0.5,2.5
*TEMPERATURE,FILE=15,BSTEP=8(INC=2),ESTEP=8(INC=6)
*END STEP
*STEP,INC=5
LIFT 3, CONTINUE CALC AT T=26.0 DAYS, RUN DAYS 26-30, DT=1.0
*STATIC,PTOL=10.,DIRECT=NOSTOP
1.0,5.0
*TEMPERATURE,FILE=15,BSTEP=9(INC=1),ESTEP=9(INC=5)
*END STEP
*STEP,INC=5
LIFT 3, CONTINUE CALC AT T=30. DAYS, RUN DAYS 31-40, DT=2.D
*STATIC,PTOL=10.,DIRECT=NOSTOP
2.,10.
*TEMPERATURE,FILE=15,BSTEP=10(INC=1),ESTEP=10(INC=5)
*END STEP
*STEP,INC=5
LIFT 3, CONTINUE CALC AT T=40. DAYS, RUN DAYS 41-65, DT=5.
*STATIC,PTOL=10.,DIRECT=NOSTOP
5.,25.
*TEMPERATURE,FILE=15,BSTEP=11(INC=1),ESTEP=11(INC=5)
*END STEP
*STEP,INC=5
LIFT 3, CONTINUE CALC AT T=65. DAYS, RUN DAYS 66-106, DT=10
*STATIC,PTOL=10.,DIRECT=NOSTOP
10.,50.
*TEMPERATURE,FILE=15,BSTEP=12(INC=1),ESTEP=12(INC=5)
*END STEP
```

APPENDIX C

UMAT SUBROUTINE

```
        SUBROUTINE UMAT(STRESS,STATEV,HH,SSE,SPD,SCD,DUM1,
     $  DUM2,DUM3,DUM4,TEPS,DEP,TYME,DELTM,TEMP,DTEMP,
     $  PREDEF,DPRED,CMAT,NDI,NSHR,NTENS,NSTATV,PROPS,
     $  NPROPS,COORDS,DUM5)
C
C   FOR DOUBLE PRECISION VERSIONS OF ABAQUS 4.7
C
       IMPLICIT REAL*8 (A-H,O-Z)
C
C      UMAT86:  COMBINED UMAT FOR HANDLING
C
C        JPROPS=1  2-D CONCRETE WITH ELASTIC CRACKING ONLY
C        JPROPS=2  2-D CONCRETE WITH CRACKINC, PLASTICITY, CREEP & AGING
C        JPROPS=3  3-D CONCRETE WITH CRACKING, PLASTICITY, CREEP & AGING
C
C
C       THE FOLLOWING ARE ABAQUS COMMON BLOCKS.  THESE ARE FOR 4.5 ONLY
C
       COMMON /CSP/SINT(513)
       COMMON /CELGI/ IDUM(7),IEDBR,JDUM(110)
       COMMON /CEL/ LCEL(75)
       COMMON /COUNT/ ICOUNT(4),ACOUNT(14),JCOUNT(6),BCOUNT,KSTIF,
    $                 KDUM,DDUM(3),LDUM(2),EDUM(6),NDUM(8),FDUM(4),
    $                 MDUM(2),GDUM(3)
C
       COMMON /RSDINF/ NOUT,JELNO,INT,NSTPAB,INCRAB,NPASS
       COMMON /RSDPR/ IPRINT
       COMMON /RSDDBG/ NBUG
       LOGICAL IPRINT
C
       COMMON /FLTNUM/ ZERO,ONE,TWO,THREE,FOUR,FIVE,SIX,SEVEN,EIGHT,
     $   NINE,TEN,HALF,THIRD,FOURTH,FIFTH,SIXTH,SEVNTH,EIGHTH,NINETH,
     $   TENTH,HUNDRD,THOU,MILLON,PI,PIFAC,PIFAC1,EXPN
       REAL*8 NINE,NINETH,MILLON
       CHARACTER*8 CMAT
C
       PARAMETER (IAR=78,MAXSV=57)
C
       DIMENSION STRESS(NTENS),STATEV(NSTATV),PROPS(NPROPS),NPRINT(100),
     $ TEPS(NTENS),DEP(NTENS),HH(NTENS,NTENS),IORDER(6),DTIM(2),
     $ PROPI(100),AR(IAR),EP(6),PH(6,6),COORDS(3),DEPS(4),SIG(4),D(11),
     $ LORDER(4),DE(4,4),DDEPS(4),DSIG(4),KRK1(3),KRK2(3),JORDER(6),
     $ IARSV(IAR),DUM2(NTENS),DUM3(NTENS),DUM5(3,3)
C
       LOGICAL FIRST(100),FIRSTE,BAD,XAGE
C
       SAVE
       DATA FIRST/100*.TRUE./, IORDER/1,2,3,6,5,4/,
     $ MPRINT/0/, FIRSTE/.TRUE./, BAD/.FALSE./, NPRINT/100*0/,
     $ LORDER/1,2,4,3/, KRK1/3*0/, KRK2/3*0/, NUMITR/1/, NCRACK/0/,
     $ IARSV/12*0,1,2,3,9*0,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,
     $  20,21,22,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,39,40,
     $ 41,42,43,44,45,46,47,48,49,50,51,52,53,54,55,56,57/
      EPSMACH=1.E-9
      NOUT=6
      JPROPS=NEAR(PROPS(1))
C
C         CONVERT MATERIAL NAME TO NUMBER
C
      IF(CMAT(1:1).NE.'M')WRITE(6,888)
888   FORMAT('ERROR IN MATERIAL NAME. MUST BE MXX WHERE XX IS',
     $     /'NUMBER FROM 1 TO 99.')
      IF(CMAT(3:3).EQ.' ')THEN
        READ(CMAT(2:2),'(I1)')MATERL
      ELSE
        READ(CMAT(2:3),'(I2)')MATERL
      END IF
C
C         DETERMINE WHICH UMAT MODEL IS TO BE USED
C
      IF (JPROPS.NE.2.OR.JPROPS.NE.3)THEN
        WRITE(NOUT,977)
 977    FORMAT('0ERROR IN USER SUBROUTINE CARD')
      ELSEIF (JPROPS.EQ.2.OR.JPROPS.EQ.3) THEN
       XAGE=.TRUE.
       IF (NSTATV.LT.MAXSV) THEN
        WRITE(NOUT,11) JPROPS,NSTATV,MAXSV,IAR
   11   FORMAT('0BAD CONTROL DATA TO UMAT.  JPROPS,NSTATV,MAXSV,IAR =',
     $   I9,3I5)
        STOP 'BAD CONTROL DATA FOR UMATAGE'
       ENDIF
      ENDIF
C
C            SET CONTROL PARAMETERS FROM ABAQUS
C
      CALL ACOPDI(SINT(IEDBR+1),JELNO,1)
      NINT=LCEL(23)
      INT=LCEL(5)
      NSEC=LCEL(8)
      INCRAB=ICOUNT(1)
      NSTPAB=JCOUNT(1)
      IF (FIRSTE) THEN
       IF (NSTPAB.EQ.1.AND.INCRAB.EQ.1) THEN
        INCRMT=0
       ELSE
        INCRMT=1
       ENDIF
       CALL FLOATN
       AGEMIN=0.
       JELNO1=JELNO
       FIRSTE=.FALSE.
C      CALL SECOND(TZERO)
       NPASS=0
       WRITE(NOUT,12)
```

```
12     FORMAT('0ANATECH UMAT87 VERSION = SEP 16, 1987')
      ENDIF
C
      IF (JELNO.EQ.JELNO1.AND.INT.EQ.1) THEN
C
C         FIRST ELEMENT & FIRST INTEGRATION PT ONLY
C
       NPASS=NPASS+1
C
       IF (KSTIF.EQ.1) THEN
        IF (NSTPAB.EQ.1.AND.INCRAB.EQ.1.AND.INCRMT.GT.0) THEN
         INCRMT=1
         WRITE(NOUT,21)
         PRINT 21
   21    FORMAT(' WARNING NSTPAB & INCRAB = 1, BUT INCRMT .GT. 0')
        ELSE
         MPASS=NPASS
         NPASS=0
         INCRMT=INCRMT+1
         PRINT 25, NSTPAB,INCRAB
   25    FORMAT(' START STEP=',I4,' INCR=',I4)
         IF (NCRACK.NE.0) WRITE(NOUT,26)
     $    (KRK2(I),I=1,3),(KRK1(I),I=1,3)
   26    FORMAT(' NUMBER OF INTEGRATION POINTS IN PREVIOUS INCREMENT ',
     $    'WITH OPEN   1 2 & 3 CRACKS = ',3I5/
     $    ' NUMBER OF INTEGRATION POINTS IN PREVIOUS INCREMENT ',
     $    'WITH CLOSED 1 2 & 3 CRACKS = ',3I5)
        ENDIF
       ENDIF
C
       NT=MIN(2,INCRMT)
C
       DO 27 I=1,3
       KRK2(I)=0
27     KRK1(I)=0
C
      ENDIF
C
C     CONCRETE PROPERTIES
C
      IF (NPROPS.EQ.0) THEN
       ECONC=ZERO
       IPROPS=4
       GO TO 560
      ENDIF
C
      IFLAG=0
      IF (JPROPS.LT.10.AND.JPROPS.GT.0) IFLAG=1
      IPROPS=NPROPS-IFLAG
      IPROPS=MIN(IPROPS,13)
      IF (IPROPS.EQ.1 .AND. PROPS(1).LT.ONE) THEN
       IPROPS=0
       GO TO 560
      ENDIF
      GO TO (559,558,557,556,555,554,553,552,551,550,549,548,547),IPROPS
  547 IHANOP=NEAR(PROPS(13+IFLAG))
  548 IPRUM=NEAR(PROPS(12+IFLAG))
  549 TIMREF=PROPS(11+IFLAG)
  550 EPSHRK=PROPS(10+IFLAG)
  551 CREEP=PROPS(9+IFLAG)
  552 SHRINK=PROPS(8+IFLAG)
  553 AGE=PROPS(7+IFLAG)
  554 TREF=PROPS(6+IFLAG)
  555 ALFAC=PROPS(5+IFLAG)
  556 EPFRAC=PROPS(4+IFLAG)
  557 CRUSH=PROPS(3+IFLAG)
  558 XVC=PROPS(2+IFLAG)
  559 ECONC=PROPS(1+IFLAG)
  560 IPROPS=IPROPS+1
C
C      ALL VALUES ARE IN PSI, DEG F & DAY UNITS.
C
      GO TO (561,562,563,564,565,566,567,568,569,570,571,572,573,574),
     $ IPROPS
  561 ECONC=3.E6
  562 XVC=0.15
  563 CRUSH=(ECONC/57600.)**2
  564 EPFRAC=(6.7*SQRT(CRUSH))/ECONC
  565 ALFAC=ZERO
  566 TREF=ZERO
  567 AGE=THOU
  568 SHRINK=ZERO
  569 CREEP=ZERO
  570 EPSHRK=ZERO
  571 TIMREF=ZERO
  572 IPRUM=2
  573 IHANOP=0
C
  574 G=ECONC/(ONE+XVC)
C
      IF (ECONC.NE.ZERO) THEN
       IF (XVC.LT.ZERO.OR.XVC.GT.0.49.OR.ABS(CRUSH/ECONC).LT.1.E-4
     $  .OR.EPFRAC.LE.ZERO.OR.CRUSH.LE.ZERO) BAD=.TRUE.
      ENDIF
C
      IPRUM=MAX(1,MIN(4,IPRUM))
      IHANOP=MAX(0,MIN(2,IHANOP))
      IF (IPRUM.NE.4) IHANOP=0
C
      IF (MATERL.GT.100) GO TO 123
      IF (.NOT.FIRST(MATERL)) GO TO 123
C
      IF (AGE.LT.AGEMIN) THEN
       WRITE(NOUT,56) AGE,AGEMIN
```

```
   56  FORMAT('0THE UMAT AGE IS TOO SMALL. AGE,AGEMIN=',1P2E11.3,
     $  ' DAYS')
       STOP 'UMAT AGE TOO SMALL'
      ENDIF
C
      WRITE(NOUT,57) CMAT,NPROPS,ECONC,XVC,CRUSH,EPFRAC,ALFAC,
     $ TREF,AGE,SHRINK,CREEP,EPSHRK,TIMREF,IPRUM,IHANOP
   57 FORMAT('0UMAT PROPERTIES:'/'  MATERL = ',A/'  NPROPS = ',
     $ I5/'  ECONC = ',1PE11.3,' (PSI)'/'  XVC = ',0PF7.3/'  CRUSH = ',
     $ 1PE11.3,' (PSI)'/'  EPFRAC = ',1PE11.3,' (IN/IN)'/'  ALFAC = ',
     $ 1PE11.3,' (IN/IN/DEG)'/'  TREF = ',1PE11.3,' (DEG)'/'  AGE = ',
     $ 1PE11.3,' (DAYS)'/'  SHRINK FACTOR = ',1PE11.3/
     $ '  CREEP FACTOR = ',1PE11.3/'  INITIAL SHRINKAGE = ',1PE11.3,
     $ ' (IN/IN)'/'  TIMREF = ',1PE11.3,' (DAYS)'/'  IPRUM = ',I5/
     $ '  IHANOP = ',I5)
C
C
C
      IF (BAD) THEN
       WRITE(NOUT,70)
   70  FORMAT('0BAD CONCRETE MATERIAL PROPERTIES IN UMAT')
       STOP 'BAD CONCRETE MATERIAL PROPERTIES IN UMAT'
      ENDIF
C
      FIRST(MATERL)=.FALSE.
      JPRINT=0
C
      IAE=7
      MPROPS=7 + IAE
C
C
C             PRINTING CONTROL
C
      IF (NPROPS.GE.MPROPS+1) THEN
       IPR=NEAR(PROPS(MPROPS))
       IF (IPR.EQ.999) THEN
        MPROPS=MPROPS+1
        JPRINT=NEAR(PROPS(MPROPS))
        IF (JPRINT.LE.0.OR.NPROPS.LT.(MPROPS+JPRINT).AND.FIRST(MATERL))
     $  THEN
         WRITE(NOUT,80) NPROPS,JPRINT,MPROPS
   80    FORMAT('0BAD VALUES FOR PRINT CONTROL IN UMAT.  ',
     $     'NPROPS,JPRINT,MPROPS=',3I5)
         STOP 'BAD VALUE OF PRINT CONTROL IN UMAT'
        ELSE
         MPRINT=MIN(JPRINT,100)
         DO 90 I=1,MPRINT
         MPROPS=MPROPS+1
   90    NPRINT(I)=NEAR(PROPS(MPROPS))
         WRITE(NOUT,112) MPRINT,(NPRINT(I),I=1,MPRINT)
  112    FORMAT('0UMAT INFORMATION PRINTED FOR THE FOLLOWING',
     $     ' ELEMENTS.  MPRINT = ',I5/(5X,10I8))
```

```
        ENDIF
       ENDIF
      ENDIF
      IF (MPROPS.LT.NPROPS.AND.FIRST(MATERL)) WRITE(NOUT,122)
     $ (PROPS(I),I=MPROPS+1,NPROPS)
  122 FORMAT('OUNRECOGNIZED USER PROPERTIES IN UMAT.'/
     $ (1P10E11.3))
C
  123 IPRINT=.FALSE.
      IF (MPRINT.GT.0.AND.INT.EQ.1) THEN
       DO 130 I=1,MPRINT
       IF (JELNO.NE.NPRINT(I)) GO TO 130
       IPRINT=.TRUE.
       GO TO 135
  130  CONTINUE
  135  CONTINUE
      ENDIF
C
C  CALL TO AGING CREEP & CRACKING MODEL
C
      DO 142 I=1,IAR
      N=IARSV(I)
      IF (N.GT.0) THEN
       AR(I)=STATEV(N)
      ELSE
       AR(I)=ZERO
      ENDIF
  142 CONTINUE
C
      IBUG=0
      IF (IPRINT) THEN
       IBUG=1
       WRITE(NOUT,146) JELNO,JELNO1,NPASS,INCRMT,KSTIF,NSTPAB,INCRAB,
     $  INT,NT,NDI
  146  FORMAT('OBEFORE STRN3D CALL. JELNO,JELNO1,NPASS,INCRMT,',
     $  'KSTIF,NSTPAB,INCRAB,INT,NT,NDI'/10I5)
      ENDIF
C
      DTIM(1)=TYME-TIMREF
      EPSMACH=1.E-14
      IF(DABS(DTIM(1)).LT.EPSMACH)NT=1
      IF (DTIM(1).LT.ZERO) THEN
       DO 120 I=1,NTENS
       STRESS(I)=ZERO
       DO 110 J=1,NTENS
  110  HH(I,J)=ZERO
  120  HH(I,I)=ECONC
       RETURN
      ENDIF
C
      DTIM(2)=DTIM(1)+DELTM
C
```

```
      DO 721 I=1,6
      AR(I)=ZERO
      AR(I+6)=ZERO
      AR(I+18)=ZERO
      EP(I)=ZERO
721   JORDER(I)=0
      N=0
      DO 722 I=1,NDI
      N=N+1
722   JORDER(N)=IORDER(I)
      DO 723 I=1,NSHR
      N=N+1
723   JORDER(N)=IORDER(I+3)
      NTEN=6
      DO 126 I=1,NTEN
      J=JORDER(I)
      IF (J.NE.0) THEN
       AR(J)=STRESS(I)
       AR(J+6)=STRESS(I)
       AR(J+18)=TEPS(I)
       EP(J)=DEP(I)
      ENDIF
126   CONTINUE
C
      MAXITR=NUMITR
      IF (KSTIF.EQ.1) MAXITR=1
      DO 127 NITER=1,MAXITR
      KITER=1
      IF (NITER.EQ.MAXITR) KITER=2
      CALL STRN3D(AR,ALFAC,CRUSH,TREF,ECONC,EP,NT,XVC,PH,EPFRAC,TEMP,
     $ DTEMP,DTIM,AGE,SHRINK,CREEP,EPSHRK,KITER,NDI,IPRUM,IHANOP,KSTIF)
127   CONTINUE
C
      DO 175 I=1,NTENS
      II=JORDER(I)
      STRESS(I)=AR(II+6)
      DO 175 J=1,NTENS
      JJ=JORDER(J)
175   HH(I,J)=PH(II,JJ)
C
      DO 180 I=1,IAR
      N=IARSV(I)
      IF (N.GT.0) STATEV(N)=AR(I)
180   CONTINUE
C
      KRAK=NEAR(AR(26))
      IF (KRAK.GT.0) THEN
       NCRACK=1
       KMOD=10
       KDIV=1
       DO 190 I=1,3
       K=MOD(KRAK,KMOD)/KDIV
       KMOD=KMOD*10
       KDIV=KDIV*10
       IF (K.EQ.2) THEN
        KRK2(I)=KRK2(I)+1
       ELSEIF (K.EQ.1) THEN
        KRK1(I)=KRK1(I)+1
       ENDIF
190    CONTINUE
      ENDIF
C
C CRACKING REPORT
C
      IF(AR(26).NE.0.)WRITE(6,888)DTIM(2),AR(26),COORDS(1),COORDS(2),
     & (AR(I),I=37,42)
888   FORMAT('T,CRK.FLG,X,Y,DIR.COS:',F7.2,9E11.4)
C
      RETURN
      END

      FUNCTION NEAR(X)
      REAL*8 X
      NEAR=NINT(X)
      RETURN
      END

      SUBROUTINE FLOATN
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON /FLTNUM/ ZERO,ONE,TWO,THREE,FOUR,FIVE,SIX,SEVEN,EIGHT,
     $  NINE,TEN,HALF,THIRD,FOURTH,FIFTH,SIXTH,SEVNTH,EIGHTH,NINETH,
     $  TENTH,HUNDRD,THOU,MILLON,PI,PIFAC,PIFAC1,EXPN
      REAL*8 NINE,NINETH,MILLON
C
      ZERO=0.0D0
      ONE=1.0D0
      TWO=ONE+ONE
      THREE=TWO+ONE
      FOUR=TWO*TWO
      FIVE=FOUR+ONE
      SIX=FIVE+ONE
      SEVEN=SIX+ONE
      EIGHT=FOUR*TWO
      NINE=THREE*THREE
      TEN=FIVE*TWO
      HUNDRD=TEN*TEN
      THOU=HUNDRD*TEN
      MILLON=THOU*THOU
      HALF=ONE/TWO
      THIRD=ONE/THREE
      FOURTH=ONE/FOUR
      FIFTH=ONE/FIVE
      SIXTH=ONE/SIX
      SEVNTH=ONE/SEVEN
      EIGHTH=ONE/EIGHT
      NINETH=ONE/NINE
      TENTH=ONE/TEN
      PIFAC=ATAN(ONE)
      PI=PIFAC*FOUR
      PIFAC=PIFAC/(NINE*FIVE)
      PIFAC1=ONE/PIFAC
      EXPN=EXP(ONE)
      RETURN
      END

      SUBROUTINE SYMINV(H,NDIM,NN)
C
      IMPLICIT REAL*8 (A-H,O-Z)
C
      COMMON /FLTNUM/ ZERO,ONE,TWO,THREE,FOUR,FIVE,SIX,SEVEN,EIGHT,
     $  NINE,TEN,HALF,THIRD,FOURTH,FIFTH,SIXTH,SEVNTH,EIGHTH,NINETH,
     $  TENTH,HUNDRD,THOU,MILLON,PI,PIFAC,PIFAC1,EXPN
      REAL*8 NINE,NINETH,MILLON
C
      DIMENSION H(NDIM,NDIM)
C
      IF (NN.LT.3.OR.NN.GT.6) STOP 'BAD SIZE TO SYMINV'
C
      H11=H(1,1)
      H22=H(2,2)
      H33=H(3,3)
      H12=H(1,2)
      H13=H(1,3)
      H23=H(2,3)
C
      DET=H11*(H22*H33-H23*H23)+H12*(H23*H13-H12*H33)+
     $ H13*(H12*H23-H22*H13)
C
      IF (DET.LE.ZERO) THEN
       WRITE(6,10) DET,((H(I,J),J=1,3),I=1,3)
10     FORMAT(' BAD DET IN SYMINV = ',1PE11.3/(1P3E11.3))
       STOP 'BAD DET IN SYMINV'
      ENDIF
C
      H(2,1)=-(H12*H33-H23*H13)/DET
      H(3,1)=(H12*H23-H22*H13)/DET
      H(3,2)=-(H11*H23-H12*H13)/DET
      H(1,2)=H(2,1)
      H(1,3)=H(3,1)
      H(2,3)=H(3,2)
C
      H(1,1)=(H22*H33-H23*H23)/DET
      H(2,2)=(H11*H33-H13*H13)/DET
      H(3,3)=(H11*H22-H12*H12)/DET
C
      IF (NN.GE.4) H(4,4)=ONE/H(4,4)
      IF (NN.GE.5) H(5,5)=ONE/H(5,5)
      IF (NN.GE.6) H(6,6)=ONE/H(6,6)
C
      RETURN
      END

      SUBROUTINE INVERT(AA,NDIM,NN)
C
C          GENERAL MATRIX INVERSION SUBROUTINE
C
      IMPLICIT REAL*8 (A-H,O-Z)
C
      COMMON /FLTNUM/ ZERO,ONE,TWO,THREE,FOUR,FIVE,SIX,SEVEN,EIGHT,
     $  NINE,TEN,HALF,THIRD,FOURTH,FIFTH,SIXTH,SEVNTH,EIGHTH,NINETH,
     $  TENTH,HUNDRD,THOU,MILLON,PI,PIFAC,PIFAC1,EXPN
      REAL*8 NINE,NINETH,MILLON
C
      DIMENSION AA(1),A(36),M(6),C(6)
C
      IF (NN.LE.0.OR.NN.GT.6) STOP 'BAD CALL TO INVERT'
C
      N=0
      DO 10 J=1,NN
      L=(J-1)*NDIM
      DO 10 I=1,NN
      N=N+1
      L=L+1
   10 A(N)=AA(L)
C
      DO 90 I=1,NN
   90 M(I)=-I
      DO 140 I=1,NN
C     LOCATE LARGEST ELEMENT
      D=0.0
      DO 112 L=1,NN
      IF (M(L)) 100,100,112
  100 J=L
      DO 110 K=1,NN
      IF (M(K)) 103,103,108
  103 IF (ABS(D) - ABS(A(J))) 105,105,108
  105 LD=L
      KD=K
      D=A(J)
  108 J=J+NN
  110 CONTINUE
  112 CONTINUE
C     INTERCHANGE ROWS
      TEMP=-M(LD)
      M(LD)=M(KD)
      M(KD)=TEMP
      L=LD
      K=KD
      DO 114 J=1,NN
      C(J)=A(L)
      A(L)=A(K)
      A(K)=C(J)
      L=L+NN
  114 K=K+NN
C     DIVIDE COLUNM BY LARGEST ELEMENT
      NR=(KD-1)*NN+1
      NH=NR+NN-1
      DO 115 K=NR,NH
  115 A(K)=A(K)/D
C     REDUCE REMAINING ROWS AND COLUMNS
      L=1
      DO 135 J=1,NN
      IF (J-KD) 130,125,130
  125 L=L+NN
      GO TO 135
  130 DO 134 K=NR,NH
      A(L)=A(L)-C(J)*A(K)
  134 L=L+1
  135 CONTINUE
C     REDUCE ROW
      C(KD)=-1.0
      J=KD
      DO 140 K=1,NN
      A(J)=-C(K)/D
  140 J=J+NN
C     INTERCHANGE COLUMNS
      DO 200 I=1,NN
      L=0
  150 L=L+1
      IF(M(L)-I) 150,160,150
  160 K=(L-1)*NN+1
      J=(I-1)*NN+1
      M(L)=M(I)
      M(I)=I
      DO 200 L=1,NN
      TEMP=A(K)
      A(K)=A(J)
      A(J)=TEMP
      J=J+1
  200 K=K+1
C
      N=0
      DO 210 J=1,NN
      L=(J-1)*NDIM
      DO 210 I=1,NN
      N=N+1
      L=L+1
  210 AA(L)=A(N)
C
      RETURN
      END

      SUBROUTINE MATCON(TEMP2,EE,YIELD,FACTR,EPSEFF,CON,ESEC,EPFRAC,
     $ AGEFAC,CRUSH,ECONC,IPROP,IHANOP,DTIM,ULTTEN,BAND,TEMP1,INCRMT,
     $ IHARD)
      IMPLICIT REAL*8 (A-H,O-Z)
C
      COMMON /RSDDBG/ IBUG
C
      COMMON /FLTNUM/ ZERO,ONE,TWO,THREE,FOUR,FIVE,SIX,SEVEN,EIGHT,
     $  NINE,TEN,HALF,THIRD,FOURTH,FIFTH,SIXTH,SEVNTH,EIGHTH,NINETH,
     $  TENTH,HUNDRD,THOU,MILLON,PI,PIFAC,PIFAC1,EXPN
      REAL*8 NINE,NINETH,MILLON
C
      DIMENSION DTIM(2),A(4,4),X(4),DX(4),XPR(4),BAND(6)
C
C     DATA FOR ELASTIC MODULUS CURVE FIT FOR NEW HANFORD CONCRETE
C
      DATA ((A(I,J),J=I,4),I=1,4),A1,A2,A3,A4,S/
     1 3.0226E-1,-3.5888E-2,-3.1970E-4,-1.0128E-2,1.0323E-2,
     2 -4.6499E-6,-2.1691E-4,8.9565E-7,3.2632E-6,2.0813E-3,
     3 5.3947,1.233E-1,-6.751E-3,-1.786E-1,3.5/
C
      IF (IPROP.EQ.4) GO TO 100
C
C        GENERAL, WES & OLD HANFORD CONCRETE
C
      IF (ECONC.GT.ONE) THEN
C
C          USER DEFINED CONCRETE PROPERTIES
C
       EE=ECONC*AGEFAC
       IF (CRUSH.LE.ZERO) THEN
        WRITE(6,10)
10      FORMAT('0CRUSH .LE.0 IN MATCON.  EXECUTION TERMINATED.')
        STOP 'CRUSH .LE. 0 IN MATCON'
       ENDIF
       SIGULT=CRUSH*AGEFAC
       IF (EPFRAC.LE.ZERO) THEN
        EPFRAC=SIGULT/EE/TEN
       ENDIF
       ETA0=0.1
       FACTR=-0.02
       IHARD=1
      ELSEIF (ECONC.EQ.ZERO) THEN
C
C     STANDARD CONCRETE PROPERTIES:
C                    E=3.35E6
C                    ULTIMATE(CRUSH)=4650.
C                    YIELD=0.5*CRUSH=2325.
C                    FRACTURE STRAIN(EPFRAC)=138.8 MICRONS
C                    FACTR=-0.02
```

```
C                     EEFF1=2240. MICRONS
C
       IHARD=1
       EE=3.350E6*AGEFAC
       SIGULT=4650.*AGEFAC
       EPFRAC=138.8E-6
       EEFF1=2240.E-6
       E1=SIGULT/EEFF1
       EODE1=EE/E1
       ETA0=TWO-EODE1
       ETA0=MIN(ETA0,ONE)
       ETAMIN=0.05
       ETA0=MAX(ETA0,ETAMIN)
       FACTR=-0.02
      ELSEIF (ECONC.LT.-ONE) THEN
C
C          USER DEFINED ELASTIC-PLASTIC PROPERTIES WITH FRACTURE
C
       IHARD=0
       EE=-ECONC*AGEFAC
       ESEC=EE
       CON=ESEC*EE
       IF (CRUSH.LE.ZERO) THEN
        WRITE(6,10)
        STOP 'CRUSH .LE. 0 IN MATCON'
       ENDIF
       IF (EPFRAC.LT.ZERO) THEN
        WRITE(6,15)
15      FORMAT('0BAD EPFRAC IN MATCON.  EXECUTION TERMINATED.')
        STOP 'EPFRAC .LT. 0 IN MATCON'
       ENDIF
       YIELD=CRUSH*AGEFAC
       FACTR=-0.02
       IF (IBUG.EQ.1) WRITE (6,98) IHARD,ECONC,CRUSH,EPFRAC,AGEFAC,
     $  EPSEFF,EE,EEFF1,FACTR,ETA0,YIELD,SPLUS,ESEC
C
       RETURN
      ELSE
       WRITE(6,20)
20     FORMAT('0BAD ECONC IN MATCON.  EXECUTION TERMINATED.')
       STOP 'BAD ECONC IN MATCON'
      ENDIF
C
      EPSULT=SIGULT/EE
      EEFF1=(TWO-ETA0)*EPSULT
      PSI=EPSEFF/EEFF1
      IF (PSI.LE.ETA0) THEN
       YIELD=ETA0*EEFF1*EE
       ESEC=EE
       CON=EE*EE
      ELSE
       SPLUS=EE/(ONE-ETA0*PSI+PSI**2)
```

```
        EPS=(SPLUS)**2
        YIELD=SPLUS*EPSEFF
        EPN=(YIELD/EEFF1)**2
        ETAN=(EPS-EPN)/EE
        CON=ETAN*EE
        CON=CON/(EE-ETAN)
        ETOL=EE/THOU
        IF (ABS(CON).LT.ETOL) CON=CON*ETOL
        IF (CON.LT.ETOL) CON=ETOL
        ESEC=YIELD/EPSEFF
       ENDIF
C
       IF (EPSEFF.GT.EEFF1) EE=EE*((YIELD/SIGULT)**2)
       ESEC=(TWO*ESEC+EE)/THREE
C
       IF (IBUG.EQ.1) WRITE (6,98) IHARD,ECONC,CRUSH,EPFRAC,AGEFAC,
      $ EPSEFF,EE,EEFF1,FACTR,ETA0,YIELD,SPLUS,ESEC
98     FORMAT (' IN MATCON: IHARD,ECONC,CRUSH,EPFRAC,AGEFAC,EPSEFF,',
      $ 'EE,EEFF1,FACTR,ETA0,YIELD,SPLUS,ESEC=',I3/(1P10E11.3))
C
       RETURN
C
C         NEW HANFORD CONCRETE - MAY 1987
C
100    IHARD=0
       DTEMP=TEMP2-TEMP1
       IF (IBUG.NE.0) WRITE(6,101) EE,BANMOD,TEMP2,DTEMP,DTIM
101    FORMAT(' IN MATCON BEFORE 1ST CALL TO CRVFIT: EE,BANMOD,TEMP2,',
      $ 'DTEMP,DTIM'/(1P10E11.3))
C
       TEMMAX=EPSEFF
       IF (TEMP2.LT.TEMMAX) GO TO 170
       DTEMP=MIN(DTEMP,TEMP2-TEMMAX)
       EE=EE/MILLON
       CALL CRVFIT(EE,BAND(1),BAND(2),TEMP2,ZERO,DTEMP,ZERO,DTIM,INCRMT,
      $ IHANOP,1)
       EE=EE*MILLON
       TA=MAX(ZERO,TEMP2-350.0)
       DTA=TA-MAX(ZERO,TEMP1-350.0)
       DTA=MAX(DTA,ZERO)
       CALL CRVFIT(ULTTEN,BAND(5),BAND(6),TA,ZERO,DTA,ZERO,DTIM,INCRMT,
      $ IHANOP,3)
C
       TEMLOW=250.0
       TEMP3=MAX(TEMP2,TEMLOW)
       TEMP4=MAX(TEMP1,TEMLOW)
       TB=MAX(ZERO,350.0-TEMP3)
       TBM1=MAX(ZERO,350.0-TEMP4)
       DTB=TB-TBM1
       DTB=MAX(DTB,ZERO)
       CALL CRVFIT(YIELD,BAND(3),BAND(4),TA,TB,DTA,DTB,DTIM,INCRMT,
      $ IHANOP,2)
```

```
C
      TEMMAX=TEMP2
170   EPFRAC=ULTTEN/ECONC
      ESEC=EE
      CON=EE*EE
C
      IF (IBUG.NE.0) WRITE (6,198) EE,YIELD,ULTTEN,EPFRAC,ESEC,ECONC
198   FORMAT (' MATCON(IPROP=4): EE,YIELD,ULTTEN,EPFRAC,ESEC,ECONC'/
     $ (1P10E11.3))
C
      RETURN
      END

      SUBROUTINE USHRNK(TIME,SHRINK)
C
      IMPLICIT REAL*8 (A-H,O-Z)
C
      COMMON /FLTNUM/ ZERO,ONE,TWO,THREE,FOUR,FIVE,SIX,SEVEN,EIGHT,
     $  NINE,TEN,HALF,THIRD,FOURTH,FIFTH,SIXTH,SEVNTH,EIGHTH,NINETH,
     $  TENTH,HUNDRD,THOU,MILLON,PI,PIFAC,PIFAC1,EXPN
      REAL*8 NINE,NINETH,MILLON
C
      COMMON /RSDDBG/ IBUG
C
      SHRINK=204.91E-6*(ONE-EXP(-0.15*TIME))+
     $ 145.09E-6*(ONE-EXP(-0.0226348*TIME))
      SHRINK=SHRINK*.32
      RETURN
      END

      SUBROUTINE CRPROP(GE,DTIM,IN,CREEP,TEMP,IPROP)
C
      IMPLICIT REAL*8 (A-H,O-Z)
C
      COMMON /RSDDBG/ IBUG
C
      COMMON /FLTNUM/ ZERO,ONE,TWO,THREE,FOUR,FIVE,SIX,SEVEN,EIGHT,
     $  NINE,TEN,HALF,THIRD,FOURTH,FIFTH,SIXTH,SEVNTH,EIGHTH,NINETH,
     $  TENTH,HUNDRD,THOU,MILLON,PI,PIFAC,PIFAC1,EXPN
      REAL*8 NINE,NINETH,MILLON
C
      DIMENSION DTIM(2),GE(4)
C
      DTM=DTIM(1)
      DTP=DTIM(2)
      DELTM=DTP-DTM
      XK=DTM
      TS1=ZERO
      TS2=ZERO
      TIM=DTP-XK
      XKN=XK+DELTM*HALF
      IF(IPROP.NE.4) THEN
```

```
       CALL UAGE(XKN,TEMP,AFACT,TEMFAC,IPROP)
       CON=CREEP
       CALL SHIFT1(TEMP,R,PRIME,SECOND,CON,XK,IN,IPROP)
C
C         THE FOLLOWING TWO LINES ARE TO BE USED WITH THE
C       STANDARD FIT
C
       IF(AFACT.GT.1.)THEN
        A=PRIME/(AFACT**2)
        D=SECOND/(AFACT**2)
       ELSE
C
C       THE FOLLOWING TWO LINES ARE TO BE USED WITH
C       LOW AGING FACTOR FOR EARLY TIME
C
       A=PRIME/AFACT
       D=SECOND/AFACT
      END IF
C
      ELSE
       CON=ONE
       CALL SHIFT1(TEMP,R,A,D,CON,XK,IN,IPROP)
      ENDIF
      TIM=MIN(DELTM,0.63/R)
      TS=TIM
      CONST=R*TS
      IF (CONST.GT.85.0) CONST=85.0
      GE4=EXP(-CONST)
      GE(4)=GE4
      TS1=A*(ONE-GE4)
      TS2=D*DELTM
      GE(1)=3.0*TS1
      GE(2)=ZERO
      IF (IN.EQ.1.AND.IPROP.NE.4) GE(2)=3.0*TS2
      GE(3)=DELTM
      RETURN
      END

      SUBROUTINE SHIFT1(TEMP,R,A,D,CON,XK,IN,IPROP)
C
      IMPLICIT REAL*8 (A-H,O-Z)
C
      COMMON /FLTNUM/ ZERO,ONE,TWO,THREE,FOUR,FIVE,SIX,SEVEN,EIGHT,
     $  NINE,TEN,HALF,THIRD,FOURTH,FIFTH,SIXTH,SEVNTH,EIGHTH,NINETH,
     $  TENTH,HUNDRD,THOU,MILLON,PI,PIFAC,PIFAC1,EXPN
      REAL*8 NINE,NINETH,MILLON
C
C       IPROP=1  CREEP DATA FOR GENERAL CONCRETE
C       IPROP=2  WES CREEP DATA FOR YOUNG CONCRETE
C       IPROP=3  CREEP DATA FOR OLD HANFORD CONCRETE - 1979
C       IPROP=4  CREEP DATA FOR NEW HANFORD CONCRETE - 1987
C
```

```
      T=TEMP
C
      IF (TEMP.LT.60.0) TEMP=60.0
C
      IF (IPROP.NE.1) GO TO 200
C
C       GENERAL CONCRETE
C
      AP0=0.10425
      AP1=-0.02775
      AP2=0.0062667
      CON=ONE
      FF=(AP0+AP1*LOG10(XK))*0.625
      F=FF*EXP(AP2*T)
      F=F*CON*1.E-6
      R=0.07
      DT=T-200.0
      IF (DT.LT.ZERO) DT=ZERO
      D1=0.00075+DT*1.0E-5
      D2=0.0015
      D=MIN(D1,D2)*F
      IF (IN.EQ.1) THEN
       A=HALF*F
       R=0.6
       D=ZERO
      ELSEIF (IN.EQ.2) THEN
       A=1.5*F
       R=0.07
       D=ZERO
      ELSEIF (IN.EQ.3) THEN
       A=(0.056*DT-DT*DT*1.511E-4)*F
       R=0.0046
      ENDIF
C
      RETURN
C
200   IF (IPROP.NE.2) GO TO 300
C
C        WES CONCRETE
C
      TK=(T-32.0)*FIVE/NINE+273.0
      RTK=(70.0-32.0)*FIVE/NINE+273.0
      EQRT=EXP(-4345.0/(1.98*TK))/EXP(-4345.0/(1.98*RTK))
      CONN=EQRT*1.E-6
C
C MODIFIED FOR RED RIVER MIX A11 (NO LINEAR TERM) 1/25/89
C
C     D=5.2477E-4*CON*CONN
      IF (IN.EQ.1) THEN
       A=CONN*CON*.13887
       R=1.7661058
       D=ZERO
      ELSEIF (IN.EQ.2) THEN
       A=CONN*CON*.15890
       R=.18922563
       D=ZERO
      ELSEIF (IN.EQ.3) THEN
       A=CONN*CON*.10576
       R=.05887019
       D=ZERO
      ENDIF
      RETURN
C
C        MATERIAL DATA FOR OLD HANFORD CONCRETE - 1979
C
300   IF (IPROP.NE.3) GO TO 400
      TK=(T-32.0)*FIVE/NINE + 273.0
      EQRT=EXP(-4345.0/(1.98*TK))
      A=1.11E-4*EQRT
      D=3.8265E-7*EQRT
      R=0.23
      ET=A*SIG
      EM=D*SIG
      RETURN
C
C        MATERIAL DATA FOR NEW HANFORD CONCRETE - 1987
C
  400 XT=226.09 - 0.00429*T + 147.52*T**(-0.367) - 309.26*T**(-0.044)
      XT=XT*1.0E-6
      D=ZERO
      IF(IN.EQ.1) THEN
       A=0.000934*XT
       R=6.9
      ELSEIF(IN.EQ.2) THEN
       A=0.097*XT
       R=0.69
      ELSEIF(IN.EQ.3) THEN
       A=0.0957*XT
       R=0.069
      ELSEIF(IN.EQ.4) THEN
       A=0.28*XT
       R=0.0069
      ELSEIF(IN.EQ.5) THEN
       A=0.375*XT
       R=0.00069
      ELSEIF(IN.EQ.6) THEN
       A=0.348*XT
       R=0.000069
      ENDIF
      RETURN
      END

      SUBROUTINE COEF(TIME,TEMPT,AGEFAC,TEMFAC,IPROP)
C
```

```
C        CALCULATION OF AGE AND TEMPERATURE DEPENDENT COEFFICIENTS
C        OF THE CREEP FORMULA
C
      IMPLICIT REAL*8 (A-H,O-Z)
C
      COMMON /FLTNUM/ ZERO,ONE,TWO,THREE,FOUR,FIVE,SIX,SEVEN,EIGHT,
     $  NINE,TEN,HALF,THIRD,FOURTH,FIFTH,SIXTH,SEVNTH,EIGHTH,NINETH,
     $  TENTH,HUNDRD,THOU,MILLON,PI,PIFAC,PIFAC1,EXPN
      REAL*8 NINE,NINETH,MILLON
C
      IF (IPROP.EQ.4) RETURN
      XKN=ONE/TIME
C
C        WES CONCRETE
C
      IF (IPROP.NE.2) GO TO 200
      TEMP=TEMPT
      IF (TEMP.LT.60.0) TEMP=60.0
      T=(TEMP-32.)*FIVE/NINE
      XK=ONE/28.0
      FF1=ZERO
      DO 10 I=1,2
      F1=FF1
       A0=.201881+T*(-1.98117E-4+T*(5.31521E-5-T*4.23376E-7))
       A1=-5.41464+T*(.40337+T*(-5.36256E-3+T*2.33622E-5))
       A2=69.245+T*(-3.38663+T*(-6.42465E-3+2.34411E-4*T))
       A3=-583.108+T*(19.8205+T*(.683703-.67766E-2*T))
       A4=3356.7+T*(-83.0216+T*(-5.91922+5.39942E-2*T))
       A5=-7664.4+T*(.132439E3+T*(17.0764-.150322*T))
      FF1=A0+XK*(A1+XK*(A2+XK*(A3+XK*(A4+XK*A5))))
      T=(70.0-32.0)*FIVE/NINE
   10 CONTINUE
      F1=F1*1.E-6
      FF1=FF1*1.E-6
      ET28=ONE/F1
      ERT28=ONE/FF1
      TSHAPE=ET28/ERT28
      TM=ONE/XKN-ONE
      IF(TM.LT.0.)TM=0.
      ETATAU=1.E6*(3.58802*(ONE-EXP(-0.03250502*TM))+1.29715*
     . (ONE-EXP(-0.40756288*TM))+.332774*(ONE-EXP(-2.649*TM))+
     .  .6)*TSHAPE
      IF(TM.EQ.0.)THEN
        ETATAU=ONE/XKN*ETATAU
      END IF
      TM=TWO
      ETA3=1.E6*(3.58802*(ONE-EXP(-0.03250502*TM))+1.29715*
     . (ONE-EXP(-0.40756288*TM))+.332774*(ONE-EXP(-2.649*TM))+
     .  0.6)*TSHAPE
      AGEFAC=ETATAU/ETA3
      TEMFAC=TSHAPE
      RETURN
```

```
C
C        REGULAR & OLD HANFORD CONCRETE
C
200   ALFA=0.926+4.444*XKN
      ESTAR=ET28/ALFA
      AGEFAC=ESTAR/ERT28
      F1=F1*ALFA
C
      T=(TEMP-32.)*FIVE/NINE
      BETA=0.56+12.245*XKN
      XK= ONE/28.0
      TIME=ONE/XK
C
   15  AP0=-.076945+T*(7.70542E-3+T*(-8.38733E-5+T*3.82484E-7))
       AP1= 14.0236+T*(-.713801+T*( 1.12008E-2-5.71309E-5*T))
       AP2=-422.681+T*( 21.8111+T*(-.347742+1.78503E-3*T))
       AP3= 6016.54+T*(-304.577+T*( 4.84830-2.48844E-2*T))
       AP4=-39844.4+T*( 1986.93+T*(-31.4952+.161156*T))
       AP5= 98581.7+T*(-4865.40+T*( 76.8028-.391529*T))
      F2= 1.E-6*(AP0+XK*(AP1+XK*(AP2+XK*(AP3+XK*(AP4+XK*AP5)))))
      IF (F2.GT.ZERO) GO TO 20
      TIME=TIME+HALF
      XK=ONE/TIME
      GO TO 15
   20 CONTINUE
      F2=F2*BETA
      RETURN
      END

      SUBROUTINE UAGE(TIME,TEMPT,AGEFAC,TEMFAC,IPROP)
C
C         CALCULATION OF AGING & TEMPERATURE FACTORS
C
      IMPLICIT REAL*8 (A-H,O-Z)
C
      COMMON /FLTNUM/ ZERO,ONE,TWO,THREE,FOUR,FIVE,SIX,SEVEN,EIGHT,
     $  NINE,TEN,HALF,THIRD,FOURTH,FIFTH,SIXTH,SEVNTH,EIGHTH,NINETH,
     $  TENTH,HUNDRD,THOU,MILLON,PI,PIFAC,PIFAC1,EXPN
      REAL*8 NINE,NINETH,MILLON
C
C     S1(X,A)=ONE-THREE*(X/A)**2+TWO*(X/A)**3
C     S2(X,A)=X-TWO*X*(X/A)+X*(X/A)**2
C     S3(X,A)=THREE*(X/A)**2-TWO*(X/A)**3
C     S4(X,A)=-X*(X/A)+X*(X/A)**2
C
      IF (IPROP.EQ.4) RETURN
C
C     LINEAR VARIATION OF AGEFAC FROM 0.033 AT .25 DAYS TO
C     STANDARD VALUE AT 1 DAY (0.8)
C
C       IF (TIME.LT.ONE) THEN
C        CALL COEF(ONE,TEMPT,AFAC1,TEMFAC,IPROP)
```

```
C       AFAC0=HUNDRD/(THREE*THOU)
C       TIME0=0.25
C       AGEFAC=AFAC0+(AFAC1-AFAC0)*(TIME-TIME0)/(ONE-TIME0)
C       IF (TIME.LE.TIME0) AGEFAC=AFAC0
C       CALL COEF(TIME,TEMPT,AGEDUM,TEMFAC,IPROP)
C      ELSE
        CALL COEF(TIME,TEMPT,AGEFAC,TEMFAC,IPROP)
C      ENDIF
C
C     CUBIC FIT OF AGEFAC FROM 0.1 AT 0.2 DAYS TO 1.0 AT 3 DAYS
C
C     IF (TIME.LT.THREE) THEN
C      TYME=TIME-0.2
C      XLEN=THREE-0.2
C      AGEFAC=0.1*S1(TYME,XLEN)+0.8*S2(TYME,XLEN)+S3(TYME,XLEN)+
C    $  0.03*S4(TYME,XLEN)
C      CALL COEF(TIME,TEMPT,AGEDUM,TEMFAC,IPROP)
C     ELSE
C      CALL COEF(TIME,TEMPT,AGEFAC,TEMFAC,IPROP)
C     ENDIF
      RETURN
      END

      SUBROUTINE CRVFIT(PBAND,P,TSTRIN,TA,TBIN,DTA,DTBIN,DTIM,
     $ INCRMT,IFIT,ICRV)
      RETURN
      END

      SUBROUTINE STRN3D(AR,ALFAC,CRUSH,TREF,ECONC,DEP,INCRMT,XVC,PH,
     $ EPFR,TEMP1,DTEMP,DTIME,AGE,SHRINK,CREEP,EPSHRK,ITER,NDI,IPROP,
     $ IHANOP,KSTIF)
C
      IMPLICIT REAL*8 (A-H,O-Z)
C
      COMMON /FLTNUM/ ZERO,ONE,TWO,THREE,FOUR,FIVE,SIX,SEVEN,EIGHT,
     $  NINE,TEN,HALF,THIRD,FOURTH,FIFTH,SIXTH,SEVNTH,EIGHTH,NINETH,
     $  TENTH,HUNDRD,THOU,MILLON,PI,PIFAC,PIFAC1,EXPN
      REAL*8 NINE,NINETH,MILLON
C
      COMMON /RSDDBG/ IBUG
      COMMON /RSDINF/ NOUT,JELNO,INT,NSTPAB,INCRAB,NPASS
C
      DIMENSION EPP(6),DTIM(2),AR(1),PH(6,6),GE(4,6),EP(6),DTIME(2),
     $ STRESS(6),DEP(6),BAND(6)
C
      LOGICAL ABAQUS
C
      DATA ABAQUS/.TRUE./
C
C     ECONC    = ELASTIC MODULUS
C     XVC      = POISSON'S RATIO
C     CRUSH    = COMPRESSIVE STRENGTH
```

```
C     ALFAC   = COEFFICIENT OF CONCRETE THERMAL EXPANSION
C     TREF    = REFERENCE (STRESS-FREE) TEMPERATURE
C     EPFRAC  = TENSION FRACTURE STRAIN
C     TEMP1   = TEMPERATURE AT START OF STEP
C     DTEMP   = CHANGE IN TEMPERATURE
C     DTIME(1)= TIME AT START OF STEP
C     DTIME(2)= TIME AT END OF STEP
C     PH      = TANGENT CONSTITUTIVE MATRIX
C     INCRMT  = LOAD STEP NUMBER
C     ITER    = 1  INTERMEDIATE ITERATION
C               2  FINAL ITERATION
C     KSTIF   = 1  FOR IST ITERATION WHERE DEP=0
C             > 1  FOR SUBSEQUENT ITERATIONS
C     ABAQUS  = .TRUE.  MEANS TOTAL STRAINS ARE STORED IN AR
C               .FALSE. MEANS MECHANICAL STRAINS ARE STORED IN AR
C
C     AR      = STATE VARIABLES
C       (1-6)      STRESSES AT START OF STEP
C       (7-12)     STRESSES AT END OF STEP
C       (13-15)    CLOSED CRACK STRAIN (LOCAL)
C       (19-24)    TOTAL STRAINS
C       (25)       PLASTICITY FLAG
C       (26)       CRACKING FLAG
C       (27)       EFFECTIVE STRAIN (MAX TEMP FOR IPROP=4)
C       (28)       MODULUS
C       (29)       YIELD STRESS (ULTIMATE COMPRESSION FOR IPROP=4)
C       (30)       TENSION STRENGTH FOR IPROP=4 ONLY (ULTTEN)
C       (31-36)    BAND HISTORIES FOR IPROP=4 ONLY
C       (37-42)    DIRECTION COSINES OF FIRST TWO CRACKS
C       (43-78)    CREEP HISTORY PARAMETERS
C
C     STRESS & STRAIN ORDERING: 11,22,33,23,31,12
C     SHEAR STRAINS ARE GAMMAS: DU/DY+DV/DX
C
      NCOS=36
      NCREEP=42
      JBAND=6
      NBAND=30
      JCREEP=3
      IF (IPROP.EQ.4) JCREEP=6
      IF (CREEP.LE.1.0E-6) JCREEP=0
      IF (IBUG.NE.0) WRITE(NOUT,13) INCRMT,ITER,NDI,IPROP,IHANOP,JCREEP
13    FORMAT(' START STRN3D: INCRMT,ITER,NDI,IPROP,IHANOP,JCREEP=',6I5)
      NTENS=6
      EPFRAC=EPFR
      TEMP2=TEMP1+DTEMP
      AVGTEM=TEMP1+HALF*DTEMP
      DELTM=DTIME(2)-DTIME(1)
      IF (JCREEP.NE.0.AND.DELTM.LT.1.E-6) DELTM=1.E-6
      DTIM(1)=DTIME(1)+ACE
      DTIM(2)=DTIM(1)+DELTM
      EPSEFF=AR(27)
      AGEFAC=ONE
      DAGE=ONE
C
C          1ST INCREMENT INITIALIZATION
C
      IF (INCRMT.EQ.1) THEN
       DELTM=DTIM(2)
C
C          AGING (1ST INCREMENT)
C
       IF (AGE.LT.999.0) THEN
        CALL UAGE(DTIM(2),TEMP1,AGEFAC,TEMFAC,IPROP)
        AGEFAC=AGEFAC*TEMFAC
       ENDIF
C
C         PRESET OPEN CRACKS FOR NDI .LT. 3
C         ONLY 1ST TWO VECTOR DIRECTIONS SET
C
       N=NCOS
       DO 50 I=1,6
       N=N+1
50     AR(N)=ZERO
       AR(NCOS+1)=ONE
       AR(NCOS+5)=ONE
       KRACK=0
       DO 60 I=NDI+1,3
60     KRACK=KRACK+2*(10**(I-1))
       AR(26)=KRACK
      ELSE
C
C          AGING (GENERAL INCREMENT)
C
       IF (AGE.LT.999.0) THEN
        CALL UAGE(DTIM(2),TEMP2,AGEFAC,TEMFAC,IPROP)
        AGEFAC=AGEFAC*TEMFAC
        CALL UAGE(DTIM(1),TEMP1,AGEM1,TEMFAC,IPROP)
        AGEM1=AGEM1*TEMFAC
        DELAGE=AGEFAC-AGEM1
        DAGE=ONE+DELAGE/AGEM1
       ENDIF
      ENDIF
C
C          CONCRETE PROPERTIES
C
      IF (IPROP.EQ.4) THEN
       EI=AR(28)
       YIELD=AR(29)
       ULTTEN=AR(30)
       N=NBAND
       DO 61 I=1,JBAND
       N=N+1
61     BAND(I)=AR(N)
```

```
      ENDIF
C
      CALL MATCON(TEMP2,EI,YIELD,FACTR,EPSEFF,CON,ESEC,EPFRAC,AGEFAC,
     $ CRUSH,ECONC,IPROP,IHANOP,DTIM,ULTTEN,BAND,TEMP1,INCRMT,IHARD)
      YIELD=YIELD*SQRT(ONE+THREE*FACTR)
      IF (INCRMT.EQ.1) THEN
       AR(28)=EI
       AR(29)=YIELD
      ENDIF
      IF (IPROP.EQ.4.AND.ITER.NE.1) THEN
       AR(27)=EPSEFF
       AR(28)=EI
       AR(29)=YIELD
       AR(30)=ULTTEN
       N=NBAND
       DO 62 I=1,JBAND
       N=N+1
62     AR(N)=BAND(I)
      ENDIF
C
C         INCREMENTAL SHRINKAGE (TOTAL FOR 1ST INCREMENT)
C
      SHRNK1=ZERO
      SHRNK2=ZERO
      IF (AGE.LT.999.0.AND.IPROP.NE.4) THEN
       CALL USHRNK(DTIM(1),SHRNK1)
       CALL USHRNK(DTIM(2),SHRNK2)
      ENDIF
C
      EXPND1=-SHRINK*SHRNK1-EPSHRK+ALFAC*(TEMP1-TREF)
      IF (INCRMT.EQ.1) EXPND1=ZERO
      EXPND2=-SHRINK*SHRNK2-EPSHRK+ALFAC*(TEMP2-TREF)
      ADT=EXPND2-EXPND1
C
C           SET CREEP PARAMETERS
C
      GE(2,1)=ZERO
      DO 130 J=1,JCREEP
      DO 120 I=1,NTENS
120   GE(I,J)=ZERO
      CALL CRPROP(GE(1,J),DTIM,J,CREEP,AVGTEM,IPROP)
130   CONTINUE
C
      IF (IBUG.NE.0) THEN
C       WRITE(NOUT,131) AGE,DTIM(1),DTIM(2),DELTM,TREF,TEMP1,DTEMP,
C     $ TEMFAC,AGEM1,DELAGE.AGEFAC,DAGE,DSHRNK,ADT,EPSHRK,SHRINK,SHRNK1,
C     $ SHRNK2,EXPND1,EXPND2
C131    FORMAT(' STRN3D AFTER 130: AGE,DTIM(1),DTIM(2),DELTM,TREF,TEMP1',
C     $  ',DTEMP,TEMFAC,AGEM1,DELAGE'/' AGEFAC,DAGE,DSHRNK,ADT,',
C     $  'EPSHRK,SHRINK,SHRNK1,SHRNK2,EXPND1,EXPND2'/(1P10E11.3))
       WRITE(NOUT,132) (AR(I),I=1,NCOS)
132    FORMAT(' AR'/(1P10E11.3))
```

```
      ENDIF
C
C           DEP  -  INCRMENT IN STRAIN (DU/DX)
C           EP   -  INCRMENT IN MECHANICAL STRAIN (DU/DX-ADT)
C           EPP  -  OLD TOTAL MECHANICAL STRAIN (DU/DX-ADT) GOING TO CONC3D
C           AR(1-5) & STRESS  -  STRESSES AT START OF INCREMENT
C           AR(7-12)          -  STRESSES AT END OF INCREMENT
C           AR(19-24)         -  TOTAL MECHANICAL STRAINS
C
      IF (.NOT.ABAQUS) EXPND1=ZERO
      SUM=ZERO
      DO 140 I=1,3
      STRESS(I)=AR(I)
      STRESS(I+3)=AR(I+3)
      EP(I)=DEP(I)-ADT
      EP(I+3)=DEP(I+3)
      EPP(I)=AR(I+18)-EXPND1
140   EPP(I+3)=AR(I+21)
      IF (IBUG.NE.0) WRITE(NOUT,141) EP,EPP,STRESS
141   FORMAT(' STRN3D AFTER 140: EP/EPP/STRESS/'/(1P6E11.3))
C
      CALL CONC3D(AR,PH,EPFRAC,EP,EPP,GE,EI,CON,FACTR,XVC,YIELD,
     $ ITER,STRESS.NTENS,JCREEP,NCOS,NCREEP,KSTIF,ECONC,NDI,DEP,
     $ DAGE,TEMP2.AGEFAC,CRUSH,IPROP,IHANOP,DTIM,ULTTEN,BAND,TEMP1,
     $ INCRMT,IHARD,ABAQUS)
C
      RETURN
      END

      SUBROUTINE CONC3D(AR,PH,EPFRAC,EP,EPP,GE,EI,CON,FACTR,XVC,YIELD,
     $ ITER,STRESS,NTENS,JCREEP,NCOS,NCREEP,KSTIF,ECONC,NDI,DEP,
     $ DAGE,TEMP2,AGEFAC,CRUSH,IPROP,IHANOP,DTIM,ULTTEN,BAND,TEMP1,
     $ INCRMT,IHARD,ABAQUS)
C
      IMPLICIT REAL*8 (A-H,O-Z)
C
      COMMON /FLTNUM/ ZERO,ONE,TWO,THREE,FOUR,FIVE,SIX,SEVEN,EIGHT,
     $  NINE,TEN,HALF,THIRD,FOURTH,FIFTH,SIXTH,SEVNTH,EIGHTH,NINETH,
     $  TENTH,HUNDRD,THOU,MILLON,PI,PIFAC,PIFAC1,EXPN
      REAL*8 NINE,NINETH,MILLON
C
      COMMON /RSDDBG/ IBUG
      COMMON /RSDINF/ NOUT,JELNO,INT,NSTPAB,INCRAB,NPASS
C
      DIMENSION ECC(6),EPP(6),BF(6),HH(6,6),Q(6,6),DF(6),SR(6),STN(6),
     $ HK(6,6),AS(6,6).SIJ(6),H2(6,6),HB(6,6),DIJ(6),GE(4,6),AR(1),
     $ PH(6,6),EP(6),STR(6),TF(6),SG(6),H(6,6),KRK(3),PSTRS(6),PSTRN(6),
     $ PSTRNO(6),DPSTRN(6),IPERM(3),A(3,3),STRESS(6),EPSFAC(6),TAU(6),
     $ SIGFAC(6),PSTRSO(6),ECLOSE(3),KOC(3),DEP(6)
C
      LOGICAL PLAST,ABAQUS
C
```

```
      DATA IPERM/2,3,1/, SQ3/1.732/
C
C       NPL(1) THE PLASTICITY FLAGS
C
C         NPL = 0  MEANS STEP IS ELASTIC
C               1  MEANS STEP IS PLASTIC
C               2  MEANS ELASTIC PREDICTOR IS PLASTIC BUT STEP ELASTIC
C               3  MEANS STEP IS ELASTIC BUT CRACKING STRESSES PLASTIC
C
C       KRK = K1 + 10*K2 + 100*K3
C
C         K1,K2,K3  .EQ. 0 MEANS POINT IS UNCRACKED
C                        1 MEANS PREVIOUSLY OPENED CRACK IS CLOSED
C                        2 MEANS CRACK IS OPEN
C
C        RETRIVE TWO DIRECTIONS OF PREVIOUS CRACKING
C
      N=NCOS
      DO 40 J=1,2
      DO 40 I=1,3
      N=N+1
40    A(I,J)=AR(N)
C
      KRKFLG=NEAR(AR(26))
      KRKOLD=MOD(KRKFLG,10000)
C
C        COMPUTE THIRD CRACK DIRECTION
C
50    CALL CROSS(A(1,1),A(1,2),A(1,3),DET,IPERM)
C
C        STORE PREVIOUS STEP CRACK STATUS
C
      MMOD=10
      MDIV=1
      DO 55 I=1,3
      KRK(I)=MOD(KRKOLD,MMOD)/MDIV
      MMOD=MMOD*10
55    MDIV=MDIV*10
      K123=0
      NK=0
      NOPEN=0
      NCLOSE=0
C
C        ESTABLISH STIFFNESS(ECC), STRESS(SIGFAC) & STRAIN(EPSFAC)
C        FACTORS FOR CRACKING FOR NORMAL DIRECTIONS
C
      DO 80 I=1,3
      K=KRK(I)
      K123=MAX(K123,K)
      IF (K.NE.0) NK=NK+1
      IF (K.EQ.2) THEN
       NOPEN=NOPEN+1
```

```
        ECC(I)=1.E-3
        SIGFAC(I)=ZERO
        EPSFAC(I)=ZERO
       ELSE
        IF (K.EQ.1) NCLOSE=NCLOSE+1
        ECC(I)=ONE
        SIGFAC(I)=1.E20
        EPSFAC(I)=ONE
       ENDIF
C
C         CHECK FOR CONSISTENCY OF CRACKING ORDER
C
       IF (I.GT.1.AND.K.NE.0) THEN
        DO 70 II=1,I-1
        IF (KRK(II).EQ.0) THEN
         KRK(II)=K
         KRK(I)=0
         DO 60 J=1,3
         SUM=A(J,I)
         A(J,I)=A(J,II)
60       A(J,II)=SUM
         KRKOLD=KRK(1)+10*KRK(2)+100*KRK(3)
         GO TO 50
        ENDIF
70      CONTINUE
       ENDIF
80     CONTINUE
C
C         ROTATE OLD & NEW TOTAL MECHANICAL STRAINS
C         THESE STRAINS ARE NOT PURIFIED
C         TO LOCAL (PRINCIPAL OR CRACKED) DIRECTIONS
C
       DO 85 I=1,NTENS
       STN(I)=EPP(I)
85     EPP(I)=EPP(I)+EP(I)
       CALL PVAL3D(1,NK,EPP,PSTRN,A)
C
C         STORE NEW PRINCIPAL DIRECTIONS
C
       N=NCOS
       DO 88 J=1,2
       DO 88 I=1,3
       N=N+1
88     AR(N)=A(I,J)
       CALL PVAL3D(1,3,STN,PSTRNO,A)
C
C         ESTABLISH STIFFNESS(ECC), STRESS(SIGFAC) & STRAIN(EPSFAC)
C         FACTORS FOR CRACKING FOR SHEAR
C
       SIGO=EI*EPFRAC
       DO 90 K=1,3
       KP3=K+3
```

```
      I=IPERM(K)
      J=IPERM(I)
      KIJ=MAX(KRK(I),KRK(J))
      IF (KIJ.EQ.2) THEN
       EPSFAC(KP3)=ZERO
       SIGFAC(KP3)=SIGO/MAX(ONE,ABS(PSTRN(K))/EPFRAC)
       EPPMAX=MAX(PSTRN(I),PSTRN(J))
       GRATIO=MAX(EPPMAX/EPFRAC,ONE)
       ECC(KP3)=0.4/GRATIO
      ELSE
       SIGFAC(KP3)=1.E20
       EPSFAC(KP3)=ONE
       ECC(KP3)=ONE
      ENDIF
90    CONTINUE
      IF (IBUG.NE.0) WRITE(NOUT,95) KRKOLD,KRK,K123,NOPEN,NCLOSE,EPP,
     $ PSTRN,SIGFAC,EPSFAC,ECC
95    FORMAT(' CONC3D AFTER 90: KRKOLD,K1,K2,K3,K123,NOPEN,NCLOSE/EPP/',
     $ 'PSTRN/SIGFAC/EPSFAC/ECC=',7I5/(1P12E11.3))
C
C         INITIALIZATION
C
      XVCP1=ONE+XVC
      DO 110 I=1,NTENS
      TF(I)=ZERO
      DF(I)=ZERO
      BF(I)=ZERO
      STN(I)=ZERO
      STR(I)=ZERO
      DO 110 J=1,NTENS
      HH(I,J)=ZERO
      HB(I,J)=ZERO
      H(I,J)=ZERO
      H2(I,J)=ZERO
      Q(I,J)=ZERO
110   HK(I,J)=ZERO
      DO 130 I=1,3
      DO 120 J=1,3
      H2(I,J)=-XVC/THREE
120   HK(I,J)=-XVC
      H2(I,I)=ONE/THREE
      HK(I,I)=ONE
      H2(I+3,I+3)=TWO*XVCP1/THREE
130   HK(I+3,I+3)=TWO*XVCP1
C
      C1=XVC*EI/(XVCP1*(ONE-TWO*XVC))
      C2=(ONE-XVC)*EI/(XVCP1*(ONE-TWO*XVC))
      C3=HALF*EI/XVCP1
      H(1,1)=C2
      H(1,2)=C1
      H(2,1)=C1
      H(1,3)=C1
      H(3,1)=C1
      H(2,2)=C2
      H(2,3)=C1
      H(3,2)=C1
      H(3,3)=C2
      H(4,4)=C3
      H(5,5)=C3
      H(6,6)=C3
C
C        MODIFICATIONS TO CONSTITUTIVE MATRIX DUE TO CHANGE IN MODULUS
C        AND CREEP
C
      DELTAE=EI-AR(28)
      IF(AR(28).LT.0.01E6)DEE=-DELTAE/(EI*EI+ONE)
      IF(AR(28).GE.0.01E6)DEE=-DELTAE/(EI*EI+ONE)*EI/AR(28)
      GEN=ZERO
      IF (JCREEP.NE.0) GEN=GE(2,1)
      DO 180 J=1,JCREEP
180   GEN=GEN+GE(1,J)
      DO 190 I=1,NTENS
      DO 190 J=1,NTENS
      PH(I,J)=H(I,J)
      HH(I,J)=HK(I,J)
190   AS(I,J)=DEE*HK(I,J)+GEN*H2(I,J)
C
C        MODIFICATIONS DUE TO CRACKING
C
      IF (K123.NE.2) GO TO 250
      DO 220 K=1,3
      IF (KRK(K).EQ.2) THEN
       DO 210 L=1,3
       HH(L,K)=ZERO
210    HH(K,L)=ZERO
      ENDIF
      I=IPERM(K)
      J=IPERM(I)
      KIJ=MAX(KRK(I),KRK(J))
      IF (KIJ.EQ.2) THEN
       HH(K+3,K+3)=TWO/ECC(K+3)
      ELSE
       HH(K+3,K+3)=TWO*XVCP1
      ENDIF
220   HH(K,K)=ONE/ECC(K)
      DO 230 I=1,NTENS
      DO 230 J=1,NTENS
230   HK(I,J)=HH(I,J)
      IF (IBUG.NE.0) WRITE(NOUT,236) ((HK(I,J),J=1,6),I=1,6)
236   FORMAT(' CONC3D AFTER 230: HK'/(1P6E11.3))
C
      CALL SYMINV(HH,6,NTENS)
C
      DO 240 I=1,NTENS
      DO 240 J=1,NTENS
      AS(I,J)=DEE*HK(I,J)+GEN*H2(I,J)
240   H(I,J)=HH(I,J)*EI
C
250   CALL TRNS3D(A,Q,HB)
C
      IF (K123.NE.2) GO TO 310
      DO 270 K=1,NTENS
      DO 270 L=1,NTENS
      AS(K,L)=ZERO
      HH(K,L)=ZERO
      DO 270 M=1,NTENS
      HH(K,L)=HH(K,L)+HK(K,M)*HB(M,L)
270   AS(K,L)=AS(K,L)+H(K,M)*Q(M,L)
      DO 280 K=1,NTENS
      DO 280 L=1,NTENS
      HK(K,L)=ZERO
      H(K,L)=ZERO
      DO 280 M=1,NTENS
      HK(K,L)=HK(K,L)+HB(M,K)*HH(M,L)
280   H(K,L)=H(K,L)+Q(M,K)*AS(M,L)
      DO 290 K=1,NTENS
      DO 290 L=1,NTENS
290   AS(K,L)=DEE*HK(K,L)+GEN*H2(K,L)
C
C        STRESSES DUE TO CREEP
C
310   N=NCREEP
      DO 320 J=1,JCREEP
      DO 320 I=1,NTENS
      N=N+1
320   TF(I)=TF(I)+GE(4,J)*AR(N)*GE(3,J)
      IF (IBUG.NE.0) WRITE(6,321) DEE,GEN,TF,GE
321   FORMAT(' CONC3D AFTER 320: DEE,GEN/TF/GE=',1P3E11.3/
     $ 1P6E11.3/(1P12E11.3))
      IF (ABS(DEE).LT.1.E-9.AND.GEN.LT.1.E-9) GO TO 410
      DO 350 I=1,NTENS
      DO 350 J=1,NTENS
      HH(I,J)=ZERO
      DO 350 K=1,NTENS
350   HH(I,J)=HH(I,J)+H(I,K)*AS(K,J)
      DO 360 I=1,NTENS
360   HH(I,I)=HH(I,I)+ONE
      IF (IBUG.NE.0) WRITE(6,361) H,AS,HH
361   FORMAT(' CONC3D AFTER 360: H/AS/HH'/(1P6E11.3))
C
      CALL INVERT (HH,6,NTENS)
C
      DO 370 I=1,NTENS
      DO 370 J=1,NTENS
      PH(I,J)=ZERO
      DO 370 K=1,NTENS
```

```
370   PH(I,J)=PH(I,J)+HH(I,K)*H(K,J)
      DO 380 I=1,NTENS
      DO 380 J=1,NTENS
380   H(I,J)=PH(I,J)
      IF (IBUG.NE.0) WRITE(6,381) PH
381   FORMAT(' CONC3D AFTER 380: PH'/(1P6E11.3))
C
410   DO 420 I=1,NTENS
      BF(I)=ZERO
      DO 420 J=1,NTENS
      BF(I)=BF(I)-HK(I,J)*AR(J)*DEE-H2(I,J)*TF(J)
420   PH(I,J)=H(I,J)
C
C        ELASTIC PREDICTION OF THE NEW STRESS STATE
C
      SIGM=ZERO
      DO 430 I=1,NTENS
      SUM=STRESS(I)
      DO 425 J=1,NTENS
425   SUM=SUM+H(I,J)*(EP(J)+BF(J))
      IF (I.LE.3) SIGM=SIGM+SUM
430   SG(I)=SUM
      SIGM=SIGM/THREE
      IF (IBUG.NE.0) WRITE(NOUT,431) ((H(I,J),J=1,6),SG(I),EP(I),
     $ BF(I),I=1,6)
431   FORMAT(' CONC3D AFTER 430: H,SG,EP,BF'/(1P9E11.3))
C
C        PLASTICITY CALCULATIONS
C        SKIP FOR STIFFNESS ONLY RECOVERY  ( EP = 0 )
C
      IF (KSTIF.EQ.1) GO TO 1155
C
C        GET UPDATED PROPERTIES BASED ON ELASTIC PREDICTOR STRESSES
C        AND UPDATED EFFECTIVE STRAIN
C
      DO 450 I=1,3
450   ECLOSE(I)=AR(I+12)
      CALL PVAL3D(0,3,STRESS,PSTRSO,A)
      IF (IHARD.NE.0) THEN
       CALL PVAL3D(0,3,SG,PSTRS,A)
       ECONCA=ABS(ECONC)
       CALL ESTRN(NTENS,XVC,EI,PSTRN,PSTRS,EPSFAC,NOPEN,EPSEFF,KRK,
     $  ECLOSE,ECONCA)
       IF (SIGM.GT.ZERO) EPSEFF=ZERO
       CALL MATCON(TEMP2,EI,YIELD,FACTR,EPSEFF,CON,ESEC,EPFRAC,AGEFAC,
     $  CRUSH,ECONC,IPROP,IHANOP,DTIM,ULTTEN,BAND,TEMP1,INCRMT,IHARD)
       YIELD=YIELD*SQRT(ONE+THREE*FACTR)
      ENDIF
C
C        TEST YIELD CONDITION - IF EITHER THE OLD OR NEW YIELD
C        CONDITION IS EXCEEDED THE STEP IS PLASTIC
C
```

```
      NPL=NEAR(AR(25))
      NPL1=NPL/100
      NPL=MOD(NPL,100)
      ECONCA=ABS(ECONC)
      EK=AR(29)/SQ3
      EK2=EK*EK
      PP=ZERO
      CALL DEVIAT(SG,SIJ,SIGM)
C
      IF (SIGM.GT.ZERO) THEN
       NPL1=0
       YIELD=AR(29)*DAGE
       EI=AR(28)*DAGE
       GO TO 890
      ENDIF
      CALL YFUN(SG,SIJ,SIGM,FACTR,EK2,K123,RADFAC,PLAST)
      EK=YIELD/SQ3
      EK2=EK*EK
      NPL1=0
      IF (PLAST) NPL1=1
      CALL YFUN(SG,SIJ,SIGM,FACTR,EK2,K123,RADFAC,PLAST)
      IF (PLAST) THEN
       NPL1=1
      ELSE
       IF (NPL1.NE.0) NPL1=2
       IF (NPL1.EQ.0) THEN
        YIELD=AR(29)*DAGE
        EI=AR(28)*DAGE
       ENDIF
       GO TO 890
      ENDIF
C
      CALL RADRET(SG,SIJ,SIGM,K123,RADFAC)
      IF (IBUG.NE.0) WRITE(6,461) NPL,NPL1,RADFAC,SG
461   FORMAT(' IN CONC3D AFTER 460: NPL,NPL1,RADFAC,SG=',
     $  3I3,1PE11.3/(1P6E11.3))
C
C        CURRENT INCREMENT IS PLASTIC
C
600   FA=FACTR
      CI1=TWO/THREE*CON
      SUM=THREE*SIGM
      FA=TWO*FA*SUM
      RT=ONE-PP
      DO 610 I=1,3
      STR(I)=SIJ(I)+FA
610   STR(I+3)=SIJ(I+3)
      IF (K123.NE.2) GO TO 670
      DO 620 K=1,NTENS
      DIJ(K)=ZERO
      DO 620 L=1,NTENS
620   DIJ(K)=DIJ(K)+HB(K,L)*SG(L)
```

```
      DO 630 M=1,NTENS
630   DIJ(M)=DIJ(M)*EPSFAC(M)
      DO 640 I=1,NTENS
640   DIJ(I)=DIJ(I)
      DO 650 K=1,NTENS
      STR(K)=ZERO
      DO 650 L=1,NTENS
650   STR(K)=STR(K)+Q(L,K)*DIJ(L)
      SUM=STR(1)+STR(2)+STR(3)
      DO 660 K=1,3
660   STR(K)=STR(K)-SUM*(ONE/THREE-TWO*FACTR)
C
670   DO 680 I=1,NTENS
      STN(I)=ZERO
      DO 680 J=1,NTENS
680   STN(I)=STN(I)+PH(I,J)*STR(J)
      RS=ZERO
      DO 690 I=1,NTENS
      RS=RS+STR(I)*STN(I)+CI1*STR(I)*STR(I)
      DO 690 J=1,NTENS
690   HH(I,J)=STN(I)*STN(J)
      IF (IBUG.NE.0) WRITE(NOUT,691) NPL,NPL1,STR,STN,CI1,RT,RS,
     $ SUM
691   FORMAT(' CONC3D AFTER 690: NPL,NPL1/STR,STN/CI1,RT,RS,SUM',
     $ 3I3/(1P12E11.3))
      IF (RS.EQ.ZERO) RS=ONE
      DO 700 I=1,NTENS
      DO 700 J=1,NTENS
700   PH(I,J)=PH(I,J)-HH(I,J)*RT/RS
C
C        MODIFY CREEP & PLASTIC CONSTITUTIVE MATRIX FOR CRACKING
C
      IF (K123.NE.2) GO TO 890
      DO 730 I=1,NTENS
      DO 730 J=1,NTENS
      SUM=ZERO
      DO 720 K=1,NTENS
720   SUM=SUM+HB(I,K)*PH(K,J)
730   H(I,J)=SUM
      DO 750 I=1,NTENS
      DO 750 J=1,NTENS
      SUM=ZERO
      DO 740 K=1,NTENS
740   SUM=SUM+H(I,K)*HB(J,K)
750   HK(I,J)=SUM
      DO 780 K=1,3
      IF (KRK(K).NE.2) GO TO 780
      DO 770 I=1,NTENS
      IF (I.EQ.K) GO TO 770
      DO 760 J=1,NTENS
      IF (J.EQ.K) GO TO 760
      HK(I,J)=HK(I,J)-HK(I,K)*HK(K,J)/HK(K,K)
```

```
760   CONTINUE
770   CONTINUE
780   CONTINUE
      DO 790 I=1,NTENS
      DO 790 J=1,NTENS
790   H(I,J)=HK(I,J)
      DO 810 I=1,NTENS
      DO 810 J=1,NTENS
      SUM=ZERO
      DO 800 K=1,NTENS
800   SUM=SUM+H(I,K)*Q(K,J)
810   HH(I,J)=SUM
      DO 830 I=1,NTENS
      DO 830 J=1,NTENS
      SUM=ZERO
      DO 820 K=1,NTENS
820   SUM=SUM+Q(K,I)*HH(K,J)
830   PH(I,J)=SUM
C
890   IF (KSTIF.EQ.1) RETURN
C
C        ROT\TE STRESSES TO LOCAL (PRINCIPAL OR CRACKED) DIRECTIONS
C
      CALL PVAL3D(0,3,SG,PSTRS,A)
C
C        ROTATE MECHANICAL STRAIN INCREMENTS TO LOCAL
C
      CALL PVAL3D(1,3,EP,DPSTRN,A)
C
C        CRACKING CRITERIA FOR NEW CRACK STATUS
C        USING INTERACTION CURVE BASED ON ORIGINAL MODULUS
C
      CRKCLO=-.1E-6
      NOPEN=0
      NCLOSE=0
      NOPING=0
      NCLING=0
      K123=0
      NCPERM=3-NDI
      DO 950 I=1,3
      IF (I.LE.NCPERM) THEN
       K=2
       GO TO 940
      ENDIF
      PSIG=PSTRS(I)
      PEPS=PSTRN(I)
      DPEPS=DPSTRN(I)
      K=KRK(I)
      KOC(I)=0
      IF (K.EQ.0) THEN
       EPSF=MAX((TWO*EPFRAC-PSIG/ECONCA),ZERO)
       SIGF=MAX((TWO*SIGO-PEPS*ECONCA),ZERO)
```

```
C
C        MULTIAXIAL TENSION STRENGTH CUTOFF
C
       J=IPERM(I)
       L=IPERM(J)
       SIGT=MAX(PSTRS(J)*EPSFAC(J),PSTRS(L)*EPSFAC(L),ZERO)
       SIGT=MIN(SIGT,SIGO)
       SIGT=TWO*SIGO-SIGT
       SIGF=MIN(SIGF,SIGT)
      ELSE
       EPSF=ZERO
       SIGF=ZERO
      ENDIF
      SIGFAC(I)=1.E20
      EPSFAC(I)=ONE
C
C        CLOSED CRACK
C
      IF ((K.EQ.2 .AND. DPEPS.LE.CRKCLO) .OR.
     $ (K.EQ.1 .AND. PSIG.LE.ZERO .AND.
     $ (DPEPS.LE.CRKCLO .OR. PEPS.LE.ZERO))) THEN
       IF (K.EQ.2) THEN
        KOC(I)=1
        ECLOSE(I)=PEPS
        NCLING=NCLING+1
       ENDIF
       K=1
       NCLOSE=NCLOSE+1
       GO TO 940
      ENDIF
C
C        UNCRACKED
C
      IF (K.EQ.0.AND.(PSIG.LT.SIGF.OR.PEPS.LT.EPSF)) GO TO 940
C
C        OPEN CRACK
C
      IF (K.NE.2) THEN
       NOPING=NOPING+1
       KOC(I)=-1
       ECLOSE(I)=ZERO
      ENDIF
      K=2
      NOPEN=NOPEN+1
      SIGFAC(I)=ZERO
      EPSFAC(I)=ZERO
C
940   KRK(I)=K
      K123=MAX(K123,K)
950   CONTINUE
C
C        UPDATE CRACK STATUS
```

```
C
      KRKNEW=KRK(1)+10*KRK(2)+100*KRK(3)
      AR(26)=KRKOLD+10000*KRKNEW
      IF (ITER.NE.1) AR(26)=KRKNEW+10000*KRKNEW
C
C         SANITIZE LOCAL STRESSES & STRAINS
C
      DO 955 K=1,3
      KP3=K+3
      I=IPERM(K)
      J=IPERM(I)
      KIJ=MAX(KRK(I),KRK(J))
      IF (KIJ.EQ.2) THEN
       SIGFAC(KP3)=SIGO/MAX(ONE,ABS(PSTRN(K))/EPFRAC)
       EPSFAC(KP3)=EPFRAC/MAX(ONE,ABS(PSTRN(K)))
      ELSE
       SIGFAC(KP3)=1.E20
       EPSFAC(KP3)=ONE
      ENDIF
955   CONTINUE
      IF (IBUG.NE.0) WRITE(NOUT,956) KRK,ECLOSE,((A(I,J),J=1,3),I=1,3),
     $ PSTRS,PSTRN,DPSTRN,SIGFAC
956   FORMAT(' CONC3D AFTER 955: K1,K2,K3,ECLOSE=',3I5,1P3E11.3,
     $ ' /A/PSTRS/PSTRN/DPSTRN/SIGFAC'/1P3E11.3/1P3E11.3/1P3E11.3/
     $ (1P6E11.3))
      DO 960 I=1,NTENS
      PSTRN(I)=(PSTRNO(I)+DPSTRN(I))*EPSFAC(I)
960   PSTRS(I)=SIGN(MIN(ABS(PSTRS(I)),SIGFAC(I)),PSTRS(I))
C
C         ADJUST TRANSVERSE STRESSES FOR OPENING CRACKS
C
      IF (NOPING.GT.0.AND.NOPEN.LT.3) THEN
       DO 965 I=1,3
       DO 965 J=1,3
965    H(I,J)=PH(I,J)
       CALL SYMINV(H,6,3)
       IF (NOPEN.EQ.1) THEN
        IF (KRK(1).EQ.2) THEN
         I=1
        ELSEIF(KRK(2).EQ.2) THEN
         I=2
        ELSE
         I=3
        ENDIF
        J=IPERM(I)
        K=IPERM(J)
        DEPSJ=H(J,I)*PSTRSO(I)
        DEPSK=H(K,I)*PSTRSO(I)
        PSTRS(J)=PSTRS(J)-PH(J,I)*DPSTRN(I)+PH(J,J)*DEPSJ+PH(J,K)*DEPSK
        PSTRS(K)=PSTRS(K)-PH(K,I)*DPSTRN(I)+PH(K,J)*DEPSJ+PH(K,K)*DEPSK
       ELSE
        IF (KRK(1).NE.2) THEN
```

```
         K=1
        ELSEIF (KRK(2).NE.2) THEN
         K=2
        ELSE
         K=3
        ENDIF
        I=IPERM(K)
        J=IPERM(I)
        DEPSJ=ZERO
        DEPSK=H(K,I)*PSTRSO(I)+H(K,J)*PSTRSO(J)
        PSTRS(K)=PSTRS(K)-PH(K,I)*DPSTRN(I)-PH(K,J)*DPSTRN(J)+
     $   PH(K,K)*DEPSK
       ENDIF
       IF (IBUG.NE.0) WRITE(6,967) NOPING,NOPEN,I,J,K,H(J,I),H(K,I),
     $  DEPSJ,DEPSK,PH(J,J),PH(J,K),PH(K,K),(PSTRSO(L),L=1,3),
     $  (PSTRS(L),L=1,3),(DPSTRN(L),L=1,3)
967    FORMAT(' CONC3D AFTER TRANS. CORR: NOPING,NOPEN,I,J,K,',
     $  'H(J,I K,I),DEPSJ,DEPSK,PH(J,J J,K K,K)/PSTRSO,PSTRS,DPSTRN='/
     $  5I3,1P7E11.3/(1P9E11.3))
      ENDIF
C
C        FURTHER PLASTICITY CHECK (LOCAL)
C        ROTATE OLD STRESSES TO LOCAL SYSTEM
C
      CALL DEVIAT(PSTRS,SIJ,SIGM)
      IF (SIGM.LE.ZERO) THEN
       CALL YFUN(PSTRS,SIJ,SIGM,FACTR,EK2,K123,RADFAC,PLAST)
      ELSE
       PLAST=.FALSE.
       RADFAC=ONE
      ENDIF
C
C        UPDATE PLASTICITY FLAG
C
      IF (PLAST .AND. NPL1.NE.1) NPL1=3
      AR(25)=NPL+100*NPL1
      IF (IBUG.NE.0) WRITE(6,983) NPL1,RADFAC
983   FORMAT(' CONC3D AFTER FINAL CHECK: NPL1,RADFAC=',I2,1PE11.3)
C
C        PERFORM RADIAL RETURN FOR CRACKED STRESSES
C
      IF (NPL1.EQ.1.AND.RADFAC.LT.ONE)
     $ CALL RADRET(PSTRS,SIJ,SIGM,K123,RADFAC)
C
C        ROTATE LOCAL SANITIZED STRESSES TO GLOBAL
C
      CALL PVAL3D(0,4,PSTRS,SG,A)
C
C        UPDATE THE EFFECTIVE STRAIN ONLY FOR HARDENING MATERIALS
C
      IF (IHARD.NE.0) THEN
       IF (SIGM.GT.ZERO) EPSEFF=ZERO
       AR(27)=EPSEFF
      ENDIF
C
C        UPDATE NEW STRESSES FOR ITER=1
C
1155  IF (ITER.EQ.1.OR.KSTIF.EQ.1) THEN
       DO 1160 I=1,NTENS
1160   AR(I+6)=SG(I)
       GO TO 1200
      ENDIF
C
C        UPDATE ALL STRAINS & STRESSES
C
      DO 1170 I=1,NTENS
      IF (ABAQUS) THEN
       AR(I+18)=AR(I+18)+DEP(I)
      ELSE
       AR(I+18)=AR(I+18)+EP(I)
      ENDIF
      SR(I)=SG(I)-AR(I)
      AR(I+6)=SG(I)
1170  AR(I)=SG(I)
      DO 1175 I=1,3
1175  AR(I+12)=ECLOSE(I)
C
C        UPDATE CREEP PARAMETERS
C
      N=NCREEP
      DO 1180 J=1,JCREEP
      DO 1180 I=1,NTENS
      N=N+1
1180  AR(N)=AR(N)*GE(4,J)+SR(I)*GE(1,J)/GE(3,J)
C
      AR(28)=EI
      AR(29)=AR(29)*DAGE
      IF (NPL1.GT.0) AR(29)=YIELD
C
C        UPDATE PLASTICITY FLAG
C
      AR(25)=NPL1+100*NPL1
C
1200  IF (IBUG.NE.0) WRITE(NOUT,1210) (AR(I),I=1,NCOS)
1210  FORMAT(' CONC3D BEFORE RETURN: AR'/(1P10E11.3))
C
      RETURN
      END

      SUBROUTINE ESTRN(NTENS,XVC,ESEC,PEPS,PSTRS,EPSFAC,NK,EPSEFF,KRK,
     $ ECLOSE,ECONC)
      IMPLICIT REAL*8 (A-H,O-Z)
C
      COMMON /FLTNUM/ ZERO,ONE,TWO,THREE,FOUR,FIVE,SIX,SEVEN,EIGHT,
     $  NINE,TEN,HALF,THIRD,FOURTH,FIFTH,SIXTH,SEVNTH,EIGHTH,NINETH,
     $  TENTH,HUNDRD,THOU,MILLON,PI,PIFAC,PIFAC1,EXPN
      REAL*8 NINE,NINETH,MILLON
C
      COMMON /RSDDBG/ IBUG
C
      DIMENSION H(6,6),PSTRN(6),PSTRS(6),EPSFAC(6),EPSE(6),EPSP(6),
     $ KRK(3),ECLOSE(3),PEPS(6)
C
      DATA PTOL/1.0E-14/, SIZTOL/0.1E-6/
C
      IF (NK.GE.3) THEN
       EPSEFF=ZERO
       RETURN
      ENDIF
C
C        COMPUTE EFFECTIVE STRAIN
C
      EI=ESEC
      XVCP1=ONE+XVC
10    DO 20 I=1,NTENS
      PSTRN(I)=PEPS(I)
      DO 20 J=1,NTENS
20    H(I,J)=ZERO
      DO 40 I=1,3
      DO 30 J=1,3
30    H(I,J)=-XVC/EI
      H(I,I)=ONE/EI
40    H(I+3,I+3)=TWO*XVCP1/EI
C
C        SANATIZE STRAINS FOR CLOSED CRACKS
C
      DO 45 I=1,3
      IF (KRK(I).EQ.1) PSTRN(I)=PSTRN(I)-ECLOSE(I)
45    CONTINUE
C
C        COMPUTE PURIFIED MECHANICAL ELASTIC & PLASTIC STRAINS
C
      EPSEM=ZERO
      EPSPM=ZERO
      ESTR=ZERO
      PSTR=ZERO
      ESIZE=ZERO
      PSIZE=ZERO
      PTEST=ZERO
      DO 60 I=1,NTENS
      SUM=ZERO
      DO 50 J=1,NTENS
50    SUM=SUM+H(I,J)*PSTRS(J)
      ESTR=SUM*EPSFAC(I)
      EPSE(I)=ESTR
      PSTR=PSTRN(I)*EPSFAC(I)-ESTR
      EPSP(I)=PSTR
      ESIZE=MAX(ESIZE,ABS(ESTR))
      PSIZE=MAX(PSIZE,ABS(PSTR))
      IF (I.LE.3) THEN
       PTEST=PTEST+ESTR*PSTR
       EPSEM=EPSEM+ESTR
       EPSPM=EPSPM+PSTR
      ELSE
       PTEST=PTEST+TWO*ESTR*PSTR
      ENDIF
60    CONTINUE
C
      IF (ESIZE.LT.SIZTOL .AND. PSIZE.LT.SIZTOL) THEN
       EPSEFF=ZERO
       RETURN
      ELSEIF (ESIZE.GT.TEN .OR. PSIZE.GT.TEN) THEN
       WRITE(6,61) PSTRS,PSTRN
61     FORMAT(' ESTRN-STRAINS TOO LARGE: /PSTRS/PSTRN'/(1P6E11.3))
       EPSEFF=ZERO
       RETURN
      ENDIF
      EPSEM=EPSEM/THREE
      EPSPM=EPSPM/THREE
C
      FE=THREE/TWO
      FP=FE
      IF (NK.EQ.0) THEN
       FE=FE/(XVCP1**2)
       FP=TWO/THREE
      ELSEIF (NK.EQ.1) THEN
       FE=FE/(XVCP1+XVC*XVC)
       FP=SIX/SEVEN
      ELSEIF (NK.GE.3) THEN
       FE=ZERO
       FP=ZERO
      ENDIF
C
      EFFP=ZERO
      EFFE=ZERO
      DO 70 I=1,3
      EFFP=EFFP+(EPSP(I)-EPSPM)**2+TWO*(EPSP(I+3)**2)
70    EFFE=EFFE+(EPSE(I)-EPSEM)**2+TWO*(EPSE(I+3)**2)
      IF (PSIZE.GT.SIZTOL) THEN
       IF (PTEST.LT.PTOL) THEN
        IF (ABS(EI/ECONC-ONE).GT.0.01) THEN
         EI=ECONC
         GO TO 10
        ENDIF
        IF (IBUG.NE.0) WRITE(6,69) ESIZE,PSIZE,PTEST,ESEC,ECONC,EPSE,
     $   EPSP,PSTRS,PSTRN
69      FORMAT(' ESTRN BAD PLASTIC STRAIN TEST: ESIZE,PSIZE,PTEST,',
     $   'ESEC,ECONC/EPSE/EPSP/PSTRS/PSTRN='/1P5E11.3/(1P6E11.3))
```

```
         FP=ZERO
        ENDIF
       ELSE
        FP=ZERO
       ENDIF
       EPSEFF=SQRT(FP*EFFP)+SQRT(FE*EFFE)
       IF (IBUG.NE.0) WRITE(6,71) NK,EPSFAC,FE,FP,EPSEFF,EFFE,EFFP,PSTRN,
     $ ECLOSE
71     FORMAT(' END ESTRN: NK,EPSFAC/FE,FP,EPSEFF,EFFE,EFFP/PSTRN/ECLOSE'
     $ /I5,1P6E11.3/1P5E11.3/(1P6E11.3))
C
       RETURN
       END

      SUBROUTINE DEVIAT(SG,SIJ,SIGM)
      IMPLICIT REAL*8 (A-H,O-Z)
C
      COMMON /FLTNUM/ ZERO,ONE,TWO,THREE,FOUR,FIVE,SIX,SEVEN,EIGHT,
     $  NINE,TEN,HALF,THIRD,FOURTH,FIFTH,SIXTH,SEVNTH,EIGHTH,NINETH,
     $  TENTH,HUNDRD,THOU,MILLON,PI,PIFAC,PIFAC1,EXPN
      REAL*8 NINE,NINETH,MILLON
C
      COMMON /RSDDBG/ IBUG
C
      DIMENSION SG(6),SIJ(6)
C
      SIGM=(SG(1)+SG(2)+SG(3))/THREE
      DO 10 I=1,3
      SIJ(I)=SG(I)-SIGM
10    SIJ(I+3)=SG(I+3)
      RETURN
      END

      SUBROUTINE YFUN(SG,SIJ,SIGM,FACTR,EK2,K123,RADFAC,PLAST)
      IMPLICIT REAL*8 (A-H,O-Z)
C
      COMMON /FLTNUM/ ZERO,ONE,TWO,THREE,FOUR,FIVE,SIX,SEVEN,EIGHT,
     $  NINE,TEN,HALF,THIRD,FOURTH,FIFTH,SIXTH,SEVNTH,EIGHTH,NINETH,
     $  TENTH,HUNDRD,THOU,MILLON,PI,PIFAC,PIFAC1,EXPN
      REAL*8 NINE,NINETH,MILLON
C
      COMMON /RSDDBG/ IBUG
C
      DIMENSION SG(6),SIJ(6)
C
      LOGICAL UNCON,PLAST
C
      DATA PTOL/1.E-20/
C
      IF (IBUG.NE.0) WRITE(6,1) K123,FACTR,EK2,SIGM,SIJ
1     FORMAT(' START YFUN: K123/FACTR,EK2,SIGM,SIJ=',I5/(1P9E11.3))
      PLAN=ZERO
      PLAS=ZERO
      DO 10 I=1,3
      PLAN=PLAN+HALF*(SIJ(I)**2)
10    PLAS=PLAS+SIJ(I+3)**2
      PLA=PLAN+PLAS
      PLAKK=NINE*FACTR*(SIGM**2)
      UNCON=.FALSE.
C
      IF (K123.EQ.2.OR.UNCON) THEN
C
C        WITH AT LEAST ONE OPEN CRACK OR LOW CONFINING STRESS
C        PERFORM THE RADIAL RETURN ON THE TOTAL STRESSES
C
       IF (EK2.LE.ZERO) THEN
        RADFAC=ZERO
       ELSE
        RADFAC=ONE
        PLA=PLA+PLAKK
        PLAST=.FALSE.
        IF (PLA.GT.PTOL) THEN
         RADFAC=SQRT(EK2/PLA)
         IF (RADFAC.LE.ONE) PLAST=.TRUE.
        ENDIF
       ENDIF
      ELSE
C
C        WITH NO OPEN CRACKS PERFORM THE RADIAL RETURN
C        ON THE DEVIATORIC STRESSES
C
       EKEK=EK2-PLAKK
       IF (EKEK.LE.ZERO) THEN
        RADFAC=ZERO
       ELSE
        RADFAC=ONE
        PLAST=.FALSE.
        IF (PLA.GT.PTOL) THEN
         RADFAC=SQRT(EKEK/PLA)
         IF (RADFAC.LE.ONE) PLAST=.TRUE.
        ENDIF
       ENDIF
      ENDIF
C
      IF (IBUG.NE.0) WRITE(6,99) UNCON,PLAST,PLA,PLAN,PLAS,PLAKK,RADFAC
99    FORMAT(' END YFUN: UNCON,PLAST,PLA,PLAN,PLAS,PLAKK,RADFAC= ',
     $ L1,1X,L1/(1P10E11.3))
      RETURN
      END

      SUBROUTINE RADRET(SG,SIJ,SIGM,K123,RADFAC)
      IMPLICIT REAL*8 (A-H,O-Z)
C
      COMMON /FLTNUM/ ZERO,ONE,TWO,THREE,FOUR,FIVE,SIX,SEVEN,EIGHT,
     $ NINE,TEN,HALF,THIRD,FOURTH,FIFTH,SIXTH,SEVNTH,EIGHTH,NINETH,
     $ TENTH,HUNDRD,THOU,MILLON,PI,PIFAC,PIFAC1,EXPN
      REAL*8 NINE,NINETH,MILLON
C
      COMMON /RSDDBG/ IBUG
C
      DIMENSION SG(6),SIJ(6)
C
      IF (K123.EQ.2) THEN
C
C        WITH AT LEAST ONE OPEN CRACK PERFORM THE RADIAL RETURN
C        ON THE TOTAL STRESSES
C
       DO 20 I=1,6
       SIJ(I)=SIJ(I)*RADFAC
20     SG(I)=SG(I)*RADFAC
       SIGM=SIGM*RADFAC
      ELSE
C
C        WITH NO OPEN CRACKS PERFORM THE RADIAL RETURN
C        ON THE DEVIATORIC STRESSES
C
       DO 30 I=1,3
       S=SIJ(I)*RADFAC
       SIJ(I)=S
       SG(I)=S+SIGM
       S=SIJ(I+3)*RADFAC
       SIJ(I+3)=S
30     SG(I+3)=S
      ENDIF
C
      IF (IBUG.NE.0) WRITE(6,99) SIGM,SIJ,SG
99    FORMAT(' AFTER RADRET: SIGM,SIJ/SG='/1P7E11.3/(1P6E11.3))
      RETURN
      END

      SUBROUTINE TRNS3D(A,Q,B)
      IMPLICIT REAL*8 (A-H,O-Z)
C
C     THIS ROUTINE CREATES Q AND B (BOTH 6X6) OUT OF THE TRANSFORMATON
C     MATRIX A. Q AND B MAY THEN BE USED TO TRANSFORM A 6X6 TENSOR.
C
      COMMON /FLTNUM/ ZERO,ONE,TWO,THREE,FOUR,FIVE,SIX,SEVEN,EIGHT,
     $ NINE,TEN,HALF,THIRD,FOURTH,FIFTH,SIXTH,SEVNTH,EIGHTH,NINETH,
     $ TENTH,HUNDRD,THOU,MILLON,PI,PIFAC,PIFAC1,EXPN
      REAL*8 NINE,NINETH,MILLON
C
      DIMENSION A(3,3),Q(6,6),B(6,6)
C
      DO 200 K=1,3
      DO 100 I=1,3
100   B(K,I)=A(I,K)*A(I,K)
```

```
      B(K,4)=A(2,K)*A(3,K)*TWO
      B(K,5)=A(1,K)*A(3,K)*TWO
      B(K,6)=A(1,K)*A(2,K)*TWO
      B(4,K)=A(K,2)*A(K,3)
      B(5,K)=A(K,1)*A(K,3)
200   B(6,K)=A(K,1)*A(K,2)
C
      B(4,4)=A(2,2)*A(3,3)+A(3,2)*A(2,3)
      B(4,5)=A(1,2)*A(3,3)+A(3,2)*A(1,3)
      B(4,6)=A(1,2)*A(2,3)+A(2,2)*A(1,3)
      B(5,4)=A(2,1)*A(3,3)+A(3,1)*A(2,3)
      B(5,5)=A(1,1)*A(3,3)+A(3,1)*A(1,3)
      B(5,6)=A(1,1)*A(2,3)+A(2,1)*A(1,3)
      B(6,4)=A(2,1)*A(3,2)+A(3,1)*A(2,2)
      B(6,5)=A(1,1)*A(3,2)+A(3,1)*A(1,2)
      B(6,6)=A(1,1)*A(2,2)+A(2,1)*A(1,2)
C
      DO 300 I=1,6
      DO 300 J=1,6
300   Q(I,J)=B(I,J)
      DO 400 I=1,3
      DO 400 J=4,6
400   Q(I,J)=HALF*Q(I,J)
      DO 500 I=4,6
      DO 500 J=1,3
500   Q(I,J)=TWO*Q(I,J)
C
      RETURN
      END

      SUBROUTINE PVAL3D(ISTRN,IOPT,SIG,PSIG,A)
      IMPLICIT REAL*8 (A-H,O-Z)
C
C            CALCULATE PRINCIPAL STRESSES AND DIRECTIONS
C
C            ISTRN
C                    EQ.0 - STRESSES
C                    NE.0 - STRAINS
C
C            IOPT
C                    EQ.0 - COMPUTE PRINC. VALS./DIRECTS.
C                    EQ.1 - COMPUTE PRINC. VALS./DIRECTS.
C                           (1ST COL. OF A GIVEN)
C                    EQ.2 - COMPUTE PRINC. VALS./DIRECTS.
C                           (1ST & 2ND COLS. OF A GIVEN)
C                    EQ.3 - ROTATE GLOBAL TO LOCAL
C                           (A GIVEN)
C                    EQ.4 - ROTATE LOCAL TO GLOBAL
C                           (A GIVEN)
C
C            ORDER OF SIG & PSIG
C
```

```
C                    1 - 1,1
C                    2 - 2,2
C                    3 - 3,3
C                    4 - 2,3
C                    5 - 3,1
C                    6 - 1,2
C
C            A(I,J)=COS(X(I),X'(J))
C
C                    WHERE X IS GLOBAL & X' IS LOCAL(PRINCIPAL)
C                    SO THAT COLUMNS IN A CORRESPOND TO THE LOCAL
C                    SYSTEM
C
      COMMON /FLTNUM/ ZERO,ONE,TWO,THREE,FOUR,FIVE,SIX,SEVEN,EIGHT,
     $  NINE,TEN,HALF,THIRD,FOURTH,FIFTH,SIXTH,SEVNTH,EIGHTH,NINETH,
     $  TENTH,HUNDRD,THOU,MILLON,PI,PIFAC,PIFAC1,EXPN
      REAL*8 NINE,NINETH,MILLON
C
      COMMON /RSDINF/ NOUT,JELNO,INT,NSTPAB,INCRAB,NPASS
C
      DIMENSION SIG(6),PSIG(6),A(3,3),S(3,3),XU(3),SP(3,3),
     $ IPERM(3),AT(3,3)
C
      EQUIVALENCE (S1,S(1,1)),(S2,S(2,2)),(S3,S(3,3)),(S6,S(1,2)),
     $ (S5,S(1,3)),(S4,S(2,3))
C
      DATA TOL/1.E-6/, CTOL/1.E-3/, IPERM/2,3,1/, NERROR/0/
C
      X27=THREE*NINE
      X120=FOUR*THREE*TEN*PIFAC
      XNS=ONE
      IF (ISTRN.NE.0) XNS=TWO
      DO 10 N=1,3
      I=IPERM(N)
      J=IPERM(I)
      SUM=SIG(N+3)/XNS
      S(I,J)=SUM
      S(J,I)=SUM
10    S(N,N)=SIG(N)
C
      IGO=IOPT+1
      GO TO (20,100,200,300,400), IGO
C
C               COMPUTE ALL VALUES & DIRECTIONS
C
20    T1=S4*S4
      T2=S5*S5
      T3=S6*S6
      P=-(S1+S2+S3)
      Q=S1*S2+S2*S3+S3*S1-T1-T2-T3
      R=-(S1*S2*S3+TWO*S4*S5*S6-S1*T1-S2*T2-S3*T3)
      Z=Q-P*P/THREE
      IF (ISTRN.EQ.0) THEN
        ZTOL=TEN**(-6)
      ELSE
        ZTOL=TEN**(-10)
      ENDIF
      IF (ABS(Z).LT.ZTOL) GO TO 42
      IF (Z.LE.-ZTOL) GO TO 30
      WRITE(NOUT,26) ISTRN,IOPT,Z,ZTOL,SIG
 26   FORMAT(' NEGATIVE ROOT NEEDED IN PVAL3D.  ISOTROPIC',
     $ ' STATE RETURNED.'/'ISTRN,IOPT,Z,ZTOL,SIG='2I6,1P8E11.3)
      GO TO 42
C
 30   B=TWO*P*P*P/X27-P*Q/THREE+R
      C=SQRT(-X27*B*B/(FOUR*Z*Z*Z))
      IF (ABS(C).GT.ONE) C=ABS(C)/C
      PHI=-SIGN(C,B)
      PHI=ACOS(PHI)
      C=TWO*SQRT(-Z/THREE)
C
C               THE PRINCIPAL VALUES
C
      CRIT=ZERO
      DO 35 K=1,3
      PSIG(K+3)=ZERO
      ANG=X120*FLOAT(K-1)+PHI/THREE
      X=C*COS(ANG)-P/THREE
 35   XU(K)=X
      CRIT=TOL*(P/THREE)**2
      IF (CRIT.LE.ZERO) CRIT=TOL*(C**2)
C
      PSIG(1)=MAX(XU(1),XU(2),XU(3))
      PSIG(3)=MIN(XU(1),XU(2),XU(3))
      PSIG(2)=XU(1)+XU(2)+XU(3)-PSIG(1)-PSIG(3)
C
C               TEST FOR EQUAL ROOTS
C
      EQUAL=MAX(ABS(PSIG(1)),ABS(PSIG(3)))*TOL
      NE=0
      N=0
      IF (ABS(PSIG(1)-PSIG(2)).LT.EQUAL) THEN
       NE=NE+1
       N=3
      ENDIF
      IF (ABS(PSIG(2)-PSIG(3)).LT.EQUAL) THEN
       NE=NE+1
       N=1
      ENDIF
      IF (ABS(PSIG(3)-PSIG(1)).LT.EQUAL) THEN
       NE=NE+1
       N=2
      ENDIF
C
```

```
C                 ISOTROPIC STATE
C
      IF (NE.LT.2) GO TO 46
42    DO 45 I=1,3
      PSIG(I)=-P/THREE
      PSIG(I+3)=ZERO
      DO 44 J=1,3
44    A(J,I)=ZERO
45    A(I,I)=ONE
      GO TO 500
46    IF (NE.EQ.0) THEN
C
C                 COMPUTE ALL 3 DIRECTION COSINES
C
        DO 60 I=1,3
        CALL DIRCOS(PSIG(I),S,A(1,I),CRIT,CTOL,DET)
        IF (DET.LE.CRIT) GO TO 600
60      CONTINUE
      ELSE
C
C                 1 PAIR OF EQUAL PRINCIPAL VALUES
C
        CALL DIRCOS(PSIG(N),S,A(1,N),CRIT,CTOL,DET)
        IF (DET.LE.CRIT) GO TO 600
        CALL TWOVEC(N,A,XU,IPERM,CTOL,DET)
        IF (DET.LE.CTOL) GO TO 600
      ENDIF
      GO TO 500
C
C                 FIRST DIRECTION KNOWN
C                 COMPUTE LARGEST IN-PLANE "PRINCIPAL" VALUES
C
100   CALL TWOVEC(1,A,XU,IPERM,CTOL,DET)
      IF (DET.LE.CTOL) GO TO 600
      CALL RSDROT(S,SP,A,PSIG,IPERM)
      PSIG(4)=ZERO
      CEN=(SP(2,2)+SP(3,3))/TWO
      DIF=(SP(2,2)-SP(3,3))/TWO
      TAU=SP(2,3)
      RAD=SQRT(DIF*DIF+TAU*TAU)
      PSIG(2)=CEN+RAD
      PSIG(3)=CEN-RAD
      ANG=ZERO
      IF (RAD.GT.ZERO) ANG=ATAN2(TAU,DIF)/TWO
      CA=COS(ANG)
      SA=SIN(ANG)
      DO 110 I=1,3
      SP(I,1)=ZERO
      SP(1,I)=ZERO
      DO 110 J=1,3
110   S(I,J)=A(I,J)
      SP(1,1)=ONE
      SP(2,2)=CA
      SP(3,3)=CA
      SP(2,3)=-SA
      SP(3,2)=SA
      DO 130 I=1,3
      DO 130 J=1,3
      SUM=ZERO
      DO 120 K=1,3
120   SUM=SUM+S(I,K)*SP(K,J)
130   A(I,J)=SUM
      GO TO 500
C
C               TWO DIRECTIONS KNOWN
C
200   CALL CROSS(A(1,1),A(1,2),A(1,3),DET,IPERM)
      IF (DET.LE.CTOL) GO TO 600
C
C               ALL 3 DIRECTIONS KNOWN
C               ROTATE GLOBAL-TO-LOCAL
C
300   CALL RSDROT(S,SP,A,PSIG,IPERM)
      GO TO 500
C
C               ROTATE LOCAL-TO-GLOBAL
C
400   DO 410 I=1,3
      DO 410 J=1,3
410   AT(J,I)=A(I,J)
      CALL RSDROT(S,SP,AT,PSIG,IPERM)
C
C               RESET STRAINS
C
500   IF (ISTRN.EQ.0) RETURN
      DO 510 N=4,6
510   PSIG(N)=XNS*PSIG(N)
      RETURN
C
C           DIRECTION ERRORS
C
600   WRITE(NOUT,601) JELNO,INT,NSTPAB,INCRAB,NPASS,IOPT,ISTRN,CRIT,
     $ CTOL,DET,(SIG(I),I=1,6),(PSIG(I),I=1,3),A
601   FORMAT('0PROBLEMS COMPUTING PRINCIPAL VALUES OR DIRECTIONS.  ',
     $ 'ISOTROPIC VALUES & UNIT VECTORS RETURNED.'/' JELNO,INT,NSTPAB',
     $ ',INCRAB,NPASS,IOPT,ISTRN,CRIT,CTOL,DET='/7I7,1P3E11.3/
     $ ' SIG/EPS=',1P6E11.3/' PSIG/EPS=',1P3E11.3/' A=',1P9E11.3)
      NERROR=NERROR+1
      IF (NERROR.LT.50) GO TO 42
      STOP 'PVAL3D ERROR TERMINATION'
C
      END

      SUBROUTINE RSDROT(S,SP,A,PSIG,IPERM)
      IMPLICIT REAL*8 (A-H,O-Z)
C
      COMMON /FLTNUM/ ZERO,ONE,TWO,THREE,FOUR,FIVE,SIX,SEVEN,EIGHT,
     $  NINE,TEN,HALF,THIRD,FOURTH,FIFTH,SIXTH,SEVNTH,EIGHTH,NINETH,
     $  TENTH,HUNDRD,THOU,MILLON,PI,PIFAC,PIFAC1,EXPN
      REAL*8 NINE,NINETH,MILLON
C
      DIMENSION S(3,3),SP(3,3),A(3,3),PSIG(6),IPERM(3)
C
      DO 20 I=1,3
      DO 20 J=I,3
      SUM=ZERO
      DO 10 K=1,3
      AKI=A(K,I)
      DO 10 L=1,3
10    SUM=SUM+AKI*A(L,J)*S(K,L)
      SP(J,I)=SUM
20    SP(I,J)=SUM
      DO 30 N=1,3
      PSIG(N)=SP(N,N)
      I=IPERM(N)
      J=IPERM(I)
30    PSIG(N+3)=SP(I,J)
      RETURN
      END

      SUBROUTINE CROSS(A,B,C,DET,IPERM)
      IMPLICIT REAL*8 (A-H,O-Z)
C
      COMMON /FLTNUM/ ZERO,ONE,TWO,THREE,FOUR,FIVE,SIX,SEVEN,EIGHT,
     $  NINE,TEN,HALF,THIRD,FOURTH,FIFTH,SIXTH,SEVNTH,EIGHTH,NINETH,
     $  TENTH,HUNDRD,THOU,MILLON,PI,PIFAC,PIFAC1,EXPN
      REAL*8 NINE,NINETH,MILLON
C
      DIMENSION A(3),B(3),C(3),IPERM(3)
C
      DET=ZERO
      DO 10 N=1,3
      I=IPERM(N)
      J=IPERM(I)
      CC=A(I)*B(J)-A(J)*B(I)
      DET=DET+CC*CC
10    C(N)=CC
      IF (DET.LE.ZERO) RETURN
      DET=SQRT(DET)
      DO 20 N=1,3
20    C(N)=C(N)/DET
      RETURN
      END

      SUBROUTINE DIRCOS(PVAL,S,XU,CRIT,CTOL,DET)
      IMPLICIT REAL*8 (A-H,O-Z)
```

```
C
      COMMON /FLTNUM/ ZERO,ONE,TWO,THREE,FOUR,FIVE,SIX,SEVEN,EIGHT,
     $  NINE,TEN,HALF,THIRD,FOURTH,FIFTH,SIXTH,SEVNTH,EIGHTH,NINETH,
     $  TENTH,HUNDRD,THOU,MILLON,PI,PIFAC,PIFAC1,EXPN
      REAL*8 NINE,NINETH,MILLON
C
      DIMENSION S(3,3),SP(3,3),XU(3)
C
      NP=0
      DO 20 I=1,3
      XU(I)=ZERO
      DO 10 J=1,3
10    SP(J,I)=S(J,I)
20    SP(I,I)=S(I,I)-PVAL
C
30    DO 50 I=1,2
      IP1=I+1
      DO 40 J=IP1,3
      OFF=SP(I,J)
      DET=(SP(I,I)*SP(J,J)-OFF*OFF)
      IF (ABS(DET).LT.CRIT) GO TO 40
C
C               NORMAL RANK 2 MATRIX
C
      K=6-I-J
      XU(K)=ONE
      F1=-SP(I,K)
      F2=-SP(J,K)
      XU(I)=(SP(J,J)*F1-OFF*F2)/DET
      XU(J)=(SP(I,I)*F2-OFF*F1)/DET
      GO TO 60
C
40    CONTINUE
50    CONTINUE
C
C               REDUCE CRIT & TRY AGAIN
C
      DET=ABS(DET)
      IF (NP.NE.0) RETURN
      NP=1
      CRIT=CTOL*CRIT
      GO TO 30
C
60    FAC=SQRT(XU(1)**2+XU(2)**2+XU(3)**2)
      DO 70 I=1,3
70    XU(I)=XU(I)/FAC
      DET=ABS(DET)
      RETURN
      END

      SUBROUTINE TWOVEC(N,A,XU,IPERM,CTOL,DET)
      IMPLICIT REAL*8 (A-H,O-Z)
```

```
C
      COMMON /FLTNUM/ ZERO,ONE,TWO,THREE,FOUR,FIVE,SIX,SEVEN,EIGHT,
     $  NINE,TEN,HALF,THIRD,FOURTH,FIFTH,SIXTH,SEVNTH,EIGHTH,NINETH,
     $  TENTH,HUNDRD,THOU,MILLON,PI,PIFAC,PIFAC1,EXPN
      REAL*8 NINE,NINETH,MILLON
C
      DIMENSION A(3,3),XU(3),IPERM(3)
C
      I=IPERM(N)
      J=IPERM(I)
      XU(1)=ONE
      XU(2)=ZERO
      XU(3)=ZERO
      CALL CROSS(A(1,N),XU,A(1,I),DET,IPERM)
      IF (DET.GT.CTOL) GO TO 10
      XU(1)=ZERO
      XU(2)=ONE
      CALL CROSS(A(1,N),XU,A(1,I),DET,IPERM)
      IF (DET.GT.CTOL) GO TO 10
      XU(2)=ZERO
      XU(3)=ONE
      CALL CROSS(A(1,N),XU,A(1,I),DET,IPERM)
      IF (DET.LE.CTOL) RETURN
C
10    CALL CROSS(A(1,N),A(1,I),A(1,J),DET,IPERM)
      RETURN
      END

C     SUBROUTINF DUMMY
C     IMPLICIT REAL*8 (A-H,O-Z)
C     ENTRY LEON
C     ENTRY STRNEC
C
C     WRITE(6,10)
C10    FORMAT('0***BAD CALL TO DUMMY UMAT SUBROUTINE***')
C     STOP 'BAD CALL TO DUMMY UMAT SUBROUTINE'
C     END
^Z
```