The Representation of Inconsistent Knowledge in Advanced Knowledge Based Systems

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Abstract. Contradiction handling is one of the central problems in AI. There are different approaches to dealing with contradictions and other types of inconsistency. We describe an approach based on logical varieties, which are complex structures constructed from logical calculi. Being locally isomorphic to a logical calculus, globally logical varieties allow representation of contradictory knowledge in a consistent way, providing much more flexibility and efficacy for AI than standard logical methods. Problems of logical variety immersion into a logical calculus are studied. Such immersions extend the local structure of a logical calculus to the global structure of a logical variety, demonstrating when it is possible to use standard logical tools, such as logical calculi, and when it is necessary to go beyond this traditional technique. Finally a particular logical variety, the Logic of Reasonable Inferences, applied to the design of legal knowledge based systems is described.

Keywords: Inconsistent knowledge, Logical Varieties, Logic of Reasonable Inferences, Legal Knowledge Based Systems.

1 Introduction

Minsky [20] was one of the first researchers in AI who attracted attention to the problem of inconsistent knowledge. He wrote that consistency is a delicate concept that assumes the absence of contradictions in systems of axioms. Minsky also suggested that in artificial intelligence (AI) systems this assumption was superfluous because there were no completely consistent AI systems. In his opinion, it is important to understand how people solve paradoxes, find a way out of a critical situation, learn from their own or others' mistakes or how they recognize and exclude different inconsistencies. Minsky [21] suggested that consistency and effectiveness may well be incompatible. He also writes [22]: “An entire generation of logical philosophers has thus wrongly tried to force their theories of mind to fit the rigid frames of formal logic. In doing that, they cut themselves off from the powerful new discoveries of computer science. Yes, it is true that we can describe the operation of a computer's hardware in terms of simple logical expressions. But no, we cannot use the same expressions to describe the meanings of that computer's output -- because that

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would require us to formalize those descriptions inside the same logical system. And this, I claim, is something we cannot do without violating that assumption of consistency." Then Minsky [22] continues, "In summary, there is no basis for assuming that humans are consistent - not is there any basic obstacle to making machines use inconsistent forms of reasoning". Moreover, it has been discovered that not only human knowledge but also representations/models of human knowledge (e.g., large knowledge bases) are inherently inconsistent (Delgrande, *et al*, [9]).

It is necessary to remark that inconsistencies bothered logicians from the time of Aristotle. However, the first logical systems treating contradictions appeared only in the 20th century. At first, they had the form of multivalued logics developed by Vasil'év and Łukasiewicz. Then the first relevant logics were built by Orlov. However, their work did not make any impact at the time and the first logician to have developed formal paraconsistent logic was a student of Łukasiewicz, Jaśkowski, [14]. Starting from this time, a diversity of different paraconsistent logics, including fuzzy logics, multivalued logics and relevant logics, has been elaborated (cf., for example, Routley, *et al*,[28]).

The perspective-bound character of information and information processing often results in natural inconsistency coming from different perspectives or from a faulty perception or from faulty information processing, such as processing on the basis of incomplete knowledge from a single perspective. As a result, now many understand that contradiction handling is one of the central problems in AI. Inconsistent knowledge/belief systems exist in many areas of AI, such as distributed knowledge base and databases, defeasible reasoning, dynamic expert systems, merging ontologies, ontology evolution, knowledge transition from one formalism to another, and belief revision.

There are three basic approaches to dealing with inconsistency. The first one is aimed at restoring consistency of an inconsistent knowledge system, e.g., a database (Rescher and Manor, [26]). Another approach is to tolerate inconsistency (Bertossi, *et al*, [5]) by including an inconsistent knowledge system into a paraconsistent or fuzzy logic (Priest, *et al*, [25]; Ross, [27]) and using this logic for inference in the given knowledge system. The third way is based on implicit or explicit utilization of logical varieties, quasi-varieties and prevarieties.

In comparison with non-monotonic logics, which form the base for the first approach, logical varieties, quasi-varieties and prevarieties provide tools for preserving all points of view, approaches and positions even when some of them taken together lead to contradiction. Due to their flexibility, logical varieties, quasi-varieties and prevarieties allow treating any form of logical contradictions in a rigorous and consistent way.

Paraconsistent logics, which form the base for the second approach, are inferentially *weaker* than classical logic; that is, they deem *fewer* inferences valid. Thus, in comparison with paraconsistent logics, logical varieties, quasi-varieties and prevarieties allow utilization of sufficiently powerful means of logical inference, for example, deductive rules of the classical predicate calculus. Besides, Weinzierl [31] explains why paraconsistent reasoning is not acceptable for many real-life scenarios and other approaches are necessary. In addition, paraconsistent logics attempts to deal with contradictions in a discriminating way, while logical varieties, quasi-varieties and prevarieties treat contradictions and other inconsistencies by a separation technique.

Although conventional logical systems based on logical calculi have been successfully used in mathematics and beyond, they have definite limitations that often restrict their applications. For instance, the principal condition for any logical calculus is its consistency. At the same time, knowledge about large object domains (in science or in practice) is essentially inconsistent (Burgin, [7]; de Vey Mestdagh, [19]; Nguen, [23]). From this perspective, Partridge and Wilks ([24]) write, "because of privacy and discretionary concerns, different knowledge bases will contain different perspectives and conflicting beliefs. Thus, all the knowledge bases of a distributed AI system taken together will be perpetually inconsistent." Consequently, when conventional logic is used for formalization, it is possible to represent only small fragments of the object domain. Otherwise, contradictions appear.

To eliminate these limitations in a logically correct way, logical prevarieties and varieties were introduced (Burgin, [7]). Logical varieties represent the natural development of logical calculi, being more advanced systems of logic, and thus, they show the direction in which mathematical logic will inevitably go. Including logical calculi as the simplest case, logical varieties and related systems offer several advantages over conventional logic:

1. Logical varieties, prevarieties and quasi-varieties give an exact and rigorous structure to deal with all kinds of inconsistencies.
2. Logical varieties allow modeling/realization of all other approaches to inconsistent knowledge. For instance, it is possible to use any kind of paraconsistent logics as components of logical varieties. In (Burgin, [7]), it is demonstrated how logical varieties realize non-monotonic inference.
3. Theoretical results on logical varieties provide means for more efficient application of logical methods to problems in different areas (e.g. the application of the Logic of Reasonable Inferences (a logical variety) to represent and process contradicting opinions in the legal domain as described below).
4. Logical varieties allow partitioning of an inconsistent knowledge system into consistent parts and to use powerful tools of classical logic for reasoning.
5. Logical varieties allow utilization of different kinds of logic (multifunctionality) in the same knowledge system. For instance, it is possible to use a combination of the classical predicate calculus and non monotonic calculus to represent two perspectives one of which is based on complete knowledge and the other on incomplete knowledge
6. Logical varieties allow separation of different parts in a knowledge system and working with them separately.
7. Logical varieties provide means to reflect change of beliefs, knowledge and opinions without loss of previously existed beliefs, knowledge and opinions even in the case when new beliefs, knowledge and opinions contradict to what was before. In (Burgin, [8]), they are applied to temporal databases.

These qualities of logic varieties are especially important for normative, in particular, legal, knowledge because this knowledge consists of a collection of formalized systems, a collection of adopted laws, a collection of existing traditions and precedents, and a collection of people's opinions. In addition, in the process of functioning, normative (legal) knowledge involves a variety of situational knowledge, beliefs and opinions. To analyze and use this diversity, it is necessary to have a flexible system that allows one to make sense of all different approaches without discarding them in an attempt to build a unique consistent system. To formalize these characteristics of normative knowledge, a form of a logical variety, the Logic of Reasonable Inferences (LRI) was developed (de Vey Mestdagh [18]). The LRI was subsequently used as specification for the implementation of a knowledge based system shell, Argumentator (de Vey Mestdagh [19]). This shell has consequently been used to acquire and represent legal knowledge. The resulting legal knowledge based system has been successfully used to test the empirical validity of the theory about legal reasoning and decision making modeled by the LRI (de Vey Mestdagh [19]).

It is interesting that several other systems used for inconsistency resolution, e.g., Multi-Context Systems (Weinzierl [31]), are also logical varieties and prevarieties. For instance, bridge rules used in Multi-Context Systems for non-monotonic information exchange are functions that glue together components of a logical variety or prevariety (cf. Definition 2.1).

In section 2 we define the concepts of quasi varieties, prevarieties and varieties formally. In section 3 we describe the immersion of logical variety into a logical calculus. The mathematical results presented in this section are new. In section 4 we describe the Logic of reasonable Inferences as a form of logical variety and its use to represent inconsistent legal knowledge.

2 Logical Quasi-Varieties, Prevarieties, and Varieties

There are different types and kinds of logical varieties and prevarieties: Deductive or syntactic varieties and prevarieties, Functional or semantic varieties and prevarieties and Model or pragmatic varieties and prevarieties. Semantic logical varieties and prevarieties are formed by separating those parts that represent definite semantic units. In contrast to syntactic and semantic varieties, model varieties are essentially formal structures.

Syntactic varieties, quasi-varieties and prevarieties are built from logical calculi as buildings are built from blocks. That is why, we, at first remind the concept of a logical calculus.

Let us consider a logical language L and an inference language R.

Definition 2.1. A *syntactic* or *deductive logical calculus*, usually called *logical calculus*, is a triad (a named set) of the form $\boldsymbol{C} = (A, H, T)$ where $H \subseteq R$ and $A, T \subseteq L$, A is the set of axioms, H consists of inference rules (rules of deduction) by which from axioms the theorems of the calculus are deduced, and the set of theorems T is obtained by applying algorithms/procedures/rules from H to elements from A.

Let $\mathbf{K}$ be some class of syntactic logical calculi, R be a set of inference rules, and $\mathbf{F}$ be a class of partial mappings from L to L.

Definition 2.2. A triad $\mathbf{M} = (A, H, M)$, where A and M are sets of expressions that belong to L (A consists of axioms and M consists of theorems) and H is a set of inference rules, which belong to the set R, is called:

(1) a *projective syntactic* (**K**,**F**)-*quasi-prevariety* if there exists a set of logical calculi $\boldsymbol{C}_i = (A_i, H_i, T_i)$ from **K** and a system of mappings $f_i : A_i \to L$ and $g_i : M_i \to L$ ($i \in I$) from **F** in which A_i consists of axioms and M_i consists of some (not necessarily all) theorems of the logical calculus $\boldsymbol{C}_i$, and for which the equalities $A = \bigcup_{i\in I} f_i(A_i)$, $H = \bigcup_{i\in I} H_i$ and $M = \bigcup_{i\in I} g_i(M_i)$ are valid (it is possible that $\boldsymbol{C}_i = \boldsymbol{C}_j$ for some $i \neq j$).

(2) a *syntactic* **K**-*quasi-prevariety* if it is a projective syntactic (**K**,**F**)- quasi-prevariety where all mappings f_i and g_i that define **M** are inclusions, i.e., $A = \bigcup_{i\in I} A_i$ and $M = \bigcup_{i\in I} M_i$;

(3) a *projective syntactic* (**K**,**F**)-*quasi-variety* with the depth k if it is a projective syntactic (**K**,**F**)-quasi-prevariety and for any $i_1, i_2, i_3, \dots, i_k \in I$ either the intersections $\bigcap_{j=1}^{k} f_{ij}(A_{ij})$ and $\bigcap_{j=1}^{k} g_{ij}(T_{ij})$ are empty or there exists a calculus $\boldsymbol{C} = (A, H, T)$ from **K** and projections $f: A \to \bigcap_{j=1}^{k} f_{ij}(A_{ij})$ and $g: N \to \bigcap_{j=1}^{k} g_{ij}(M_{ij})$ from **F** where $N \subseteq T$;

(4) a *syntactic* **K**-*quasi-variety* with the depth k if it is a projective syntactic (**K**,**F**)-quasi-variety with depth k in which all mappings f_i and g_i that define **M** are bijections on the sets A_i and M_i, correspondingly.

(5) a (*full*) *projective syntactic* (**K**,**F**)-*quasi-variety* if for any $k > 0$, it is a projective syntactic (**K**,**F**)-*quasi-variety* with the depth k;

(6) a (*full*) *syntactic* **K**-*quasi-variety* if for any $k > 0$, it is a **K**-quasi-variety with the depth k;

(7) a *projective syntactic* (**K**,**F**)-prevariety if it is a projective syntactic (**K**,**F**)-quasi-prevariety in which $M_i = T_i$ for all $i \in I$;

(8) a *syntactic* **K**-*prevariety* if it is a syntactic (**K**,**F**)-quasi-prevariety in which $M_i = T_i$ for all $i \in I$;

(9) a *projective syntactic* (**K**,**F**)-*variety* with the depth k if it is a projective syntactic (**K**,**F**)-quasi-prevariety in which $M_i = T_i$ for all $i \in I$;

(10) a *syntactic* **K**-*variety* with the depth k if it is a projective syntactic (**K**,**F**)-quasi-variety with depth k in which $M_i = T_i$ for all $i \in I$;

(11) a (*full*) *projective syntactic* (**K**,**F**)-*variety* if for any $k > 0$, it is a projective syntactic (**K**,**F**)-variety with the depth k;

(12) a (*full*) *syntactic* **K**-*variety* if for any $k > 0$, it is a **K**-variety with the depth k.

We see that the collection of mappings f_i and g_i makes a unified system called a prevariety or quasi-prevariety out of separate logical calculi $\boldsymbol{C}_i$, while the collection of the intersections $\bigcap_{j=1}^{k} f_{ij}(A_{ij})$ and $\bigcap_{j=1}^{k} g_{ij}(T_{ij})$ makes a unified system called a variety out of separate logical calculi $\boldsymbol{C}_i$. For instance, mappings f_i and g_i allow one to establish a correspondence between norms/laws that were used in one country during different periods of time or between norms/laws used in different countries.

The main goal of syntactic logical varieties is in presenting sets of formulas as a structured logical system using logical calculi, which have means for inference and other logical operations. Semantically, it allows one to describe a domain of interest, e.g., a database, knowledge of an individual or the text of a novel, by a syntactic logical variety dividing the domain in parts that allow representation by calculi.

In comparison with varieties and prevarietis, logical quasi-varieties and quasi-prevarieties are not necessarily closed under logical inference. This trait allows better flexibility in knowledge representation.

Definition 2.4. The calculi $\boldsymbol{C}_i$ used in the formation of the prevariety (variety) **M** are called *components* of **M**.

An example of a logical variety is a distributed database or knowledge base, each component of which consists of consistent knowledge/data. Then components of this knowledge/database are naturally represented by components of a logical variety. Besides, in one knowledge base different object domains may be represented. In these domains some object may have properties that contradict properties of an object from another domain. As an example let us consider a knowledge base containing mathematical information. Suppose that this information concerns some large mathematical field like algebra or even its part - theory of groups. Mathematical logics are frequently considered to be the basis of mathematics and logical calculi are viewed as precise models and formalizations of real mathematical theories. But the theory of groups does not coincide with elementary (logical) theory of groups that is a deductive calculus. The field that is called in mathematics "the Theory of Groups" contains various subtheories (Hall, [13]).

In the theory of groups, such mathematical objects as finite and torsion-free groups are studied. In any finite group, the formula $\forall x\ \exists n\ (\ x^n = e)$ is valid where e is the identity element. At the same time, in torsion-free groups another formula $\forall x \forall n\ \rceil (x^n = e)$ is true. Thus, if theory of groups with its subtheories, such as the theory of finite groups and theory of torsion-free groups, is represented as a single calculus, then both these formulae produce a contradiction. At the same time, a relevant logical variety in which subtheories are represented by its components provides means for consistent representation of the theory of groups.

Inference in a logical variety **M** is restricted to inference in its components because at each step of inference, it is permissible to use only rules from one set H_i applying these rules only to elements from the set T_i . This allows one to better model non-monotonicity of human thinking.

Indeed, the main difference between monotonic and non-monotonic reasoning arises from the different kinds of knowledge used in the process of inference. For instance, in the case of non-monotonic reasoning an inference rule of the following type can be used: "*A* is true if *B* cannot be proved", i.e. to prove *A* the system relies on its ignorance of *B*. The statement *B* is not included in the system of initial axioms. That is why by the given above rule of inference, the statement A becomes true in the intellectual system. However, it is possible that *B* becomes proved at some stage of the inference. So in this situation, the intellectual system must invalidate *A* and even more - to revise each piece of knowledge depending on *A*. In this way the monotonic property of the consequence relation is violated. Usually, the statement *A* is excluded and the knowledge/belief revision takes place. Logical varieties allow not to eliminate knowledge/beliefs in the process of revision but to build a new component from which all knowledge/beliefs that contradict *B* are eliminated. In such a way, all previously obtained knowledge/beliefs are preserved.

Although any logical calculus is a logical variety, this particular case does not give anything new in logic because logical calculi already exist in logic. A non-trivial example of logical varieties is given by *many-sorted logics* (Meinke and Tucker, [17]; Abadi, *et al*, [1]). In these logics, the variables range over different domains. Consequently, logical variables are "typed" as variables in many computer programming languages. Many-sorted logics allow one not to work with the domain of discourse as a homogeneous collection of objects, but to partition this domain into several parts with various functions and relations connecting them. In this case, these parts being formalized form a model variety, while the system of logics that describe these parts forms a syntactic variety.

For instance, semantics of computer languages employ different types (domains) of data, such as the integers and the real numbers. Each domain has its own equality, relations, identities, and arithmetical operations. The logical language that describes the union of these domains will have two sorts of variables, real variables and integer variables. The meaning of a quantifier would be determined by the type of the variable it binds. The corresponding logic will be a logical variety built of two calculi. Intersection of these calculi will include such formulas as the commutative law

$$x + y = y + x$$

and the associative law

$$x + (y + z) = (x + y) + z$$

Any big mathematical theory, such as group theory, ring theory or topology (theory of topological spaces), forms a syntactic logical variety. For instance, group theory as the set of all consistent formulas in the formal language of group theory contains many subtheories, such as the theory of finite groups, the theory of toirsion-free groups, the theory of abelian groups, the theory of nilpotent groups, and the theory of prime groups. In the theory of finite groups, the formula that says that any element of a finite group, there is a number *n* such that taken to the power *n*, this element is equal to the unit of the group, is valid, while in the theory of toirsion-free groups, the negation of this formula is valid. This negation says that any element of a finite group, there is no number *n* such that taken to the power *n*, this element is equal to the unit of the group. Thus, group theory, as it is understood in mathematics and not the formal theory of groups in the logical sense, is not a calculus because it is incompatiple, but it is a logical variety, which has weight > 1.

We have a similar situation in the ring theory, which contains the theory of commutative rings and the theory of Lie rings. These two subtheories contain formalas such that one of them is the nagation of the other.

A similar situation also exists in other disciplines, for example, in archeology (VanPool and VanPool, [30]).

One more example of naturally formed logical varieties is the technique *Chunk and Permeate* built by Brown and Priest [6]. This technique suggests to begin reasoning from inconsistent premisses and proceeds by separating the assumptions into consistent theories (called by the authors *chunks*). These chunks are components of the logical variety shaped by them. After this, appropriate consequences arederived in one component (chunk). Then those consequences are transfered to a different component (chunk) for further consequences to be derived. This is exactly the way how logical varieties are used to realize and model nonmonotonic reasoning (Burgin, [7]). Brown and Priest suggest that Newton's original reasoning in taking derivates in the calculus, was of this form.

An interesting type of logical varieties was developed in artificial intelligence and large knowledge bases. As Amir and McIlraith write ([3], [15], [16]), there is growing interest in building large knowledge bases of everyday knowledge about the world, comprising tens or hundreds of thousands of assertions. However working with large knowledge bases, general-purpose reasoning engines tend to suffer from combinatorial explosion when they answer user's queries. A promising approach to grappling with this complexity is to structure the content into multiple domain- or task-specific partitions. These partitions generate a logical variety comprising the knowledge base content. For instance, a first-order predicate theory or a propositional theory is partitioned into tightly coupled subtheories according to the language of the axioms in the theory. This partitioning induces a graphical representation where a node represents a particular partition or subtheory and an arc represents the shared language between subtheories.

The technology of content partitioning allows reasoning engines to improve the efficiency of theorem proving in large knowledge bases by identifying and exploiting the implicit structure of the knowledge (Amir and McIlraith, [3]; McIlraith and Amir, [16]; MacCartney, *et al*, [15]),. The basic approach is to convert a graphical representation of the problem into a tree-structured representation, where each node in the tree represents a tightly-connected subproblem, and the arcs represent the loose coupling between subproblems. To maximize the effectiveness of partition-based reasoning, the coupling between partitions is minimized, information being passed between nodes is reduced, and local inference within each partition is also minimized.

Additional advantage of portioning is a possibility to reason effectively with multiple knowledge bases that have overlap in content (Amir and McIlraith, [3]).

The tools and methodology of content partitioning and thus, implicitly of logical varieties are applied for the design of logical theories describing the domain of robot motion and interaction (Amir, [2]).

Concepts of logical varieties and prevarieties provide further formalization for local logics of Barwise and Seligman ([4]), many-worlds model of quantum reality of Everett (Everett, [11]; DeWitt, [10]), and pluralistic quantum field theory of Smolin related to the many-worlds theory (Smolin, [29]).

3 Compatibility in Logical Varieties

An important problem of logic is to combine logics including all of them into one calculus. Gabbay ([12]) writes that "the problem of combining logics and systems is central for modern logic, both pure and applied. The need to combine logics starts both from applications and from within logic itself as a discipline. As logic is being used more and more to formalize field problems in philosophy, language, artificial intelligence, logic programming, and computer science, the kind of logics required becomes more and more complex." Logical varieties and prevarieties give a relevant context for solving this problem because it is possible to treat any system of logics as a logical variety or prevariety. Here we consider only deductive varieties.

Let us take a class **K** of logical calculi and a deductive **K**-variety $\mathbf{M} = \{C_i\,;\, i \in I\}$.

Definition 3.1. A logical variety **M** is called:

1) *Discrete* if its components are disjoint;
2) *Classical* if all its components are classical deductive calculi;
3) *Connected* if any two of its components have a non-void intersection;
4) *Compatible* if it is a subset of a consistent calculus.
5) *Provably compatible* if it is possible to prove by classical methods that it is a subset of a consistent calculus.

Definition 3.2. A set of components $\{C_i\,;\, i \in J\}$ of **M** is called (provably) compatible if the subvariety of **M** generated by these components is (provably) compatible

Lemma 1.

a) For any deductive variety M, there is a discrete deductive variety DM such that their upper levels are equal, i.e., T(M) = T(DM).
b) The discrete counterpart DM of a variety M preserves consistency.

Proposition 1. If **M** is compatible (**K**-compatible), then **DM** is compatible (**K**-compatible).

Theorem 1. For any number $n > 1$ there is a classical connected deductive logical variety M with n components such that any n - 1 components of **M** are provably compatible, **M** is compatible, but it is not provably compatible.

Remark 1. The condition that the variety is classical is essential. However, for limit ordinals similar results are not valid.

Theorem 2. For any classical deductive logical variety M, if any finite subset of components of **M** is compatible, then **M** is compatible.

Corollary 1. For any classical deductive logical variety **M** with a countable number of components, if any finite subset of components of **M** is compatible, then **M** is compatible.

The compatibility of a logical variety means that it is possible to immerse all components of this variety into one calculus from the class **K** of logical calculi. Thus, the obtained results show that the possibility of logic system immersion into one calculus is undecidable for a finite number of logics (Theorem 1), while for an infinite number of logics, the decidability problem is reducible to the finite case (Theorem 2) and thus, is undecidable in general.

4 Knowledge Representation and Logical Inference in the Legal Domain

Although each domain of knowledge is more or less affected by the problem of inconsistent knowledge, this issue is particularly intense in the domain of legal knowledge, since it consists of the rules and procedures used to describe and solve legal conflicts, which presupposes contradictory and hence inconsistent perspectives. Human processors of legal knowledge follow formal and informal problem-solving methods in order to reduce the number of legal perspectives and eventually to decide, temporally and within a specific context, on a common perspective. The formal methods are based on universal properties of formally valid legal argument. The informal methods are based on legal heuristics consisting in tentative legal decision principles. The first category can be formalized by logic because it applies peremptorily to all legal perspectives. The second category cannot be fully formalized by logic because, although it is commonly applicable, it can always be refuted by a contradictory decision principle and even by the mere existence of an underlying contradictory argument.

Legal opinions range from informal to formal. On the informal side we find moral principles, social scripts, protocols, (technical) instructions, rules of thumb, rules of play etc. On the formal side we find legislation, legal principles, jurisprudence, policy rules etc. Legal opinions can be of a general (uninstantiated) and of a specific (instantiated) character. Legal procedures consist of (1) procedures to list all the normative opinions about a given situation that can be inferred from the given situation combined with the set of pre-existing normative opinions of the parties concerned and (2) procedures to reduce the number of normative opinions about the given situation to a (local and temporal) common opinion for (not necessarily *of*) the parties concerned. Both procedures involve *legal reasoning*. The second procedure also involves *legal decision-making*. Legal reasoning in the first class of procedures is concerned with the inference of normative opinions about the given situation (the object level). Legal reasoning in the second class of procedures is concerned with the inference of normative opinions about the reduction of normative opinions (the metalevel, e.g. “the judge is obliged to decide for a legally valid opinion”). However, the decision principles applied at this level also represent opinions, so there is no exhaustive or non-contradictory set of decisive opinions at this level either.

These properties of legal knowledge should be taken into consideration in order to be able to develop a tenable computational model of the application of legal knowledge. To achieve this aim logical varieties are used as a foundation for the Logic of Reasonable Inferences (LRI), which provides means for legal knowledge representation and models legal reasoning, using the language of classical first-order predicate calculus, as this language seems powerful enough to express legal rules and factual situations without losing any relevant information (de Vey Mestdagh, [18]). Being a logical variety the LRI allows for the representation of internally consistent, but mutually inconsistent, alternative opinions in one system, thus preserving all legal perspectives. Each of these opinions (called a *position* below) from the logical variety of LRI is represented by a single classical calculus.

LRI uses the language $\mathcal{L}$ of the first order predicate calculus and is constructed as a logical variety **V** in $\mathcal{L}$. Expressions from $\mathcal{L}$ are called beliefs and components of **V** are called positions or convictions. The common part of all components (positions) is a set A of well formed formulas (wwfs) of $\mathcal{L}$ which are called axioms of **V**.

Definition 4.1. A *reasonable base* in $\mathcal{L}$ is a pair Δ defined as $\Delta = (A, H)$ where H is sets of *wffs* in $\mathcal{L}$, which are called (tentative) *assumptions* (*hypotheses* or *beliefs*).

The *assumptions* can model the rules of law that may or may not be applied in a given factual situation to derive a conclusion and contain all normative or subjective classifications of the factual situation. The *axioms* are intended to be valid in every justification and thus, restrict the number of possible justifications. These axioms represent the ascertained facts and previously ascertained conclusions (the permanent database in any implementation).

Definition 4.2. A *position* (or *conviction*) ϕ within a domain of rules $\Delta = (A, H)$ is a consistent set of wffs defined as $\phi = A \cup H'$ where $H' \subseteq H$.

Definition 4.3. A position (conviction) ϕ within a domain of rules $\Delta = (A, H)$ is called *logically closed* if it is a predicate calculus.

Thus, a position is a set of rules taken from the domain of rules and represents a conviction of an individual or a group of people. Note that all positions should at least contain all axioms of the domain of rules and each position is consistent by definition. This shows that all logically closed positions form a logical variety in which all intersection are equal to A.

Let Δ be a domain of rules. Define a new semantic derivability-relation $\vDash_r$ as $\Delta \vDash_r \varphi$ *iff* there exists a position ϕ within Δ which satisfies $\phi \vDash \varphi$ where $\vDash$ is the normal predicate calculus semantic derivability relation. If $\Delta \vDash_r \varphi$ holds, φ is said to be a *reasonable inference* from the domain of rules Δ. This the exact form of inference in logical varieties.

We can paraphrase this definition by stating that a *wff* can reasonably be inferred from an inconsistent set of *wff* iff it is derivable (in the normal predicate calculus sense) from a consistent subset of this set which contains at least the axioms. Note that if a domain of rules $\Delta = (A, H)$ is consistent (i.e. if $A \cup H = \Gamma$ is consistent), then $\Delta \vDash_r \varphi \Leftrightarrow \Gamma \vDash \varphi$ behaves exactly like $\vDash$ when applied to consistent theories.

Definition 4.4. A *justification* for a conclusion φ derived from a domain of rules Δ is a minimal position (with respect to set-inclusion) J within Δ such that $J \vDash \varphi$.

This definition is based on the intuitive concept of a set of rules and statements about the factual situation used to draw the conclusion. Note that a justification needs not be unique but it is always consistent, thus, satisfying our constraints.

Definition 4.5. A *context* in Δ is the union of n simultaneously derived conclusions ψ_i and their justifications J_i derived from Δ, i.e. a context is the set of tuples $\{ (\psi_i , J_i) \mid 1 \leq i \leq n\}$.

Definition 4.6. A context in Δ is called *consistent* if the justifications J_i derived from Δ satisfy the following condition:

$$\text{The union } \bigcup_{i=1}^{n} J_i \text{ is consistent}$$

This guarantees that simultaneously derived conclusions are not based on mutually inconsistent positions.

Definition 4.7. A *reasonable theory* Th Δ with a base $\Delta = (A, H)$ is the set of all wffs deducible in LRI from Δ, i.e., Th $\Delta = \{ \varphi \in \mathcal{L}$; there is a position $\phi \in \Delta$, such that $\phi \vDash \varphi\}$.

The construction of a reasonable theory shows that any reasonable theory Th Δ is a deductive logical variety V that has the form

$$V = \{C_i \,;\, i \in I \text{ and there is a position } \phi \text{, such that } C_i = (\phi, d, T_\phi) \}$$

where d is the set of all deduction rules of classical the first-order predicate calculus and T_ϕ is the set of all formulas deducible from the position ϕ by rules from d. V is a first order predicate variety.

An earlier version of the LRI (de Vey Mestdagh, [18]) has been used to specify a knowledge based system shell, Argumentator (de Vey Mestdagh, [19]). This shell has consequently been used to acquire and represent legal knowledge. The resulting legal knowledge based systems have been successfully used to test the empirical validity of the theory about legal reasoning and decision making modeled by the LRI (de Vey Mestdagh, [19]).

5 Conclusion

Logical varieties, quasi-varieties and prevarieties eliminate certain limitations of conventional logical systems based on logical calculi. In comparison with paraconsistent logics, they allow utilization of sufficiently powerful means of logical inference, for example, deductive rules of the classical predicate calculus. In comparison with non-monotonic logics, logical varieties, quasi-varieties and prevarieties provide tools for preserving all points of view, approaches and positions even when some of them taken together lead to contradiction. Due to their flexibility, logical varieties, quasi-varieties and prevarieties allow treating any form of logical contradictions in a rigorous and consistent way.

The mathematical results of this paper explore the problem of applicability of the classical logic to knowledge systems. Indeed, it is possible to represent any logical system expressed in a logical language as a classical logical variety, prevariety or, at least, quasi-variety M. Then this system is embeddable in a classical logic if and only if all components of this variety, prevariety or quasi-variety M are compatible. However, our results show that this important problem is undecidable in a general case.

The LRI is a form of a logical variety that is aimed at building legal decision support systems and legal expert systems, which can be used by judges, jurors, lawyers, detectives, and attorneys. The legal system is characterized by different often contradictory opinions and the need to decide temporally within a certain legal context but to preserve the different points of view. The results within the legal domain can of course be generalized to any domain with similar characteristics. The LRI has been successfully implemented and applied in a legal knowledge based system to accommodate for these characteristics and has been empirically verified, by comparing the decisions made by the system with decisions made by human decision makers.

Generally, Legal Decision Support System based on logical varieties can help jurors and judges to find if witnesses are consistent in their depositions, if statements of different witnesses are compatible, if versions of persecution and defenders are consistent, and which of these versions is more grounded. It can help a judge to find what laws and/or what precedence cases are more compatible with the given case. Finally it can help detectives and attorneys to find which conjectures are compatible with evidence and with one another and which of these conjectures are more grounded.

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