Quantum Key Distribution, cellular automata based large S-Box and almost key homomorphic block cipher for long term secret storage

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LONG TERM SECRET STORAGE

Issue and challenges

Why long term secret storage

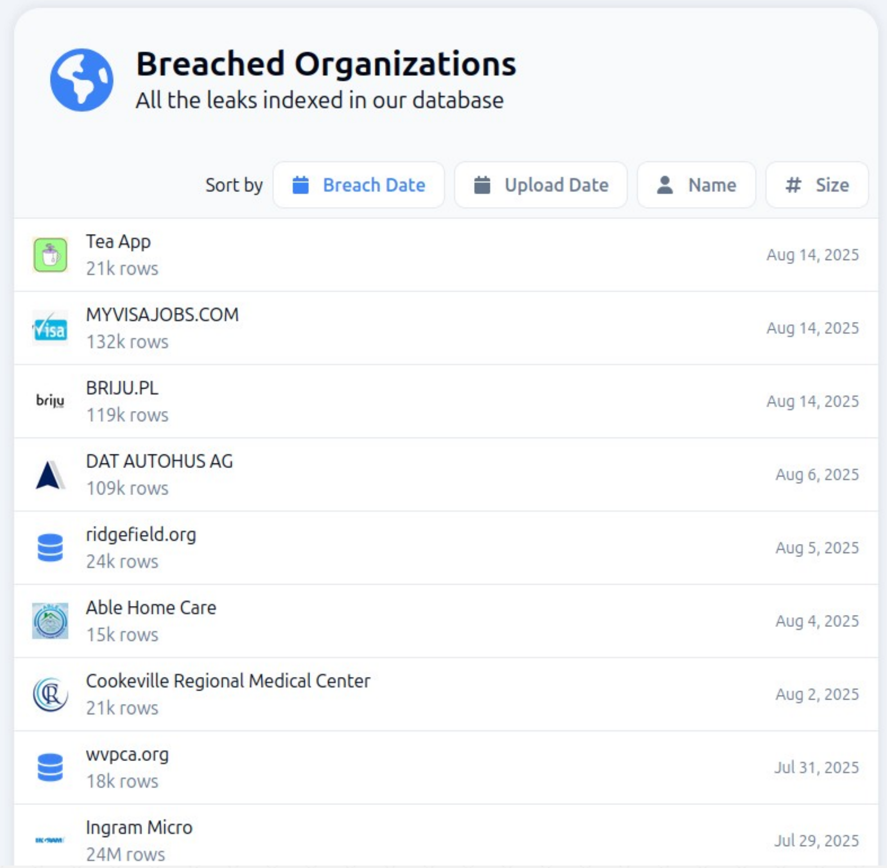
Some data could require confidentiality over decades

Medical data

Trade secrets

State secrets

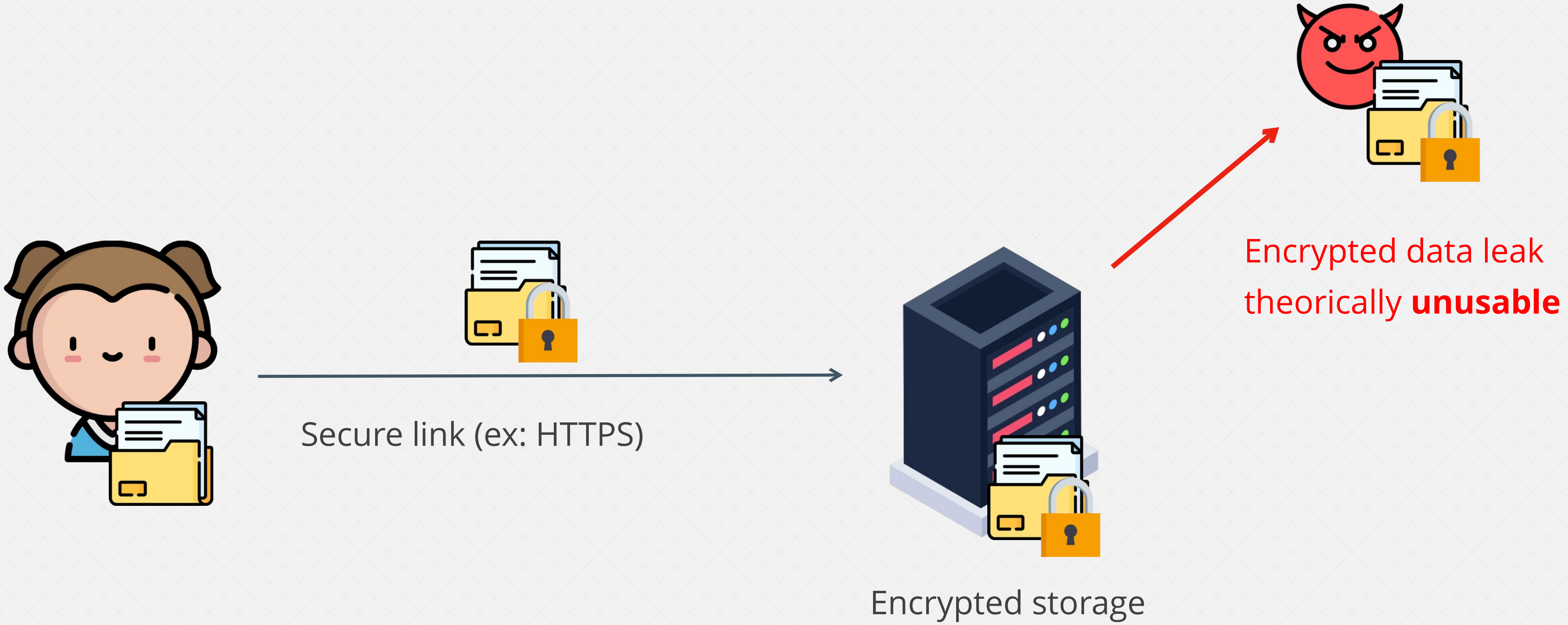
Issues with classical storage



Source: *https://databreach.com/breach*

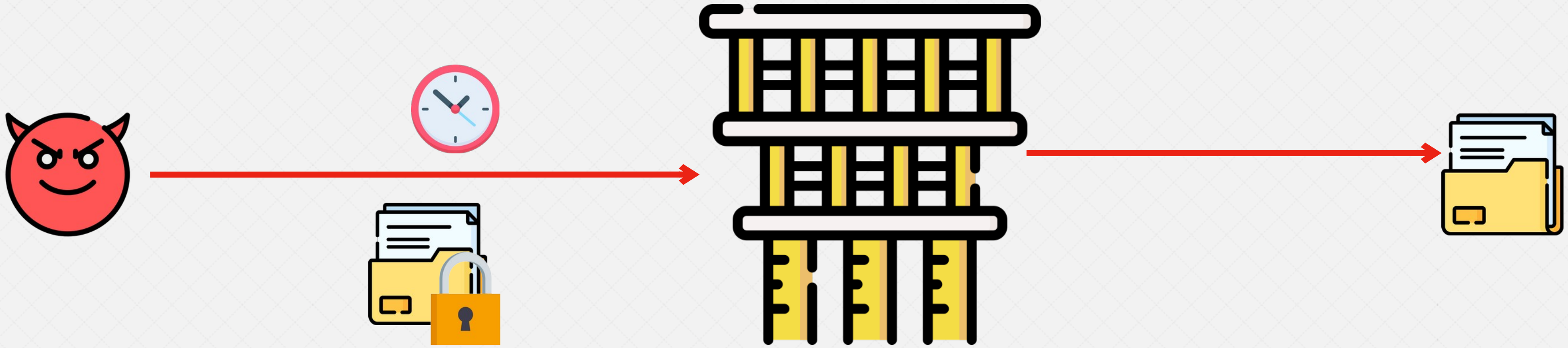
Source: *https://bonjourlafuite.eu.org/*

Solution: cryptography



Long term cryptography ?

Increased computing power, advances in cryptanalysis and the arrival of the quantum computer will allow an attacker to break current encryption



«*Harvest now, decrypt later*» attack

What about «post-quantum» cryptograhy?

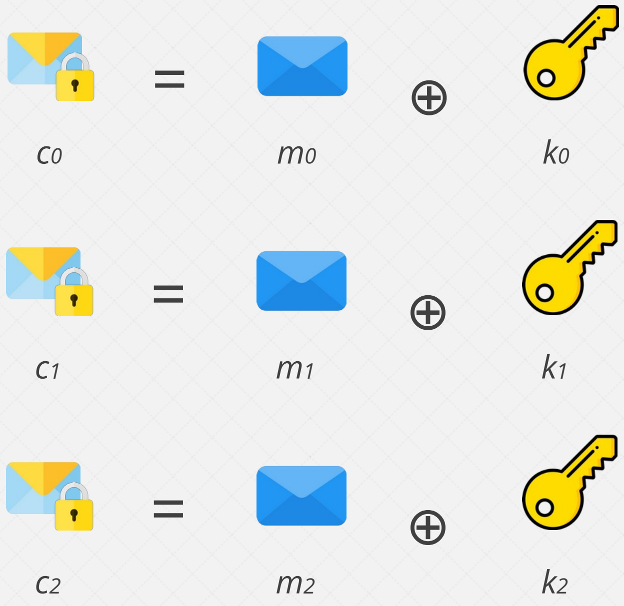
Mathemathical foundations are too recent for «long term security»

Solution: perfect secrecy

*(Loosely speaking, we will interchange the term "perfect secrecy" with "Information Theoretic Security" (**ITS**), although these two terms do not precisely designate the same thing)*

One Time Pad (OTP) : **random, non-reused** key, the **same size as the original message**

Unbreakable even in the long term («it is as hard to break the encryption as guessing the message by chance»)



Drawback: the key is hard to carry between the participants...

We should find a way to exchange the key securely

etc.

QUANTUM KEY DISTRIBUTION (QKD)

Possibilities and limitations

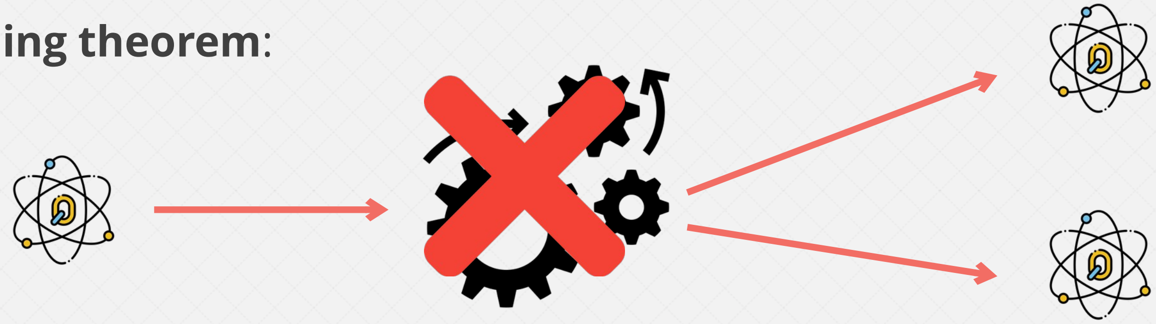
Uncertainty principle

A qubit (like a photon in an optical fiber) exists in a **superposition of states**:

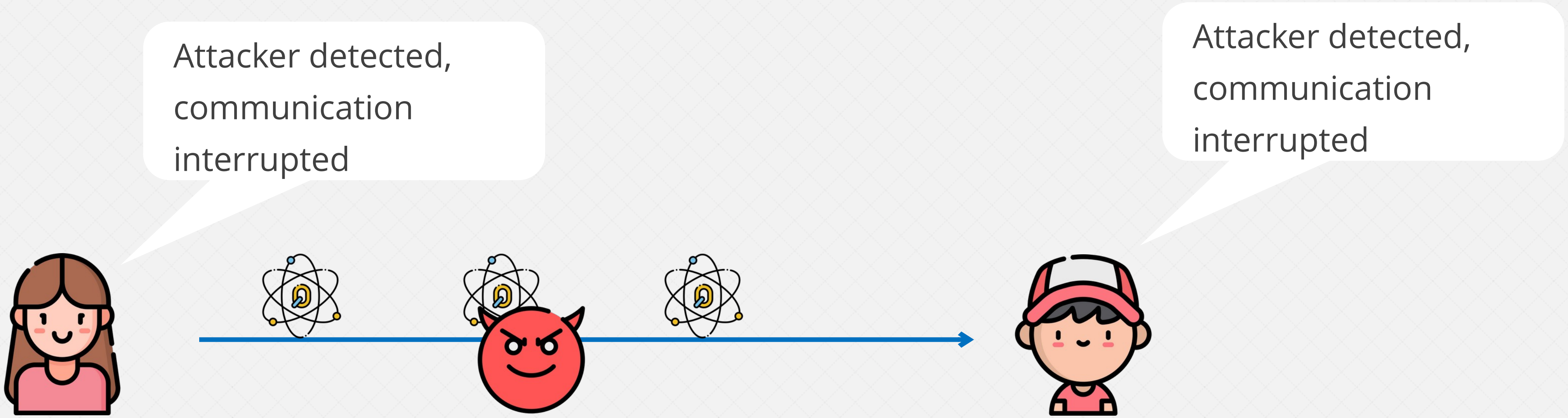
$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

Reading (ie measuring) the qubit collapse its state randomly:

Consequently: **no-cloning theorem**:



Principles of QKD



Advantages:

- The attacker has no information at all
- Theorically perfect security

Drawbacks:

- Costly hardware (for now)
- Limited geographical reach (few hundreds kms, as no-cloning theorem forbids to use optical amplifiers)

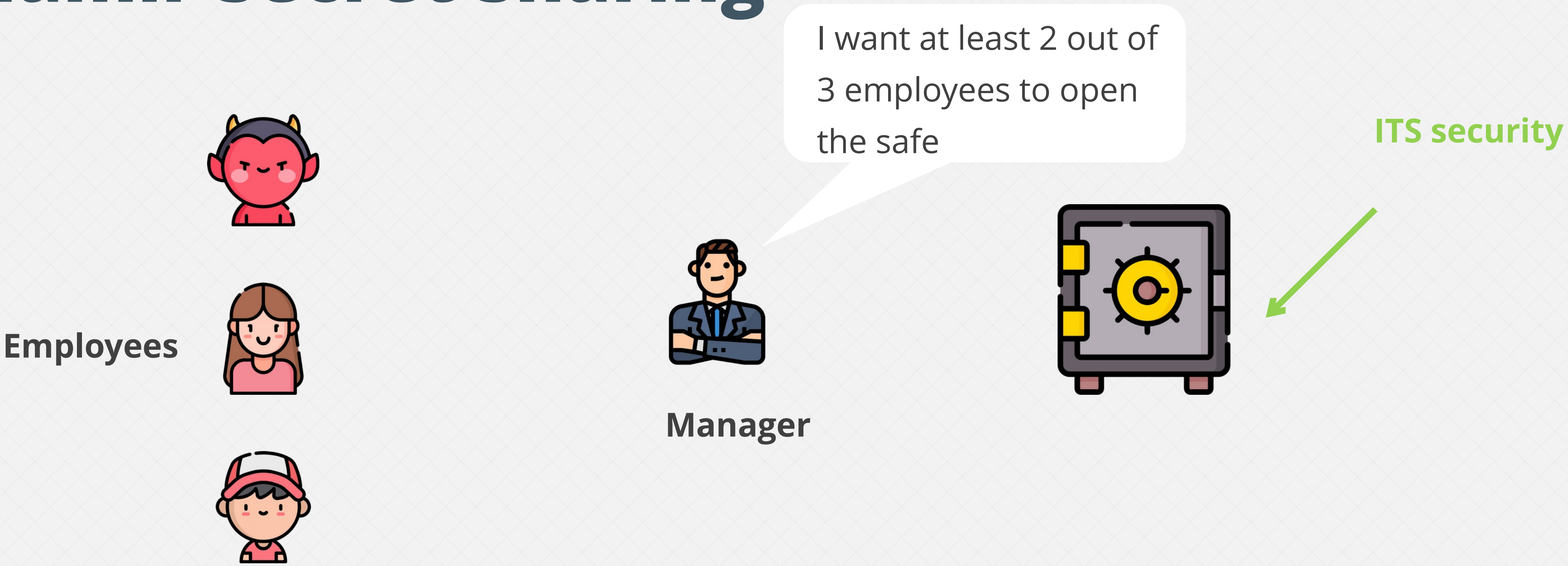
Progressive deployment of metropolitan QKD networks

MULTISS PROTOCOL

Long term secret storage across multiple QKD networks

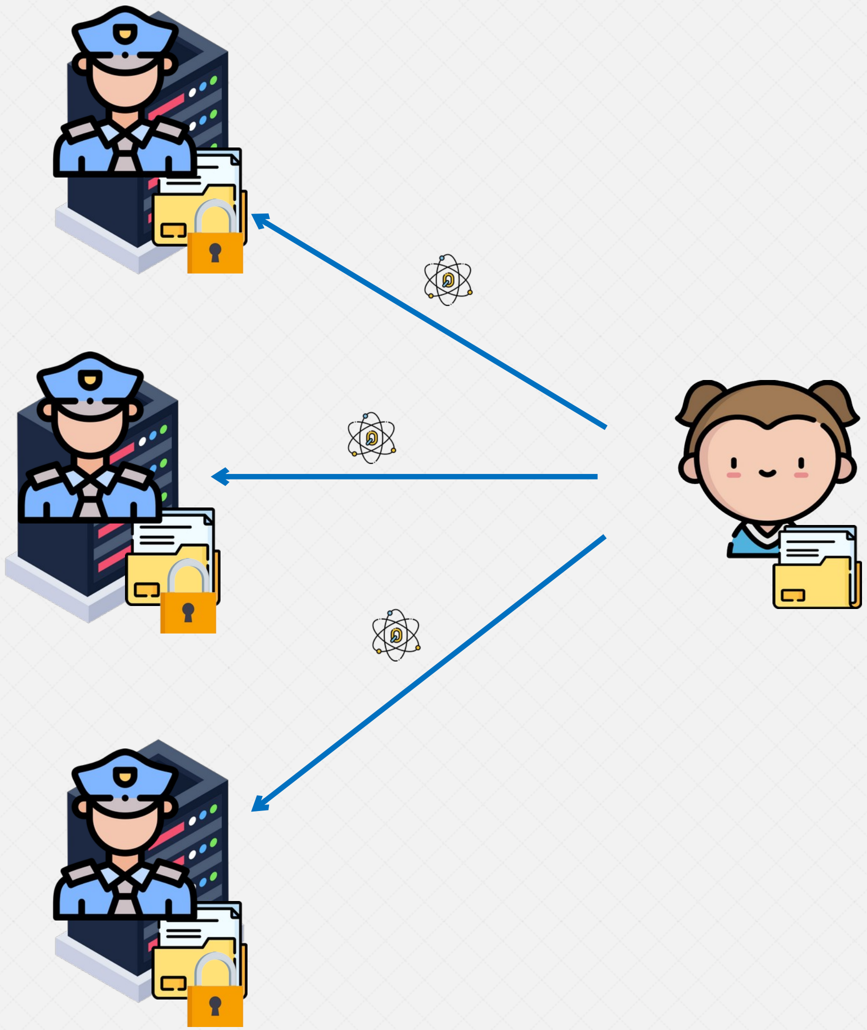
VeriQloud

Shamir secret sharing



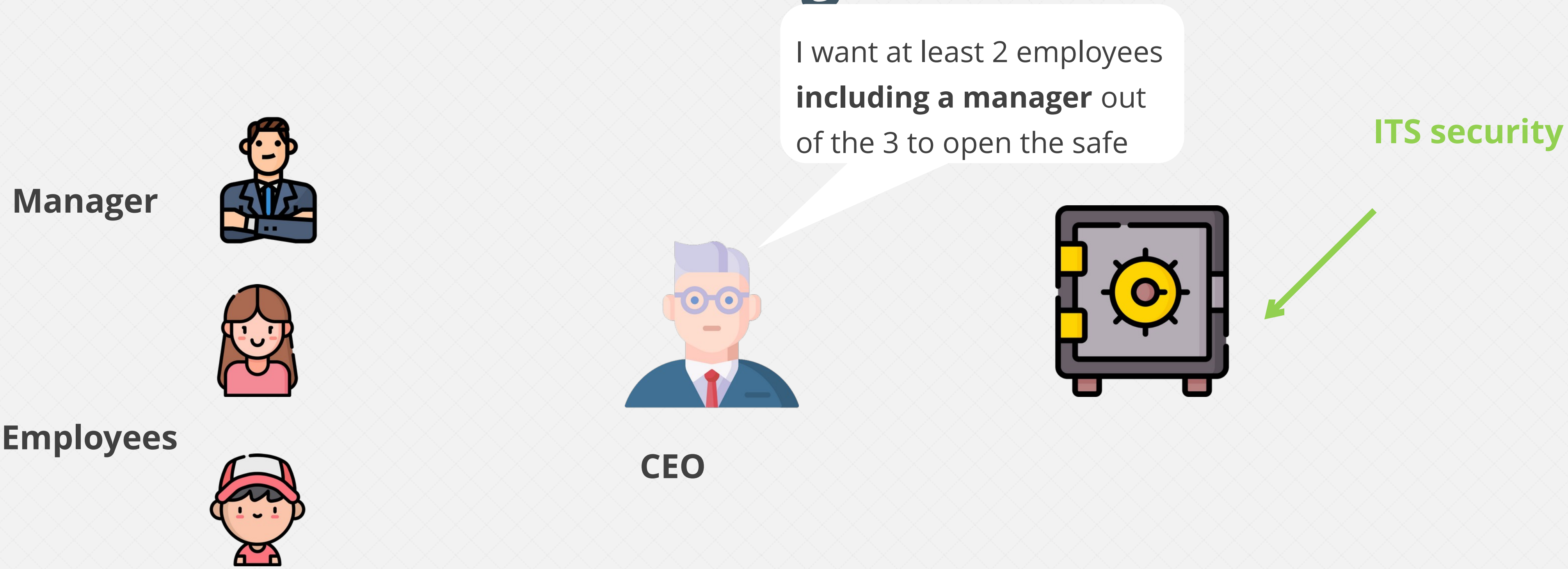
- Generate random polynomial $P \in F_q[x]$, such that deg(P) = threshold - 1 and $P(0)$ = secret
- Distribute to participants $P(1)$ mod q, $P(2)$ mod q ... , q being prime > secret

LINCOS protocol



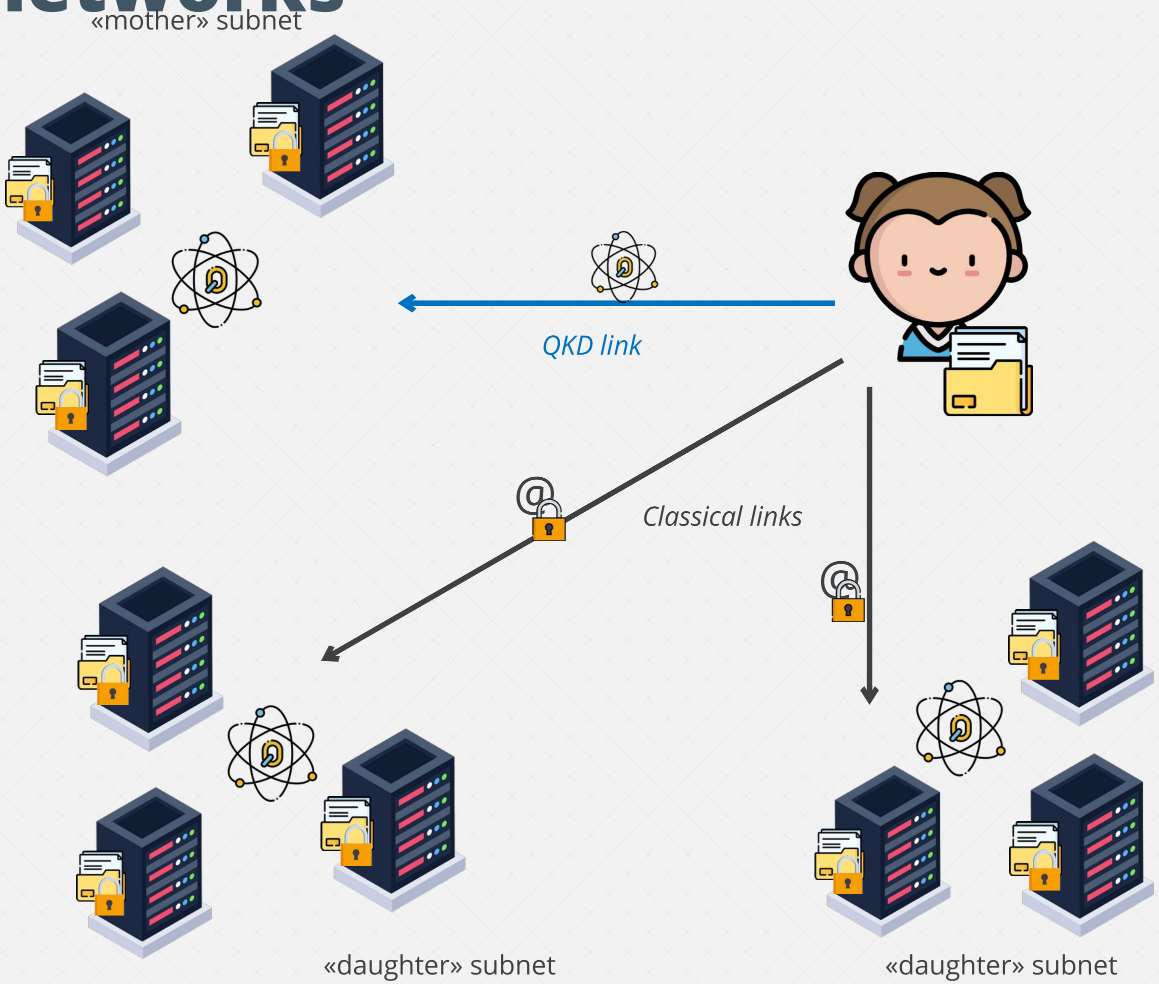
- Existence of a quantum link between the owner of the document and the nodes of the metropolitan network
- Document sharing between the different nodes using the Shamir primitive (perfect theoretical security)
- Vulnerable if the entire metropolitan network is compromised

Hierarchical secret sharing



- The managers are given evaluations *P(1)* mod *q*, *P(2)* mod *q* ...
- The employees are given evaluations of derivative polynomial *P'(1)* mod *q*, *P'(2)* mod *q* ...

MULTISS: Confidential secret storage across multiple QKD networks



- Secret distribution by hierarchical secret sharing (Birkhoff interpolation)
- Primitive shares on «mother subnet», derived on «daughter subnets»

- Retains security properties strictly greater than those of LINCOS
- Ensures security against an adversary who manages to take control of a QKD network

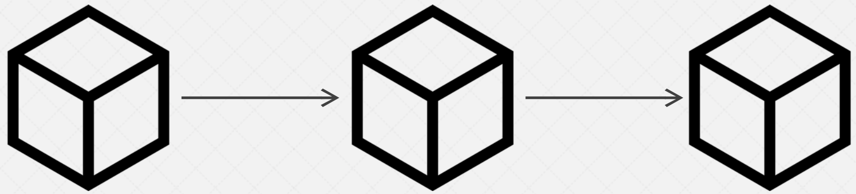
- Currently experimenting between Nice and Paris QKD networks

CELLULAR AUTOMATA BASED LARGE S-BOX

And comparison with AES S-Box

Block cipher encryption

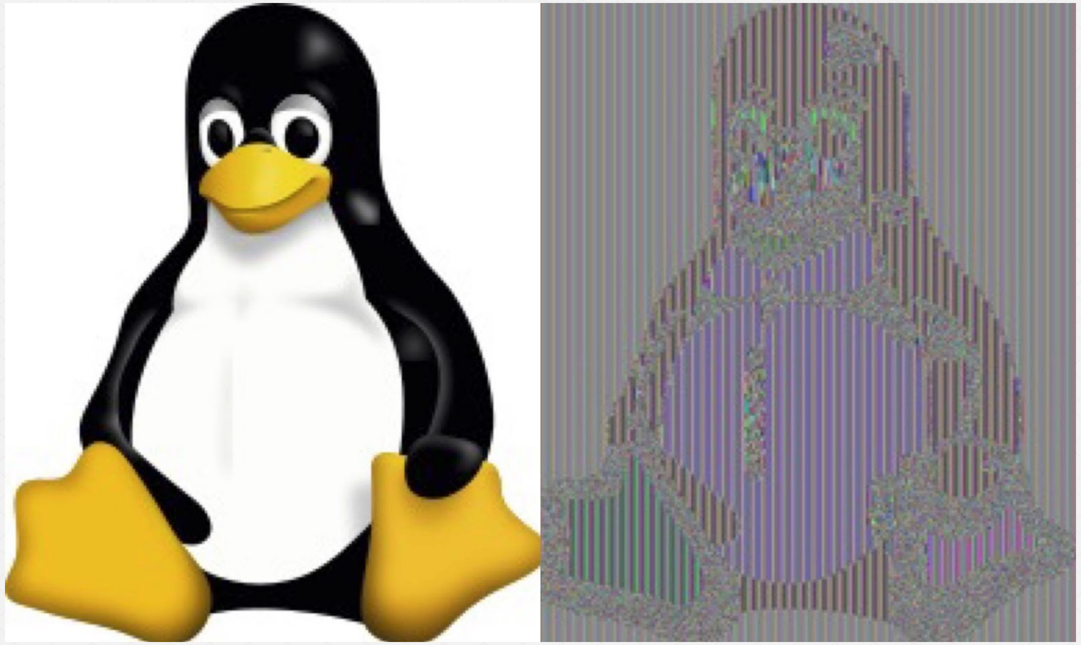
- Commonly used symmetric encryption
- Slicing the message into equal sized blocks



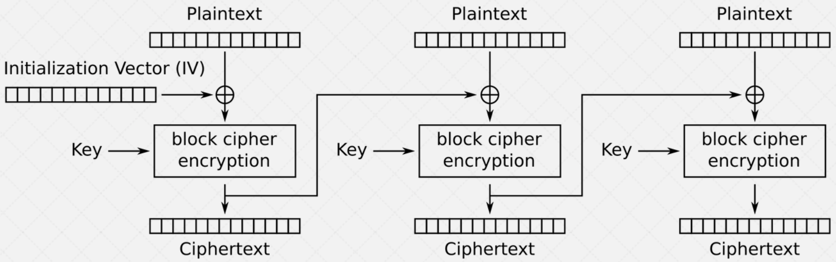
Example: **A**dvanced **E**ncryption **S**tandard (AES),
NIST standardized algorithm for symmetric cryptograohy

Blocks interdependecy

If each block was encrypted independently:

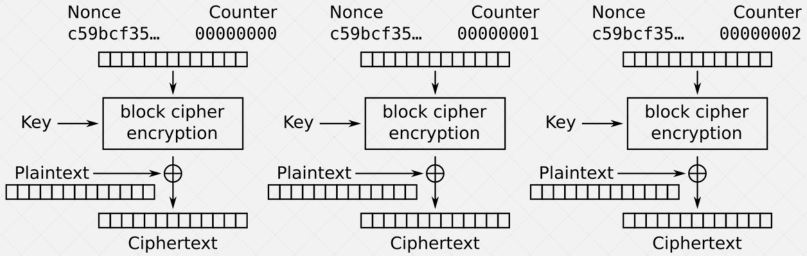


Solution 1: block chaining (CBC): not parallelisable



Cipher Block Chaining (CBC) mode encryption

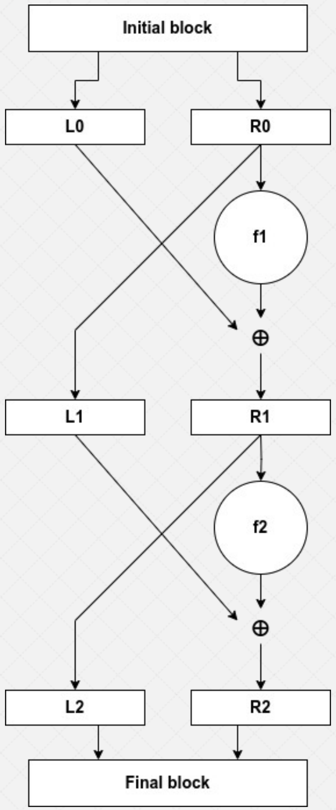
Solution 2: use a counter (GCM, CTR...)



Counter (CTR) mode encryption

Illustration of block encryption structure: Feistel networks

Used in some block cipher algorithms, like Blowfish
(AES uses another similar construction)



With:

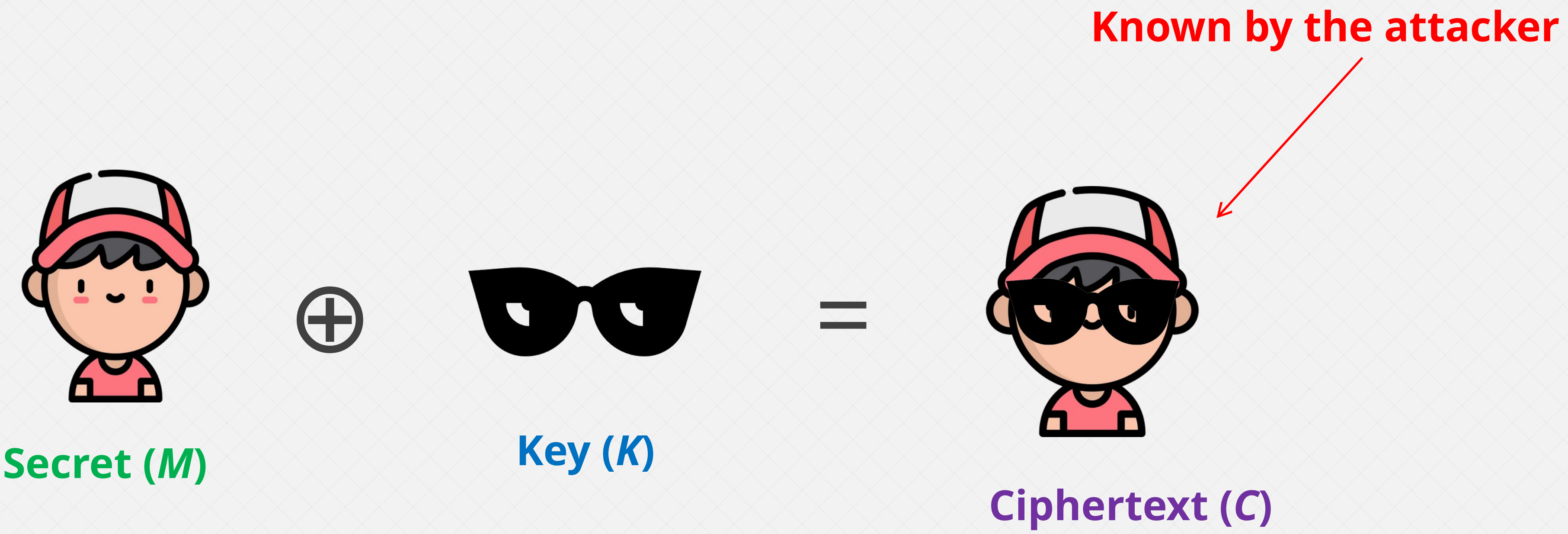
- f_1 and f_2: pseudo-random permutations
- ⊕ XOR operator (exclusive OR)
- Feistel network depth = 2

«pseudo-random» permutation:
Permutation that indistinguishable from a truly random permutation by a «*polynomial time adversary*» (an adversary with a computer with limited computing power)

But what are the subpermutations (f_1, f_2) made of?

Why do we need S-Boxes?

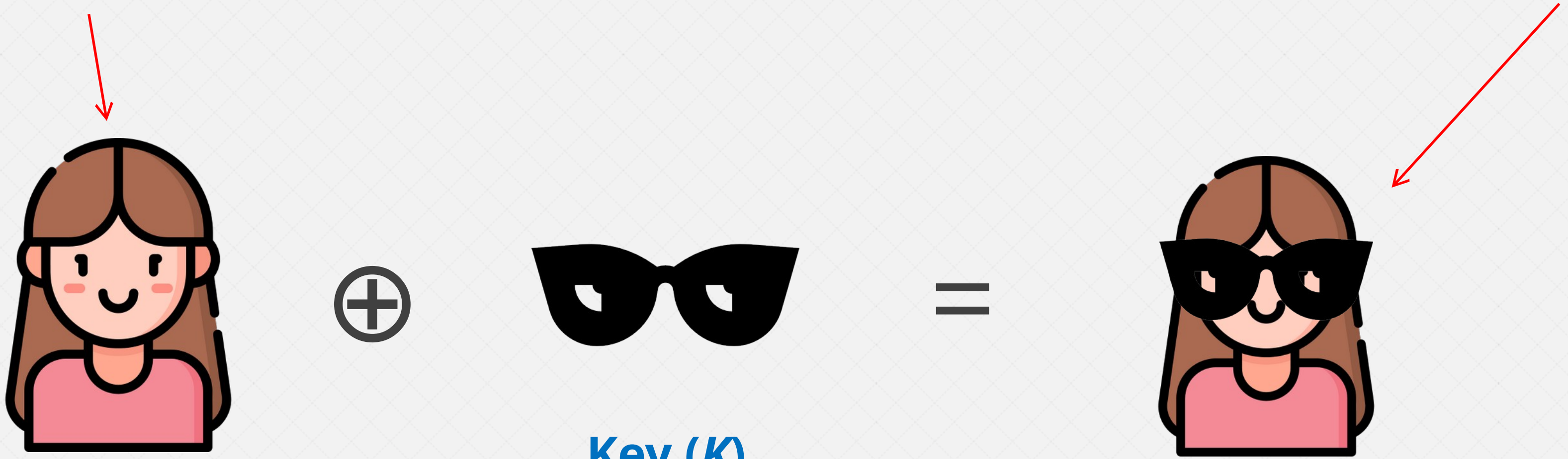
If block cipher was linear:



Why do we need S-Boxes?

If block cipher was linear:

Known by the attacker **Known by the attacker**



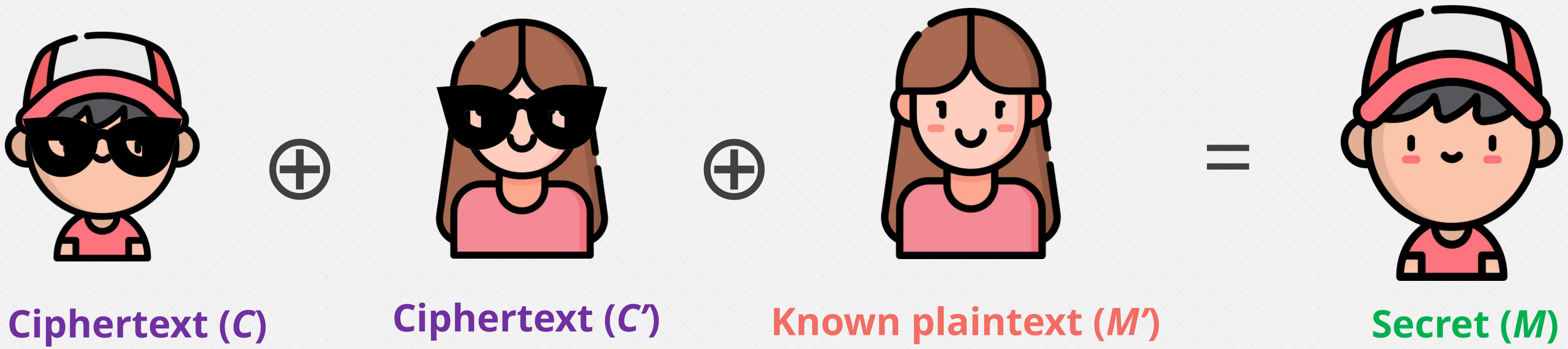
Key (K)

Known plaintext (M') **Ciphertext (C')**

Example of known plaintext: home page of bank website, before filling your credentials

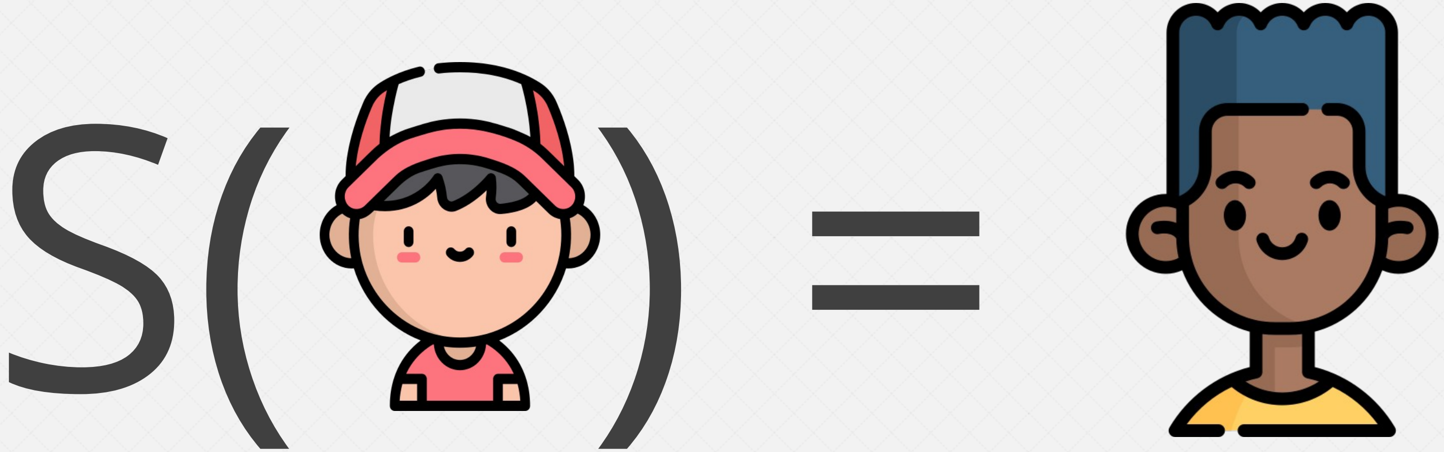
Why do we need S-Boxes?

If block cipher was linear:



This is a **known plaintext attack**

S-Box principle



So a simplified subpermutation round is **the S-Box action combined with a linear operation with the key**

A S-Box is a **public substitution table** that must be as far as possible from a finear function.
As we will see, there are other expected mathematical properties

S-Box example: PRESENT

x	0	1	2	3	4	4	6	7
S(x)	12	5	6	11	9	0	10	13

x	8	9	10	11	12	13	14	15
S(x)	3	14	15	8	4	7	1	2

A S-Box is a **public bijective* function $B^n \rightarrow B^n$ that is as far as possible from a linear function**

**There are non-bijective S-Boxes but this is not what we need here*

Boolean functions

$f(x_1, x_2, \ldots, x_n) = y$, with $x_1, x_2, \ldots, x_n, y \in \boldsymbol{B}$

Algebraic Normal Form (ANF):

$y = x_1*x_2*x_0 \oplus x_2*x_4 \oplus x_5 \oplus 1$

Here deg(f) = 3: size of the largest monomial

Linear function:

if degree = 1 ou degree = 0 (constant function)

There are $2^{\wedge(2^{\wedge}n)}$ possible n-variable Boolean functions

S-Box component functions

For $S(x_1, x_2, \ldots, x_n) = y_1, y_2, \ldots, y_n$, with $x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_n \in \mathbf{B}$

There are 2^n-1 component Boolean functions of S-Box S:

- $f_1(x_1, x_2, \ldots, x_n) = y_1$
- $f_2(x_1, x_2, \ldots, x_n) = y_2$
- …
- $f_{n+1}(x_1, x_2, \ldots, x_n) = y_1 \oplus y_2$
- …
- $f_{2^\wedge n\text{-}1+1}(x_1, x_2, \ldots, x_n) = y_1 \oplus y_2 \oplus \ldots \oplus y_n$

S-Box component functions

Example:

For *S* defined as:

x	00	01	10	11
$S(x)$	10	00	11	01

We have:

x	$f_1(x) = y_1$
00	1
01	0
10	1
11	0

x	$f_2(x) = y_2$
00	0
01	0
10	1
11	1

x	$f_2(x) = y_1 \oplus y_2$
00	1
01	0
10	0
11	1

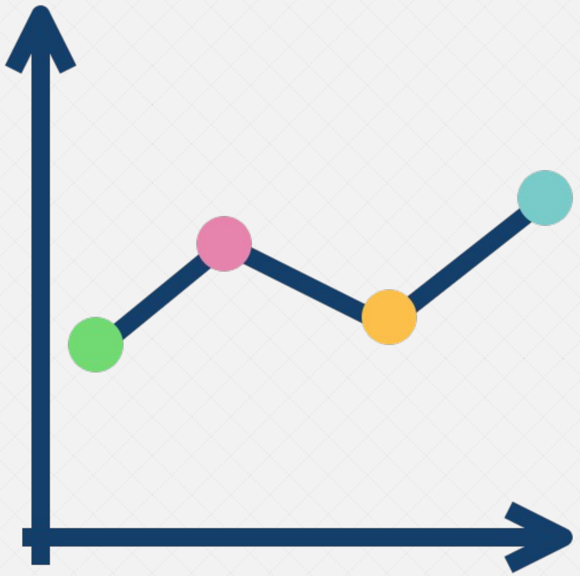
S-Box Mathematical properties

Exhaustive list:

- Min and max algebraic degree
- Algebraic complexity
- Nonlinearity
- Strict Avalanche Criterion (SAC)
- Bit Independence Criterion (BIC)
- Linear Approximation Probability (LAP)
- Differential Approximation Probability (DAP)
- Differential Uniformity (DU)
- Boomerang Uniformity (BU)

Nonlinearity

- For each component function, number of bits that should be switched to have a linear function
- The worst value is the metric



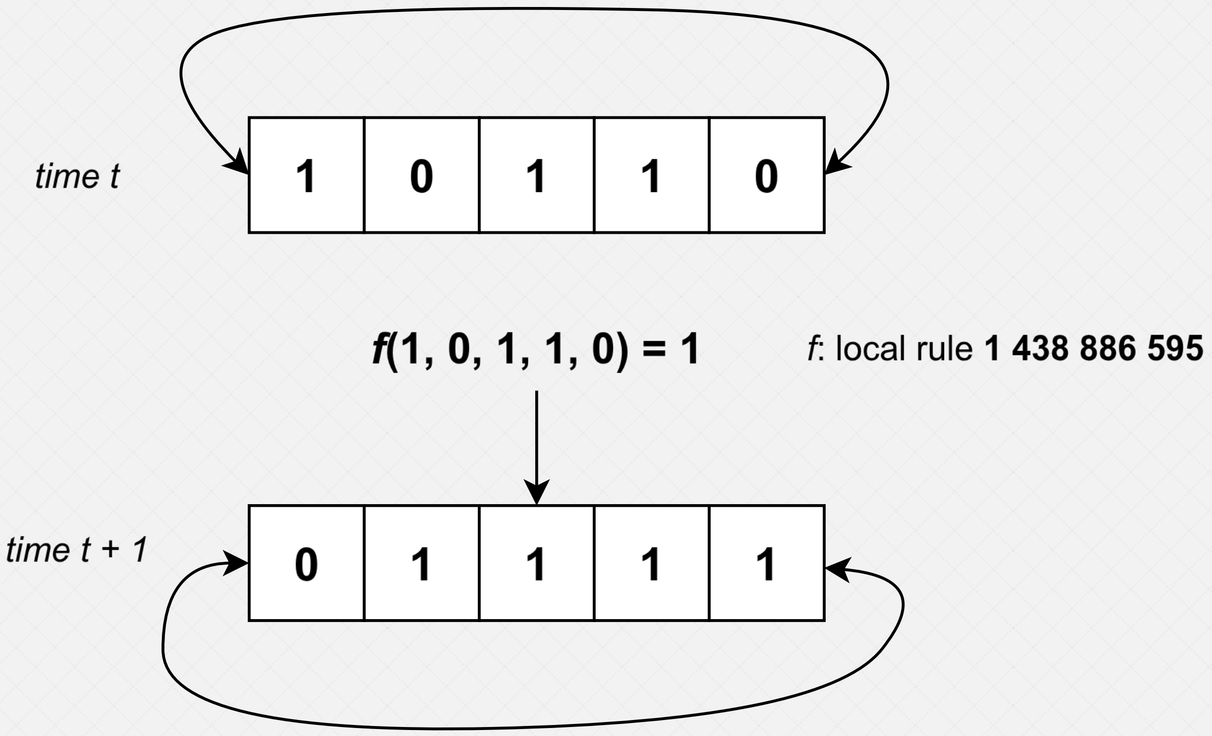
- A **high value enables linear cryptanalysis resistance**

Bit Independence Criterion

- BIC is satisfied when for all input bit *k*, for all output bits *i*, *j*, flipping k^{th} input bit flips i^{th} and j^{th} output bits independently

- The metric is a number between 0 and 1 (closest to satisfy the BIC), **1 the worst and 0 the best**

Uniform cellular automaton



- Ring* of Boolean cells
- At each **discrete** time step, each cell is updated according to its value and the values of its neighbors, according to a weel chosen **local transition function**

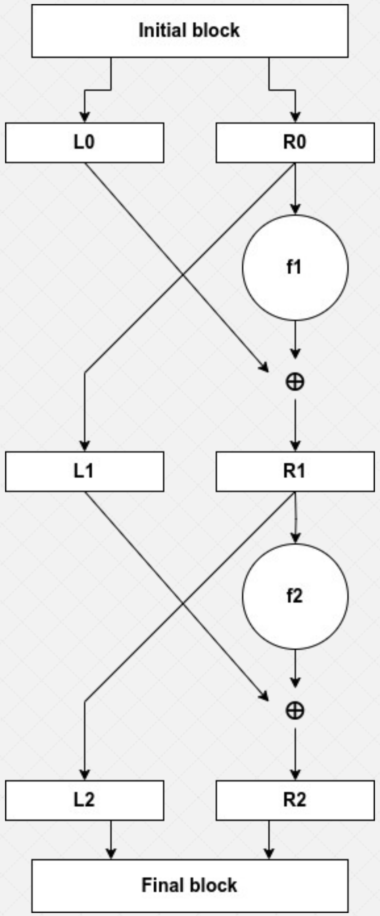
In this specific case

With $f(x) = x_0{*}x_3 \oplus x_1{*}x_3 \oplus x_1 \oplus x_2{*}x_3 \oplus x_2 \oplus x_3{*}x_4 \oplus x_3 \oplus 1$

1 438 886 595 is the **decimal representation** of the truth table

Construction of our 10-bit S-Box

Our S-Box itself is a **sub 10-bit Feistel network**, of depth 11



Empirical construction based on cryptanalysis:

- f_1: affine function: f(x) = 5x+3 mod 31
- f_2 to f_5: 1 generation of our automaton
- f_6: affine function: f(x) = 7x+11 mod 31
- f_7 to f_9: 1 generation of our automaton
- f_{10}: affine function: f(x) = 13x+17 mod 31
- f_{11}: 1 generation of our automaton

Results

Comparison with AES S-Box *(values are normalized to compare a 10-bit S-Box with a 8-bit S-Box)*

Property	Our 10-bit S-Box	8-bit AES S-Box
Min algebraic degree	**8**	7
Max algebraic degree	**9**	7
Algebraic complexity	**1023**	255
Nonlinearity	**434 (= 108.5 * 4)**	112
Strict Avalanche Criterion	**0.44 - 0.5 - 0.57**	0.45 - 0.5 - 0.56

Results

Comparison with AES S-Box *(values are normalized to compare a 10-bit S-Box with a 8-bit S-Box)*

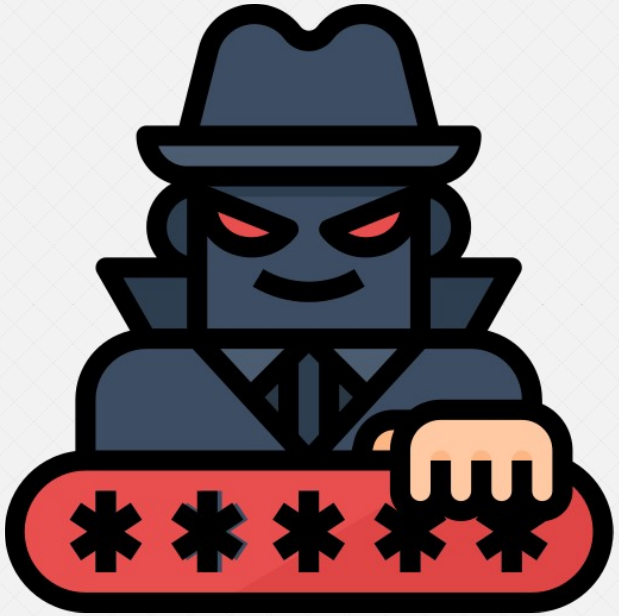
Property	Our 10-bit S-Box	8-bit AES S-Box
Bit Independence Citerion	**0.124**	0.134
Linear Approximation Probability	**9.28%**	6.25%
Differential Approximation Probability	**1.37%**	1.56%
Differential Uniformity	**14 (= 3.5 * 4)**	4
Boomerang Uniformity	**24 (= 6 * 4)**	6

ALMOST KEY-HOMOMORPHIC BLOCK CIPHER

Key rotation and security level update

Security levels

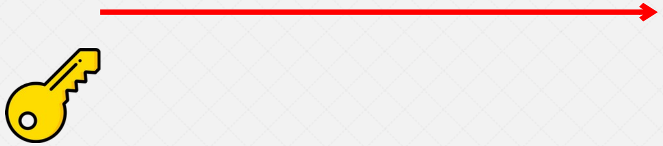
- Indication of the number of operations required to break the cipher (using bruteforce attack)
- 256 bits of security level: $O(2^{256}) \approx 10^{177}$ operations needed to break the cipher
- While 80 bits of security were considered «enough» in the 2000's, the standard today is 256 bits
- For a given ciphertext, we may want to update security level «on-the-fly»...



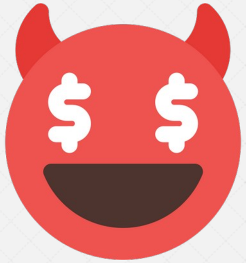
Key rotation ?

Top secret encrypted document

Fired employee



Encryption key

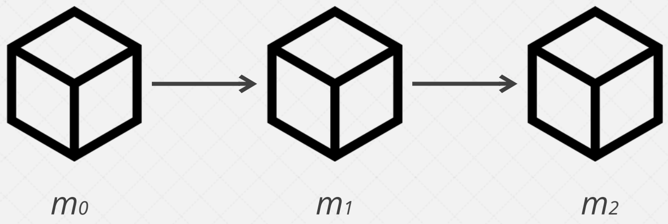


Competitor

Mandatory for a company that manages credit card numbers: **PCI-DSS standard** (Payment Card Industry Data Security Standard)

«Almost Key-Homomorphic» symmetric block cipher

Slicing of message m into equal sized blocks m_0, m_1, m_2 ...



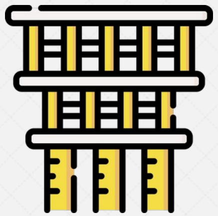
For each block, the cipher c_i is given by:

$$c_i = m_i + a_i\, k + e$$

With a_i **public** random polynomial vector, k **secret key**, e **secret** random error (with small coefficients)

(a_i, k, $e \in \mathbb{Z}q[x] / (x^n + 1)$, q prime)

Supposedly quantum-proof...



Simple key rotation

Knowledge of token Δ is **useless for the adversary**

Token Δ

=

New key k_2

-

Old key k_1

security_level(k_1) = security_level(k_2)

For each block, the new cipher c_i' is given by:

$$c_i' = c_i + a_i \Delta + e'$$

With a_i random **public** vector, Δ **jeton**, e' new **secret** random error *(needed only for indistinguishability between old and new ciphertext)*

We can then decrypt using new key k_2: the ciphertext as been **updated without decryption**

On-the-fly security level update

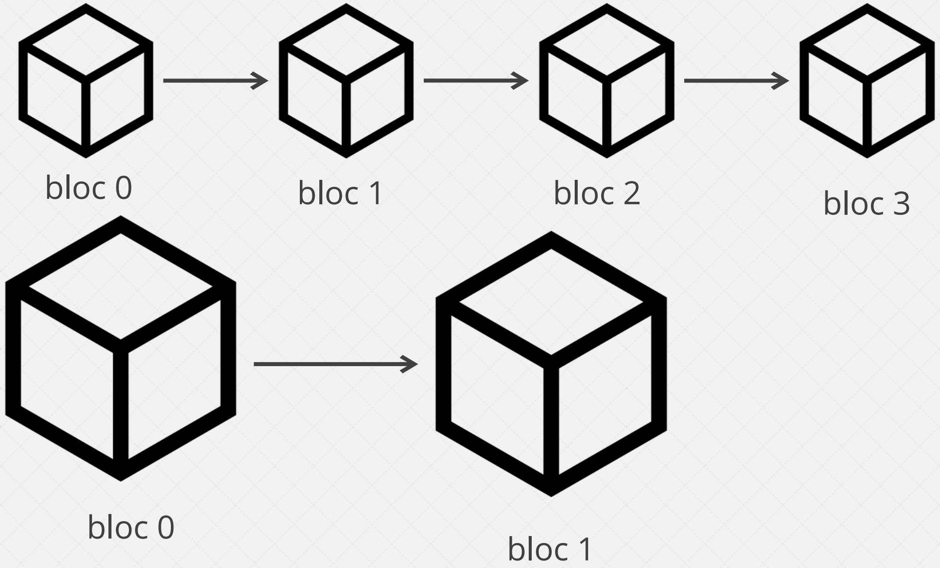
Old block encrypted with key k_1 (security level = n)

k_1: n bits of security

$$c_i = m_i + k_1\, \boldsymbol{a}^{i+1} + e$$

Neighbor blocks merging:

$$c_i' = c_{2i} + X^n\, c_{2i+1}$$



Key rotation toward new key k_2 (security level = $2n$)

k_2: $2n$ bits of security

CONCLUSION

Conclusion

QKD

Ensure perfect secrecy on limited geographical reach

MULTISS

Long term confidential secret storage across multiple remote QKD networks

Strong 10-bit S-Box

Gains time against actual cryptanalysis capabilities

Upgradable security level

Increases ciphertext security against evolving attacker's computing power

THANK YOU

Questions ?

Min and max algebraic degree

Size of the largest monomial of each function:

- If $f1(x_1, x_2, \ldots, x_n) = x_1 * x_2 * x_4 \oplus x_1 * x_2 \oplus x_3$ then $\deg(f1) = 3$
- Largest and lowest degree of each component function

Large values avoid «Low order approximation attack»

Strict avalanche criterion

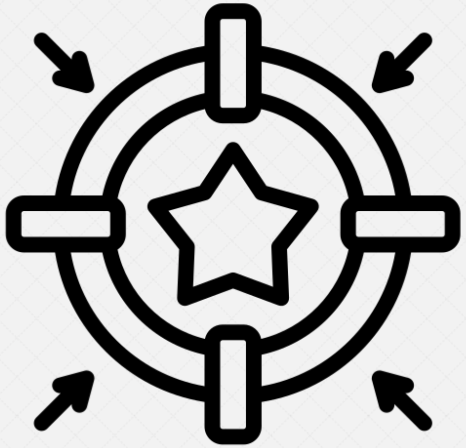
- When an input bit is flipped, 50% of the output bits must be flipped on average
- The ideal value is 50%

We define a table of size *n*n*:

- When the i^{th} input bit is flipped, in which proportion is the j^{th} output bit flipped?

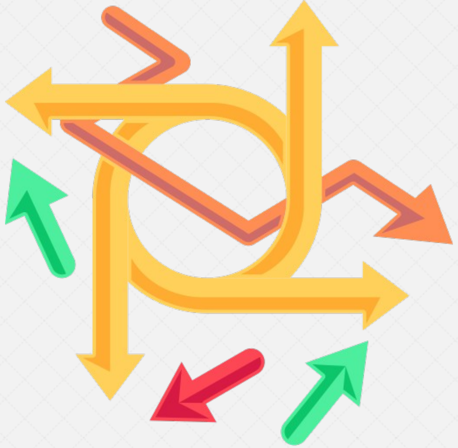
Each table value should be as close as possible of 50%

Differential uniformity



- Gives proximity to a perfectly nonlinear S-Box (impossible for bijectivity)
- For each combination *(a, b)*, differential uniformity table δ gives the number of inputs x such that $S(x) \oplus S(x \oplus a) = b$
- The metric is then $U = \max(\delta)$
- The **lowest value is the best**

Algebraic complexity



Our S-Box is represented over $\mathbb{N}$:

$S(x) = a_0 + a_1 * x + \ldots + a_{(2\hat{}n)-1} * x^{(2\hat{}n)-1}$ mod 2^n avec $x, a_0, a_1, \ldots \in [\![0, 2^n-1]\!]$

Algebraic complexity is the number of monomials in the univariate polynomial

A **large value protects against interpolation attacks**

Linear Approximation probability

- Gives an indication about S-Box resistance against linear cryptanalysis
- Defined as the maximum correlation between $\alpha * x$ et $\beta * S(x)$, pour tout α et $\beta \in [\![1, 2^n]\!]$
- **Lowest value is the best**

Differential Approximation probability

Given by the XOR distribution between input and output

- For each combination *(Δx, Δy)*, differential probability table DP gives the number of inputs *x* such that $S(x) \oplus S(x \oplus \Delta x) = \Delta y$
- So DAP = max(DP)

A **low value ensures resistance** against differential cryptanalysis

Boomerang Uniformity

- Defines S-Box resistance against boomerang attacks (a variant of differential cryptanalysis)
- For each combination *(a, b)*, Boomerang Connectivity Table (BCT) gives the number of inputs *x* such that:

$$S\text{^-}1(S(x) \oplus b) \oplus S\text{^-}1(S(x \oplus a) \oplus b) = a$$

- *BU* = max(BCT)
- The **lowest value is the best** against boomerang attacks