

# Identification Methods for Structural Systems

**Prof. Dr. Eleni Chatzi**

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The work is done either in

the frequency domain - **Modal ID methods**

using the Frequency Response Function (FRF) information

or in the

time domain - **Time Domain ID methods**

where an analytical or numerical (FE) model of the system or information from the Impulse Response Function (IRF) are used.

**Reminder:** The IRF is the inverse Fourier Transform of the FRF.

## Applications

- Troubleshooting
- Finite Element model updating
- Structural Modification
- Reduction of the Order of mathematical models
- Response Prediction
- Structural Damage Detection
- Active Vibration Control

## Modal Id Methods

aim at determining the inherent dynamic characteristics of a system in forms of natural frequencies, damping factors and mode shapes, and using them to formulate a mathematical model for its dynamic behavior.

The formulated mathematical model is referred to as **modal model** and the information for the characteristics are known as its **modal data**.

Modal analysis is based upon the fact that the vibration response of a linear time-invariant (LTI) dynamic system can be expressed as the linear combination of a set of simple harmonic motions called the natural modes of vibration.

This concept is akin to the use of a Fourier combination of sine and cosine waves to represent a complicated waveform.

source: Jimin He, Zhi-Fang Fu, "Modal Analysis", Butterworth-Heinemann

- 1 Classification based on the number of **input and output locations**:

SISO - SIMO - MIMO - MISO

- 2 Based on the type of **identified properties**:

Direct Methods	Indirect Methods
Mechanical k,m,c	Modal ( $\omega_i, \zeta_i$ , etc)

## Basic Concepts

Assume an  $N$ -degree of freedom system with damping:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{F}$$

$\mathbf{x}$  is the  $N \times 1$  state space vector of the system and  $\mathbf{M}$ ,  $\mathbf{C}$ ,  $\mathbf{K}$  are the system mass, damping and stiffness matrices.

In steady state conditions (i.e. once the free response has died down), we showed that one can establish the relationship between the complex amplitude of the response  $\mathbf{X}$  and the input  $\mathbf{F} = \mathbf{F}_0 e^{i\omega t}$  as:

$$\mathbf{X} = \mathbf{H}(i\omega) \cdot \mathbf{F}$$

where  $\mathbf{H}$  is the FRF matrix

As already mentioned in Lecture 5 (slide 5), the FRF can be written in partial fraction expansion. (i.e poles/residues form):

$$H(\omega) = \sum_{k=1}^N \frac{A_k}{i\omega - \lambda_k} + \frac{A_k^*}{i\omega - \lambda_k^*}$$

where  $A_k$  is the modal constant matrix with components  ${}_k A_{ij}$  related to components  $\phi_{ik}$ ,  $\phi_{jk}$  of eigenvector  $k$  through the formula:

$${}_k A_{ij} = q_k \phi_{ik} \phi_{jk} \quad \text{and } q_k \text{ is a modal participation factor}$$

In addition, index  $k$  corresponds to the  $k$ -th mode and the poles of the denominator are simply the roots of the characteristic polynomial:

$$\lambda_k, \lambda_k^* = -\omega_k \zeta_k \pm i \left( \omega_k \sqrt{1 - \zeta_k^2} \right)$$

## The Frequency Response Function

We already discussed that the component  $H_{ij}$  of the FRF matrix,  $H$ , corresponds to a particular output response at point  $i$  due to an input force at point  $j$ .

Based on this, **Modal testing** is an experimental technique used to derive the modal model of a LTI vibratory system.

Combinations of excitation and response at different locations lead to a complete set of frequency response functions (FRFs) which can be collectively represented by an FRF matrix of the system.

This matrix is usually symmetric, reflecting the structural reciprocity of the system i.e.  $H_{ij} = H_{ji}$ .



# Basic Concepts

The Impulse Response Function  $h_{ij}(\omega)$ , is the equivalent of  $H_{ij}(\omega)$  in the time domain, hence it can be obtained using IFT and the property

$$\mathcal{F}^{-1} \left\{ \frac{1}{a + i\omega} \right\} = e^{-at}, \quad t > 0$$

$$\text{or } h_{ij}(t) = ({}_k A_{ij} e^{\lambda_k t} + {}_k A_{ij}^* e^{\lambda_k^* t})$$
$$\text{with } \lambda_k = -\omega r \zeta_k + i \omega_k \sqrt{1 - \zeta_k^2}$$

Ultimately this can also be compactly written as:

$$h_{ij}(t) = \sum_{k=1}^{2N} {}_k A_{ij} e^{\lambda_k t}$$

(with a slightly different notation for  ${}_k A_{ij}$ ,  $\lambda_k$ )

## Modal ID Fundamentals

Return to the definition of the FRF, since the matrices of residuals  $A_k$ , are of rank one, meaning that they can be decomposed as:

$$A_k = \Phi_k \Phi_k^T = \begin{bmatrix} \Phi_{1k} \\ \vdots \\ \Phi_{Nk} \end{bmatrix} \begin{bmatrix} \Phi_{1k} & \cdots & \Phi_{Nk} \end{bmatrix}$$

$\Phi_k$  : eigenvector of mode  $k$

then it follows that essentially, the FRF of a linear mdof system with  $N$  dofs is the sum of  $N$  sdof FRFs (modal superposition) and the transfer function matrix is completely characterized by the modal parameters i.e the roots of the character polynomial  $\lambda_m$  and the mode shape vectors  $\Phi_m$ ,  $m = 1, \dots, N$

## Mode Shapes and Operating Deflection Shapes

At or near the natural frequency of a mode, the overall vibration shape will be dominated by the mode shape of the resonance.

Applying a harmonic force at one of the degrees of freedom, say  $r$ , at a frequency  $i\omega_{dn}$  (damped frequency of the  $n$ th mode) results in a displacement vector,  $X(\omega_{dn})$ , which is approximately proportional to the mode shape vector of mode  $n$ , i.e.  $\Phi_n$ .

In reality there is always a small contribution of the other modes.

Within this context of properly exciting the structure, modal testing was also known as resonance testing, as it initiated using various shakers properly tuned in order to put the structure into resonance.

# Modal Testing

The practice of **modal testing** involves measuring the FRFs or impulse responses of a structure. The FRF measurement can simply be done by asserting a measured excitation at one location of the structure in the absence of other excitations and measure vibration responses at one or more location(s). The excitation can be of a selected frequency band, stepped sinusoid, transient, random or white noise. It is usually measured by a force transducer at the driving point while the response is measured by accelerometers or other probes.

Given input and output time histories the FRF can be calculated through the output/input Fourier Transform ratio:

$$Y(\omega) = H(\omega)X(\omega) \Rightarrow \\ H(\omega) = \frac{Y(\omega)}{X(\omega)} \quad \text{where} \quad X(\omega) = \mathcal{F}\{x(t)\}, Y(\omega) = \mathcal{F}\{y(t)\}$$

**Important Note:** When the Input corresponds to an impulse i.e.

$x(t) = \delta(t)$ , the Fourier transform is  $\mathcal{F}\{x(t)\} = 1 \Rightarrow H(\omega) = Y(\omega)$

# Modal Testing

For a structural system what is usually measured as input is the force ( $F$ ) inputted to the system. However the measured output can be displacement ( $U$ ), velocity ( $\dot{U}$ ) or acceleration ( $\ddot{U}$ ).

Therefore, based on the available measurements we distinguish between the following cases:

$$H(\omega) = \frac{U(\omega)}{F(\omega)} \quad \text{Receptance FRF}$$

$$H(\omega) = \frac{\dot{U}(\omega)}{F(\omega)} \quad \text{Mobility FRF}$$

$$H(\omega) = \frac{\ddot{U}(\omega)}{F(\omega)} \quad \text{Accelerance FRF}$$

Also, when the response and excitation coordinates coincide ( $i = j$ ), the FRF component  $H_{ij}$  is referred to as a **point FRF**. For  $i \neq j$  we have a **transfer FRF**.

As it turns out, using fundamental properties from signal processing, the previous formulation for the estimation of the FRF is equivalent to the following ratio:

$$H(\omega) = \frac{S_{yx}(\omega)}{S_{xx}(\omega)} \quad \text{where}$$

$S_{xx}$  stands for **Power Spectral Density** and  
 $S_{yx}$  stands for **Cross Spectral Density**

This calculation proves to yield more smoothed results in the presence of noise and is therefore preferable.

In MATLAB the FRF is estimated via the built-in *tfestimate* function.

## Definitions -Power Spectral Density

The PSD is the FT of autocorrelation function (ACF) of a signal. It refers to the amount of power per unit (density) of frequency (spectral) as a function of the frequency.

$$S_{xx}(\omega) = \int_{-\infty}^{\infty} R_{xx}(t)e^{-i\omega t} dt = F(R_{xx})$$

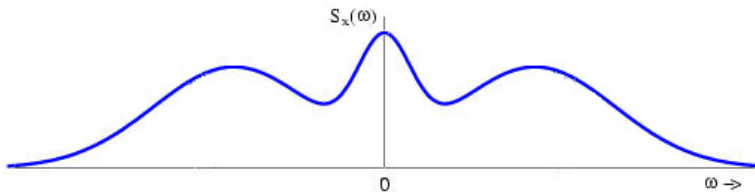
This is a positive, real valued function of frequency, often called the spectrum of a signal.

**Note:** For the PSD to exist, the Random Process (signal) has to be at least Wide Sense Stationary (WSS).

## PSD Properties

- 1 PSD of a real valued process is symmetric  $S(-x) = S(x)$
- 2 Continuous and differentiable on  $[-\pi, \pi]$
- 3 Has a derivative equal to 0 at  $x = 0$
- 4 Describes the mean signal power:

$$\text{Var}(X_t) = R_{xx}(0) = \frac{1}{\pi} \int_0^{\pi} S(\omega) d\omega \quad (df = \frac{d\omega}{2\pi})$$

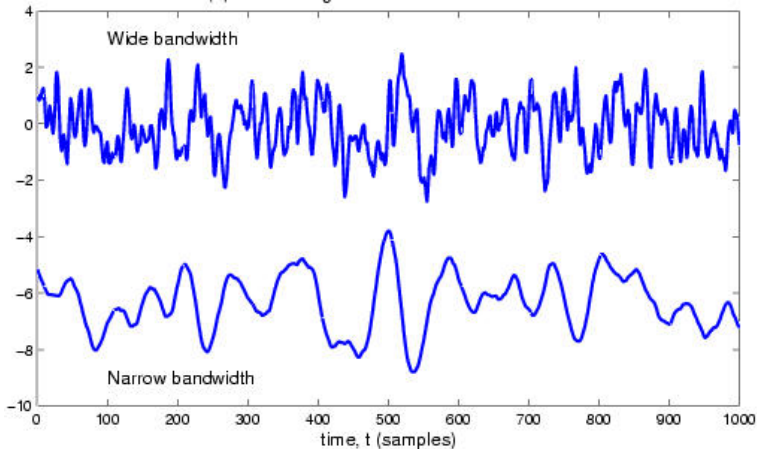




# The PSD

## Examples:

Consider the following random signals with different bandwidths

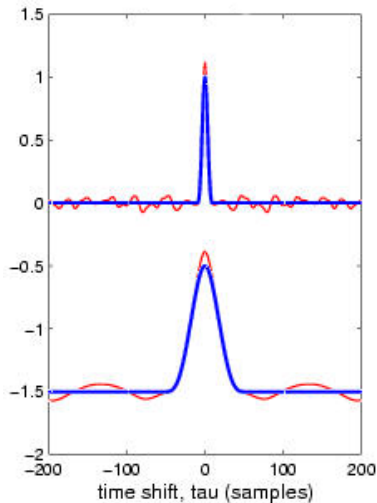


(source:<http://cnx.org>)

## ACF

The width of the ACF tells us how slowly a signal is fluctuating or how band-limited it is.

In the upper plot, the ACF decays quickly to zero as  $t$  increases, whereas, for a slowly fluctuating signal (lower plot) the ACF decays much more slowly.

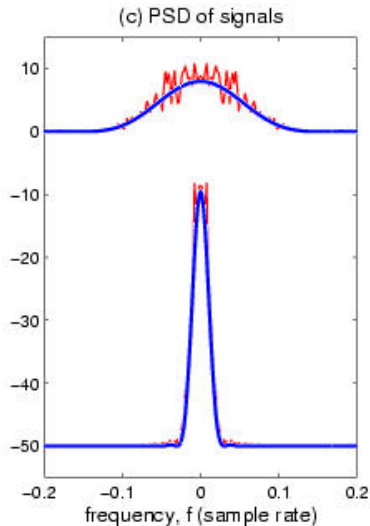


## PSD

Here you see the corresponding PSDs

In the upper plot energy is scattered on a broader band of frequencies

In the lower plot, it is concentrated on a narrow band



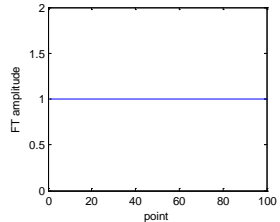
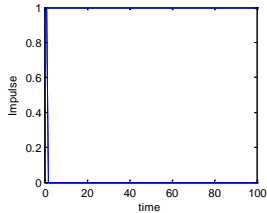
## Important FT and PSD Properties:

### FT for Impulse

#### MATLAB code

```
% Generate Impulse
imp=zeros(1,100);
imp(1)=1;
hold on
plot(imp)
xlabel('time')
ylabel('Impulse')

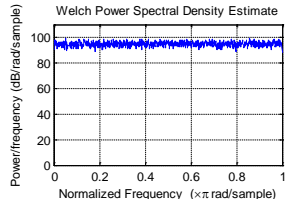
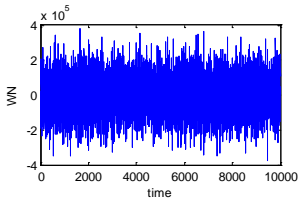
% Generate the Fourier
Transform
figure
plot(abs(fft(imp)))
xlabel('point')
ylabel('FT amplitude')
```



### PSD for White Noise signal

#### MATLAB code

```
% Generate 10000 point White
Noise
WN=wgn(1,10000, 100);
plot(WN)
xlabel('time')
ylabel('WN')
figure
% Generate the Spectrum
pwelch(WN)
ylim([0 110])
```

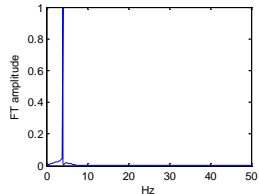
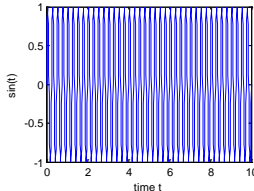


## Important FT and PSD Properties:

### FT for Sinusoid

#### MATLAB code

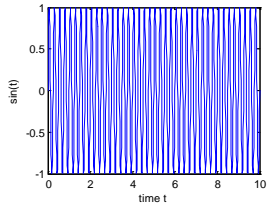
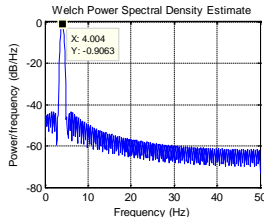
```
% Generate Sinusoid, freq=8*pi rad/s ->
8*pi/(2*pi)=4Hz
fs=100; %sampling freq.
t=[0:1/fs:10];
s=sin(8*pi*t);
plot(t,s)
xlabel('time t')
ylabel('sin(t)')
figure
%Plot the FFT of the signal
L=length(s);
NFFT= 2*nextpow2(L); % Next power of 2 from length of x; %FFT points need to be larger than the length of the
signal
Y = fft(s,NFFT)/L;
f = fs/2*linspace(0,1,NFFT/2+1); %Horizontal axis frequency equivalent in Hz
figure(2)
plot(f,2*abs(Y(1:NFFT/2+1)));hold on %Plot only half
xlabel('Hz')
ylabel('FT amplitude')
```



### PSD for Sinusoid

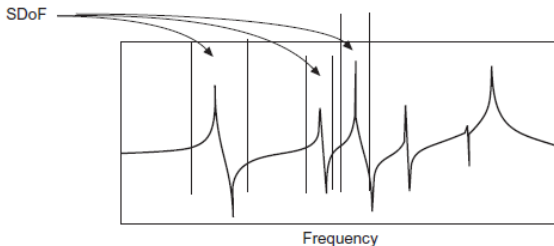
#### MATLAB code

```
% Generate Sinusoid, freq=8*pi
rad/s -> 8*pi/(2*pi)=4Hz
fs=100; %sampling freq.
t=[0:1/fs:10];
s=sin(8*pi*t);
plot(t,s)
xlabel('time t')
ylabel('sin(t)')
figure
% Generate the Spectrum
pwelch(s,[],[],1024,fs)
```

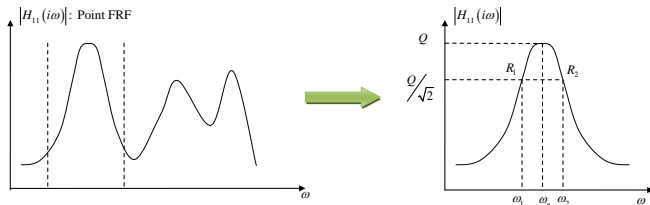


## Peak Picking Method

At the vicinity of a resonance, the FRF is dominated by the contribution of that vibration mode and the contributions of other vibration modes are negligible. If this assumption holds, then the FRF from an MDoF system or a real structure can be treated as the FRF from an SDoF system momentarily. The SDoF mathematical model can then be used with curve fitting in order to derive the modal parameters.



## Peak Picking method - Half Power method (Lecture 3)



### Identified Quantities (per peak)

- Natural Frequency  $\omega_n = \omega_{peak}$     &    Damping  $\zeta_n = \frac{\omega_2 - \omega_1}{2\omega_n}$
- Modal constant (related to the mode shape) can be proven to be equal to:  $A_n = 2Q\zeta_n\omega_n^2$

## Peak Picking method - Half Power method

The latter is derived from the SDoF system TF, (see Lecture 3) written for mode n:

$$H(\omega) = \frac{A_n}{\omega_n^2 - \omega^2 + 2i\zeta_n\omega\omega_n}$$

hence, for  $\omega = \omega_n$

$$Q = |H(\omega_n)| = \frac{A_n}{2\zeta_n\omega_n^2} \Rightarrow A_n = 2Q\zeta_n\omega_n^2$$



## Mode Shape Extraction

As already stated, the modal constant components are (per mode  $k$ ):  ${}_kA_{ij} = q_k \phi_{ij} \phi_{jk}$ . Therefore when looking at the FRF point component  $H_{ij}$ , for excitation at  $i$  and response at  $j$ , we have that for each mode  $k = 1, \dots, N$ :

$${}_kA_{ii} = q_k \phi_{ik} \phi_{ik} \Rightarrow \phi_{ik} \text{ is defined}$$

( $q_k$  is a scaling factor and does not affect the estimate of the mode)  
Then the  $j$ -th component of mode  $k$  can be obtained either from:

$$\text{the point FRF: } H_{jj} \Rightarrow \phi_{jk} = \sqrt{{}_kA_{jj}} \quad \text{or}$$

$$\text{the transfer FRF: } H_{ij} \Rightarrow \phi_{jk} = \frac{{}_kA_{ij}}{\phi_{ik}}$$

## Disadvantages

- This method relies on the peak FRF value, which is very difficult to measure accurately
- Therefore, it is not capable of producing accurate modal data
- Damping is estimated from half power points only. No other FRF data points are used. The half power points have to be interpolated, as it is unlikely that they are two of the measured data points
- Requires knowledge (recorded data) of the input signal (excitation)
- Cannot handle noise efficiently
- The assumption of SDoF behavior in the vicinity of resonance is not accurate, in practice the nearby modes will most likely also contribute.

## The Frequency Domain Decomposition Method (FDD)

This method which is very similar to the peak picking method is used for the identification of the modal characteristics of the structure, given only ambient measurements.

Assume the standard input  $X(t)$  - output  $Y(t)$  relationship, based on the Fourier Transform:

$$Y(\omega) = H(\omega)X(\omega)$$

When assuming wide sense stationary processes one can equivalently write the following using PSDs:

$$G_{yy}(\omega) = H^*(\omega)G_{xx}(\omega)H^T(\omega) \quad (1)$$

where  $G_{xx}$  is the  $(r \times r)$  power spectral density (PSD) matrix of the input,  $r$  is the number of inputs,  $G_{yy}$  is the  $(m \times m)$  PSD matrix of the responses,  $m$  is the number of responses,  $H(\omega)$  is the  $(m \times r)$  frequency response function (FRF) matrix and the overbar and superscript T denote the complex conjugate and transpose, respectively.

The method is applicable for cases where the Power Spectral Density of the input is constant, i.e.  $G_{xx}(\omega) = C$ . This applies for the case of broadband random input excitations like **white noise** (such as ambient excitations). In this way FDD is developed for output only modal identification where only measurements of the response are available.

## Proof

Remember that the PSD  $G_{xx}(\omega)$  is the FT of the autocorrelation function  $R_{xx}(t)$ . The autocorrelation function essentially indicates the correlation between values of the process at different points in time, as a function of the two times  $t_1$ ,  $t_2$  or of the time difference (lag -  $\tau$ ).

## By definition:

A continuous time random process  $w(t)$  where  $t \in \mathbb{R}$  is a white noise process if and only if its mean function and autocorrelation function satisfy the following:

$$\begin{aligned}\mu_w(t) &= E\{w(t)\} = 0 \\ R_{ww}(t_1, t_2) &= E\{w(t_1)w(t_2)\} = C\delta(t_1 - t_2)\end{aligned}$$

i.e. it is a zero mean process for all time and has infinite power at zero time shift since its autocorrelation function is the Dirac delta function. The above autocorrelation function implies the following power spectral density, since the FT of Dirac Delta is unity:

$$G_{ww}(i\omega) = C$$

# The FDD method

As already mentioned the FRF can be written in partial fraction expansion.  
(i.e poles/residues form)

$$H(i\omega) = \sum_{m=1}^N \frac{A_m}{i\omega - \lambda_m} + \frac{A_m^*}{i\omega - \lambda_m^*}$$

where  $A_m = \Phi_m \gamma_m^T$

$\Phi_m$  : modal vector

$\gamma_m$  : modal participation vector

Suppose the input  $y(t)$  is white noise, then the PSD is a constant matrix  
 $\Rightarrow G_{xx}(i\omega) = C$ . Hence,

$$\begin{aligned} (??) \Rightarrow G_{yy}(i\omega) &= \sum_{m=1}^N \sum_{k=1}^N \left[ \frac{A_m}{i\omega - \lambda_m} + \frac{A_m^*}{i\omega - \lambda_m^*} \right] C \\ &\quad \left[ \frac{A_k}{i\omega - \lambda_k} + \frac{A_k^*}{i\omega - \lambda_k^*} \right]^H \end{aligned}$$

where  $^H$  denotes the conjugate transpose (Hermitian transpose) matrix  $(^*)^T$ .

# The FDD method

After some math manipulations, this can be further simplified to:

$$G_{yy}(\omega) = \sum_{m=1}^N \frac{A_m}{i\omega - \lambda_m} + \frac{A_m^*}{i\omega - \lambda_m^*} + \frac{B_m}{-i\omega - \lambda_m} + \frac{B_m^*}{-i\omega - \lambda_m^*}$$

where  $A_m$  is the  $m$ th residue matrix of the output PSD and it is an  $N \times N$  matrix itself:

$$A_m = R_m C \left( \sum_{K=1}^N \frac{R_K^{*T}}{-\lambda_m - \lambda_K} + \frac{R_K^T}{-\lambda_m - \lambda_K} \right) \quad (2)$$

Therefore the contribution of only the  $m$ th mode in expression (2) is:

$$A_m = \frac{R_m C R_m^{*T}}{-2\operatorname{Re}(\lambda_m)}$$

# The FDD method

When the damping is light, the residue becomes proportional to the mode shape vector: (dominating term)

$$D_m \propto A_m C A_m^{*T} = \Phi_m \gamma_m^T C \gamma_m \Phi_m^T = d_m \Phi_m \Phi_m^T$$

In fact, at a certain frequency  $\omega$ , only a limited number of modes will contribute significantly. Let this set be denoted by  $\text{Sub}(\omega)$ . This, the response spectral density can be written:

$$G_{yy}(\omega) = \sum_{m \in \text{Sub}(\omega)} \frac{d_m \Phi_m \Phi_m^T}{i\omega + \lambda_m} + \frac{d_m^* \Phi_m^* \Phi_m^{*T}}{i\omega - \lambda_m^*}$$

**Source:** Brincker R., Zhang L., Palle A., "Modal Identification from Ambient Responses using Frequency Domain Decomposition"



## Steps

- 1 From a set of given output measurements  $y(t)$ ,  $\in \mathbb{R}^{m \times 1}$  obtain the PSD  $G_{yy}(\omega) \in \mathbb{R}^{m \times m}$ . This can be done using Matlab's function *cpsd*( $x, x$ )

**Note:** *cpsd* stands for cross spectral density. It is in general written for two separate signals  $x(t)$ ,  $y(t)$  and it is equal to the power spectral density when  $x = y$ .

- 2  $G_{yy}(\omega)$  is “decomposed” by taking the Singular value Decomposition (SVD) of the  $G_{yy}(\omega)$  matrix

## Basics: Singular value Decomposition - SVD

Suppose  $\mathbf{A}$  is an  $m \times n$  matrix whose entries  $\in \mathcal{C}$  (complex) or  $\mathbb{R}$  then:

$$\mathbf{A} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H$$

where

$\mathbf{U}$  is a  $m \times m$  unitary matrix:  $\mathbf{U}^H\mathbf{U} = \mathbf{U}\mathbf{U}^H = \mathbf{I}_m$

$\mathbf{\Sigma}$  is a  $m \times n$  diagonal matrix with  $\Sigma_{ii} \geq 0$

$\mathbf{V}^H$  is a  $n \times n$  unitary matrix

$\Sigma_{ii}$  are called the singular values of  $\mathbf{A}$

( $^H$  denotes conjugate transpose)

## Properties:

$$\mathbf{A}^H \mathbf{A} = (\mathbf{V} \Sigma^H \mathbf{U}^H)(\mathbf{U} \Sigma \mathbf{V}^H) = \mathbf{V}(\Sigma^H \Sigma) \mathbf{V}^H$$

$$\mathbf{A} \mathbf{A}^H = \mathbf{U} \Sigma \mathbf{V}^H \mathbf{V} \Sigma^H \mathbf{U}^H = \mathbf{U}(\Sigma \Sigma^H) \mathbf{U}^H$$

Hence, the columns of  $\mathbf{V}$  (right singular vectors) are eigenvectors of  $\mathbf{A}^H \mathbf{A}$

So we can write at each discrete frequency  $\omega_i$ :

$$\mathbf{G}_{yy}(\omega_i) = \mathbf{U}_i \Sigma_i \mathbf{U}_i^H$$

$$\text{where } \mathbf{U}_i = [\mathbf{U}_{i1}, \dots, \mathbf{U}_{im}]$$

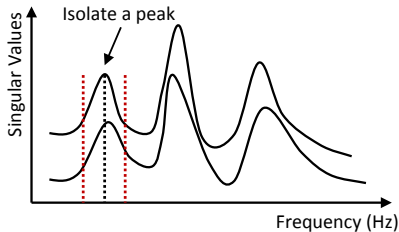
Note that here,  $\mathbf{V} = \mathbf{U}$  because  $\mathbf{G}_{yy}$  is a normal matrix (i.e. it is square and  $\mathbf{G}_{yy}^H \mathbf{G}_{yy} = \mathbf{G}_{yy} \mathbf{G}_{yy}^H$ )

- 3 Near a peak corresponding to the k-th mode in the spectrum, that mode (or maybe a closely spaced mode) will be dominating. Assume the case of no closely spaced modes then the first singular vector is an estimate of that mode shape:  
 $\hat{\phi} = \mathbf{U}_{i1}$  since only the k-th term dominates in Equation 1.

The corresponding singular value defines the PSD of the corresponding single degree of freedom (sdof) system. As a note, this (partial) PSD is identified by isolating that peak and comparing the mode shape estimate  $\hat{\phi}$  with the singular vectors obtained for frequency lines around the peak. Based on a modal criterion (MAC) we can assess the similarity of that vector to the ones around the peak and set an appropriate threshold, according to which the virtual **“SDoF”** area is obtained.

# Identification Algorithm

## How to properly isolate the SDoF vicinity?



Using the **Modal Assurance Criterion (MAC)**. This essentially compares the relationship between 2 mode shapes by comparing their linearity.

Assume two modal vectors  $\phi, \psi$ . One expression for the MAC criterion is:

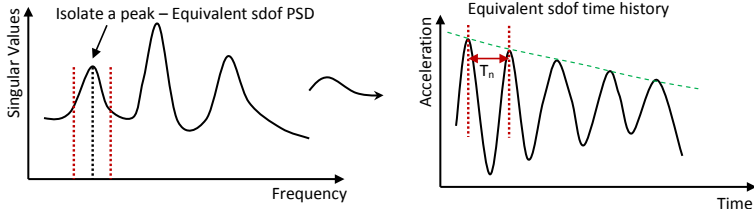
$$MAC = \frac{|\psi^T \phi|^2}{(\psi^T \psi)(\phi^T \phi)}$$

for  $MAC = 1 \Rightarrow$  the vectors are consistent (similar)

$MAC = 0 \Rightarrow$  not consistent

As long as the SVDs around that peak singular value give high MAC values  
 $\Rightarrow$  that singular vector belongs to the SDoF PSD.

# Identification Algorithm



- 4 After we define the limits of the sdof, we can either apply the  $1/2$  power method to find  $\zeta$  or we can take the signal back to the frequency domain. There, we define  $f_d$  (damped frequency) as  $1/T_d$  from the zero crossings (or peak distance) and  $\zeta$  from the logarithmic decrement method.
- 5 In case of closely spaced modes, the first singular vector will always be a good estimate of the strongest mode.  
In the case though that these two closely spaced modes are orthogonal, the first two singular vectors can give unbiased estimates of the corresponding mode shapes.

## Variations of the method

- Enhanced Frequency Domain Decomposition (EFDD)  
This method uses the transformation in time domain for the derivation of  $\omega_n, \zeta_n$
- Curve Fit frequency Domain Decomposition (CFDD)  
This method does not go to time domain but uses a SDoF curve fitter in the frequency domain in order to obtain,  $\omega_n, \zeta_n$

## Consideration

### Selection of model order - Stabilization Chart

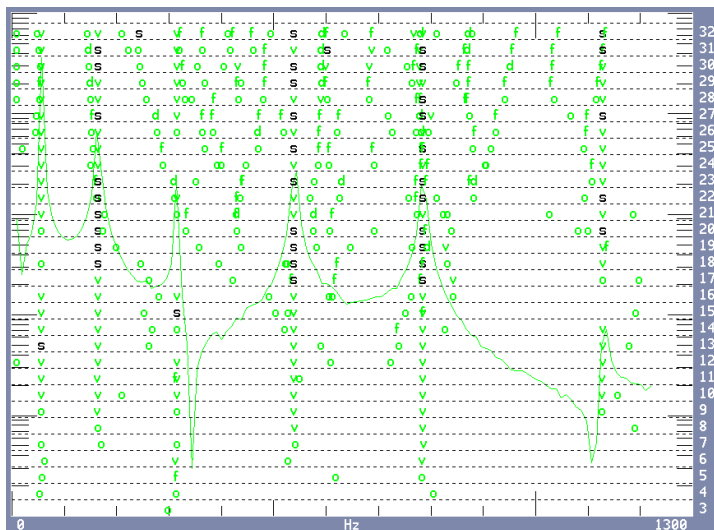
Estimated poles corresponding to physically relevant system modes tend to appear, for all estimation orders (i.e. assumed frequency peaks), at nearly identical locations

While the so-called mathematical poles, i.e. poles resulting from the mathematical solution of the normal equations but meaningless with respect to the physical interpretation, tend to vary in location.

The occurrence of this last class of poles is mainly due to the presence of noise in the measurements.



# Stabilization Chart



Frequency vs Model Order