

# Modal Damping Estimates from Modally Filtered Data using Log Decrement

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## ABSTRACT

Obtaining accurate estimates of modal damping is vital to many structural dynamics applications. However, modal damping estimation has historically presented many challenges to engineers. The most widely used and accepted method is currently modal parameter estimation, which produces good results under the right circumstances – when clean frequency response functions (FRFs) can be obtained and all modes of interest can be adequately excited. Unfortunately, there are many applications where modal parameter estimation cannot be employed due to being unable to obtain high quality FRFs, such as when testing wind turbine blades or similarly sized structures where appropriate excitation cannot be applied and measured. For these cases operational modal analysis can be used, but damping estimation results from these techniques have a wide range of variability. This work seeks to develop a procedure to achieve modal damping estimates with lower variability than operational modal analysis methods without requiring high quality FRFs.

The logarithmic decrement technique has been considered one of the most accurate methods for modal damping estimation, but is rarely used because it requires a dissipating time response consisting of only one mode. Real-world structures will almost never provide a response of only one mode, so logarithmic decrement has remained a seldom used technique. This work shows that a modal filtering technique can be used to decompose the time response into individual modal contributions, to which the logarithmic decrement technique can be applied to estimate modal damping. The modal filtering technique uses finite element (FE) mode shapes, experimental mode shapes, or operational shapes, allowing accurate modal damping estimates to be achieved in cases where FRFs cannot be obtained. This work shows that damping estimates can now be achieved without needing to acquire FRFs, but with lower variability than results from operational modal analysis.

KEYWORDS: Logarithmic Decrement, Modal Filtering, Modal Damping, Operational Modal Analysis

## INTRODUCTION

Achieving accurate damping estimates for large structures such as wind turbine blades and bridges presents many challenges. For structures too large to adequately excite in laboratory conditions, modal parameter estimation must be conducted from operational tests rather than experimental modal tests [1]. There are a variety of methods used to achieve damping estimates from operational tests including the Peak-Picking Method, the Complex Mode Indicator Function, and Data-Driven Stochastic Subspace Identification [1, 2]. Natural frequencies and mode shapes can be well characterized using techniques for operational modal analysis, but damping estimation with these methods does not produce results comparable in accuracy to experimental modal analysis [3].

The logarithmic decrement method utilizes a decay in amplitude in the time domain to estimate modal damping, and requires the time response to consist of a single mode [4]. While this technique is considered highly accurate, in most scenarios time responses consist of multiple modes, and so the technique is rarely used. Several techniques have been utilized to decompose the time response to isolate modes individually for the purpose of applying logarithmic decrement method. These include band-pass filtering [5], the Hilbert Transform [6], and a singular value decomposition in the frequency domain [7]. However, applying a modal filtering technique to achieve modal coordinates in the time domain for use with the logarithmic decrement method has yet to be addressed.

Many modal filtering techniques focus on filtering response measurements to achieve estimates of modal coordinates [8-12]. Some of these applications have explored using the pseudoinverse of the mode shape matrix as a modal filter in the

frequency domain [13, 14]. The application of the pseudoinverse of the mode shape matrix to filter time response vectors has been discussed [15], but potential applications of this technique have not yet been fully realized. The approach presented in this work uses the pseudoinverse of the mode shape matrix to decompose decaying time responses from a sparsely instrumented structure, then applies the logarithmic decrement technique to the resulting estimates of modal coordinates to achieve modal damping estimates. The mode shapes used in the pseudoinverse can be finite element modes shapes, experimental mode shapes, or operational mode shapes, allowing the technique to be used for structures that are too large for experimental modal analysis to be performed. An analytical case study will be presented, then future work and experimental validation will be discussed.

## THEORETICAL BACKGROUND

The procedure utilizes a modal filter to decompose the time response into the individual modal estimates, then applies the logarithmic decrement method to these responses. The structure's time response is obtained at a reduced set of  $a$  degrees-of-freedom to form physical response matrix  $\{X_a\}$ . Then, the decomposition is performed as

$$p = [U_a]^T \{X_a\} \quad (1)$$

where the modal filter  $[U_a]^T$  can contain experimental mode shape values, operating shapes, or finite element mode shape values from a well-correlated finite element model. The resulting matrix  $p$  contains modal estimates in the time domain for each mode included in the modal filter. In order to achieve modal estimates that can be used to estimate damping, all modes that have a strong contribution to the response must be included in the modal filter. Otherwise, the modal estimates may include more than one mode and logarithmic decrement cannot be applied.

The logarithmic decrement is calculated from two peak amplitudes separated by  $n$  periods using

$$\delta = \frac{1}{n} \ln \frac{x(t)}{x(t+nT)} \quad (2)$$

where  $T$  is the period of the mode. For analytical case studies, a single cycle should be sufficient for accurate damping estimates, but experimental data will benefit from calculating  $\delta$  over many periods to reduce effects from noise. Once the logarithmic decrement is calculated, the damping ratio can be found using

$$\zeta = \frac{1}{\sqrt{1 + \left(\frac{2\pi}{\delta}\right)^2}} \quad (3)$$

The damping ratio is calculated using for each modal estimate separately, to achieve damping estimates for each mode contributing to the time response.

## ANALYTICAL CASE STUDY

A finite element model of a thin aluminum plate with dimensions 24"x24"x0.06" was formulated using plate elements, with a total of 2500 nodes and six degrees of freedom per node. All nodes located at the edge of the plate were pinned. To form a reduced set, referred to as aDOF, an evenly spaced grid of 25 nodes were selected and the transverse degree of freedom was used, resulting in twenty-five aDOF. The full field and reduced set of aDOF are shown in Figure 1.

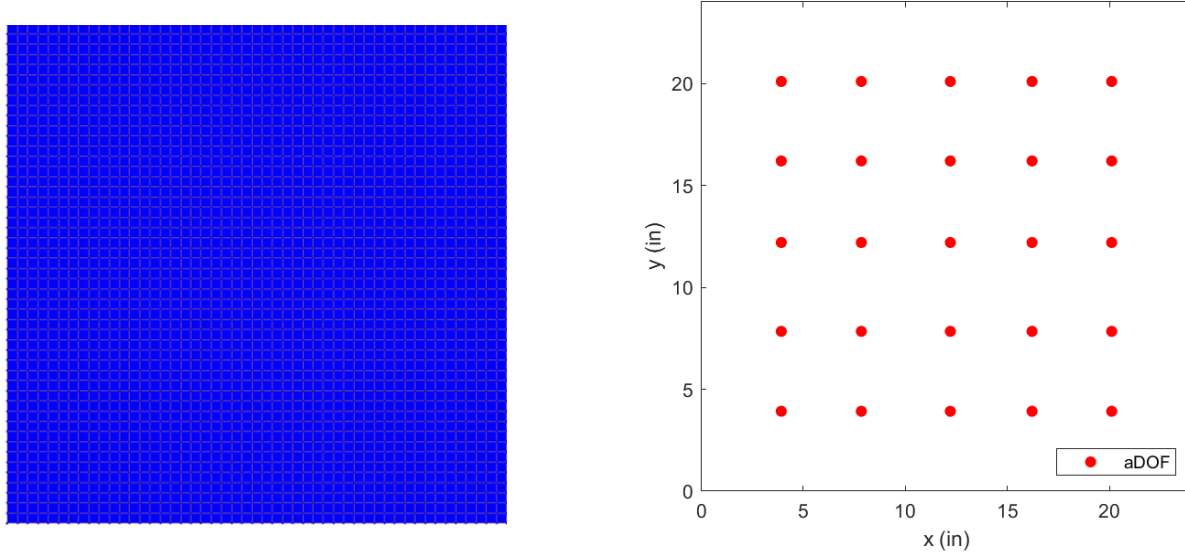


Figure 1. Full field finite element model (left) and reduced set of aDOF (right).

Physical displacement in response to an input force pulse was calculated using modal superposition to achieve response data at aDOF. The first six modes of the plate were included in the modal superposition calculation, whose frequencies range between 19.7 Hz and 98.8 Hz. Each mode was assigned a unique critical damping value between 2% and 3%, to confirm that the damping value for each mode can be independently extracted. A force input pulse exciting up to 400 Hz was used as excitation, shown in Figure 2. The calculated response for the aDOF in the lower left corner of the plate is shown in Figure 3.

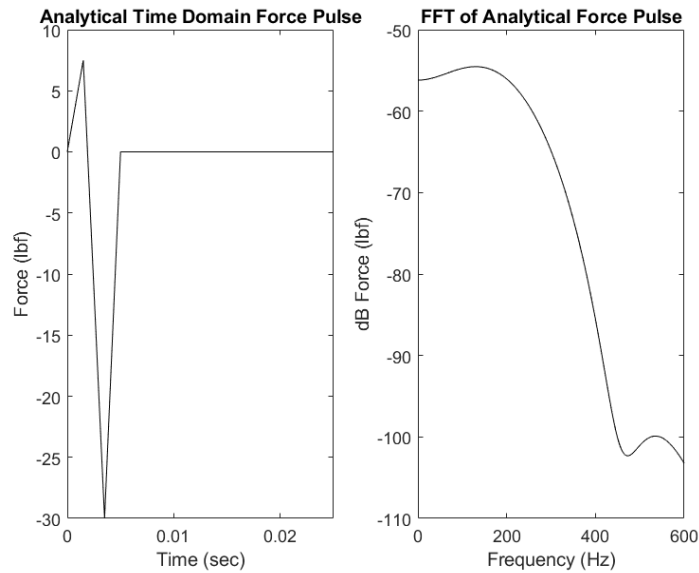


Figure 2. Force input in the time (left) and frequency (right) domains.

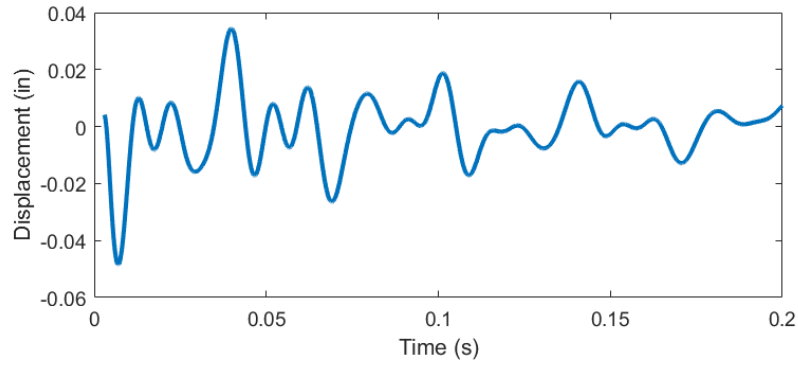


Figure 3. Displacement Response for a single DOF of the analytical plate model

Using the generalized inverse of the mode shape matrix at aDOF, the time response was decomposed as described in Eq. (1) to achieve estimated modal coordinates for the six modes excited in the time response. Figure 4 shows that the estimated modal coordinates replicate the original modal contributions from the modal superposition process, which implies that the logarithmic decrement results should be accurate. The logarithmic decrement process was applied to the first two cycles of the decaying time responses to achieve damping estimates, which are listed in Table 1. For this highly simplified analytical case, the logarithmic decrement process correctly reproduced the defined damping values with no error.

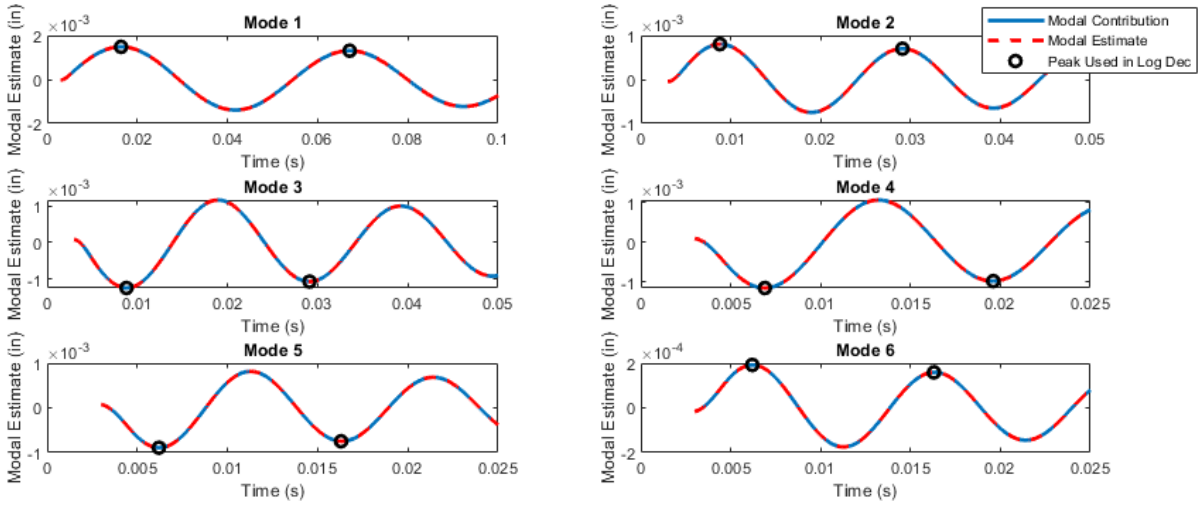


Figure 4. Estimated modal coordinates compared to the modal contributions, marked with the peak amplitudes used in the logarithmic decrement process to estimate damping

Table 1. Comparison of defined modal damping to modal damping estimated using log decrement.

Mode	$\zeta$	$\zeta$ Estimate	Error (%)
1	2%	2%	0
2	2.2%	2.2%	0
3	2.4%	2.4%	0
4	2.6%	2.6%	0
5	2.8%	2.8%	0
6	3.0%	3.0%	0

## EXPERIMENTAL DATA AND FUTURE WORK

Experimental data has been collected and will be the subject of future publications. In general, the technique appears to perform well on data collected thus far. Additional cases need to be studied with less pristine conditions where noise and other contaminants are present to allow for a complete evaluation of a more realistic test environment.

Future work will provide experimental validation for the technique on a real-world structure. Operational mode shapes, experimental mode shapes, and finite element mode shapes will be obtained for use in the modal filtering process. Damping estimates from the logarithmic decrement process will be compared to damping estimates from more traditional modal estimation programs commercially available.

## CONCLUSION

A method for damping estimation using the logarithmic decrement method on estimated modal coordinates achieved using a modal filtering process was presented. The underlying theory was discussed, and an analytical case study was shown to demonstrate the procedure. The technique allows for damping estimates to be achieved when experimental modal analysis cannot be employed due to size or location constraints. Future work will provide experimental validation for the technique, as well as study the susceptibility to noise.

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