A Unified Thruster Configuration for Pure Translation and Pure Rotation with Automatic Center-of-Mass Compensation

1. Goal

We seek a *single, fixed thruster configuration* mounted on a rigid body that can:

(a) Translate in any direction with **zero torque**,

(b) Generate torque about any axis with **zero net force**,

(c) Continue to achieve (a–b) even when the center of mass (CoM) **moves inside the vehicle**.

The solution is to use a combination of:

- 6 **radial thrusters** (pure force, zero torque),
- 6 **tangential thrusters** arranged in pairs (pure torque, zero net force).

At runtime, the CoM motion is compensated mathematically by updating the moment arms for each thruster.

2. Thruster Geometry in Body Frame

All thrusters are mounted in a **body-fixed** frame B. Their positions and directions never change.

Let $L > 0$ be a fixed distance from the origin of the body frame.

2.1 Radial Thrusters (T1–T6): Pure Force, No Torque

$$
\begin{aligned}
\text{T1: } r_1^B &= (L, 0, 0), & a_1^B &= (1, 0, 0),\\
\text{T2: } r_2^B &= (-L, 0, 0), & a_2^B &= (-1, 0, 0),\\
\text{T3: } r_3^B &= (0, L, 0), & a_3^B &= (0, 1, 0),\\
\text{T4: } r_4^B &= (0, -L, 0), & a_4^B &= (0, -1, 0),\\
\text{T5: } r_5^B &= (0, 0, L), & a_5^B &= (0, 0, 1),\\
\text{T6: } r_6^B &= (0, 0, -L), & a_6^B &= (0, 0, -1).
\end{aligned}
$$

These thrusters produce pure forces because $r_i^B \parallel a_i^B$.

2.2 Torque Thrusters (T7–T12): Pure Torque, Zero Net Force

Three orthogonal torque pairs:

$$
\begin{aligned}
\text{T7: } r_7^B &= (0, L, 0), & a_7^B &= (0, 0, 1), \\
\text{T8: } r_8^B &= (0, -L, 0), & a_8^B &= (0, 0, -1), \\
\text{T9: } r_9^B &= (0, 0, L), & a_9^B &= (1, 0, 0), \\
\text{T10: } r_{10}^B &= (0, 0, -L), & a_{10}^B &= (-1, 0, 0), \\
\text{T11: } r_{11}^B &= (L, 0, 0), & a_{11}^B &= (0, 1, 0), \\
\text{T12: } r_{12}^B &= (-L, 0, 0), & a_{12}^B &= (0, -1, 0).
\end{aligned}
$$

Firing each pair with equal magnitude yields zero net force but nonzero torque.

3. CoM-Dependent Moment Arms

Let $c^B \in \mathbb{R}^3$ be the **current center of mass in the body frame**. This point can move as fuel is consumed or loads shift.

The moment arm for thruster i relative to the CoM is:

$$\tilde{r}_i = r_i^B - c^B.$$

4. Force and Torque Equations

Let $T_i \geq 0$ be the thrust magnitude of thruster i.

Force from thruster i:

$$f_i = T_i \, a_i^B.$$

Total force:

$$F = \sum_{i=1}^{12} T_i \, a_i^B.$$

Torque from thruster i:

$$\tau_i = \tilde{r}_i \times (T_i \, a_i^B).$$

Total torque:

$$\tau = \sum_{i=1}^{12} T_i \left(\tilde{r}_i \times a_i^B \right).$$

5. Wrench Matrix Representation

Define

$$A = [a_1^B \; \ldots \; a_{12}^B] \in \mathbb{R}^{3\times 12},$$

$$B(c^B) = [\tilde{r}_1 \times a_1^B \; \ldots \; \tilde{r}_{12} \times a_{12}^B] \in \mathbb{R}^{3\times 12}.$$

Stack thrust vector:

$$T = (T_1, \ldots, T_{12})^\top.$$

Then:

$$\begin{bmatrix} F \\ \tau \end{bmatrix} = W(c^B)\,T, \qquad W(c^B) = \begin{bmatrix} A \\ B(c^B) \end{bmatrix}.$$

If $W(c^B)$ has rank 6, we can generate arbitrary force+torque commands.

6. Mode 1: Pure Translation (Zero Torque)

Given a desired force F_{des} and zero torque:

$$\tau_{\text{des}} = 0,$$

solve:

$$W(c^B)\,T = \begin{bmatrix} F_{\text{des}} \\ 0 \end{bmatrix}, \qquad 0 \le T_i \le T_{\text{max},i}.$$

This yields translation in any direction with zero torque, even when CoM shifts.

7. Mode 2: Pure Rotation (Zero Net Force)

Given desired torque τ_{des} and zero force:

$$F_{\text{des}} = 0,$$

solve:

$$W(c^B)\,T = \begin{bmatrix} 0 \\ \tau_{\text{des}} \end{bmatrix}, \qquad 0 \le T_i \le T_{\text{max},i}.$$

This produces rotation about any axis with zero translation for the current CoM.

8. CoM Adaptation

When mass shifts:

(1) Measure or estimate new CoM c^B.

(2) Recompute moment arms:

$$\tilde{r}_i = r_i^B - c^B.$$

(3) Recompute torque matrix $B(c^B)$.

(4) Resolve the force/torque allocation problem for the new CoM.

No hardware changes are required.

9. Summary

This single 12-thruster configuration:

- Provides full 3D translation: any $F = (F_x, F_y, F_z)$ with 0 torque,
- Provides full 3D rotation: any $\tau = (\tau_x, \tau_y, \tau_z)$ with 0 force,
- Works for any CoM location inside the body by updating the moment arms,
- Enables general 6-DOF control by solving $W(c^B)T = (F, \tau)$.