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NEUR 603 Assignment 5: The Occulomotor System

1. a)

Given the 1-order simplification to Robinson's (1964) plant mechanics 4th order linear differential equation, we can express the amplitude of eye rotation as a function of time from saccade onset to saccade end.

$$F = k\theta + r\frac{d\theta}{dt}$$

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$$\frac{F}{r} = \frac{k}{r}\theta + \frac{d\theta}{dt}$$

Now, we can write the integrating factor as:

$$\mu(t) = e^{\int \frac{k}{r}dt}$$

$$\mu(t) = e^{\frac{kt}{r}}$$

This gives:

$$\frac{F}{r}e^{\frac{kt}{r}} = \frac{k}{r}\theta e^{\frac{kt}{r}} + \frac{d\theta}{dt}e^{\frac{kt}{r}}$$

$$\frac{F}{r}e^{\frac{kt}{r}} = \frac{d}{dt}\theta e^{\frac{kt}{r}}$$

Now, integrating both sides, we get (combining the constants into c):

$$\frac{F}{r}\int e^{\frac{kt}{r}}dt = \frac{d}{dt}\int \theta e^{\frac{kt}{r}}dt$$

$$\frac{F}{r}\cdot\frac{r}{k}e^{\frac{kt}{r}} = \theta e^{\frac{kt}{r}} + c$$

Solving for θ, we have:

$$\theta(t) = ce^{\frac{-kt}{r}} + \frac{F}{k}$$

Given our initial conditions, we can determine the constant c. We have $\theta(0) = 0$, and so:

$$0 = c + \frac{F}{k}$$

$$c = -\frac{F}{k}$$

Therefore, we have $\theta(t)$ as:

$$\boldsymbol{\theta(t) = \frac{F}{k}(1 - e^{\frac{-kt}{r}})}$$

1. b)

Now, we'll use the Sylvester & Cullen (1999) values for the stiffness, k (k = 4.2g/°) and the viscosity, r (r = 0.42gs/°), to find F during saccades of magnitude and duration *10^{o} (45ms), 20^{o} (68ms),* and *40° (110ms*), respectively. Rearranging our expression for $\theta(t)$, we get*:*

$$F = \frac{k \cdot \theta(t)}{(1 - e^{\frac{-kt}{r}})}$$

10^{o} (45ms):

$$F = \frac{(4.2g/°) \cdot 10°}{(1 - e^{\frac{-(4.2g/°)(0.045\ s)}{(0.42\ gs/°)}})}$$

$$F = 115.9\ g$$

20^{o} (68ms):

$$F = \frac{(4.2g/°) \cdot 20°}{(1 - e^{\frac{-(4.2g/°)(0.068\ s)}{(0.42\ gs/°)}})}$$

$$F = 170.2\ g$$

40^{o} (110ms):

$$F = \frac{(4.2g/°) \cdot 40°}{(1 - e^{\frac{-(4.2g/°)(0.110\ s)}{(0.42\ gs/°)}})}$$

$$F = 251.8\ g$$

Therefore, during saccades of magnitude and duration *10^{o} (45ms); 20^{o} (68ms); 40° (110ms),* we find F to be *115.9 g, 170.2 g*, and 251.8 *g*, respectively.

1. c)

For the period after the saccade in which the eye is immobile, the change in amplitude of eye rotation will be zero. So, for the period after the saccade, we have $\frac{d\theta}{dt} = 0$ and so:

$$F = k\theta$$

Now, for the saccades of magnitude and duration *10⁰ (45ms); 20⁰ (68ms); 40° (110ms),* we find F to be *-43.3 g, -106.4 g*, and *-336.0 g,* we have the following forces:

10^{o} (45ms):

$$F = (4.2g/°) \cdot 10°$$

$$F = 42\ g$$

20^{o} (68ms):

$$F = (4.2g/°) \cdot 20°$$

$$F = 84\ g$$

40^{o} (110ms):

$$F = (4.2g/°) \cdot 40°$$

$$F = 168\ g$$

Therefore, *after* saccades of magnitude and duration *10^{o} (45ms); 20^{o} (68ms); 40° (110ms),* we find F to be *42 g, 84 g*, and 168 *g*, respectively.

1. d)

$\theta(t)$ and F are plotted as a function of time in Figure 1 and Figure 2, respectively, for saccades of magnitude and duration of 10^{o} (45ms), 20^{o} (68ms), and 40° (110ms). Figure 2 illustrates that to generate saccades of increasing amplitude, the motorneuron uses pulses that increase in both magnitude and duration.

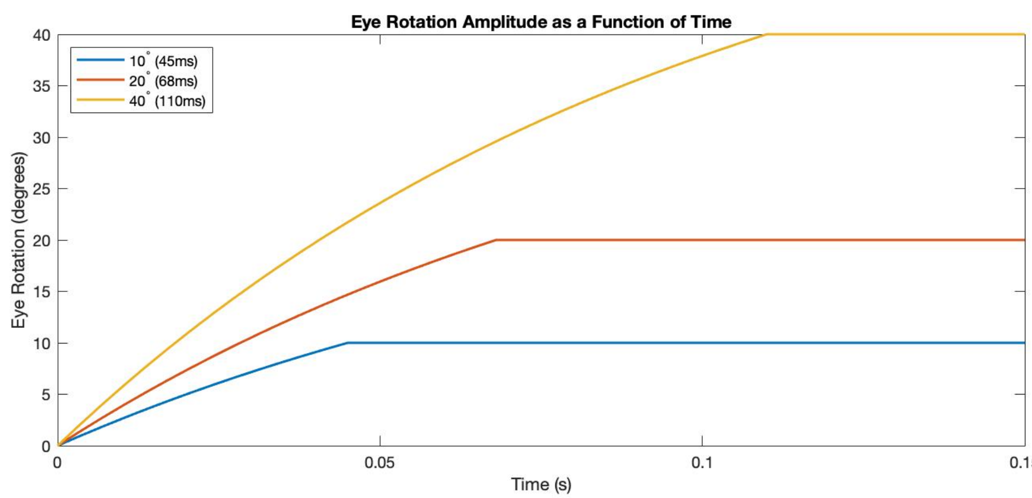

Figure 1 - Eye rotation amplitude (i.e. eye position) as a function of time for saccades of various amplitudes and durations

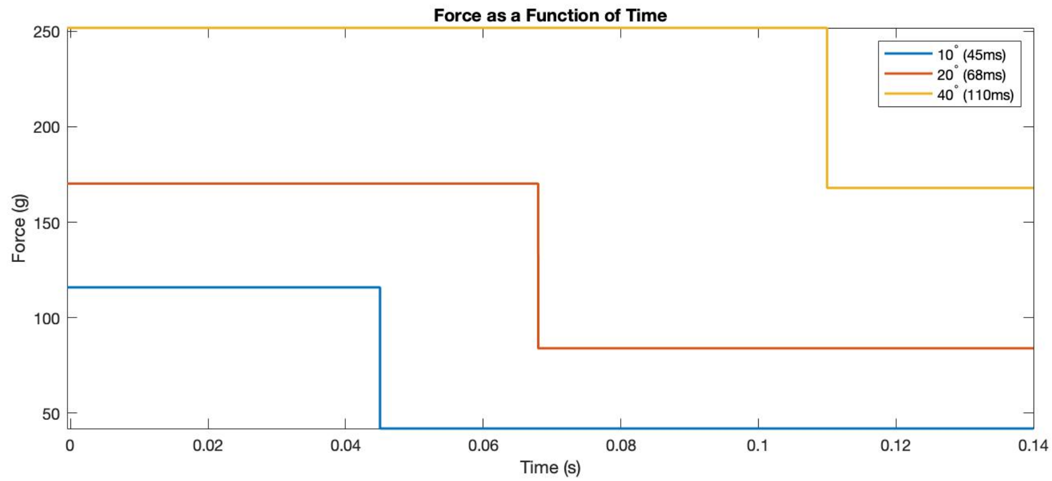

Figure 2 - Force pulse steps for saccades of various amplitudes and durations

2. a)

Box 2 (see handout) clearly implements the following:

$$A(F - B\frac{d\theta}{dt}) = \theta$$

We can re-write equation 2 similarly as follows:

$$F = k\theta + r\frac{d\theta}{dt}$$

$$\frac{1}{k}(F - r\frac{d\theta}{dt}) = \theta$$

Therefore, we have the expressions that relate constants A and B to r and k as follows:

$$\boldsymbol{A = \frac{1}{k}}$$

$$\boldsymbol{B = r}$$

2. b)

Now, the equation implemented by Box 1 (see handout), relating burst generator (BG) output to firing frequency (ff), is the following:

$$R \cdot BG + \ K\int BG\, dt = ff$$

Given that $F \equiv ff$, we have:

$$r \cdot BG + \ k\int BG\, dt = F$$

Where, from equation 2, we can write:

$$r \cdot BG + k \int BG\, dt = k\theta + r\frac{d\theta}{dt}$$

Thus, we find that the "?" in Box 3 is the time integral operator, and we can write:

$$\int BG\, dt = \theta$$

$$"?" = \int \boldsymbol{dt}$$

This makes sense – the time-integral of the burst generator determines eye position θ in degrees.

2. c)

The relationship between ff and θ_{des} was implemented in MATLAB as *run_controlsys()*. The estimated eye position signal (estimated by the neural circuit with constant BG output), the BG output, and the force (ff)) are plotted in Figure 3).

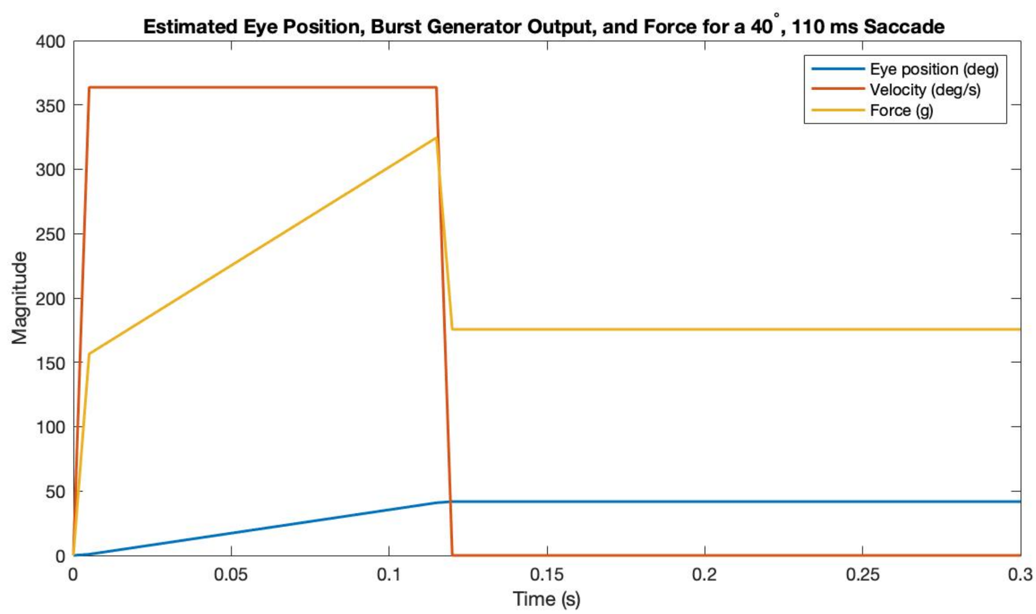

Figure 3 - Estimated eye position signal, the BG output and the force as functions of time for a 40 deg, 110 ms saccade

2. d)

The plant equation 2 was solved in MATLAB using *ode45(...)*. The predicted eye rotation θ_{out} for the force temporal profile calculated in 2.c is plotted in Figure 4, and compared against the estimated eye rotation θ_{est} (see 2.e).

2. e)

θ_{out} is compared with θ_{est} in Figure 4. We can observe from the plot that θ_{out} and θ_{est} are very similar, suggesting that Box 2 is effectively the inverse of Box 1, and that the firing frequency (ff) is approximately equal to the force (F). Further, the time course of the 40^o saccade obtained with a constant force in question 1.d is plotted in Figure 4. We can see that approximating the force as being constant reasonably approximates the eye movements estimated when the pulse height and duration are controlled by Robinson's feedback loop.

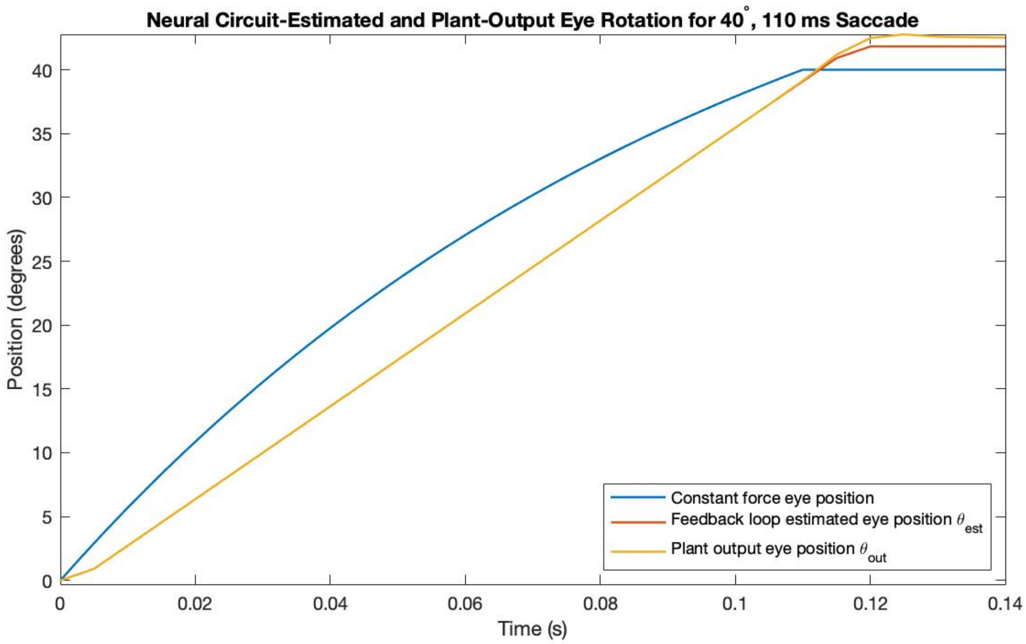

Figure 4 - Plot illustrating predicted eye rotation, ϑ_{out} for the force temporal profile calculated in 2.c, the estimated eye position signal, ϑ_{est}, for the same force temporal profile, and the time course of the 40° saccade obtained with a constant force in question 1.d

2. f)

If we assume the integrator in the loop is deficient and has $k_{lesion} = 0.5 * k$, the resulting eye trajectories θ_{out} for a 40° saccade are plotted in Figure 5.

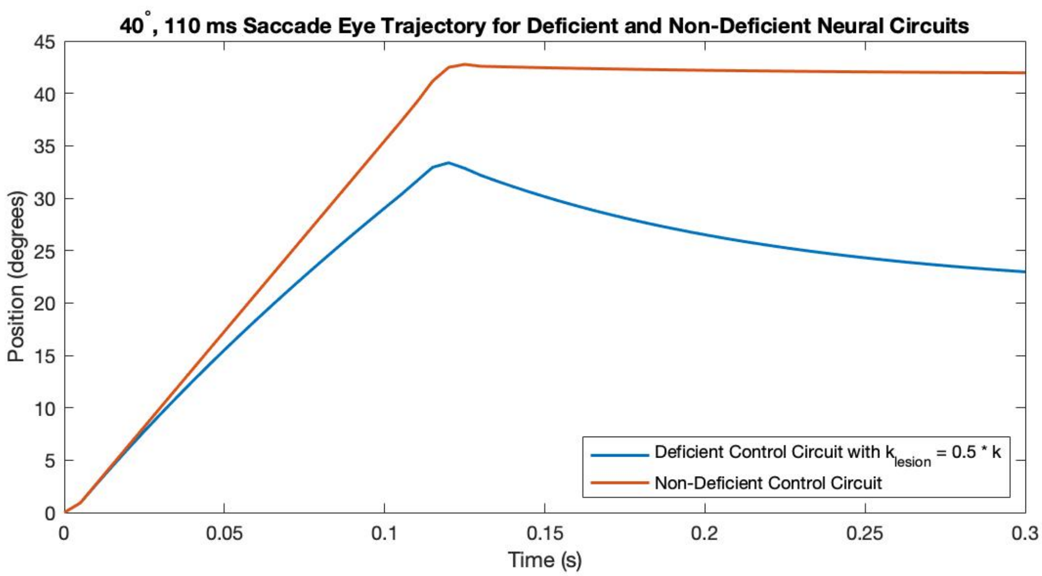

Figure 5 - Eye trajectories for deficient and non-deficient loop circuits. Note that the lesioned integrator fails to produce the proper firing frequency to attain the 40° saccade