

140.30

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Problem. Suppose $T \in \mathcal{L}(\mathbf{R}^3)$ and $-4, 5$, and $\sqrt{7}$ are eigenvalues of T . Prove that there exists $x \in \mathbf{R}^3$ such that $Tx - 9x = (-4, 5, \sqrt{7})$.

Claim. 9 is not an eigenvalue of T .

Proof. T can have at most $\dim \mathbf{R}^3 = 3$ eigenvalues by 5.13, and we already have three distinct eigenvalues: $-4, 5$, and $\sqrt{7}$. \square

Claim. $T - 9I$ is invertible.

Proof. Otherwise, 9 would be an eigenvalue of T by 5.6. \square

Claim. $x = (T - 9I)^{-1}((-4, 5, \sqrt{7}))$ works.

Proof. Note that

$$\begin{aligned}Tx - 9x &= (T - 9I)x \\ &= (-4, 5, \sqrt{7})\end{aligned}$$

as desired. \square

Note. You can view the source code for this solution [here](#).