267.1

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Problem. Suppose $T \in \mathcal{L}(\mathbf{C}^4)$ is such that the eigenvalues of T are 3, 5, 8. Prove that $(T - 3I)^2(T - 5I)^2(T - 8I)^2 = 0$.

Suppose $p(z)$ is the characteristic polynomial of T.

Claim. $p(z) = (z - 3)^a(z - 5)^b(z - 8)^c$, where one of a, b, c equals 2 and the others equal 1.

Proof. Since each eigenvalue has multiplicity at least 1, and the sum of the multiplicities is 4, we see that the multiplicities are 1, 1, 2. □

Suppose $q(z) = (z - 3)^2(z - 5)^2(z - 8)^2$.

Claim. $q(z)$ is a multiple of the minimal polynomial.

Proof. By *8.48*, we know that $p(z)$ is a multiple of the minimal polynomial. Since $q(z)$ is a multiple of $p(z)$, it follows that $q(z)$ is a multiple of the minimal polynomial. □

Finally, by *8.46*, $q(T) = 0$.

Note. You can view the source code for this solution here.