223.8

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Problem. Give an example of an operator T on a complex vector space such that $T^9 = T^8$ but $T^2 \neq T$.

The key idea is to construct a T such that $T^2, T^3, \ldots, T^9$ are all the zero map, while T itself is not.

Consider the complex vector space $\mathbf{C}$ with basis $1+0i, 0+1i$. Let T be the linear map defined by

$$T(a+bi) = b+0i$$

for all $a+bi \in \mathbf{C}$.

Claim. $T^2 \equiv 0$.

Proof. For all $a+bi \in \mathbf{C}$,

$$\begin{aligned} T^2(a+bi) &= T(b+0i) \\ &= 0+0i. \end{aligned}$$

Thus, T^2 is the zero map. □

Claim. $T^9 \neq T^8$.

Proof. It follows from the previous claim that $T^8 \equiv 0$ and $T^9 \equiv 0$, so $T^8 = T^9$. □

Claim. $T^2 \neq T$.

Proof. Since

$$T(1+1i) = 1+0i$$

while

$$T^2(1+1i) = 0+0i,$$

the two maps are different. □

Note. You can view the source code for this solution here.