268.8

THOMAS BREYDO

Problem. Suppose $T \in \mathcal{L}(V)$ and

$$p(z) = z^n + a_{n-1}z^{n-1} + \cdots + a_0$$

is the minimal polynomial of T. Prove that T is invertible if and only if $a_0 \neq 0$.

Claim. T is invertible if and only if $a_0 \neq 0$.

Proof. We will prove the claim by a series of "if-and-only-ifs:"

$$\begin{aligned} T \text{ is invertible} &\iff 0 \text{ is not an eigenvalue of } T \\ &\iff 0 \text{ is not a root of } p(z) \\ &\iff p(0) \neq 0 \\ &\iff a_0 \neq 0 \end{aligned}$$

□

Note. You can view the source code for this solution here.