267.1

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> **Problem.** Suppose $T \in \mathcal{L}(\mathbf{C})$ is such that the eigenvalues of T are 3, 5, 8.
> Prove that $(T - 3I)^2(T - 5I)^2(T - 8I)^2 = 0$.

Suppose $p(z)$ is the characteristic polynomial of T.

> **Claim.** $p(z) = (z - 3)^a(z - 5)^b(z - 8)^c$, where one of a, b, c equals 2 and the
> others equal 1.

Proof. Since each eigenvalue has multiplicity at least 1, and the sum of the multiplicities is 4, we see that the multiplicities are 1, 1, 2. □

Suppose $q(z) = (z - 3)^2(z - 5)^2(z - 8)^2$.

> **Claim.** $q(z)$ is a multiple of the minimal polynomial.

Proof. By *8.48*, we know that $p(z)$ is a multiple of the minimal polynomial. Since $q(z)$ is a multiple of $p(z)$, it follows that $q(z)$ is a multiple of the minimal polynomial. □

Finally, by *8.46*, $q(T) = 0$.

> **Note.** You can view the source code for this solution here.