250.9

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Problem. Suppose $S, T \in \mathcal{L}(V)$ and ST is nilpotent. Prove that TS is nilpotent.

Claim. TS is nilpotent.

Proof. Since ST is nilpotent, then there exists some k for which

$$(ST)^k = 0.$$

Note that

$$\begin{aligned}(TS)^{k+1} &= T(ST)^k S \\ &= 0,\end{aligned}$$

and thus TS is nilpotent. □

Note. You can view the source code for this solution here.