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Road vehicles — Objective rating metric for non- ambiguous signals

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TECHNICAL SPECIFICATION

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Road vehicles — Objective rating metric for non-ambiguous signals

*Véhicules routiers — Mesures pour l'évaluation objective de signaux
non ambigus*



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Road vehicles — Objective rating metric for non-ambiguous signals

1 Scope

This Technical Specification (TS) provides validation metrics and rating procedures to be used to calculate the level of correlation between two non-ambiguous signals obtained from a physical test and a computational model, and is aimed at vehicle safety applications. The objective comparison of time-history signals of model and test is validated against various loading cases under different types of physical loads such as forces, moments, and accelerations. However, other applications might be possible too, but are not within the scope of this Technical Specification.

2 Normative references

There are no normative references used in this document.

3 Terms and definitions

For the purposes of this document, the following terms and definitions apply.

3.1

filtering

smoothing of signals by using standardized algorithms

3.2

goodness or level of correlation

similarity of two signals

3.3

interval of evaluation

time domain that is used to calculate the correlation between two signals

3.4

rating

rating score

calculated value that represents a certain level of correlation (objective rating)

3.5

sampling rate

recording frequency of a signal

3.6

time sample

pair values (e.g. time and amplitude) of a recorded signal

3.7

time-history signal

physical value recorded in a time domain; those signals are non-ambiguous

4 Symbols and abbreviated terms

CAE

Computer-aided engineering

CORA	Correlation and analysis
DTW	Dynamic time warping
EEARTH	Enhanced error assessment of response time histories
SME	Subject matter expert
a_0	Relative half width of the inner corridor
b_0	Relative half width of the outer corridor
$C, C(t)$	Analysed signal (CAE signal)
$C^{ts}, C^{ts}(i)$	Truncated and shifted CAE curve
C^{ts+d}	Derivative CAE curve, C^{ts}
C^{ts+w}	Warped CAE curve, C^{ts}
DTW	Dynamic time warping distance
$DTW_{opt}(i, j)$	Cost of the optimal warping path
d	Local cost matrix to perform the dynamic time warping
$d(i, j)$	Local cost function to perform the dynamic time warping
$dtw[i, j]$	Cumulative cost matrix
Δt	Interval between two time samples
δ_i	Half width of the inner corridor
$\delta_i(t)$	Lower/upper bounds of the inner corridor at time, t , (curve)
δ_o	Half width of the outer corridor
$\delta_o(t)$	Lower/upper bounds of the outer corridor at time, t , (curve)
E_M	Magnitude score
E_P	Phase score
E_S	Slope (topology) score
ε_M^*	Maximum allowable magnitude error
ε_P^*	Maximum allowable percentage of time shift
ε_S^*	Maximum allowable slope error
ε_{mag}	Magnitude error
ε_{slope}	Slope error
i	Index number of time shifted and truncated CAE curve, C^{ts}
i_k	Index number of k -th warping path of curve, C^{ts}

j_w	Index number of warping path of CAE curve, C^{ts}
j	Index number of time shifted and truncated test curve, T^{ts}
j_k	Index number of k -th warping path of curve, T^{ts}
j_w	Index number of warping path of test curve, T^{ts}
k	Index number
k_M	Exponent factor for calculating the magnitude score, E_M
k_P	Exponent factor for calculating the phase score, E_P
k_S	Exponent factor for calculating the slope score, E_S
k_Z	Exponent factor for calculating the corridor score between the inner and outer corridors
m	Time steps moved to evaluate the phase error
N	Total number of sample points (e.g. time steps) between the starting time, t_{start} , and ending time, t_{end}
$N > 0$	All natural numbers without zero
n	Number of data samples of time shifted and truncated curves (C^{ts} and T^{ts})
n_w	Number of data samples of the optimal warping path
n_E	Number of time shifts to get ρ_E
ρ_E	Maximum cross correlation of all $\rho_L(m)$ and $\rho_R(m)$
$\rho_L(m)$	Cross correlation (signal is moved to the left)
$\rho_R(m)$	Cross correlation (signal is moved to the right)
R	Overall ISO rating
r	Rank of the sliding scale of the ISO metric
$SC_{lower}(r)$	Lower threshold of rank, r
$SC_{upper}(r)$	Upper threshold of rank, r
$T, T(t)$	Reference signal (test signal)
T_{norm}	Absolute maximum amplitude of the reference signal, T
$T^{ts}, T^{ts}(j)$	Truncated and shifted test curve
T^{ts+d}	Derivative test curve, T^{ts}
T^{ts+w}	Warped test curve, T^{ts}
t	Time signal (axis of abscissa)
t_{end}	Ending time of the interval of evaluation
t_{start}	Starting time of the interval of evaluation

t_0	Time zero of an event (e.g. test, crash, impact etc.)
w	Warping path
w_M	Weighting factor of the magnitude score, E_M
w_P	Weighting factor of the phase score, E_P
w_S	Weighting factor of the slope score, E_S
w_Z	Weighting factor of the corridor score, Z
w_k	The k -th warping path cell
Z	Corridor score
$Z(t)$	Corridor score at time, t , (curve)

5 General data requirements

The metric described in this Technical Specification requires non-ambiguous curves (e.g. time-history curves). Furthermore, it is required that the reference curve, $T(t)$, and the evaluated curve, $C(t)$, are both defined between starting time, t_{start} , and ending time, t_{end} . Both curves shall have the same number of sample points, N , with a constant time interval, Δt , within the evaluation interval.

6 ISO metric

The approach of this Technical Specification is to combine different types of algorithms to get reliable and robust assessments of the correlation of two signals. The calculated score must provide fair assessment for poor and for good correlations of two signals. The two most promising metrics are identified in Reference [10] and they are CORA corridor method and EEARTH. A combined metric based on the improved CORA corridor method and EEARTH is then proposed for an ISO Technical Specification which has been fully validated using responses from multiple vehicle passive safety applications.

Figure 1 shows the structure of the overall ISO metric. While the corridor method calculates the deviation between curves with the help of automatically generated corridors, the EEARTH method analyses specific curve characteristics such as phase shift, magnitude, and shape. Hence, the ISO metric consists of the two best available algorithms.

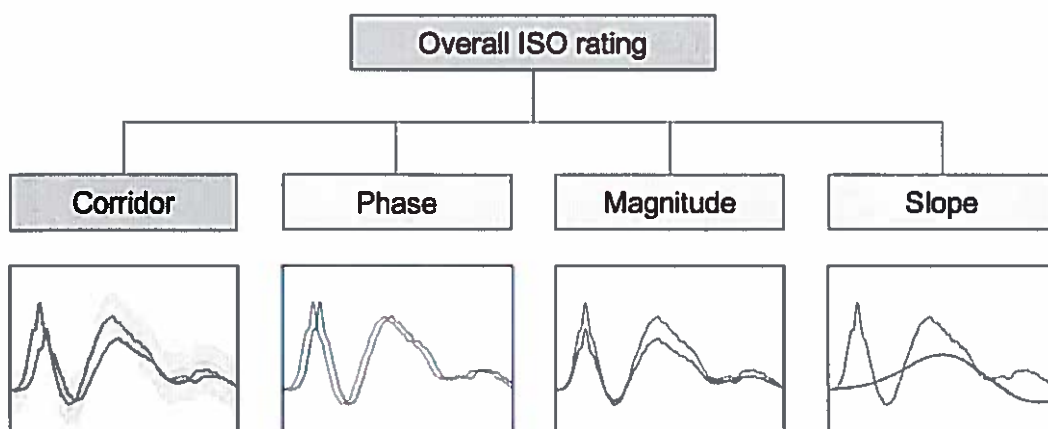


Figure 1 — ISO metric structure

6.1 Calculation of the overall ISO rating

The combination of the four metric ratings (corridor, phase, magnitude, and slope) will provide a single number, R , for the correlation of the analysed signals which represents the final overall objective rating. The overall objective rating, R , is calculated by combining the separate sub-ratings of corridor (Z), phase (E_P), magnitude (E_M), and slope (E_S). Four individual weighting factors are defining the influence of each metric on the overall rating [see Formulae (1) and (2)]. The corresponding weighting factors are shown in Table 1.

$$R = w_Z \cdot Z + w_P \cdot E_P + w_M \cdot E_M + w_S \cdot E_S \quad (1)$$

$$w_Z + w_P + w_M + w_S = 1 \quad (2)$$

Table 1 — Weighting factors of the ISO sub-ratings

Parameter	Value	Description
w_Z	0,4	Weighting factor of the corridor score
w_P	0,2	Weighting factor of the phase score
w_M	0,2	Weighting factor of the magnitude score
w_S	0,2	Weighting factor of the slope score

6.2 Corridor score

The corridor metric calculates the deviation between two signals by means of corridor fitting. The two sets of corridors, the inner and the outer corridors, are defined along the mean curve. If the evaluated curve, C , is within the inner corridor bounds, a score of "1" is given and if it is outside the outer corridor bounds, the score is set to "0". The assessment declines from "1" to "0" between the bounds of inner and outer corridors resulting in three different rating zones as shown in Figure 2. The compliance with the corridors is calculated at each specific time, t , and the final corridor score, Z , of a signal is the average of all scores, $Z(t)$, at specific times, t .

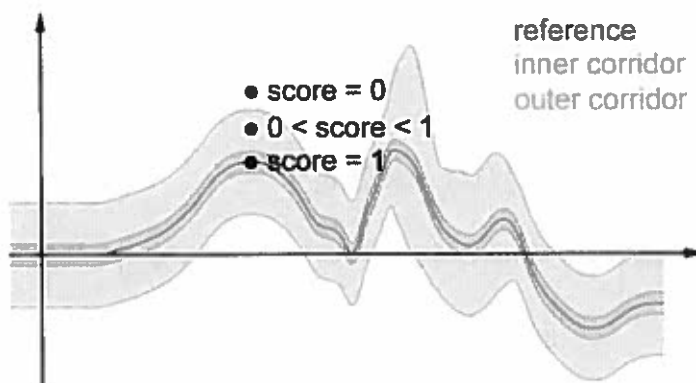


Figure 2 — Rating zones of the corridor metric (corridors of constant width)[9]

The philosophy of the ISO approach is to use a narrow inner corridor and a wide outer corridor.[14] It limits the number of "1" ratings to only good correlations and gives the opportunity to distinguish between poor and fair correlations. If the outer corridor is too narrow, too many curves of a fair or moderate correlation would get the same poor rating of "0", like signals of almost no correlation with the reference. Basically the width of the corridors can be adjusted in order to reflect the specific signal characteristic. The width can be constant for the whole duration of the dynamic responses or vary at the different time intervals. This Technical Specification applies the most common approach of using constant corridor widths for the whole duration of the dynamic response.[10] [26]

6.2.1 Calculation

The parameters a_0 and b_0 define the relative half widths of the inner and the outer corridors. Both shall be between "0" and "1", and a_0 must be less than b_0 . The absolute half widths of both corridors are defined as the product of relative half width and the absolute maximum amplitude, T_{norm} , of the reference signal, T . Formula (3) shows the calculation of T_{norm} and it is calculated within the interval of evaluation.

$$T_{norm} = \max\{|\min(T)|, |\max(T)|\} \quad (3)$$

The absolute half width of the inner corridor (absolute distance from the reference signal to the outer bounds of the inner corridor) is defined by Formula (4). The calculation of the absolute half width of the outer corridors [see Formula (5)] is similar to that of the inner corridors.

$$\delta_i = a_0 \cdot T_{norm} \quad 0 \leq a_0 \leq 1 \quad (4)$$

$$\delta_o = b_0 \cdot T_{norm} \quad 0 \leq b_0 \leq 1 \text{ and } a_0 < b_0 \quad (5)$$

Based on these definitions, the lower and upper bounds of the inner corridor are defined by Formula (6) and the lower and upper bounds of the outer corridor are defined by Formula (7).

$$\delta_i(t) = T(t) \pm \delta_i \quad (6)$$

$$\delta_o(t) = T(t) \pm \delta_o \quad (7)$$

Formula (8) shows the calculation of the corridor score for the correlation between the reference signal, T , and the analysed signal, C , at each evaluation time, t . If the absolute difference between the signals, T and C , is less than the half width of the inner corridor (δ_i), then the score is set to "1". The score is calculated by Formula (8) when the absolute difference between both signals is in between $\delta_i \leq |T(t) - C(t)| \leq \delta_o$. If the absolute difference between both signals is greater than the half width of the outer corridor (δ_o), then the score is set to "0". The parameter, k_Z , assesses the location of the analysed signal within the outer corridor, and it applies the appropriate penalty on the rating score. A linear ($k_Z = 1$), quadratic ($k_Z = 2$), cubical ($k_Z = 3$) or any other regression relationship can be defined accordingly.

$$Z(t) = \begin{cases} 1 & \text{if } |T(t) - C(t)| < \delta_i \\ \left(\frac{\delta_o - |T(t) - C(t)|}{\delta_o - \delta_i} \right)^{k_Z} & \text{if } \delta_i \leq |T(t) - C(t)| \leq \delta_o \\ 0 & \text{if } |T(t) - C(t)| > \delta_o \end{cases} \quad k_Z \in N_{>0} \quad (8)$$

The final corridor score, Z , is calculated by averaging all single time step score $Z(t)$ as shown in Formula (9). The parameter, N , represents the total number of sample points (e.g. time steps) between starting and ending times of the interval of evaluation.

$$Z = \frac{\sum_{t=t_{start}}^{t_{end}} Z(t)}{N} \quad (9)$$

One of the advantages of the corridor metric is the simplicity and the clearness of the algorithm. It reflects criteria which are used intuitively in engineering judgment. Sometimes this simplicity may be the disadvantage of the method. For example, a small distortion of the phase can lead to a very undesirable rating.^[10]

Based on a sensitivity study of CORA^[14] and as described in Reference [10], fixed width corridors are employed and the most appropriate metric parameters are identified as shown in Table 2.

Table 2 — Parameters of the corridor metric

Parameter	Value	Description
a_0	0,05	Relative half width of the inner corridor
b_0	0,50	Relative half width of the outer corridor
k_z	2	Transition between ratings of "1" and "0" (progression)

6.2.2 Step by step procedure

First of all, the signals shall be pre-processed as described in Clause 8. After preparing the signals for the analysis and defining the interval of evaluation, the maximum absolute amplitude, T_{norm} , of the reference signal, T , shall be determined within this interval. It is used to calculate the inner and outer corridors. The actual corridor assessment shall be executed within this defined interval. The total score ranges between "0" and "1". A score of "1" does not mean that both signals are identical. Solely their correlation is mathematically perfect within the defined tolerances.

To summarize, the following step-by-step procedures shall be followed to calculate corridor score.

- Pre-process both signals according to Clause 8.
- Calculate T_{norm} within the interval of evaluation by using the reference signal.
- Calculate the inner and the outer corridors.
- Calculate the corridor score, $Z(t)$, at every specific time t within the interval of evaluation.
- Calculate the total corridor score, Z , based on $Z(t)$ and the number, N , of time samples.

6.3 Phase, magnitude, and slope scores

Phase, magnitude, and slope (or so-called topology) error assessments between the time history curves, T and C , are used as objective rating metrics^{[24] [28]} in addition to the corridor metric described earlier. The enhanced error assessment of response time histories (EEARTH) metric combines these three assessments to the global response error.^[28] It is defined as the error associated with the complete time history with equal weight on each point. Quantifying the errors associated with these features of phase, magnitude, and slope (topology) separately is challenging because there are strong interactions among them. For example, to quantify the error associated with magnitude, the presence of a phase difference between the time histories may result in a misleading measurement. A unique feature dynamic time warping (DTW)^[22] is used to separate the interaction of phase, magnitude, and slope (topology) errors. It aligns peaks and valleys of the two signals as much as possible by expanding and compressing the time axis according to a given cost (distance) function.^[8]

The ranges of the three errors are quite different and there is no single rating that can provide a quantitative assessment alone. Therefore, a numerical optimization method is employed to identify the appropriate parameters so that the resulted phase, magnitude, and slope sub-ratings can match with SME's ratings closely.^{[7] [21]} Figure 3 shows the workflow of the procedures and the details of the algorithms are described in the following subsections.

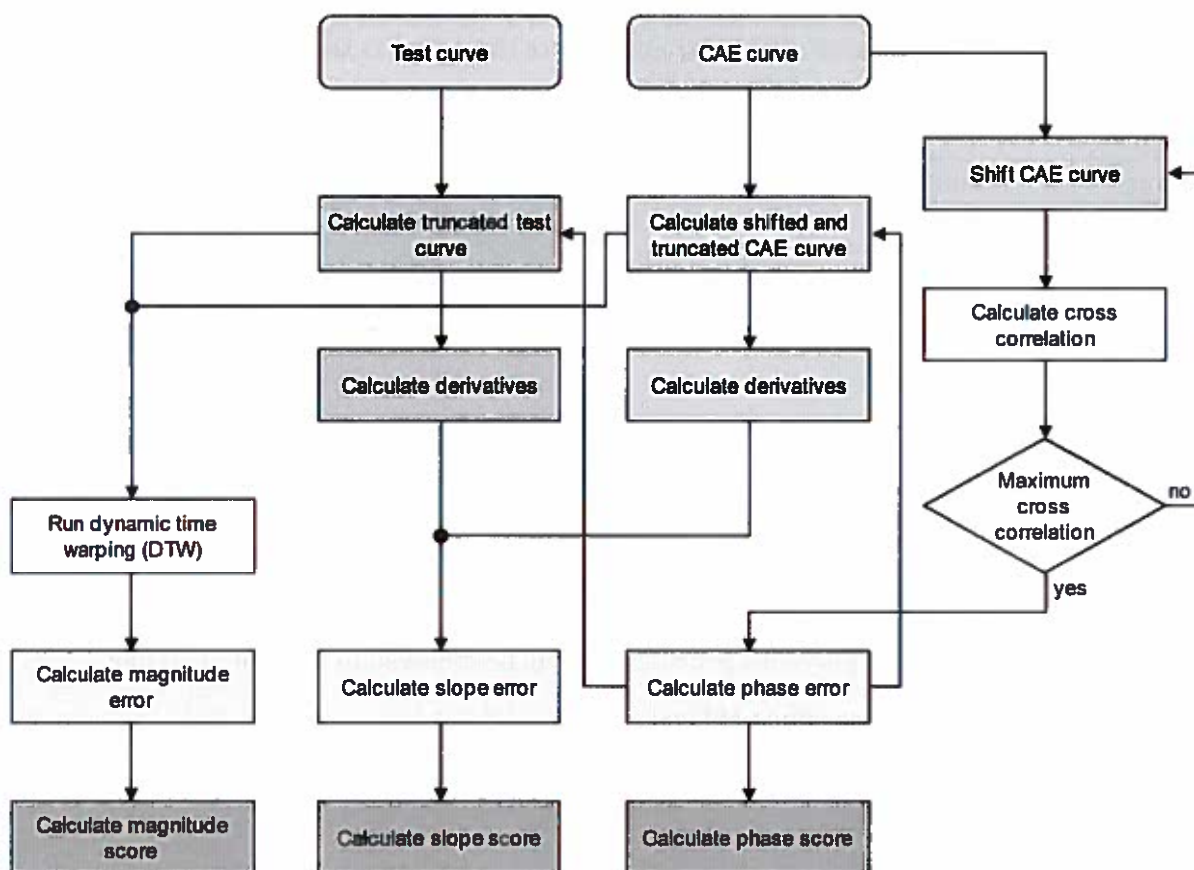


Figure 3 — Workflow of the calculation of phase, magnitude, and slope scores

6.3.1 Phase score

The phase score, E_p , is used to measure the phase lag between the two time histories, T and C . The maximum allowable percentage of time shift is ε_p^* and it is pre-defined. In this step, the initial curve, C , is shifted left then right one step at a time to the original test data, curve T , and the cross correlation between the truncated test curve T , and shifted and truncated curve, C , are calculated until reaching the maximum allowable time shift limits $\varepsilon_p^* \cdot (t_{end} - t_{start})$.

When the initial curve, C , is moved to the left by m time steps, the number of overlapping points of the two time histories after time shift, $m \cdot \Delta t$, is reduced to n ($n = N - m$) and the corresponding cross correlation value, $\rho_L(m)$, is calculated by Formula (10).

$$\rho_L(m) = \frac{\sum_{i=0}^{n-1} [C(t_{start} + (m+i) \cdot \Delta t) - \bar{C}(t)] \cdot [T(t_{start} + i \cdot \Delta t) - \bar{T}(t)]}{\sqrt{\sum_{i=0}^{n-1} [C(t_{start} + (m+i) \cdot \Delta t) - \bar{C}(t)]^2} \cdot \sqrt{\sum_{i=0}^{n-1} [T(t_{start} + i \cdot \Delta t) - \bar{T}(t)]^2}} \quad (10)$$

When the initial curve C is moved to the right by m time steps, the number of overlapping points after time shift, $m \cdot \Delta t$, is reduced to n ($n = N - m$) and the corresponding cross correlation value, $\rho_R(m)$, is calculated by Formula (11).

$$\rho_R(m) = \frac{\sum_{i=0}^{n-1} [C(t_{start} + i \cdot \Delta t) - \bar{C}(t)] \cdot [T(t_{start} + (m+i) \cdot \Delta t) - \bar{T}(t)]}{\sqrt{\sum_{i=0}^{n-1} [C(t_{start} + i \cdot \Delta t) - \bar{C}(t)]^2} \cdot \sqrt{\sum_{i=0}^{n-1} [T(t_{start} + (m+i) \cdot \Delta t) - \bar{T}(t)]^2}} \quad (11)$$

The maximum cross correlation, ρ_E , is the maximum of all $\rho_L(m)$ and $\rho_R(m)$. The number of the time shifting steps that yields the maximum cross correlation, ρ_E , is defined as the phase error, n_E . The corresponding shifted and truncated CAE curve, C , is recorded as C^{ts} and the corresponding truncated test curve, T , is recorded as T^{ts} .

The phase score, E_P , is calculated by Formula (12). The best phase score is "1", which means there is no need to shift the CAE curve to reach the maximum cross correlation between the initial test and CAE curves. If the time shift, n_E , is equal to or greater than the maximum allowable time shift threshold $\varepsilon_P^* \cdot N$, then the phase score is "0". In between, the phase score is calculated by a regression method. It is either linear ($k_P = 1$), quadratic ($k_P = 2$), or cubical ($k_P = 3$).

$$E_P = \begin{cases} 1 & \text{if } n_E = 0 \\ \left(\frac{\varepsilon_P^* \cdot N - n_E}{\varepsilon_P^* \cdot N} \right)^{k_P} & \text{if } 0 < n_E < \varepsilon_P^* \cdot N \\ 0 & \text{if } n_E \geq \varepsilon_P^* \cdot N \end{cases} \quad k_P \in \{1, 2, 3\} \quad (12)$$

The pre-defined parameters shown in Table 3 are identical to the definition in Reference [10].

Table 3 — Fixed parameters of the phase score

Parameter	Value	Description
k_P	1	Exponent factor for calculating the phase score
ε_P^*	0,2	Maximum allowable percentage of time shift

6.3.2 Magnitude score

The magnitude error is a measure of discrepancy in the amplitude of the two time histories. It is defined as the difference in amplitude of the two time histories when there is no time lag between them. Before calculating the magnitude error, the difference between the time histories caused by error in phase and slope (topology) are minimized by using dynamic time warping (DTW).

The definition of DTW is based on the notion of warping path. Let d be the matrix $n \times n$ of pair-wise squared distances between samples of C^{ts} and T^{ts} . This matrix, d , is called the local cost matrix. The

function used to calculate the value for each cell of the matrix is called local cost function, $d(i, j)$. It is shown in Formula (13).

$$d(i, j) = (C^{ts}(i) - T^{ts}(j))^2 \quad (13)$$

Once the local cost matrix is built, the algorithm finds the alignment path which runs through the low-cost areas on the cumulated cost matrix. A warping path, w , [see Formula (14)] is a sequence of k matrix cells, w_k , [see Formula (15)].

$$w = \langle w_1, w_2, \dots, w_k \rangle \quad n \leq k \leq (2n-1) \quad (14)$$

$$w_k = [i_k, j_k] \quad i_k \geq 1 \quad \text{and} \quad j_k \leq n \quad (15)$$

The local cost matrix must meet the following three conditions:

— boundary conditions

$w_1 = [1, 1]$ and $w_k = [n, n]$, i.e. w starts in the lower left cell and ends in the upper right cell.

— continuity

Given $w_{k-1} = [i_{k-1}, j_{k-1}]$ and $w_k = [i_k, j_k]$, then $i_k - i_{k-1} \leq 1$ and $j_k - j_{k-1} \leq 1$. This ensures that the cells of the warping path are adjacent.

— monotonicity

Given $w_{k-1} = [i_{k-1}, j_{k-1}]$ and $w_k = [i_k, j_k]$, then $i_k - i_{k-1} \geq 0$ and $j_k - j_{k-1} \geq 0$, with at least one strict inequality. This enforces w to progress over time.

The DTW distance is recursively computed using a dynamic programming approach that fills the cells of a cumulative cost matrix, $dtw[i, j]$, and recurrence relation [see Formula (16)]. Then the DTW distance is evaluated as shown in Formula (17).

$$dtw[i, j] = \begin{cases} d(i, j) & i = 1, j = 1 \\ d(i, j) + dtw[i, j-1] & i = 1 \\ d(i, j) + dtw[i-1, j] & j = 1 \\ d(i, j) + \min(dtw[i-1, j-1], dtw[i-1, j], dtw[i, j-1]) & \text{otherwise} \end{cases} \quad (16)$$

$$DTW = dtw[n, n] \quad (17)$$

The warping path which has a minimal cost associated with alignment is called the optimal warping path. It is found by following the definition that every possible warping path between C^{ts} and T^{ts} should be tested which could be computationally challenging due to the exponential growth of the number of optimal paths as the lengths of C^{ts} and T^{ts} grow linearly.

Any warping path w defines an alignment between C^{ts} and T^{ts} and, consequently, a cost to align the two histories. $DTW_{opt}(C^{ts}, T^{ts})^2$ is the minimum of such costs, i.e. the cost of the optimal warping path [see Formula (18)].

$$DTW_{opt}(C^{ts}, T^{ts})^2 = \min_w \left(\sum_{[i_k, j_k] \in w} d(i_k, j_k) \right) \quad (18)$$

Let i^w and j^w represent the index of warping path of CAE and test data. A warping path index w is formed as shown in Formula (19). It starts with $i^w = n$ and $j^w = n$, then it records the optimal warping

path of test and CAE by recording each time step from $[n, n]$ to $[1, 1]$. The algorithms can be expressed as shown in Formula (20).

$$w = \begin{bmatrix} j^w & j^w - 1 & \dots & 1 \\ i^w & i^w - 1 & \dots & 1 \end{bmatrix}^T \quad (19)$$

$$[i^w - 1, j^w - 1] = \begin{cases} [i-1, 1] & j=1 \\ [1, j-1] & i=1 \\ [i-1, j] & dtw[i-1, j] = \min(dtw[i-1, j-1], dtw[i-1, j], dtw[i, j-1]) \\ [i, j-1] & dtw[i, j-1] = \min(dtw[i-1, j-1], dtw[i-1, j], dtw[i, j-1]) \\ [i-1, j-1] & dtw[i-1, j-1] = \min(dtw[i-1, j-1], dtw[i-1, j], dtw[i, j-1]) \end{cases} \quad (20)$$

Hence, the index matrix of warping path, w , can be expressed by the index of CAE (C^{ts}) and test (T^{ts}) curves as shown in Formula (21).

$$w = \begin{bmatrix} j^w & j^w - 1 & \dots & 1 \\ i^w & i^w - 1 & \dots & 1 \end{bmatrix}^T = \begin{bmatrix} n & n-1 & \dots & 1 \\ n & n-1 & \dots & 1 \end{bmatrix}^T \quad w \in \mathbb{R}^{n_w \times 2} \text{ and } n_w > n \quad (21)$$

Then the truncated and warped test curve T^{ts+w} and the shifted, truncated, and warped CAE curve, C^{ts+w} , are formed as shown in Formulae (22) and (23).

$$[T^{ts+w}(i^w), T^{ts+w}(i^w - 1), \dots, T^{ts+w}(1)] = [T^{ts}(n), T^{ts}(n-1), \dots, T^{ts}(1)] \quad T^{ts+w} \in \mathbb{R}^{n_w} \quad (22)$$

$$[C^{ts+w}(i^w), C^{ts+w}(i^w - 1), \dots, C^{ts+w}(1)] = [C^{ts}(n), C^{ts}(n-1), \dots, C^{ts}(1)] \quad C^{ts+w} \in \mathbb{R}^{n_w} \quad (23)$$

The magnitude error, ε_{mag} , is calculated by Formula (24).

$$\varepsilon_{mag} = \frac{\|C^{ts+w} - T^{ts+w}\|_1}{\|T^{ts+w}\|_1} \quad (24)$$

Formula (25) is used to calculate the magnitude score (E_M), where ε_M^* is the maximum allowable magnitude error and k_M defines the order of the regression. The best magnitude score is "1", which means there is no difference in the amplitudes after phase shift and dynamic time warping. If the magnitude error, ε_{mag} , is equal to or greater than the maximum allowable magnitude error threshold (ε_M^*), then the magnitude score is "0". In between, the magnitude score is calculated by regression method.

$$E_M = \begin{cases} 1 & \text{if } \varepsilon_{mag} = 0 \\ \left(\frac{\varepsilon_M^* - \varepsilon_{mag}}{\varepsilon_M^*} \right)^{k_M} & \text{if } 0 < \varepsilon_{mag} < \varepsilon_M^* \\ 0 & \text{if } \varepsilon_{mag} \geq \varepsilon_M^* \end{cases} \quad k_M \in \{1, 2, 3\} \quad (25)$$

The pre-defined parameters shown in Table 4 are identical to the definition in Reference [10].

Table 4 — Fixed parameters of the magnitude score

Parameter	Value	Description
k_M	1	Exponent factor for calculating the magnitude score
ε_M^*	0,5	Maximum allowable magnitude error

6.3.3 Slope score

The slope error is a measure of discrepancy in slope (topology) of the two time histories. The slope of a time history is defined by the slope at each point. In order to ensure that the effect of global time shift is minimized, the slope is calculated from the time shifted histories, T^{ts} and C^{ts} .

The time shifted histories, T^{ts} and C^{ts} , are first divided into multiple intervals with a fixed length of 10 data points (1 ms sampling rate of 10 kHz). If the total number of data points of the whole signal is not a multiple of 10, then the last interval shall be calculated with the remaining data points. Next, the average slope is calculated in each interval to generate the slope curves (C^{ts+d} and T^{ts+d}). Therefore, the slope curves are used to calculate the slope error directly without performing dynamic time warping. Both curves are then used to calculate the slope error, ε_{slope} , by Formula (26).

$$\varepsilon_{slope} = \frac{\|C^{ts+d} - T^{ts+d}\|_1}{\|T^{ts+d}\|_1} \quad (26)$$

Formula (27) is used to calculate the slope score (E_S), where ε_S^* is the maximum allowable slope error and k_S defines the order of the regression. The best slope score is "1", which means there is no difference between the two slope curves. If the slope error, ε_{slope} , is equal to or greater than the maximum allowable slope error (ε_S^*), then the slope score is "0". In between, the slope score is calculated by regression method.

$$E_S = \begin{cases} 1 & \text{if } \varepsilon_{slope} = 0 \\ \left(\frac{\varepsilon_S^* - \varepsilon_{slope}}{\varepsilon_S^*} \right)^{k_S} & \text{if } 0 < \varepsilon_{slope} < \varepsilon_S^* \\ 0 & \text{if } \varepsilon_{slope} \geq \varepsilon_S^* \end{cases} \quad k_S \in \{1, 2, 3\} \quad (27)$$

The pre-defined parameters shown in Table 5 are identical to the definition in Reference [10].

Table 5 — Fixed parameters of the slope score

Parameter	Value	Description
k_S	1	Exponent factor for calculating the slope score
ε_S^*	2,0	Maximum allowable slope error

6.3.4 Step by step procedure

The following step by step process shall be followed to calculate the phase, magnitude, and slope scores.

- Pre-process both signals according to Clause 8 (T and C).
- Calculate the phase error in terms of time steps, n_e , by maximizing cross correlation.
- Calculate the phase score, E_P .
- Calculate the shifted and truncated time history curves, T^{ts} and C^{ts} .

- e) Perform dynamic time warping on the shifted and truncated time history curves to generate the shifted, truncated, and warped time history curves, T^{ts+w} and C^{ts+w} .
- f) Calculate the magnitude error, ε_{mag} , between T^{ts+w} and C^{ts+w} .
- g) Calculate the magnitude score, E_M .
- h) Generate the shifted and truncated derivative time history curves, T^{ts+d} and C^{ts+d} .
- i) Calculate the slope error, ε_{slope} , between T^{ts+d} and C^{ts+d} .
- j) Calculate the slope score, E_S .

7 Meaning of the overall ISO rating

The objective rating score, R , ranges from "0" to "1". The higher the score the better the correlation between the two signals. This single-rating number can be transferred to a grade that represents the goodness of the correlation by using a sliding scale (see Table 6).

Table 6 — Sliding scale of the overall ISO rating

Rank, r	Grade	Rating, R	Description
1	Excellent	$R > 0,94$	Almost perfect characteristics of the reference signal is captured
2	Good	$0,80 < R \leq 0,94$	Reasonably good characteristics of the reference signal is captured, but there are noticeable differences between both signals
3	Fair	$0,58 < R \leq 0,80$	Basic characteristics of the reference signal is captured but there are significant differences between the two signals
4	Poor	$R \leq 0,58$	Almost no correlation between the two signals

The lower and upper bounds of the different scales are calculated by using Formulae (28) and (29). Every grade is bounded by $[SC_{lower}(r), SC_{upper}(r)]$ except the fourth grade "poor" because there is no lower threshold $SC_{lower}(r=4)$ defined.

$$SC_{lower}(r) = 1 - \frac{1}{25}r^2 - \frac{1}{50}r \quad r \in \{1, 2, 3\} \quad (28)$$

$$SC_{upper}(r) = 1 - \frac{1}{25}(r-1)^2 - \frac{1}{50}(r-1) \quad r \in \{1, 2, 3, 4\} \quad (29)$$

However, the thresholds of R in each grade are only valid if all the parameters (e.g. weighting factors, regression schemes, sampling rates, etc.) described in the previous sections are not altered.

8 Pre-processing of the data

During the evaluation and validation of the ISO metric, it was concluded that basic conditions for the compared signals such as, starting and ending times of the signals, sampling rate, and filtering class must be kept the same in order to obtain correct results. This must be done by the user.

8.1 Synchronization of the signals

Initially, the signals must be synchronized by physical meanings (t_0) and by its timing. At each time step of the test signal, a value of the CAE signal is required.

8.2 Sampling rate

The ISO metric was validated with signals of 10 kHz sampling rate. The magnitude and slope scores are sensitive to the signal's sampling rate.

8.3 Filtering

The algorithms do not modify the original signals. It should be noted that the calculation of the correlation could be difficult when using noisy signals. Figure 4 shows an example of the filtering effect. Signals A and B are derived from the same unfiltered signal and differ only by the applied filter class. The overall rating of the signal B is 6 % higher than that of signal A due to the application of a higher filter class. High-frequency oscillations could lead to misleading results.

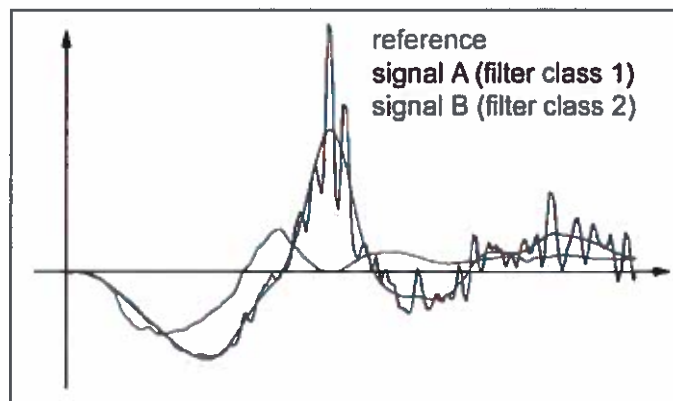


Figure 4 — Signals with different filter classes[9]

8.4 Interval of evaluation

The assessment of the correlation should be focused on the relevant parts of the given signals. Typically, crash signals include pre- and post-crash phases that are not of interest and should be excluded from the rating calculation. Therefore, an interval of evaluation shall be defined where the part of the signals are to be assessed. The interval starts at t_{start} and ends at t_{end} . An assessment of using ratings of different sub-intervals of the same pair of signals is not allowed.

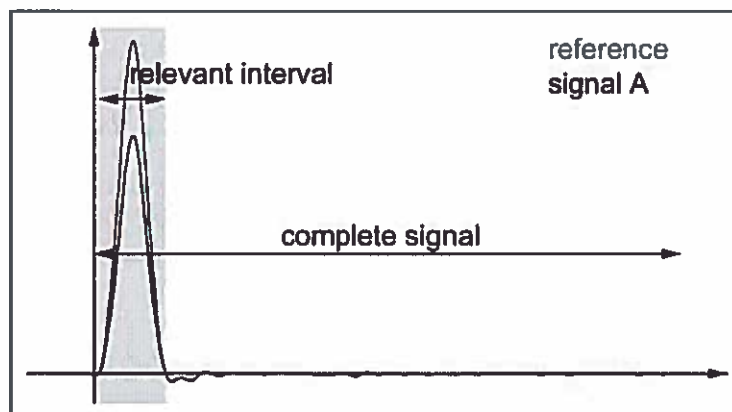


Figure 5 — Different intervals of evaluation[9]

Figure 5 depicts an example of this problem. The rating increases by 35 % when extending the interval of evaluation from the relevant part to the whole time domain.

The ISO metric requires a minimum length of the interval of evaluation of 10 ms.

9 Limitations

This Technical Specification describes a method to apply an objective metric to calculate the goodness of the correlation between two signals. As described previously, the application of such a metric requires some basic conditions. Below is a list of a few known limitations that must be considered when applying this metric.

9.1 Type of signals

The application of this metric is limited to non-ambiguous signals obtained in all kinds of tests of the passive safety of vehicles and the corresponding numerical simulations (CAE). The most commonly used signals in this field are time-history curves.

9.2 Metric validation

The metric is validated with time-history signals obtained from different data channel types such as, forces, moments, accelerations, velocities and displacements. It is also validated with time-history signals of various correlation qualities.

9.3 Meaning of the results

As described in this Technical Specification, the presented sliding scale (see [Clause 7](#)) is only valid for the comparison of two signals. Any modification to the parameters such as weighting factors, sampling rates, etc. requires a revision of the grade's thresholds. Furthermore, the defined scale shall only be applied to the overall objective rating, *R*, and not to its metrics.

9.4 Multiple responses

This ISO metric is defined to calculate the level of the goodness of correlation between two signals only. If more than one pair of signals (e.g. whole set of signals from various channels of a test) are considered, the defined thresholds of the sliding scale are no longer valid.

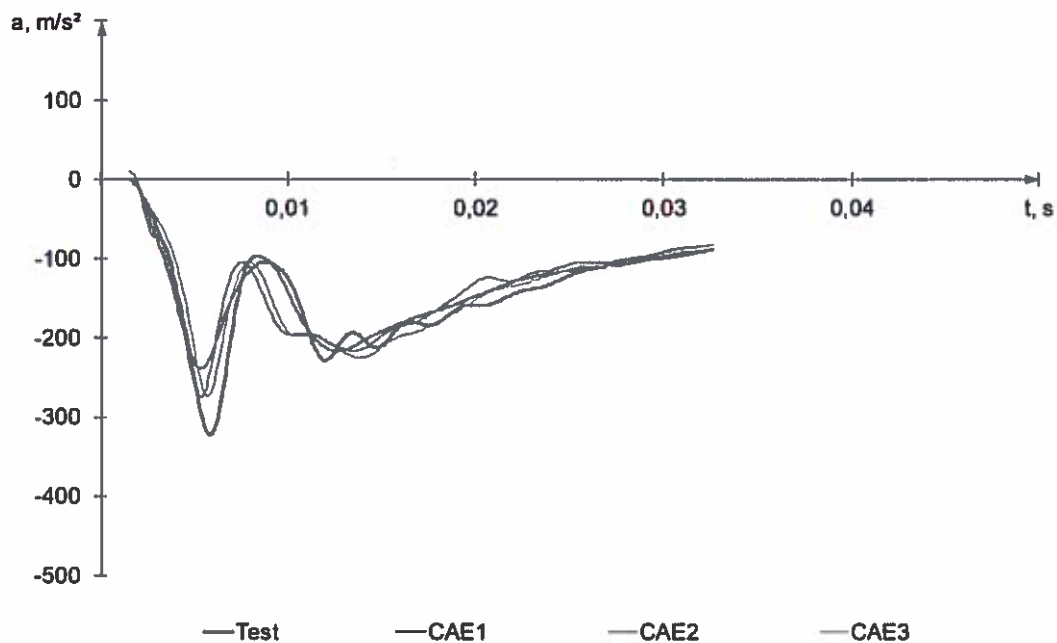
Annex A (informative)

Case studies

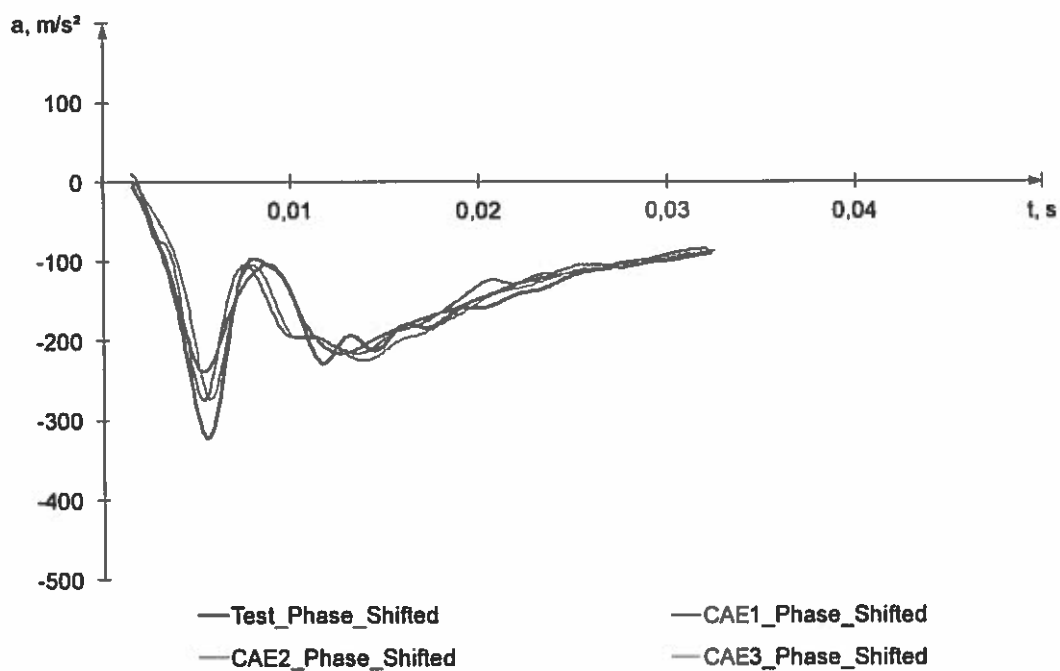
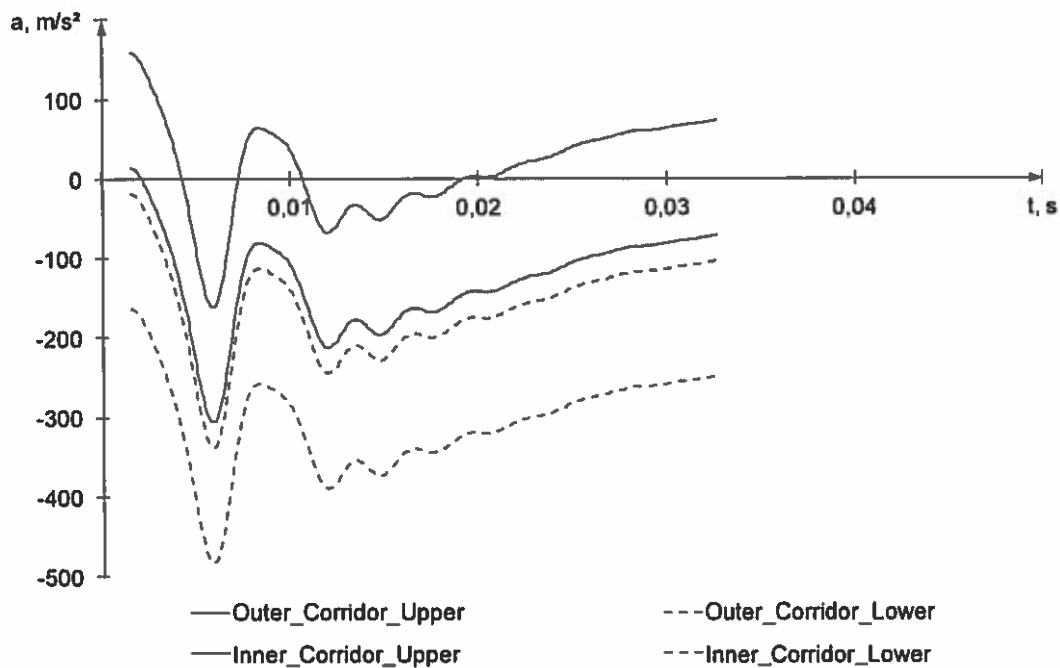
This Annex provides some examples of the application of this Technical Specification. It is identical to ISO/TR 16250:2013, Annex C^[10] but shows intermediate result curves too. All responses are obtained from various tests related to passive safety of vehicles. The focus is on filtered and anonymized dummy responses. In each case three CAE signals are compared with a test signal. As the quality of the CAE signals differs, different levels of correlation are covered by these case studies.

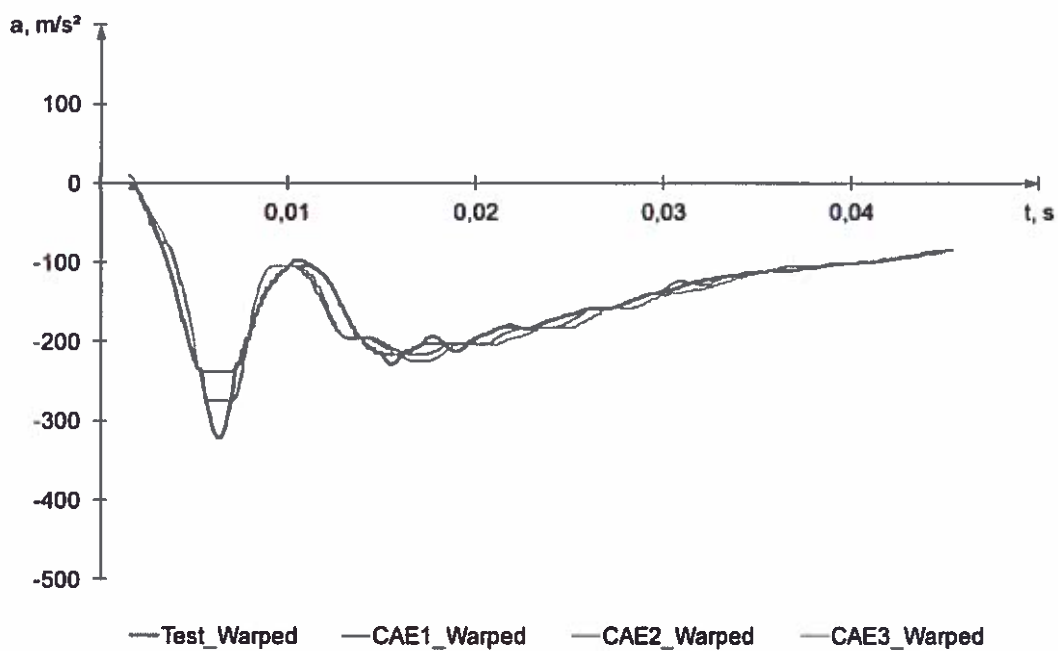
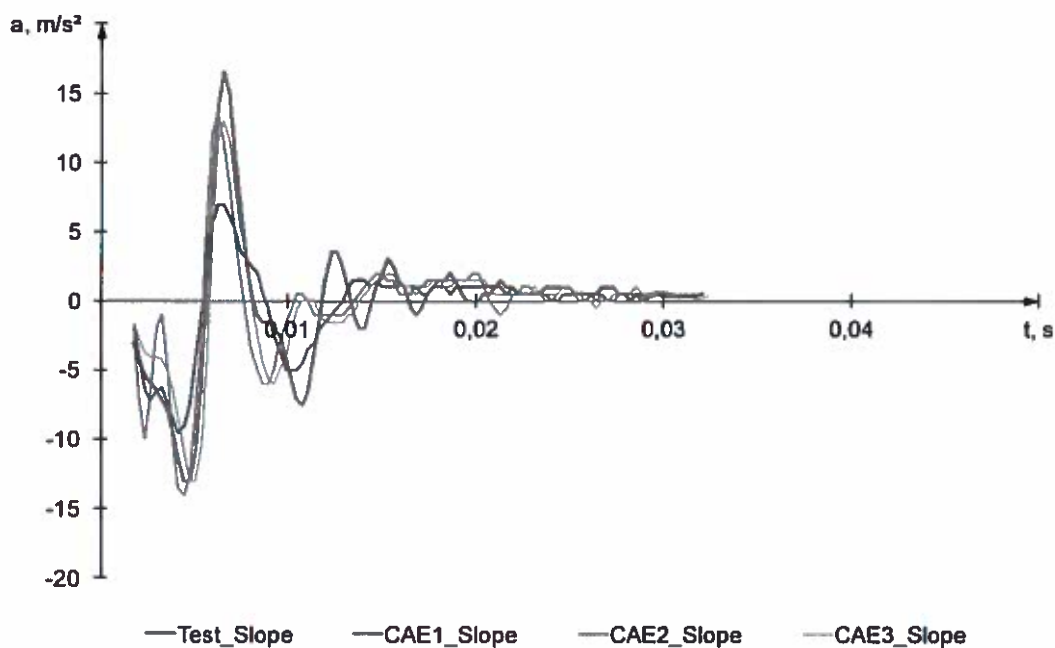
A.1 Accelerations

A.1.1 Acceleration 1

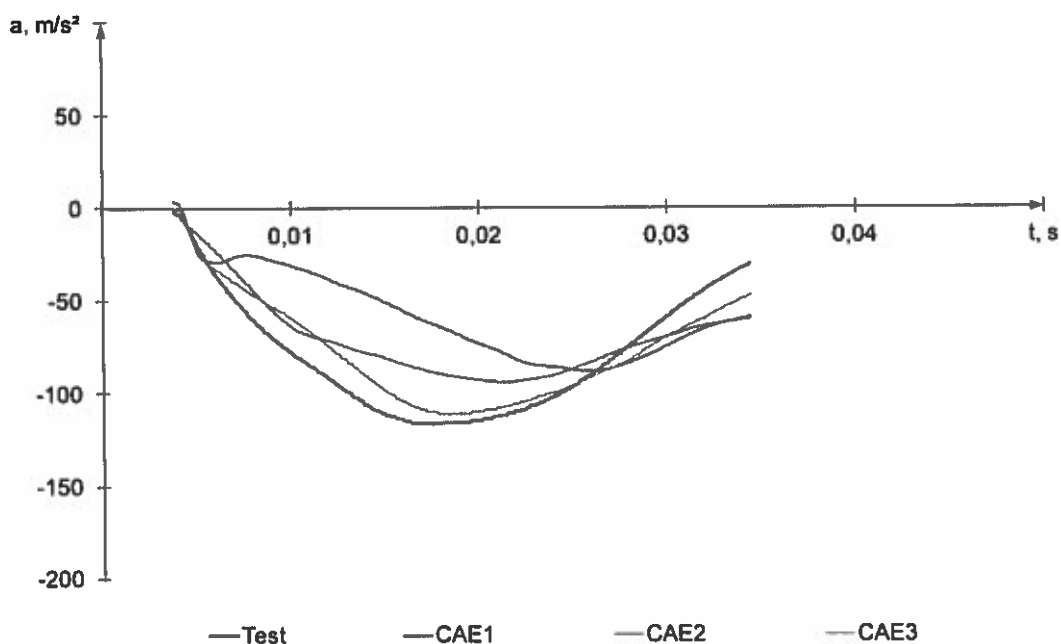


	CAE1 (C_1)	CAE2 (C_2)	CAE3 (C_3)
Grade	Good	Good	Good
Overall rating, R	0,910	0,877	0,906
Corridor score, Z	0,956	0,898	0,922
Phase score, E_P	0,936	0,904	0,968
Magnitude score, E_M	0,952	0,964	0,972
Slope score, E_S	0,751	0,721	0,745

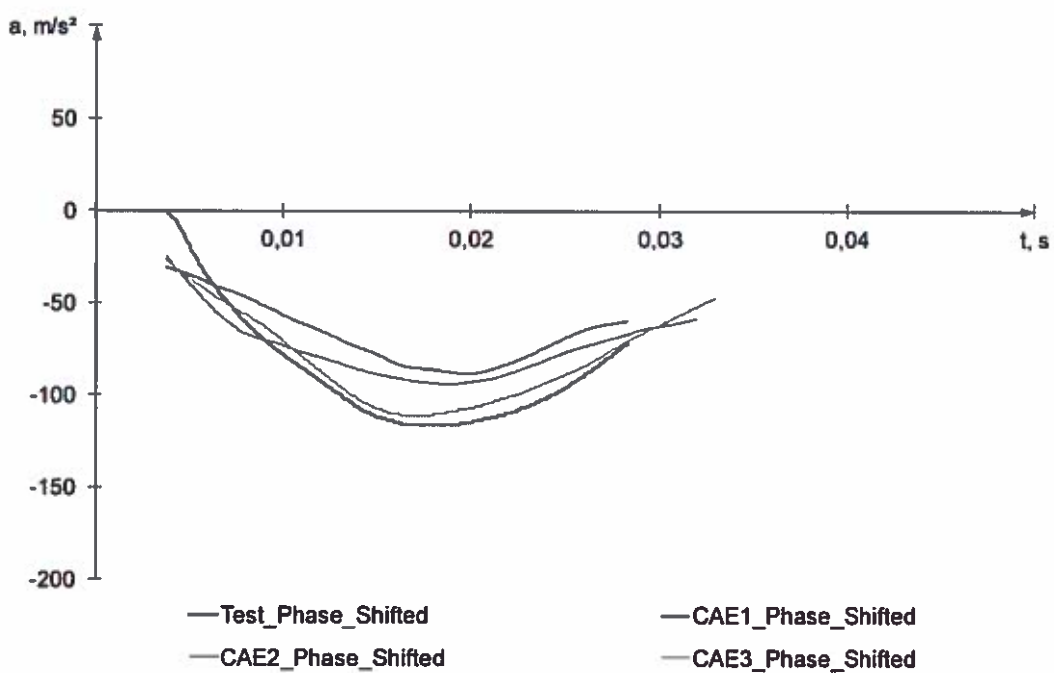
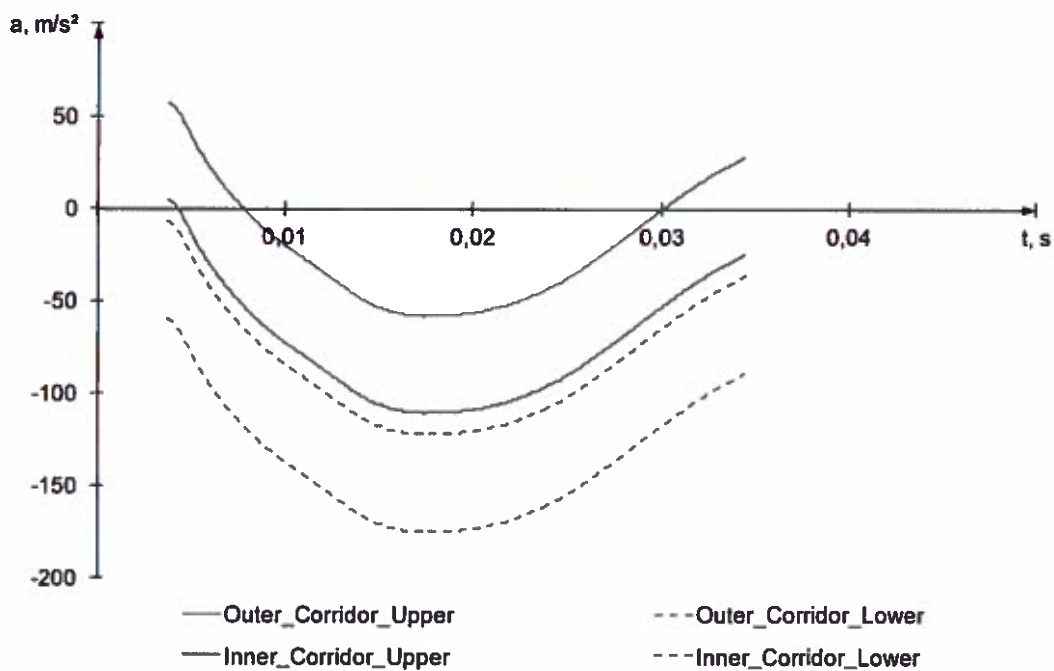


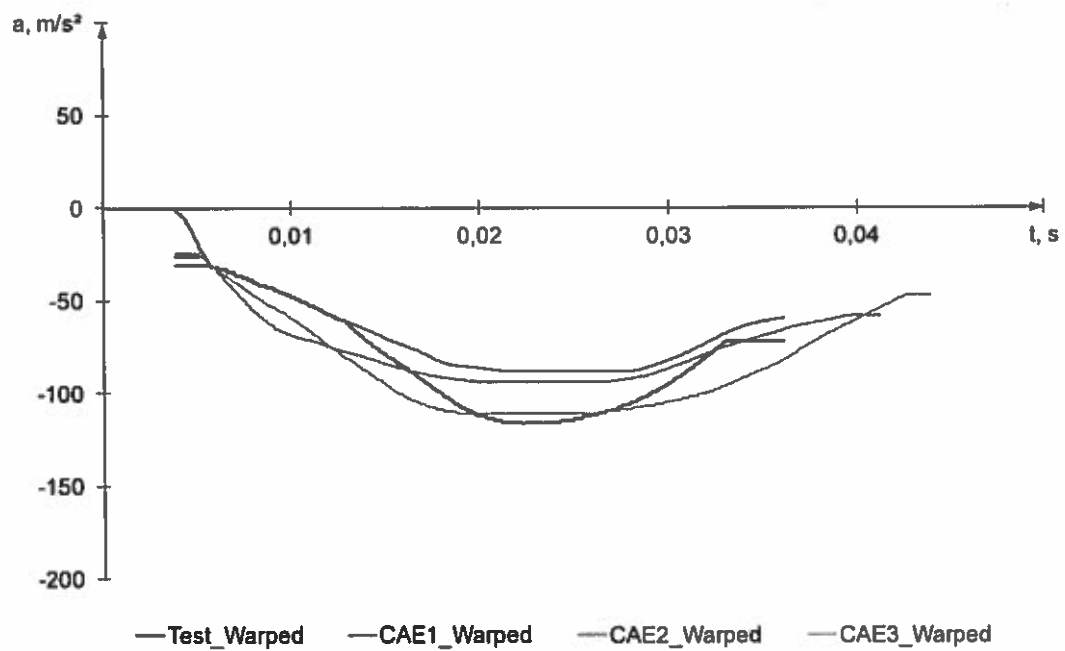
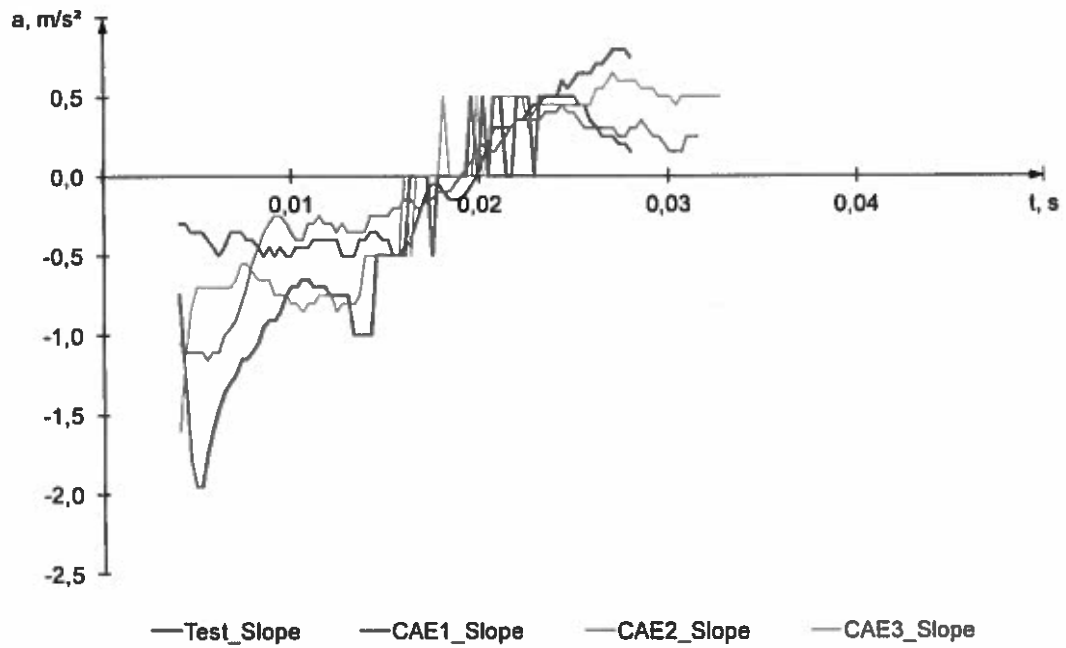


A.1.2 Acceleration 2

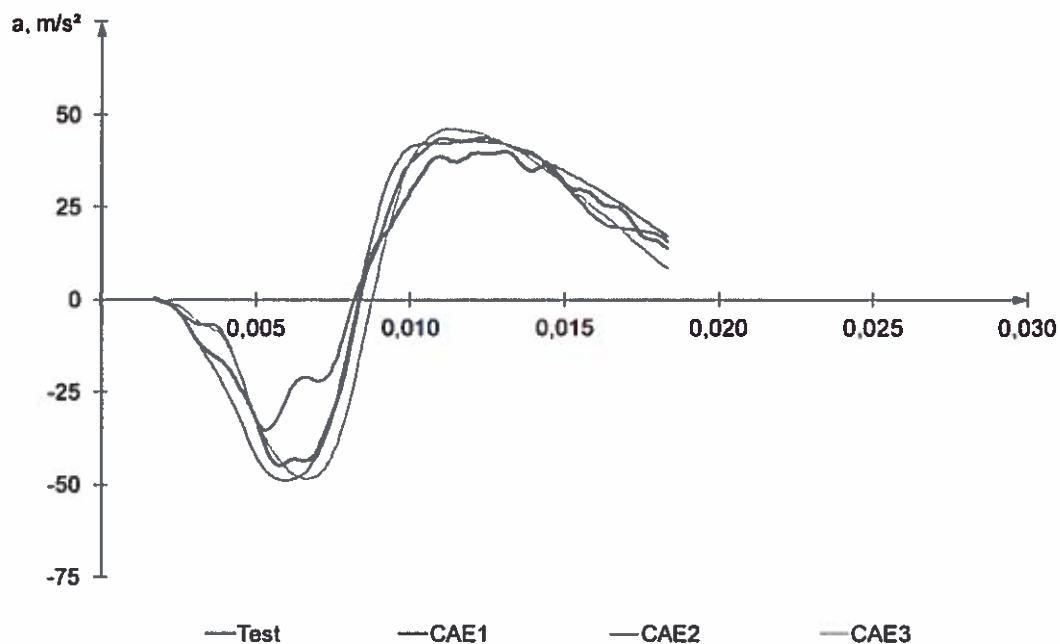


	CAE1 (C_1)	CAE2 (C_2)	CAE3 (C_3)
Grade	Poor	Fair	Good
Overall rating, R	0,439	0,683	0,841
Corridor score, Z	0,406	0,642	0,845
Phase score, E_P	0,000	0,577	0,739
Magnitude score, E_M	0,679	0,825	0,965
Slope score, E_S	0,704	0,729	0,811

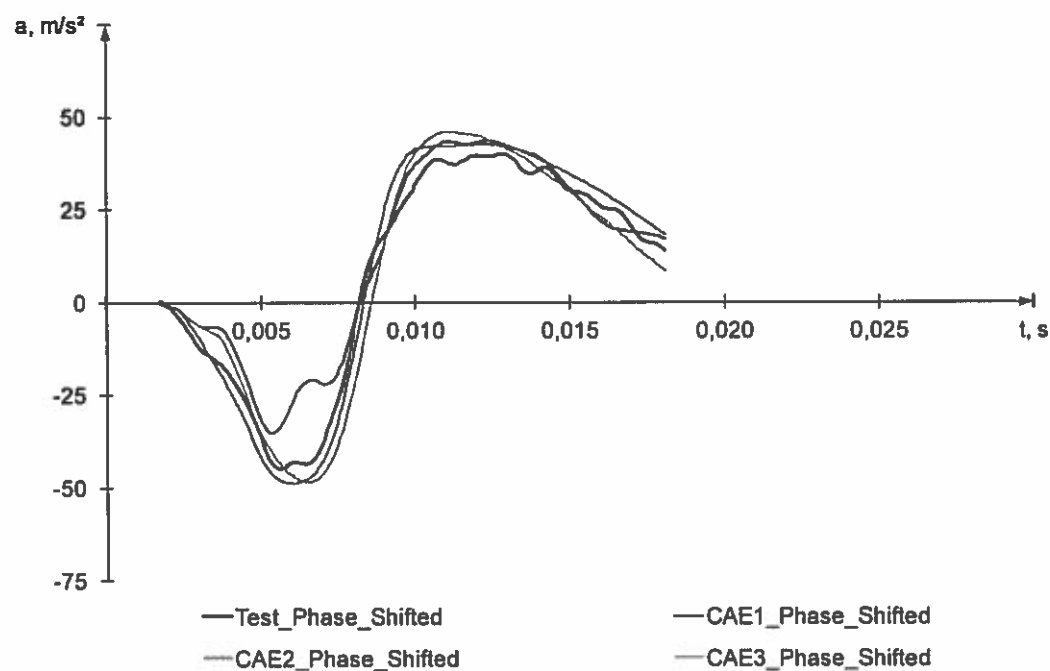
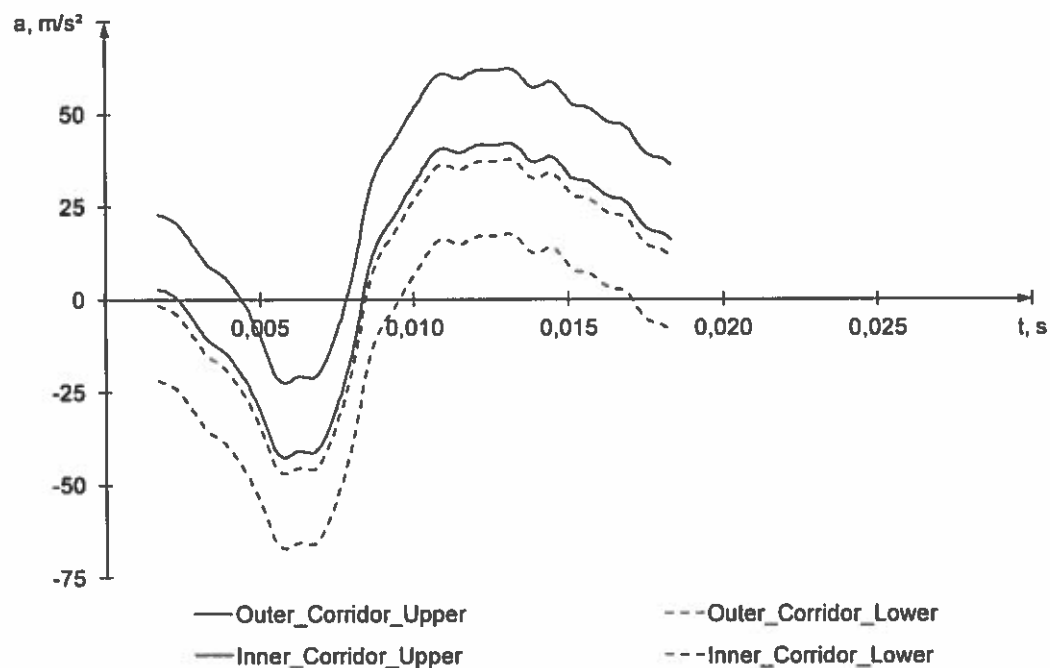


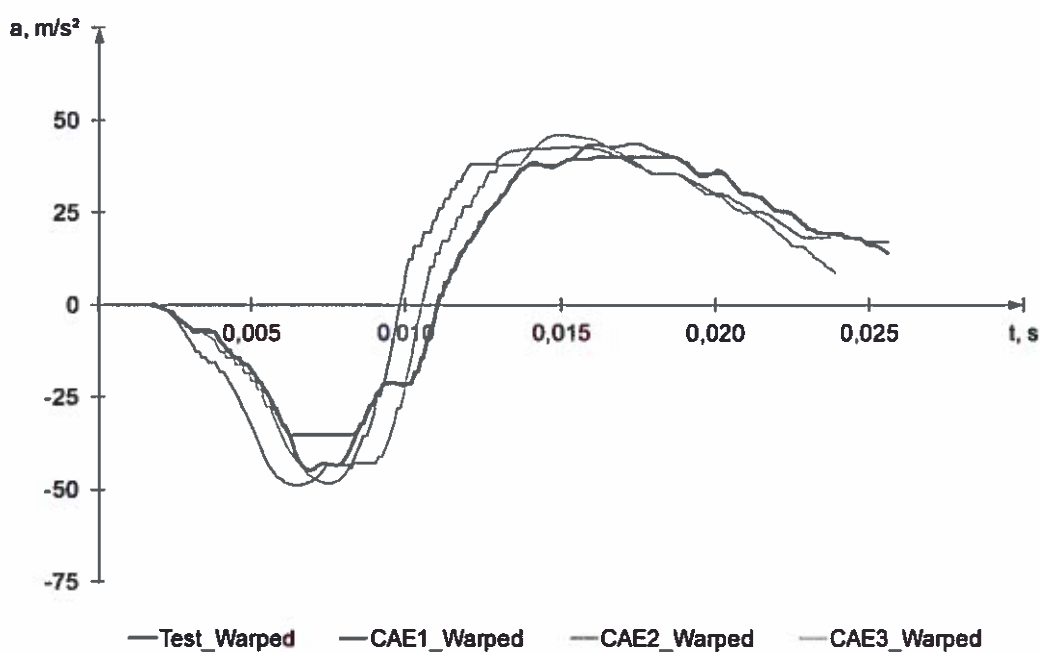
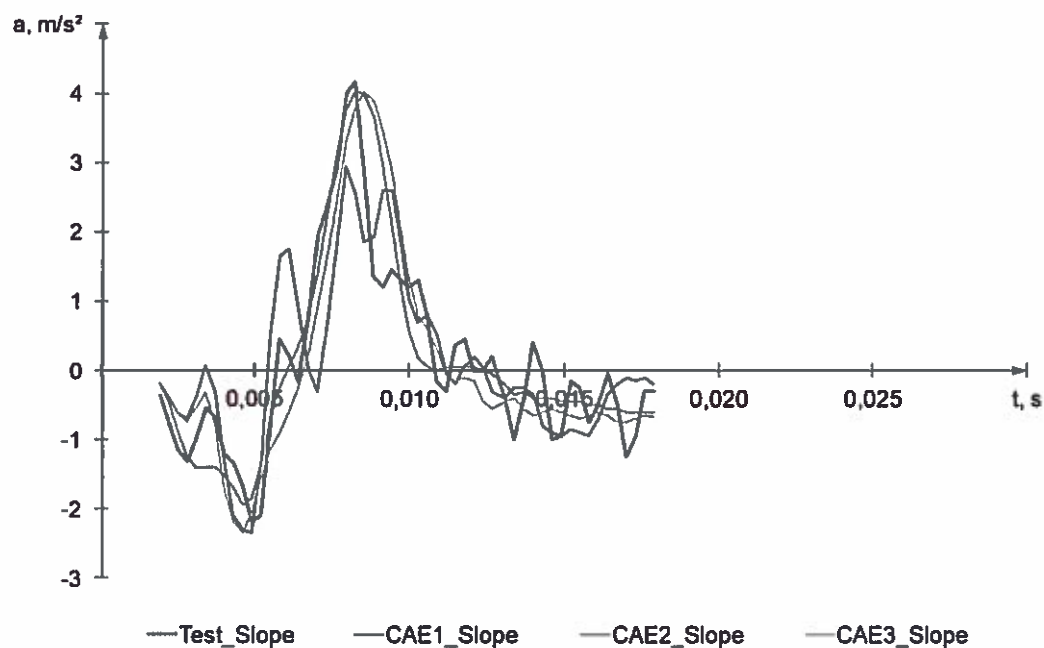


A.1.3 Acceleration 3

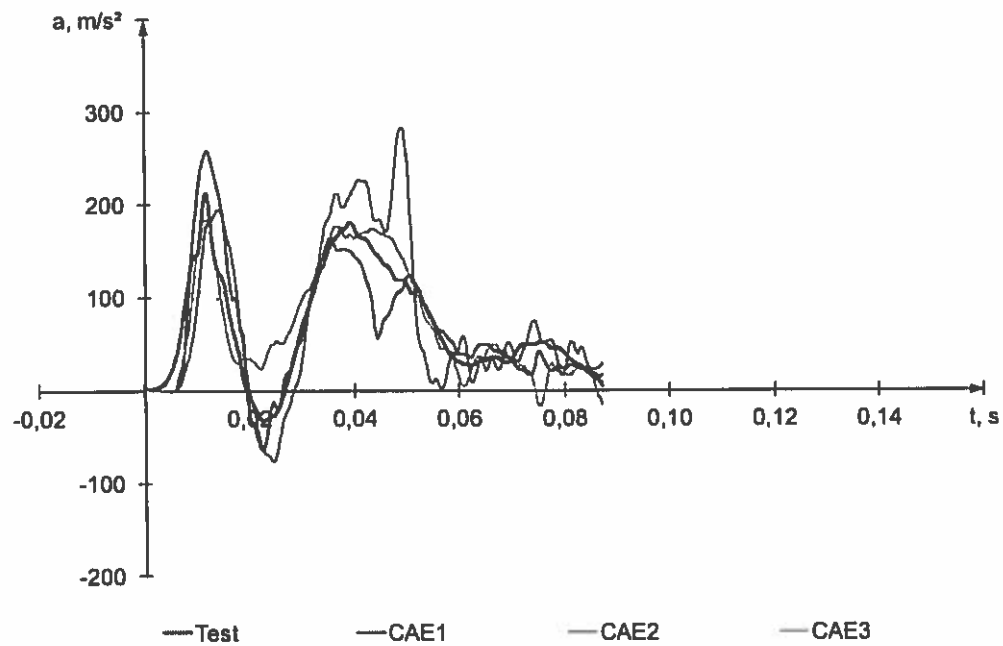


	CAE1 (C_1)	CAE2 (C_2)	CAE3 (C_3)
Grade	Fair	Good	Good
Overall rating, R	0,788	0,850	0,813
Corridor score, Z	0,725	0,816	0,749
Phase score, E_P	0,910	0,910	0,910
Magnitude score, E_M	0,898	0,931	0,909
Slope score, E_S	0,682	0,780	0,747

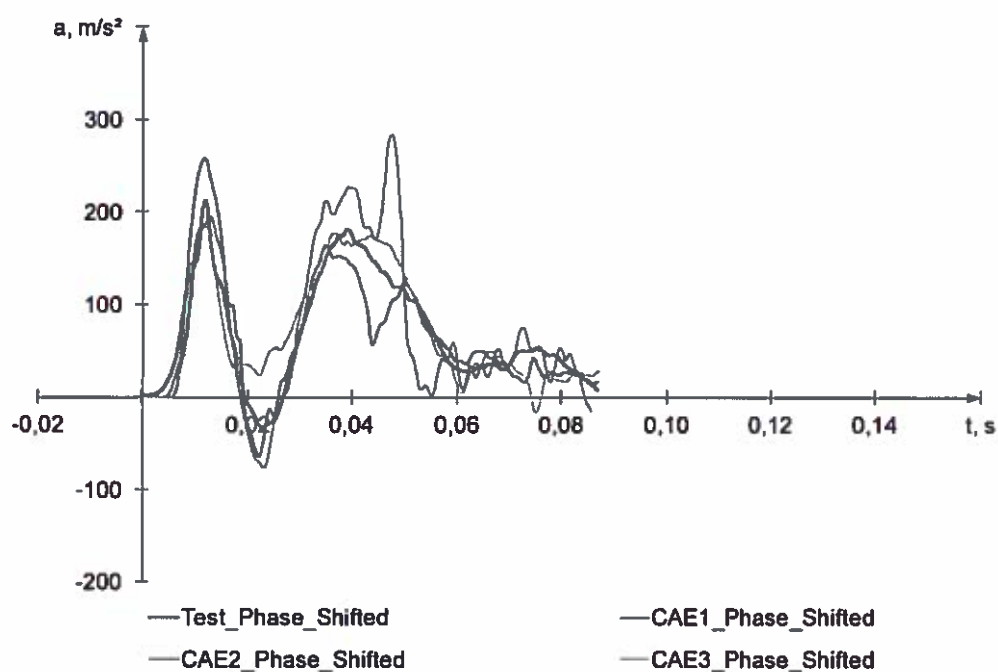
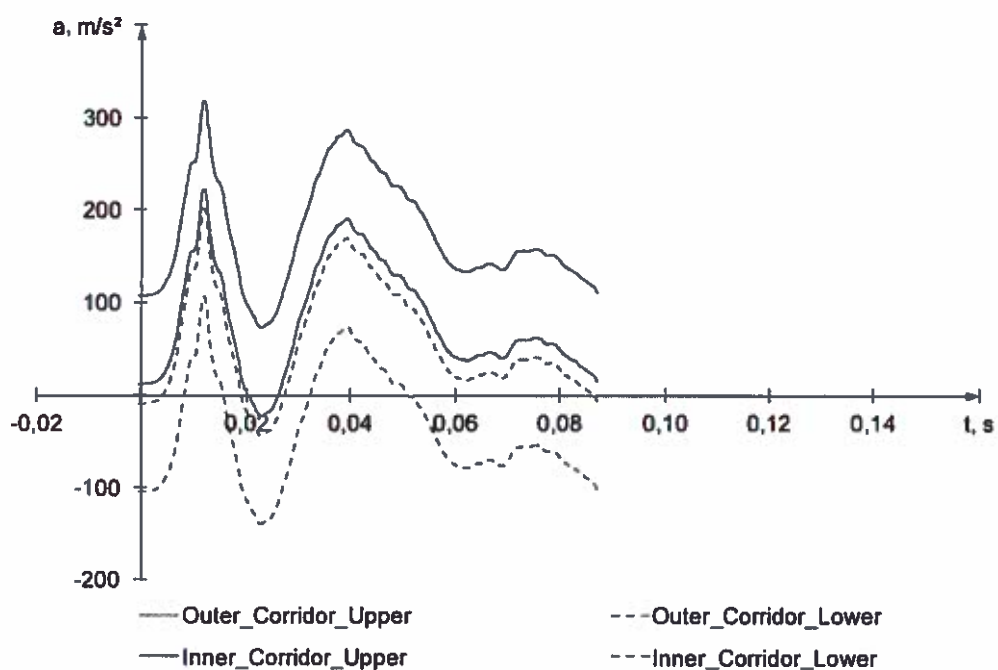


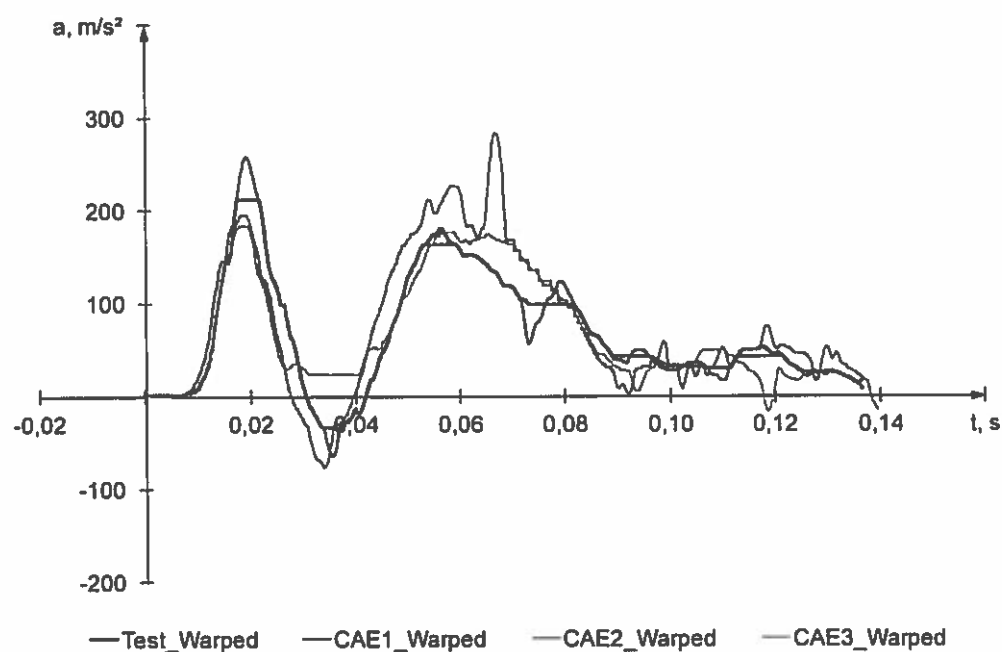
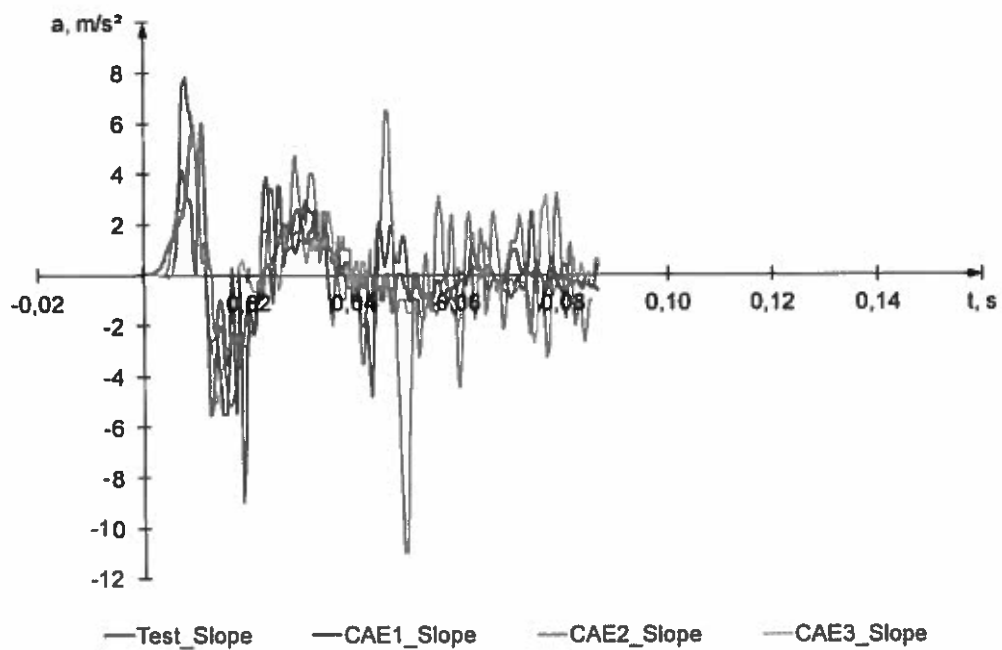


A.1.4 Acceleration 4



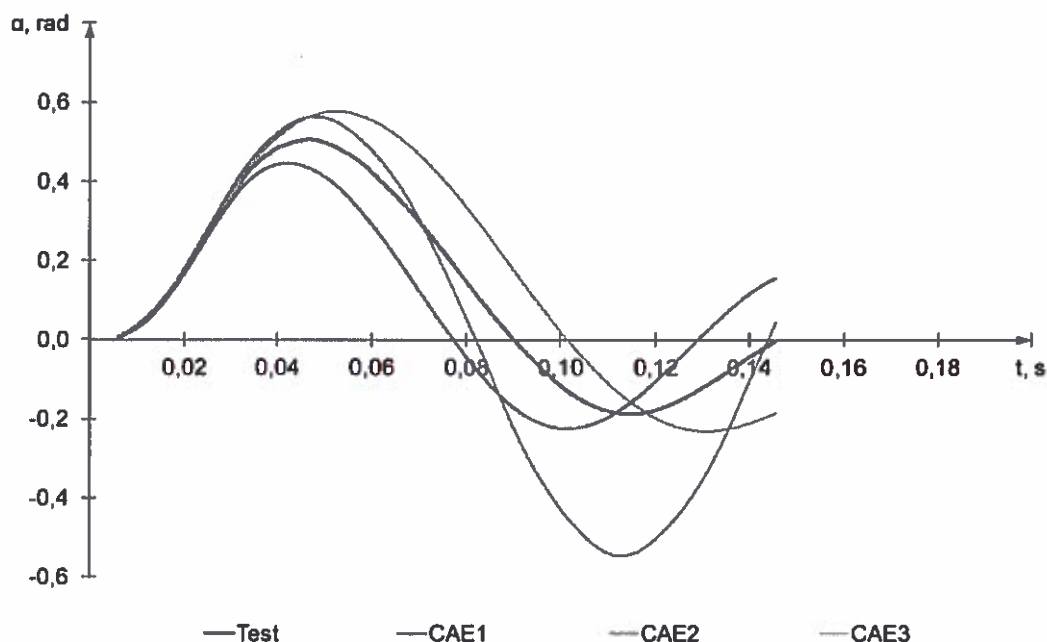
	CAE1 (C_1)	CAE2 (C_2)	CAE3 (C_3)
Grade	Fair	Fair	Fair
Overall rating, R	0,777	0,642	0,786
Corridor score, Z	0,793	0,647	0,784
Phase score, E_P	0,971	0,909	0,989
Magnitude score, E_M	0,871	0,793	0,849
Slope score, E_S	0,455	0,213	0,526



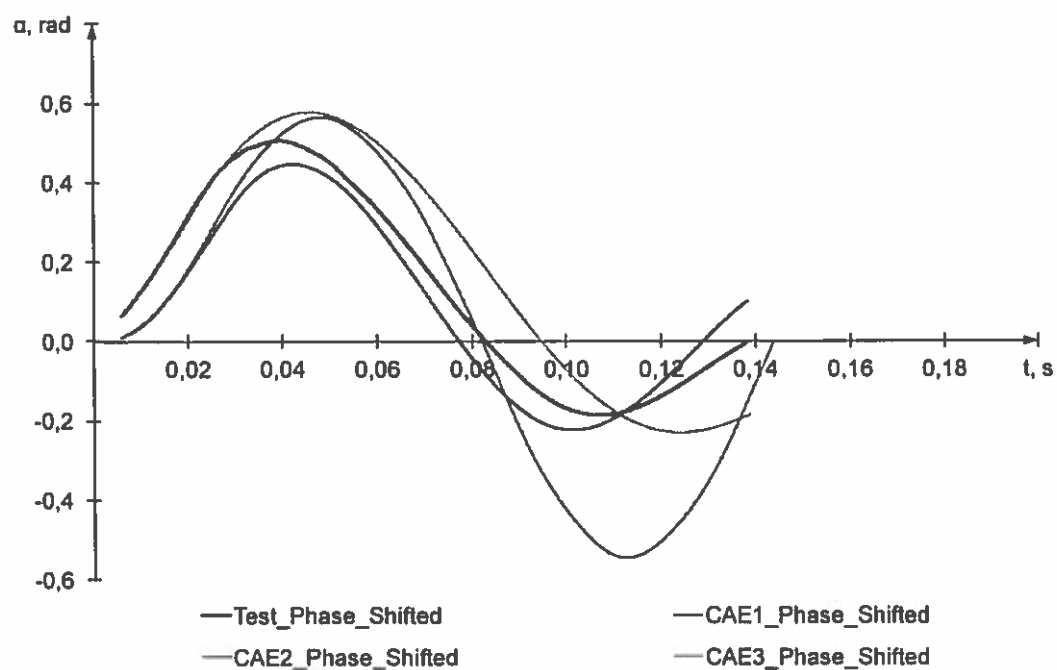
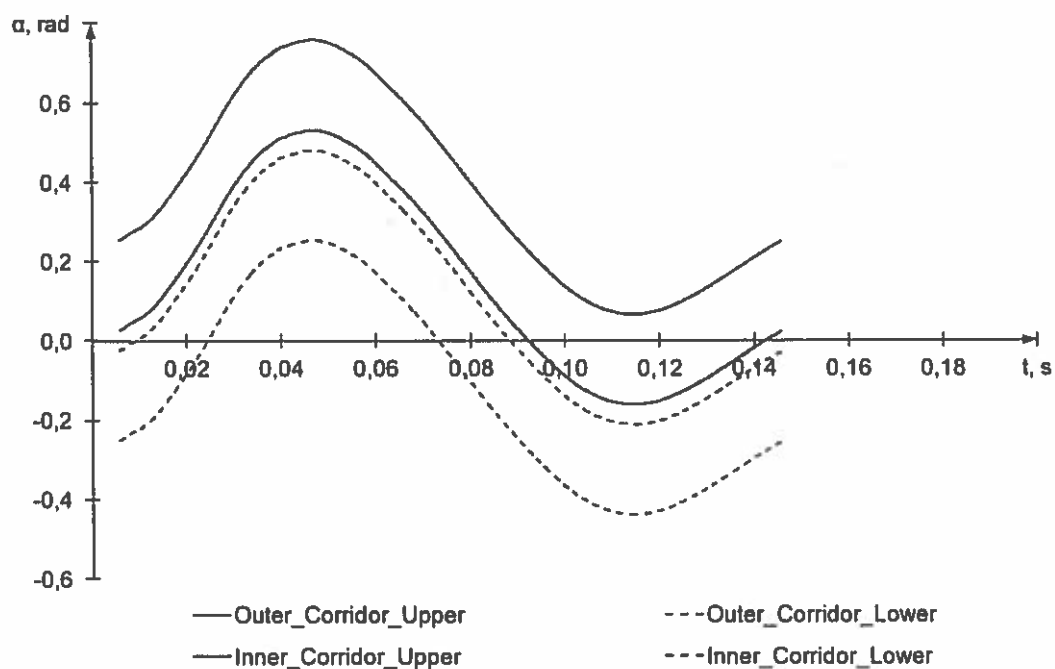


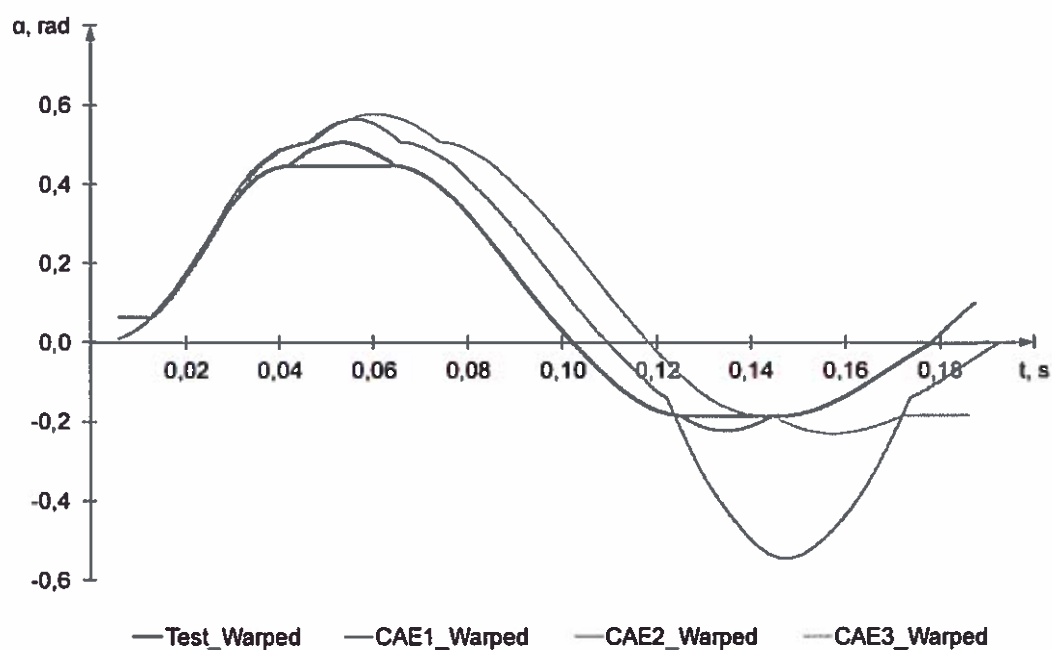
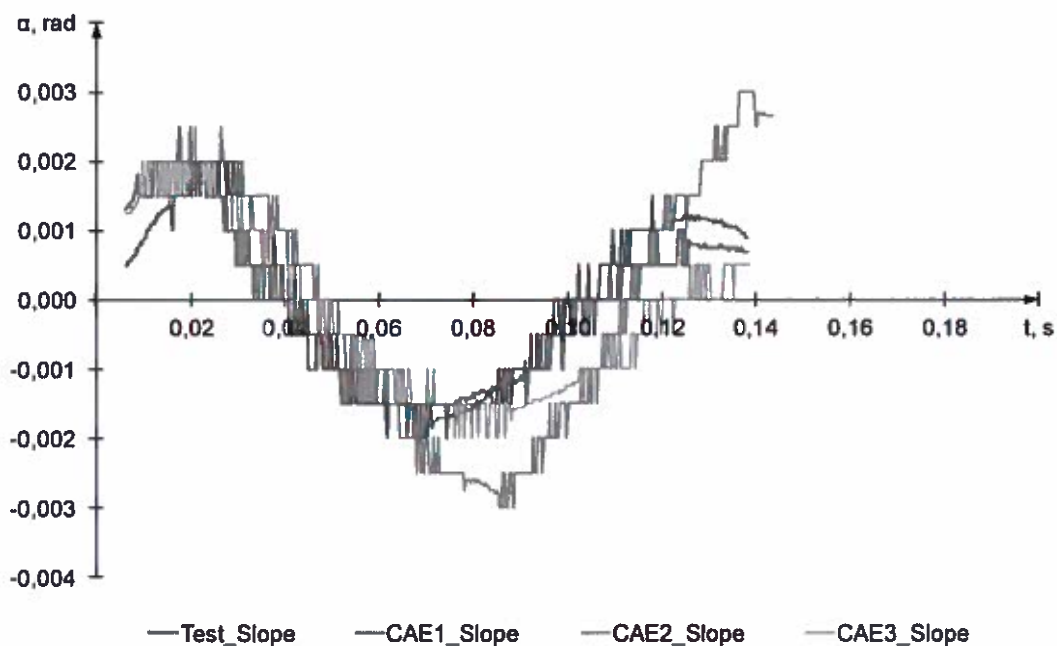
A.2 Angles

A.2.1 Angle 1

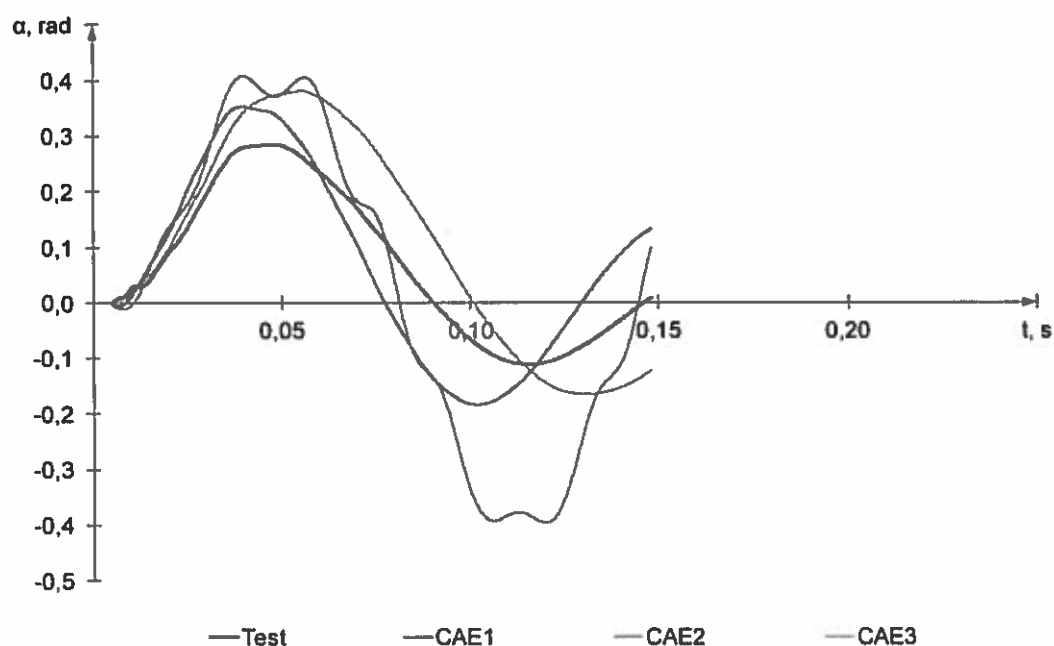


	CAE1 (C_1)	CAE2 (C_2)	CAE3 (C_3)
Grade	Fair	Fair	Fair
Overall rating, R	0,701	0,628	0,700
Corridor score, Z	0,530	0,541	0,538
Phase score, E_p	0,732	0,928	0,753
Magnitude score, E_M	0,895	0,466	0,851
Slope score, E_S	0,821	0,665	0,822

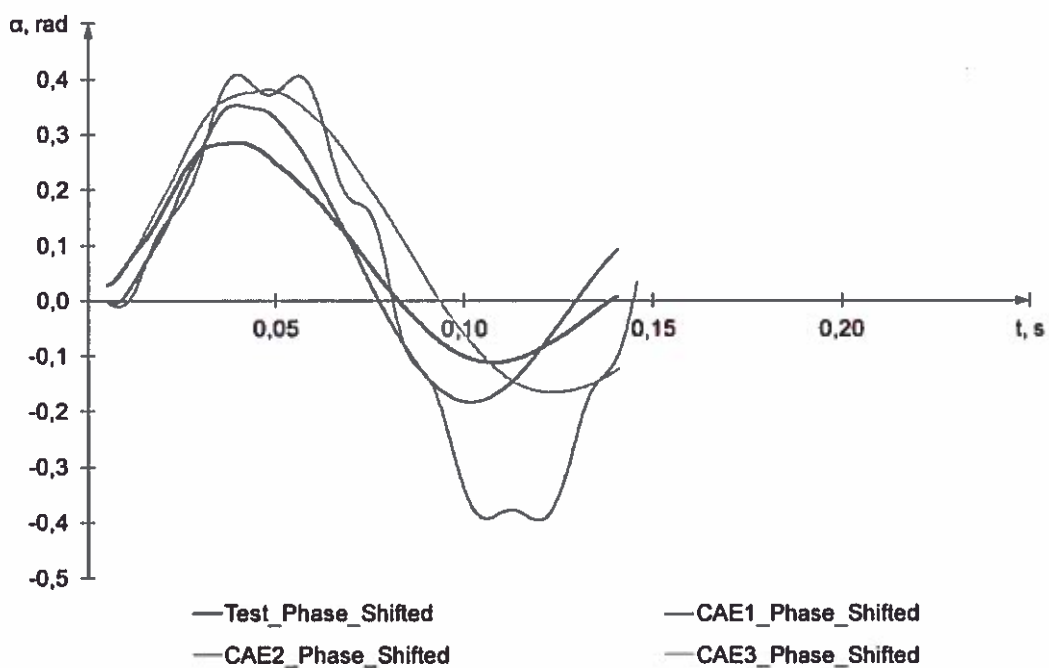
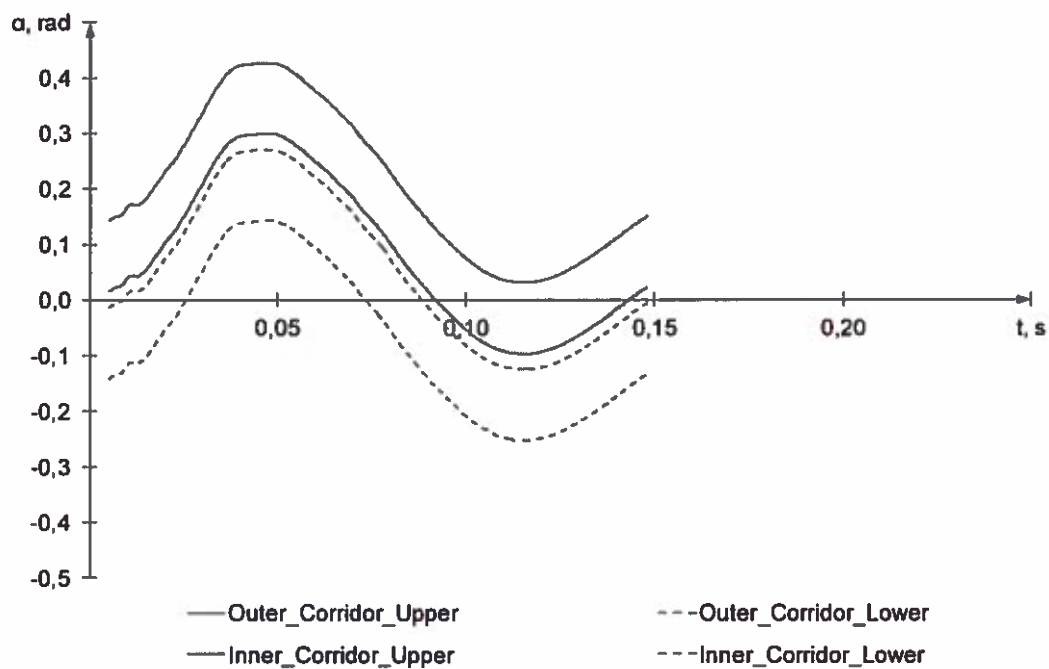


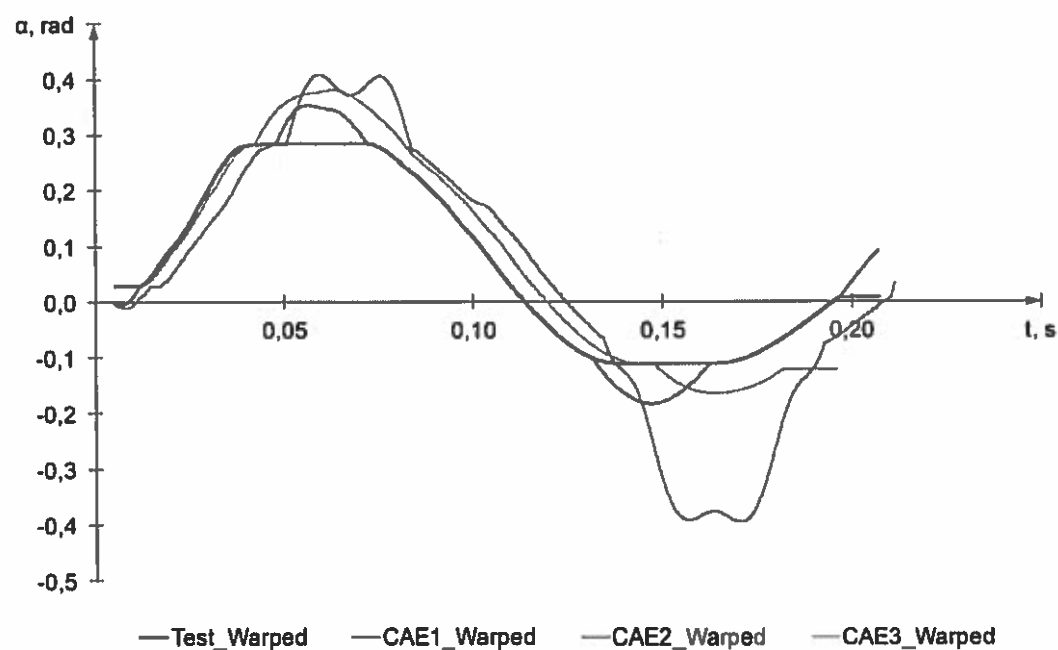
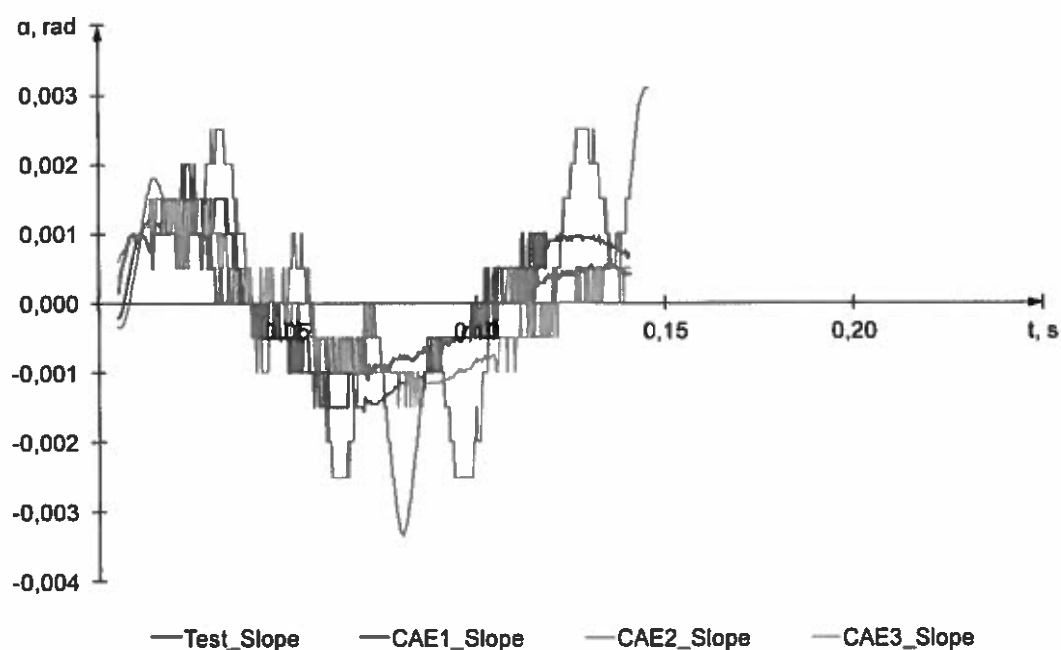


A.2.2 Angle 2



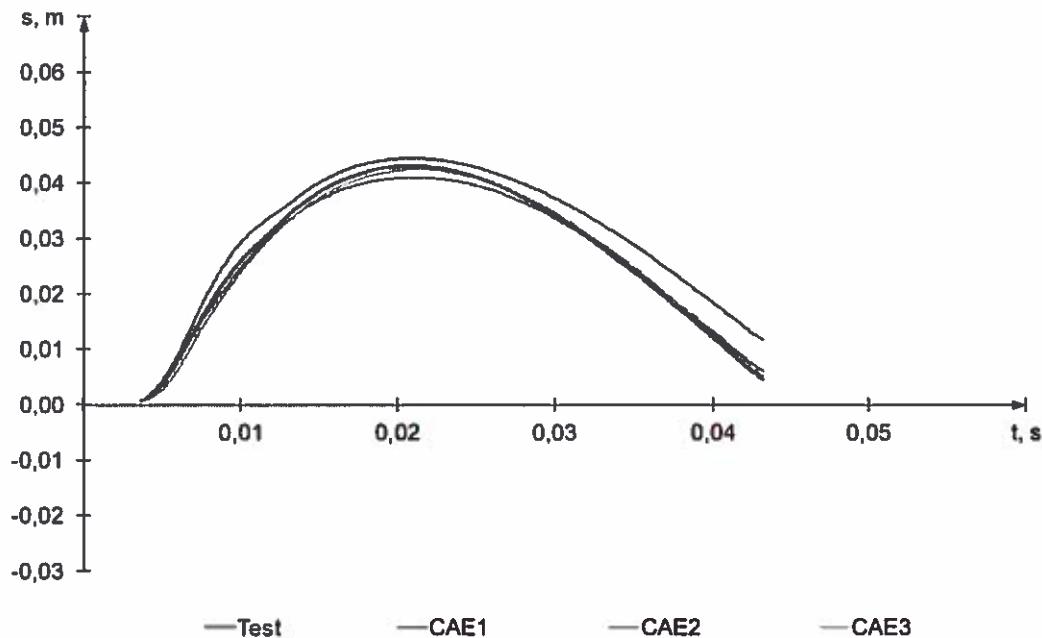
	CAE1 (C_1)	CAE2 (C_2)	CAE3 (C_3)
Grade	Fair	Poor	Fair
Overall rating, R	0,592	0,363	0,580
Corridor score, Z	0,403	0,287	0,374
Phase score, E_P	0,734	0,916	0,748
Magnitude score, E_M	0,763	0,105	0,648
Slope score, E_S	0,658	0,219	0,759



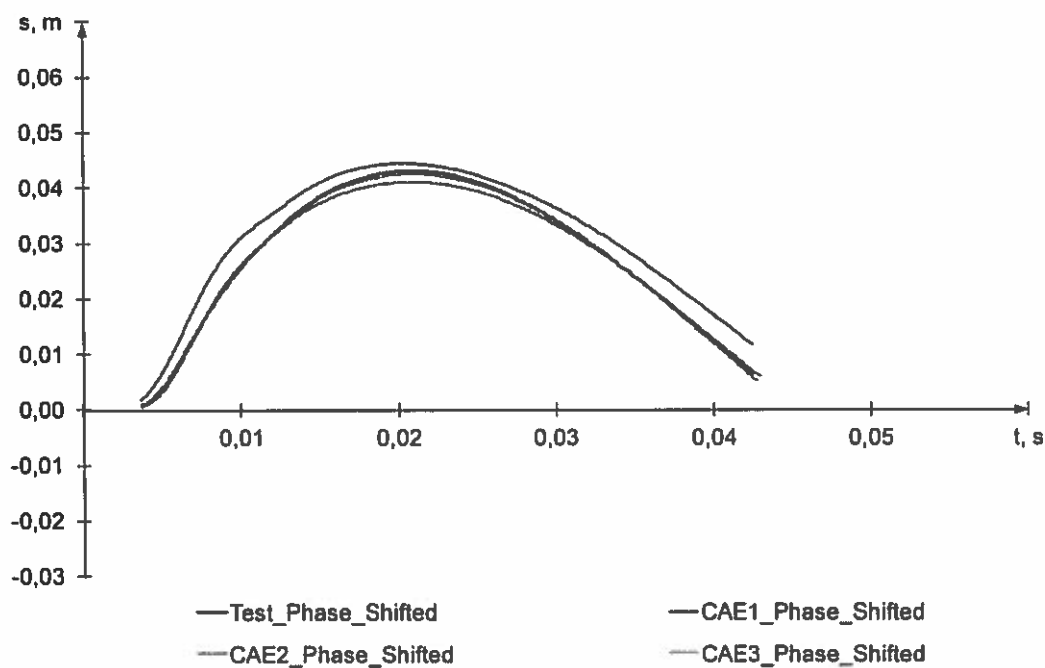
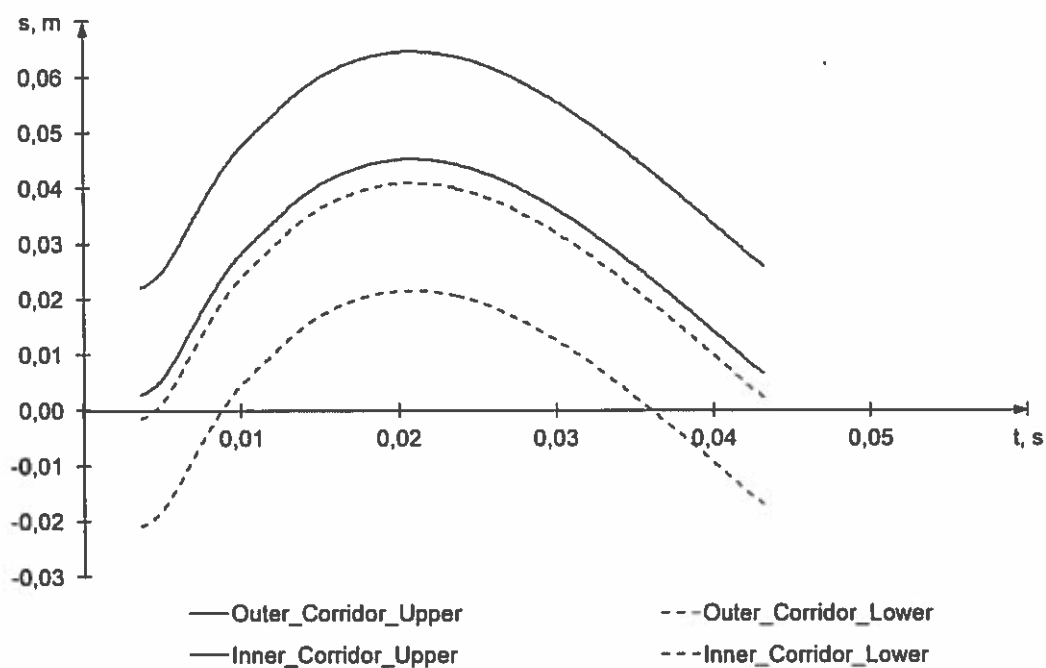


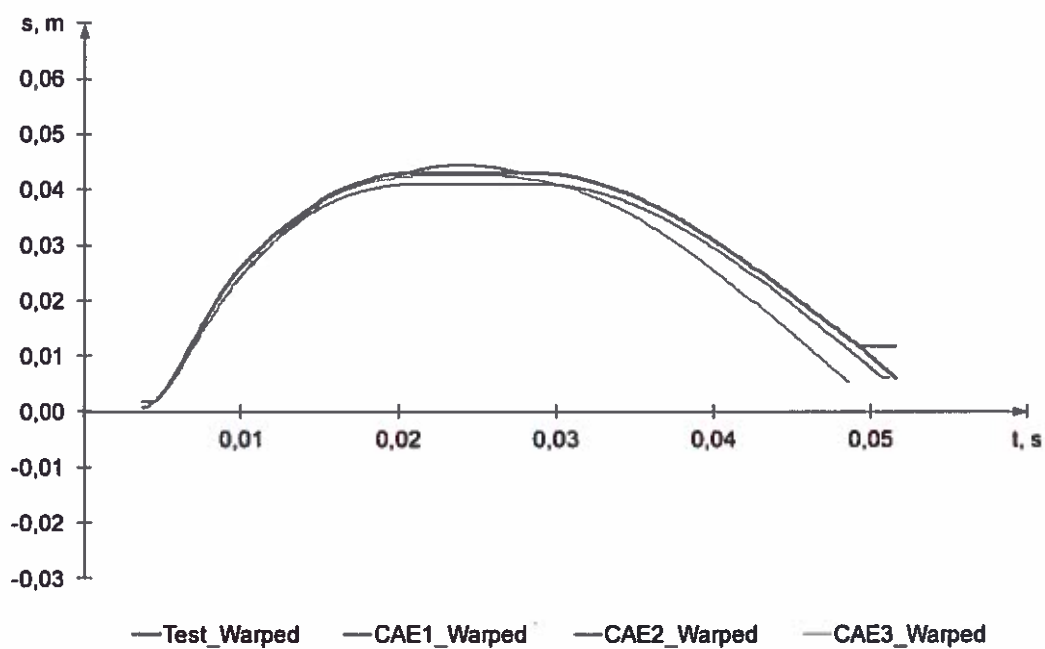
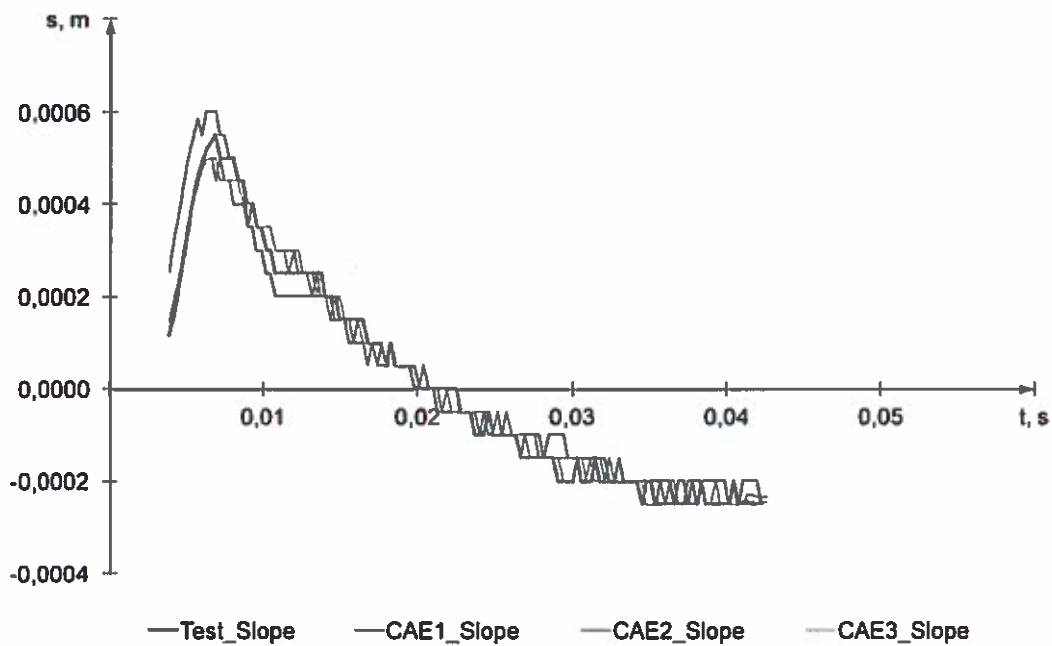
A.3 Displacements

A.3.1 Displacement 1

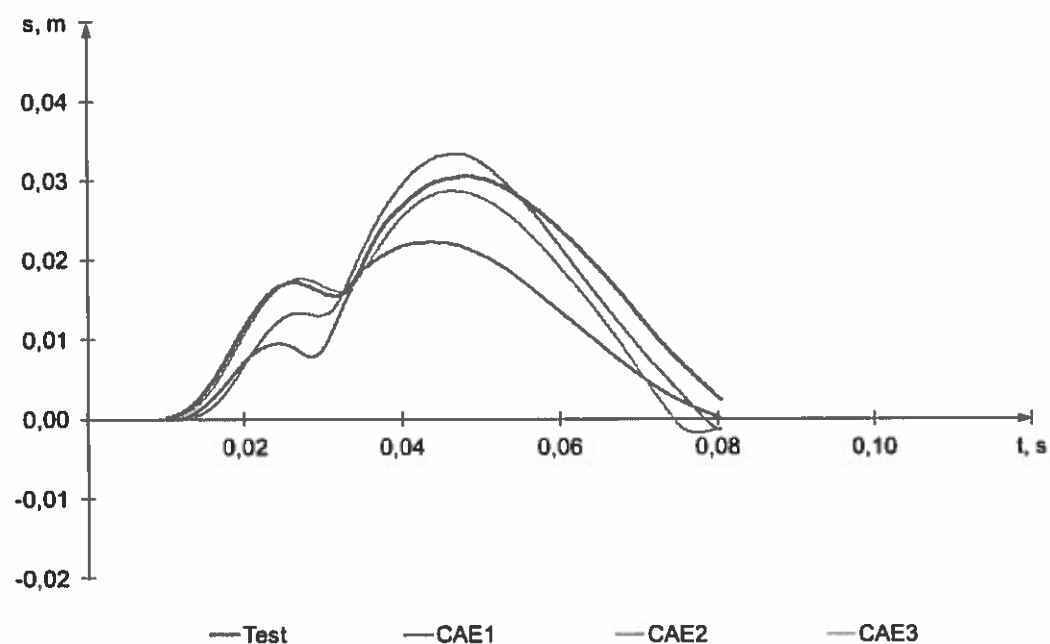


	CAE1 (C_1)	CAE2 (C_2)	CAE3 (C_3)
Grade	Good	Excellent	Excellent
Overall rating, R	0,915	0,978	0,979
Corridor score, Z	0,889	1,000	0,999
Phase score, E_P	0,911	0,962	0,937
Magnitude score, E_M	0,978	0,981	0,995
Slope score, E_S	0,910	0,948	0,965

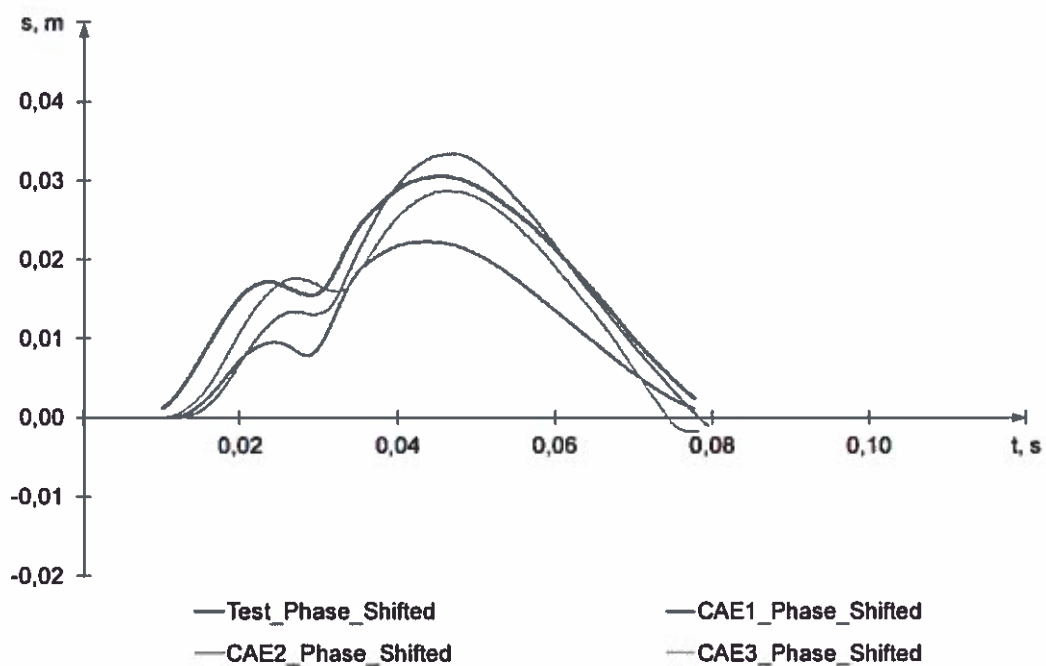
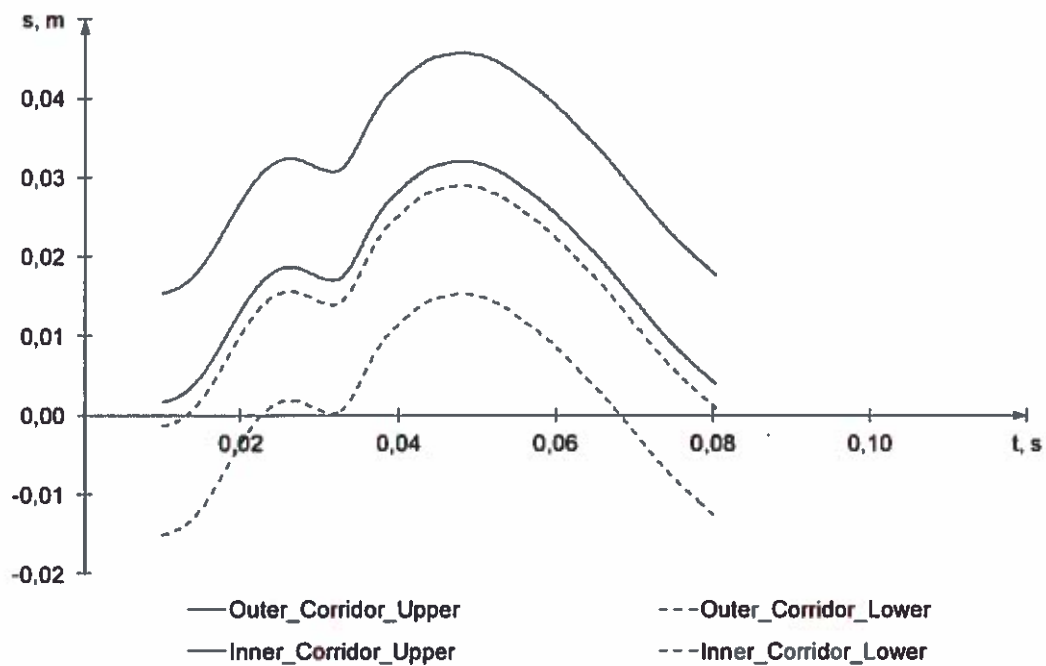


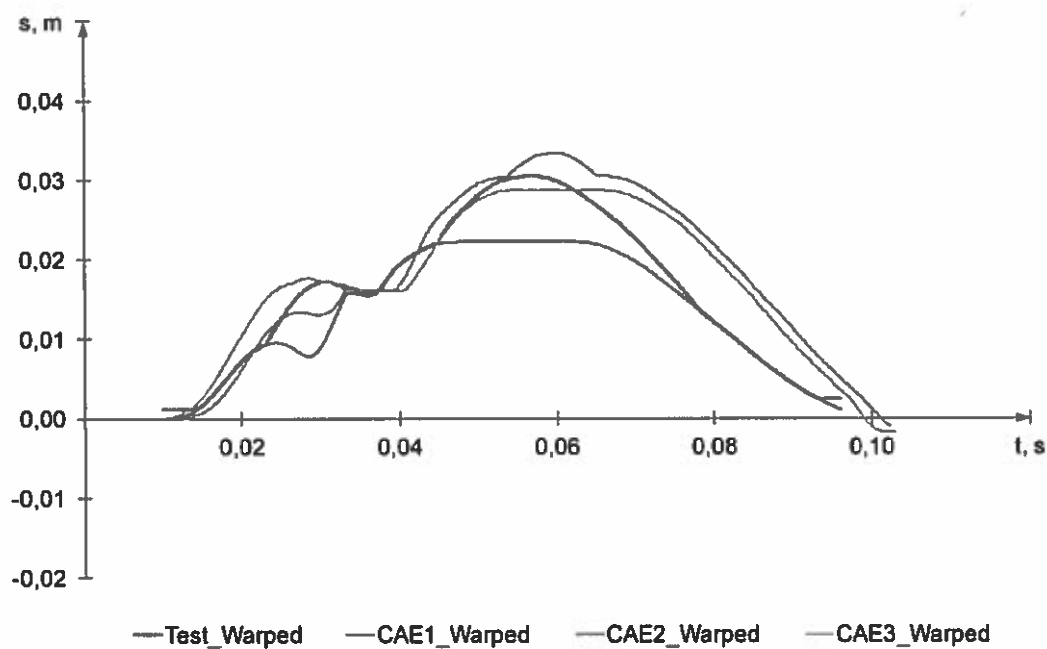
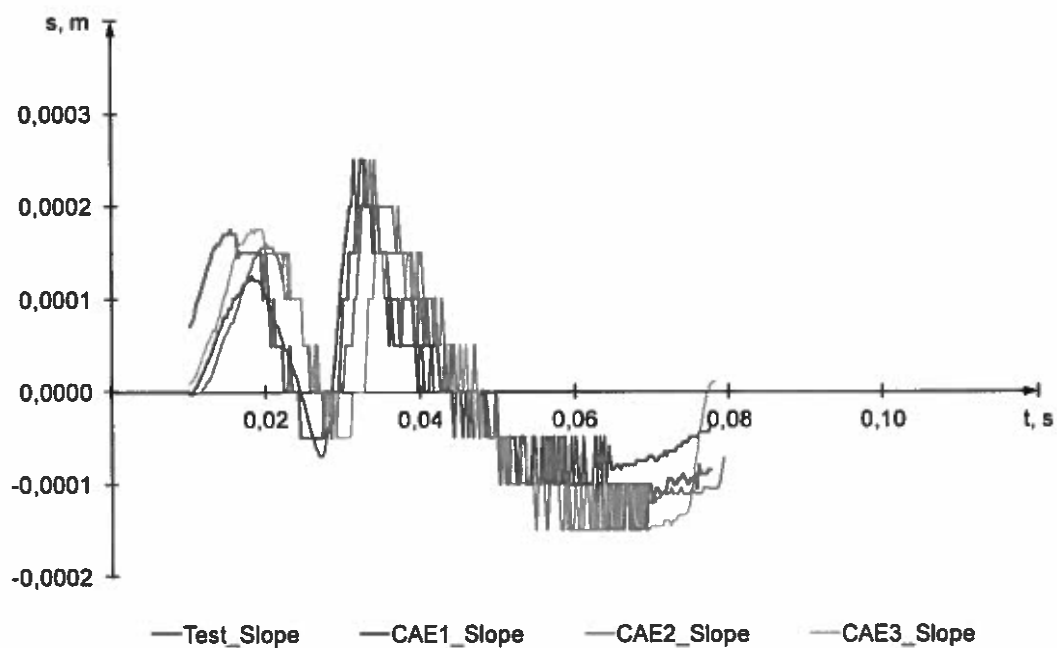


A.3.2 Displacement 2



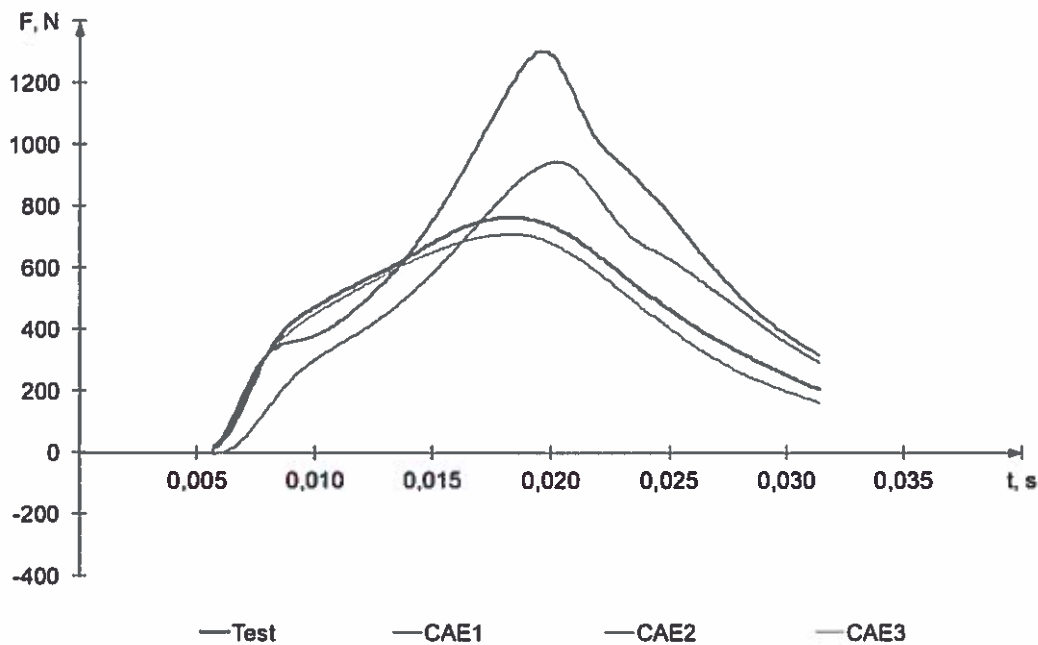
	CAE1 (C_1)	CAE2 (C_2)	CAE3 (C_3)
Grade	Fair	Good	Good
Overall rating, R	0,647	0,859	0,829
Corridor score, Z	0,470	0,807	0,791
Phase score, E_p	0,809	0,929	0,844
Magnitude score, E_M	0,689	0,943	0,956
Slope score, E_S	0,797	0,811	0,763



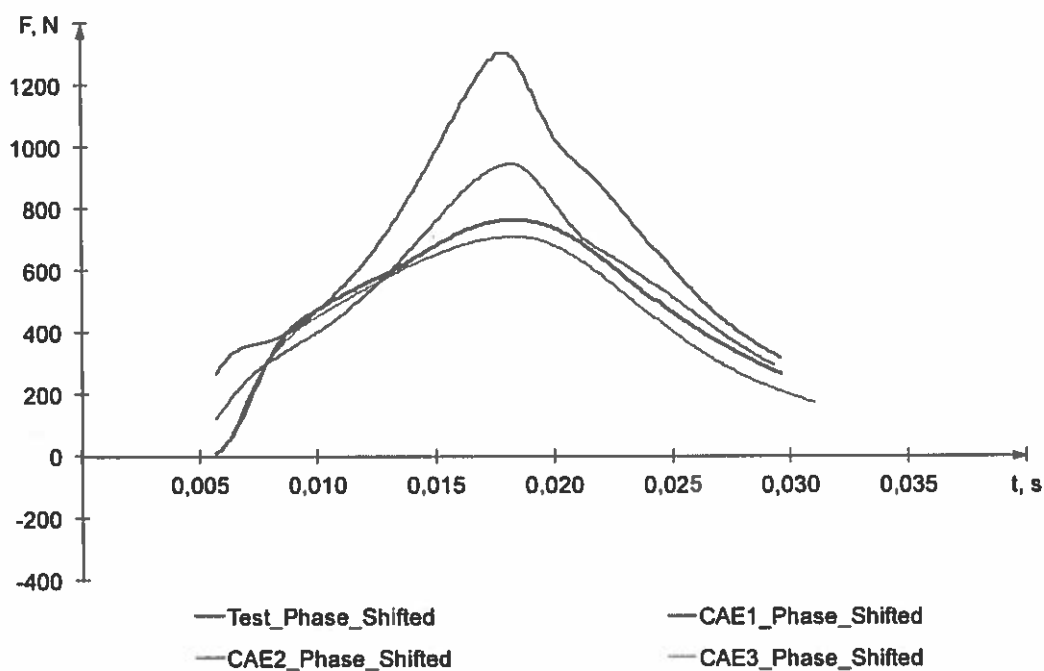
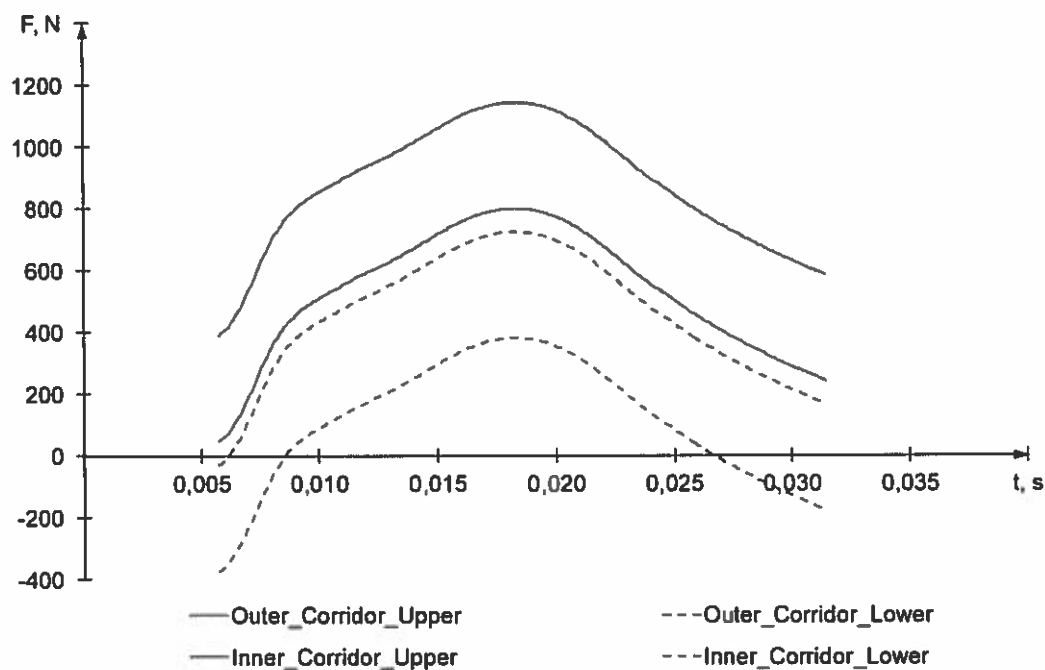


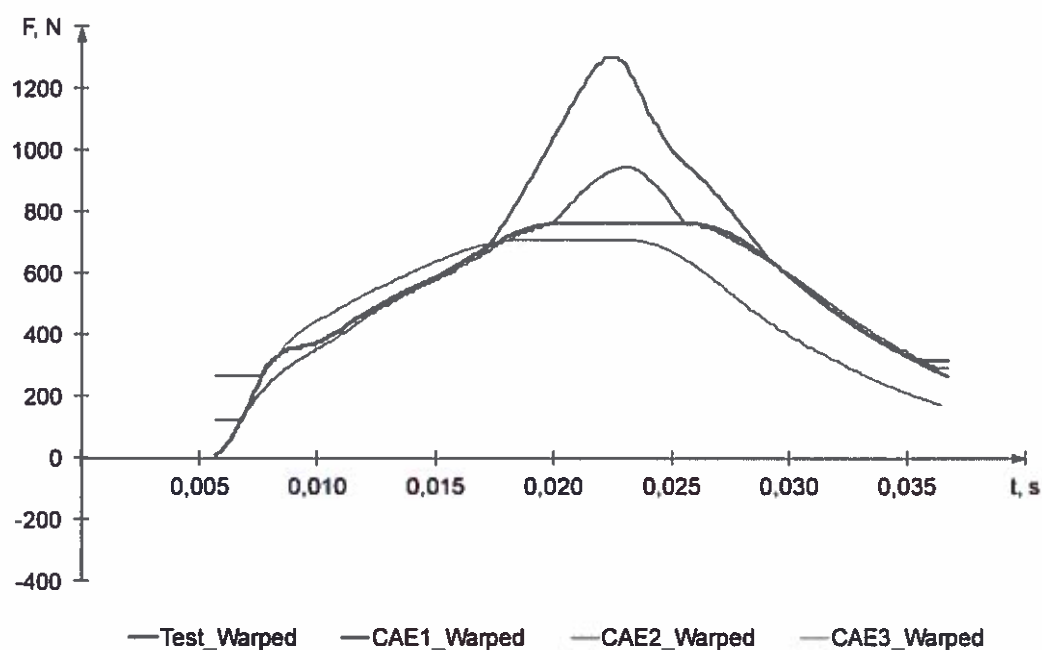
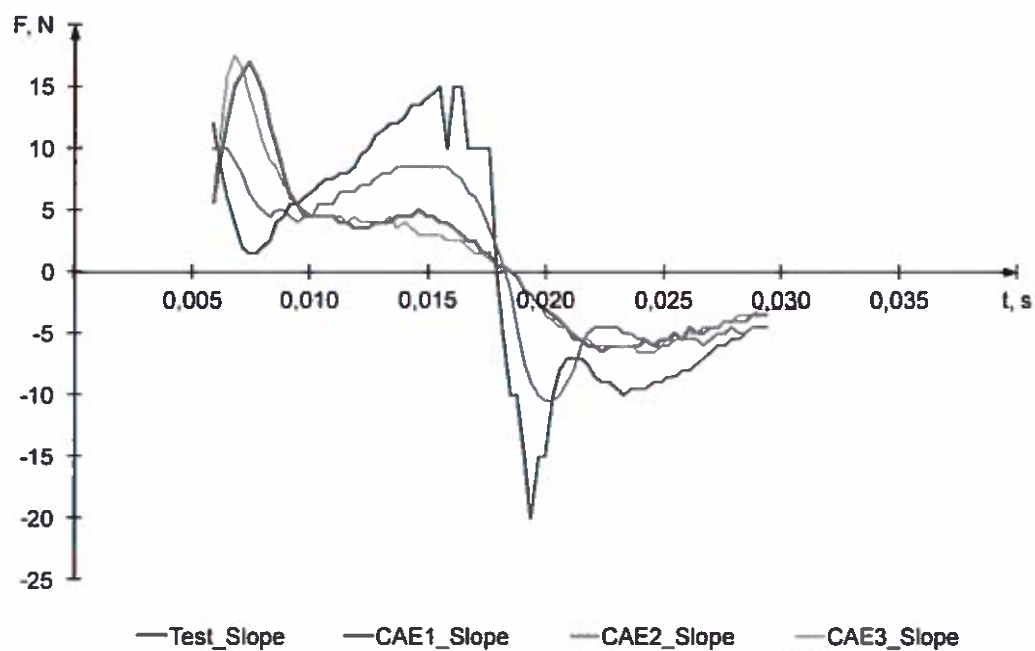
A.4 Forces

A.4.1 Force 1

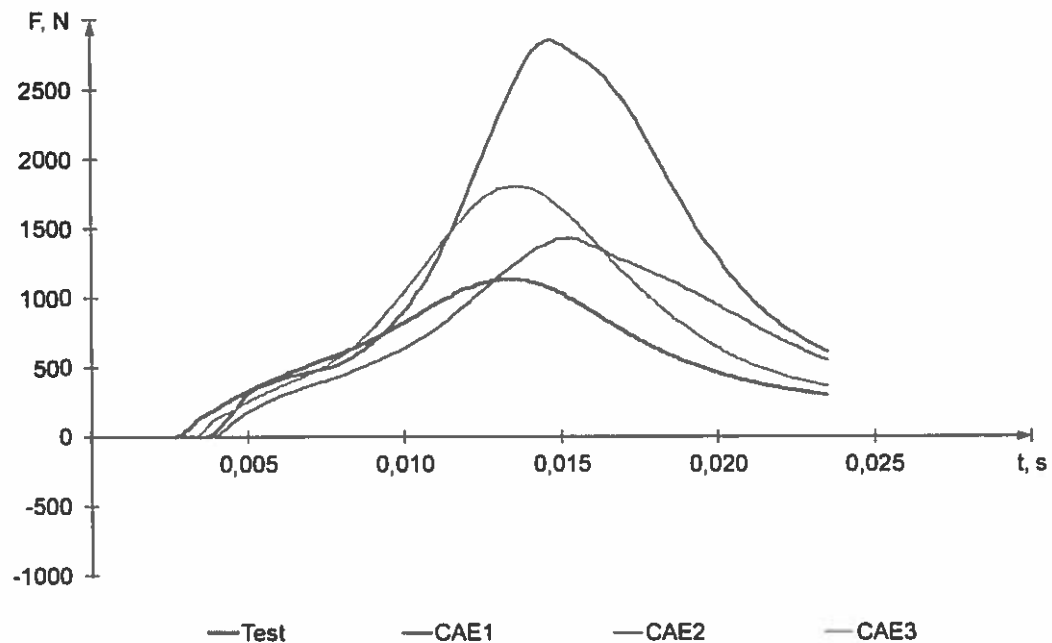


	CAE1 (<i>C</i> ₁)	CAE2 (<i>C</i> ₂)	CAE3 (<i>C</i> ₃)
Grade	Poor	Fair	Good
Overall rating, <i>R</i>	0,523	0,651	0,937
Corridor score, <i>Z</i>	0,452	0,527	0,938
Phase score, <i>E_P</i>	0,630	0,572	0,903
Magnitude score, <i>E_M</i>	0,621	0,911	0,967
Slope score, <i>E_S</i>	0,462	0,719	0,939

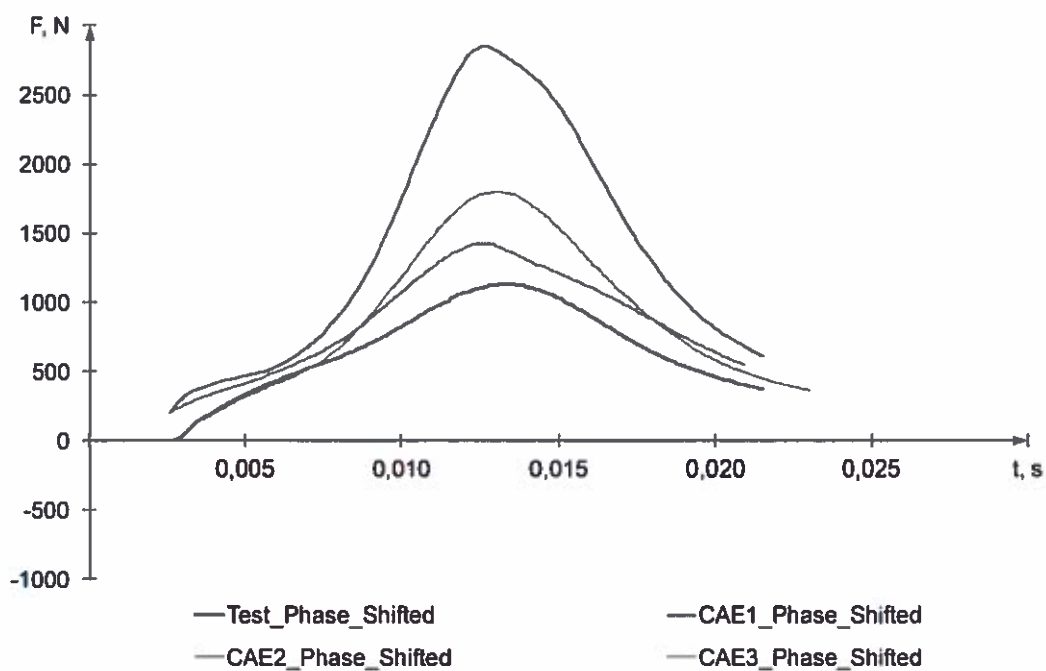
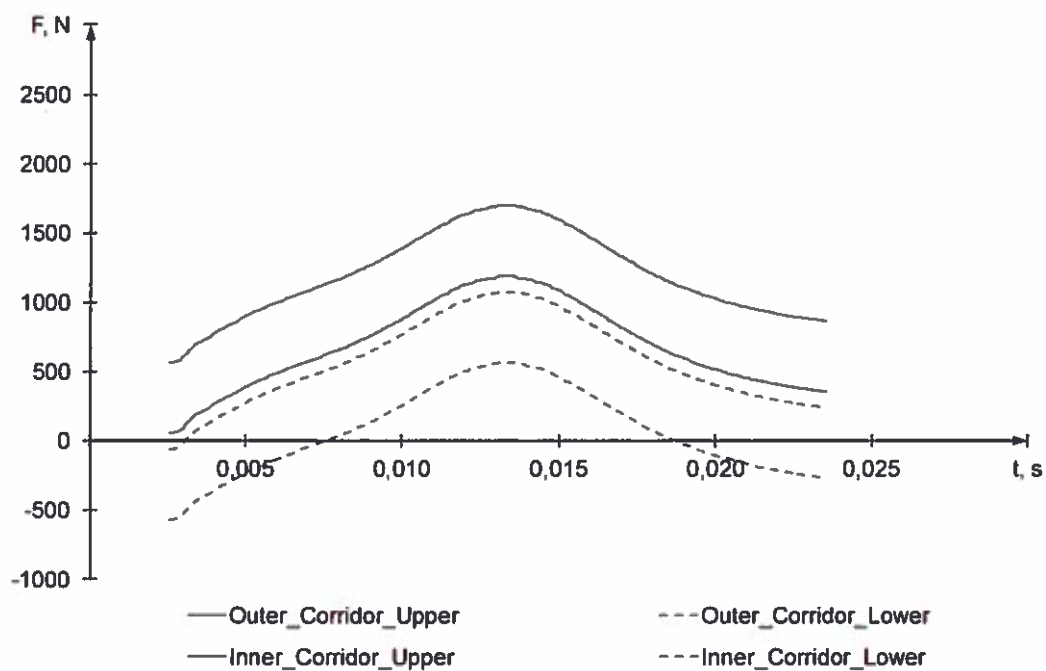


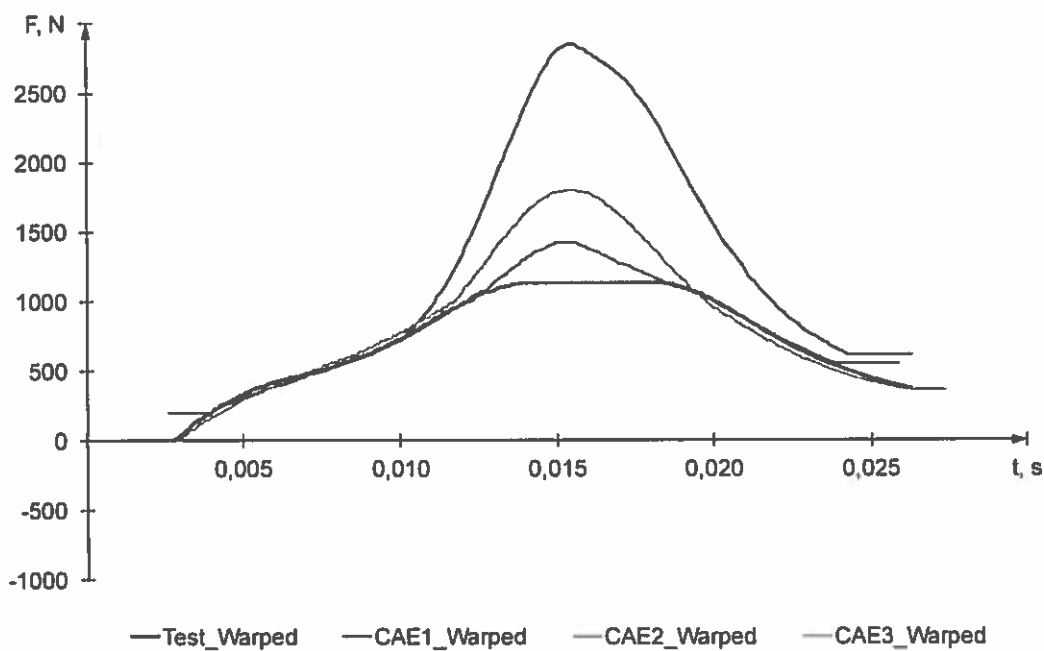
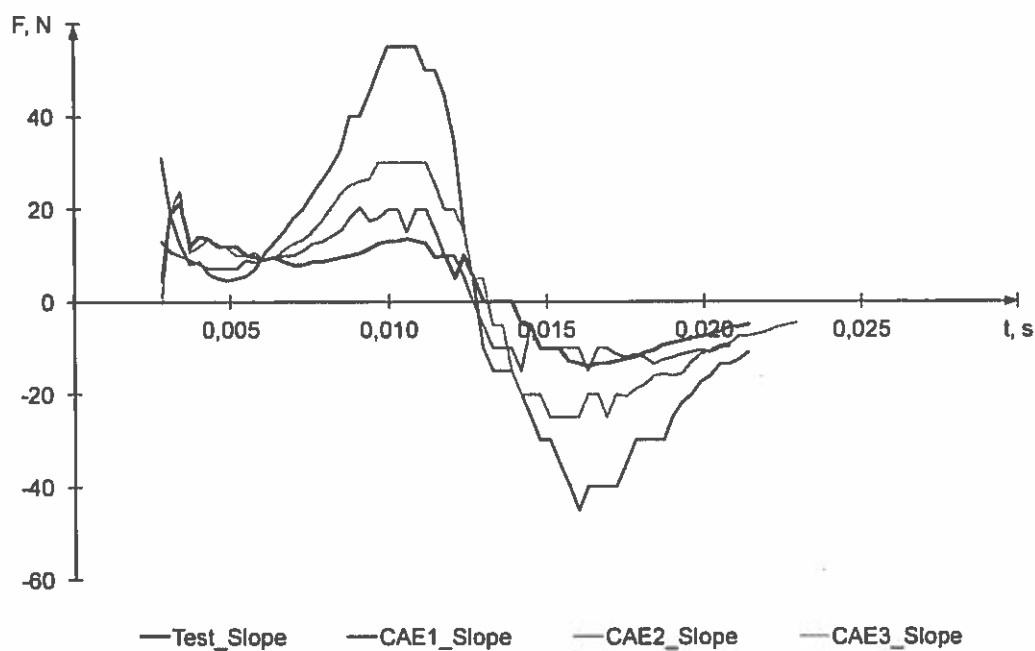


A.4.2 Force 2

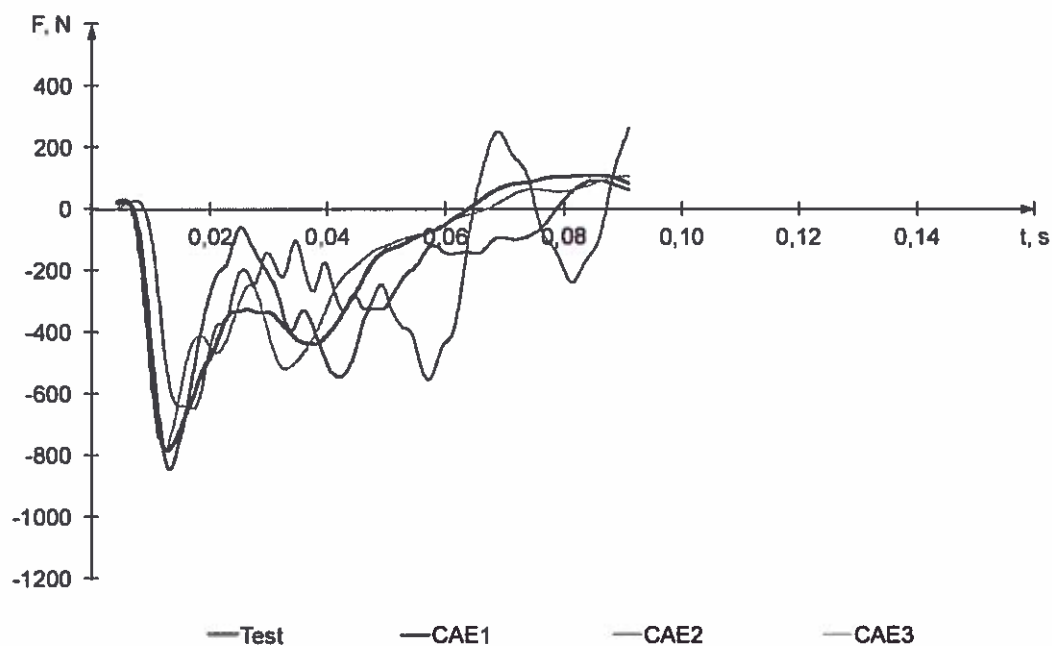


	CAE1 (C_1)	CAE2 (C_2)	CAE3 (C_3)
Grade	Poor	Poor	Fair
Overall rating, R	0,261	0,569	0,632
Corridor score, Z	0,375	0,428	0,521
Phase score, E_P	0,500	0,357	0,857
Magnitude score, E_M	0,000	0,850	0,658
Slope score, E_S	0,055	0,782	0,602

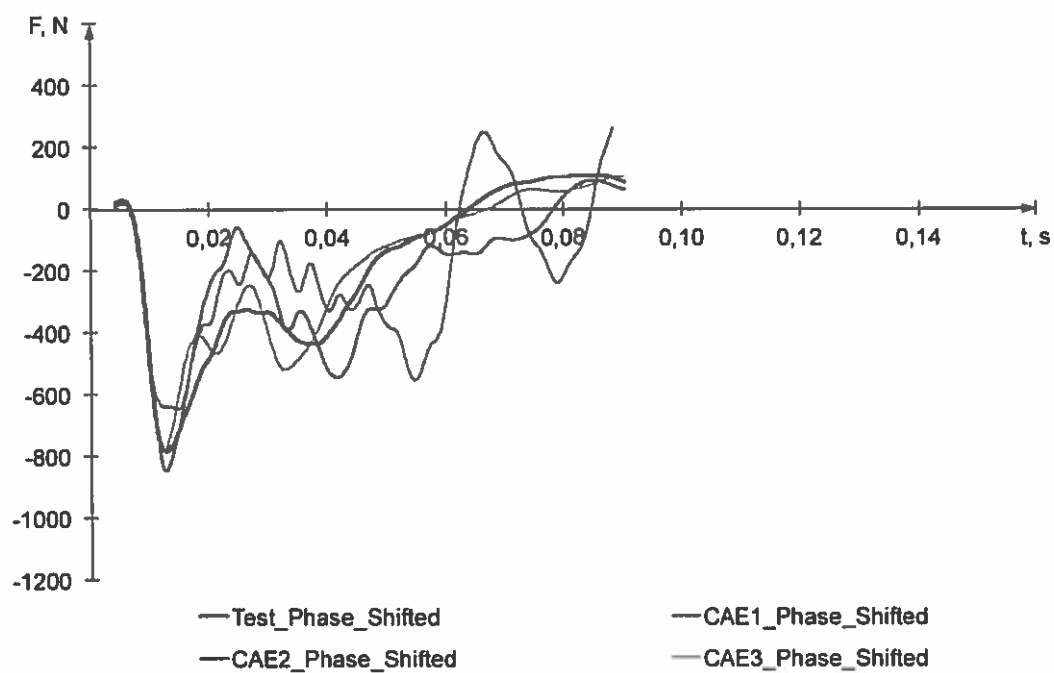
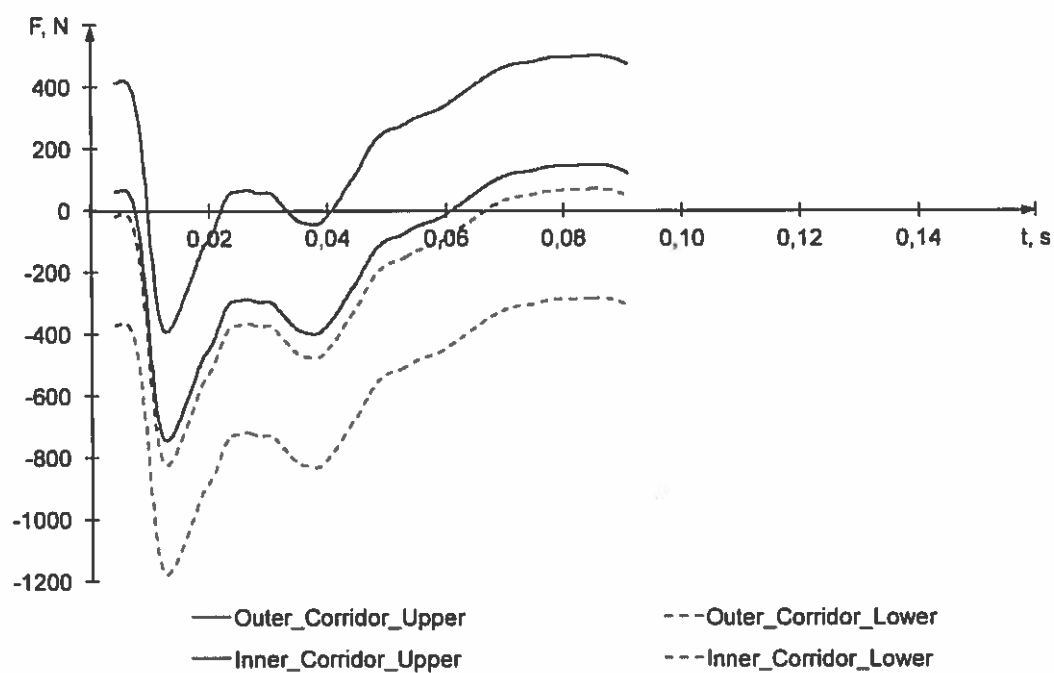


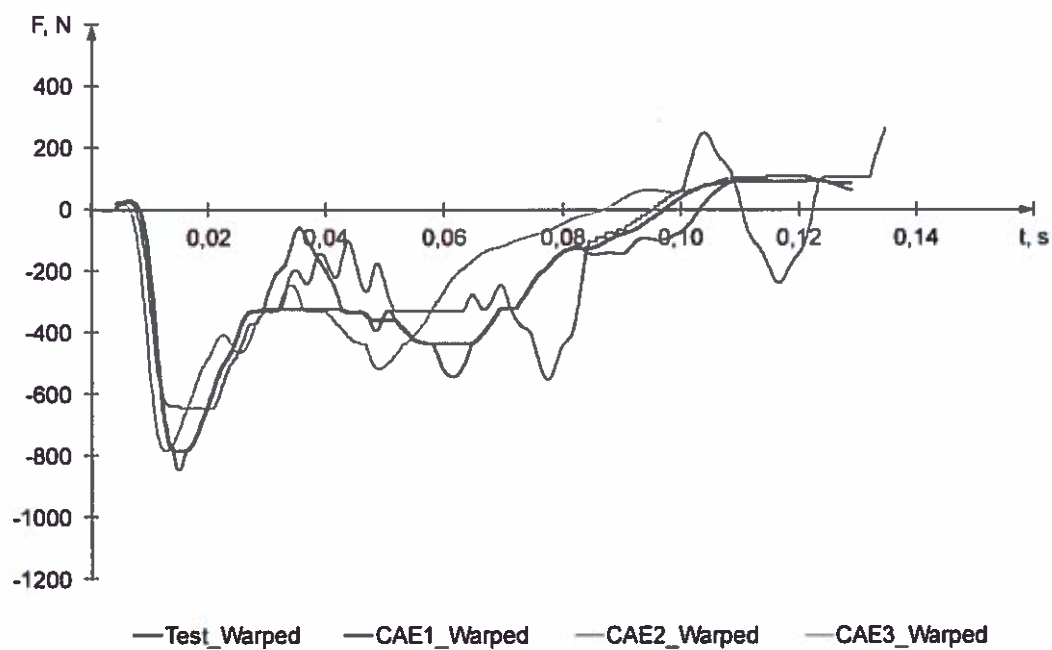
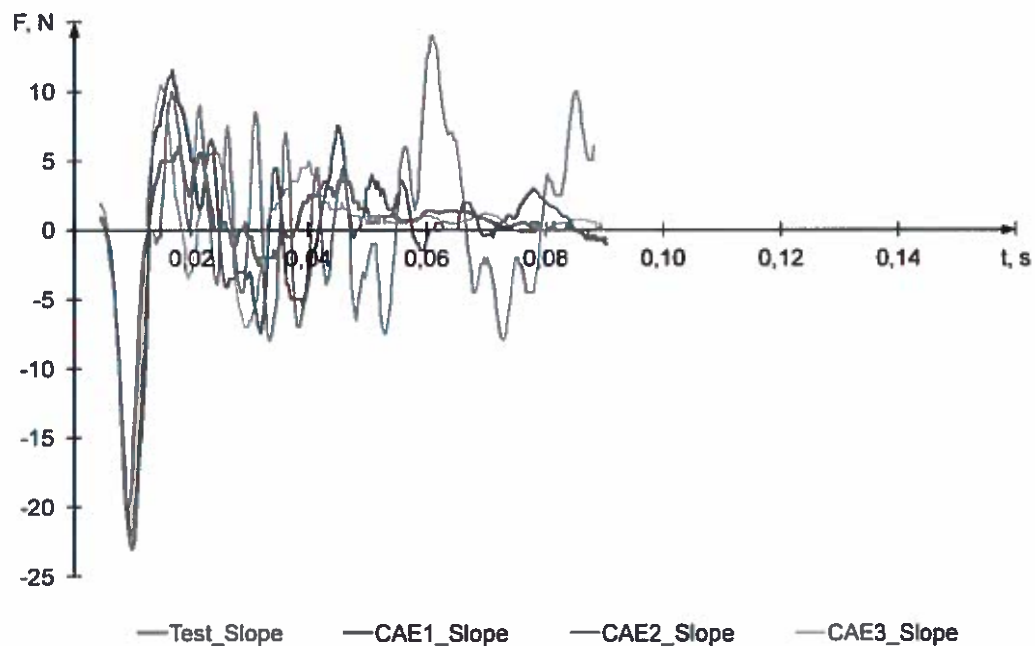


A.4.3 Force 3



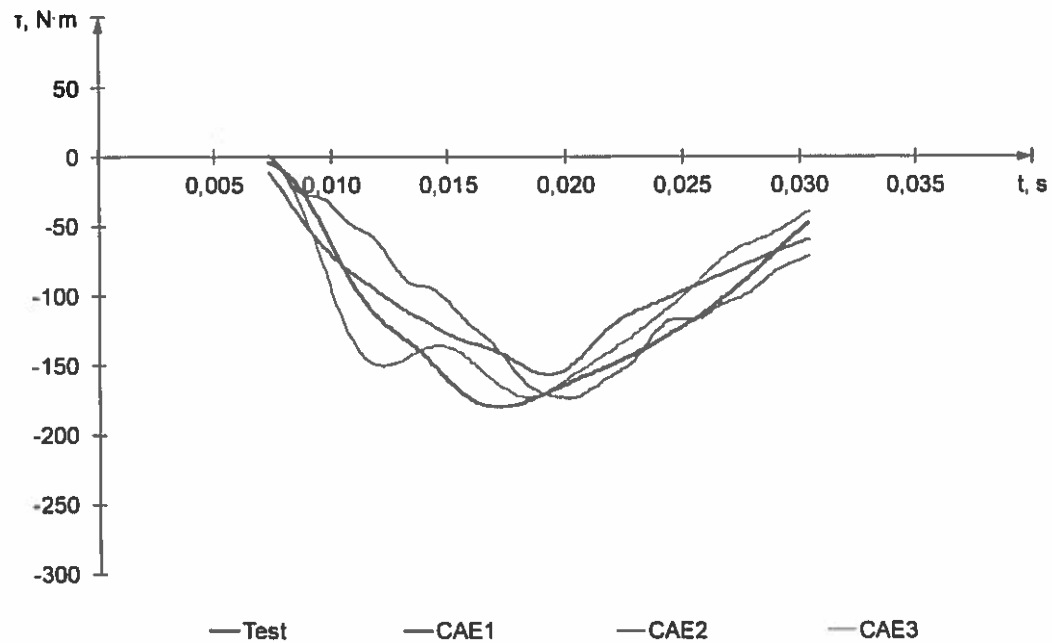
	CAE1 (C_1)	CAE2 (C_2)	CAE3 (C_3)
Grade	Fair	Poor	Good
Overall rating, R	0,710	0,460	0,862
Corridor score, Z	0,654	0,492	0,904
Phase score, E_P	0,971	0,856	0,954
Magnitude score, E_M	0,738	0,372	0,929
Slope score, E_S	0,533	0,087	0,621



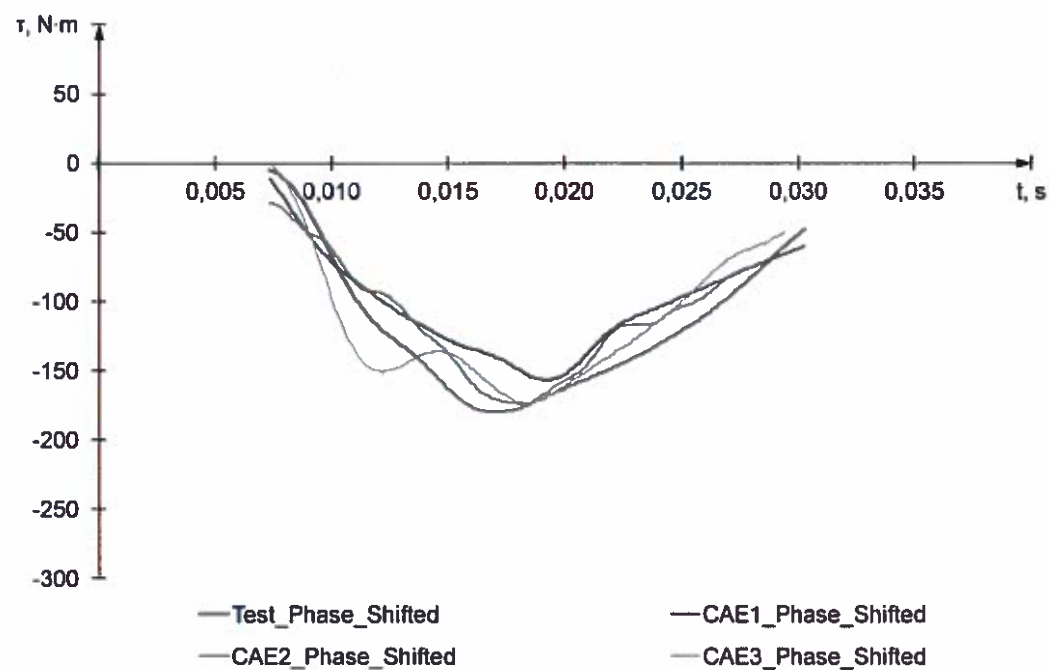
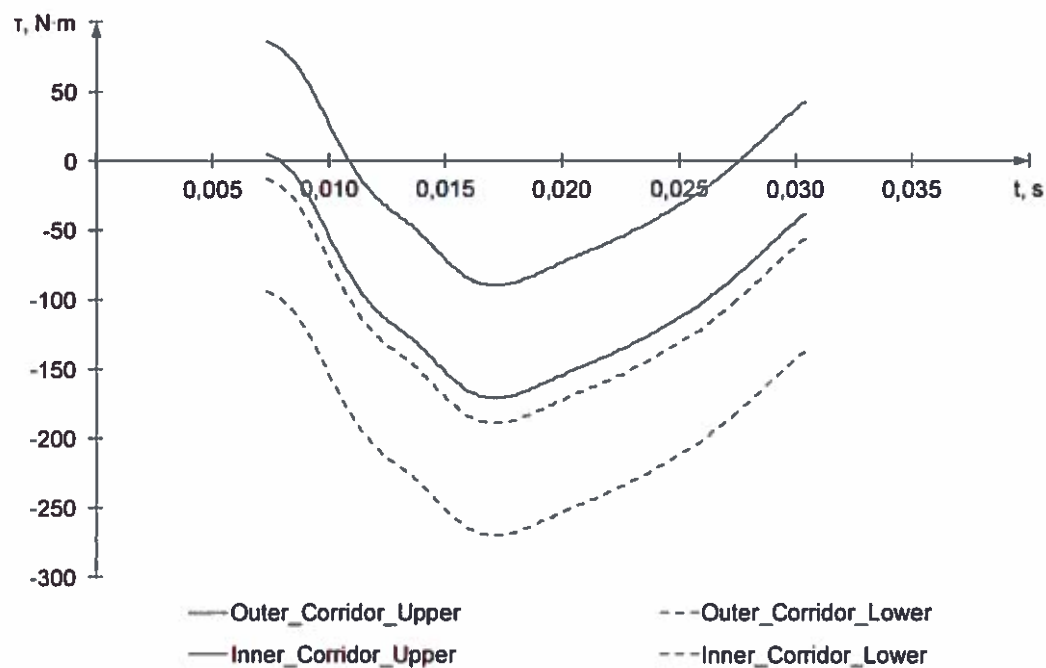


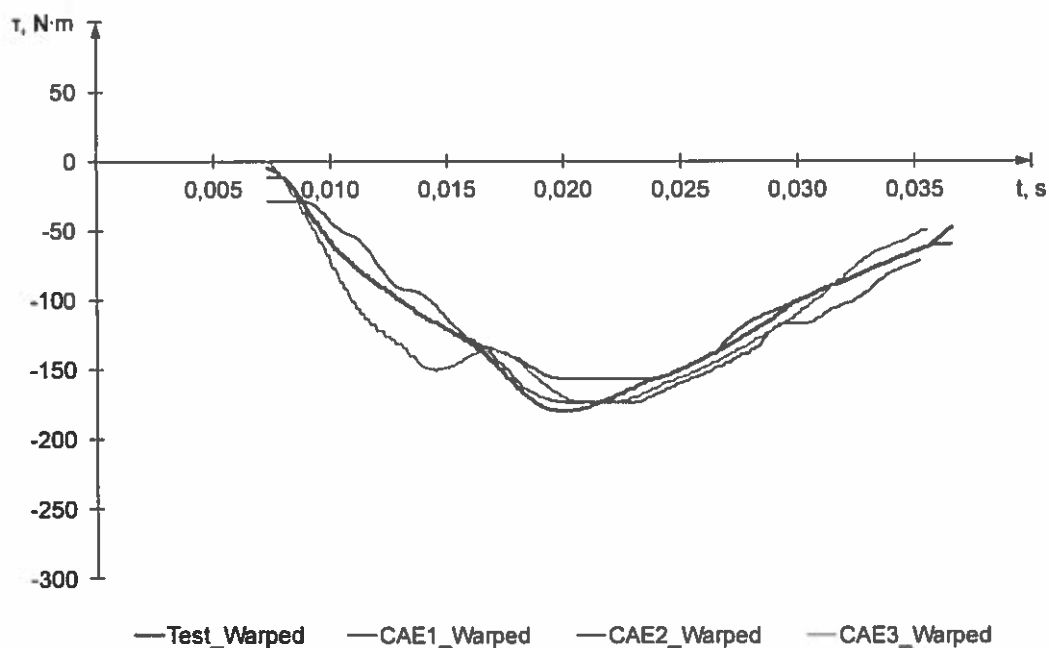
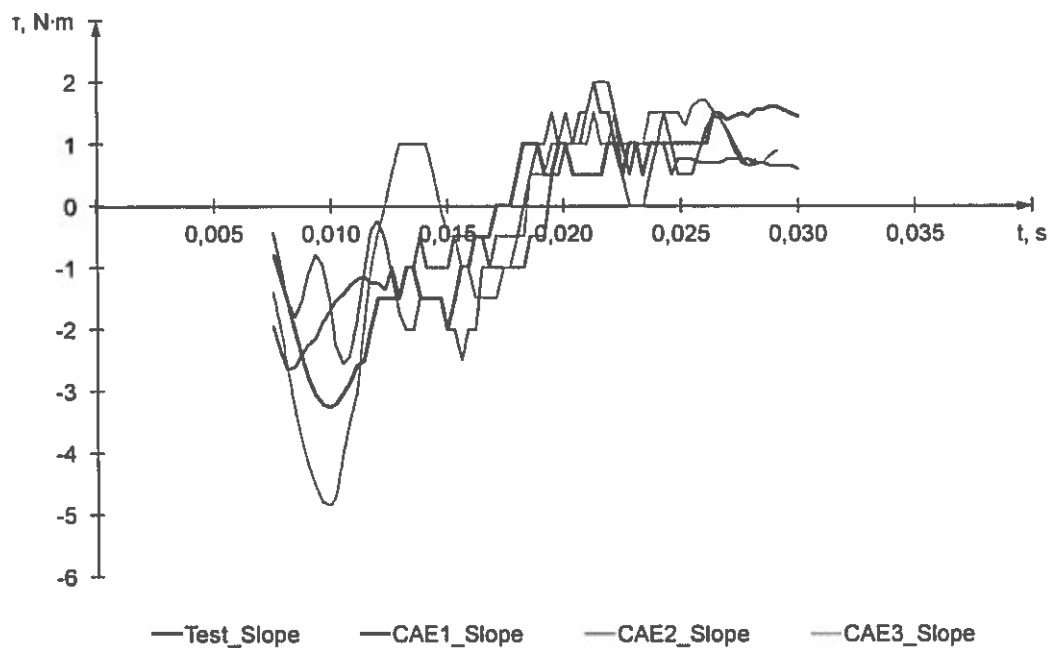
A.5 Moments

A.5.1 Moment 1

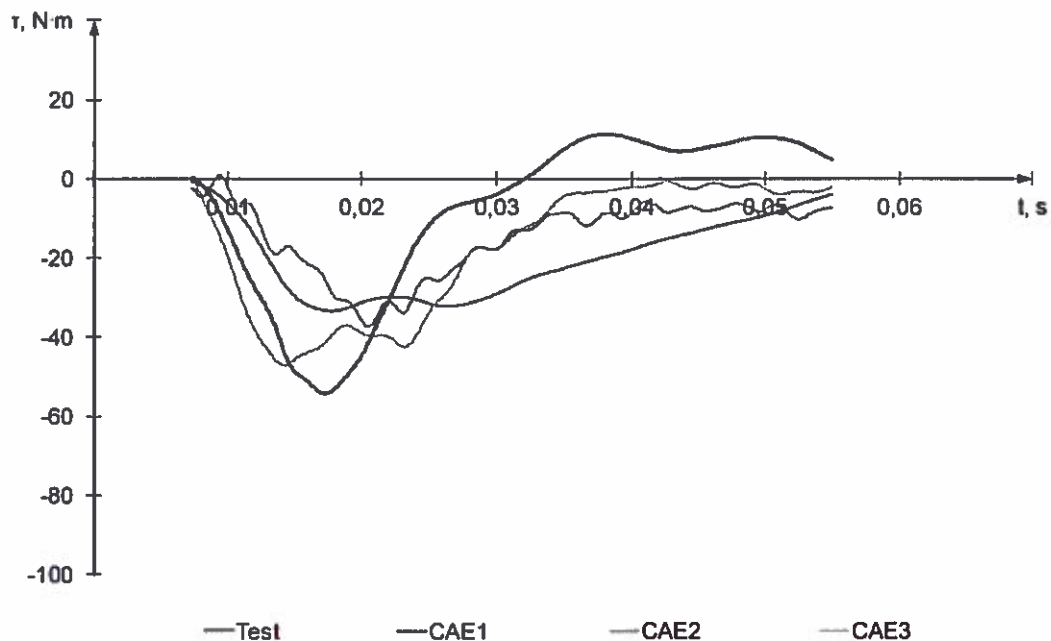


	CAE1 (C_1)	CAE2 (C_2)	CAE3 (C_3)
Grade	Good	Fair	Fair
Overall rating, R	0,815	0,747	0,794
Corridor score, Z	0,740	0,723	0,790
Phase score, E_p	0,957	0,547	0,763
Magnitude score, E_M	0,909	0,967	0,960
Slope score, E_S	0,730	0,772	0,664

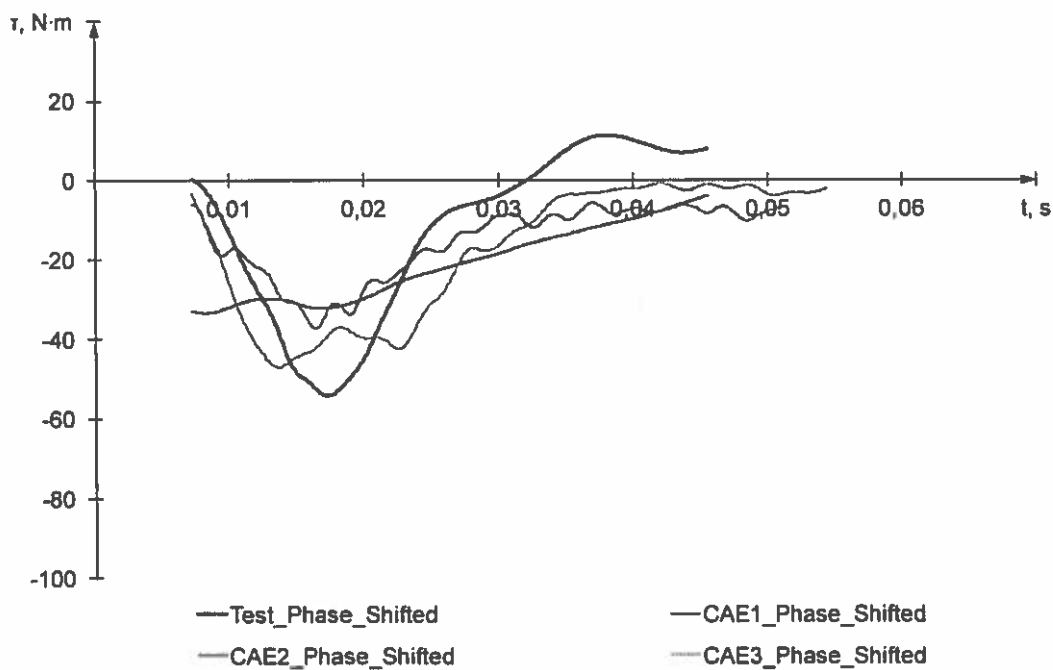
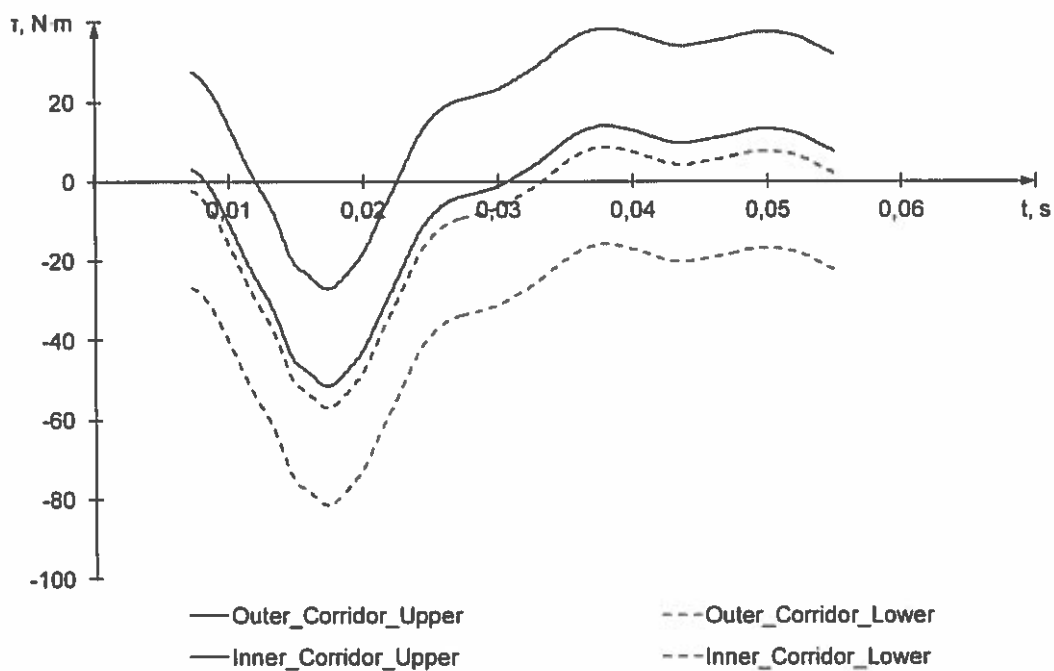


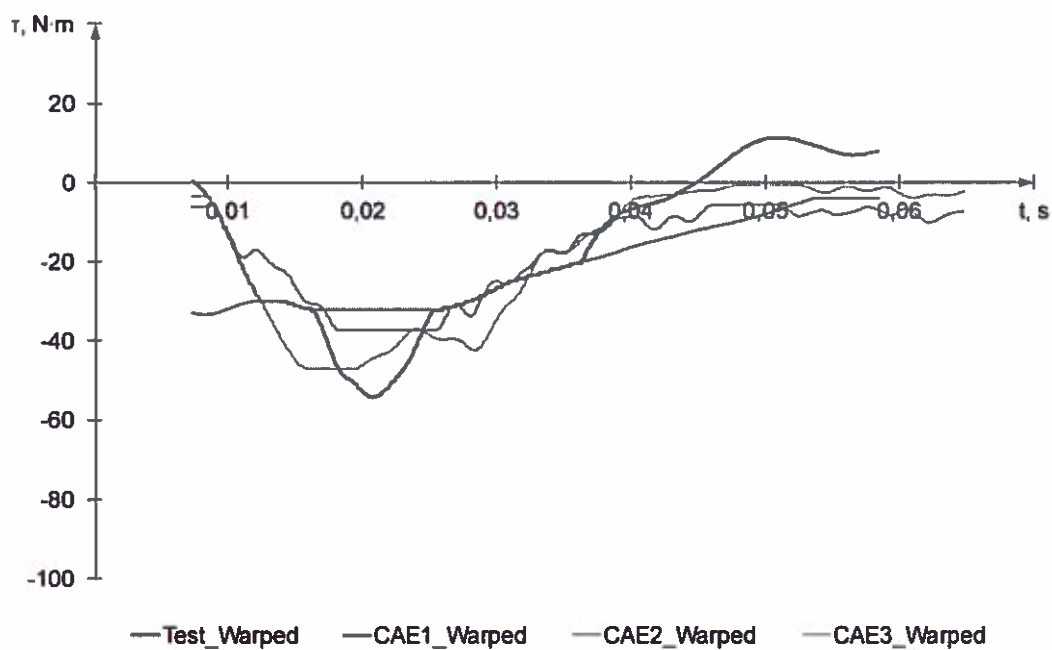
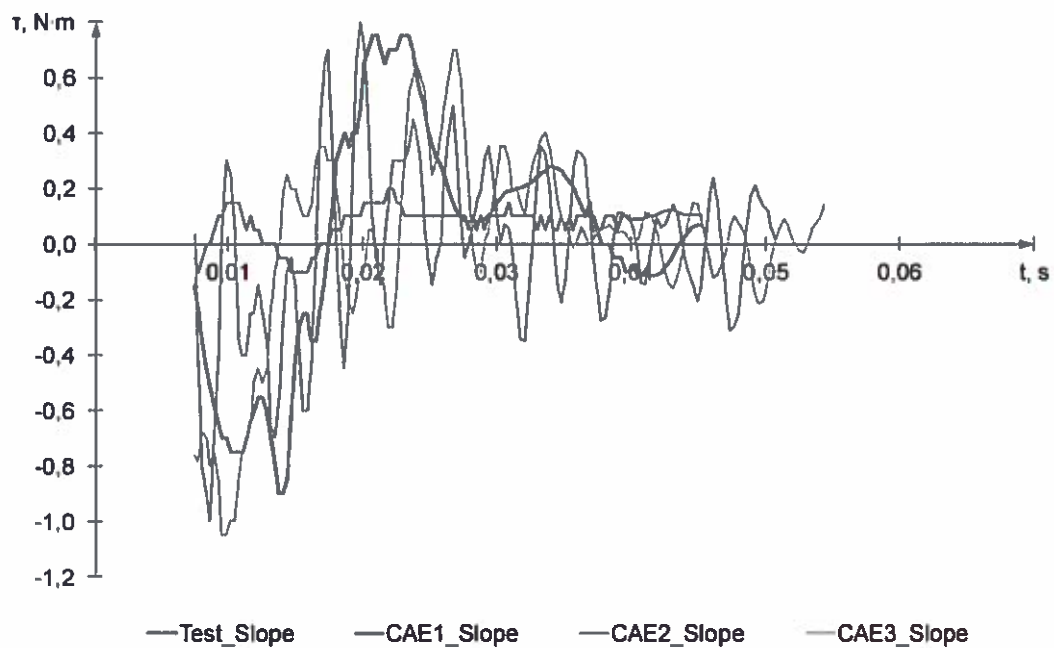


A.5.2 Moment 2

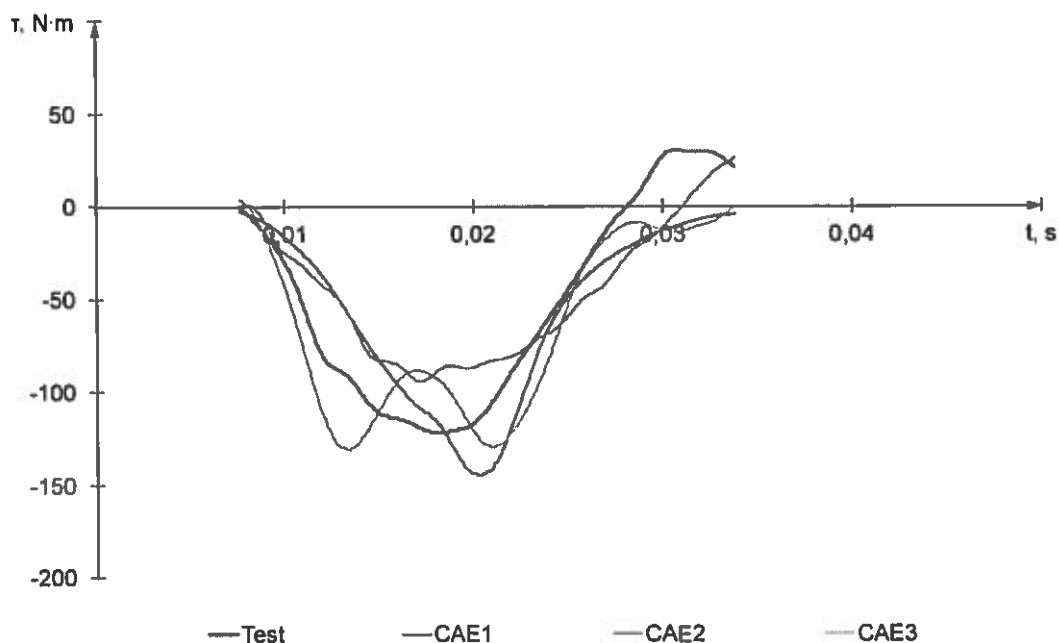


	CAE1 (C_1)	CAE2 (C_2)	CAE3 (C_3)
Grade	Poor	Poor	Poor
Overall rating, R	0,203	0,370	0,577
Corridor score, Z	0,205	0,280	0,448
Phase score, E_P	0,000	0,561	0,927
Magnitude score, E_M	0,054	0,210	0,500
Slope score, E_S	0,551	0,519	0,562





A.5.3 Moment 3



	CAE1 (C_1)	CAE2 (C_2)	CAE3 (C_3)
Grade	Fair	Fair	Fair
Overall rating, R	0,654	0,655	0,665
Corridor score, Z	0,539	0,538	0,556
Phase score, E_P	0,677	0,696	0,962
Magnitude score, E_M	0,840	0,798	0,735
Slope score, E_S	0,674	0,707	0,515

A.6 ASCII data of the case studies

The data shown in the previous sections of this Annex is also available in a digital format to verify the implementation of the algorithms. All test, CAE, and intermediate result curves are provided as comma-separated values (CSV) in plain-text form. Semicolons (;) are used to separate the columns. The numbers use a point (".") as decimal separator.

The first and second row of each file describe the data of each column. The name of the row is specified in the first row and the corresponding physical unit in the second row. The third to the last row contain the data. An overview of all columns is given in the following table.

Column	Name	Description
1	Generic_Time	Generic time: enumerator (abscissa values)
2	Time	Real time of the signals (abscissa values)
3	Test	Test signal (ordinate values)
4	CAEx	CAE signal x (ordinate values)
5	Outer_Corridor_Upper	Outer corridor, upper curve (ordinate values)
6	Inner_Corridor_Upper	Inner corridor, upper curve (ordinate values)
7	Inner_Corridor_Lower	Inner corridor, lower curve (ordinate values)
8	Outer_Corridor_Lower	Outer corridor, lower curve (ordinate values)
9	Test_Phase_Shifted	Phase shifted test curve (ordinate values)
10	CAEx_Phase_Shifted	Phase shifted CAE curve x (ordinate values)
11	Test_Slope	Slope test curve (ordinate values)
12	CAEx_Slope	Slope CAE curve x (ordinate values)
13	Test_Warped	Warped test curve (ordinate values)
14	CAEx_Warped	Warped CAE curve x (ordinate values)

The following table shows the link between the sub-sections of this Annex and the names of the ASCII files. Every file contains the CAE signal, the corresponding test signal, and intermediate result curves.

Section of Annex A	CAE signal	File name
A.1.1	CAE1	annex_a_1_1_ac1_cae1.csv
	CAE2	annex_a_1_1_ac1_cae2.csv
	CAE3	annex_a_1_1_ac1_cae3.csv
A.1.2	CAE1	annex_a_1_2_ac2_cae1.csv
	CAE2	annex_a_1_2_ac2_cae2.csv
	CAE3	annex_a_1_2_ac2_cae3.csv
A.1.3	CAE1	annex_a_1_3_ac3_cae1.csv
	CAE2	annex_a_1_3_ac3_cae2.csv
	CAE3	annex_a_1_3_ac3_cae3.csv
A.1.4	CAE1	annex_a_1_4_ac4_cae1.csv
	CAE2	annex_a_1_4_ac4_cae2.csv
	CAE3	annex_a_1_4_ac4_cae3.csv
A.2.1	CAE1	annex_a_2_1_an1_cae1.csv
	CAE2	annex_a_2_1_an1_cae2.csv
	CAE3	annex_a_2_1_an1_cae3.csv
A.2.2	CAE1	annex_a_2_2_an2_cae1.csv
	CAE2	annex_a_2_2_an2_cae2.csv
	CAE3	annex_a_2_2_an2_cae3.csv
A.3.1	CAE1	annex_a_3_1_ds1_cae1.csv
	CAE2	annex_a_3_1_ds1_cae2.csv
	CAE3	annex_a_3_1_ds1_cae3.csv
A.3.2	CAE1	annex_a_3_2_ds2_cae1.csv
	CAE2	annex_a_3_2_ds2_cae2.csv
	CAE3	annex_a_3_2_ds2_cae3.csv
A.4.1	CAE1	annex_a_4_1_fo1_cae1.csv
	CAE2	annex_a_4_1_fo1_cae2.csv
	CAE3	annex_a_4_1_fo1_cae3.csv
A.4.2	CAE1	annex_a_4_2_fo2_cae1.csv
	CAE2	annex_a_4_2_fo2_cae2.csv
	CAE3	annex_a_4_2_fo2_cae3.csv
A.4.3	CAE1	annex_a_4_3_fo3_cae1.csv
	CAE2	annex_a_4_3_fo3_cae2.csv
	CAE3	annex_a_4_3_fo3_cae3.csv
A.5.1	CAE1	annex_a_5_1_mo1_cae1.csv
	CAE2	annex_a_5_1_mo1_cae2.csv
	CAE3	annex_a_5_1_mo1_cae3.csv

Section of Annex A	CAE signal	File name
A.5.2	CAE1	annex_a_5_2_mo2_cae1.csv
	CAE2	annex_a_5_2_mo2_cae2.csv
	CAE3	annex_a_5_2_mo2_cae3.csv
A.5.3	CAE1	annex_a_5_3_mo3_cae1.csv
	CAE2	annex_a_5_3_mo3_cae2.csv
	CAE3	annex_a_5_3_mo3_cae3.csv

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