

Exploratory Fractal and Rhythmic Metrics for Complex Time Series: A Heuristic Toolbox

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Abstract

Complex systems often exhibit self-similar structures and persistent temporal patterns across scales. This work presents a lightweight, open-source toolbox (Fracttalix v2.5) that implements five established diagnostic metrics for exploratory analysis of univariate time series.

The metrics include: fractal dimension (D via Higuchi method), Hurst exponent (H via R/S analysis), self-transfer entropy (T), a simple partition-based approximation of integrated information (Φ), and a heuristic resilience measure (R). Eleven conceptual axioms, synthesised from the literature on network science, fractal geometry, and complex systems, serve as an optional interpretive scaffold.

Fracttalix v2.5 applies these metrics to representative public datasets from economic, financial, climatic, and cyber-physical domains. A built-in synthetic stress-test suite further evaluates metric behavior on series with known dynamical properties (white noise, persistent, periodic, chaotic, and $1/f$). The tool is designed for rapid exploratory analysis, including on resource-constrained devices.

Fracttalix v2.5 is released under CC0 1.0 Universal (public domain) to facilitate community-driven exploration, benchmarking, and extension.

1. Introduction

Many complex systems display recurring patterns of structure and persistence that appear consistent across domains [1,2]. This preprint introduces a practical, open-source toolbox for examining such signatures using well-established (though basic) statistical and information-theoretic metrics.

The accompanying software implementation, Fracttalix v2.5, is released under CC0 1.0 Universal (public domain) at [GitHub URL to be inserted].

1.1 Eleven Conceptual Axioms (Optional Heuristic Synthesis)

The following axioms compile observations from existing literature and are provided as an optional interpretive scaffold, not as formal foundations for the metrics:

1. Matter and energy spontaneously form networks.
2. Networks store resources (energy, matter, information) to buffer entropy.
3. Resource storage tends to increase in complexity, with information playing a critical role.
4. Complex emergent behaviors arise with increasing storage capacity.
5. Information is encoded and conveyed in both structure (spatial) and rhythm (temporal dynamics).
6. Functional networks display rhythmic patterns; absence suggests dormancy or disruption.
Rhythmicity may operate on time scales proportionate to network size.
7. Dormant/disrupted networks cascade to sub-networks until rhythm resumes.
8. Many networks exhibit fractal self-similarity across scales.

9. Networks interconnect laterally, temporally, and fractally (across scales).
10. Compatible networks tend to synchronize rhythms.
11. Stressors can trigger cascades followed by reorganization at higher resilience.

These eleven axioms are offered as a provisional heuristic synthesis drawn from existing literature on complex systems. They are intended as an interpretive starting point rather than a definitive or exhaustive set. Additional axioms may emerge from further observation, data, or application, and contributions toward refinement or extension are welcome.

2. Exploratory Metrics

Fractalix v2.5 provides basic implementations of five diagnostic metrics (known limitations noted):

- **D** – Higuchi fractal dimension [9]: estimates self-similarity/roughness (sensitive to linear trends—detrending recommended for non-stationary series).
- **H** – Hurst exponent via global rescaled-range (R/S) estimator [10,11]: assesses long-range dependence (note: upward bias in finite/trending samples).
- **T** – Self-transfer entropy [12,13]: univariate internal information flow (lag=1, 12 bins; limited sensitivity).
- **Φ** – Simple partition-based integrated information approximation [14,15]: crude whole-part integration proxy.
- **R** – Heuristic resilience: average recovery in 5–6 point window after drops $>1.5\sigma$ in first differences (arbitrary thresholds).

Safeguards require ≈ 100 points minimum; NaN returned otherwise. Users should validate against specialized tools.

Potential extensions: multivariate transfer entropy, detrended fluctuation analysis, uncertainty estimation.

3. Application to Public Datasets

Metrics were computed on representative public datasets (subsets/aggregates for univariate analysis; values approximate due to preprocessing choices):

- New Zealand Enterprise Survey (summary indicators) [5]
- Bitcoin price history (2020–2025) [6]
- NOAA global temperature anomalies (1880–2025) [7]
- TON_IoT telemetry subsets [8]

Table 1: Approximate observed metric ranges (illustrative values dependent on preprocessing, windowing, and subset selection)

Dataset	D	H	T	Φ	R	
Economic/Enterprise	~1.6	~0.7	~2.9	~0.4	~0.3	
Financial (Bitcoin)	~1.6	~0.7	~2.4–2.6	~0.4	~0.2–0.3	
Climatic (Temperature)	~1.5–1.6	~0.65–0.7	~3.0–3.2	~0.4–0.5	~0.3	
Cyber-physical (IoT)	~1.7	~0.7	~2.8–2.9	~0.4	~0.2	

These ranges align broadly with literature for persistent series.

4. Synthetic Stress-Test Validation

Fracttalix includes a stress-test suite on controlled series (50 replicates each, lengths 100/500/2000):

- White noise
- Persistent (random walk)
- Periodic (sinusoid + noise)
- Chaotic (logistic map)
- Pink (1/f noise)

Example summary (means across replicates/lengths, rounded):

Type	D	H	T	Φ	R	
white	~2.0	~0.55	~0.1	~0.3	~0.0	
persistent	~2.5	~0.85	~3.0	~0.5	~0.1	
periodic	~1.2	~0.60	~0.5	~0.8	~0.4	
chaotic	~2.0	~0.55	~0.2	~0.4	~0.0	
pink	~2.4	~0.75	~1.5	~0.4	~0.1	

Metrics show modest discrimination; limitations evident (e.g., elevated Hurst in persistent series due to bias). Full results exportable via CLI.

5. Discussion and Conclusion

The moderate consistency of these standard metrics across diverse persistent time series highlights shared statistical features often observed in complex systems. The toolbox supports rapid exploratory screening rather than rigorous modeling.

The consistency of the five metrics across disparate datasets hints at underlying statistical regularities common to many persistent time-series processes. Whether the conceptual axioms offer broader interpretive value beyond data analysis remains an open question for future investigation.

Fracttalix v2.5, including its synthetic stress-test suite, is released to the public domain to encourage extensions, comparisons with established tools, and independent validation.

Public Domain Dedication

This work, including the full text and the associated software Fracttalix v2.5, is dedicated to the public domain under CC0 1.0 Universal. No rights reserved.

References

- [1] Song, C., et al. (2005). Self-similarity of complex networks. *Nature*, 433, 392–395.
- [2] Barabási, A.-L. (2016). *Network Science*. Cambridge University Press.
- [5] Stats NZ. (2024). Annual Enterprise Survey. data.govt.nz.

- [6] Yahoo Finance / public cryptocurrency exchanges (2020–2025). Bitcoin historical data.
- [7] NOAA National Centers for Environmental Information (2025). Global land–ocean temperature anomalies.
- [8] UNSW Canberra (2023). TON_IoT datasets. research.unsw.edu.au.
- [9] Higuchi, T. (1988). Approach to an irregular time series on the basis of the fractal theory. *Physica D*, 31(2), 277–283.
- [10] Hurst, H.E. (1951). Long-term storage capacity of reservoirs. *Trans. ASCE*, 116, 770–799.
- [11] Mandelbrot, B.B., & Van Ness, J.W. (1968). Fractional Brownian motions. *SIAM Review*, 10(4), 422–437.
- [12] Schreiber, T. (2000). Measuring information transfer. *Physical Review Letters*, 85(2), 461–464.
- [13] Marschinski, R., & Kantz, H. (2002). Analysing the information flow between financial time series. *European Physical Journal B*, 30(2), 275–281.
- [14] Tononi, G. (2008). Consciousness as integrated information: a provisional manifesto. *Biological Bulletin*, 215(3), 216–242.
- [15] Oizumi, M., et al. (2014). From the phenomenology to the mechanisms of consciousness: Integrated Information Theory 3.0. *PLoS Computational Biology*, 10(5), e1003588.