274.5

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Problem. Suppose $T \in \mathcal{L}(V)$ and $v_1, \ldots, v_n$ is a basis of V that is a Jordan basis for T. Describe the matrix of T^2 with respect to this basis.

Since $v_1, \ldots, v_n$ is a Jordan basis for T,

$$\mathcal{M}(T) = \begin{pmatrix} A_1 & & 0 \\ & \ddots & \\ 0 & & A_p \end{pmatrix},$$

where

$$A_j = \begin{pmatrix} \lambda_j & 1 & & 0 \\ & \ddots & \ddots & \\ & & \ddots & 1 \\ 0 & & & \lambda_j \end{pmatrix},$$

Claim.

$$\mathcal{M}(T^2) = \begin{pmatrix} A_1^2 & & 0 \\ & \ddots & \\ 0 & & A_p^2 \end{pmatrix},$$

Proof. If you do the matrix multiplication $\mathcal{M}(T) \cdot \mathcal{M}(T)$, you will get the desired result. Intuitively, if T acts independently on the p subspaces, then T^2 will act exactly as described by $A_1^2, \ldots, A_p^2$. $\square$

As a result of the claim above, we see that

$$\mathcal{M}(T^2) = \begin{pmatrix} B_1 & & 0 \\ & \ddots & \\ 0 & & B_p \end{pmatrix},$$

where

$$
\begin{aligned}
B_j &= A_j^2 \\
&= \begin{pmatrix}
\lambda_j^2 & 2\lambda_j & 1 & & 0 \\
& \ddots & \ddots & \ddots & \\
& & \ddots & \ddots & 1 \\
& & & \ddots & 2\lambda_j \\
0 & & & & \lambda_j
\end{pmatrix}.
\end{aligned}
$$

Note. You can view the source code for this solution here.