Bounded Preemptive Simplex with Priority Goals (Full Tableaux)

Geometry (updated):

$r_1 = (0, 2, 0), \quad a_1 = (-100, 0, 0), \qquad r_2 = (0, -4, 0), \quad a_2 = (-100, 0, 0), \qquad 0 \le t_1, t_2 \le 1.$

Force and torque maps:

$$A = \begin{bmatrix} -100 & -100 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 200 & -400 \end{bmatrix}.$$

Thus

$$F = \begin{bmatrix} -100(t_1 + t_2) \\ 0 \\ 0 \end{bmatrix}, \qquad T = \begin{bmatrix} 0 \\ 0 \\ 200\, t_1 - 400\, t_2 \end{bmatrix}.$$

Stage P_1: Minimize Torque (L1)

Introduce deviational variables $\tau^+, \tau^- \ge 0$:

$$200t_1 - 400t_2 + \tau^- - \tau^+ = 0, \qquad \min z_1 = \tau^+ + \tau^-.$$

P1 Tableau 0 (start)

Make τ^+ basic:

Basic	RHS	t_1	t_2	τ^-	τ^+	Status
τ^+	0	+200	−400	+1	1	B
z_1	0	**+200**	**−400**	**+2**	0	

Nonbasic variables at start:

$$t_1 : \ \mathsf{LB}, \qquad t_2 : \ \mathsf{LB}, \qquad \tau^- : \ \mathsf{LB}.$$

Minimization rule: - A nonbasic at LB can enter if $r_j < 0$. Here $r_{t_2} = -400 < 0 \Rightarrow t_2$ enters.

P1 Tableau 1 (degenerate pivot: t_2 enters, τ^+ leaves)

Solve the goal row for t_2:

$$t_2 = \tfrac{1}{2}t_1 + \tfrac{1}{400}\tau^- - \tfrac{1}{400}\tau^+.$$

Basic	RHS	t_1	t_2	τ^-	τ^+	Status
t_2	0	$\frac{1}{2}$	1	$\frac{1}{400}$	$-\frac{1}{400}$	B
z_1	0	**0**	0	**+1**	**+1**	

This shows:

$$z_1 = 0 \quad \text{whenever} \quad \tau^{\pm} = 0, \text{ and } 200t_1 - 400t_2 = 0,$$

i.e.

$$\boxed{t_1 = 2t_2}.$$

So torque can be driven to zero: $Z_1^* = 0$. Lock this as a hard relation for P_2.

Stage P_2: Force objective with P_1 locked

With $t_1 = 2t_2$ and $0 \le t_1, t_2 \le 1$, the effective bound is

$$0 \le t_2 \le 0.5, \qquad t_1 = 2t_2.$$

Variant A: Maximize $+x$ force

$$z_2 = F_x = -100(t_1 + t_2) = -300\, t_2, \qquad \max z_2.$$

P2-A Tableau

Var	Value	Status	Reduced cost
t_2	0.0	LB ($[0, 0.5]$)	-300
t_1	0.0	B ($t_1 = 2t_2$)	–

Maximization rule: - Nonbasic at LB can improve only if $r_j > 0$. Here $r_{t_2} = -300 < 0$ $\Rightarrow$ increasing t_2 makes z_2 worse.

$$\boxed{t_1 = t_2 = 0, \quad F = (0,0,0),\ T = (0,0,0).}$$

Variant B: Maximize thrust in $-x$ direction

Instead maximize $-F_x = +300t_2$.

P2-B Tableau A (start)

Var	Value	Status	Reduced cost
t_2	0.0	LB ($[0, 0.5]$)	$+300$
t_1	0.0	B ($t_1 = 2t_2$)	–

Bounded ratio test (preemptive):

$$t_1 = 2t_2 \le 1 \Rightarrow t_2 \le 0.5.$$

So opposite-bound distance is $0.5 - 0 = 0.5$. No pivot needed: *preemptive UB hit.*

P2-B Tableau B (after UB hit)

Var	Value	Status
t_2	0.5	UB ($[0, 0.5]$)
t_1	1.0	B ($t_1 = 2t_2$)

Optimality (max): - Nonbasic at UB could only help if $r_j < 0$. Here $r_{t_2} = +300 > 0 \Rightarrow$ decreasing t_2 reduces z_2.

$$\boxed{t_1 = 1, \quad t_2 = 0.5, \quad F = (-150, 0, 0),\ T = (0, 0, 0).}$$