Optimal fiscal policy in a second-best climate economy model with heterogeneous agents

Thomas Douenne * † Albert Jan Hummel *‡ Marcelo Pedroni *§

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Abstract

This paper studies optimal fiscal policy in a climate economy model with heterogeneous agents and distortionary taxes on labor and capital income. We derive optimal tax rules and show how they are modified relative to first-best and relative to the case without heterogeneity. We also explore quantitatively the role of heterogeneity for optimal carbon taxation in a version of the model that is calibrated to the US economy.

JEL classification: E62, H21, H23, Q5

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* University of Amsterdam, Amsterdam School of Economics. Present address: University of Amsterdam, Roetersstraat 11, 1018 WB Amsterdam, Netherlands.

† email: t.r.g.r.douenne@uva.nl

‡ email: a.j.hummel@uva.nl

§ email: m.pedroni@uva.nl

1 Introduction

Economic inequality and environmental degradation are certainly two of the most critical issues facing societies today. In order to address these two problems, economists have long argued for the use of fiscal instruments: labor and capital taxes can be used to provide redistribution, and following the Pigouvian principle a pollution tax can be used to internalize environmental externalities. However, pollution taxes also have distributional implications as they reduce purchasing power and because individuals are heterogeneously affected by environmental degradation. Conversely, capital and labor taxes also affect the costs and benefits of improving the environment by reducing incentives to work and invest. The goal of this study is to analyze how these instruments should be jointly optimized if society wishes to reduce both inequality and environmental degradation.

We address these questions from both a theoretical and a quantitative perspective. To do so, this paper presents a dynamic second-best climate-economy model with heterogeneous agents. We use the technique introduced by Werning (2007) to extend the climate-economy model of Barrage (2019) to heterogeneous agents. In our model, individuals derive utility from consumption, leisure, and environmental quality. The final consumption good is produced using energy as one of its inputs. Energy production is polluting, and pollution leads to environmental degradation that affects productivity and households' utility. As in Barrage (2019), energy producers can reduce the carbon intensity of their output by engaging in costly abatement activities. Because of these costs, positive abatement will occur only if producers also need to pay for their pollution, for example through a pollution tax. The government thus faces multiple tasks at once: mitigating the pollution externality, providing redistribution, and financing some exogenous government spending.

We model this as a Ramsey problem in which the government chooses the level of linear taxes—in particular, taxes on labor and capital income and on energy usage—and a uniform lump-sum transfer to maximize aggregate welfare. Because agents are heterogeneous but tax instruments are anonymous, the government must rely on distortionary instruments to provide redistribution. We analytically characterize optimal tax formulas and study the implications of heterogeneity for optimal pollution taxation.

[To be included: results]

Our paper contributes to two strands of the literature. On the normative side, it contributes to the literature on the optimal taxation of pollution. Since the pioneering work of Pigou (1920), an extensive body of literature has studied pollution taxes in second best environments. Important papers in that literature include (among others) Sandmo (1975), Bovenberg and de Mooij (1994), Bovenberg and van der Ploeg (1994), Cremer et al. (1998), Kaplow (2012), Jacobs and de Mooij (2015), and Jacobs and van der Ploeg (2019). These papers usually focus on static partial equilibrium settings (with some general equilibrium results) and model the pollution externality in a stylized manner. By contrast, we develop a dynamic general equilibrium (DGE) model of the climate-economy, which enables us to study second-best environmental taxation in a richer setting. In doing so, our paper closely relates to Barrage (2019) who creates a critical bridge between the climate-economy literature and the dynamic public finance literature. Her framework integrates a climate-economy model in the spirit of Golosov et al. (2014) into the representative agent Ramsey model of Chari and Kehoe (1999). Our main innovation relative to Barrage (2019) is to introduce heterogeneous agents, which we see as critical for two reasons. First, this allows us to jointly study environmental and equity issues. In addition of the importance of equity in normative analysis, recent experience has shown that the distributional effects of environmental policies were also critical to ensure their public support.[1] Second, agents' heterogeneity provides a sound foundation for the study of second-best policies. In representative agent settings, the second-best environment arises because lump-sum transfers are assumed unfeasible: governments therefore *need to* rely on distortionary taxes to finance their expenditures. Yet, in practice lump-sum transfers are feasible as they simply correspond to the intercept on a tax scheme, and recent policy proposals such as the carbon tax and dividend advocated by the Climate Leadership Council even call for using such instruments to redistribute the carbon tax revenue.[2] With heterogeneous agents, lump-sum transfers are no longer excluded as long as they do not discriminate between agents. Although this non-distortionary source of public income is available, governments now *want to* use distortionary taxes to provide redistribution. Thus, the rationale behind distortionary taxation is entirely different in heterogeneous agents model, leading to reconsider the implications of the optimal tax results.

On the positive side, this paper contributes to the analysis of the distributive effects of environmental taxes in general equilibrium. An extensive literature has analyzed the distributional effects of environmental taxes through the consumption channel (for a recent survey, see Pizer and Sexton, 2019), generally pointing to regressive effects since the consumption share of polluting goods tends to decrease with income (Levinson and O'Brien, 2019). More recently, several authors have also analyzed the heterogeneous incidence of environmental taxes on households' income (see *e.g.* Rausch et al., 2011; Fullerton and Monti, 2013; Williams et al., 2015; Goulder et al., 2019), generally pointing to progressive effects due to the larger negative impact of the policy on capital income relative to labor income and transfers. In a recent paper, Fried et al. (2018) study the effect of introducing a carbon tax with three alternative revenue-recycling schemes in a quantitative OLG model with heterogeneity within-generations. They focus on the non-environmental benefits of the policies, and investigate their effects both at the steady state and during transition. They show that while a uniform lump-sum rebate is more costly than reductions of the labor or capital tax rates in steady state, it is more favorable to the current generation and leads to less adverse distributional effects. In a working paper (Fried et al., 2021), the same authors focus exclusively on the steady-state and study the optimal recycling policy, but the carbon tax remains exogenous and the analysis abstracts from environmental effects. Relative to this literature, our objective is to develop a framework to analytically and quantitatively study optimal carbon taxation in

[1] Public protests against policy-induced increases in energy prices have recently occurred in many countries worldwide. For instance, in France the Yellow Vests movement strongly opposed carbon tax increases due to the expected impact on households' purchasing power, leading to the abandonment of the scheduled carbon tax reforms (Douenne and Fabre, 2022).

[2] See Wall Street Journal 2019's column signed by 3,354 American economists in support of carbon pricing with lump-sum rebates.

a dynamic general equilibrium setting with a rich representation of agents' heterogeneity. Because our model endogenizes the environmental externality, it also allows us to study the heterogeneous welfare impacts arising from the environmental effects of climate policies.

The rest of the paper is organized as follows. Section 2 presents the model, and section 3 the optimal tax formulas. Section 4 presents our main quantitative exercise. Extensions of our main framework are provided in Section 5. Section 6 Concludes.

2 Model

The model builds on Barrage (2019): one sector of the economy produces a final good using capital, labor, and energy that is produced in the second sector. Energy production generates pollution that leads to environmental degradation, which in turn affects productivity and households' utility. The government finances an exogenous stream of expenditures using taxes on labor income, capital income, energy, and pollution, as well as a lump-sum tax. The key differences with Barrage (2019) are that in our model, agents are heterogeneous and the government has access to a (non-individualized) lump-sum tax. Consequently, although the government has access to a non-distortionary source of revenue, it uses distortionary taxes for redistributive purposes.

2.1 Households

We consider an economy populated by a continuum of infinitely-lived agents divided into types $i \in I$ of size π_i. Each agent of type $i \in I$ ranks streams of consumption of a final good $c_{i,t}$, labor supply $h_{i,t}$, and environmental degradation Z_t according to the preferences

$$\sum_{t=0}^{\infty} \beta^t u\left(c_{i,t}, h_{i,t}, Z_t\right). \tag{1}$$

Agents are assumed to differ in two ways: their productivity levels, e_i, and their initial asset holdings, $a_{i,0}$. Productivity levels are normalized such that $\sum_i \pi_i e_i = 1$. Agents' assets are composed of government debts and capital and we denote respectively $b_{i,t}$ and $k_{i,t}$ the number of units of these assets held by agents of type i between periods $t-1$ and t, with $a_{i,t} = b_{i,t} + k_{i,t}$. Aggregates are denoted without the subscript i: $C_t = \sum_i \pi_i c_{i,t}$, $H_t = \sum_i \pi_i e_i h_{i,t}$, $B_t = \sum_i \pi_i b_{i,t}$, and $K_t = \sum_i \pi_i k_{i,t}$.

Let p_t denote the price of the consumption good in period t in terms of consumption in period 0 (so that $p_0 = 1$), w_t and r_t denote the real wage and the rental rate of capital in period t, and R_t its gross return (between $t-1$ and t). Finally, let $\tau_{H,t}$ and $\tau_{K,t}$ represent the labor and capital income taxes, and T_t the uniform lump-sum transfer received by all households in period t. Given $k_{i,0}$, $b_{i,0}$, prices $\{p_t, w_t, R_t\}_{t=0}^{\infty}$ and policies $\{\tau_{H,t}, \tau_{K,t}, T_t\}_{t=0}^{\infty}$, the agent chooses $\{c_{i,t}, h_{i,t}, k_{i,t+1}, b_{i,t+1}\}$ to maximize (1) subject to the intertemporal budget constraint

$$\sum_{t=0}^{\infty} p_t \left(c_{i,t} + k_{i,t+1} + b_{i,t+1}\right) \leq \sum_{t=0}^{\infty} p_t \left(\left(1 - \tau_{H,t}\right) w_t e_i h_{i,t} + R_t \left(k_{i,t} + b_{i,t}\right) + T_t\right),$$

where $R_t \equiv 1 + \left(1 - \tau_{K,t}\right)\left(r_t - \delta\right)$, for $t \geq 0$. Here, we use the convention that the capital income tax is levied on the rate of return net of depreciation, but none of our results depend on it. Ensuring no arbitrage opportunities requires $p_t = R_{t+1} p_{t+1}$, and defining $T \equiv \sum_{t=0}^{\infty} p_t T_t$, the budget constraint is equivalent to

$$\sum_{t=0}^{\infty} p_t \Big(c_{i,t} - \left(1 - \tau_{H,t}\right) w_t e_i h_{i,t}\Big) \leq R_0 a_{i,0} + T. \tag{2}$$

From the first order conditions of agent i's problem we have

$$\begin{aligned} \beta^t \frac{u_c\left(c_{i,t}, h_{i,t}, Z_t\right)}{u_c\left(c_{i,0}, h_{i,0}, Z_0\right)} &= p_t, \quad \forall\, t \geq 0, \\ \frac{u_h\left(c_{i,t}, h_{i,t}, Z_t\right)}{u_c\left(c_{i,t}, h_{i,t}, Z_t\right)} &= -\left(1 - \tau_{H,t}\right) e_i w_t, \quad \forall\, t \geq 0, \end{aligned}$$

which holds across all agents.

2.2 Final-good sector

As in Barrage (2019), there are two production sectors. In the final-good sector, indexed by 1, consumption-capital good is produced with a concave, constant returns to scale technology, $F\left(K_{1,t}, H_{1,t}, E_t\right)$, that uses capital $K_{1,t}$, labor $H_{1,t}$, and energy E_t. The total factor productivity is given by $A_{1,t}$ and the function $D\left(Z_t\right)$ controls the damages to production implied by environmental degradation, with $D'(Z_t) > 0$. The output $Y_{1,t}$ is given by

$$Y_{1,t} = (1 - D(Z_t)) A_{1,t} F(K_{1,t}, H_{1,t}, E_t).$$

The first order conditions for the firm problem are:

$$r_t = \left(1 - D\left(Z_t\right)\right) A_{1,t} F_K\left(K_{1,t}, H_{1,t}, E_t\right), \quad \forall\, t \geq 0, \tag{3}$$

$$w_t = \left(1 - D\left(Z_t\right)\right) A_{1,t} F_H\left(K_{1,t}, H_{1,t}, E_t\right), \quad \forall\, t \geq 0, \tag{4}$$

$$p_{E,t} = \left(1 - D\left(Z_t\right)\right) A_{1,t} F_E\left(K_{1,t}, H_{1,t}, E_t\right), \quad \forall\, t \geq 0. \tag{5}$$

Here, $p_{E,t}$ denotes the relative price of energy in period t. Because there are constant returns to scale and inputs are paid according to their marginal productivity, final goods producers make zero profits.

2.3 Energy sector

The energy sector, indexed by 2, produces energy E_t using capital $K_{2,t}$, and labor $H_{2,t}$ with a constant returns to scale technology so that

$$E_t = A_{2,t} G\left(K_{2,t}, H_{2,t}\right), \quad \forall\, t \geq 0. \tag{6}$$

Energy producers can provide a fraction μ_t of energy from clean technologies, at additional cost $\Theta_t\left(\mu_t E_t\right)$, which satisfies $\Theta_t'(\mu_t), -\Theta_t''(\mu_t) > 0$ and $\Theta_t(0) = 0$. Convexity in $\Theta_t(\cdot)$ captures decreasing returns to abatement. Total profits from energy production are thus given by

$$\Pi_t = \left(p_{E,t} - \tau_{I,t}\right) E_t - \tau_{E,t}\left(1 - \mu_t\right) E_t - w_t H_{2,t} - r_t K_{2,t} - \Theta_t\left(\mu_t E_t\right),$$

where $\tau_{I,t}$ denotes the excise intermediate-goods tax on total energy, and $\tau_{E,t}$ denotes the excise tax on pollution emissions $E_t^M = \left(1 - \mu_t\right) E_t$. Firms maximize profits subject to the technology constraint given by (6) by choosing the abatement term μ_t, capital $K_{2,t}$, and labor $H_{2,t}$. The first order conditions are

$$r_t = \left(p_{E,t} - \tau_{I,t} - \tau_{E,t}\right) A_{2,t} G_K\left(K_{2,t}, H_{2,t}\right), \quad \forall\, t \geq 0, \tag{7}$$

$$w_t = \left(p_{E,t} - \tau_{I,t} - \tau_{E,t}\right) A_{2,t} G_H\left(K_{2,t}, H_{2,t}\right), \quad \forall\, t \geq 0, \tag{8}$$

$$\tau_{E,t} = \Theta_t'\left(\mu_t E_t\right), \quad \forall\, t \geq 0. \tag{9}$$

If there are decreasing returns to abatement (i.e., if $\Theta_t(\cdot)$ is strictly convex) and firms abate a positive fraction $\mu_t > 0$, profits in the energy sector will be positive. For simplicity, we assume that these profits are taxed at a confiscatory rate $\tau_{\pi,t} = 1$.[3]

Capital and labor are mobile across sectors, so the market clearing conditions give

$$K_{1,t} + K_{2,t} = K_t, \quad \forall\, t \geq 0, \tag{10}$$

$$H_{1,t} + H_{2,t} = H_t, \quad \forall\, t \geq 0. \tag{11}$$

2.4 Government

Each period the government finances the expenses G_t and lump sum transfers T_t with proportional income taxes on capital $\tau_{K,t}$ and labor $\tau_{H,t}$, total energy taxes $\tau_{I,t}$, emissions taxes $\tau_{E,t}$ and profit taxes $\tau_{\pi,t}$. The government's intertemporal budget constraint is

$$R_0 B_0 + T + \sum_t p_t G_t = \sum_t p_t \left(\tau_{H,t} w_t H_t + \tau_{K,t}\left(r_t - \delta\right) K_t + \tau_{I,t} E_t + \tau_{E,t} E_t^M + \tau_{\pi,t} \Pi_t\right). \tag{12}$$

[3]Doing so is typically optimal, as taxing pure profits does not generate distortions and income from shareholdings tends to be unequally distributed.

2.5 Environmental degradation

The environmental variable is affected by the history of pollution emissions $E_t^M = (1 - \mu_t) E_t$, initial conditions S_0, and the history of exogenous shifters η_t according to

$$Z_t = J\left(S_0, E_0^M, ..., E_t^M, \eta_0, ..., \eta_t\right), \quad \forall\, t \geq 0. \tag{13}$$

In our calibration below, Z represents the global mean temperature that is the outcome of the climate model J. In this section and the next, we do not further specify this function and our theoretical results can apply to any kind of pollution externality affecting production and households' utility.

2.6 Competitive equilibrium

Definition 1 *Given $\{a_{i,0}\}$, K_0, and B_0, a competitive equilibrium is a policy $\{\tau_{H,t}, \tau_{K,t}, \tau_{I,t}, \tau_{E,t}, T_t\}_{t=0}^{\infty}$, a price system $\{p_t, w_t, r_t, p_{E,t}\}_{t=0}^{\infty}$ and an allocation $\left\{(c_{i,t}, h_{i,t})_i, Z_t, E_t, K_{1,t}, K_{2,t}, K_{t+1}, H_{1,t}, H_{2,t}, H_t\right\}_{t=0}^{\infty}$ such that: (i) agents choose $\left\{(c_{i,t}, h_{i,t})_i\right\}_{t=0}^{\infty}$ to maximize utility subject to budget constraint (2) taking policies and prices (that satisfy $p_t = R_{t+1}p_{t+1}$) as given; (ii) firms maximize profits; (iii) the government's budget constraint (12) holds; (iv) markets clear: the resource constraints (6), (10), (11), and (13) hold, and*

$$C_t + G_t + K_{t+1} + \Theta_t\left(\mu_t E_t\right) = \left(1 - D\left(Z_t\right)\right) A_{1,t} F\left(K_{1,t}, H_{1,t}, E_t\right) + (1 - \delta) K_t, \quad \forall\, t \geq 0. \tag{14}$$

3 Optimal tax rules

In this section, we use the technique introduced by Werning (2007) to express agents' equilibrium allocations as a function of aggregate variables, and solve the Ramsey problem as a function of aggregates instead of their full distributions.

3.1 A simple characterization of equilibrium

Suppose agents have preferences over consumption, leisure and environmental degradation, with the following period utility function

$$u\left(c_i, h_i, Z\right) = \frac{\left(c_i(1 - \varsigma h_i)^{\gamma}\right)^{1-\sigma}}{1 - \sigma} + \frac{\left(1 + \alpha_0 Z^2\right)^{-(1-\sigma)}}{1 - \sigma}.$$

Because the government sets linear tax rates, all individuals face the same marginal rate of substitution between consumption and leisure. A direct implication is that the distribution of individual allocations (c_{it}, h_{it}) is efficient *given* aggregates (C_t, H_t). Another way of stating this is that taxation is distortionary only to the extent it affects aggregates. Following Werning (2007), it is therefore possible to split up the optimal tax problem in two steps. The first is to determine individual allocation given aggregates, and the second is to determine the aggregates. Starting with the first step, denote by $\varphi \equiv \{\varphi_i\}$ a set of market weights normalized so that

$$\sum_j \pi_j \left(\varphi_j e_j^{\gamma(\sigma-1)}\right)^{\frac{1}{\sigma-(1-\sigma)\gamma}} = 1,$$

with $\varphi_i \geq 0$. Using the property that individual allocations are efficient given aggregates, we can characterize these allocations by solving the following static sub-problem for each period t:

$$\begin{aligned} U\left(C_t, H_t, Z_t; \varphi\right) &\equiv \max_{c_{i,t}, h_{i,t}} \sum_i \pi_i \varphi_i u\left(c_{i,t}, h_{i,t}, Z_t\right), \\ \text{s.t.} \quad &\sum_i \pi_i c_{i,t} = C_t \quad \text{and} \quad \sum_i \pi_i z_{i,t} h_{i,t} = H_t. \end{aligned} \tag{15}$$

Here, $U\left(C_t, H_t, Z_t; \varphi\right)$ denotes the indirect aggregate utility function, computed using market weights and aggregates.

As shown in appendix, given our functional form assumptions, the equilibrium individual allocations can be expressed as simple functions of φ and aggregate variables. In particular, we have

$$\begin{aligned} c_{i,t}^m\left(C_t, H_t, Z_t; \varphi\right) &= \omega_i C_t, \\ 1 - \varsigma h_{i,t}^m\left(C_t, H_t, Z_t; \varphi\right) &= \frac{\omega_i}{e_i}(1 - \varsigma H_t), \end{aligned}$$

where

$$\omega_i = \left(\varphi_i e_i^{\gamma(\sigma-1)}\right)^{\frac{1}{1-(1+\gamma)(1-\sigma)}}.$$

From our normalization of market weights, we have $\sum_i \pi_i \omega_i = 1$, and ω_i can be understood as the relative consumption of an agent of type i. Going back to (15), we can now express $U\left(C_t, H_t, Z_t; \varphi\right)$ in terms of the aggregates C_t, H_t, and Z_t and market weights φ

$$U\left(C_t, H_t, Z_t, \varphi\right) = \frac{\left(C_t(1-\varsigma H_t)^\gamma\right)^{1-\sigma}}{1-\sigma} + \Gamma \frac{\left(1+\alpha_0 Z_t^2\right)^{-(1-\sigma)}}{1-\sigma}, \tag{16}$$

with $\Gamma \equiv \sum_i \pi_i \varphi_i$.

3.2 Implementability condition

Using the simple characterization from the previous section we can now derive the implementability condition. Applying the envelope theorem to problem (15) and using consumers' first order conditions we get

$$\frac{U_H\big(C_t, H_t\big)}{U_C\big(C_t, H_t\big)} = \frac{u_h\left(c_{i,t}, h_{i,t}\right)}{u_c\left(c_{i,t}, h_{i,t}\right) e_i} = -w_t\left(1 - \tau_{H,t}\right),$$

and

$$\frac{U_C\big(C_t,H_t\big)}{U_C\big(C_0,H_0\big)}=\frac{u_c\left(c_{i,t},h_{i,t}\right)}{u_c\left(c_{i,0},h_{i,0}\right)}=\frac{p_t}{\beta^t},$$

where the variable Z has been omitted from the list of arguments in partial derivatives given the strong separability with consumption and labor in (16). Using these relationships to substitute out for prices in agents' budget constraints, for any agent i we can arrive at the implementability condition that depends only on the aggregates C_t and H_t and market weights φ

$$U_C\big(C_0,H_0\big)\left(R_0a_{i,0}+T\right)\geq\sum_{t=0}^{\infty}\beta^t\left(U_C\big(C_t,H_t\big)c^m_{i,t}\Big(C_t,H_t;\varphi\Big)+U_H\big(C_t,H_t\big)e_ih^m_{i,t}\left(C_t,H_t;\varphi\right)\right),\;\;\forall\; i. \tag{17}$$

The following Proposition follows immediately from the arguments above.

Proposition 1 *An aggregate allocation $\{C_t,H_{1,t},H_{2,t},K_{1,t},K_{2,t},E_t,Z_t,\mu_t\}_{t=0}^{\infty}$ can be supported by a competitive equilibrium if and only if the market clearing conditions (10), and (11) hold, the resource constraints (6), (13), (14) hold and there exist market weights φ and a lump-sum tax T such that the implementability conditions (17) hold for all $i\in I$. Individual allocations can then be computed using functions $c^m_{i,t}$ and $h^m_{i,t}$, prices and taxes can be computed using the usual equilibrium conditions.*

3.3 Ramsey Problem

Let $\lambda\equiv\{\lambda_i\}$ be the planner's welfare weight on type i, with $\sum_i\pi_i\lambda_i=1$. The Ramsey planner problem is

$$\max_{\substack{\{C_t,H_{1,t},H_{2,t},K_{1,t},K_{2,t},\\ E_t,Z_t,\mu_t\}_{t=0}^{\infty},T,\varphi,\tau_0^k\leq 1}}\sum_{t,i}\beta^t\pi_i\lambda_iu\Big(c^m_{i,t}\big(C_t,H_t;\varphi\big),h^m_{i,t}\big(C_t,H_t;\varphi\big),Z_t\Big)$$

subject to

$$\begin{aligned}
U_C\big(C_0,H_0\big)\big(R_0a_{i,0}+T\big)&\geq\sum_{t=0}^{\infty}\beta^t\left(U_C\big(C_t,H_t\big)c^m_{i,t}\Big(C_t,H_t;\varphi\Big)+U_H\big(C_t,H_t\big)e_ih^m_{i,t}\left(C_t,H_t;\varphi\right)\right),\quad\forall\; i,\\
\frac{F_K(K_{1,t}H_{1,t},E_t)}{F_H(K_{1,t}H_{1,t},E_t)}&=\frac{G_K(K_{2,t}H_{2,t})}{G_H(K_{2,t}H_{2,t})},\\
C_t+G_t+K_{t+1}+\Theta_t\left(\mu_tE_t\right)&=\left(1-D\left(Z_t\right)\right)A_{1,t}F\left(K_{1,t},H_{1,t},E_t\right)+\left(1-\delta\right)K_t,\quad\forall\; t\geq 0,\\
E_t&=A_{2,t}G\left(K_{2,t},H_{2,t}\right),\quad\forall\; t\geq 0,\\
Z_t&=J\left(S_0,E_0^M,...,E_t^M,\eta_0,...,\eta_t\right),\quad\forall\; t\geq 0,\\
K_{1,t}+K_{2,t}&=K_t,\quad\forall\; t\geq 0,\\
H_{1,t}+H_{2,t}&=H_t,\quad\forall\; t\geq 0,\\
\sum_j\pi_j\left(\varphi_je_j^{\gamma(\sigma-1)}\right)^{\frac{1}{\sigma-(1-\sigma)\gamma}}&=1.
\end{aligned}$$

The first of these is the implementability condition, which must hold for each agent i. It is written solely in terms of allocation variables and states that the present value of consumption equals the present value of labor income, initial assets and lump-sum transfers. The second constraint states that the marginal rate of technical substitution between capital and labor is the same in both sectors. It is a restriction imposed on the allocation which reflects that the government does not use sector-specific instruments and factors are mobile across sectors. The other constraints reflect market clearing for capital, labor and goods, technological constraints and a normalization of market weights.

3.3.1 Initial capital taxes

The first order condition with respect to τ_0^k is given by

$$U_C\left(C_0, H_0\right)\big(\left(1-D\left(Z_0\right)\right) A_{1,0} F_K\left(K_{1,0}, H_{1,0}, E_0\right)-\delta\big) \sum_i \pi_i \theta_i a_{i,0}=0$$

So, it is optimal to expropriate initial asset holdings until $\sum_i \pi_i \theta_i a_{i,0}=0$. If this is not feasible, then the best the government can do is to raise the period-0 capital tax until $R_0=0$, which implies all wealth is appropriated. As explained by Werning (2007), tighter restrictions on initial capital taxation are difficult to justify because a wealth tax can be mimicked using consumption taxes. Hence, abstracting from consumption taxes, as we have done throughout, is only without loss of generality if we allow for wealth expropriation.

3.3.2 Capital and Labor income taxes

From the planner's first order conditions and using our functional form assumption, the labor and capital income taxes are determined by

$$\tau_{H,t}=\frac{\Psi \varsigma\left(1-\varsigma H_t\right)^{-1}}{\Phi+\Psi \varsigma(1-\gamma(1-\sigma))\left(1-\varsigma H_t\right)^{-1}}, \tag{18}$$

and

$$\frac{R_{t+1}}{R_{t+1}^*}=\frac{\Phi-\Psi \varsigma \gamma(1-\sigma)\left(1-\varsigma H_{t+1}\right)^{-1}}{\Phi-\Psi \varsigma \gamma(1-\sigma)\left(1-\varsigma H_t\right)^{-1}}, \tag{19}$$

with

$$\begin{aligned}
\Phi &= \sum_j \pi_j \frac{\lambda_j}{\varphi_j}+\big(1-(1+\gamma)(1-\sigma)\big) \operatorname{cov}\left(\lambda_i / \varphi_i, \omega_i\right), \\
\Psi &= -\frac{\operatorname{cov}\left(\lambda_i / \varphi_i, e_i\right)}{\varsigma}.
\end{aligned}$$

From the previous formulas, we see that both the labor and the capital income tax rates are zero in three special cases: *(i)* when there is no agent heterogeneity, *(ii)* when the planner's and the market's weights are perfectly aligned, and *(iii)* when agents' productivity are uncorrelated with the relative social weights. Intuitively, the first case corresponds to the outcome of a representative agent model in which lump-sum taxation is allowed: since there is no need to redistribute, the government can rely only on non-distortionary taxes to finance its expenditures. The second case corresponds to the situation in which the market allocation happens to be the one preferred by the planner: although there might be inequalities due to differences in productivity and asset holdings, they are consistent with the relative weight the planner gives to each type of individual. The third situation encompasses the two previous ones, but also includes situations in which the planner would want to redistribute but faces a targeting problem, *i.e.* it cannot reach a better allocation than the market one using anonymous linear instruments due to the absence of correlation between the source of inequalities and its relative preference over agents' types.

3.3.3 Excise taxes on energy and emissions

The planner's first order conditions together with firms equilibrium conditions give

$$\tau_{I,t} = 0.$$

Thus, as long as labor, capital, profits and pollution can be taxed, there is no point in distorting production decisions. This result can also be found in Barrage (2019) and goes back to the production efficiency theorem of Diamond and Mirrlees (1971). Turning to the pollution tax we have

$$\tau_{E,t} = \Theta_t' \left(\mu_t E_t\right) = \frac{1}{\nu_{1,t}} \sum_{j=0}^{\infty} \beta^j \nu_{3,t+j} J_{E_t^M,t+j}, \tag{20}$$

where $\nu_{1,t}$ is the multiplier on the planner's resource constraint (14), $\nu_{3,t}$ the multiplier on the environmental constraint (13), and $J_{E_t^M,t+j}$ the derivative with respect to current emissions of the environmental variable in j periods. Thus, the optimal corrective tax corresponds to the discounted sum of all future marginal damages from current emissions—captured by $\nu_{3,t}$, the shadow cost of the environmental variable—relative to one unit of final good as valued by the resource constraint today.

3.3.4 Pigouvian taxation and the marginal cost of funds

Using our functional form assumption and first order conditions from the planner's problem, we can re-write the tax formula given by (20) as

$$\tau_{E,t} = \frac{1}{\nu_{1,t}} \sum_{j=0}^{\infty} \beta^j \left(\nu_{1,t+j} D' \left(Z_{t+j}\right) A_{1,t+j} F \left(K_{1,t+j}, H_{1,t+j}, E_{t+j}\right) - \frac{U_Z(Z_{t+j})}{\Gamma} \right) J_{E_t^M,t+j},$$

where we used the normalization of the planner's weights to get $\frac{\sum_i \pi_i \lambda_i}{\sum_i \pi_i \varphi_i} = \frac{1}{\Gamma}$.

To further examine our second-best pollution tax rule, we express it as a function of the first best « Pigouvian » tax. As shown in appendix, the first best tax—that is obtained in the case where individualized lump-sum transfers are available—corresponds to the social cost of pollution (SCP), *i.e.* the discounted sum of marginal costs from emitting one additional unit of pollution valued at agents' marginal utility of consumption. Following Barrage (2019), we decompose it into two elements: $\tau_{E,t}^{Pigou,Y}$ and $\tau_{E,t}^{Pigou,U}$ denote the level of Pigouvian taxes corresponding to the SCP arising respectively from aggregate losses on production and agents utility:

$$\begin{aligned}
\tau_{E,t}^{Pigou,Y} &= \sum_{j=0}^{\infty} \beta^j \frac{U_C(C_{t+j}, H_{t+j})}{U_C(C_t, H_t)} \Big(D'\left(Z_{t+j}\right) A_{1,t+j} F\left(K_{1,t+j}, H_{1,t+j}, E_{t+j}\right) \Big) J_{E_t^M, t+j} \ , \\
\tau_{E,t}^{Pigou,U} &= (-1) \sum_{j=0}^{\infty} \beta^j \frac{U_Z(Z_{t+j})}{U_C(C_t, H_t)} J_{E_t^M, t+j} \ ,
\end{aligned}$$

with the total Pigouvian tax defined as $\tau_{E,t}^{Pigou} \equiv \tau_{E,t}^{Pigou,Y} + \tau_{E,t}^{Pigou,U}$. Define the marginal cost of funds (MCF) as the ratio between the social marginal value of public income and the average marginal value of private income using the planner's weights:

$$\mathrm{MCF}_t \equiv \frac{\nu_{1,t} \Gamma}{U_C\left(C_t, H_t; \varphi\right)}.$$

The term $\nu_{1,t}$ measures the increase in social welfare if government consumption G_t of the final good in period t decreases by one unit. The term $U_C(C_t, H_t; \varphi)/\Gamma$, in turn, measures by how much social welfare increases if all individuals receive an additional unit of consumption. Now, if we define Δ_{t+s} as the share of marginal production damages occurring at time $t+s$ due to a marginal change in emissions at time t, i.e.

$$\Delta_{t+s} \equiv \frac{\beta^j D'\left(Z_{t+s}\right) A_{1,t+s} F\left(K_{1,t+s}, H_{1,t+s}, E_{t+s}\right) J_{E_t^M, t+s}}{\sum_{j=0}^{\infty} \beta^j \Big(D'\left(Z_{t+j}\right) A_{1,t+j} F\left(K_{1,t+j}, H_{1,t+j}, E_{t+j}\right) J_{E_t^M, t+j} \Big)} \ ,$$

the second-best pollution tax rule in this economy can be expressed as the following modified Pigouvian rule

$$\tau_{E,t} = \sum_{j=0}^{\infty} \frac{\mathrm{MCF}_{t+j}}{\mathrm{MCF}_t} \Delta_{t+j} \tau_{E,t}^{Pigou,Y} + \frac{\tau_{E,t}^{Pigou,U}}{\mathrm{MCF}_t}.$$

This tax formula resembles the one derived by Barrage (2019) who generalizes to a dynamic setting similar results previously obtained in static environments (see *e.g.,* Sandmo, 1975; Bovenberg and van der Ploeg, 1994). In second best, the optimal corrective tax depends on the social cost of pollution and the marginal cost of funds. Utility losses from pollution are internalized at the level of the SCP weighted by the inverse of the MCF; production damages from pollution are internalized at the level of the SCP weighted by the ratios of the MCF at the time damages occur over the current MCF. As shown in appendix, the latter ratio can be expressed as a function of the optimal capital tax rates in future periods. When the capital tax is optimally set to zero in all future periods in which damages from current emissions occur, this ratio equals 1 and the second-best corrective tax simplifies to

$$\tau_{E,t} = \tau_{E,t}^{Pigou,Y} + \frac{\tau_{E,t}^{Pigou,U}}{\text{MCF}_t}.$$

The key difference between our tax formula and the ones derived by the previous authors is the mechanism driving the MCF. These previous studies consider representative agent models in which the government is assumed unable to use lump-sum taxation—even though this would be optimal to use such taxes—hence distortionary taxes have to be used to finance government's expenditures. By contrast, in our model we consider heterogeneous agents who differ in their earning abilities. Thus, although we allow the planner to use lump-sum taxation, distortionary taxes are used to provide redistribution and the MCF is not generally equal to 1. From the planner's first order condition with respect to C_t, we can express the MCF as

$$\text{MCF}_t = \Gamma\Big(\Phi - \Psi\frac{\varsigma\gamma(1-\sigma)}{(1-\varsigma H_t)}\Big),$$

from which we see that the MCF depends on the model's parameters, aggregates, and agents heterogeneity summarized by covariance terms. In the special case where there is no agent heterogeneity, or where the weights of the market and the planner are equal, then one can show that $\Gamma = 1$, $\Phi = 1$, and $\Psi = 0$ so that MCF $= 1$, and the second best corrective tax is set at the Pigouvian level. Outside this special case, we show in appendix that inequalities affect the MCF in an ambiguous way, such that an increase in inequalities may in theory move the second best tax either above or below the first best level depending on parameters' values.

[To be included: mechanisms]

4 Quantitative analysis

This section explores quantitatively the implications of heterogeneity in productivity and asset holdings for the optimal taxation of carbon, capital income and labor income.

4.1 Calibration

The calibration of the climate economy model is taken from Barrage (2019). For a detailed explanation, we refer to her paper. The main modification is that we allow for heterogeneity in productivity and asset holdings. Because in our baseline model we also allow for an expropriatory wealth tax (i.e., we allow the planner to set $R_0 = 0$), we first demonstrate results without heterogeneity in asset holdings. A potential micro-foundation is that wealth and productivity are positively correlated, which makes it optimal for the planner to tax all initial wealth at a confiscatory rate. In Section 5, we consider further constrained environments where the planner cannot optimize over all fiscal instruments. When the capital income tax is exogenously fixed, the planner is no longer able to fully expropriate wealth in the first period and the initial distribution of assets matters.

We calibrate the ability distribution on the basis of hourly wage data that we obtain from the Survey of Consumer Finances (SCF). For each of the 5,777 households in the 2019 wave of the survey, we sum the hours worked on their main job and potential additional job(s) in a normal week. Annual labor supply of the respondent and their partner is then calculated by multiplying weekly hours worked by 52 minus the number of weeks they have spent unemployed during the past 12 months minus the number of weeks spend on holidays (which we assume equals 3 for each worker). The household hourly wage is then obtained as the household annual income from wages and salaries before taxes, divided by the household total annual labor supply (i.e., the sum of the respondent and their partner's labor supply). This number reflects how much households members were paid on average for each hour of work they supplied in the past year.

To obtain the hourly wage distribution, we make a few additional adjustments. We first drop all households with an hourly wage below \$1 or above \$1,000. We also restrict the sample to households who have worked at least 1 week over the past 12 months, who work at least 1 hour on a normal week, and with no member working above 100 hours. Finally we restrict the sample to households whose respondent is at least 18 years old, and at most 65 years old. Using this sub-sample, we divide households in ten groups of hourly wage weighted deciles (the weights are those of the SCF). These correspond to $I = 10$ groups with size $\pi_i = 0.10$. For each group, we compute the average hourly wage.[4] The lowest hourly wage is \$7.04 and the highest hourly wage is \$103, with an average of approximately \$32.

4.2 Results

Figure 1 shows the path of optimal carbon taxes under three different scenarios. All these results are obtained under a utilitarian welfare criterion (i.e., $\lambda_i = 1$ for all i). In the first scenario (First Best), we abstract from all sources of heterogeneity and allow the planner to optimize carbon taxes in addition to taxes on capital income, labor income and, importantly, a lump-sum transfer. Naturally, this instrument set allows the planner to achieve the first best allocation. The carbon tax is set at the Pigouvian level, so that all utility and production externalities are internalized, and all the additional revenue that is required to finance government spending is raised through a lump-sum tax. These results can also be found in Barrage (2019), who characterizes the path of optimal carbon taxes both in the case where the government does and where the government does not have access to a lump-sum tax.

[4] Because agents in our model are infinitely lived but hourly wage is positively correlated with age, we control for generational heterogeneity. To do so, we divide households in ten generations based on the age of the respondent, and compute the average hourly wage of each hourly wage decile group within each generation. We then obtain the average hourly wage for each decile group as the average hourly wage of that decile group over all generations.

In the second scenario (First Best with Productivity Heterogeneity), we introduce productivity heterogeneity as outlined above. To make the economies comparable, in both cases the average productivity is normalized to one. Importantly, to achieve a first-best allocation, the planner also requires *individualized* lump-sum taxes T_i in this scenario. These taxes (or transfers if negative) provide a non-distortionary source of revenue which the government can use to achieve its redistributive goals. The optimal carbon tax is then again set at its Pigouvian level. As can be seen from the figure, the levels are very close to the first scenario where there is no heterogeneity. This should come as no surprise. Because the government can use individualized lump-sum transfers or taxes to *de facto* undo all differences driven by productivity heterogeneity, the economy is very similar as in the representative agent case.

The third scenario (Second Best with Productivity Heterogeneity) features differences in productivity as in the second scenario, but *not* for individualized lump-sum transfers. Instead, the instrument set is as in the first scenario. Hence, the government can optimize carbon taxes in addition to taxes on capital income, labor income and a *non-individualized* lump-sum transfer. Because the transfer cannot be conditioned on individual types, the first best is no longer attainable and the government needs to rely on distortionary taxes to generate revenues and achieve its distributional goals. Figure 1 shows that, when the first best is no longer attainable, the optimal carbon tax is significantly reduced. This result is reminiscent of Barrage (2019), who also finds that accounting for tax distortions lowers the optimal carbon tax. An important difference is that in her framework, tax distortions occur because the government does not have access to a non-distortionary source of revenue. By contrast, in our framework the government can generate all revenue with a lump-sum tax, but doing so is sub-optimal because it comes at the expense of redistribution.

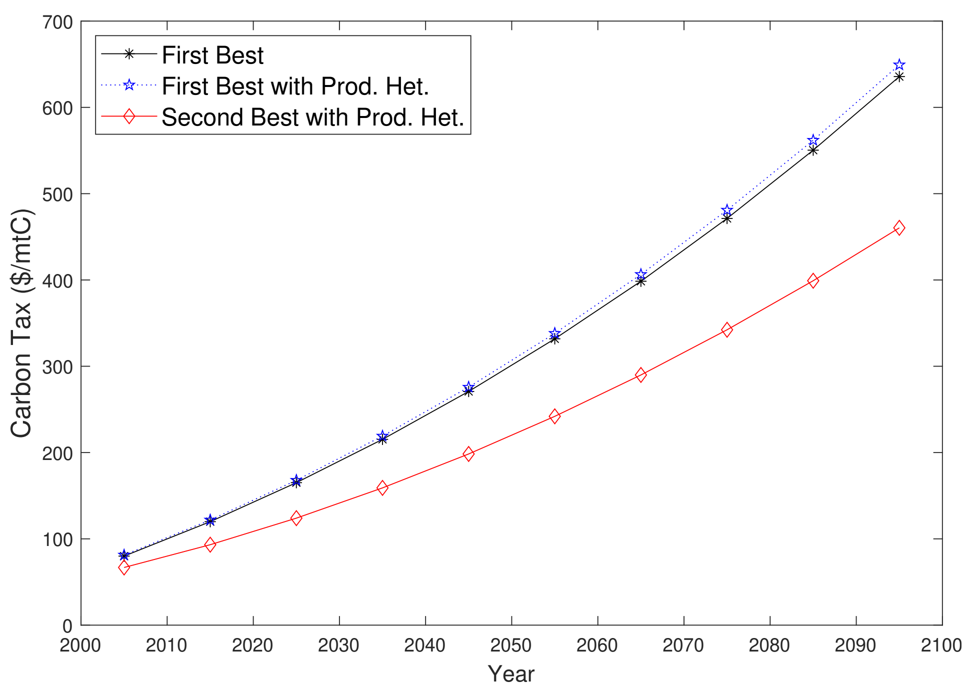

Figure 1: Optimal Carbon Taxes

The difference between the first and second best tax levels can be attributed to two reasons. First, as shown in the previous section, the second best pollution tax formula differs from the Pigouvian tax. Second, the two formulas are evaluated at different allocations. [to be included: figure of the first best tax evaluated at the second best allocation, and discussion]

5 Extensions

5.1 Business-as-usual scenarios

We have considered a Ramsey problem in which the government faces two key constraints: only linear and anonymous instruments can be used. Still, this set of fiscal instruments confers a lot of power to the government, arguably more than what most governments have. When introducing a carbon tax policy, a government may not have complete freedom to adjust labor or capital income taxes. In particular, the full expropriation of asset holdings in the initial period that is optimal in our benchmark is not a realistic policy option. To explore these issues, we now turn to fiscal environments with additional constraints on the set of available instruments.

As in Barrage (2019), we consider the cases where either the labor or the capital income tax is fixed at an exogenous rate. Because our framework allows for lump-sum taxation, we can also consider the case where both instruments are exogenously fixed.

5.1.1 Exogenous labor income tax

Let us assume that the planner cannot choose the labor income tax, that is exogenously fixed at a level $\bar{\tau}_H$ in all periods $t \geq 0$. The planner now faces additional constraints on allocations: in every period $t \geq 0$, it must ensure that

$$\frac{U_{H,t}}{U_{C,t}} = -\left(1-\tau_H\right)\left(1-D_t\right)A_{1,t}F_{H,t}, \tag{21}$$

Let $\beta^t \Lambda_t^H$ denote the multiplier on the constraint (21). In addition, let $\beta^t \Omega_t^H$ be the multiplier on the constraint that the marginal rate of technical substitution between capital and labor is the same across both sectors.[5] As shown in appendix, the constrained second-best optimal capital income tax in this scenario is given by

[to be included]

[5]Without condition (21), one can show that the multiplier associated with this constraint is optimally zero: the government does not wish to distort production decisions, which also explains why $\tau_{I,t} = 0$. With additional restrictions on the tax system, this is no longer generally true.

and the constrained second-best optimal pollution tax becomes

[to be included]

5.1.2 Exogenous capital income tax

Let us now assume that the planner cannot choose the capital income tax, that is exogenously fixed at a level $\bar{\tau}_K$ in all periods $t \geq 0$. The new constraint faced by the planner are such that in every period $t \geq 0$

$$\frac{U_{C,t}}{U_{C,t+1}} = \beta \left(1 + (1 - \tau_K) \left((1 - D_{t+1}) A_{1,t+1} F_{K,t+1} - \delta\right)\right) \tag{22}$$

Let $\beta^t \Lambda^K_{t+1}$ be the multiplier on this constraint and $\beta^t \Omega^K_t$ the multiplier on the constraint that the marginal rate of technical substitution between capital and labor is the same across sectors. As shown in appendix, the constrained second-best optimal labor income tax in this scenario becomes

[to be included]

and the constrained second-best optimal pollution tax becomes

[to be included]

5.1.3 Exogenous labor and capital income taxes

...

5.2 Additional sources of heterogeneity

5.2.1 Optimal tax rules

Our benchmark model considers heterogeneous agents who differ in productivity and initial asset holdings. The presence of inequalities in the absence of individualized lump-sum transfers leads the planner to use distortionary taxation, which affects the optimal pollution tax. To further explore the role of agents heterogeneity on optimal fiscal policy, we now introduce two additional ingredients to our benchmark model: a second consumption good, and heterogeneous preferences. We assume that an household of type i derives utility from the consumption of a final good $c_{i,t}$, labor supply $h_{i,t}$, environmental degradation Z_t, and energy consumption $d_{i,t}$ according to the preferences

$$\sum_{t=0}^{\infty} \beta^t u\left(c_{i,t}, d_{i,t}, h_{i,t}, Z_t\right), \tag{23}$$

where the second "dirty" good d is produced from a linear technology that uses energy as its only input. To further simplify notations, we assume that energy produced in the energy sector (E_t) is now used in the final good sector and directly consumed by households, such that

$$E_t = E_{1,t} + D_t, \tag{24}$$

with $E_{1,t}$ the quantity of energy used as an input in the final good sector and $D_t = \sum_i \pi_i d_{i,t}$ the households' aggregate energy consumption. In order to match empirically observed budget shares for energy (or alternatively, polluting goods) for different income groups, we assume households utility can be represented by the following period utility function

$$u\left(c_i, d_i, h_i, Z\right) = \frac{\left(c_i (d_i - \bar{d}_i)^{\epsilon} (1 - \varsigma h_i)^{\gamma}\right)^{1-\sigma}}{1-\sigma} + \chi_i \frac{\left(1 + \alpha_0 Z^2\right)^{-(1-\sigma)}}{1-\sigma}.$$

Thus, in line with previous studies in this literature (*e.g.* Fried et al., 2018; Jacobs and van der Ploeg, 2019) preferences for consumption are modeled with a Stone-Geary utility function, so that an agent of type i experiences positive utility from energy consumption only after consuming its first $\bar{d}_i$ units of energy. $\bar{d}_i$ therefore denotes the subsistence consumption level of energy for an agent of type i, which we allow to be type (and time) specific. This specification allows us to consider households with non-homothetic preferences to better capture the heterogeneous impact of pollution taxes on households' budget. Assuming type-specific values for $\bar{d}_i$, this specification also allows us to consider non-linear aggregate Engel curves as well as horizontal heterogeneity.[6,7] In addition, we assume that agents' relative sensitivity to the environmental variable Z is also type specific and denoted χ_i, normalized such that $\sum_i \pi_i \chi_i = 1$.

Because there is an additional consumption good, the planner uses an additional instrument: it levies an excise tax $\tau_{D,t}$ on households' consumption of energy. Households' budget constraint can thus be expressed as

$$\sum_{t=0}^{\infty} p_t \Big(c_{i,t} + d_{i,t}(p_{E,t} + \tau_{D,t}) - \left(1 - \tau_{H,t}\right) w_t e_i h_{i,t} \Big) \leq R_0 a_{i,0} + T. \tag{25}$$

[6] With Stone-Geary preferences, agents' Engel curves are linear. When preferences are heterogeneous, the aggregate distribution of expenditures may however be a non-linear function of income.

[7] Horizontal heterogeneity arises when individuals with the same income do not consume goods in the same proportions. Recent studies have shown the importance of horizontal heterogeneity on the distributive impacts of energy taxes (Cronin et al., 2019; Pizer and Sexton, 2019), and their implications for the design of tax reforms (Sallee, 2019).

We apply the same solution method as in our benchmark model. Using the method of Werning (2007), we can express individual allocations as a function of aggregate variables and market weights as follows

$$c_{i,t}^m\left(C_t, D_t, H_t; \varphi\right) = \omega_i C_t, \tag{26}$$

$$d_{i,t}^m\left(C_t, D_t, H_t; \varphi\right) = \bar{d}_{i,t} + \omega_i\big(D_t - \bar{D}_t\big), \tag{27}$$

$$1 - \varsigma h_{i,t}^m\left(C_t, D_t, H_t; \varphi\right) = \frac{\omega_i}{e_i}(1 - \varsigma H_t), \tag{28}$$

with $\bar{D}_t = \sum_i \pi_i \bar{d}_{i,t}$ the aggregate subsistence level and where

$$\omega_i = \left(\varphi_i e_i^{\gamma(\sigma-1)}\right)^{\frac{1}{1-(1+\epsilon+\gamma)(1-\sigma)}} \tag{29}$$

normalized such that $\sum_i \pi_i \omega_i = 1$. These expressions allow us to write the aggregate utility function $U(C_t, D_t, H_t, Z_t, \varphi)$ and individual implementability conditions necessary to solve the Ramsey problem based on aggregate variables and market weights only. As shown in appendix, the second best labor income tax in this extended framework is

$$\tau_{H,t} = \frac{\Psi\varsigma(1 - \varsigma H_t)^{-1}}{\Phi + \Psi\frac{\varsigma\left(1-\gamma(1-\sigma)\right)}{(1-\varsigma H_t)} - \Lambda_t\frac{\epsilon(\sigma-1)}{(D_t-\bar{D}_t)}}, \tag{30}$$

the capital income can be obtained from

$$\frac{R_{t+1}}{R_{t+1}^*} = \frac{\Phi + \Psi\frac{\varsigma\gamma(\sigma-1)}{(1-\varsigma H_{t+1})} - \Lambda_{t+1}\frac{\epsilon(\sigma-1)}{(D_{t+1}-\bar{D}_{t+1})}}{\Phi + \Psi\frac{\varsigma\gamma(\sigma-1)}{(1-\varsigma H_t)} - \Lambda_t\frac{\epsilon(\sigma-1)}{(D_t-\bar{D}_t)}}, \tag{31}$$

the excise tax on energy remains unchanged at $\tau_{I,t} = 0$, and the households energy consumption excise tax is

$$\tau_{D,t} = \frac{\Lambda_t \epsilon C_t}{\Phi + \Psi\varsigma\gamma(\sigma - 1)(1 - \varsigma H_t)^{-1} - \Lambda_t\epsilon(\sigma - 1)(D_t - \bar{D}_t)^{-1}}, \tag{32}$$

with

$$\Phi = \sum_j \pi_j \frac{\lambda_j}{\varphi_j} + \Big(1 - (1 + \epsilon + \gamma)(1 - \sigma)\Big)\mathrm{cov}(\lambda_i/\varphi_i, \omega_i), \tag{33}$$

$$\Psi = -\frac{\mathrm{cov}(\lambda_i/\varphi_i, e_i)}{\varsigma}, \tag{34}$$

$$\Lambda_t = -\mathrm{cov}(\lambda_i/\varphi_i, \bar{d}_{i,t}), \tag{35}$$

and market weights are given by

$$\varphi_i = \frac{1}{e_i^{\gamma(\sigma-1)}}\left(\frac{U_C(C_0,D_0,H_0)(R_0a_{i,0}+T)+\sum_t\beta^t\Big(U_H(C_t,D_t,H_t)\frac{e_i}{\varsigma}-U_D(C_t,D_t,H_t)\bar{d}_{i,t}\Big)}{(1-\sigma)(1+\epsilon+\gamma)\sum_t\beta^tU(C_t,D_t,H_t)}\right)^{1-(1+\epsilon+\gamma)(1-\sigma)} \tag{36}$$

Turning to the pollution tax, we can once again express it as a function of its Pigouvian level decomposed into a production and a utility component

$$\tau_{E,t} = \sum_{j=0}^{\infty}\frac{\text{MCF}_{t+j}}{\text{MCF}_t}\Delta_{t+j}\tau_{E,t}^{Pigou,Y} + \frac{\Gamma}{\Gamma_\chi}\frac{\tau_{E,t}^{Pigou,U}}{\text{MCF}_t}\ . \tag{37}$$

with

$$\Gamma_\chi \equiv \frac{\sum_i\pi_i\varphi_i\chi_i}{\sum_i\pi_i\lambda_i\chi_i} = \frac{\sum_i\pi_i\varphi_i+\text{cov}(\varphi_i,\chi_i)}{\sum_i\pi_i\lambda_i+\text{cov}(\lambda_i,\chi_i)} \tag{38}$$

and with the marginal cost of funds now equal to

$$\text{MCF}_t = \Gamma\Big(\Phi-\Psi\frac{\varsigma\gamma(1-\sigma)}{(1-\varsigma H_t)}-\Lambda_t\frac{\epsilon(\sigma-1)}{D_t-\bar{D}_t}\Big).$$

Compared to our benchmark, these optimal tax rules are modified for three reasons: households can now consume two goods, their relative preferences for these two goods may differ, and they may differ in their sensitivity (or exposure) to environmental damages. We analyze the implication of these three elements in turn.

Stone-Geary utility with identical preferences Abstracting from preference heterogeneity (*i.e.* assuming that for $t \geq 0$ and for all i, $\bar{d}_{i,t} = \bar{d}_t$ and $\chi_i = 1$), we have $\Lambda_t = 0$ and $\Gamma_\chi = \Gamma$. Then, all tax formulas remain unchanged relative to our benchmark. This result is reminiscent of Jacobs and van der Ploeg (2019) who show that as long as Engel curves are linear—which is the case with Stone-Geary utility—corrective taxation should not serve to address redistributive objectives, even when non-linear income taxation is not available. Still, as shown in equation (36) the distribution of market weights is affected by the consumption of a second good: having a second good modeled as a necessity generates a fixed-cost to households welfare which affects the whole distribution of welfare, and thus affects the tax rates although the tax formulas are preserved.

Stone-Geary utility with heterogeneous preferences for energy consumption With heterogeneous preferences for energy consumption, Λ_t is not generally 0 anymore. When the consumption threshold ($\bar{d}_i$) varies positively with the relative planner's weight (λ_i/φ_i)—*i.e.* when individuals who are relatively more valued by the planner are also the ones with higher energy needs—then Λ_t is negative. In this situation, for $\sigma > 1$ the labor income tax will be lower, the marginal cost of funds will be higher, so the second best pollution tax will be lower, and the excise tax on energy consumption will be negative. The logic behind the previous results is that aggregate Engel curves being non-linear with heterogeneous preferences, commodity taxes offer an additional levy for redistribution. When the agents who are valued relatively more by the planner also have higher energy needs, the planner can target these agents by subsidizing the energy good.

Heterogeneous exposure to environmental damages Abstracting from heterogeneous preferences for energy consumption, the second best pollution tax given by (37) resembles the benchmark formula, with utility damages now adjusted by an additional term $\frac{\Gamma}{\Gamma_\chi}$. Assuming $\lambda_i = \lambda = 1$, the decomposition of this term given by equation (38) shows that when exposure to environmental damages χ_i is negatively correlated with market weights, $\Gamma_\chi < \Gamma$ and the second best pollution tax is set at a higher level. Indeed, such a negative correlation corresponds to the situation in which agents who are relatively worse off (lower φ_i) are also more exposed to environmental damages, which leads the planner to set the corrective tax at a higher level than what it would do in the first best where individualized lump-sum transfers are set such that market weights are equal across types.

5.2.2 Quantitative analysis

[To be included: quantitative analysis based on CEX data.]

6 Conclusion

What are the implications of heterogeneity in productivity and asset holdings for optimal carbon pricing? This paper attempts to shed light on this question in a climate-economy model, where environmental degradation generates both production and utility externalities. We extend the analysis from Barrage (2019) by including agent heterogeneity, which provides a micro-foundation for the use of distortionary taxes on labor and capital income. We study both theoretically and quantitatively how different sources of heterogeneity and a concern for redistribution affects the optimal carbon tax relative to the first-best (Pigouvian) scenario and relative to the case with a representative agent. [To be included: results.]

References

Barrage, Lint (2019) "Optimal Dynamic Carbon Taxes in a Climate–Economy Model with Distortionary Fiscal Policy," *The Review of Economic Studies*, 87 (1), 1–39, 10.1093/restud/rdz055.

Bovenberg, Lans and Ruud de Mooij (1994) "Environmental Levies and Distortionary Taxation," *American Economic Review*, 84 (4), 1085–89.

Bovenberg, Lans and Frederick (Rick) van der Ploeg (1994) "Environmental policy, public finance and the labour market in a second-best world," *Journal of Public Economics*, 55 (3), 349–390, `https://EconPapers.repec.org/RePEc:eee:pubeco:v:55:y:1994:i:3:p:349-390`.

Chari, V.V. and Patrick J. Kehoe (1999) "Chapter 26 Optimal fiscal and monetary policy," 1 of Handbook of Macroeconomics, 1671–1745: Elsevier, https://doi.org/10.1016/S1574-0048(99)10039-9.

Cremer, Helmuth, Firouz Gahvari, and Norbert Ladoux (1998) "Externalities and optimal taxation," *Journal of Public Economics*, 70 (3), 343–364, https://doi.org/10.1016/S0047-2727(98)00039-5.

Cronin, Julie Anne, Don Fullerton, and Steven Sexton (2019) "Vertical and Horizontal Redistributions from a Carbon Tax and Rebate," *Journal of the Association of Environmental and Resource Economists*, 6 (S1), 169–208.

Diamond, Peter A and James A Mirrlees (1971) "Optimal taxation and public production I: Production efficiency," *The American economic review*, 61 (1), 8–27.

Douenne, Thomas and Adrien Fabre (2022) "Yellow Vests, Pessimistic Beliefs, and Carbon Tax Aversion," *American Economic Journal: Economic Policy*.

Fried, Stephie, Kevin Michael Novan, and William Peterman (2021) "Recycling Carbon Tax Revenue to Maximize Welfare."

Fried, Stephie, Kevin Novan, and William Peterman (2018) "The Distributional Effects of a Carbon Tax on Current and Future Generations," *Review of Economic Dynamics*, 30, 30–46, `https://EconPapers.repec.org/RePEc:red:issued:16-217`.

Fullerton, Don and Holly Monti (2013) "Can pollution tax rebates protect low-wage earners?" *Journal of Environmental Economics and Management*, 66 (3), 539–553.

Golosov, Mikhail, John Hassler, Per Krusell, and Aleh Tsyvinski (2014) "OPTIMAL TAXES ON FOSSIL FUEL IN GENERAL EQUILIBRIUM," *Econometrica*, 82 (1), 41–88, `http://www.jstor.org/stable/24029171`.

Goulder, Lawrence H, Marc AC Hafstead, GyuRim Kim, and Xianling Long (2019) "Impacts of a carbon tax across US household income groups: What are the equity-efficiency trade-offs?" *Journal of Public Economics*, 175, 44–64.

Jacobs, Bas and Ruud A. de Mooij (2015) "Pigou meets Mirrlees: On the irrelevance of tax distortions for the second-best Pigouvian tax," *Journal of Environmental Economics and Management*, 71, 90–108, https://doi.org/10.1016/j.jeem.2015.01.003.

Jacobs, Bas and Frederick (Rick) van der Ploeg (2019) "Redistribution and pollution taxes with non-linear Engel curves," *Journal of Environmental Economics and Management*, 95 (C), 198–226, `https://EconPapers.repec.org/RePEc:eee:jeeman:v:95:y:2019:i:c:p:198-226`.

Kaplow, Louis (2012) "OPTIMAL CONTROL OF EXTERNALITIES IN THE PRESENCE OF INCOME TAXATION," *International Economic Review*, 53 (2), 487–509, `http://www.jstor.org/stable/23251596`.

Levinson, Arik and James O'Brien (2019) "Environmental Engel curves: Indirect emissions of common air pollutants," *Review of Economics and Statistics*, 101 (1), 121–133.

Pigou, A.C. (1920) *The Economics of Welfare*: Macmillan.

Pizer, William A and Steven Sexton (2019) "The Distributional Impacts of Energy Taxes," *Review of Environmental Economics and Policy*, 13 (1), 104–123.

Rausch, Sebastian, Gilbert E Metcalf, and John M Reilly (2011) "Distributional impacts of carbon pricing: A general equilibrium approach with micro-data for households," *Energy economics*, 33, S20–S33.

Sallee, James M (2019) "Pigou Creates Losers: On the Implausibility of Achieving Pareto Improvements from Efficiency-Enhancing Policies," Working Paper 25831, National Bureau of Economic Research, 10.3386/w25831.

Sandmo, Agnar (1975) "Optimal Taxation in the Presence of Externalities," *The Swedish Journal of Economics*, 77 (1), 86–98, `http://www.jstor.org/stable/3439329`.

Werning, Iván (2007) "Optimal Fiscal Policy with Redistribution*," *The Quarterly Journal of Economics*, 122 (3), 925–967, 10.1162/qjec.122.3.925.

Williams, Roberton, Hal Gordon, Dallas Burtraw, Jared Carbone, and Richard D. Morgenstern (2015) "The Initial Incidence of a Carbon Tax Across Income Groups," *National Tax Journal*, 68 (1), 195–214.

Appendices

A Optimal tax rules in the benchmark model

A.1 Characterization of equilibrium

Let $\varphi \equiv \{\varphi_i\}$ be the market weights normalized so that

$$\sum_j \pi_j \left(\varphi_j e_j^{\gamma(\sigma-1)}\right)^{\frac{1}{\sigma-(1-\sigma)\gamma}} = 1,$$

with $\varphi_i \geq 0$. Then, given aggregate levels C_t, H_t and Z_t, the individual levels can be found by solving the following static subproblem for each period t:

$$U\left(C_t, H_t, Z_t; \varphi\right) \equiv \max_{c_{i,t}, h_{i,t}} \sum_i \pi_i \varphi_i u\left(c_{i,t}, h_{i,t}, Z_t\right), \quad \text{s.t.} \sum_i \pi_i c_{i,t} = C_t, \quad \text{and} \quad \sum_i \pi_i e_i h_{i,t} = H_t. \tag{39}$$

In what follows, we obtain a simple formula for the aggregate indirect utility $U\left(C_t, H_t, Z_t; \varphi\right)$. The Lagrangian for this problem is

$$L = \sum_i \pi_i \varphi_i \left[\frac{\left(c_{i,t}\left(1-\varsigma h_{i,t}\right)^{\gamma}\right)^{1-\sigma}}{1-\sigma} + \frac{\left(1+\alpha_0 Z_t^2\right)^{-(1-\sigma)}}{1-\sigma}\right] + \theta_t^c \left(C_t - \sum_i \pi_i c_{i,t}\right) - \theta_t^h \left(H_t - \sum_i \pi_i e_i h_{i,t}\right),$$

where θ_t^c and θ_t^h are Lagrange multipliers. The first order conditions are

$$[c_{i,t}] : \varphi_i \left(c_{i,t}\left(1-\varsigma h_{i,t}\right)^{\gamma}\right)^{1-\sigma} c_{i,t}^{-1} = \theta_t^c, \quad \forall\, t \geq 0, \tag{40}$$

$$[h_{i,t}] : \varphi_i \left(c_{i,t}\left(1-\varsigma h_{i,t}\right)^{\gamma}\right)^{1-\sigma} \gamma\varsigma \left(1-\varsigma h_{i,t}\right)^{-1} = e_i \theta_t^h, \quad \forall\, t \geq 0, \tag{41}$$

rearranging yields

$$c_{i,t} = \frac{\theta_t^h}{\theta_t^c} \frac{e_i\left(1-\varsigma h_{i,t}\right)}{\gamma\varsigma},$$

so that

$$c_{i,t} = \left(\frac{\theta_t^c}{\varphi_i}\left(\frac{\theta_t^h}{\theta_t^c}\frac{e_i}{\gamma\varsigma}\right)^{\gamma(1-\sigma)}\right)^{-\frac{1}{\sigma-(1-\sigma)\gamma}}$$

$$1-\varsigma h_{i,t} = \frac{\theta_t^c}{\theta_t^h}\frac{\gamma\varsigma}{e_i}\left(\frac{\theta_t^c}{\varphi_i}\left(\frac{\theta_t^h}{\theta_t^c}\frac{e_i}{\gamma\varsigma}\right)^{\gamma(1-\sigma)}\right)^{-\frac{1}{\sigma-(1-\sigma)\gamma}},$$

and summing across types (given that $C_t = \sum_i \pi_i c_{i,t}$, and $H_t = \sum_i \pi_i e_i h_{i,t}$)

$$C_t = \left(\theta_t^c\left(\frac{\theta_t^h}{\theta_t^c}\frac{1}{\gamma\varsigma}\right)^{\gamma(1-\sigma)}\right)^{-\frac{1}{\sigma-(1-\sigma)\gamma}} \sum_i \pi_i \left(\frac{e_i^{\gamma(1-\sigma)}}{\varphi_i}\right)^{-\frac{1}{\sigma-(1-\sigma)\gamma}} \tag{42}$$

$$1-\varsigma H_t = \frac{\theta_t^c}{\theta_t^h}\gamma\varsigma\left(\theta_t^c\left(\frac{\theta_t^h}{\theta_t^c}\frac{1}{\gamma\varsigma}\right)^{\gamma(1-\sigma)}\right)^{-\frac{1}{\sigma-(1-\sigma)\gamma}} \sum_i \pi_i \left(\frac{e_i}{\varphi_i}^{\gamma(1-\sigma)}\right)^{-\frac{1}{\sigma-(1-\sigma)\gamma}} \tag{43}$$

It follows that

$$c_{i,t}^{m}\left(C_t,H_t;\varphi\right)=\omega_i C_t, \tag{44}$$

$$1-\varsigma h_{i,t}^{m}\left(C_t,H_t;\varphi\right)=\frac{\omega_i}{e_i}\left(1-\varsigma H_t\right), \tag{45}$$

where

$$\omega_i=\frac{\left(\varphi_i\left(e_i\right)^{\gamma(\sigma-1)}\right)^{\frac{1}{\sigma-(1-\sigma)\gamma}}}{\sum_i \pi_i\left(\varphi_j e_j^{\gamma(\sigma-1)}\right)^{\frac{1}{\sigma-(1-\sigma)\gamma}}}=\left(\varphi_i\left(e_i\right)^{\gamma(\sigma-1)}\right)^{\frac{1}{\sigma-(1-\sigma)\gamma}}$$

Hence, we can write aggregate indirect utility $U\left(C_t,H_t,Z_t;\varphi\right)$ in terms of the aggregates C_t, H_t, and Z_t

$$U\left(C_t,H_t,Z_t\right)=\sum_j \pi_j\varphi_j\left(\frac{\omega_j^{1+\gamma}}{e_j^{\gamma}}\right)^{1-\sigma}\frac{\left(C_t\left(1-\varsigma H_t\right)^{\gamma}\right)^{1-\sigma}}{1-\sigma}+\sum_i \pi_i\varphi_i\frac{\left(1+\alpha_0 Z_t^2\right)^{-(1-\sigma)}}{1-\sigma}, \tag{46}$$

$$=\frac{\left(C_t\left(1-\varsigma H_t\right)^{\gamma}\right)^{1-\sigma}}{1-\sigma}+\Gamma\frac{\left(1+\alpha_0 Z_t^2\right)^{-(1-\sigma)}}{1-\sigma}, \tag{47}$$

since from the normalization of market weights we have

$$\sum_j \pi_j\varphi_j\left(\frac{\omega_j^{1+\gamma}}{e_j^{\gamma}}\right)^{1-\sigma}=\sum_j \pi_j\left(\varphi_j e_j^{\gamma(\sigma-1)}\right)^{\frac{1}{\sigma-(1-\sigma)\gamma}}=1,$$

and with $\Gamma\equiv\sum_i \pi_i\varphi_i$.

A.2 Implementability condition

Applying the envelope theorem to problem (39) we get

$$U_C\left(C_t,H_t\right)=\theta_t^c,\quad \text{and}\quad U_H\left(C_t,H_t\right)=-\theta_t^h.$$

From the first order conditions of problem (39), we also have

$$\varphi_i u_c\left(c_{i,t},h_{i,t}\right)=\theta_t^c,\quad \text{and}\quad \varphi_i u_h\left(c_{i,t},h_{i,t}\right)=-e_i\theta_t^h.$$

It follows that

$$U_C\left(C_t,H_t\right)=\varphi_i u_c\left(c_{i,t},h_{i,t}\right), \tag{48}$$

$$U_H\left(C_t,H_t\right)=\frac{\varphi_i u_h\left(c_{i,t},h_{i,t}\right)}{e_i}. \tag{49}$$

In any competitive equilibrium these optimality conditions must hold for every agent i. Hence, using (48), (49), and agents' first order conditions given by

$$\beta^t \frac{u_c\left(c_{i,t}, h_{i,t}, Z_t\right)}{u_c\left(c_{i,0}, h_{i,0}, Z_0\right)} = p_t, \quad \forall\, t \geq 0, \tag{50}$$

$$\frac{u_h\left(c_{i,t}, h_{i,t}, Z_t\right)}{u_c\left(c_{i,t}, h_{i,t}, Z_t\right)} = -\left(1-\tau_{H,t}\right) e_i w_t, \quad \forall\, t \geq 0, \tag{51}$$

we obtain

$$\frac{U_H\left(C_t, H_t\right)}{U_C\left(C_t, H_t\right)} = \frac{u_h\left(c_{i,t}, h_{i,t}\right)}{u_c\left(c_{i,t}, h_{i,t}\right) e_i} = -w_t\left(1-\tau_{H,t}\right) \tag{52}$$

and

$$\frac{U_C\left(C_t, H_t\right)}{U_C\left(C_0, H_0\right)} = \frac{u_c\left(c_{i,t}, h_{i,t}\right)}{u_c\left(c_{i,0}, h_{i,0}\right)} = \frac{p_t}{\beta^t}. \tag{53}$$

Given the relationships above we can derive the implementation condition which relies only on the aggregates C_t, and H_t, and market weights φ. Let $c^m_{i,t}\left(C_t, H_t; \varphi\right)$ and $h^m_{i,t}\left(C_t, H_t; \varphi\right)$ be the $\arg\max$ of problem (39) given by (44) and (45) respectively. The budget constraint of agent i implies

$$\sum_{t=0}^{\infty} p_t\left(c^m_{i,t}\left(C_t, H_t; \varphi\right) - \left(1-\tau_{H,t}\right) w_t e_i h^m_{i,t}\left(C_t, H_t; \varphi\right)\right) \leq R_0 a_{i,0} + T,$$

which using (52) and (53) can be restated as

$$U_C\left(C_0, H_0\right)\left(R_0 a_{i,0} + T\right) \geq \sum_{t=0}^{\infty} \beta^t\left(U_C\left(C_t, H_t\right) c^m_{i,t}\left(C_t, H_t; \varphi\right) + U_H\left(C_t, H_t\right) e_i h^m_{i,t}\left(C_t, H_t; \varphi\right)\right), \quad \forall\, i. \tag{54}$$

A.3 Ramsey problem

A.3.1 Problem

Let $\lambda \equiv \{\lambda_i\}$ be the planner's welfare weight on type i, with $\sum_i \pi_i \lambda_i = 1$. Define

$$\begin{aligned} W\left(C_t, H_t, Z_t; \varphi, \theta, \lambda\right) \equiv & \sum_i \pi_i \lambda_i u\left(c^m_{i,t}\left(C_t, H_t; \varphi\right), h^m_{i,t}\left(C_t, H_t; \varphi\right), Z_t\right) \\ & + \sum_i \pi_i \theta_i\left[U_C\left(C_t, H_t\right) c^m_{i,t}\left(C_t, H_t; \varphi\right) + U_H\left(C_t, H_t\right) e_i h^m_{i,t}\left(C_t, H_t; \varphi\right)\right] \end{aligned}$$

where $\pi_i \theta_i$ is the Lagrange multiplier on the implementability constraint of agent i, and $\theta \equiv \{\theta_i\}$. The Ramsey problem can be written as

$$\max_{\substack{\{C_t, H_{1,t}, H_{2,t}, K_{1,t}, K_{2,t}, \\ E_t, Z_t, \mu_t\}_{t=0}^{\infty}, T, \varphi, \tau_0^k \leq 1}} \sum_{t,i} \beta^t W\left(C_t, H_t, Z_t; \varphi, \theta, \lambda\right) - U_C\left(C_0, H_0\right) \sum_i \pi_i \theta_i\left(R_0 a_{i,0} + T\right)$$

subject to

$$
\begin{aligned}
C_t + G_t + K_{t+1} + \Theta_t\left(\mu_t E_t\right) &= \left(1 - D\left(Z_t\right)\right) A_{1,t} F\left(K_{1,t}, H_{1,t}, E_t\right) + \left(1-\delta\right) K_t, \quad \forall\, t \geq 0, \\
E_t &= A_{2,t} G\left(K_{2,t}, H_{2,t}\right), \quad \forall\, t \geq 0, \\
Z_t &= J\left(S_0, E_0^M, ..., E_t^M, \eta_0, ..., \eta_t\right), \quad \forall\, t \geq 0, \\
\frac{F_K(K_{1,t} H_{1,t}, E_t)}{F_H(K_{1,t} H_{1,t}, E_t)} &= \frac{G_K(K_{2,t} H_{2,t})}{G_H(K_{2,t} H_{2,t})}, \\
K_{1,t} + K_{2,t} &= K_t, \quad \forall\, t \geq 0, \\
H_{1,t} + H_{2,t} &= H_t, \quad \forall\, t \geq 0, \\
\sum_j \pi_j \left(\varphi_j e_j^{\gamma(\sigma-1)}\right)^{\frac{1}{\sigma-(1-\sigma)\gamma}} &= 1,
\end{aligned}
$$

where $\beta^t \nu_{jt}$ for $j \in \{1, 2, 3\}$ are the Lagrange multipliers on the feasibility constraints in the order above, and ζ is the multiplier on the normalization constraint on $\{\varphi_i\}$.

A.3.2 First order conditions

The first order conditions are

$$[C_t] : W_C\left(C_t, H_t; \varphi, \theta, \lambda\right) - \nu_{1,t} = 0, \quad \forall\, t \geq 0, \tag{55}$$

$$[H_{1,t}] : W_H\left(C_t, H_t; \varphi, \theta, \lambda\right) + \nu_{1,t}\left(1 - D\left(Z_t\right)\right) A_{1,t} F_H\left(K_{1,t}, H_{1,t}, E_t\right) = 0, \quad \forall\, t \geq 0, \tag{56}$$

$$[H_{2,t}] : W_H\left(C_t, H_t; \varphi, \theta, \lambda\right) + \nu_{2,t} A_{2,t} G_H\left(K_{2,t}, H_{2,t}\right) = 0, \quad \forall\, t \geq 0, \tag{57}$$

$$[K_{1,t+1}] : -\nu_{1,t} + \left[\left(1 - D\left(Z_{t+1}\right)\right) A_{1,t+1} F_K\left(K_{1,t+1}, H_{1,t+1}, E_{t+1}\right) + (1-\delta)\right] \beta \nu_{1,t+1} = 0, \quad \forall\, t \geq 0, \tag{58}$$

$$[K_{2,t+1}] : -\nu_{1,t} + A_{2,t+1} G_K\left(K_{2,t+1}, H_{2,t+1}\right) \beta \nu_{2,t+1} + \left(1-\delta\right) \beta \nu_{1,t+1} = 0, \quad \forall\, t \geq 0, \tag{59}$$

$$
\begin{aligned}
[E_t] : &-\nu_{1,t}\left(\mu_t \Theta_t'\left(\mu_t E_t\right) - \left(1 - D\left(Z_t\right)\right) A_{1,t} F_E\left(K_{1,t}, H_{1,t}, E_t\right)\right) - \nu_{2,t} \\
&- \sum_{j=0}^{\infty} \beta^j \nu_{3,t+j} J_{E_t^M, t+j}\left(1 - \mu_t\right) = 0, \quad \forall\, t \geq 0,
\end{aligned} \tag{60}
$$

$$[Z_t] : W_Z\left(C_t, H_t, Z_t; \varphi, \theta, \lambda\right) - \nu_{1,t} D'\left(Z_t\right) A_{1,t} F\left(K_{1,t}, H_{1,t}, E_t\right) + \nu_{3,t} = 0, \quad \forall\, t \geq 0, \tag{61}$$

$$[\mu_t] : -\nu_{1,t} E_t \Theta_t'\left(\mu_t E_t\right) + \sum_{j=0}^{\infty} \beta^j \nu_{3,t+j} J_{E_t^M, t+j} E_t = 0, \quad \forall\, t \geq 0, \tag{62}$$

$$[T] : \sum_i \pi_i \theta_i = 0, \tag{63}$$

$$[\varphi_i] : \sum_t \beta^t W_{\varphi_i}\left(C_t, H_t, Z_t; \varphi, \theta, \lambda\right) - \frac{\zeta}{\sigma - \left(1-\sigma\right)\gamma} \frac{\pi_i \omega_i}{\varphi_i} = 0, \tag{64}$$

and at $t = 0$,

$$\left[\tau_0^k\right] : U_C\left(C_0, H_0\right)\left(\left(1 - D\left(Z_0\right)\right) A_{1,0} F_K\left(K_{1,0}, H_{1,0}, E_0\right) - \delta\right) \sum_i \pi_i \theta_i a_{i,0} = 0 \tag{65}$$

$$\left[K_{1,0}\right] : \left[\left(1 - D\left(Z_0\right)\right) A_{1,0} F_K\left(K_{1,0}, H_{1,0}, E_0\right) + (1-\delta)\right] \nu_{1,0} - \kappa = 0 \tag{66}$$

$$\left[K_{2,0}\right] : A_{2,0} G_K\left(K_{2,0}, H_{2,0}\right) \nu_{2,0} + (1-\delta)\nu_{1,0} - \kappa = 0 \tag{67}$$

where κ is the Lagrange multiplier on the constraint $K_{1,0} + K_{2,0} = K_0$, and it follows that

$$\left(1 - D\left(Z_0\right)\right) A_{1,0} F_K\left(K_{1,0}, H_{1,0}, E_0\right) \nu_{1,0} = A_{2,0} G_K\left(K_{2,0}, H_{2,0}\right) \nu_{2,0},$$

which together with (56) and (57), implies that

$$\frac{F_K\left(K_{1,0}, H_{1,0}, E_0\right)}{F_H\left(K_{1,0}, H_{1,0}, E_0\right)} = \frac{G_K\left(K_{2,0}, H_{2,0}\right)}{G_H\left(K_{2,0}, H_{2,0}\right)}. \tag{68}$$

A.4 Optimal taxes

A.4.1 Capital and Labor income taxes

From (55) and (56) we obtain

$$\left(1 - D\left(Z_t\right)\right) A_{1,t} F_H\left(K_{1,t}, H_{1,t}, E_t\right) = -\frac{W_H\left(C_t, H_t; \varphi, \theta, \lambda\right)}{W_C\left(C_t, H_t; \varphi, \theta, \lambda\right)}, \quad \forall\, t \geq 0, \tag{69}$$

and using the intertemporal condition (58) we get

$$R_{t+1}^* \equiv 1 + r_{t+1} - \delta = \frac{1}{\beta} \frac{W_C\left(C_t, H_t; \varphi, \theta, \lambda\right)}{W_C\left(C_{t+1}, H_{t+1}; \varphi, \theta, \lambda\right)}, \quad \forall\, t \geq 0, \tag{70}$$

These two equations can be used to back out the optimal taxes on labor and capital income.

Plugging (69) into (52) implies

$$\frac{U_H\left(C_t, H_t\right)}{U_C\left(C_t, H_t\right)} = \frac{W_H\left(C_t, H_t; \varphi, \theta, \lambda\right)}{W_C\left(C_t, H_t; \varphi, \theta, \lambda\right)} \left(1 - \tau_{H,t}\right),$$

which can be rearranged into

$$\tau_{H,t} = 1 - \frac{U_H\left(C_t, H_t\right)}{U_C\left(C_t, H_t\right)} \frac{W_C\left(C_t, H_t; \varphi, \theta, \lambda\right)}{W_H\left(C_t, H_t; \varphi, \theta, \lambda\right)}. \tag{71}$$

In any competitive equilibrium (53) holds, which together with $p_t = R_{t+1} p_{t+1}$ implies

$$\frac{U_C\left(C_{t+1}, H_{t+1}\right)}{U_C\left(C_t, H_t\right)} \beta R_{t+1} = 1.$$

Substituting this into (70), it follows that

$$\frac{R_{t+1}}{R_{t+1}^*} = \frac{W_C\left(C_{t+1}, H_{t+1}; \varphi, \theta, \lambda\right)}{W_C\left(C_t, H_t; \varphi, \theta, \lambda\right)} \frac{U_C\left(C_t, H_t\right)}{U_C\left(C_{t+1}, H_{t+1}\right)}. \tag{72}$$

A.4.2 Excise taxes of energy and emissions

From the abatement first-order condition (62) we have that

$$\Theta_t'(\mu_t E_t) = \frac{1}{\nu_{1,t}} \sum_{j=0}^{\infty} \beta^j \nu_{3,t+j} J_{E_t^M, t+j}.$$

From the climate variable first-order condition (61) we have that

$$\nu_{3,t} = \nu_{1,t} D'(Z_t) A_{1,t} F(K_{1,t}, H_{1,t}, E_t) - W_Z(C_t, H_t, Z_t; \varphi, \theta, \lambda).$$

From the energy first-order condition (60) we have that

$$(1 - D(Z_t)) A_{1,t} F_E(K_{1,t}, H_{1,t}, E_t) - \frac{\nu_{2,t}}{\nu_{1,t}} = \Theta_t'(\mu_t E_t).$$

Combining the first-order conditions for sectoral labor supplies (56) and (57), it follows that

$$\frac{\nu_{2,t}}{\nu_{1,t}} = \frac{(1 - D(Z_t)) A_{1,t} F_H(K_{1,t}, H_{1,t}, E_t)}{A_{2,t} G_H(K_{2,t}, H_{2,t})}$$

and, therefore

$$(1 - D(Z_t)) A_{1,t} F_E(K_{1,t}, H_{1,t}, E_t) = \Theta_t'(\mu_t E_t) + \frac{(1 - D(Z_t)) A_{1,t} F_H(K_{1,t}, H_{1,t}, E_t)}{A_{2,t} G_H(K_{2,t}, H_{2,t})}$$

Then, from (9) we have that

$$\tau_{E,t} = \Theta_t'(\mu_t E_t) = \frac{1}{\nu_{1,t}} \sum_{j=0}^{\infty} \beta^j \nu_{3,t+j} J_{E_t^M, t+j} \tag{73}$$

and from (4), (5), and (8) we have that

$$(1 - D(Z_t)) A_{1,t} F_H(K_{1,t}, H_{1,t}, E_t) = ((1 - D(Z_t)) A_{1,t} F_E(K_{1,t}, H_{1,t}, E_t) - \tau_{I,t} - \tau_{E,t}) A_{2,t} G_H(K_{2,t}, H_{2,t})$$

and therefore

$$\tau_{I,t} = 0. \tag{74}$$

Finally, using (61) in (73) we get

$$\tau_{E,t} = \frac{1}{\nu_{1,t}} \sum_{j=0}^{\infty} \beta^j \left(\nu_{1,t+j} D'(Z_{t+j}) A_{1,t+j} F(K_{1,t+j}, H_{1,t+j}, E_{t+j}) - W_Z(C_{t+j}, H_{t+j}, Z_{t+j}; \varphi, \theta, \lambda) \right) J_{E_t^M, t+j}. \tag{75}$$

A.4.3 Explicit formulas

From (54), substituting the derivatives of U into the definition of $W(C_t, H_t, Z_t; \varphi, \theta, \lambda)$ we get

$$
\begin{aligned}
W(C_t, H_t, Z_t; \varphi, \theta, \lambda) = & \sum_i \pi_i \lambda_i \left(\frac{\omega_i}{\varphi_i} \frac{\left(C_t (1 - \varsigma H_t)^\gamma\right)^{1-\sigma}}{1-\sigma} + \frac{\left(1 + \alpha_0 Z_t^2\right)^{-(1-\sigma)}}{1-\sigma} \right) \\
& + \sum_i \pi_i \theta_i \left[\left(C_t (1 - \varsigma H_t)^\gamma\right)^{1-\sigma} \omega_i - \gamma \left(C_t (1 - \varsigma H_t)^\gamma\right)^{1-\sigma} (1 - \varsigma H_t)^{-1} (e_i - \omega_i (1 - \varsigma H_t)) \right]
\end{aligned}
\tag{76}
$$

Collecting terms and simplifying we obtain

$$
W(C_t, H_t, Z_t; \varphi, \theta, \lambda) = \Phi \frac{\left(C_t (1 - \varsigma H_t)^\gamma\right)^{1-\sigma}}{1-\sigma} + \frac{\left(1 + \alpha_0 Z_t^2\right)^{-(1-\sigma)}}{1-\sigma} + \Psi U_H (C_t, H_t). \tag{77}
$$

where

$$
\Phi \equiv \sum_i \pi_i \omega_i \left(\frac{\lambda_i}{\varphi_i} + (1-\sigma)(1+\gamma)\theta_i \right), \tag{78}
$$

$$
\Psi \equiv \sum_i \frac{\pi_i \theta_i e_i}{\varsigma}. \tag{79}
$$

Substituting the derivatives into equation (71) we get

$$
\tau_{H,t} = \frac{\Psi \varsigma (1 - \varsigma H_t)^{-1}}{\Phi + \Psi \varsigma (1 - \gamma (1 - \sigma)) (1 - \varsigma H_t)^{-1}}, \tag{80}
$$

substituting the derivatives into (72) yields

$$
\frac{R_{t+1}}{R^*_{t+1}} = \frac{\Phi - \Psi \varsigma \gamma (1-\sigma) (1 - \varsigma H_{t+1})^{-1}}{\Phi - \Psi \varsigma \gamma (1-\sigma) (1 - \varsigma H_t)^{-1}}, \tag{81}
$$

and substituting the derivatives into (75) we get

$$
\tau_{E,t} = \frac{1}{\nu_{1,t}} \sum_{j=0}^{\infty} \beta^j \left(\nu_{1,t+j} D'(Z_{t+j}) A_{1,t+j} F(K_{1,t+j}, H_{1,t+j}, E_{t+j}) - \frac{U_Z(Z_{t+j})}{\Gamma} \right) J_{E_t^M, t+j}, \tag{82}
$$

Notice that from (64) we have that

$$
\sum_{t=0}^{\infty} \beta^t \frac{\left(C_t (1 - \varsigma H_t)^\gamma\right)^{1-\sigma}}{1-\sigma} \frac{(1-\sigma)(1+\gamma)}{\sigma - (1-\sigma)\gamma} \frac{\pi_i \omega_i}{\varphi_i} \left(\frac{\lambda_i}{\varphi_i} + \theta_i \right) - \frac{\zeta}{\sigma - (1-\sigma)\gamma} \frac{\pi_i \omega_i}{\varphi_i} = 0, \quad \forall\, i,
$$

and therefore

$$
\frac{\lambda_i}{\varphi_i} + \theta_i = \frac{\zeta}{(1-\sigma)(1+\gamma) V}, \quad \forall\, i,
$$

where

$$V \equiv \sum_{t=0}^{\infty} \beta^t U(C_t, H_t).$$

Using the fact that

$$\sum_i \pi_i \theta_i = 0, \quad \sum_i \pi_i \omega_i = 1, \quad \text{and} \quad \sum_i \pi_i e_i = 1$$

it follows that

$$\sum_j \frac{\pi_j \lambda_j}{\varphi_j} = \frac{\zeta}{(1-\sigma)(1+\gamma)V},$$

and, therefore

$$\theta_i = \sum_j \frac{\pi_j \lambda_j}{\varphi_j} - \frac{\lambda_i}{\varphi_i}. \tag{83}$$

This allows us to rewrite

$$\begin{aligned}
\Phi &= \sum_i \pi_i \omega_i \left(\frac{\lambda_i}{\varphi_i} + (1-\sigma)(1+\gamma) \left(\sum_j \frac{\pi_j \lambda_j}{\varphi_j} - \frac{\lambda_i}{\varphi_i} \right) \right) \\
&= \sum_j \pi_j \frac{\lambda_j}{\varphi_j} + \big(1 - (1+\gamma)(1-\sigma)\big) \operatorname{cov}(\lambda_i/\varphi_i, \omega_i), \\
\Psi &= \frac{1}{\varsigma} \sum_j \pi_j \frac{\lambda_j}{\varphi_j} (1 - e_j) \\
&= -\frac{\operatorname{cov}(\lambda_i/\varphi_i, e_i)}{\varsigma},
\end{aligned}$$

where the last result is obtained using the normalization of productivity levels, $\sum_i \pi_i e_i = 1$. The implementability conditions can be rewritten as

$$\omega_i = \frac{U_C(C_0, H_0)(R_0 a_{i,0} + T) + M e_i}{(1-\sigma)(1+\gamma)V}, \quad \forall\, i,$$

with

$$M \equiv \sum_{t=0}^{\infty} \beta^t \gamma \left(C_t (1 - \varsigma H_t)^{\gamma} \right)^{1-\sigma} (1 - \varsigma H_t)^{-1}.$$

Since

$$\omega_i = \left(\varphi_i e_i^{\gamma(\sigma-1)} \right)^{\frac{1}{\sigma-(1-\sigma)\gamma}}$$

we can express market weights as

$$\varphi_i = \frac{\omega_i^{\sigma-(1-\sigma)\gamma}}{e_i^{\gamma(\sigma-1)}} = \frac{1}{e_i^{\gamma(\sigma-1)}} \left(\frac{U_C(C_0, H_0)(R_0 a_{i,0} + T) + M e_i}{(1-\sigma)(1+\gamma)V} \right)^{\sigma-(1-\sigma)\gamma}$$

A.4.4 Comparison with first best

To compare our second-best results with the first best, we solve the same Ramsey problem except that we now allow for individualized lump-sum transfers. All first order conditions remain the same except for the one with respect to T given by (63): we now have

$$\theta_i = 0, \quad \forall i. \tag{84}$$

From (83), this implies that

$$\frac{\lambda_i}{\varphi_i} = \sum_j \frac{\pi_j \lambda_i}{\varphi_i}, \quad \forall i, \tag{85}$$

and as a consequence we have $\Gamma^{-1} = \Phi$ and $\Psi = 0$, so that for all t, $\tau_{H,t} = 0$ and $\tau_{K,t} = 0$. Substituting for $\nu_{1,t}$ and W_Z in (75), we can express the Pigouvian tax as

$$\tau_{E,t}^{Pigou} = \sum_{j=0}^{\infty} \beta^j \left(\frac{U_C(C_{t+j}, H_{t+j})}{U_C(C_t, H_t)} D'\left(Z_{t+j}\right) A_{1,t+j} F\left(K_{1,t+j}, H_{1,t+j}, E_{t+j}\right) - \frac{U_Z(Z_{t+j})}{U_C(C_t, H_t)} \right) J_{E_t^M, t+j} \ , \tag{86}$$

since in the first best $\nu_{1,t} = W_C = \Phi U_C$ and $W_Z = U_Z \Gamma^{-1}$. This leads to the following decomposition of the Pigouvian tax rule into a production damage component and a utility damage component:

$$\tau_{E,t}^{Pigou,Y} = \sum_{j=0}^{\infty} \beta^j \frac{U_C(C_{t+j}, H_{t+j})}{U_C(C_t, H_t)} \Big(D'\left(Z_{t+j}\right) A_{1,t+j} F\left(K_{1,t+j}, H_{1,t+j}, E_{t+j}\right) \Big) J_{E_t^M, t+j} \ ,$$

$$\tau_{E,t}^{Pigou,U} = (-1) \sum_{j=0}^{\infty} \beta^j \frac{U_Z(Z_{t+j})}{U_C(C_t, H_t)} J_{E_t^M, t+j} \ .$$

If we define the marginal cost of funds as

$$\mathrm{MCF}_t \equiv \frac{\nu_{1,t}\Gamma}{U_C\left(C_t, H_t; \varphi\right)},$$

the share of marginal production damages occurring at time $t+s$ due to a marginal change in emissions at time t, as

$$\Delta_{t+s} \equiv \frac{\beta^j D'\left(Z_{t+s}\right) A_{1,t+s} F\left(K_{1,t+s}, H_{1,t+s}, E_{t+s}\right) J_{E_t^M, t+s}}{\sum_{j=0}^{\infty} \beta^j \Big(D'\left(Z_{t+j}\right) A_{1,t+j} F\left(K_{1,t+j}, H_{1,t+j}, E_{t+j}\right) J_{E_t^M, t+j} \Big)} \ ,$$

then the second best tax given by (82) can be re-written

$$\tau_{E,t} = \sum_{j=0}^{\infty} \frac{\mathrm{MCF}_{t+j}}{\mathrm{MCF}_t} \Delta_{t+j} \tau_{E,t}^{Pigou,Y} + \frac{\tau_{E,t}^{Pigou,U}}{\mathrm{MCF}_t} \ .$$

From (72), we can also write the ratio of MCFs as

$$\frac{\mathrm{MCF}_{t+j}}{\mathrm{MCF}_t} = \prod_{k=1}^{j} \frac{R_{t+k}}{R_{t+k}^*} \ ,$$

from which we see that the ratio is equal to 1 if the capital tax is null for all future periods where current emissions generate production damages. From the planner's first order condition and using the derivative of U with respect to C, we can express the MCF as

$$\begin{aligned}\mathrm{MCF}_t =&\Gamma\Big(\Phi - \Psi\frac{\varsigma\gamma(1-\sigma)}{(1-\varsigma H_t)}\Big)\\ =&\frac{\sum_i \pi_i \frac{\lambda_i}{\varphi_i} + \big(1-(1+\gamma)(1-\sigma)\big)\mathrm{cov}(\lambda_i/\varphi_i,\omega_i) + \mathrm{cov}(\lambda_i/\varphi_i,e_i)\frac{\gamma(1-\sigma)}{(1-\varsigma H_t)}}{\frac{\sum_i \pi_i\lambda_i}{\sum_i \pi_i\varphi_i}}.\end{aligned}$$

Assuming that the planner has utilitarian preferences (*i.e.*, $\forall i$, $\lambda_i = \lambda = 1$), we know from Jensen's inequality that $\sum_i \pi_i \frac{\lambda_i}{\varphi_i} \geq \frac{\sum_i \pi_i\lambda_i}{\sum_i \pi_i\varphi_i}$, so that inequalities lead to a MCF larger than one through the first term. The other two terms however lead to different conclusions, so that the overall effect of inequalities on the MCF remains ambiguous. Indeed, assuming $\sigma > \frac{\gamma}{\gamma+1}$, the second term is negative since $\mathrm{cov}(\lambda_i/\varphi_i,\omega_i)$ is negative as higher market weights φ_i are associated with higher consumption ratios ω_i. As higher productivity levels e_i are also associated with higher market weights, $\mathrm{cov}(\lambda_i/\varphi_i,e_i)$ is negative and for $\sigma < 1$ the third term is positive in the presence of heterogeneity. Whether the MCF is above or below one—and as a consequence, whether the second best pollution tax is above or below the Pigouvian level—is therefore an empirical question.

B Optimal tax rules with Stone-Geary utility and heterogeneous preferences

B.1 Characterization of equilibrium

The derivation of optimal tax rules in this extended version of the model closely follows the method applied to solve the benchmark model. This appendix highlights the differences with the benchmark presented in appendix (A).

Let $\varphi \equiv \{\varphi_i\}$ be the market weights normalized so that

$$\sum_j \pi_j \left(\varphi_j e_j^{\gamma(\sigma-1)}\right)^{\frac{1}{1-(1+\epsilon+\gamma)(1-\sigma)}} = 1,$$

with $\varphi_i \geq 0$. Then, given aggregate levels C_t, D_t, H_t and Z_t, the individual levels can be found by solving the following static subproblem for each period t:

$$\begin{gathered} U\left(C_t, D_t, H_t, Z_t; \varphi\right) \equiv \max_{c_{i,t}, d_{i,t}, h_{i,t}} \sum_i \pi_i\varphi_i u_i\left(c_{i,t}, d_{i,t}, h_{i,t}, Z_t\right), \\ \text{s.t. } \sum_i \pi_i c_{i,t} = C_t, \quad \text{and} \quad \sum_i \pi_i d_{i,t} = D_t, \quad \text{and} \quad \sum_i \pi_i e_i h_{i,t} = H_t. \end{gathered} \tag{87}$$

Following the same steps as in appendix (A), we obtain the following solutions for this problem

$$
\begin{aligned}
c_{i,t}^{m}\left(C_t, D_t, H_t; \varphi\right) &= \omega_i C_t, && (88)\\
d_{i,t}^{m}\left(C_t, D_t, H_t; \varphi\right) &= \bar{d}_{i,t} + \omega_i\big(D_t - \bar{D}_t\big), && (89)\\
1 - \varsigma h_{i,t}^{m}\left(C_t, D_t, H_t; \varphi\right) &= \frac{\omega_i}{e_i}(1 - \varsigma H_t), && (90)
\end{aligned}
$$

where

$$
\omega_i = \left(\varphi_i e_i^{\gamma(\sigma-1)}\right)^{\frac{1}{1-(1+\epsilon+\gamma)(1-\sigma)}} \tag{91}
$$

which enables us to write the aggregate indirect utility in terms of the aggregates and market weights

$$
U\left(C_t, D_t, H_t, Z_t\right) = \frac{\left(C_t(D_t - \bar{D}_t)^{\epsilon}\left(1 - \varsigma H_t\right)^{\gamma}\right)^{1-\sigma}}{1-\sigma} + \Gamma_{\chi}\frac{\big(1 + \alpha_0 Z_t^2\big)^{-(1-\sigma)}}{1-\sigma}, \tag{92}
$$

with $\Gamma_{\chi} \equiv \sum_i \pi_i \varphi_i \chi_i$.

B.2 Implementability condition

From the first order conditions of problem (87) and applying the envelope theorem we have

$$
\begin{aligned}
U_C\left(C_t, D_t, H_t\right) &= \varphi_i u_c\left(c_{i,t}, d_{i,t}, h_{i,t}\right), && (93)\\
U_D\left(C_t, D_t, H_t\right) &= \varphi_i U_D\left(c_{i,t}, d_{i,t}, h_{i,t}\right), && (94)\\
U_H\left(C_t, D_t, H_t\right) &= \frac{\varphi_i u_h\left(c_{i,t}, d_{i,t}, h_{i,t}\right)}{e_i}, && (95)
\end{aligned}
$$

which together with the first order conditions of individual agents' problems give

$$
\frac{U_H\big(C_t, D_t, H_t\big)}{U_C\big(C_t, D_t, H_t\big)} = \frac{u_h\left(c_{i,t}, d_{i,t}, h_{i,t}\right)}{u_c\left(c_{i,t}, d_{i,t}, h_{i,t}\right)e_{i,t}} = -w_t\left(1 - \tau_{H,t}\right), \tag{96}
$$

$$
\frac{U_D\big(C_t, D_t, H_t\big)}{U_C\big(C_t, D_t, H_t\big)} = \frac{U_D\left(c_{i,t}, d_{i,t}, h_{i,t}\right)}{u_c\left(c_{i,t}, d_{i,t}, h_{i,t}\right)} = p_{E,t} + \tau_{D,t}, \tag{97}
$$

and

$$
\frac{U_C\big(C_t, D_t, H_t\big)}{U_C\big(C_0, D_0, H_0\big)} = \frac{u_c\left(c_{i,t}, d_{i,t}, h_{i,t}\right)}{u_c\left(c_{i,0}, e_{i,0}, h_{i,0}\right)} = \frac{p_t}{\beta^t}. \tag{98}
$$

Using (96), (97), and (98) to substitute in households' budget constraint (25), we obtain the implementability condition

$$
\begin{aligned}
U_C\big(C_0, D_0, H_0\big)\left(R_0 a_{i,0} + T\right) \geq \sum_{t=0}^{\infty} \beta^t \Big(& U_C\big(C_t, D_t, H_t\big) c_{i,t}^m\big(C_t, D_t, H_t; \varphi\big) \\
& + U_D\big(C_t, D_t, H_t\big) d_{i,t}^m\left(C_t, D_t, H_t; \varphi\right) \\
& + U_H\big(C_t, D_t, H_t\big) e_i h_{i,t}^m\left(C_t, D_t, H_t; \varphi\right) \Big), \;\; \forall\, i. \qquad (99)
\end{aligned}
$$

B.3 Ramsey problem

B.3.1 Problem

Let again $\lambda \equiv \{\lambda_i\}$ be the planner's welfare weight on type i, with $\sum_i \pi_i \lambda_i = 1$. Define the pseudo-utility function

$$
\begin{aligned}
W\big(C_t, D_t, H_t, Z_t; \varphi, \theta, \lambda\big) \equiv & \sum_i \pi_i \lambda_i u_i\big(c_{i,t}^m\big(C_t, D_t, H_t; \varphi\big), d_{i,t}^m\big(C_t, D_t, H_t; \varphi\big), h_{i,t}^m\big(C_t, D_t, H_t; \varphi\big), Z_t\big) \\
& + \sum_i \pi_i \theta_i \Big[U_C\big(C_t, D_t, H_t\big) c_{i,t}^m\big(C_t, D_t, H_t; \varphi\big) \\
& + U_D\big(C_t, D_t, H_t\big) d_{i,t}^m\big(C_t, D_t, H_t; \varphi\big) + U_H\big(C_t, D_t, H_t\big) e_{i,t} h_{i,t}^m\big(C_t, D_t, H_t; \varphi\big) \Big],
\end{aligned}
$$

where $\pi_i \theta_i$ is the Lagrange multiplier on the implementability constraint of agent i, and $\theta \equiv \{\theta_i\}$. The new Ramsey problem can be written as

$$
\max_{\substack{\{C_t, H_{1,t}, H_{2,t}, K_{1,t}, K_{2,t}, \\ D_t, E_{1,t}, Z_t, \mu_t\}_{t=0}^{\infty}, T, \varphi, \tau_0^k \leq 1}} \sum_{t,i} \beta^t W\left(C_t, D_t, H_t, Z_t; \varphi, \theta, \lambda\right) - U_C\left(C_0, D_0, H_0, Z_0\right) \sum_i \pi_i \theta_i \left(R_0 a_{i,0} + T\right),
$$

subject to

$$
\begin{aligned}
C_t + G_t + K_{t+1} + \Theta_t(\mu_t E_t) &= (1 - D(Z_t)) A_{1,t} F(K_{1,t}, H_{1,t}, E_{1,t}) + (1-\delta) K_t, \quad \forall\, t \geq 0, \\
E_t &= A_{2,t} G(K_{2,t}, H_{2,t}), \quad \forall\, t \geq 0, \\
Z_t &= J\left(S_0, E_0^M, ..., E_t^M, \eta_0, ..., \eta_t\right), \quad \forall\, t \geq 0, \\
\frac{F_K(K_{1,t} H_{1,t}, E_{1,t})}{F_H(K_{1,t} H_{1,t}, E_{1,t})} &= \frac{G_K(K_{2,t} H_{2,t})}{G_H(K_{2,t} H_{2,t})}, \\
K_{1,t} + K_{2,t} &= K_t, \quad \forall\, t \geq 0, \\
H_{1,t} + H_{2,t} &= H_t, \quad \forall\, t \geq 0, \\
D_t + E_{1,t} &= E_t, \quad \forall\, t \geq 0, \\
\sum_j \pi_j \left(\varphi_j e_j^{\gamma(\sigma-1)}\right)^{\frac{1}{1-(1+\epsilon+\gamma)(1-\sigma)}} &= 1,
\end{aligned}
$$

where $D_t + E_{1,t} = E_t$ is the only additional constraint compared to the benchmark problem.

B.4 Optimal taxes

From the first order conditions of the Ramsey problem, we can show that

$$
\tau_{H,t} = 1 - \frac{U_H(C_t, D_t, H_t)}{U_C(C_t, D_t, H_t)} \frac{W_C(C_t, D_t, H_t; \varphi, \theta, \lambda)}{W_H(C_t, D_t, H_t; \varphi, \theta, \lambda)}, \tag{100}
$$

$$
\frac{R_{t+1}}{R_{t+1}^*} = \frac{W_C(C_{t+1}, D_{t+1}, H_{t+1}; \varphi, \theta, \lambda)}{W_C(C_t, D_t, H_t; \varphi, \theta, \lambda)} \frac{U_C(C_t, D_t, H_t)}{U_C(C_{t+1}, D_{t+1}, H_{t+1})}, \tag{101}
$$

$$
\tau_{E,t} = \frac{1}{\nu_{1,t}} \sum_{j=0}^{\infty} \beta^j \left(\nu_{1,t+j} D'(Z_{t+j}) A_{1,t+j} F(K_{1,t+j}, H_{1,t+j}, E_{t+j}) - W_Z(C_{t+j}, H_{t+j}, Z_{t+j}; \varphi, \theta, \lambda)\right) J_{E_t^M, t+j}, \tag{102}
$$

and

$$
\tau_{I,t} = 0. \tag{103}
$$

Using the first order conditions with respect to D_t, $E_{1,t}$ and C_t we have

$$
W_E(C_t, D_t, H_t, Z_t; \varphi, \theta, \lambda) = W_C(C_t, D_t, H_t, Z_t; \varphi, \theta, \lambda)(1 - D(Z_t)) A_{1,t} F_E(K_{1,t}, H_{1,t}, E_{1,t}),
$$

which together with (97) and the final good firm's first order condition with respect to $E_{1,t}$ (given by (5) in the benchmark model) gives

$$
\tau_{D,t} = \frac{U_D(C_t, D_t, H_t)}{U_C(C_t, D_t, H_t)} - \frac{W_E(C_t, D_t, H_t, Z_t; \varphi, \theta, \lambda)}{W_C(C_t, D_t, H_t, Z_t; \varphi, \theta, \lambda)}. \tag{104}
$$

Using our functional form assumption, we can rewrite the pseudo-utility function as follows

$$W\left(C_t, D_t, H_t, Z_t; \varphi, \theta, \lambda\right) = \Phi U(C_t, D_t, H_t) + \frac{\sum_i \pi_i \lambda_i \chi_i}{\sum_i \pi_i \varphi_i \chi_i} U(Z_t) + \Psi U_H(C_t, D_t, H_t) + \Lambda_t U_D(C_t, D_t, H_t), \tag{105}$$

with

$$U(C_t, D_t, H_t) = \frac{(C_t(D_t - \bar{D}_t)^{\epsilon}(1 - \varsigma H_t)^{\gamma})^{1-\sigma}}{1-\sigma},$$
$$U(Z_t) = \Gamma_\chi \frac{(1 + \alpha_0 Z_t^2)^{-(1-\sigma)}}{1-\sigma},$$

where

$$\Phi \equiv \sum_i \pi_i \omega_i \Big(\frac{\lambda_i}{\varphi_i} + \big(1-\sigma\big)\big(1+\epsilon+\gamma\big)\theta_i\Big), \tag{106}$$

$$\Psi \equiv \frac{1}{\varsigma} \sum_i \pi_i \theta_i e_i, \tag{107}$$

$$\Lambda_t \equiv \sum_i \pi_i \theta_i \bar{d}_{i,t}. \tag{108}$$

Substituting the derivatives into equation (100) we get

$$\tau_{H,t} = 1 - \frac{\Phi + \Psi \frac{U_{CH}}{U_C} + \Lambda_t \frac{U_{CD}}{U_C}}{\Phi + \Psi \frac{U_{HH}}{U_H} + \Lambda_t \frac{U_{DH}}{U_H}} = \frac{\Psi \varsigma (1 - \varsigma H_t)^{-1}}{\Phi + \Psi \frac{\varsigma\big(1-\gamma(1-\sigma)\big)}{(1-\varsigma H_t)} + \Lambda_t \frac{\epsilon(1-\sigma)}{(D_t - \bar{D}_t)}}, \tag{109}$$

substituting the derivatives into (101) yields

$$\frac{R_{t+1}}{R^*_{t+1}} = \frac{\Phi + \Lambda_{t+1} \frac{U_{CD_{t+1}}}{U_{C_{t+1}}} + \Psi \frac{U_{CH_{t+1}}}{U_{C_{t+1}}}}{\Phi + \Lambda_t \frac{U_{CD_t}}{U_{C_t}} + \Psi \frac{U_{CH_t}}{U_{C_t}}} = \frac{\Phi + \Lambda_{t+1} \frac{\epsilon(1-\sigma)}{(D_{t+1} - \bar{D}_{t+1})} - \Psi \frac{\varsigma\gamma(1-\sigma)}{(1-\varsigma H_{t+1})}}{\Phi + \Lambda_t \frac{\epsilon(1-\sigma)}{(D_t - \bar{D}_t)} - \Psi \frac{\varsigma\gamma(1-\sigma)}{(1-\varsigma H_t)}}, \tag{110}$$

substituting the derivatives into (102) we get

$$\tau_{E,t} = \frac{1}{\nu_{1,t}} \sum_{j=0}^{\infty} \beta^j \Bigg(\nu_{1,t+j} D'\left(Z_{t+j}\right) A_{1,t+j} F\left(K_{1,t+j}, H_{1,t+j}, E_{t+j}\right) - \frac{\sum_i \pi_i \lambda_i \chi_i}{\sum_i \pi_i \varphi_i \chi_i} U_Z(Z_{t+j})\Bigg) J_{E_t^M, t+j}, \tag{111}$$

and finally substituting the derivatives into (104) we get

$$\tau_{D,t} = \frac{\Lambda_t (D_t - \bar{D}_t)^{-1} U_D}{\Phi U_C + \Psi U_{HC} + \Lambda_t U_{DC}} = \frac{\Lambda_t \epsilon C_t}{\Phi + \Psi \varsigma \gamma (\sigma - 1)(1 - \varsigma H_t)^{-1} - \Lambda_t \epsilon (\sigma - 1)(D_t - \bar{D}_t)^{-1}}. \tag{112}$$

We can then use the first order conditions with respect to market weights to obtain

$$\theta_i = \sum_j \frac{\pi_j \lambda_j}{\varphi_j} - \frac{\lambda_i}{\varphi_i}, \tag{113}$$

from which we can rewrite

$$\Phi = \sum_i \pi_i \omega_i \Big(\frac{\lambda_i}{\varphi_i} + (1-\sigma)(1+\epsilon+\gamma) \Big(\sum_j \frac{\pi_j \lambda_j}{\varphi_j} - \frac{\lambda_i}{\varphi_i} \Big) \Big) \tag{114}$$

$$= \sum_j \pi_j \frac{\lambda_j}{\varphi_j} + \Big(1 - (1+\epsilon+\gamma)(1-\sigma) \Big) \mathrm{cov}(\lambda_i/\varphi_i, \omega_i), \tag{115}$$

$$\Psi = \frac{1}{\varsigma} \sum_i \pi_i \Big(\sum_j \frac{\pi_j \lambda_j}{\varphi_j} - \frac{\lambda_i}{\varphi_i} \Big) e_i \tag{116}$$

$$= -\frac{\mathrm{cov}(\lambda_i/\varphi_i, e_i)}{\varsigma}, \tag{117}$$

$$\Lambda_t = \sum_i \pi_i \Big(\sum_j \frac{\pi_j \lambda_j}{\varphi_j} - \frac{\lambda_i}{\varphi_i} \Big) \bar{d}_{i,t} \tag{118}$$

$$= -\mathrm{cov}(\lambda_i/\varphi_i, \bar{d}_{i,t}), \tag{119}$$

and obtain an expression for market weights

$$\varphi_i = \frac{1}{e_i^{\gamma(\sigma-1)}} \left(\frac{U_C(C_0, D_0, H_0)(R_0 a_{i,0} + T) + \sum_t \beta^t \Big(U_H(C_t, D_t, H_t) \frac{e_i}{\varsigma} - U_D(C_t, D_t, H_t) \bar{d}_{i,t} \Big)}{(1-\sigma)(1+\epsilon+\gamma) \sum_t \beta^t U(C_t, D_t, H_t)} \right)^{1-(1+\epsilon+\gamma)(1-\sigma)}$$

In order to compare the second best pollution tax with its first best level, we can solve the same Ramsey problem but allow the planner to use individualized lump-sum transfers. As in the benchmark model, this leads to $\theta_i = 0$ for all i, which implies

$$\tau_{E,t}^{Pigou} = \sum_{j=0}^{\infty} \beta^j \left(\frac{U_C(C_{t+j}, D_{t+j}, H_{t+j})}{U_C(C_t, D_t, H_t)} D'(Z_{t+j}) A_{1,t+j} F(K_{1,t+j}, H_{1,t+j}, E_{t+j}) - \frac{U_Z(Z_{t+j})}{U_C(C_t, D_t, H_t)} \right) J_{E_t^M, t+j} \ . \tag{120}$$

We can again decompose this formula into a production and a utility component, and express the second best tax as

$$\tau_{E,t} = \sum_{j=0}^{\infty} \frac{\mathrm{MCF}_{t+j}}{\mathrm{MCF}_t} \Delta_{t+j} \tau_{E,t}^{Pigou,Y} + \frac{\Gamma}{\Gamma_\chi} \frac{\tau_{E,t}^{Pigou,U}}{\mathrm{MCF}_t} \ . \tag{121}$$

with MCF_t, Δ_t, and Γ defined as in the benchmark model, *i.e.*

$$
\begin{aligned}
\mathrm{MCF}_t &\equiv \frac{\nu_{1,t}\Gamma}{U_C\left(C_t, H_t; \varphi\right)} = \Gamma\Big(\Phi - \Psi\frac{\varsigma\gamma(1-\sigma)}{(1-\varsigma H_t)} + \Lambda_t\frac{\epsilon(1-\sigma)}{D_t - \bar{D}_t}\Big), \\
\Delta_{t+s} &\equiv \frac{\beta^j D'\left(Z_{t+s}\right) A_{1,t+s} F\left(K_{1,t+s}, H_{1,t+s}, E_{1,t+s}\right) J_{E_t^M, t+s}}{\sum_{j=0}^{\infty} \beta^j \Big(D'\left(Z_{t+j}\right) A_{1,t+j} F\left(K_{1,t+j}, H_{1,t+j}, E_{1,t+j}\right) J_{E_t^M, t+j}\Big)}, \\
\Gamma &\equiv \sum_i \pi_i \varphi_i = \frac{\sum_i \pi_i \varphi_i}{\sum_i \pi_i \lambda_i}.
\end{aligned}
$$