



## Communication

## Modified water–cement ratio law for silica fume concretes

S. Bhanja<sup>a,\*</sup>, B. Sengupta<sup>b</sup><sup>a</sup>Department of Civil Engineering, Bengal Engineering College (D.U.), Howrah, West Bengal 711103, India<sup>b</sup>Department of Civil Engineering, Jadavpur University, Kolkata, West Bengal 700032, India

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**Abstract**

The application of condensed silica fume as a mineral admixture in concrete is almost a routine one nowadays for the production of tailor-made high-performance concretes. Abrams' Law, which was originally formulated for conventional concrete containing cement as the only cementitious material, is not directly applicable to these new-generation concretes. In the present paper, modified relationships have been proposed to evaluate the strength of silica fume concrete. An extensive experimentation was carried out to determine the isolated effect of silica fume on concrete, and, analyzing the 28-day strength results of 32 concrete mixes performed over a wide range of water–binder ratios and silica fume replacement percentages, simplified relationships have been proposed. These simplified models might serve as useful guides for proportioning concrete mixes incorporating silica fume.

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**1. Introduction**

Abrams' formulation of the water–cement ratio law in 1918 is still considered as a milestone in the history of concrete technology, and, up until now, it is accepted that the largest single factor that governs the strength of concrete is the water–cement ratio. The concept of concrete making has undergone radical changes since the formulation of Abrams' Law. Originally, concrete was made by mixing cement, aggregates and water, and the use of admixtures was unknown. The only cementitious material was cement. The present-day, new-generation concretes almost invariably incorporate mineral admixtures for a variety of reasons such as strength, durability, economy and ecological factors. Out of the different types of admixtures used nowadays, in spite of the very high price of silica fume, it seems to be the only answer when extremely high strengths and durability are required. Hence, nowadays, the water–cementitious material ratio should be considered instead of the water–cement ratio, and Abrams' formulation needs to be modified. Slanicka [1] proposed a modification of Abrams' Law

for silica fume concrete and reported that the contribution of silica fume to the strength of concrete is nonlinear. Analyzing the test results of previous researchers, Oluokun [2] has proposed an augmented water–cementitious material ratio law for the design of concrete mixes with fly ash. Though both fly ash and silica fume are pozzolanic materials and are used in conjunction with cement in concrete, their reactivity and mode of action in strength development of concrete are quite different. In the present study, a detailed investigation on the contribution of silica fume on concrete has been performed over a wide range of water–binder ratios and cement replacement percentages [3]. On the basis of 28-day strength results, modified strength water–cementitious material ratio relationships have been proposed for concrete containing cement plus silica fume as a supplementary cementitious material.

**2. Experimental program****2.1. Materials**

The constituent materials used in the program were tested to comply with the relevant Indian Standards. To assure uniformity of supply, materials were subjected to periodical control tests. The cement used was Ordinary

\* Corresponding author. Tel.: +91-33-466-2630.

E-mail addresses: san244in@yahoo.com, jucivbsg@iacs.res.in  
(S. Bhanja).

Portland Cement, having a 28-day compressive strength of 54 MPa. Silica fume, containing 90.9% SiO<sub>2</sub> and having a BET specific surface area of 18,000 m<sup>2</sup>/kg (Elkem Materials, Norway), was used. Natural river sand having a fineness modulus of 2.5 was used. The specific gravity and water absorption values were obtained as 2.65 and 0.8%, respectively. Crushed, angular, graded coarse aggregates with a nominal maximum size of 12.5 mm were used in the investigation. The specific gravity and the water absorption of the aggregates were 2.85 and 0.9%, respectively. Potable water and a high dosage of high-range water-reducing admixtures (SP) were employed for the mixing.

## 2.2. Experimental procedure

The experimental research program was carefully designated to determine the isolated effect of silica fume on the properties of concrete over a wide range of water–binder ratios by eliminating the effect of other mix design factors. Therefore, while performing the investigations other mix design variables like quality of ingredients, mix proportions including the dosage of SP, mixing procedures, curing conditions and testing procedures were kept constant. Since the SP content of all the mixes were kept constant, to minimize variations in workability the compaction energy had to be varied to achieve proper compaction. The strength of silica fume concrete depends on the water–binder ratio and silica fume content of the mix. The experimental program included five sets of concrete mixes, at w/cm ratios of 0.26, 0.30, 0.34, 0.38 and 0.42, prepared by partial replacement of cement by equal mass of silica fume. Each set had mixes at six different percentages of cement replacement. The dosages of silica fume were 0% (control mix), 5%, 10%, 15%, 20% and 25% of the total cementitious materials. For w/cm ratios of 0.38 and 0.42, even 30% silica fume dosage was adopted. The mix proportions adopted were C:FA:CA=1:1.28:2.2 for all the mixes. The total binder content was fixed at about 500 kg/m<sup>3</sup> varying from 520 kg/m<sup>3</sup> at w/cm ratio of 0.26 to 480 kg/m<sup>3</sup> at w/cm ratio of 0.42. According to Neville [4], superplasticizers can affect the concrete strength even at constant water cement ratio. Hence, the dosage of superplasticizer was also kept constant for all the mixes and, thus, the change in concrete properties at any constant water–binder ratio occurred primarily due to silica fume incorporation [5]. Specimens (150-mm cubes) were used for compressive strength determination. All the specimens were moist-cured under water at room temperature until testing. Each strength value was the average of three specimens.

## 3. Analysis of results

The 28-day compressive strength results are presented in Fig. 1. The strength values at different water–cementitious

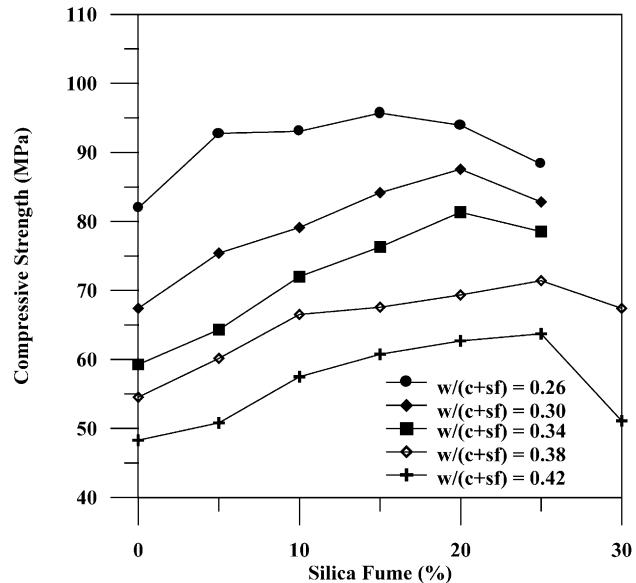


Fig. 1. Relationship between 28-day compressive strength and percentage replacement of silica fume.

material ratios have been plotted at each silica fume replacement percentage. The results indicate that the optimum silica fume replacement percentage is not constant at all the water–binder ratios but is dependent on the water content of the mix. Similar observations have been reported by Oluokun [2]. The present strength results have been analyzed to formulate statistical models relating compressive strength with water–binder ratio and silica fume contents of the mix. Oluokun [2] performed a detailed statistical analysis using the mix design variables that affect the concrete strength and reported that the variables that are the most influential in the strength development of concrete are water–cementitious material ratio, total cementitious material content and the cement admixture ratio. For the present investigation, the total binder content was fixed at about 500 kg/m<sup>3</sup> to eliminate its effect on concrete strength. Thus, for the present study, the main factors that affect the strength of concrete may be considered as the water–cementitious material ratio and the silica fume content. Hence, a modified compressive strength water–cementitious material ratio relationship may be expressed as

$$\log S = b_1 + b_2 \left( \frac{w}{cm} \right) + b_3 \left( \frac{sf}{c} \right) \quad (1)$$

where  $S$  denotes the 28-day compressive strength of concrete,  $w$ ,  $cm$ ,  $sf$  and  $c$  denote the water, cementitious material, silica fume and cement contents of the mix in kg/m<sup>3</sup> and  $b_1$ ,  $b_2$  and  $b_3$  are the constants of regression analysis. Analyzing the present strength results statistically by the principle of least squares, three normal equations involving the unknown regression parameters

are obtained [6], which, on solution, yield the following equation

$$\log S = 2.275 - 1.3366 \left( \frac{w}{cm} \right) + 0.2013 \left( \frac{sf}{c} \right) \quad (2)$$

The value of the multiple correlation coefficient ( $r$ ) has been obtained as .93 [6]. Expressed in exponential form, Eq. (2) can be written as

$$S = \frac{188.365}{21.71^\alpha} \quad (3)$$

where

$$\alpha = \frac{w}{c + sf} - 0.1506 \left( \frac{sf}{c} \right)$$

The methods of mere addition or replacement are not sufficient to properly estimate the contribution of a pozzolanic material on the properties of concrete. Its activity is often assessed in terms of the amount of cement replaced, which is termed as cementing efficiency factor. Gutierrez and Canovas [7] developed a model on the compressive strength of silica fume concrete based on a constant efficiency factor of silica fume as 4.75. Ganesh Babu and Surya Prakash [8] have reported that the efficiency of silica fume is not constant at all the percentages of replacement. A relation was proposed to determine the efficiency at different levels of replacement using which the efficiencies at 5%, 10%, 15%, 20%, 25% and 30% replacements are obtained as 6.83, 5.33, 4.06, 3.01, 2.19 and 1.59 respectively. Thus a generalized relationship involving the strength, water–binder ratio, silica fume contents and efficiency of silica fume may be expressed as

$$\log S = b_4 + b_5 \left( \frac{w}{cm} \right) + b_6 \left( \frac{ksf}{c} \right) \quad (4)$$

where  $k$  denotes the efficiency of silica fume and  $b_4$ ,  $b_5$  and  $b_6$  are the constants of regression. Using the efficiency factors as mentioned above, and solving the regression parameters for the present set of results, the following expression is obtained

$$\log S = 2.2258 - 1.299 \left( \frac{w}{cm} \right) + 0.1325 \left( \frac{ksf}{c} \right) \quad (5)$$

The value of the multiple correlation coefficient ( $r$ ) has been obtained as .98 [5]. In exponential form, Eq. (5) can be written as

$$S = \frac{168.19}{19.91^\beta} \quad (6)$$

where

$$\beta = \left( \frac{w}{cm} \right) - 0.102 \left( \frac{ksf}{c} \right)$$

Comparing the values of the correlation coefficients, it can be inferred that the incorporation of efficiency factors results in a considerable improvement of the strength model. Khedr and Abou-Zeid [9] determined the compressive strengths of silica fume concrete on 150-mm cubes at w/cm ratio of 0.3 and silica fume replacement percentages of 5, 10, 15, 20 and 25. The strength predictions as per Eq. (6) show a maximum variation of about 10% from the values obtained by Khedr and Abou-Zeid [5,8].

#### 4. Conclusions

For the present-day concretes that contain supplementary cementitious materials along with cement, Abrams' Law is not directly applicable, and modified expressions are needed to predict the compressive strengths based on the water–cementitious material ratio and mineral admixture content. In the present research, an extensive investigation was performed to determine the isolated influence of silica fume over a wide range of water–binder ratios and cement replacement percentages. Based on the 28-day strength results on 150-mm cubes, statistical models have been proposed. Though the strength of silica fume concrete depend on a wide range of parameters the present formulations might find useful applications for assessing the influence of water–binder ratio and silica fume contents on the compressive strength of silica fume concrete. The validity of the present model has been verified with the experimental results obtained by a previous researcher.

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