## 223.8

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**Problem.** Give an example of an operator T on a complex vector space such that  $T^9=T^8$  but  $T^2\neq T$ .

The key idea is to construct a T such that  $T^2, T^3, \ldots, T^9$  are all the zero map, while T itself is not.

Consider the complex vector space  $\mathbf{C}$  with basis 1+0i, 0+1i. Let T be the linear map defined by

$$T(a+bi) = b+0i$$

for all  $a + bi \in \mathbf{C}$ .

Claim.  $T^2 \equiv 0$ .

*Proof.* For all  $a + bi \in \mathbf{C}$ ,

$$T^{2}(a+bi) = T(b+0i)$$
$$= 0+0i.$$

Thus,  $T^2$  is the zero map.

Claim.  $T^9 = T^8$ .

*Proof.* It follows from the previous claim that  $T^8 \equiv 0$  and  $T^9 \equiv 0$ , so  $T^8 = T^9$ .  $\Box$ 

Claim.  $T^2 \neq T$ .

Proof. Since

$$T(1+1i) = 1+0i$$

while

$$T^2(1+1i) = 0 + 0i,$$

the two maps are different.

**Note.** You can view the source code for this solution here.