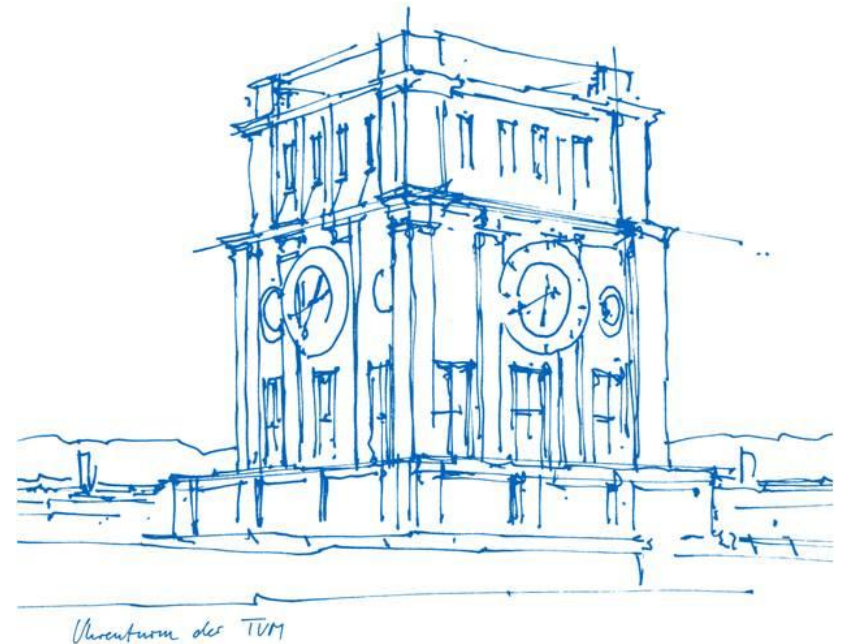


# Neural Networks for Inverse Kinematics Problems in Robotics

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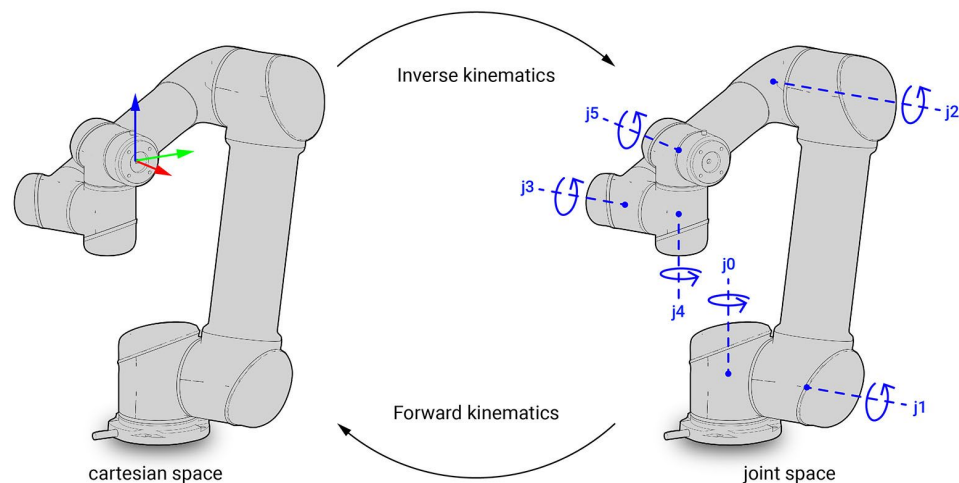


# Motivation - Inverse Kinematics

Determining the joint variables corresponding to an end-effector position and orientation

Complex problem: nonlinear, closed-form solution only for few cases

Problem can be ill-posed: single, multiple, infinite or no solutions



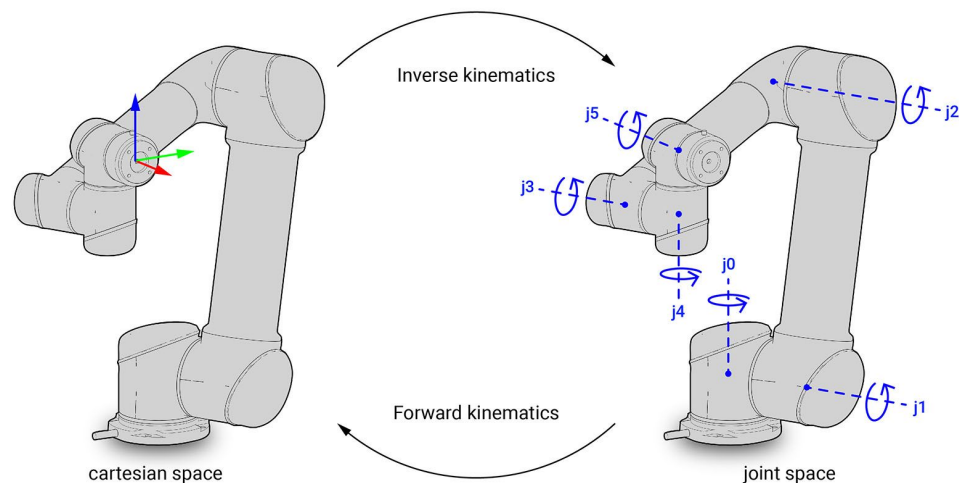
# Motivation - Inverse Kinematics

Classical approaches:

analytical solution  $\rightarrow$  not always possible

numerical solution  $\rightarrow$  expensive, limited to few solutions

Can we use neural networks to learn inverse kinematics?



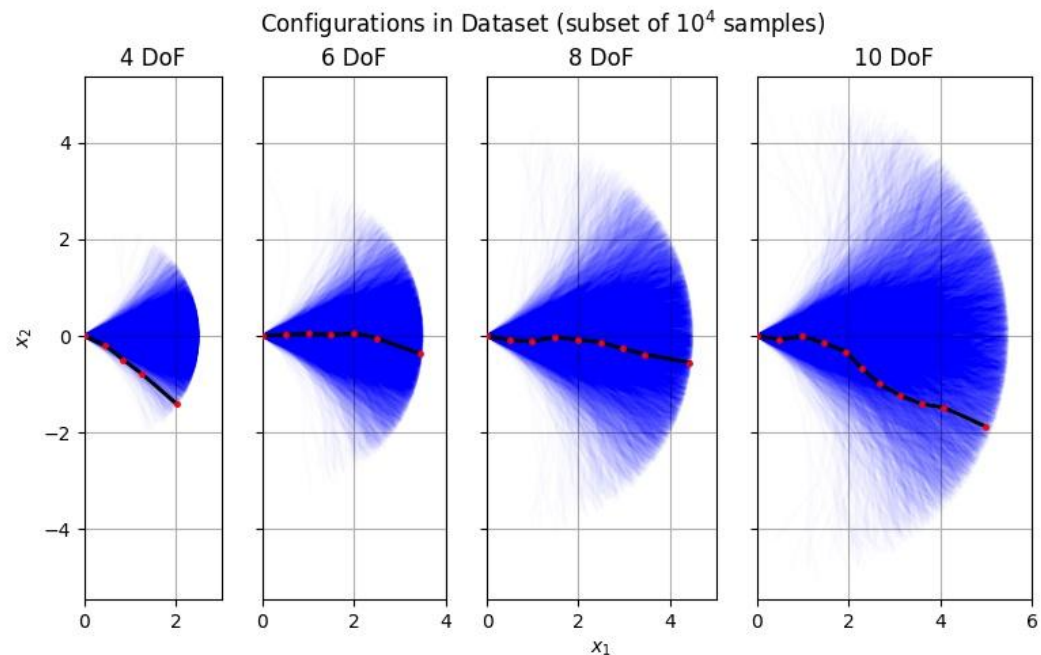
# Methods - Dataset

Joint orientations from a normal distribution:  $\theta_i \sim \mathcal{N}(\mu = 0, \sigma = 0.2)$

$10^6$  samples of joint configurations and TCP coordinates

$$x_{\text{TCP}} = \sum_{i=1}^N l_i \cos \left( \sum_{j=1}^i \theta_j \right)$$

$$y_{\text{TCP}} = \sum_{i=1}^N l_i \sin \left( \sum_{j=1}^i \theta_j \right)$$



# Methods - Hyperparameter Search

Compare performance of networks in fair setting

Hyperparameter search using Ray Tune and Scikit Optimize

Train 100 samples using subset of original dataset ( $10^4$  samples)

TABLE I  
RANGES OF INN HYPERPARAMETERS USED DURING HYPERPARAMETER SEARCH.

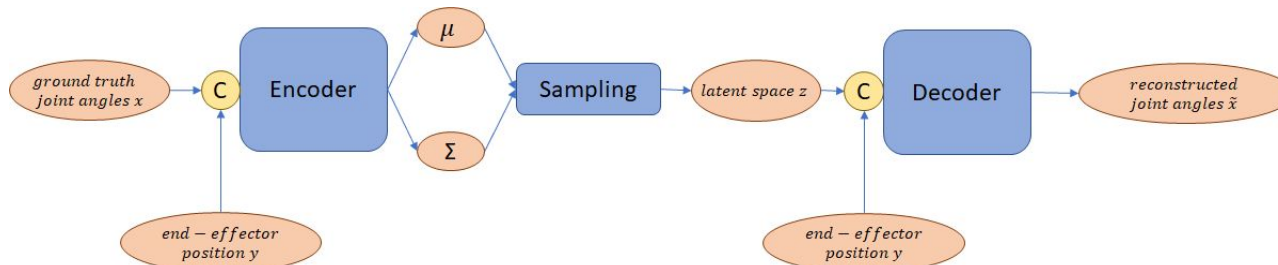
	Learning rate	# Subnet layers	# Coupling layers	# Neurons per layer	Weight decay
Min	0.0001	3	6	100	0.00001
Max	0.005	7	10	300	0.001

TABLE II  
RANGES OF CVAE HYPERPARAMETERS USED DURING HYPERPARAMETER SEARCH.

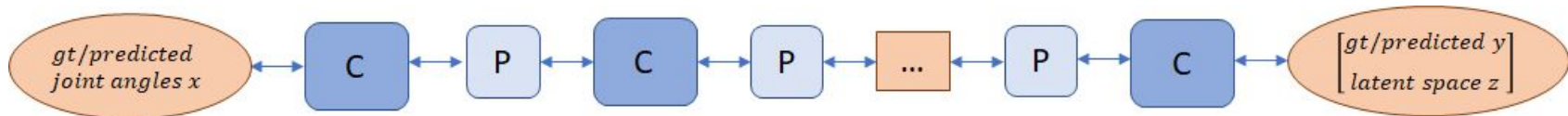
	Learning rate	# Layers	# Neurons per layer	Weight decay
Min	0.0001	3	200	0.00001
Max	0.01	15	500	0.001

# Recap: Model Architectures

## Conditional Variational Autoencoder (cVAE)



## Invertible Neural Network (INN)



# Recap: Evaluation

1. Average mismatch between true posterior and predicted posterior

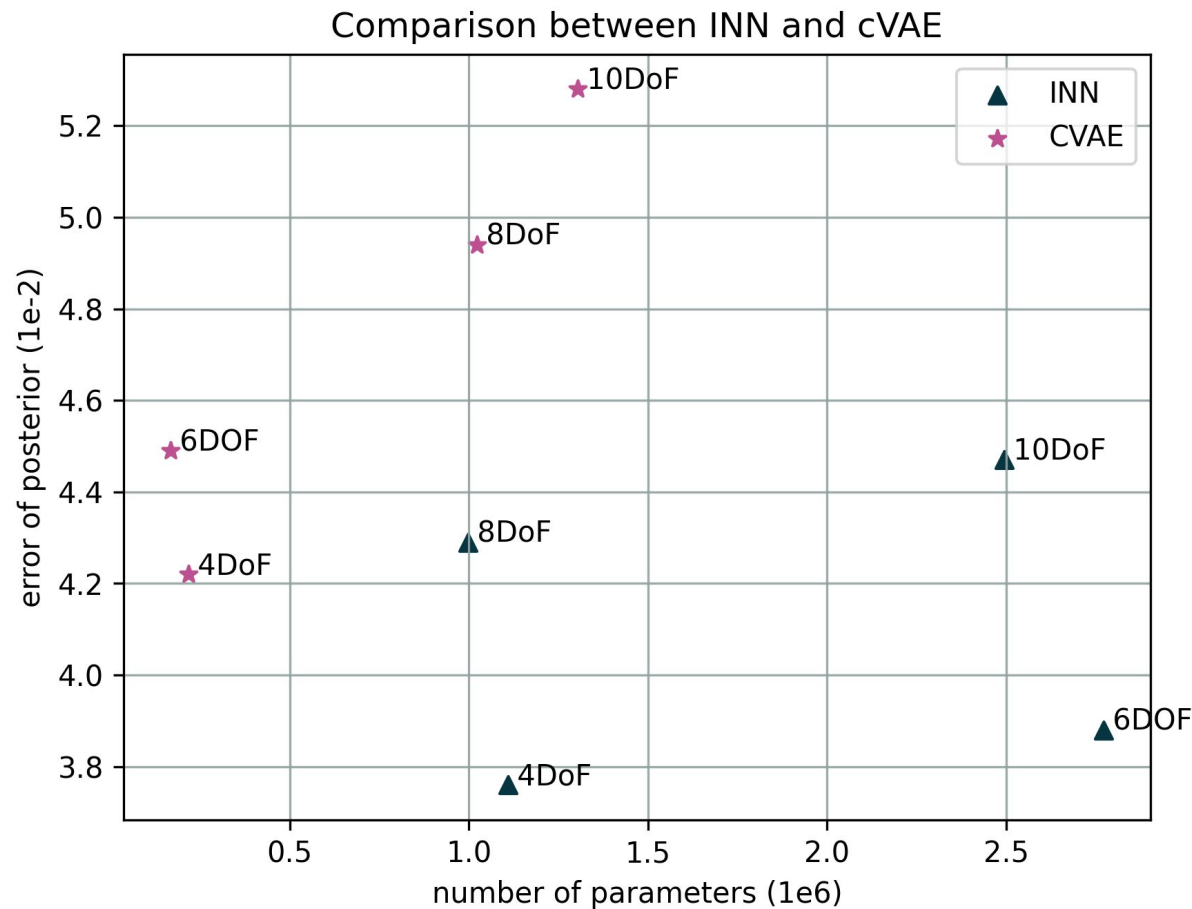
$$e_{posterior} = MMD(\tilde{p}(x|y_{gt}), p_{gt}(x|y_{gt}))$$

2. Average re-simulation error

$$e_{resim} = E_{x \sim \tilde{p}(x|y_{gt})}(\|f(x) - y_{gt}\|_2^2)$$

# Results

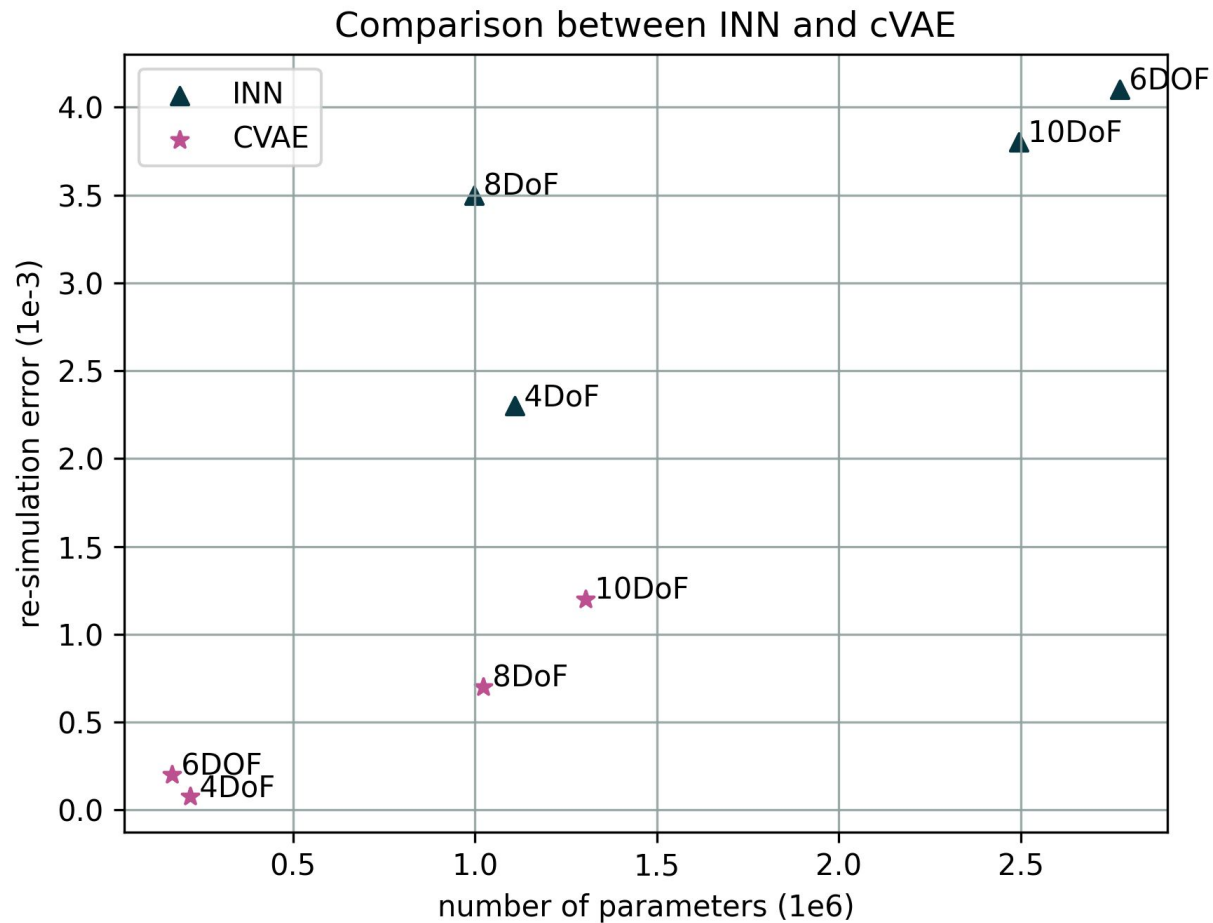
Average mismatch between true posterior and predicted posterior





# Results

## Average re-simulation error



# Results

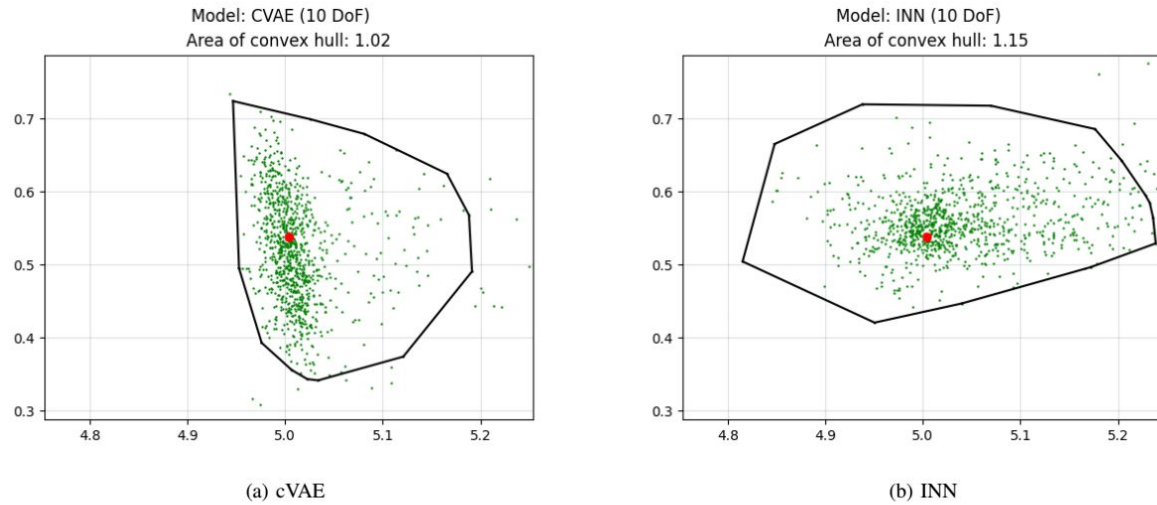


Fig. 4. Area of the convex hull of the 0.97 percentile of the re-simulated end-effector coordinates with the ground truth end-effector position at  $(x, y) = [5.00, 0.53]$  and 1000 samples. Number of DoF: 10.

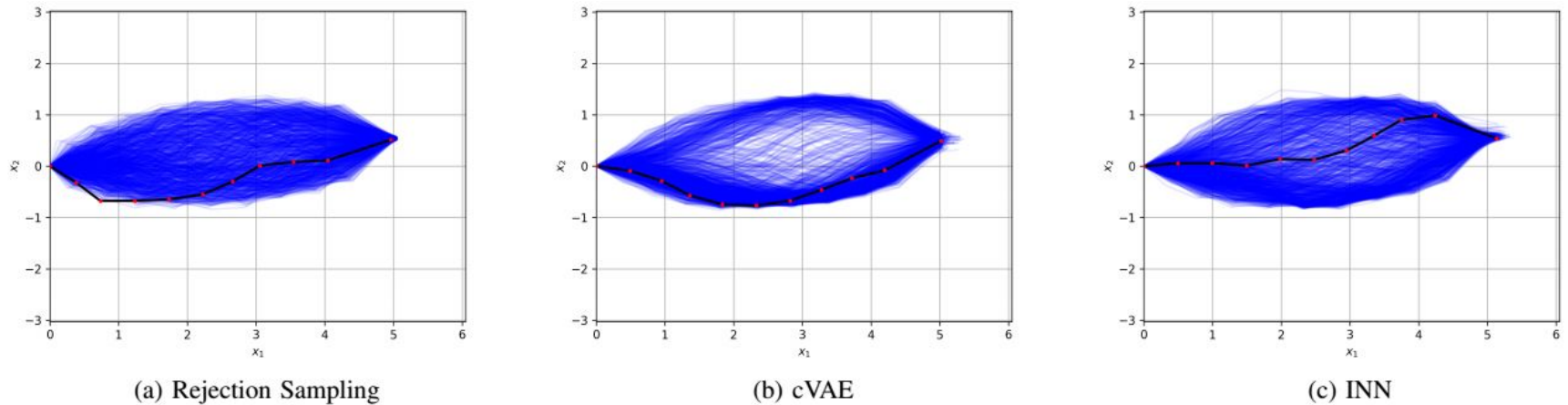


Fig. 6. Arm configuration of a planar manipulator with 10 revolute joints and end-effector position at  $(x, y) = [5.00, 0.53]$ . 1000 samples are drawn from each model's predicted posterior  $\tilde{p}(x|y_{gt})$ , one random sample configuration is highlighted.  $e_{posterior} = 0.057$  for the cVAE and  $e_{posterior} = 0.025$  for the INN.

# Conclusion

- Evaluation of INN and cVAE through simulation experiments in the context of inverse kinematics
- Finding best set of hyperparameters via Bayesian Optimization
- cVAE performed better on average re-simulation error
- INN performed better in estimating the posterior distribution of the joint angles
- INN: additional unsupervised backward training to match performance of cVAE

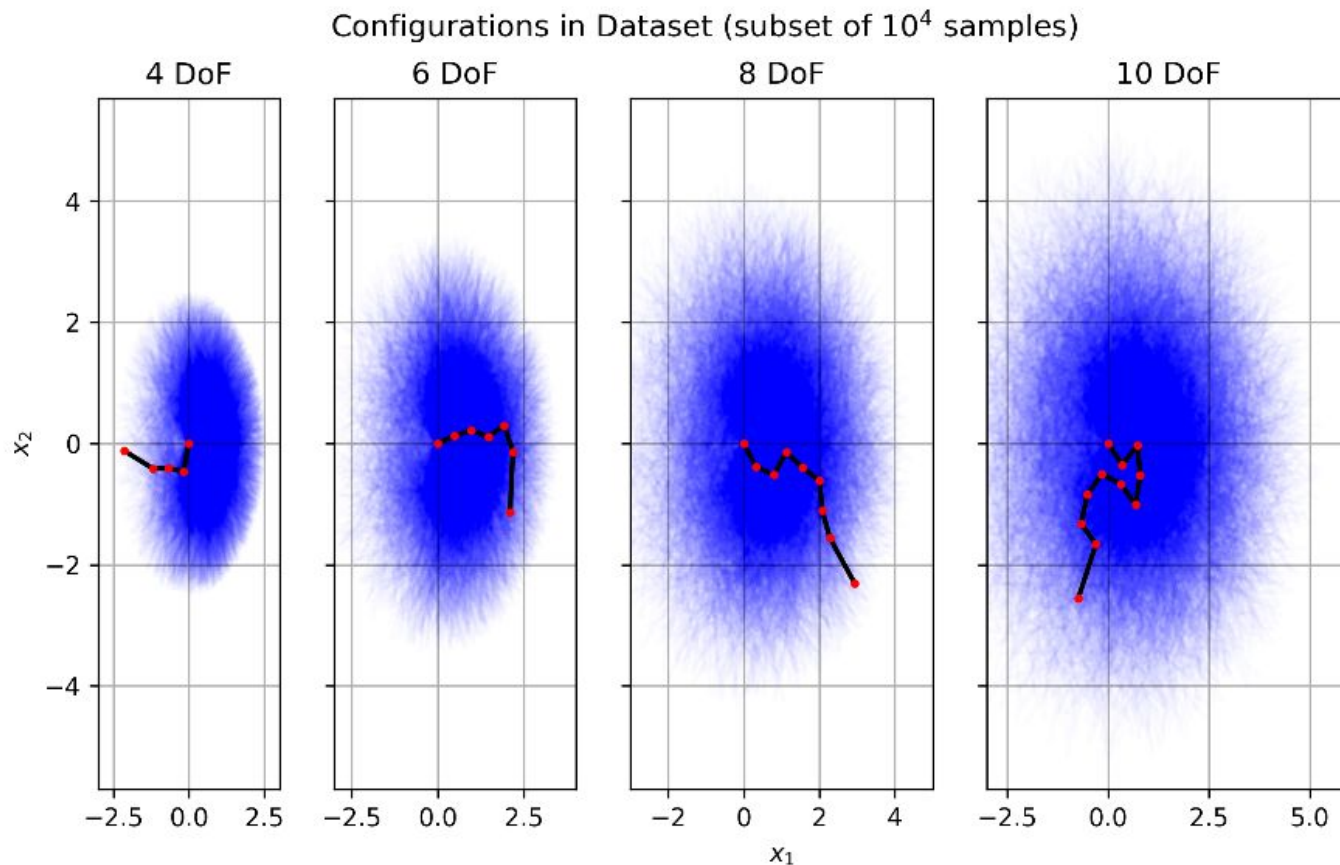
# References

- [1] Lynton Ardizzone, Jakob Kruse, Sebastian J. Wirkert, Daniel Rahner, Eric W. Pellegrini, Ralf S. Klessen, Lena Maier-Hein, Carsten Rother, and Ullrich Köthe. Analyzing inverse problems with invertible neural networks. CoRR, abs/1808.04730, 2018
- [2] Kihyuk Sohn, Xinchun Yan, and Honglak Lee. 2015. Learning structured output representation using deep conditional generative models. In Proceedings of the 28th International Conference on Neural Information Processing Systems - Volume 2 (NIPS'15). MIT Press, Cambridge, MA, USA, 3483–3491.
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- [5] Arthur Gretton, Karsten M. Borgwardt, Malte J. Rasch, Bernhard Schölkopf, and Alexander J. Smola. A kernel method for the two-sample problem. CoRR, abs/0805.2368, 2008.
- [6] Mehdi Mirza and Simon Osindero. Conditional generative adversarial nets. CoRR, abs/1411.1784, 2014.

# Questions

# Backup Slides

## Dataset with degenerated configurations (std=1.0)



# Backup Slides

## Results for HPO

TABLE III

HYPERPARAMETERS FOR CVAE TRAINED ON A PLANAR ROBOT WITH REVOLUTE JOINTS

DoF	Learning rate	# Layers	# Neurons per layer	Weight decay	# Trainable parameters
4	0.009	3	230	0.0002	217128
6	0.0041	3	200	0.00001	166814
8	0.0001	3	500	0.00001	1022020
10	0.00063	5	400	0.00029	1303226

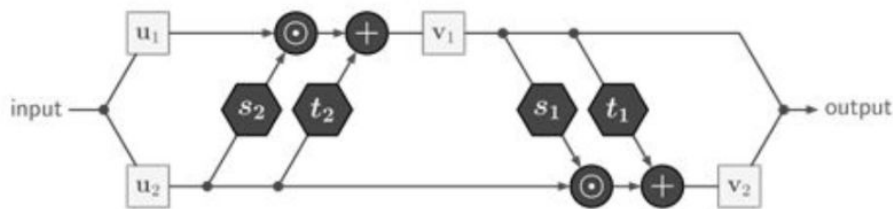
TABLE IV

HYPERPARAMETERS FOR INN TRAINED ON A PLANAR ROBOT WITH REVOLUTE JOINTS

DoF	Learning rate	# Subnet layers	# Coupling layers	# Neurons per layer	Weight decay	# Trainable parameters
4	0.0009	5	9	100	0.00008	1108872
6	0.001	5	7	180	0.0003	2772084
8	0.0014	4	9	115	0.0004	997884
10	0.002	5	7	170	0.0005	2494380

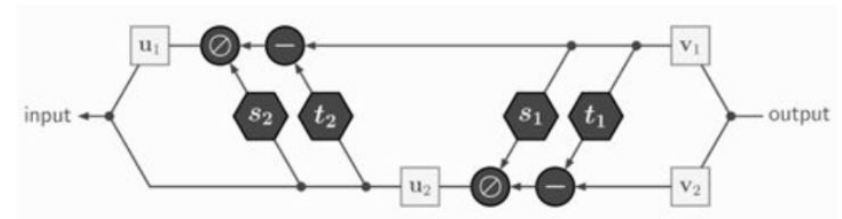
# Backup Slides

## INN: Affine Coupling Layers



$$\mathbf{v}_1 = \mathbf{u}_1 \odot \exp(s_2(\mathbf{u}_2)) + t_2(\mathbf{u}_2)$$

$$\mathbf{v}_2 = \mathbf{u}_2 \odot \exp(s_1(\mathbf{v}_1)) + t_1(\mathbf{v}_1),$$



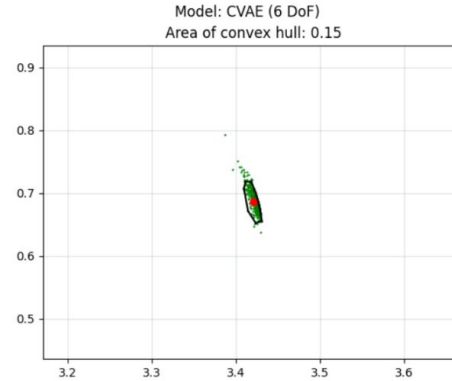
$$\mathbf{u}_2 = (\mathbf{v}_2 - t_1(\mathbf{v}_1)) \odot \exp(-s_1(\mathbf{v}_1))$$

$$\mathbf{u}_1 = (\mathbf{v}_1 - t_2(\mathbf{u}_2)) \odot \exp(-s_2(\mathbf{u}_2)).$$

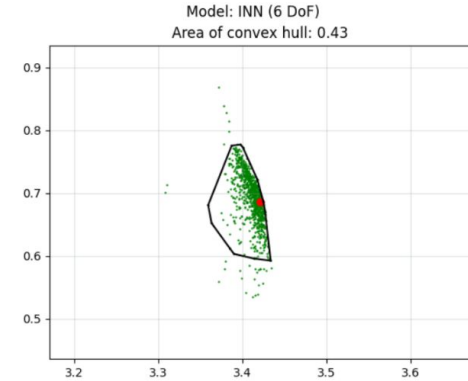


# Backup Slides

## Additional example

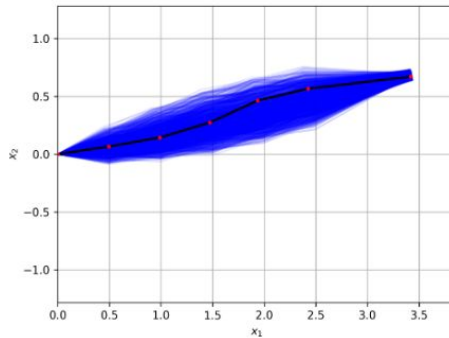


(a) cVAE

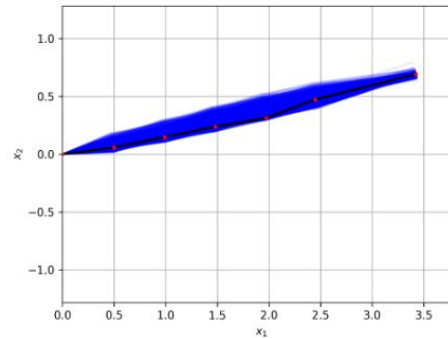


(b) INN

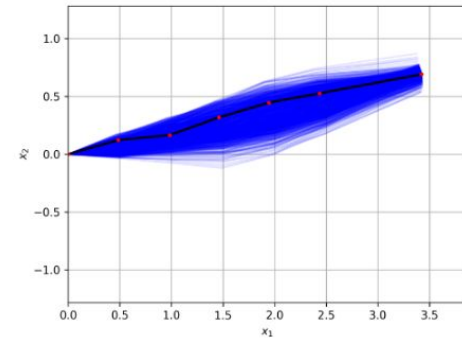
Fig. 4. Area of the convex hull of the 97th percentile of the re-simulated end-effector coordinates with the ground truth end-effector position at  $(x, y) = [3.42, 0.68]$  and 1000 samples. Number of DoF: 6.



(a) Rejection Sampling



(b) cVAE



(c) INN

Fig. 6. Arm configuration of a planar manipulator with 6 revolute joints and end-effector position at  $(x, y) = [3.42, 0.68]$ . 1000 samples are drawn from each model's predicted posterior  $\tilde{p}(x|y_{gt})$ , one random sample configuration is highlighted.  $e_{posterior} = 0.012$  for the cVAE and  $e_{posterior} = 0.0068$  for the INN.

# Backup Slides

Loss for cVAE:

$$L_y = -KL[q_{w_{enc}}(z|x, y) || p(z) \sim N(0, 1)]$$

$$L_x = E_{q_{w_{enc}}(z|x, y)}[\log(p_{w_{dec}}(x|z, y))]$$

Loss for INN:

$$L_y = MSE(y_i, f_y(x_i))$$

$$[y, z] = [f_y(x), f_z(x)]$$

$$L_x = MSE(x_i, [f_y^{-1}(y_i), f_z^{-1}(f_z(x_i))])$$

$$L_{p(z)} = MMD(p(f_z(x_i)), p(z) \sim N(0, 1))$$

$$L_{p(x)} = MMD(p([f_y^{-1}(y_i), f_z^{-1}(p(z))]), p(x))$$