Bounded Preemptive Simplex with Preemptive (Priority) Goal Programming

Thruster Geometry

Two thrusters:

$$r_1 = (0,2,0), \quad a_1 = (100,0,0), \qquad r_2 = (0,-4,0), \quad a_2 = (100,0,0).$$

Force map $F = A\,t$ and torque map $T = B\,t$:

$$A = \begin{bmatrix} 100 & 100 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -200 & 400 \end{bmatrix}.$$

Thus, for $t = (t_1, t_2)^T$,

$$F = \begin{bmatrix} 100(t_1 + t_2) \\ 0 \\ 0 \end{bmatrix}, \quad T = \begin{bmatrix} 0 \\ 0 \\ -200\,t_1 + 400\,t_2 \end{bmatrix}.$$

Throttle bounds:

$$0 \le t_1 \le 1, \qquad 0 \le t_2 \le 1.$$

Stage P_1: Minimize Torque

We minimize the L1 norm of torque. Since only T_z appears, introduce deviational variables $\tau^+, \tau^- \ge 0$:

$$-200\,t_1 + 400\,t_2 + \tau^- - \tau^+ = 0, \qquad \min z_1 = \tau^+ + \tau^-.$$

We keep variables with bounds directly in the simplex via LB/UB status tags.

Initial P_1 Tableau

Make τ^+ basic:

$$\tau^+ = -200\,t_1 + 400\,t_2 + \tau^-.$$

Basic	RHS	t_1	t_2	τ^-	τ^+	Status
τ^+	0	-200	$+400$	$+1$	1	B
z_1	0	$\mathbf{-200}$	$\mathbf{+400}$	$\mathbf{+2}$	0	

Status at start:

$$t_1 : \ \mathsf{LB}\ (0 \le t_1 \le 1), \qquad t_2 : \ \mathsf{LB}\ (0 \le t_2 \le 1), \qquad \tau^- : \ \mathsf{LB}.$$

Since $r_{t_1} = -200 < 0$, t_1 may enter (increase) from LB for minimization.

Degenerate Pivot in P_1

Pivot on t_1 entering, τ^+ leaving:

$$t_1 = 2\,t_2 + \tfrac{1}{200}\tau^- - \tfrac{1}{200}\tau^+.$$

Basic	RHS	t_1	t_2	τ^-	τ^+	Status
t_1	0	1	$+2$	$+\frac{1}{200}$	$-\frac{1}{200}$	B
z_1	0	0	$\mathbf{0}$	$+\mathbf{1}$	$+\mathbf{1}$	

This means $z_1 = 0$ is achieved whenever $\tau^\pm = 0$ and

$$\boxed{t_1 = 2\,t_2}.$$

Thus $Z_1^\star = 0$ (exact torque cancellation). We *lock* this relation for P_2.

Stage P_2: Maximize Forward Force in $+x$

$$\max z_2 = 100(t_1 + t_2) \qquad \text{s.t. } t_1 = 2t_2,\ 0 \le t_1, t_2 \le 1.$$

Substitute $t_1 = 2t_2$:

$$z_2 = 300\,t_2.$$

Bounds require $2t_2 \le 1$ and $0 \le t_2 \le 1$, so $0 \le t_2 \le 0.5$.
Treat t_1 as basic; t_2 as nonbasic.

P_2 Tableau A (start)

Variable	Value	Status
t_2	0.0	LB ($[0, 0.5]$)
t_1	0.0	B ($t_1 = 2t_2$)
z_2		reduced cost of t_2 : $+\mathbf{300}$

Since $r_{t_2} > 0$ and t_2 is at LB, we increase t_2.

Bounded Ratio (Preemptive) in P_2

Equality $t_1 = 2t_2$ does not limit t_2, but $t_1 \le 1$ gives

$$2t_2 \le 1 \Rightarrow t_2 \le 0.5.$$

Opposite-bound distance: $0.5 - 0 = 0.5$.
Thus: *preemptive bound hit* (no pivot).

P_2 Tableau B (after UB hit)

Variable	Value	Status
t_2	0.5	UB ($[0, 0.5]$)
t_1	1.0	B ($t_1 = 2t_2$)
z_2	150	

Optimality

A nonbasic at UB can only improve a maximization by *decreasing* if $r_j < 0$. Here $r_{t_2} = +300 > 0$, so decreasing t_2 lowers z_2. No other candidates.

$$\boxed{\text{Optimal for } P_2 \text{ while } P_1 \text{ remains locked.}}$$

Final Solution

$$\boxed{t_1 = 1.0, \quad t_2 = 0.5, \quad F = (150, 0, 0), \quad T = (0, 0, 0).}$$