Bounded Preemptive Simplex with Priority Goals: Full Tableau Walkthrough

Problem

Decision variables with bounds:

$$0 \le x_1 \le 4, \qquad 0 \le x_2 \le 3.$$

Hard constraint:

$$2x_1 + 3x_2 + s_c = 12 \quad (s_c \ge 0).$$

Goal 1 (priority P_1): meet $x_1 + x_2 = 5$ with minimal *underachievement*

$$x_1 + x_2 + d_1^- - d_1^+ = 5, \quad \text{minimize } d_1^-.$$

Goal 2 (priority P_2): keep $2x_1 + x_2 \le 8$ with minimal *overachievement*

$$2x_1 + x_2 + d_2^- - d_2^+ = 8, \quad \text{minimize } d_2^+.$$

We begin at the bound-feasible point $x_1 = x_2 = 0$ so that

$$s_c = 12, \quad d_1^- = 5, \quad d_2^- = 8.$$

Columns are ordered as

$$[x_1,\ x_2,\ s_c,\ d_1^-,\ d_1^+,\ d_2^-,\ d_2^+].$$

Tableau 1: Initial

Basic	RHS	x_1	x_2	s_c	d_1^-	d_1^+	d_2^-	d_2^+
s_c	12	2	3	1	0	0	0	0
d_1^-	5	1	1	0	1	-1	0	0
d_2^-	8	2	1	0	0	0	1	-1
$P_1 = z_1$	0	-1	-1	0	0	1	0	0

Here, at the start, $r_{x_2} = -1 < 0$ so *increasing* x_2 (from its lower bound) reduces d_1^-.

Tableau 2: After preemptive slide $x_2 : 0 \to 3$ (no pivot)

Increase x_2 until a limit is reached. Bounds allow $x_2 \le 3$, so we slide directly to 3:

$$x_2 = 3, \qquad s_c = 3, \qquad d_1^- = 2, \qquad d_2^- = 5.$$

Basic	RHS	x_1	x_2	s_c	d_1^-	d_1^+	d_2^-	d_2^+
s_c	3	2	3	1	0	0	0	0
d_1^-	2	1	1	0	1	-1	0	0
d_2^-	5	2	1	0	0	0	1	-1
$P_1 = z_1$	-2	-1	-1	0	0	1	0	0

Tableau 3: After Pivot 1 (enter x_1, leave s_c)

A pivot on the (s_c, x_1) entry gives:

Basic	RHS	x_1	x_2	s_c	d_1^-	d_1^+	d_2^-	d_2^+
x_1	1.5	1	1.5	0.5	0	0	0	0
d_1^-	0.5	0	-0.5	-0.5	1	-1	0	0
d_2^-	2	0	-2	-1	0	0	1	-1
$P_1 = z_1$	-0.5	0	0.5	0.5	0	1	0	0

We have $x_1 = 1.5$, $x_2 = 3$, $d_1^- = 0.5$. To push $d_1^- \downarrow 0$, we must *decrease* x_2 off its upper bound (its reduced cost $+0.5$ for P_1 means that decreasing it will lower z_1).

Intermediate Tableau: Bound-Exchange Pivot (enter x_2 by decreasing)

We now pivot on the (d_1^-, x_2) entry (value -0.5) to bring x_2 into the basis and send d_1^- out:

Basic	RHS	x_1	x_2	s_c		d_1^-	d_1^+	d_2^-	d_2^+
x_1	3	1	0	0		-1	1	1	-1
x_2	2	0	1	0		2	-2	-1	1
d_2^-	0	0	0	1		-2	2	1	-1
$P_1 = z_1$	0	0	0	0	(locked)				

This tableau represents the point $(x_1, x_2) = (3, 2)$ with

$$d_1^- = 0, \quad d_1^+ = 0, \quad d_2^- = 0, \quad d_2^+ = 0, \quad s_c = 0.$$

Thus the goal of priority P_1 (minimizing d_1^-) has reached the best possible value of 0.

Final P_1-Optimal Tableau (one clean basis)

A convenient P_1-optimal basis is $\{x_1, x_2, s_c\}$:

Basic	RHS	x_1	x_2	s_c		d_1^-	d_1^+	d_2^-	d_2^+
x_1	3	1	0	0		-1	1	1	-1
x_2	2	0	1	0		2	-2	-1	1
s_c	0	0	0	1		-4	4	1	-1
$P_1 = z_1$	0	0	0	0	(locked)				

Since $d_1^- = 0$ is fixed (optimal for P_1), we check P_2:

$$2x_1 + x_2 = 2 \cdot 3 + 2 = 8 \quad \Rightarrow \quad d_2^+ = 0,$$

which is optimal subject to keeping $d_1^- = 0$.

Conclusion

The lexicographically optimal solution is

$$x_1 = 3, \qquad x_2 = 2, \qquad s_c = 0,$$

with all deviational variables zero. Thus both goals are satisfied at their best possible levels.