238.2

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Problem. Give an example of $T \in \mathcal{L}(\mathbf{C}^2)$ such that 0 is the only eigenvalue of T and the singular values are $5, 0$.

Suppose

$$\mathcal{M}(T) = \begin{pmatrix} 0 & a \\ b & 0 \end{pmatrix}.$$

Since the standard basis is orthonormal,

$$\mathcal{M}(T^*) = \begin{pmatrix} 0 & b \\ a & 0 \end{pmatrix}.$$

Thus,

$$\begin{aligned} \mathcal{M}(T^*T) &= \mathcal{M}(T^*)\mathcal{M}(T) \\ &= \begin{pmatrix} 0 & b \\ a & 0 \end{pmatrix} \begin{pmatrix} 0 & a \\ b & 0 \end{pmatrix} \\ &= \begin{pmatrix} b^2 & 0 \\ 0 & a^2 \end{pmatrix} \end{aligned}$$

Since we need T to have singular values 0 and 5, we need T^*T to have eigenvalues 0 and 25. If we let $b = 0$ and $a = 5$, we get

$$\mathcal{M}(T) = \begin{pmatrix} 0 & 5 \\ 0 & 0 \end{pmatrix}.$$

Not sure how to prove that 0 is the only eigenvalue of $T(z_1, z_2) = (5z_2, 0)$. Clearly it *is* an eigenvalue, though.

Note. You can view the source code for this solution here.