285.7

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Problem. Suppose V is a real vector space and $N \in \mathcal{L}(V)$. Prove that N_C is nilpotent if and only if N is nilpotent.

Suppose $k = \dim V$.

Claim. $N^k = 0 \quad \implies \quad (N_C)^k = 0$

Proof. Suppose $N^k = 0$. Then, for any $u + iv \in V_C$,

$$\begin{aligned}(N_C)^k(u+iv) &= N^k u + iN^k v && \text{(by 9.9)}\\ &= 0,\end{aligned}$$

and thus $(N_C)^k = 0$. □

Claim. $(N_C)^k = 0 \quad \implies \quad N^k = 0$

Proof. Suppose $(N_C)^k = 0$. Then, for any $u \in V$,

$$\begin{aligned}N^k u &= N^k u + i \cdot 0\\ &= N^k u + iN^k 0\\ &= (N_C)^k(u + i \cdot 0) && \text{(by 9.9)}\\ &= 0,\end{aligned}$$

and thus $N^k = 0$. □

Combining the two claims above, we get that

$$N^k = 0 \quad \iff \quad (N_C)^k = 0.$$

Note. You can view the source code for this solution here.