

# Milestone Report

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## INTRODUCTION

As explained in the proposal, in our project we are evaluating the feasibility of using neural networks for inverse kinematics problems in robotics. In this report, we outline the progress made since the start and discuss the results we have obtained. Lastly, we explore possible next steps for our project.

## METHODS

### Robot Simulations

For this project, we have implemented simulations of robotic arms with different complexities. In total, we have four simulations of planar robots with open kinematic chains. Three of these are composed solely of revolute joints and are of increasing complexity (with two, three and four joints). The fourth simulation also contains a prismatic joint. However, we decided to focus only on robot simulations with revolute joints and, thus, we limit ourselves to the first three simulations in this report. From now on, these simulations will be designated by their degrees of freedom (2 DOF, 3 DOF and 4 DOF). For an illustration of the robot configurations, refer to Figure 1.

In our current setup, we do not take the angle of the tool center point (TCP) into consideration. Thus, the location of the TCP is defined by its  $x, y$  coordinates. As a result of this, both the 3 DOF and 4 DOF robot arms can have up to infinite solutions of the inverse kinematics for a given TCP position. The 2 DOF robot arm can have up to two solutions.

### Dataset

The dataset used for training is composed of one million samples of robot configurations for each robot simulation (2 DOF, 3 DOF and 4 DOF). Figure 1 shows an illustration of the datasets. Each configuration was sampled from a normal distribution  $\theta_i \sim \mathcal{N}(\mu = 0, \sigma = 0.5)$ . Thus, the dataset is composed mostly of configurations in which the robot arm is extended. This reflects real-world use cases of robot arms, which have limited workspaces and whose tasks are focused on one section of the workspace. Moreover, limiting the range of the joints improved the performance of the networks.

### Conditional Variational Autoencoder

In the lecture, we have already discussed Variational Autoencoders (VAE) [1] in detail. One drawback of this type of generative architecture is that there is no control over the data generation process. In our case this would mean that although we can generate random sample configurations we cannot control in which end-effector position these configurations would result. Conditional Variational Autoencoders (cVAE)

[2] solve this problem by conditioning the latent space  $z$ . For inverse kinematics this means that random samples are drawn from  $p(z) \sim N(0, 1)$  and the predicted posterior of the joint angles is then generated conditioned on the end-effector position. During training, the position  $(x, y)$  is concatenated with both  $x$  and  $z$  and then fed into the encoder and decoder, respectively.

The cVAE is trained in the same manner as the VAE based on the ELBO (Evidence Lower Bound) loss  $L_{ELBO} = L_y + L_x$ :

$$L_y = -KL[q_\phi(z|x, y) || p(z) \sim N(0, 1)] \quad (1)$$

The Kullback-Leibler divergence measures the distance between the predicted probability distribution  $q_\phi(z|x, y)$  of the latent space  $z$  and the standard normal distribution. The reconstruction loss is defined as follows:

$$L_x = E_{q_\phi(z|x)}[\log(p_\theta(x|z, y))] = \sum_{i=0}^N MSE(v_i \cdot \tilde{v}_i, 1) \quad (2)$$

Here,  $N$  is denoted as the number of joints. For representing the joint angles of the robot, we use a vector-based representation:  $V = (\sin(\theta), \cos(\theta))$  to avoid singularities at the boundaries of the joint angles.  $v_i \cdot \tilde{v}_i$  is denoted as the scalar product between the normalized predicted posterior point estimate  $\tilde{v}_i = (\sin(\tilde{\theta}), \cos(\tilde{\theta}))$  and the ground truth vector  $v_i = (\sin(\theta), \cos(\theta))$  of the  $i$ th joint.

### Invertible Neural Network

The motivation of Invertible Neural Networks (INN) [3] is to model a bijective mapping from the end-effector position to the joint angles. This is done by introducing *coupling layers* (see Fig. ...) as the basic building unit inspired from [4]. The input is split in two halves and then transformed by affine functions with learned coefficients  $s_i, t_i$  and concatenated at the end again to form the output (see Fig. ...). These expressions are easily invertible (see Fig. ...) such that given the output the inverse process is again an affine transformation with the same learned coefficients  $s_i, t_i$ . These coefficients do not have to be invertible and can be arbitrarily complex functions. For the applications to inverse kinematics, they are represented by fully connected neural networks.

The INN is then created by stacking coupling layers and permutation layers (mix the data in a random but fixed way to enhance the interaction among individual variables) together in an alternate manner. As every single unit is invertible, the

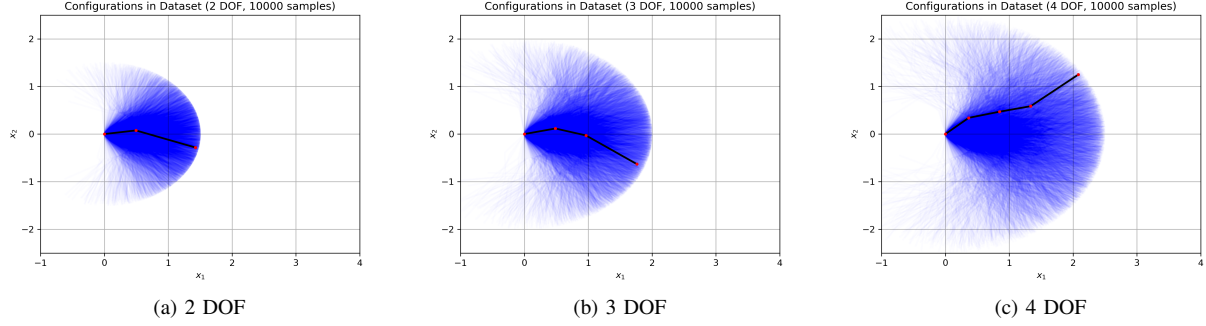


Fig. 1. Illustration of datasets used during training of models. Only a subset of the samples contained in the datasets is shown here. One configuration in the dataset is highlighted to illustrate the configuration of the robot arm.

whole INN is invertible, as well where the input  $x$  (joint angles) has the same dimensionality as the concatenated output  $[y, z]$  to ensure bijectivity. The latent space  $z$  contains the information which is lost during the forward kinematics and not contained in  $y$ . The predicted posterior of the joint angles can be then generated in the same way as with cVAE by sampling from  $p(z) \sim N(0, 1)$  and then predicting the joint angles conditioned on  $y$ .

The INN is trained based on four losses:

$$L_y = \text{MSE}(y_i, f_y(x_i)) \quad (3)$$

$$L_x = \text{MSE}(x_i, [f_y^{-1}(y_i), f_z^{-1}(f_z(x_i))]) \quad (4)$$

$$L_{p(z)} = \text{MMD}(p(f_z(x_i)), p(z) \sim N(0, 1)) \quad (5)$$

$$L_{p(x)} = \text{MMD}(p([f_y^{-1}(y_i), f_z^{-1}(p(z))]), p(x)) \quad (6)$$

where we denote  $[y, z] = [f_y(x), f_z(x)]$  as a bijective mapping between joint angles  $x$  and end-effector position  $y$  and latent space  $z$ . The losses (3) and (4) are computed by the Mean Squared Error (MSE) and losses (5) and (6) are computed by the Maximum Mean Discrepancy (MMD) [5]. This kernel-based method measures the distance between two distributions based on a batch of samples.

#### Implementation

The previous two model architectures (cVAE and INN) have been implemented in PyTorch and are inspired by existing implementations in [6], [7]. We utilized the Google Compute Engine to train our models.

### EXPERIMENTAL EVALUATION

#### Evaluation protocol

#### Results

### CONCLUSION

In this work, we have introduced the methods and robot simulations used to evaluate the feasibility of using neural networks for inverse kinematics problems in robotics. We

DoF	$e_{\text{posterior}}$	$e_{\text{resim}}$	Trainable Parameters	Model
2	0.077	<b>0.003</b>	164,808	cVAE
3	<b>0.045</b>	0.045	370,214	
4	0.063	<b>0.006</b>	373,220	
2	<b>0.061</b>	0.012	169,632	INN
3	0.066	<b>0.036</b>	369,660	
4	<b>0.044</b>	0.075	374,960	

TABLE I  
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have shown that the networks are able to learn the inverse transformations of planar robot simulations.

Our next steps will be to apply the networks to more complex and challenging scenarios, with the goal of evaluating the limits of the INN and cVAE architectures for inverse kinematics problems. In a first step, we will apply these networks to planar robots with longer kinematic chains and more degrees of freedom. As was previously discussed, our current setup uses robot simulations composed of revolute joints. In the future we plan on also using prismatic joints to evaluate the performance of the networks for different joint configurations. We also plan on eventually moving to three-dimensional space to evaluate the performance of these networks on a higher dimensional space.

Currently, the performance of our INN model is somewhat worse than the performance described in the paper [3]. One possible explanation for this mismatch is a suboptimal choice of hyperparameters. Thus, to improve the performances of our models, we plan on implementing hyperparameter optimization using random search.

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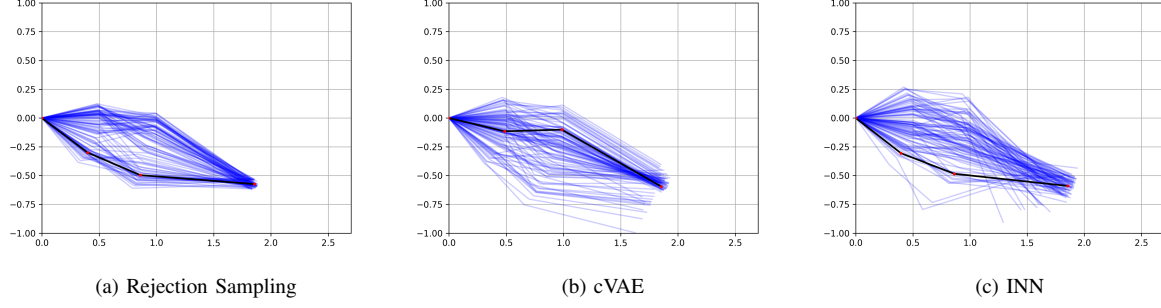


Fig. 2. Arm configuration of a planar manipulator with 3 revolute joints and end-effector position at  $(x, y) = [1.83, -0.57]$ . 100 samples are drawn from each model's predicted posterior  $\tilde{p}(x|y_{gt})$ , one random sample configuration is highlighted.

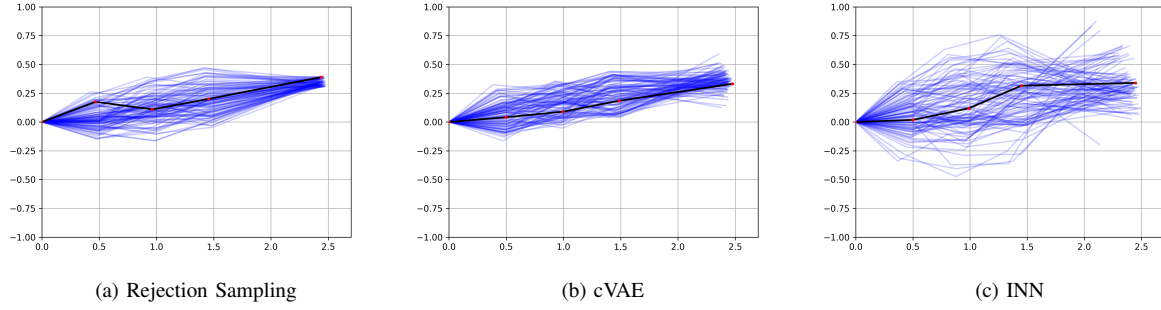


Fig. 3. Arm configuration of a planar manipulator with 4 revolute joints and end-effector position at  $(x, y) = [2.44, 0.35]$ . 100 samples are drawn from each model's predicted posterior  $\tilde{p}(x|y_{gt})$ , one random sample configuration is highlighted.

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