Direct-Bounded Simplex (No Substitution): Full Tableau with LB/UB Status

Problem

Maximize

$$z = 3x_1 - x_2$$

subject to

$$\begin{aligned} x_1 + 2x_2 + s_1 &= 10, \\ 2x_1 + x_2 + s_2 &= 12, \end{aligned} \qquad s_1, s_2 \geq 0,$$

with bounds

$$\boxed{-3 \leq x_1 \leq 6}, \qquad \boxed{0 \leq x_2 \leq 5}.$$

We will keep x_1, x_2 directly with their bounds inside the simplex. At every step, each variable is either B (basic), LB (nonbasic at lower bound), or UB (nonbasic at upper bound).

Initial bound-feasible point and statuses

Choose $x_1 = -3$ (LB), $x_2 = 0$ (LB). Then

$$s_1 = 10 - (-3 + 0) = 13, \qquad s_2 = 12 - (-6 + 0) = 18.$$

Status summary: x_1 : LB(-3), x_2 : LB(0), s_1 : B, s_2 : B.

Initial tableau (basis $\{s_1, s_2\}$, columns $[x_1, x_2, s_1, s_2]$).

Basic	RHS	x_1	x_2	s_1	s_2
s_1	13	1	2	1	0
s_2	18	2	1	0	1
z	0	**+3**	**−1**	0	0

Reduced costs (maximization rule with bounds): $r_{x_1} = +3 > 0 \Rightarrow$ since x_1 is at LB, *increasing* x_1 is improving. $r_{x_2} = -1 < 0 \Rightarrow$ since x_2 is at LB, *increasing* x_2 would hurt.

Ratio test for $x_1 \uparrow$ (include bound distance)

Row limits: $13/1 = 13$, $18/2 = 9$. Upper-bound distance for x_1: $u_1 - x_1 = 6 - (-3) = 9$. Hence

$$\theta = \min(13,\ 9,\ \underline{9}) = 9,$$

a *tie* between row s_2 and the *upper bound* of x_1.

We can handle this tie in two equivalent ways:

2A. Preemptive slide (no pivot)

Move x_1 up by 9 directly to its upper bound: $x_1 \leftarrow 6$ (UB), update RHSs:

$$s_1 \leftarrow 13 - 1 \cdot 9 = 4, \qquad s_2 \leftarrow 18 - 2 \cdot 9 = 0.$$

Basis stays $\{s_1, s_2\}$. Status now:

$$x_1 : \mathsf{UB}(6), \quad x_2 : \mathsf{LB}(0), \quad s_1 : \mathsf{B}(4), \quad s_2 : \mathsf{B}(0).$$

Objective value becomes $z = 3 \cdot 6 - 0 = 18$. No reduced costs change (basis unchanged), and bound-status rules show no improving move remains (x_1 at UB would need $r_{x_1} < 0$ to decrease; here $r_{x_1} > 0$). Thus this already certifies optimality.

2B. Pivot at the tied row (canonical update)

Alternatively, perform the pivot on the tied row (s_2, x_1) to bring x_1 into the basis and land exactly at $x_1 = 6$. Divide row s_2 by 2 and eliminate x_1 elsewhere.

Basic	RHS	x_1	x_2	s_1	s_2
x_1	6	1	0.5	0	0.5
s_1	4	0	1.5	1	0.5
z	18	0	$\mathbf{-2.5}$	0	$\mathbf{-1.5}$

Current (basic) solution from this tableau:

$$x_1 = 6\ (\mathsf{B}{=}\mathsf{UB}), \quad x_2 = 0\ (\mathsf{LB}), \quad s_1 = 4,\ s_2 = 0, \quad z = 18.$$

Optimality with bounds: - x_2 is LB with $r_{x_2} = -2.5 < 0$: increasing x_2 would *decrease* z. - x_1 is at its UB; an improving move would require $r_{x_1} < 0$ to decrease x_1, but $r_{x_1} = 0$ in this canonical row. No improving move exists; hence optimal.

Conclusion

The direct-bounded simplex (without substitutions) reaches

$$\boxed{x_1 = 6, \quad x_2 = 0, \quad z^* = 18}$$

in a single step where the ratio test ties a constraint with the *upper bound* of the entering variable. You may record this either as a *preemptive slide to the bound* (no pivot) or as a *pivot* that lands at the same bound.

Sign/ratio rules used (maximization):

- If x_j is LB: it may *increase* only when $r_j > 0$.
- If x_j is UB: it may *decrease* only when $r_j < 0$.
- Ratio test includes the opposite bound of the entering variable: $\theta = \min\{$row limits in the allowed dir $x_j\}$ when increasing from LB; $\theta = \min\{$row limits, $x_j - l_j\}$ when decreasing from UB.
- If the minimum is a bound distance, you hit that bound: update values and status (LB/UB) with *no pivot* unless a row ties; a tied row pivot is also valid.