Optimal Fiscal Policy in a Second-Best Climate-Economy Model with Heterogeneous Agents

Thomas Douenne *† Albert Jan Hummel *‡ Marcelo Pedroni *§

June 9, 2022

Disclaimer: This document is a preliminary draft.

Link to most recent version

Abstract

This paper studies optimal fiscal policy in a climate-economy model with heterogeneous agents. When individualized lump-sum taxation is not available, distortionary taxes on labor and capital income are levied to provide redistribution. The optimal pollution tax rule is then a modified Pigouvian formula that accounts for tax distortions, where the Pigouvian level depends on the degree of inequalities and tax distortions play an ambiguous role. In a quantitative analysis where the climate model is calibrated to DICE and the fiscal system to the one of the U.S., we show that tax distortions have a negligible effect on the optimal carbon tax, but inequalities reduce it by 4% in our baseline calibration. Optimal carbon taxation generates moderately negative but progressive welfare effects in the 21st century, and large positive but regressive effects thereafter.

JEL classification: E62, H21, H23, Q5

Keywords: Climate policy; Carbon taxes; Social cost of carbon; Optimal taxation; Double dividend; Heterogeneous agents.

*University of Amsterdam, Amsterdam School of Economics. Present address: University of Amsterdam, Roetersstraat 11, 1018 WB Amsterdam, Netherlands.

†email: t.r.g.r.douenne@uva.nl

‡email: a.j.hummel@uva.nl

§email: m.pedroni@uva.nl

1 Introduction

Economic inequality and environmental degradation are certainly two of the most critical issues facing societies today. In order to address these two problems, economists have long argued for the use of fiscal instruments: labor and capital taxes can be used to provide redistribution, and following the Pigouvian principle a pollution tax can be used to internalize environmental externalities. However, pollution taxes also have distributional implications as they reduce purchasing power and because individuals are heterogeneously affected by environmental degradation. Conversely, capital and labor taxes also affect the costs and benefits of improving the environment by reducing incentives to work and invest. The goal of this study is to analyze how these instruments should be jointly optimized if society wishes to tackle both inequality and environmental degradation.

We address this question from both a theoretical and a quantitative perspective. To do so, this paper presents a dynamic second-best climate-economy model with heterogeneous agents. We use the technique introduced by Werning (2007) to extend the climate-economy model of Barrage (2019) to heterogeneous agents. In our model, individuals derive utility from consumption, leisure, and environmental quality. The final consumption good is produced using energy as one of its inputs. Energy production is polluting, and pollution leads to environmental degradation that affects productivity and households' utility. As in Barrage (2019), energy producers can reduce the emission intensity of their output by engaging in costly abatement activities. Because of these costs, positive abatement will occur only if producers also need to pay for their pollution, for example through a pollution tax. The government thus faces multiple tasks at once: mitigating the pollution externality, providing redistribution, and financing some exogenous government spending.

We model this as a Ramsey problem in which the government chooses the level of linear taxes—in particular, taxes on labor and capital income, energy, and pollution—and a uniform lump-sum transfer to maximize aggregate welfare. Because agents are heterogeneous but tax instruments are anonymous, the government must rely on distortionary instruments to provide redistribution. We analytically characterize optimal tax formulas and study the implications of heterogeneity for optimal pollution taxation. We then use our model to examine how inequalities and distortionary taxation affect the social cost of carbon (SCC) and the optimal carbon tax. We calibrate our climate model following DICE 2016 (Nordhaus, 2017). On the economic side, we calibrate the fiscal system and agent heterogeneity (first in productivity, later in wealth and preferences for energy consumption) to match U.S. data. Conceptually, our quantitative analysis examines the optimal fiscal policy of the U.S. if they accounted for the negative global impact of their emissions.[1]

Theoretically, we find that the optimal pollution tax is a modified Pigouvian rule that accounts for tax distortions via the marginal cost of public funds (MCF). However, because uniform lump-sum taxation is available, the MCF is no longer unambiguously above one. In fact, for the utility specification we use in our quantitative analysis we show that the MCF is "on average" equal to 1, *i.e.*

[1]Specifically, we consider the problem of the U.S. government with its emissions scaled up to the global level. An equivalent interpretation is that the world consists of a number of U.S. economies coordinating on their climate policies.

it moves around 1 in the optimal tax system pushing the optimal pollution tax temporarily above and below the Pigouvian level. These temporary tax distortions are driven by the costs the planner faces when implementing its preferred allocation. Although these costs are on average null in the presence of lump-sum taxation, they may not be in each period. We provide conditions under which these costs are always null, and discuss the determinants of temporal variations in tax distortions when they are not. Our theoretical results also highlight the role of consumption inequalities. When the MCF is equal to unity, the second-best pollution tax is Pigouvian, but the Pigouvian tax is evaluated at the second-best allocation. We show that consumption heterogeneity affects the Pigouvian tax ambiguously through the opportunity cost of emission abatement. On the one hand, consumption is valued less in the presence of inequalities because it disproportionately goes in the hand of richer agents with lower marginal utilities of consumption. On the other hand, by concavity of the utility function, consumption inequalities increase the average marginal utility of consumption, and thus the opportunity cost of abatement. We show that with iso-elastic preferences, the latter effect dominates if and only if the intertemporal elasticity of substitution is lower than 1, in which case inequalities reduce the value of the Pigouvian tax.

Quantitatively, we find that the MCF plays an insignificant role. The second-best carbon tax starts at about 0.5% below the SCC, and then fluctuates at about 0.1% above or below. The SCC is, however, significantly affected by the presence of inequalities: consumption heterogeneity drives the SCC down by about 4% in our baseline calibration. We then compare our optimal policy to the one of a "climate skeptic" planner who optimizes fiscal instruments assuming climate change is exogenous, thus setting the carbon tax to zero. We find that the additional revenue raised by the carbon tax is about equally split between increasing transfers and reducing the labor income tax. Turning to welfare, we find that the optimal carbon tax policy has moderately negative but progressive effects in the 21$^{\text{st}}$ century, and very large positive but regressive effects afterwards.

We also examine extensions to our benchmark model. First, we show that the role of tax distortions and inequalities are robust to a more severe calibration of the damage function that drives the social cost of carbon about three times higher. Second, we theoretically characterize and quantitatively compute third-best fiscal policies, *i.e.* optimal fiscal policies when either the labor or the capital income tax is exogenously fixed at its current level. The effect of inequalities on the social cost of carbon remains similar to our benchmark, although it becomes larger when the planner cannot reduce inequalities as much as it would like to. Tax distortions still play an insignificant role through the MCF, but the additional constraint on fiscal instruments now generates a new fiscal interaction term which enters additively into the pollution tax formula. When the labor or capital income tax is exogenously fixed below (resp. above) its optimal value, this term is negative (resp. positive) and the third-best tax rule is set below (resp. above) the second-best level. Finally, we present a version of the model where households consume an additional dirty good that uses energy as its only input. In order to capture heterogeneous budget shares that vary with income, we assume agents' preferences over these two goods are non-homothetic. We show that as long as agents preferences are identical, the optimal tax formulas remain unaffected. When agents display heterogeneous preferences over the dirty good however, the planner chooses to add a tax (resp. subsidy) to the dirty good if the agents it values relatively more consume relatively less (resp. more) of it.

Our paper contributes to two strands of the literature. First, it contributes to the literature on the optimal taxation of pollution. In a pioneering work, Pigou (1920) established that the first-best policy response to an externality was to implement a tax equal to its social cost. An extensive literature has then investigated optimal pollution taxation in a second-best environment. In a representative agent framework, when the government does not have access to lump-sum transfers to finance public expenditures, distortionary taxes typically raise the MCF above 1, and it becomes optimal to set the pollution tax below the Pigouvian level (see *e.g.*, Sandmo, 1975; Bovenberg and de Mooij, 1994; Bovenberg and van der Ploeg, 1994).[2] More recently, other papers have considered this problem with heterogeneous agents and uniform lump-sum taxation (see *e.g.*, Jacobs and de Mooij, 2015; Jacobs and van der Ploeg, 2019), arguing that in this set-up the MCF is equal to 1 and the second-best tax is set at the Pigouvian level.[3] While these papers focus on static settings and model the pollution externality in a stylized manner, the recent work of Barrage (2019) creates a critical bridge between the climate-economy literature and the dynamic public finance literature. Her framework integrates a climate-economy model in the spirit of Golosov et al. (2014) into the representative agent Ramsey model of Chari and Kehoe (1999). In this setting, tax distortions again call for lower taxes. Our main innovation relative to Barrage (2019) is to introduce heterogeneous agents, which we see as critical for two reasons. First, this allows us to jointly study environmental and equity issues. In addition of the importance of equity in normative analysis, recent experience has shown that the distributional effects of environmental policies were also critical to ensure their public support.[4] Second, agents' heterogeneity provides a sound foundation for the study of second-best policies. In representative agent settings, the second-best environment arises because lump-sum transfers are assumed unfeasible: governments therefore *need to* rely on distortionary taxes to finance their expenditures. Yet, in practice lump-sum transfers are feasible as they simply correspond to the intercept on a tax scheme.[5] With heterogeneous agents, lump-sum transfers are no longer excluded as long as they do not discriminate between agents. Although this non-distortionary source of public income is available, governments now *want to* use distortionary taxes to provide redistribution. While our optimal tax formulas resemble the ones in Barrage (2019), the effect of tax distortions is now more ambiguous. Quantitatively, we find that variations in the MCF play an insignificant role, hence the optimal pollution tax is almost Pigouvian. However, we also highlight how inequalities affect the social cost of pollution, and find that

[2] For further references on second-best pollution taxation in representative agents models, see Barrage (2019).

[3] Other papers jointly studying optimal pollution taxation and redistribution include, among others, Pirttilä and Tuomala (1997), Cremer et al. (1998, 2003), Micheletto (2008), and Kaplow (2012).

[4] Public protests against policy-induced increases in energy prices have recently occurred in many countries worldwide. For instance, in France the Yellow Vests movement strongly opposed carbon tax increases due to the expected impact on households' purchasing power, leading to the abandonment of the scheduled carbon tax reforms (Douenne and Fabre, 2022).

[5] Recent policy proposals such as the carbon tax and dividend advocated by the Climate Leadership Council even call for using such instruments to redistribute the carbon tax revenue. See Economists Statement on Carbon Dividends signed by 3,354 American economists in the Wall Street Journal (2019) in support of carbon pricing with lump-sum rebates.

the optimal carbon tax is at least 4% lower than what it would be absent inequalities. Finally, we show that with heterogeneous agents the weak double dividend hypothesis (see *e.g.*, Goulder, 1995) needs to be qualified. At the optimum, the welfare gains from a marginal reduction in tax distortions is equal to the marginal cost from increasing inequalities, hence the optimal policy divides the carbon tax revenue about equally between reducing tax distortions and providing redistribution.[6]

Second, this paper contributes to the analysis of the distributional effects of environmental taxes in general equilibrium. An extensive literature has analyzed the distributional effects of environmental taxes through the consumption channel (for a recent survey, see Pizer and Sexton, 2019), generally pointing to regressive effects since the consumption share of polluting goods tends to decrease with income (Levinson and OBrien, 2019). More recently, several authors have also analyzed the heterogeneous incidence of environmental taxes on households' income. While a number of papers found progressive effects due to the larger negative impact of the policy on capital income relative to labor income and transfers (see *e.g.* Rausch et al., 2011; Fullerton and Monti, 2013; Williams et al., 2015; Goulder et al., 2019), the recent work of Känzig (2021) shows—exploiting exogenous shocks to the EU-ETS price—that carbon taxation has a larger impact on poor households' income in the U.K. because these households are over-represented in pro-cyclical sectors that are more impacted by the tax. Many papers have also shown that the incidence of carbon taxation largely depends on how the tax revenue is recycled. In particular, Fried et al. (2018) study the economic impact of introducing a carbon tax with three alternative revenue-recycling schemes in a quantitative OLG model with heterogeneity within-generations. They show that while a uniform lump-sum rebate is more costly than reductions of the labor or capital tax rates in steady state, it is more favorable to the current generation and leads to less adverse distributional effects.[7] Finally, a few papers have considered the heterogeneous environmental benefits of climate change mitigation, between generations (*e.g.*, Leach, 2009; Kotlikoff et al., 2021) or between regions (*e.g.*, Hassler and Krusell, 2012; Krusell and Smith, 2015; Cruz and Rossi-Hansberg, 2021). In this paper, we jointly study the financial and environmental impacts from optimal pollution taxation, both over time and between heterogeneous agents. We find that accounting for environmental benefits, current rich households lose the most from carbon taxation, but future rich households win the most.

The rest of the paper is organized as follows. Section 2 presents the model, and Section 3 the optimal tax formulas. Section 4 describes our calibration and Section 5 presents our main quantitative exercise. Extensions of our main framework are provided in Section 6. Section 7 concludes.

[6]This result echoes the recent findings of Fried et al. (2021) who study the optimal recycling policy for an exogenous carbon tax introduced in a sub-optimal tax system. In their model with heterogeneity between and within generations, they find that two-third of the carbon tax revenue should be used to reduce taxes on capital income, one third to provide redistribution.

[7]Leach (2009), Rausch (2013), and Rausch and Yonezawa (2018) also quantitatively investigate the distributional effects from revenue recycling across generations, with a representative agent for each generation. Other papers use a dynamic model to compute the incidence of carbon tax reforms, and simulate the distributional effects across heterogeneous agents in the initial period (Williams et al., 2015) or over different time intervals (Goulder et al., 2019). All these papers consider exogenous reforms and—with the exception of Leach (2009)—ignore environmental effects.

2 Model

The model builds on Barrage (2019): one sector of the economy produces a final good using capital, labor, and energy that is produced in the second sector. Energy production generates pollution that leads to environmental degradation, which in turn affects productivity and households' utility. The government finances an exogenous stream of expenditures using taxes on labor income, capital income, energy, and pollution, as well as a lump-sum tax. The key differences with Barrage (2019) are that in our model, agents are heterogeneous and the government has access to a (non-individualized) lump-sum tax or transfer. Consequently, although the government has access to a non-distortionary source of revenue, it uses distortionary taxes for redistributive purposes.

2.1 Households

We consider an economy populated by a continuum of infinitely-lived agents divided into types $i \in I$ of size π_i. The total population size in period t is N_t. Each agent, or dynasty of type $i \in I$ ranks streams of consumption of a final good $c_{i,t}$, labor supply $h_{i,t}$, and environmental degradation Z_t according to the preferences

$$\sum_{t=0}^{\infty} \beta^t N_t u\left(c_{i,t}, h_{i,t}, Z_t\right). \tag{1}$$

In our benchmark, agents are assumed to differ in two ways: their productivity levels, e_i, and their initial asset holdings, $a_{i,0}$. Productivity levels are normalized such that $\sum_i \pi_i e_i = 1$. Agents' assets are composed of government debt and capital and we denote respectively $b_{i,t}$ and $k_{i,t}$ the number of units of these assets held by agents of type i between periods $t-1$ and t, with $a_{i,t} = b_{i,t} + k_{i,t}$. Aggregates are denoted without the subscript i: $C_t = N_t \sum_i \pi_i c_{i,t}$, $H_t = N_t \sum_i \pi_i e_i h_{i,t}$, $B_t = N_t \sum_i \pi_i b_{i,t}$, and $K_t = N_t \sum_i \pi_i k_{i,t}$. In addition, per period average consumption and hours worked are denoted by $c_t = \sum_i \pi_i c_{i,t} = C_t / N_t$ and $h_t = \sum_i \pi_i e_i h_{i,t} = H_t / N_t$.

Let p_t denote the price of the consumption good in period t in terms of consumption in period 0 (so that $p_0 = 1$), w_t and r_t denote the real wage and the rental rate of capital in period t, and R_t its gross return (between $t-1$ and t). Finally, let $\tau_{H,t}$ and $\tau_{K,t}$ represent the labor and capital income taxes, and T_t the *aggregate* uniform lump-sum transfers received by all households in period t. Given $k_{i,0}$, $b_{i,0}$, prices $\{p_t, w_t, R_t\}_{t=0}^{\infty}$ and policies $\{\tau_{H,t}, \tau_{K,t}, T_t\}_{t=0}^{\infty}$, agents of type i choose $\{c_{i,t}, h_{i,t}, k_{i,t+1}, b_{i,t+1}\}_{t=0}^{\infty}$ to maximize (1) subject to the budget constraint

$$\sum_{t=0}^{\infty} p_t N_t \left(c_{i,t} + k_{i,t+1} + b_{i,t+1}\right) \leq \sum_{t=0}^{\infty} p_t N_t \left(\left(1 - \tau_{H,t}\right) w_t e_i h_{i,t} + R_t \left(k_{i,t} + b_{i,t}\right) + T_t / N_t\right),$$

where $R_t \equiv 1 + \left(1 - \tau_{K,t}\right)\left(r_t - \delta\right)$, for $t \geq 0$. Here, we use the convention that the capital income tax is levied on the rate of return net of depreciation, but none of our results depend on it. No arbitrage requires $p_t = R_{t+1} p_{t+1}$, and defining $T \equiv \sum_{t=0}^{\infty} p_t T_t$ as the present value of lump-sum transfers, the budget constraint can equivalently be written as

$$\sum_{t=0}^{\infty} p_t N_t \Big(c_{i,t} - (1 - \tau_{H,t})\, w_t e_i h_{i,t} \Big) \leq R_0 N_0 a_{i,0} + T. \tag{2}$$

From the first order conditions of agent i's problem we have

$$\begin{aligned} \beta^t \frac{u_{c,i,t}}{u_{c,i,0}} &= p_t, \quad \forall\, t \geq 0, \\ \frac{u_{h,i,t}}{u_{c,i,t}} &= -\left(1 - \tau_{H,t}\right) e_i w_t, \quad \forall\, t \geq 0, \end{aligned}$$

which holds across all agents. To reduce notations, we use subscripts x, i, t to denote partial derivatives with respect to argument x for agent of type i at time t, and we keep the arguments of the derivatives implicit.

2.2 Final-good sector

As in Barrage (2019), there are two production sectors. In the final-good sector, indexed by 1, a consumption-capital good is produced with a concave, constant returns to scale technology, $F\left(K_{1,t}, H_{1,t}, E_t\right)$, that uses capital $K_{1,t}$, labor $H_{1,t}$, and energy E_t. The total factor productivity is given by $A_{1,t}$ and the function $D\left(Z_t\right)$ controls the damages to production implied by environmental degradation, with $D'(Z_t) > 0$. The output $Y_{1,t}$ is given by

$$Y_{1,t} = (1 - D(Z_t)) A_{1,t} F(K_{1,t}, H_{1,t}, E_t).$$

The first order conditions for the firm problem are:

$$r_t = \left(1 - D\left(Z_t\right)\right) A_{1,t} F_{K,t}, \quad \forall\, t \geq 0, \tag{3}$$

$$w_t = \left(1 - D\left(Z_t\right)\right) A_{1,t} F_{H,t}, \quad \forall\, t \geq 0, \tag{4}$$

$$p_{E,t} = \left(1 - D\left(Z_t\right)\right) A_{1,t} F_{E,t}, \quad \forall\, t \geq 0. \tag{5}$$

Here, $p_{E,t}$ denotes the price of energy in period t. Because there are constant returns to scale and inputs are paid according to their marginal productivity, final goods producers make zero profits.

2.3 Energy sector

The energy sector, indexed by 2, produces energy E_t using capital $K_{2,t}$ and labor $H_{2,t}$ with a constant returns to scale technology so that

$$E_t = A_{2,t} G\left(K_{2,t}, H_{2,t}\right), \quad \forall\, t \geq 0. \tag{6}$$

Energy producers can provide a fraction μ_t of energy from clean technologies, at additional cost $\Theta_t\left(\mu_t, E_t\right)$, which satisfies $\Theta_{\mu,t}, \Theta_{E,t}, \Theta_{\mu\mu,t} > 0$, $\Theta_{EE,t} \geq 0$ and $\Theta_t(0, E_t) = \Theta_t(\mu_t, 0) = 0$. Convexity in $\Theta_t(\cdot,\cdot)$ captures decreasing returns to abatement. This function nests the one used in Barrage (2019), where $\Theta_t(\mu_t, E_t) = \Theta_t(\mu_t E_t)$, and in Nordhaus (2017), where it is equivalent to $\Theta_t(\mu_t, E_t) = \Theta_t(\mu_t) E_t$. In our calibration, we opt for the latter specification in order to follow DICE as closely as possible. Total profits from energy production are given by

$$\Pi_t = (p_{E,t} - \tau_{I,t}) E_t - \tau_{E,t}(1-\mu_t) E_t - w_t H_{2,t} - r_t K_{2,t} - \Theta_t(\mu_t, E_t),$$

where $\tau_{I,t}$ denotes the excise intermediate-goods tax on total energy and $\tau_{E,t}$ denotes the excise tax on pollution emissions $E_t^M = (1-\mu_t) E_t$. Firms maximize profits subject to the technology constraint given by equation (6) by choosing the abatement term μ_t, capital $K_{2,t}$, and labor $H_{2,t}$. The first order conditions are

$$r_t = \big(p_{E,t} - \tau_{I,t} - \tau_{E,t}(1-\mu_t) - \Theta_{E,t}\big) A_{2,t} G_{K,t}, \quad \forall\, t \geq 0, \tag{7}$$

$$w_t = \big(p_{E,t} - \tau_{I,t} - \tau_{E,t}(1-\mu_t) - \Theta_{E,t}\big) A_{2,t} G_{H,t}, \quad \forall\, t \geq 0, \tag{8}$$

$$\tau_{E,t} = \frac{\Theta_{\mu,t}}{E_t}, \quad \forall\, t \geq 0. \tag{9}$$

If there is positive abatement and $\Theta_t(\cdot,\cdot)$ is convex in its second argument, profits in the energy sector will be positive. For simplicity, we assume that these profits are taxed at a confiscatory rate $\tau_{\pi,t} = 1$. Doing so is typically optimal, as taxing pure profits does not generate distortions and income from shareholdings tends to be unequally distributed. In our calibration in Section 4, the abatement cost function is strictly convex in its first argument and linear in the second, hence profits are null.

Capital and labor are mobile across sectors, so market clearing requires

$$K_{1,t} + K_{2,t} = K_t, \quad \forall\, t \geq 0, \tag{10}$$

$$H_{1,t} + H_{2,t} = H_t, \quad \forall\, t \geq 0. \tag{11}$$

2.4 Government

Each period the government finances its expenses G_t and lump sum transfers T_t with proportional income taxes on capital $\tau_{K,t}$ and labor $\tau_{H,t}$, total energy taxes $\tau_{I,t}$, and emissions taxes $\tau_{E,t}$. In addition, profits are taxed at a confiscatory rate: $\tau_{\pi,t} = 1$. The government's budget constraint is

$$R_0 B_0 + T + \sum_t p_t G_t = \sum_t p_t \left(\tau_{H,t} w_t H_t + \tau_{K,t}(r_t - \delta) K_t + \tau_{I,t} E_t + \tau_{E,t} E_t^M + \Pi_t\right). \tag{12}$$

Although the instruments levied are proportional, the tax system is progressive when transfers are positive. As shown in Piketty and Saez (2013) and Dyrda and Pedroni (forthcoming), an affine tax system provides a good approximation of actual tax systems such as the one of the U.S.

2.5 Environmental degradation

The environmental variable is affected by the history of pollution emissions $E_t^M = (1-\mu_t) E_t$, initial conditions S_0, and the history of exogenous shifters η_t according to

$$Z_t = J\left(S_0, E_0^M, ..., E_t^M, \eta_0, ..., \eta_t\right), \quad \forall\, t \geq 0. \tag{13}$$

In our calibration below, Z represents the global mean temperature that is the outcome of the climate model J. In this section and the next, we do not further specify this function and our theoretical results can apply to any kind of pollution externality affecting production and households' utility.

2.6 Competitive equilibrium

Definition 1 *Given a distribution of assets $\{a_{i,0}\}$, aggregate capital K_0 and aggregate bond holdings B_0, a competitive equilibrium is a policy $\{\tau_{H,t}, \tau_{K,t}, \tau_{I,t}, \tau_{E,t}, T_t\}_{t=0}^{\infty}$, a price system $\{p_t, w_t, r_t, p_{E,t}\}_{t=0}^{\infty}$ and an allocation $\left\{(c_{i,t}, h_{i,t})_i, Z_t, E_t, K_{1,t}, K_{2,t}, K_{t+1}, H_{1,t}, H_{2,t}, H_t\right\}_{t=0}^{\infty}$ such that: (i) agents choose $\left\{(c_{i,t}, h_{i,t})_i\right\}_{t=0}^{\infty}$ to maximize utility subject to budget constraint (2) taking policies and prices (that satisfy $p_t = R_{t+1}p_{t+1}$) as given; (ii) firms maximize profits; (iii) the government's budget constraint (12) holds; (iv) markets clear: the resource constraints (6), (10), (11), and (13) hold, and*

$$N_t c_t + G_t + K_{t+1} + \Theta_t(\mu_t, E_t) = (1 - D(Z_t)) A_{1,t} F(K_{1,t}, H_{1,t}, E_t) + (1-\delta) K_t, \quad \forall\, t \geq 0. \tag{14}$$

3 Optimal tax rules

In this section, we use the technique introduced by Werning (2007) to express agents' equilibrium allocations as a function of aggregate variables, and solve the Ramsey problem as a function of aggregates instead of their full distributions.

3.1 Ramsey problem

A simple characterization of equilibrium Because the government sets linear tax rates, all individuals face the same marginal rate of substitution between consumption and leisure. Consequently, the distribution of individual allocations (c_{it}, h_{it}) is efficient *given* aggregates (c_t, h_t, Z_t), where $c_t = C_t/N_t$ and $h_t = H_t/N_t$ denote the average consumption and hours worked in period t. Following Werning (2007), it is therefore possible to split up the optimal tax problem in two steps. The first is to determine individual allocations given aggregates, and the second is to determine the aggregates. Starting with the first step, denote by $\varphi \equiv \{\varphi_i\}$ a set of market weights with $\varphi_i \geq 0$. Using the property that individual allocations are efficient given aggregates, we can characterize these allocations by solving the following static sub-problem for each period t:

$$\begin{aligned} U(c_t, h_t, Z_t; \varphi) &\equiv \max_{c_{i,t}, h_{i,t}} \sum_i \pi_i \varphi_i u(c_{i,t}, h_{i,t}, Z_t), \\ \text{s.t.} \quad \sum_i \pi_i c_{i,t} &= c_t \quad \text{and} \quad \sum_i \pi_i e_i h_{i,t} = h_t. \end{aligned} \tag{15}$$

Here, $U(c_t, h_t, Z_t; \varphi)$ denotes the indirect aggregate utility function, computed using market weights and aggregates. In Section 3.4 below, we introduce a functional form for households' utility function in order to obtain expressions for $U(c_t, h_t, Z_t; \varphi)$, as well as for $c^m_{i,t}$ and $h^m_{i,t}$, solutions to problem (15). For now we choose to keep preferences unspecified to analyze optimal tax formulas with more generality. To reduce the notation burden and ease tractability, we make the simplifying assumption that utility is additively separable in Z, *i.e.* that we can write

$$u\left(c_{i,t}, h_{i,t}, Z_t\right) \equiv \tilde{u}\left(c_{i,t}, h_{i,t}\right) + \hat{u}(Z_t).$$

Implementability condition Applying the envelope theorem to problem (15) and using consumers' first order conditions we get

$$\frac{U_{h,t}}{U_{c,t}} = \frac{u_{h,i,t}}{u_{c,i,t}e_i} = -w_t\left(1-\tau_{H,t}\right),$$

and

$$\frac{U_{c,t}}{U_{c,0}} = \frac{u_{c,i,t}}{u_{c,i,0}} = \frac{p_t}{\beta^t},$$

Using these relationships to substitute out for prices in agents' budget constraints, for any agent i we can derive an implementability condition that depends only on the aggregates c_t and h_t, and market weights φ

$$U_{c,0}\big(R_0N_0a_{i,0}+T\big) \geq \sum_{t=0}^{\infty} N_t\beta^t\Big(U_{c,t}c^m_{i,t}\big(c_t,h_t;\varphi\big) + U_{h,t}e_ih^m_{i,t}\big(c_t,h_t;\varphi\big)\Big), \ \ \forall\ i. \tag{16}$$

The following Proposition follows immediately from the arguments above.

Proposition 1 *An aggregate allocation $\{c_t, H_{1,t}, H_{2,t}, K_{1,t}, K_{2,t}, E_t, Z_t, \mu_t\}_{t=0}^{\infty}$ can be supported by a competitive equilibrium if and only if the market clearing conditions (10), and (11) hold, the resource constraints* (6), (13), (14) *hold and there exist market weights φ and a lump-sum tax T such that the implementability conditions (16) hold for all $i \in I$. Individual allocations can then be computed using functions $c^m_{i,t}$ and $h^m_{i,t}$, prices and taxes can be computed using the usual equilibrium conditions.*

Problem Let $\lambda \equiv \{\lambda_i\}$ be the planner's welfare weight on type i, with $\sum_i \pi_i\lambda_i = 1$. The Ramsey planner problem is

$$\max_{\substack{\{c_t,H_{1,t},H_{2,t},K_{1,t},K_{2,t},\\ E_t,Z_t,\mu_t\}_{t=0}^{\infty},T,\varphi}} \sum_{t,i} N_t\beta^t\pi_i\lambda_i u\Big(c^m_{i,t}\big(c_t,h_t;\varphi\big), h^m_{i,t}\big(c_t,h_t;\varphi\big), Z_t\Big) \tag{17}$$

subject to

$$U_{c,0}\big(R_0N_0a_{i,0}+T\big) \geq \sum_{t=0}^{\infty} N_t\beta^t\left(U_{c,t}c^m_{i,t}\Big(c_t,h_t;\varphi\Big) + U_{h,t}e_ih^m_{i,t}\left(c_t,h_t;\varphi\right)\right), \quad \forall\ i,$$

$$F_{K,t}G_{H,t} = G_{K,t}F_{H,t}, \quad \forall\ t\geq 0,$$

$$N_tc_t + G_t + K_{t+1} + \Theta_t\left(\mu_t, E_t\right) = \left(1-D\left(Z_t\right)\right)A_{1,t}F\left(K_{1,t}, H_{1,t}, E_t\right) + \left(1-\delta\right)K_t, \quad \forall\ t\geq 0,$$

$$E_t = A_{2,t}G\left(K_{2,t}, H_{2,t}\right), \quad \forall\ t\geq 0,$$

$$Z_t = J\left(S_0, E^M_0, ..., E^M_t, \eta_0, ..., \eta_t\right), \quad \forall\ t\geq 0,$$

$$K_{1,t} + K_{2,t} = K_t, \quad \forall\ t\geq 0,$$

$$H_{1,t} + H_{2,t} = N_th_t, \quad \forall\ t\geq 0.$$

The first of these is the implementability condition, which must hold for each agent i. It is written solely in terms of allocation variables and states that the present value of consumption equals the present value of labor income, initial assets and lump-sum transfers. The second constraint states that the marginal rate of technical substitution between capital and labor is the same in both sectors. It is a restriction imposed on the allocation which reflects that the government does not use sector-specific instruments and factors are mobile across sectors. The other constraints reflect market clearing for capital, labor and goods, and technological constraints.

To simplify the exposition, we assume for now that there is no initial wealth inequality, that is $a_{i,0} = a_{j,0}$ for all i and j. An equivalent interpretation is that there is initial wealth inequality, but that all wealth is expropriated by the planner. This can be done by taxing it directly, $R_0 = 0$, or through a combination of consumption and labor taxes: see Werning (2007) for a discussion.[8] We relax the assumption that there is no initial wealth inequality, or equivalently that all wealth can be expropriated, and study the implications for optimal taxes in Section 6.2. Without initial wealth inequality and with the ability to adjust lump-sum transfers, the optimal level of $\tau_{K,0}$ is indeterminate. We therefore assume that $\tau_{K,0}$ is taken as given by the Ramsey planner.[9]

3.2 General formulas

Capital and labor income taxes From the planner's first order conditions, the labor and capital income taxes are determined by

$$\tau_{H,t} = 1 - \frac{U_{h,t}}{U_{c,t}} \frac{W_{c,t}}{W_{h,t}},$$

and

$$\frac{R_{t+1}}{R^*_{t+1}} = \frac{W_{c,t+1}}{W_{c,t}} \frac{U_{c,t}}{U_{c,t+1}},$$

where the pseudo-utility function W is defined as

$$W\left(c_t, h_t, Z_t; \varphi, \theta, \lambda\right) \equiv V\left(c_t, h_t, Z_t; \varphi, \lambda\right) + \sum_i \pi_i \theta_i IC_i(c_t, h_t, \varphi),$$

with

$$V\left(c_t, h_t, Z_t; \varphi, \lambda\right) \equiv \sum_i \pi_i \lambda_i u\left(c^m_{i,t}\left(c_t, h_t; \varphi\right), h^m_{i,t}\left(c_t, h_t; \varphi\right), Z_t\right),$$

the aggregate utility based on the planner's weights,

$$IC_i(c_t, h_t, \varphi) \equiv U_{c,t} c^m_{i,t}\left(c_t, h_t; \varphi\right) + U_{h,t} e_i h^m_{i,t}\left(c_t, h_t; \varphi\right), \tag{18}$$

[8] Levying a confiscatory tax on all initial wealth is generally optimal if assets and productivity are positively correlated. In that case, taxing wealth reduces inequality without generating any distortions.

[9] If there is initial wealth inequality and the government can adjust a lump-sum transfer, the level of $\tau_{K,0}$ is no longer indeterminate. However, when studying the impact of initial wealth inequality on optimal taxes in Section 6.2, we also treat $\tau_{K,0}$ as given. The reason for doing so is that optimizing over $\tau_{K,0}$ allows the planner to confiscate all initial wealth, which immediately gets rid of all initial wealth *inequality* as well.

the difference between agent i spending on consumption and labor income in period t as it appears in its implementability constraint, and $\pi_i\theta_i$ the Lagrange multiplier on the implementability constraint of agent i in the Ramsey problem. These formulas are therefore the same as the ones obtained in Werning (2007). The reason is that the environmental variable enters additively to the problem and does not *directly* affect the labor and capital tax rules.

Excise taxes on energy and emissions The planner's first order conditions together with firms equilibrium conditions give

$$\tau_{I,t} = 0.$$

Thus, as long as labor, capital, profits and pollution can be taxed, there is no point in distorting production decisions. This result can also be found in Barrage (2019) and goes back to the production efficiency theorem of Diamond and Mirrlees (1971). Turning to the pollution tax we have

$$\tau_{E,t} = \sum_{j=0}^{\infty} \beta^j \left(\frac{V_{c,t+j} + \sum_i \pi_i\theta_i MIC_{i,t+j}}{V_{c,t} + \sum_i \pi_i\theta_i MIC_{i,t}} D'_{t+j} A_{1,t+j} F_{t+j} - \frac{N_{t+j} V_{Z,t+j}}{V_{c,t} + \sum_i \pi_i\theta_i MIC_{i,t}} \right) J_{E_t^M,t+j}, \tag{19}$$

where $MIC_{i,t} \equiv (\partial IC_{i,t}/\partial c_t)$, and where the arguments to the production function F_t have been dropped to simplify notations. The term $V_{c,t} + \sum_i \pi_i\theta_i MIC_{i,t}$ appears from the substitution of $W_{c,t} = \nu_{1,t}$, where $\nu_{1,t}$ is the Lagrange multiplier on the planner's resource constraint. When the pollution tax increases, abatement increases, which increases the scarcity of the final good. The opportunity cost of increasing the pollution tax therefore corresponds to the marginal cost of increasing the final good's scarcity, which is equal to the marginal utility from consumption as computed using the planner's weights ($V_{c,t}$) minus a term which captures the marginal cost for the planner to implement its preferred allocation ($-\sum_i \pi_i\theta_i MIC_{i,t}$).

This formula holds both for the first and second-best. Still, the optimal pollution tax may differ between these two fiscal environments for three reasons: the value of the marginal implementation cost, the path of aggregate variables, and the distribution of individual allocations all depend on fiscal policies.

3.3 Comparison with first-best

Social cost of the externality The first potential difference between the first and second-best pollution tax lies in the value of the marginal implementation cost, $-\sum_i \pi_i\theta_i MIC_{i,t}$. In the first-best, the first order conditions with respect to individualized lump-sum transfers give

$$\theta_i = 0, \quad \forall i.$$

It follows that the planner can achieve its preferred allocation at no cost, and the optimal pollution tax simplifies to

$$\tau_{E,t}^{FB} = \sum_{j=0}^{\infty} \beta^j \left(\frac{V_{c,t+j}}{V_{c,t}} D'_{t+j} A_{1,t+j} F_{t+j} - \frac{N_{t+j} V_{Z,t+j}}{V_{c,t}} \right) J_{E_t^M,t+j}.$$

This formula illustrates the well-known Pigouvian principle according to which the optimal corrective tax is equal to the social cost of the externality: the tax corresponds to the discounted sum of marginal (utility and production) damages valued at the marginal utility of consumption. Turning to the second-best case, where only a uniform lump-sum transfer is available in each period, the first order condition with respect to the transfer gives

$$\sum_i \pi_i \theta_i = 0.$$

Thus, at the second-best, the sum of the multipliers associated with the implementability conditions is zero, but the marginal cost for the planner to implement its preferred allocation in a given period ($-\sum_i \pi_i \theta_i MIC_{i,t}$) is not necessarily zero. In particular, we have

$$-\sum_i \pi_i \theta_i MIC_{i,t} = -\mathrm{cov}(\theta_i, MIC_{i,t}), \tag{20}$$

hence the marginal implementation cost is zero if and only if θ_i and $MIC_{i,t}$ are uncorrelated. As we show in Appendix A.4.3, the sign of this term also determines the sign of the marginal cost of public funds (MCF) relative to 1. Indeed, if we define the MCF as the ratio of the public to the private marginal utility of consumption, we have

$$\mathrm{MCF}_t = 1 + \frac{\mathrm{cov}(\theta_i, MIC_{i,t})}{V_{c,t}},$$

and the MCF is above 1 if and only if the covariance term is positive.[10]

Determinants of the marginal implementation cost Intuitively, $-\theta_i$ represents the shadow cost of implementing the desired allocation for agent i, which can be understood as the increase in implementation cost resulting from an additional unit of lump-sum transfer received by this agent. While this marginal cost is on average null, it may be positive for some agents and negative for others. Using functional forms below, we show that $-\theta_i$ is negative for households who are valued relatively more by the market than by the planner as compared to an average household. The second term, $MIC_{i,t}$, represents how the difference between the agent current consumption and current labor income changes when more resources are available for consumption. At the optimum, agents' implementability conditions must be binding, hence

$$\sum_{t=0}^{\infty} N_t \beta^t IC_i(c_t, h_t, \varphi) = U_{c,0}(R_0 a_{i,0} + T). \tag{21}$$

[10] Jacobs and de Mooij (2015) and Jacobs and van der Ploeg (2019) use a definition of the marginal cost of funds that takes into account fiscal externalities resulting from income effects. They find that the marginal cost of funds equals one at the optimal tax system, owing to the fact that the government can optimize a lump-sum transfer. However, because as in Barrage (2019) we optimize over the allocation variables directly rather than over tax instruments, it is more convenient to define the marginal costs of funds as the ratio between the multiplier on the government budget constraint and the average marginal utility of consumption computed using Pareto weights, which Jacobs and de Mooij (2015) refer to as the traditional measure of the MCF.

When there is no initial wealth inequality, or when initial wealth is expropriated—which, as we have shown, is optimal as long as initial wealth and productivity are positively correlated—then for any i, j, $R_0 a_{i,0} = R_0 a_{j,0}$, which implies that the discounted sum of $IC_{i,t}$ is invariant across types. Intuitively, this condition means that with equal initial wealth (or initial wealth expropriation) and a uniform lump-sum transfer, the discounted sum of expenditures minus the discounted sum of labor incomes must be the same for everyone. From equations (20), the marginal implementation cost will differ from zero only if individuals' expenditures minus labor income are responsive to contemporaneous changes in aggregate consumption c_t, and if these responses are heterogeneous. In particular, when preferences are such that individuals' expenditures minus labor income can be expressed as a constant fraction of aggregates, *i.e.* if we can write

$$IC_i(c_t, h_t, \varphi) = m_i \tilde{IC}(c_t, h_t, \varphi), \tag{22}$$

then from (21) we have that for any types i, j and any period t, $m_i = m_j$ and $MIC_{i,t} = MIC_{j,t}$. From (20), this implies that in all periods the marginal implementation cost is null, the MCF is equal to 1, and the second-best tax is set at the Pigouvian level. The reason is that increasing the pollution tax—and thereby leaving less resources available for consumption—affects the costs from satisfying (typically) poor agents' implementability constraint just as much as the benefits from satisfying (typically) rich agents' implementability constraint, so general fiscal motives do not affect the opportunity cost from corrective taxation in this case.

Timing of abatement and damages Going back to the pollution tax formula (19), the marginal implementation cost may imply deviations from the social cost of pollution for two reasons. First, a positive cost in period t means that that the opportunity cost of pollution abatement is lower in that period, which pushes the tax above the social cost of pollution. This effect is captured by the denominator of the formula. Second, a positive cost in period $t+j$ also means that having less production damages in that period is worth less, which pushes the tax below the social cost of pollution. This effect is captured by the numerator of the term multiplying production damages.

Focusing on production damages, we see that the marginal implementation cost operates as a form of discounting. If this cost increases over time, consumption is valued relatively more in the present than in the future, hence the pollution tax is set at a lower level. Conversely, a declining path for this term implies a higher tax. Turning to the utility part, the effect is again ambiguous and the tax is set to a higher (resp. lower) level to the extent that the marginal implementation cost is positive (resp. negative) in periods where the present value of utility damages are high.

Differences in allocations When the marginal implementation cost is null, the first and second-best tax formulas coincide, and they are both equal to the social cost of pollution. Still, the actual tax levels may differ for two reasons.

The first reason is that when the tax system is different, aggregate variables generally take different values. When capital and labor are taxed, labor supply and investments are expected to be lower, hence output, consumption, and pollution are also expected to be lower along the optimal path. Since the pollution tax level is determined by the trade-off between the marginal utility of consumption and the marginal utility of pollution abatement, if both pollution and consumption are lower, the optimal tax will generally be set at a lower level since utility is concave in consumption and convex in pollution.[11]

The second reason is that the distribution of individual allocations also differs depending on the fiscal environment. Because individualized lump-sum transfers are not feasible in the second-best, there are generally more consumption inequalities. The welfare gains from leaving more resources available for agents' consumption by decreasing the pollution tax may then be higher or lower compared to the first-best depending on the curvature of agents' utility function. As shown in Appendix A.4.3, we have

$$V_{c,t} = \sum_i \pi_i \lambda_i \frac{u_{c,i,t} c_{i,t}}{c_t}.$$

In the presence of inequalities, an increase in aggregate consumption is valued more to the extent that the average marginal utility is higher (by concavity of the utility function), but it is valued less to the extent that the inflow in consumption disproportionately goes in the hands of richer households with lower marginal utilities. Our analysis of functional form expressions below provides an illustration of these two opposite mechanisms, showing that with iso-elastic preferences, they perfectly offset each other when the intertemporal elasticity of substitution (IES) is equal to 1.

3.4 Functional form expressions

Specification In the next section, we quantitatively analyze the optimal fiscal policies presented above. Before turning to these quantitative results, it is useful to investigate the theoretical predictions using the functional form for utility chosen in our quantitative analysis. Suppose agents have preferences over consumption, leisure and environmental degradation, with the following period utility function

$$u\left(c_i, h_i, Z\right) = \frac{\left(c_i(1-\varsigma h_i)^{\gamma}\right)^{1-\sigma}}{1-\sigma} + \frac{\left(1+\alpha_0 Z^2\right)^{-(1-\sigma)}}{1-\sigma}. \tag{23}$$

Capital and labor income taxes Without loss of generality, we can add a normalization constraint for the market weights to the Ramsey problem presented above (see Appendix A.4.2). We can then express the formulas for labor and capital income taxes as follows

$$\tau_{H,t} = \frac{\Psi\varsigma\left(1-\varsigma h_t\right)^{-1}}{\Phi + \Psi\varsigma\left(1-\gamma\left(1-\sigma\right)\right)\left(1-\varsigma h_t\right)^{-1}},$$

and

$$\frac{R_{t+1}}{R^*_{t+1}} = \frac{\Phi - \Psi\varsigma\gamma\left(1-\sigma\right)\left(1-\varsigma h_{t+1}\right)^{-1}}{\Phi - \Psi\varsigma\gamma\left(1-\sigma\right)\left(1-\varsigma h_t\right)^{-1}},$$

[11] This result also depends on the law of motion of environmental degradation: if each additional unit of pollution emitted increases degradation by less than the previous unit, the marginal abatement benefits could be lower for higher levels of pollution.

with

$$\Phi = \sum_j \pi_j \frac{\lambda_j}{\varphi_j} + \big(1 - (1+\gamma)(1-\sigma)\big)\text{cov}(\lambda_i/\varphi_i, \omega_i),$$
$$\Psi = -\frac{\text{cov}(\lambda_i/\varphi_i, e_i)}{\varsigma},$$

where $\forall t$, $\omega_i = c_{i,t}/c_t$. We see that both the labor and the capital income tax rates are zero in three special cases: *(i)* when there is no agent heterogeneity, *(ii)* when the planner's and the market's weights are perfectly aligned, and *(iii)* when agents' productivity are uncorrelated with the relative social weights. Intuitively, the first case corresponds to the outcome of a representative agent model in which lump-sum taxation is allowed: since there is no need to redistribute, the government can rely only on non-distortionary taxes to finance its expenditures. The second case corresponds to the situation in which the market allocation happens to be the one preferred by the planner: although there might be inequalities due to differences in productivity and asset holdings, they are consistent with the relative weight the planner gives to each type of individual. The third situation encompasses the two previous ones, but also includes situations in which the planner would want to redistribute but faces a targeting problem, *i.e.* it cannot reach a better allocation than the market one using anonymous linear instruments due to the absence of correlation between the source of inequalities and its relative preference over agents' types.

Marginal implementation cost Using our specification, we can also further examine the determinants of the marginal implementation cost that enters the pollution tax formula. First, using the first order condition of the Ramsey problem with respect to market weights, we can express the multipliers of the implementability constraints as

$$\theta_i = \sum_j \frac{\pi_j \lambda_j}{\varphi_j} - \frac{\lambda_i}{\varphi_i}, \quad \forall i,$$

which means that the cost of implementing the planner's preferred allocation is positive (i.e. $-\theta_i$ is positive) for households who are valued relatively less by the market than by the planner as compared to an average household. Second, we can now write

$$-\sum_i \pi_i \theta_i MIC_{i,t} = (\sigma - 1)\frac{\text{cov}(\theta_i, IC_{i,t})}{c_t}, \tag{24}$$

from which we see that the marginal implementation cost is always null if $\sigma = 1$, where σ is the inverse of the IES. As shown in Appendix A.4.3, when more resources are available for agents' consumption, not only their consumption and real wage go up, but the price also goes down. When $\sigma = 1$, the price decline exactly offsets the increase in volume: expenditures and labor income remain unchanged, so that the planner has no need to adjust its transfer to ensure that the implementability constraints are satisfied. When $\sigma > 1$, the price effect dominates, so that an increase in aggregate consumption reduces the total amount of transfers needed to satisfy agents' implementability constraints. In periods when poor households (low θ_i) consume relatively more compared to what they earn (high $IC_{i,t}$), the aggregate implementation cost is positive and a reduction in transfers reduces the costs. Conversely, if the volume effect dominates ($\sigma < 1$), *or* if rich households temporarily consume relatively more compared to what they earn (low $\text{cov}(\theta_i, IC_{i,t})$), an increase in aggregate consumption increases implementation costs.

As shown in Appendix A.4.3, our utility function implies that individuals' consumption is a constant fraction of aggregate consumption, but individuals' labor supply is not proportional to aggregate labor, hence we cannot write $IC_{i,t}$ as in (22). In particular, when transfers are positive (as they are in our quantitative analysis) less productive households work relatively less when aggregate labor supply is high, *i.e.*, for i, j such that $e_i > e_j$, $h_{i,t}/h_{j,t}$ is increasing in h_t. Under the assumption that higher productivity types also have a lower marginal utility of consumption, and thus a higher θ_i, the covariance term in equation (24) is positive (resp. negative) when the aggregate labor supply is relatively low (resp. high), and the marginal implementation cost is positive (resp. negative) if and only if increasing aggregate consumption decreases the amount of transfers necessary to satisfy agents' implementability conditions ($\sigma > 1$).

Marginal utility of consumption Under our functional form assumption, we can also sign the effect of inequalities on the marginal utility of consumption ($V_{c,t}$) as a function of the value of σ, which captures the utility curvature. In particular, when $\sigma = 1$, the increase in agents' average marginal utility exactly offsets the fact that higher marginal utility agents receive a lower share of aggregate consumption increases, hence an aggregate increase in consumption is valued identically in the first and second-best. When the utility is more (resp. less) concave, consumption is valued more (resp. less) to the extent that there are more inequalities, shifting the second-best pollution tax downward (resp. upward) compared to the first-best where individualized transfers are used to reduce consumption inequalities (see Appendix A.4.3).

4 Calibration

In this section, we explain how we calibrate the model to explore quantitatively the implications of heterogeneity in productivity for the optimal taxation of carbon, capital income, and labor income. As in Barrage (2019), we consider a climate-economy model based on Nordhaus' DICE model. While Barrage (2019) considers a planner setting taxes for the global economy, we adopt a slightly different approach: we consider a global economy with the economic features of the U.S. economy, *i.e.* we parametrize the income per capita, the productivity distribution, and the fiscal system to match U.S. data, but we scale our economy so that output and emissions match global data. The objective is to determine how an economy with important inequalities and responsible for a significant share of global emissions like the U.S. should design its fiscal system if it were to internalize the global impact of its emissions, assuming that the rest of the world would behave identically.

4.1 Climate model

The calibration of the climate model is based on the 2016 version of DICE, presented for example in Nordhaus (2017). The initial period is 2015, and each period lasts 5 years. The climate model is composed of three sets of equations describing the carbon cycle, radiative forcing, and climate change.

Carbon cycle The carbon cycle is represented by three reservoirs. S^{At}, S^{Up}, and S^{Lo} represent the level of carbon concentration in the atmosphere, the upper oceans and biosphere, and the deep oceans respectively. These stocks evolve according to the following laws of motion:

$$S_t^j = b_{0,j}(E_t^M + E_t^{\text{land}}) + \sum_{i=1}^{3} b_{i,j} S_{t-1}^i,$$

where the three reservoirs j are ranked as above and with E_t^{land} the exogenous land emissions. The coefficient $b_{0,j}$ is 1 for the first reservoir (S^{At}) and 0 for the others: industrial and land emissions directly flow into the atmosphere, and later affect the other two reservoirs through the communication between the carbon stocks captured by the parameters $b_{i,j}$.

Radiative forcing The accumulation of carbon in the atmosphere increases radiative forcing, *i.e.* the net radiation received by the earth. This mechanism is captured by the following equation

$$\mathcal{F}_t = \kappa\big(\ln(S_t^{At}/S_{1750}^{AT})/\ln(2)\big) + \mathcal{F}_t^{\text{ex}}.$$

where $\mathcal{F}_t^{\text{ex}}$ is exogenous forcing. A positive radiative forcing means that the earth receives more energy from the sun than it emits back to space, hence the climate warms.

Climate change The change in temperature is modeled through two equations for the mean temperature of the atmosphere (Z_t^{At}) and deep oceans (Z_t^{Lo}) that interact as follows

$$Z_t^{At} = Z_{t-1}^{At} + \zeta_1\big(\mathcal{F}_t - \zeta_2 Z_{t-1}^{At} - \zeta_3(Z_{t-1}^{At} - Z_{t-1}^{Lo})\big),$$

$$Z_t^{Lo} = Z_{t-1}^{Lo} + \zeta_4(Z_{t-1}^{At} - Z_{t-1}^{Lo}).$$

All the parameters of the climate model are taken from DICE 2016, and reported in Table IV in the appendix.

4.2 Damages

We also model production damages as in DICE 2016, with

$$D(Z_t) = a_1 Z_t^{At} + a_2 (Z_t^{At})^{a_3}. \tag{25}$$

As in DICE, we assume that $D(Z)$ is a simple quadratic function with $a_1 = 0$ and $a_3 = 2$. Since DICE does not distinguish between production and utility damages, we follow Barrage (2019) to decompose the damages from DICE into a production and a utility component. We apply her decomposition and assign 74% of damages at 2.5°C warming to output, and 26% to utility. This provides an adjusted value for the parameter a_2 in equation (25), and enables us to determine the preference parameter α_0.

To examine the robustness of our quantitative results to the level of damages, we also consider an alternative "high damage" specification. Instead of assuming quadratic damages, we consider a cubic function ($a_1 = 0$, $a_3 = 3$) and we adjust the coefficient a_2 such that damages are identical to the baseline scenario at current warming. This high damages scenario therefore assumes that the damage function in DICE correctly captures current damages, but mis-estimates damages at higher levels of warming because of the high uncertainties surrounding the impacts of climate change at these higher temperatures (see *e.g.*, Weitzman, 2009; Pindyck, 2013).

4.3 Households

Using specification (23) and market weights, the inter-temporal aggregate utility is

$$\sum_t \beta^t N_t U\left(c_t, h_t, Z_t, \varphi\right) = \sum_t \beta^t N_t \left(\frac{\left(c_t(1-\varsigma h_t)^\gamma\right)^{1-\sigma}}{1-\sigma} + \Gamma \frac{\left(1+\alpha_0 Z_t^2\right)^{-(1-\sigma)}}{1-\sigma} \right),$$

with $\Gamma \equiv \sum_i \pi_i \varphi_i$ and where $Z_t \equiv Z_t^{At}$ is the atmospheric temperature. To ensure that aggregate emissions remain consistent with DICE, we calibrate the growth rate of population accordingly. Because we also want to match the GDP per capita of the U.S., we set the population levels as U.S. population multiplied by the ratio of world to U.S. GDP in 2011-2015, the first period of the model.

Following DICE, we calibrate the utility discount factor to $\beta = 1/(1 + 0.015)$ per year, and the inverse of the IES to $\sigma = 1.45$. The parameters γ and ς are set in order to match a Frisch elasticity of labor supply of 0.75 (see Chetty et al., 2011) and an average per capita labor supply of $h_{2015} = 0.277$ in the initial period (computed from the Survey of Consumer Finances, see Appendix D.2).

We calibrate the ability distribution on the basis of hourly wage data that we obtain from the Survey of Consumer Finances (SCF). To be consistent with the initial period in DICE, we use the SCF 2013. We divide the sample of working households into ten groups of hourly wage deciles (*i.e.*, $I = 10$, and for all i, $\pi_i = 0.1$), with an hourly wage of \$6.44 for the bottom productivity group and \$101.35 for the top productivity group, and normalize productivity levels such that $\sum_i \pi_i e_i = 1$. The full procedure is described in Appendix D.1.

4.4 Production

We model production using a Cobb-Douglas technology for both sectors. We have

$$F(K_{1,t}, H_{1,t}, E_t) = K_{1,t}^{\alpha} H_{1,t}^{1-\alpha-\nu} E_t^{\nu}$$

with $\alpha = 0.3$, and $\nu = 0.04$ (from Golosov et al., 2014), and

$$G(K_{2,t}, H_{2,t}) = K_{2,t}^{1-\alpha_E} H_{2,t}^{\alpha_E}.$$

with $\alpha_E = 0.403$ (from Barrage, 2019). The initial total factor productivities $A_{1,2015}$ and $A_{2,2015}$ are set such that output in sectors one and two match world GDP (2011-2015 average from the World Bank) and aggregate industrial emissions (from DICE 2016) respectively, and their growth rate are taken from DICE 2016.[12] Our abatement cost function is also taken from DICE, with the following specification

$$\Theta(\mu_t, E_t) = c_{1,t}\mu_t^{c_2} E_t,$$

where $c_{1,t}c_2 = P_t^{\text{backstop}}$ represents the backstop price, *i.e.* the price at which it becomes economical to abate 100% of emissions. As in DICE 2016, we assume that this price is \$550/tCO_2 in the initial period, and declines at a rate of 0.5% per year. We also calibrate the exponent $c_2 = 2.6$ as in DICE.

4.5 Government

We calibrate the fiscal part of the model to match data on U.S. fiscal policy. Here we deviate from Barrage (2019) who sets tax rates, government spending, and debt to match their empirical counterparts at the *global* level. The reason for targeting the U.S. rather than the global economy is that the degree of inequality is calibrated to match the U.S. income and wealth distribution and, more importantly, in our framework and in reality fiscal policy is typically decided on at the national level. To make the model consistent with the (global) evolution of the climate, we subsequently scale up the economy such that GDP and total emissions are consistent with their global levels. By doing so, rather than ignoring negative effects from emissions on other countries, we assume that U.S. fiscal policy is set to fully internalize the negative global effects from their carbon emissions.

To calibrate fiscal policy, we first require the empirical counterparts of taxes. In the model, there are four taxes: a tax $\tau_{K,t}$ on capital income, a tax $\tau_{H,t}$ on labor income, an excise (intermediate-goods) tax $\tau_{I,t}$ on total energy and a tax $\tau_{E,t}$ on pollution emissions. We set the tax rates on capital and labor income in line with Trabandt and Uhlig (2012), who conduct a detailed analysis of fiscal policies in the U.S. and a number of European countries. Using a comprehensive measure of taxes on capital income, they find that on average, capital income in the U.S. is taxed at a rate of 41,4%, hence we set a time-invariant $\tau_K = 0.411$ in our baseline.[13] They find that labor income in turn, is taxed at a rate of 22.1%. Combined with a tax rate on consumption of 4.6%, this translates into a consumption-labor wedge of 25.5%, or $\tau_H = 1 - (1 - 0.221)/(1 + 0.046) = 0.255$. Turning to energy taxes, we follow Barrage (2019) and set the intermediate-goods tax at $\tau_I = 0$. Regarding the tax on pollution emissions τ_E, we set it at a level so that, in our calibrated economy, 3% of total energy is obtained from clean technologies (Nordhaus, 2017). This requires $\tau_E = 2.01\$/\text{t}CO_2$ in 2015.

To calibrate initial, outstanding debt B_0 at the start of the economy, we calculate the difference

[12] To calibrate the initial values of $K_{1,0}$ and $K_{2,0}$, we assume that the economy is in a balanced growth path in which temperature remains constant at the current level.

[13] Specifically, to obtain a comprehensive measure of capital tax rates, Trabandt and Uhlig (2012) adjust the personal income tax rate to account for income, profit and capital gains taxes of corporations, taxes on financial and capital transactions and recurrent taxes on immovable property. Similarly, to calculate labor income taxes, personal income taxes are adjusted to account for payroll taxes and social security contributions.

between total liabilities and financial assets from the U.S. government's balance sheet, both as a percentage of GDP.[14] Following Barrage (2019) and in order to facilitate reproducing results for other countries, these data are obtained from the IMF Government Finance Statistics. This gives an average debt-to-GDP ratio of approximately 111% over the period 2011–2015. Because in our model a period corresponds to five years, we set $B_0/Y_{1,0} = 1.11/5 = 0.222$, or 22.2%.

Lastly, we require an empirical counterpart of government spending. In our model, G_t denotes government consumption of the final good, while T captures the present value of all lump-sum transfers households receive from the government. To better align the model with the data and to analyze business-as-usual scenarios, we follow Barrage (2019) and split up total government spending into final good spending G_t^C and *exogenous* transfers G_t^T that are provided to households. The total transfers households receive thus consist of this exogenous component G_t^T and the endogenous component T.[15] To obtain the empirical counterparts of G_t^C and G_t^T, we proceed as in Barrage (2019) and collect data on U.S. government expenses from the IMF Government Finance Statistics. Averaging over the years 2011–2015, government consumption is $G_0^C/Y_{1,0} = 0.158$, or 15.8%, while government transfers are $G_0^T/Y_{1,0} = 0,145$, or 14.5%.[16] To keep the sizes comparable to GDP going forward, both government consumption and exogenous transfers grow at the sum of technological progress and population growth.

5 Quantitative results

We now present the optimal policy obtained under a utilitarian welfare criterion (i.e., $\lambda_i = 1$ for all i), and the associated welfare effects compared to a "climate skeptic" planner scenario in which the planner ignores the anthropogenic origin of climate change and consequently sets the carbon tax to zero.

5.1 Optimal policy

Optimal tax paths Figure 1 shows the path of optimal taxes on capital and labor income in our baseline scenario. The labor income tax roughly doubles in the first period, from 25% to about 50%, and stabilizes at this level. Rebating the revenue from these taxes via lump-sum transfers achieves most of the redistribution implied by the optimal tax system. Because lump-sum taxes are available and there is no initial wealth inequality, the only reason to tax capital income is to mitigate intertemporal distortions associated with labor income taxation. Since optimal labor income taxes are close to constant, the

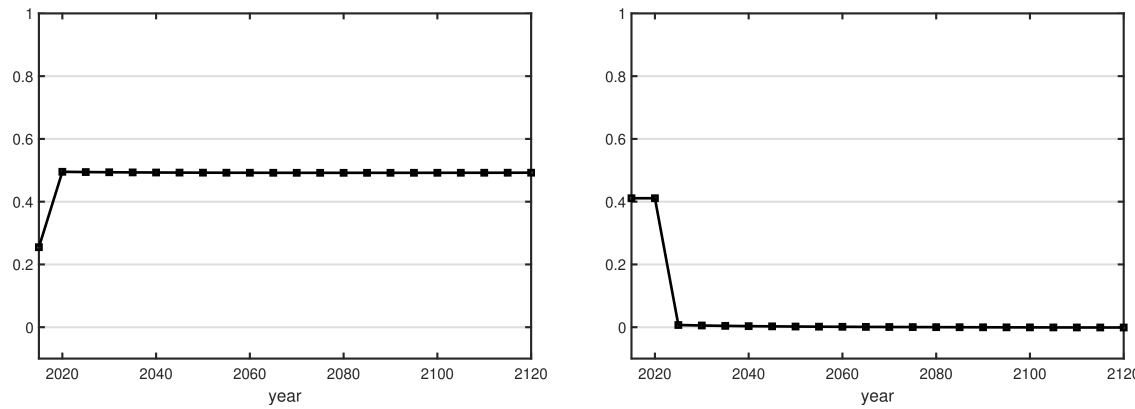
[14] The numbers are calculated at the "General Government" level.

[15] The endogenous component is set to $T = 0$ in Barrage (2019) and many other Ramsey tax models. The reason is that without heterogeneity, optimal policy would be to finance all spending through lump-sum taxes (i.e., negative transfers), in which case tax distortions become irrelevant. In our model with heterogeneity, we do not have to impose this restriction.

[16] As in Barrage (2019), we include the following categories from the expense breakdown in G_t^C: compensation of employees, use of goods and services, subsidies, grants and other expense. For transfers G_t^T, we include social benefits.

optimal capital income tax converges to zero quickly after the second period.[17] The next section examines scenarios with further constraints on policy instruments leading to deviations from this result.

Figure 2 shows the optimal path of carbon taxes: in the baseline scenario, the tax starts at 21.5\$/tCO_2 in 2020 and goes up to reach 227.3\$/tCO_2 a century later. These tax levels are consistent with the ones found in Barrage (2019) and Nordhaus (2017, 2018), but are too low to contain climate change to a level consistent with the +2°C objective of the Paris agreement. In our "high damages" scenario, the optimal income taxes remain almost the same, but the carbon tax is roughly three times as large (see Appendix E.1).



(a) Optimal labor income Taxes

(b) Optimal capital income Taxes

Figure 1: Optimal Income Taxes.

Carbon tax decomposition Figure 3 compares the second-best pollution tax normalized to 1 (black line) to what it would be if the MCF was 1 in all periods (red line)—which also corresponds to the Pigouvian tax evaluated at the second-best allocation—and to what it would be ignoring inequalities (blue line). The MCF appears to play an insignificant role: the social cost of carbon is only 0.5% above the second-best carbon tax in the initial period, a difference that becomes even smaller in subsequent periods. Thus, even in the presence of distortionary taxation, it is optimal to set the carbon tax "almost" at the social cost of carbon (*i.e.* at the Pigouvian level). However, the discrepancy between the blue and red lines indicates that the social cost of carbon itself is significantly affected by the presence of inequalities. The reason is that the social cost of carbon represents the monetary value of climate damages, and is determined by the arbitrage between reducing damages and increasing aggregate consumption. As explained in Section 3, a marginal unit of aggregate consumption is valued more in the presence of inequalities if the marginal utility is sufficiently declining in consumption. Intuitively, an increase in aggregate consumption is valued less to the extent that it disproportionately goes in the hand of richer households, but it is valued more to the extent that the average marginal

[17] Notice that, because we have lump-sum taxation, the reason for zero long-run capital income taxation is different from the usual Chamley (1986) and Judd (1985), and is not subject to the criticism in Straub and Werning (2020).

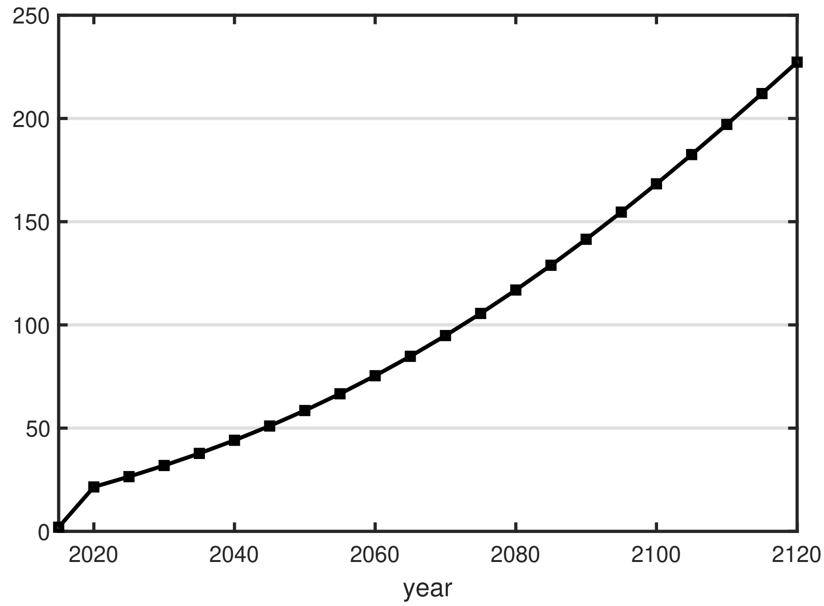


Figure 2: Optimal Carbon Taxes (\$/tCO_2).

utility becomes higher if some people have relatively low consumption levels. In particular, with our specification, consumption inequalities call for lower carbon taxes for $\sigma > 1$. With $\sigma = 1.45$, we see that ignoring consumption inequalities would lead to a social cost of carbon higher by on average 4.2% over the next century. As shown in Appendix E.1, these results do not strongly depend on the damage specification: although the social cost of carbon is about three times higher in our "high damages" scenario, the role of the MCF remains negligible and the effect of inequalities is similar, although slightly smaller (3.3% instead of 4.2% in the baseline).

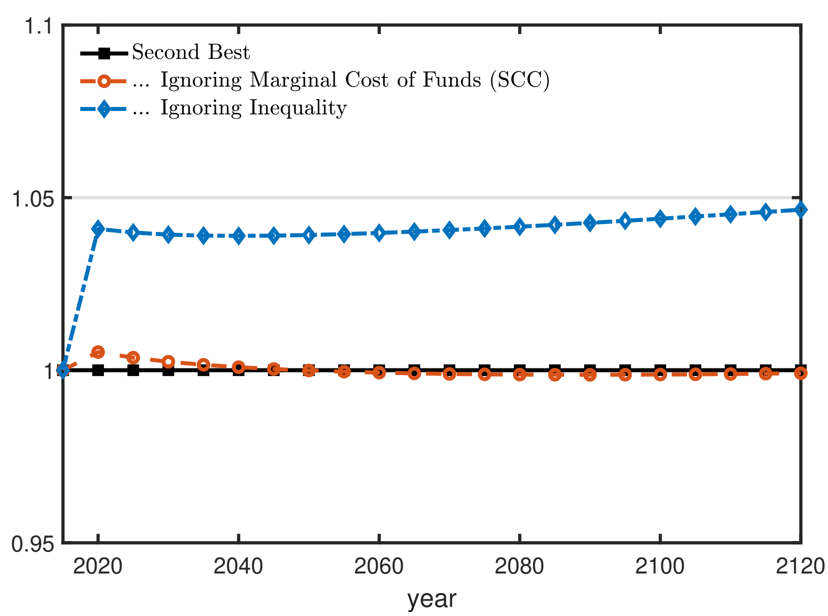


Figure 3: Carbon Tax Decomposition.

Fiscal adjustments relative to a climate skeptic planner Table I below reports the adjustments made to the government budget between our baseline second-best scenario and a "climate skeptic" planner scenario in which the planner ignores the anthropogenic origin of climate change. Specifically, this climate skeptic planner sets all taxes optimally but behave as if the climate variable was exogenous and not driven by human-made emissions. The objective of this experiment is to see how the planner should adjust the fiscal system once it acknowledges the necessity to address climate change. As shown in the table, the present value of the optimal carbon tax revenue represents 1% of the present value of GDP in our baseline calibration. This additional resource is split about equally between reducing distortionary taxes, with the present value of the labor tax decreasing by 0.6% of GDP, and increasing transfers, whose present value increases by 0.5% of GDP.[18] This finding qualifies the weak double-dividend hypothesis (for a review, see Goulder, 1995) according to which it is optimal to use the proceeds of the carbon tax to reduce distortionary taxes. With heterogeneous agents, distortionary taxes serve a redistributive purpose, hence it is not desirable to reduce them unless additional transfers can be provided through another mean. This result also gives some grounds to the popular carbon tax and dividend policy (see Economists Statement on Carbon Dividends, 2019) that calls for redistributing the proceeds of the tax lump-sum to address redistributive concerns, although we find that only half of the tax revenue should serve that purpose, the rest being aimed at improving economic efficiency.

Table I: Government Budget Adjustment.

	Revenue Source			Revenue Use		
	Labor	Capital	Carbon	Gov. Cons.	Transfer	Interest
No Carbon Tax	33.5%	0.6%	0.0%	17.2%	14.6%	2.3%
Optimal Carbon Tax	32.9%	0.6%	1.0%	17.1%	15.1%	2.3%
Change	−0.6%	0.0%	1.0%	−0.1%	0.5%	0.0%

Note: Numbers represent the present value of each component of the government budget constraint divided by the present value of GDP.

5.2 Welfare effects

Figure 4 displays the percentage increase in consumption that would be necessary in the climate skeptic scenario to make households as well-off as in the optimal scenario in each period and for each produc-

[18]Fried et al. (2021) conduct a different experiment but find similar results. Starting from exogenous tax rates, they model a continuum of potential budget-neutral recycling mechanisms after introducing an exogenous carbon tax and quantitatively determine the one maximizing aggregate welfare. They find that the strategy maximizing welfare gains involves spending about two thirds of the revenue to reduce the capital-income tax and one third to increase the labor-income tax progressivity, an option that dominates the use of transfers in their non-linear tax system. They also show that this result is relatively stable when changing the initial tax rates.

tivity group. While the average long run gains are positive for all productivity groups (the average discounted gain is 4.1% with baseline damages), the period welfare gains are heterogeneous over time and between groups. While the increase in the lump-sum transfer initially benefits poor households relatively more, in the long-run the decrease in the labor income tax benefits rich households relatively more. Overall, welfare gains are progressive but mostly negative in the 21st century and positive but regressive after 2100.[19] The reason why the optimal carbon tax is progressive initially is that the revenue gains from carbon taxation are rebated through both a higher lump-sum transfer and a reduction in the labor income tax rate: see Table I. This contributes to an increase in the progressivity of the overall tax system, which makes poorer households benefit more (or suffer less) from the initial increase in carbon taxes. In the long run, richer households are the ones who benefit more from carbon taxation. A significant share of the welfare gains from a lower temperature come from reduced utility damages. Richer households care relatively more about those damages in the sense that they are willing to give up more units of consumption for a reduction in temperature. This explains why in the long run, the welfare gains from carbon taxation are regressive when expressed in consumption units.

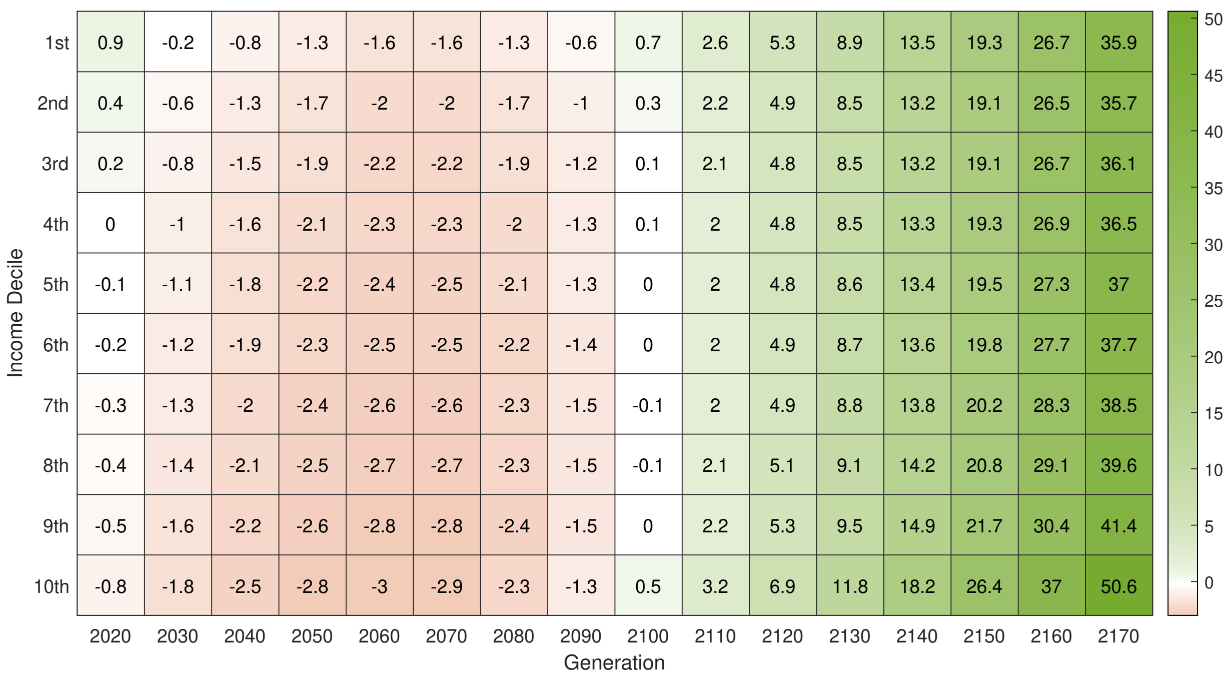


Figure 4: Period Welfare Gains (%).

These results highlight the political economy challenges associated carbon taxation. Although the policy is clearly welfare improving in the long run, the present costs outweigh the present benefits, making the implementation of carbon tax policies politically difficult.[20] Still, accounting for the distributional effects of carbon pricing and designing optimal policies accordingly, we see that the initial

[19] As shown in Dietz et al. (2021), the DICE model features too much thermal inertia, *i.e.*, the temperature response to an impulse in emissions is delayed too much compared to what climate science models predict. If this response was more immediate, welfare gains from carbon taxation could become positive earlier.

[20] Kotlikoff et al. (2021) study this question in an OLG model linked to DICE. They abstract from fiscal policies, and show that the carbon tax delivering the highest uniform welfare gains across generations implies significant inter-generational

welfare impacts are progressive and even positive for households at the bottom of the income distribution in the first period. This distribution of welfare gains could make the policy more attractive to a government concerned with redistribution, and increase public support in the first stages of the policy implementation.

6 Extensions

6.1 Third-best policies

We have considered a Ramsey problem in which the government faces two key constraints: only linear and anonymous instruments can be used. Still, this set of fiscal instruments confers a lot of power to the government, arguably more than what most governments have. When introducing a carbon tax policy, a government may not have complete freedom to adjust labor or capital income taxes. In particular, the full expropriation of asset holdings in the initial period that is optimal in our benchmark is not a realistic policy option. To explore these issues, we now turn to fiscal environments with additional constraints on the set of available instruments.

6.1.1 Third-best tax formulas

Exogenous labor income tax Let us assume that the planner cannot choose the labor income tax, that is exogenously fixed at a level $\bar{\tau}_H$ in all periods $t \geq 0$. The planner now faces additional constraints: in every period $t \geq 0$, it must ensure that

$$\frac{U_{h,t}}{U_{c,t}} = -\left(1-\bar{\tau}_H\right)\left(1-D_t\right) A_{1,t} F_{H,t}, \tag{26}$$

which pins down the wedge between the marginal rate of substitution between consumption and leisure and the marginal product of labor. For a given value of $\bar{\tau}_H$, equation (26) puts a restriction on the implementable allocations that the planner must satisfy. Let $\beta^t \Lambda_t^H$ denote the multiplier on the constraint (26). The latter is proportional to the welfare impact of raising the exogenous $\bar{\tau}_H$ in a particular period. The multiplier Λ_t^H will be positive (resp. negative) on average if the labor income tax is fixed at a sub-optimally high (resp. low) level. With the additional constraint (26) in each period t, the expression for the optimal pollution tax becomes[21]

$$\tau_{E,t} = \frac{1}{\nu_{1,t}} \sum_{j=0}^{\infty} \beta^j \left(\nu_{1,t+j} D'_{t+j} A_{1,t+j} F_{t+j} - N_{t+j} V_{Z,t+j} + \Lambda_{t+j}^H \left(1-\bar{\tau}_H\right) D'_{t+j} A_{1,t+j} F_{H,t+j} \right) J_{E_t^M,t+j}.$$

transfers.

[21]Without constraint (26), it is optimal to equalize the marginal rate of technical substitution between capital and labor across both sectors: the government does not wish to distort production decisions. In the third best, with constraint (26), this is no longer the case, and it is optimal to deviate from zero excise energy taxes, $\tau_{I,t}$. See Appendix **??** for more details.

where $\nu_{1,t}$ is the multiplier on the aggregate resource constraint in period t, which measures the scarcity of consumption goods and hence, the opportunity costs of reducing emissions.[22] Compared to equation (19), the main modification is the final component, which Barrage (2019) refers to as the fiscal interaction term. It reflects another reason for deviating from the Pigouvian tax rule. By reducing production damages, a higher pollution tax $\tau_{E,t}$ raises the marginal product of labor and hence, the before-tax wage. If τ_H is fixed at a sub-optimally low level, a further increase in the before-tax wage is welfare-reducing. The pollution tax then amplifies the costs of having a tax on labor income that is below the welfare-maximizing level. Consequently, the optimal pollution tax is reduced. The fiscal interaction term thus calls for a lower pollution tax when the labor income tax is fixed at a sub-optimally low level and *vice versa* if the labor income tax is fixed at a sub-optimally high level.

Exogenous capital income tax Let us now assume that the planner cannot choose the capital income tax, that is exogenously fixed at a level $\bar{\tau}_K$ in all periods $t \geq 0$. The new constraints faced by the planner are such that in every period $t \geq 0$

$$\frac{U_{c,t}}{U_{c,t+1}} = \beta \left(1 + (1 - \bar{\tau}_K) \left((1 - D_{t+1}) A_{1,t+1} F_{K,t+1} - \delta\right)\right), \tag{27}$$

which links the marginal rate of substitution between consumption in periods t and $t+1$ (on the left-hand side) to the after-tax interest rate (on the right-hand side). As with an exogenous labor income tax, equation (27) restricts the set of implementable allocations for a given value of $\bar{\tau}_K$. Let $\beta^t \Lambda^K_{t+1}$ be the multiplier on this constraint in period t. The multiplier is positive (negative) if the capital income tax rate is fixed at a sub-optimally high (low) level, so that raising $\bar{\tau}_K$ in a particular period lowers welfare. With the additional constraint (27), the expression for the optimal pollution tax is modified to:

$$\tau_{E,t} = \frac{1}{\nu_{1,t}} \sum_{j=0}^{\infty} \beta^j \left(\nu_{1,t+j} D'_{t+j} A_{1,t+j} F_{t+j} - N_{t+j} W_{Z,t+j} + \Lambda^K_{t+j} (1 - \bar{\tau}_K) D'_{t+j} A_{1,t+j} F_{K,t+j}\right) J_{E^M_t,t+j},$$

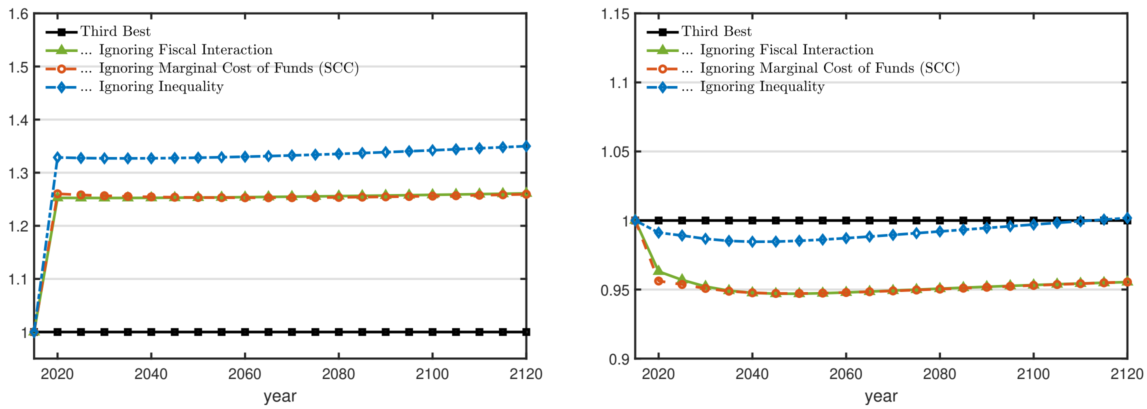
where again the last component captures the fiscal interaction term, as in Barrage (2019). The intuition is similar as before. A higher pollution tax raises the marginal product of capital by lowering production damages. The latter is beneficial if the capital income tax is fixed at a sub-optimally high level. A higher pollution tax then alleviates the savings distortion by raising the before-tax interest rate. If, by contrast, the capital income tax is fixed at a level below the one that maximizes welfare, a pollution tax amplifies the savings distortion and the fiscal interaction term reduces the optimal pollution tax.

6.1.2 Quantitative analysis

Figures 10, 11, and 12 in the appendix show the optimal path of income and carbon taxes in the previous third-best scenarios. Figure 5 below compares the third-best pollution tax normalized to 1

[22] Formally, the multiplier $\nu_{1,t}$ measures the welfare impact of decreasing government consumption G_t. In an environment without heterogeneity and distortionary taxes, the latter is equal to the representative agent's marginal utility of consumption.

(black line) with what it would be ignoring the new fiscal interaction term (green line), ignoring the MCF (red line), and ignoring inequalities (blue line). As in our benchmark scenario, the MCF plays an insignificant role but inequalities push the carbon tax downward. The effect of inequalities is slightly larger when the labor income tax is fixed: ignoring inequalities would increase the tax by around 6% in this scenario instead of 4% in the second-best and in the scenario where the capital tax is fixed. Indeed, since $\bar{\tau}_H$ is set to 25.5%, *i.e.* below the second-best tax rate, there are more consumption inequalities than in the second-best and the opportunity cost of emission abatement is higher.



(a) Given labor income tax

(b) Given capital income tax

Figure 5: Third-Best Carbon Tax Decomposition.

While the MCF still plays a negligible role, fiscal interactions now drive the carbon tax away from its Pigouvian level through the additional constraints that arise in the third-best environment. Interestingly, the fiscal interaction term lowers the optimal carbon tax when the labor income tax is fixed, whereas it raises the optimal carbon tax when the capital income tax is fixed. Recall that a carbon tax, by reducing production damages, increases *both* the marginal product of labor and the marginal product of capital and hence, the before-tax wage and interest rate. A higher before-tax wage, in turn, lowers welfare because the labor income tax is set at a sub-optimally low level (i.e., $\bar{\tau}_H = 25.5\%$ instead of around 49% at the optimum), whereas a higher before-tax interest rate raises welfare because the capital income tax is set at a sub-optimally high level (i.e., $\bar{\tau}_K = 41.1\%$ instead of virtually 0% at the optimum). A higher carbon tax thus alleviates the savings distortion, whereas it amplifies the costs of taxing labor income at a sub-optimally low level. This explains why quantitatively we find that the fiscal interaction term is positive (negative) when the capital (labor) income tax is fixed.

Appendix E.2 also provides the government budget adjustments and welfare gains in these third-best policy scenarios. These results suggest that the general pattern of the distribution of welfare gains from carbon taxation does not strongly depend on the fiscal policies currently in place, but the optimal use of the carbon tax revenue does. While this revenue is split about equally between increasing transfers and reducing the labor income tax in our baseline scenario, with additional constraints on instruments this is not the case anymore. In particular, when the government is forced to redistribute "too much" because the capital tax is set above the optimum, the carbon tax revenue is mostly targeted towards reducing tax distortions. By contrast, when the government is forced to redistribute "too little" because the labor income tax is set below the optimum, the revenue is instead targeted towards redistribution. In the latter case, introducing carbon taxation has a smaller negative impact on current generations, more progressive effects, and the least productive households experience welfare gains in all periods.

6.2 Initial wealth inequality

In this section, we consider the effect of initial wealth inequality on the optimal tax system. We first discuss its implications for the time-consistency of Ramsey policies, then discuss the optimal rules and investigate the quantitative effects given the levels of wealth inequality observed in the U.S.

6.2.1 Time-inconsistency

The tax rules we have described above apply unchanged for every period including period 0. This is the result of two features of the model considered so far. The first is the ability of the Ramsey planner to choose lump-sum transfers (or taxes), and the second is the assumption that there is no initial wealth inequality. To see this, notice that the planner's problem, see equation (17), is symmetric with respect to time except for the the last term in the objective function of the Ramsey planner, which we denote here by W_0,

$$W_0 = N_0 U_{c,0} \sum_i \pi_i \theta_i \left(R_0 a_{i,0} + T\right).$$

As argued above, the optimality condition associated with the choice of T implies that $\sum_i \pi_i \theta_i = 0$. Thus, if $a_{i,0} = a_0$ for every i, it follows that $W_0 = 0$ and that the tax rules are time invariant.

This does not mean that the tax rules are time-consistent: if the Ramsey planner was allowed to reoptimize in a future period, they would want to deviate from the choices made by the planner in period 0. The reason for the time-inconsistency is, however, different from the one in the usual representative-agent version of the Ramsey problem in which the planner cannot choose lump-sum transfers. In that case, in general $\sum_i \pi_i \theta_i \neq 0$, and $W_0 \neq 0$ regardless of initial wealth inequality, which leads to the usual reason for time-inconsistent Ramsey policies; initial capital income taxes mimic the unavailable and undistortive lump-sum taxes. In our setup, the reason for time inconsistency has to do instead with the use of capital income taxes to redistribute unequal asset income.

There is a sense in which the time-inconsistency problem in our setup is less severe than in the usual representative-agent case. If there is no initial wealth inequality, and the optimal Ramsey policy was such that the economy was in a balanced growth path starting from period 0, then there would still be no wealth inequality in every future period and the Ramsey policy would be time-consistent. In any case, in this section we address how the Ramsey policy changes in the presence of initial wealth inequality.

6.2.2 Optimal tax rules

Before characterizing the optimal pollution tax in this constrained environment, it is worth mentioning that without initial wealth inequality, the fact that we do not allow the planner to choose capital taxes in period 0, $\tau_{K,0}$, is immaterial. The planner is indifferent with respect to this choice (i.e., $\partial W_0/\partial \tau_{K,0} = 0$), because a tax on capital income in period 0 is equivalent to a reduction in lump-sum transfers T if there is no initial wealth inequality. With initial wealth inequality, this is no longer the case. In fact, if the planner could choose $\tau_{K,0}$, it would typically be optimal to expropriate all initial wealth, which by construction eliminates all initial wealth inequality as well. So, if wealth inequality is to have any effect on the tax rules, $\tau_{K,0}$ must be restricted.[23] In the quantitative results presented below we fix $\tau_{K,0}$ to be at the same level as in the current tax system.

For $t \geq 1$, the optimal tax rules are not affected by the presence of initial wealth inequality.[24] However, if there is initial wealth inequality and $\tau_{K,0}$ cannot be chosen to eliminate these differences, there is another reason for deviating from Pigouvian taxation in period 0. Specifically, the optimal carbon tax satisfies (see Appendix B):

$$\tau_{E,0} = \frac{1}{\nu_{1,0}} \sum_{j=0}^{\infty} \beta^j \left(\nu_{1,j} D'_j A_{1,j} F_j - N_j W_{Z,j}\right) J_{E_0^M,j} - N_0 \frac{U_{c,0}}{\nu_{1,0}} \Delta \left(1 - \tau_{K,0}\right) D'_0 A_{1,0} F_{K,0} J_{E_0^M,0}.$$

Compared to equation (19), the final term is new and is proportional to

$$\Delta \equiv \sum_i \pi_i \theta_i a_{i,0},$$

which captures the costs of initial wealth inequality. If initial assets and productivity are positively correlated, the costs of initial wealth inequality is typically positive as well: $\Delta > 0$.[25] These costs are amplified by a pollution tax because the latter, through a reduction in production damages, raises the marginal product of capital. An equivalent interpretation is that, by not allowing the planner to expropriate all wealth, the tax on capital income $\tau_{K,0}$ is set at a sub-optimally low level. It is then optimal to set a lower pollution tax in period 0, because additional production damages contribute to a reduction in the return to capital. This explains why *ceteris paribus*, the optimal pollution tax is lower than would be the case without initial wealth inequality.

6.2.3 Quantitative analysis of the effect of wealth inequality

We calibrate the joint distribution of productivity and initial wealth from the SCF. We divide households into 10 productivity groups, and 10 wealth groups within each productivity group, for a total of 100 different types of equal size. The full procedure is described in Appendix D.1.

[23] In this case, abstracting from consumption taxes is not inconsequential, as consumption taxes can be combined with appropriate labor income taxes to mimic wealth taxes (see Werning, 2007).

[24] The exception is the tax rule for $\tau_{K,1}$. See Appendix B for details.

[25] As explained before, the multiplier θ_i on the implementability constraint is zero on average and positive (negative) for individuals with high (low) productivity, as raising the lump-sum transfer for a rich (poor) agent would typically contribute to a reduction (increase) in welfare.

Figures 15 and 16 in the appendix show the optimal path of income and carbon taxes with initial wealth heterogeneity when the initial capital tax is fixed at its current level. Figure 17 provides a decomposition similar to the one shown in Figure 3 above. Table VIII shows the government budget adjustments made relative to the climate skeptic planner. Figure 18 displays the distribution of the lifetime welfare gains for each of the 100 groups. These gains are U-shaped with respect to income, but strictly increasing with initial wealth.

6.3 Additional sources of heterogeneity

6.3.1 Optimal tax rules

Model Our benchmark model considers heterogeneous agents who differ in productivity and initial asset holdings. To further explore the role of agents heterogeneity on optimal fiscal policy, we now introduce two additional ingredients to our benchmark model: a second consumption good, and heterogeneous preferences. We assume that a household of type i derives utility from the consumption of a final good $c_{i,t}$, labor supply $h_{i,t}$, environmental degradation Z_t, and the consumption of a "dirty" good $d_{i,t}$ according to a utility function

$$\sum_{t=0}^{\infty} \beta^t u_i\left(c_{i,t}, d_{i,t}, h_{i,t}, Z_t\right),$$

where the second dirty good d is produced from a linear technology that uses energy as its only input. To further simplify notations, we assume that energy produced in the energy sector (E_t) is now used in the final good sector or directly consumed by households, such that

$$E_t = E_{1,t} + N_t d_t,$$

with $E_{1,t}$ the quantity of energy used as an input in the final good sector and $d_t = \sum_i \pi_i d_{i,t}$ the households' average per period energy consumption. In order to match empirically observed budget shares for energy (or alternatively, polluting goods) for different income groups, we assume households utility can be represented by the following period utility function

$$u_i\left(c_i, d_i, h_i, Z\right) = \frac{\left(c_i(d_i - \bar{d}_i)^{\epsilon}(1-\varsigma h_i)^{\gamma}\right)^{1-\sigma}}{1-\sigma} + \chi_i \frac{\left(1+\alpha_0 Z^2\right)^{-(1-\sigma)}}{1-\sigma}.$$

Thus, in line with previous studies in this literature (*e.g.* Fried et al., 2018; Klenert et al., 2018; Aubert and Chiroleu-Assouline, 2019; Jacobs and van der Ploeg, 2019) preferences for consumption are modeled with a Stone-Geary utility function, so that an agent of type i experiences positive utility from energy consumption only after consuming its first $\bar{d}_i$ units of energy. $\bar{d}_i$ therefore denotes the subsistence consumption level of energy for an agent of type i, which we allow to be type (and time) specific. This specification allows us to consider households with non-homothetic preferences to better capture the heterogeneous impact of pollution taxes on households' budgets. Assuming type-specific values for $\bar{d}_i$, this specification also allows us to consider non-linear *aggregate* Engel curves as well as horizontal heterogeneity.[26,27] In addition, we assume that agents' relative sensitivity to the environmental variable Z is also type specific and denoted χ_i, normalized such that $\sum_i \pi_i \chi_i = 1$.

Because there is an additional consumption good, the planner uses an additional instrument: it levies an excise tax $\tau_{D,t}$ on households' consumption of energy. The budget constraint of agents of type i can thus be expressed as

$$\sum_{t=0}^{\infty} p_t N_t \Big(c_{i,t} + d_{i,t}(p_{E,t} + \tau_{D,t}) - (1 - \tau_{H,t})\, w_t e_i h_{i,t} \Big) \leq R_0 N_0 a_0 + T. \tag{28}$$

To focus on the additional sources of heterogeneity, we assume here that there is no initial wealth inequality, so that $a_{i,0} = a_0$ for all i.

Solution method We apply the same solution method as in our benchmark model. Using the method of Werning (2007), we can express individual allocations as a function of aggregate variables and market weights. These expressions allow us to write the aggregate utility function $U(c_t, d_t, h_t, Z_t, \varphi)$ and individual implementability conditions necessary to solve the Ramsey problem based on aggregate variables and market weights only.

Optimal taxes As shown in Appendix C.4, the second-best labor income tax in this extended framework is

$$\tau_{H,t} = \frac{\Psi \varsigma (1 - \varsigma h_t)^{-1}}{\Phi + \Psi \frac{\varsigma \left(1 - \gamma(1-\sigma)\right)}{(1 - \varsigma h_t)} - \Lambda_t \frac{\epsilon(\sigma - 1)}{(d_t - \bar{d}_t)}},$$

the capital income can be obtained from

$$\frac{R_{t+1}}{R^*_{t+1}} = \frac{\Phi + \Psi \frac{\varsigma \gamma (\sigma - 1)}{(1 - \varsigma h_{t+1})} - \Lambda_{t+1} \frac{\epsilon(\sigma-1)}{(d_{t+1} - \bar{d}_{t+1})}}{\Phi + \Psi \frac{\varsigma \gamma(\sigma-1)}{(1-\varsigma h_t)} - \Lambda_t \frac{\epsilon(\sigma-1)}{(d_t - \bar{d}_t)}},$$

the excise tax on energy remains unchanged at $\tau_{I,t} = 0$, and the households energy consumption excise tax is

$$\tau_{D,t} = \frac{\Lambda_t \epsilon c_t}{\Phi + \frac{\Psi \varsigma \gamma (\sigma-1)}{(1 - \varsigma H_t)} - \frac{\Lambda_t \epsilon (\sigma - 1)}{(D_t - \bar{D}_t)}},$$

with

$$\begin{aligned}
\Phi &= \sum_j \pi_j \frac{\lambda_j}{\varphi_j} + \Big(1 - (1 + \epsilon + \gamma)(1 - \sigma)\Big) \mathrm{cov}(\lambda_i/\varphi_i, \omega_i), \\
\Psi &= -\frac{\mathrm{cov}(\lambda_i/\varphi_i, e_i)}{\varsigma}, \\
\Lambda_t &= -\mathrm{cov}(\lambda_i/\varphi_i, \bar{d}_{i,t}).
\end{aligned}$$

[26] With Stone-Geary preferences, agents' Engel curves are linear. When preferences are heterogeneous, the aggregate distribution of expenditures may however be a non-linear function of income.

[27] Horizontal heterogeneity arises when individuals with the same income do not consume goods in the same proportions. Recent studies have shown the importance of horizontal heterogeneity on the distributional impacts of energy taxes (Cronin et al., 2019; Pizer and Sexton, 2019), and their implications for the design of tax reforms (Sallee, 2019).

Turning to the pollution tax, we obtain the same general formula as in our benchmark model

$$\tau_{E,t} = \sum_{j=0}^{\infty} \beta^j \left(\frac{V_{c,t+j} + \sum_i \pi_i \theta_i MIC_{i,t+j}}{V_{c,t} + \sum_i \pi_i \theta_i MIC_{i,t}} D'_{t+j} A_{1,t+j} F_{t+j} - \frac{N_{t+j} V_{Z,t+j}}{V_{c,t} + \sum_i \pi_i \theta_i MIC_{i,t}} \right) J_{E_t^M, t+j}.$$

Comparison with the benchmark model Besides the differences in the path of allocations, the key differences with our benchmark lie in the expressions of the marginal implementation cost and the marginal utilities.

As shown in Appendix C.4, we can again express the marginal implementation cost as the covariance between θ_i and $MIC_{i,t}$, but this last term takes a different form. While in our benchmark the covariance was positive when increasing aggregate consumption led richer households (higher θ_i) to consume relatively more or work relatively less compared to poorer households, now its value is also higher when the energy needs of richer agents increase relative to poorer households. Using our functional form for utility, we again see that this additional energy demand effect would disappear if the relative energy consumption of two agents was constant over time, or simply unaffected by changes in aggregate consumption (which is the case when $\sigma = 1$).

The expression of the marginal utility of consumption ($V_{c,t}$) is also affected by the presence of a second consumption good, since utility is not strongly separable in C and D. Regarding the marginal dis-utility from pollution ($V_{Z,t}$), the energy consumption good has no direct impact, but heterogeneity in the relative sensitivity to the environmental variable captured by the distribution of χ_i may play a role. Indeed, we now have

$$V_{Z,t} = -(1 + \text{cov}(\lambda_i, \chi_i)) 2\alpha_0 Z_t (1 + \alpha_0 Z_t^2)^{\sigma-2},$$

so that the distribution of χ_i matters in the marginal valuation of pollution to the extent that it is correlated with the planner's weights. When the agents most valued by the planner are more sensitive to pollution, the tax is set at a higher level. If we assume that the planner has utilitarian preferences however, then for all i, λ_i is constant and the distribution of χ_i has no impact on the aggregate marginal dis-utility from pollution.

The role of preferences heterogeneity In the special case where preferences are homogeneous (*i.e.* assuming that for $t \geq 0$ and for all i, $\bar{d}_{i,t} = \bar{d}_t$ and $\chi_i = 1$), we have $\Lambda_t = 0$, and all tax formulas remain unchanged relative to our benchmark. In particular, although poor households spend a larger share of their budget in energy, the pollution tax formula remains the same and the excise tax on energy consumption is null. This result is reminiscent of Jacobs and van der Ploeg (2019) who show that as long as Engel curves are linear—which is the case with Stone-Geary utility—corrective taxation should not serve to address redistributive objectives, even when non-linear income taxation is not available. Still, the distribution of market weights is affected by the consumption of a second good: having a second good modeled as a necessity generates a fixed-cost to households welfare, which affects the whole distribution of welfare. Hence, even though the optimal tax formulas are preserved, the level of the tax rates will be affected by this additional source of heterogeneity as the formulas will be evaluated at a different allocation.

With heterogeneous preferences for energy consumption, Λ_t is not generally equal to zero anymore. When the consumption threshold ($\bar{d}_i$) varies positively with the relative planner's weight (λ_i/φ_i)—*i.e.* when individuals who are relatively more valued by the planner are also the ones with higher energy needs—then Λ_t is negative. This has two effects. First, it affects the implementation cost. The sign of this effect depends on the value of σ, which once again captures the price effect discussed above. For $\sigma > 1$, a negative Λ_t will lower the labor income tax, the pollution tax, and the absolute value of the excise tax on energy consumption. The second effect is captured by the numerator of the excise tax on energy consumption: when Λ_t is negative, this tax is negative. The logic behind this result is that the *aggregate* Engel curve being non-linear with heterogeneous preferences, commodity taxes offer an additional levy for redistribution. When the agents who are valued relatively more by the planner also have higher energy needs, the planner can target these agents by subsidizing the energy good.

The sign and magnitude of the previous mechanisms therefore depend on the distribution of $\{\bar{d}_i\}_{i\in I}$, both between and within productivity types. First, as less productive types tend to have higher marginal utilities of consumption, the relative planner's weights are generally higher for these agents. Λ will therefore be lower (resp. higher) to the extent that less (resp. more) productive agents have on average higher energy needs. Second, for a given productivity level, agents with higher energy needs will also tend to have higher marginal utilities of consumption because of the higher fixed cost that they incur. This horizontal heterogeneity will therefore drive the value of Λ downward. Our quantitative analysis below uses data on U.S. households energy consumption to illustrate the impact of these two sources of heterogeneity.

6.3.2 Quantitative analysis of the extended model

For each of the ten productivity groups described above, we compute the initial distribution of energy needs from the Consumer Expenditure Surveys (CEX). The full procedure is described in Appendix D.1.

[To be included: quantitative analysis.]

7 Conclusion

Should environmental policies be less stringent in the presence of inequalities? Do inequalities increase when optimal environmental policies are implemented? This paper attempts to shed light on these questions. We develop a climate-economy model where environmental degradation generates both production and utility externalities. Our model features heterogeneous agents, which provides a micro-foundation for the use of distortionary taxes on labor and capital income. We study both theoretically and quantitatively how different sources of heterogeneity and a concern for redistribution affect the optimal carbon tax.

We show that when agents are heterogeneous but individualized lump-sum taxation is not available, the optimal carbon tax is almost equal to the social cost of carbon (SCC), but the SCC is lower than it would be absent inequalities. Indeed, tax distortions do not significantly matter for carbon taxation when distortionary taxes are optimally chosen to provide redistribution, and the optimal carbon tax is almost Pigouvian. However, inequalities call for lower carbon taxes owing to the fact that the presence of poor households increases the marginal value of consumption and increases the opportunity cost of pollution abatement. We also re-examine the double dividend hypothesis, and show that at the optimum the carbon tax revenue is divided about equally between increasing transfers and reducing distortionary taxes. This revenue recycling increases the progressivity of the tax system, making the carbon tax policy relatively more beneficial for poorer households. In the long run however, rich households experience larger welfare gains from climate change mitigation because their willingness to pay for environmental improvement is higher.

Our paper includes numerous extensions. We analyze alternative policy scenarios, and multiple sources of household heterogeneity, including heterogeneous budget shares, unequal initial assets, and differences in the sensitivity to environmental damages. Still, there are other relevant aspects that we have abstracted from. In particular, we have left for future research the role of risk—on the economic or climate side—which could interact with inequalities and be an important determinant of fiscal policies.

References

Aubert, Diane and Mireille Chiroleu-Assouline (2019) "Environmental tax reform and income distribution with imperfect heterogeneous labour markets," *European Economic Review*, 116, 60–82, https://doi.org/10.1016/j.euroecorev.2019.03.006.

Barrage, Lint (2019) "Optimal Dynamic Carbon Taxes in a ClimateEconomy Model with Distortionary Fiscal Policy," *The Review of Economic Studies*, 87 (1), 1–39, 10.1093/restud/rdz055.

Bovenberg, Lans and Ruud de Mooij (1994) "Environmental Levies and Distortionary Taxation," *American Economic Review*, 84 (4), 1085–89.

Bovenberg, Lans and Frederick (Rick) van der Ploeg (1994) "Environmental policy, public finance and the labour market in a second-best world," *Journal of Public Economics*, 55 (3), 349–390, `https://EconPapers.repec.org/RePEc:eee:pubeco:v:55:y:1994:i:3:p:349-390`.

Chamley, Christophe (1986) "Optimal taxation of capital income in general equilibrium with infinite lives," *Econometrica: Journal of the Econometric Society*, 607–622.

Chari, VV and Patrick J Kehoe (1999) "Chapter 26 optimal fiscal and monetary policy. volume 1 of Handbook of Macroeconomics."

Chetty, Raj, Adam Guren, Day Manoli, and Andrea Weber (2011) "Are micro and macro labor supply elasticities consistent? A review of evidence on the intensive and extensive margins," *American Economic Review*, 101 (3), 471–75.

Cremer, Helmuth, Firouz Gahvari, and Norbert Ladoux (1998) "Externalities and optimal taxation," *Journal of Public Economics*, 70 (3), 343–364, https://doi.org/10.1016/S0047-2727(98)00039-5.

——— (2003) "Environmental taxes with heterogeneous consumers: an application to energy consumption in France," *Journal of Public Economics*, 87 (12), 2791–2815.

Cronin, Julie Anne, Don Fullerton, and Steven Sexton (2019) "Vertical and Horizontal Redistributions from a Carbon Tax and Rebate," *Journal of the Association of Environmental and Resource Economists*, 6 (S1), 169–208.

Cruz, José-Luis and Esteban Rossi-Hansberg (2021) "The Economic Geography of Global Warming," Working Paper 28466, National Bureau of Economic Research, 10.3386/w28466.

Diamond, Peter A and James A Mirrlees (1971) "Optimal taxation and public production I: Production efficiency," *The American economic review*, 61 (1), 8–27.

Dietz, Simon, Frederick van der Ploeg, Armon Rezai, and Frank Venmans (2021) "Are economists getting climate dynamics right and does it matter?" *Journal of the Association of Environmental and Resource Economists*, 8 (5), 895–921.

Douenne, Thomas and Adrien Fabre (2022) "Yellow Vests, Pessimistic Beliefs, and Carbon Tax Aversion," *American Economic Journal: Economic Policy*, 14 (1), 81–110, 10.1257/pol.20200092.

Dyrda, Sebastian and Marcelo Pedroni (forthcoming) "Optimal fiscal policy in a model with uninsurable idiosyncratic shocks," *The Review of Economic Studies*.

Fried, Stephie, Kevin Novan, and William Peterman (2018) "The Distributional Effects of a Carbon Tax on Current and Future Generations," *Review of Economic Dynamics*, 30, 30–46, https://EconPapers.repec.org/RePEc:red:issued:16-217.

——— (2021) "Recycling Carbon Tax Revenue to Maximize Welfare," working paper, FEDS Working Paper.

Fullerton, Don and Holly Monti (2013) "Can pollution tax rebates protect low-wage earners?" *Journal of Environmental Economics and Management*, 66 (3), 539–553.

Golosov, Mikhail, John Hassler, Per Krusell, and Aleh Tsyvinski (2014) "Optimal Taxes on Fossil Fuel in General Equilibrium," *Econometrica*, 82 (1), 41–88, http://www.jstor.org/stable/24029171.

Goulder, Lawrence H (1995) "Environmental taxation and the double dividend: a reader's guide," *International tax and public finance*, 2 (2), 157–183.

Goulder, Lawrence H, Marc AC Hafstead, GyuRim Kim, and Xianling Long (2019) "Impacts of a carbon tax across US household income groups: What are the equity-efficiency trade-offs?" *Journal of Public Economics*, 175, 44–64.

Hassler, John and Per Krusell (2012) "Economics and climate change: integrated assessment in a multi-region world," *Journal of the European Economic Association*, 10 (5), 974–1000.

Jacobs, Bas and Ruud A. de Mooij (2015) "Pigou meets Mirrlees: On the irrelevance of tax distortions for the second-best Pigouvian tax," *Journal of Environmental Economics and Management*, 71, 90–108, https://doi.org/10.1016/j.jeem.2015.01.003.

Jacobs, Bas and Frederick (Rick) van der Ploeg (2019) "Redistribution and pollution taxes with non-linear Engel curves," *Journal of Environmental Economics and Management*, 95 (C), 198–226, https://EconPapers.repec.org/RePEc:eee:jeeman:v:95:y:2019:i:c:p:198-226.

Judd, Kenneth L (1985) "Redistributive taxation in a simple perfect foresight model," *Journal of public Economics*, 28 (1), 59–83.

Känzig, Diego (2021) "The unequal economic consequences of carbon pricing," working paper, London Business School.

Kaplow, Louis (2012) "Optimal Control of Externalities in the Presence of Income Taxation," *International Economic Review*, 53 (2), 487–509, http://www.jstor.org/stable/23251596.

Klenert, David, Gregor Schwerhoff, Ottmar Edenhofer, and Linus Mattauch (2018) "Environmental Taxation, Inequality and Engels Law: The Double Dividend of Redistribution," *Environmental & Resource Economics*, 71 (3), 605–624, 10.1007/s10640-016-0070-y.

Kotlikoff, Laurence, Felix Kubler, Andrey Polbin, Jeffrey Sachs, and Simon Scheidegger (2021) "Making carbon taxation a generational win win," *International Economic Review*, 62 (1), 3–46.

Krusell, Per and Anthony A Smith (2015) "Climate change around the world," in *Work in progress, Walras-Bowley Lecture. Econometric Society 2015 World Congress, Montréal*.

Kuhn, Moritz and José-Víctor Ríos-Rull (2016) "2013 Update on the US earnings, income, and wealth distributional facts: A View from Macroeconomics," *Federal Reserve Bank of Minneapolis Quarterly Review*, 37 (1), 2–73.

Leach, Andrew J (2009) "The welfare implications of climate change policy," *Journal of Environmental Economics and Management*, 57 (2), 151–165.

Levinson, Arik and James OBrien (2019) "Environmental Engel curves: Indirect emissions of common air pollutants," *Review of Economics and Statistics*, 101 (1), 121–133.

Micheletto, Luca (2008) "Redistribution and optimal mixed taxation in the presence of consumption externalities," *Journal of Public Economics*, 92 (10-11), 2262–2274.

Nordhaus, William (2018) "Projections and uncertainties about climate change in an era of minimal climate policies," *American economic journal: economic policy*, 10 (3), 333–60.

Nordhaus, William D. (2017) "Revisiting the social cost of carbon," *Proceedings of the National Academy of Sciences*, 114 (7), 1518–1523, 10.1073/pnas.1609244114.

Pigou, A.C. (1920) *The Economics of Welfare*: Macmillan.

Piketty, Thomas and Emmanuel Saez (2013) "Optimal labor income taxation," in *Handbook of public economics*, 5, 391–474: Elsevier.

Pindyck, Robert S (2013) "Climate change policy: what do the models tell us?" *Journal of Economic Literature*, 51 (3), 860–72.

Pirttilä, Jukka and Matti Tuomala (1997) "Income tax, commodity tax and environmental policy," *International Tax and Public Finance*, 4 (3), 379–393.

Pizer, William A and Steven Sexton (2019) "The Distributional Impacts of Energy Taxes," *Review of Environmental Economics and Policy*, 13 (1), 104–123.

Rausch, Sebastian (2013) "Fiscal consolidation and climate policy: An overlapping generations perspective," *Energy Economics*, 40, S134–S148.

Rausch, Sebastian, Gilbert E Metcalf, and John M Reilly (2011) "Distributional impacts of carbon pricing: A general equilibrium approach with micro-data for households," *Energy economics*, 33, S20–S33.

Rausch, Sebastian and Hidemichi Yonezawa (2018) "The intergenerational incidence of green tax reform," *Climate Change Economics*, 9 (01), 1840007.

Sallee, James M (2019) "Pigou Creates Losers: On the Implausibility of Achieving Pareto Improvements from Efficiency-Enhancing Policies," Working Paper 25831, National Bureau of Economic Research, 10.3386/w25831.

Sandmo, Agnar (1975) "Optimal Taxation in the Presence of Externalities," *The Swedish Journal of Economics*, 77 (1), 86–98, http://www.jstor.org/stable/3439329.

Straub, Ludwig and Iván Werning (2020) "Positive long-run capital taxation: Chamley-Judd revisited," *American Economic Review*, 110 (1), 86–119.

Trabandt, Mathias and Harald Uhlig (2012) "How do Laffer curves differ across countries?" *Fiscal policy after the financial crisis*, 211–249.

Weitzman, Martin L (2009) "On modeling and interpreting the economics of catastrophic climate change," *The review of economics and statistics*, 91 (1), 1–19.

Werning, Iván (2007) "Optimal Fiscal Policy with Redistribution," *The Quarterly Journal of Economics*, 122 (3), 925–967, 10.1162/qjec.122.3.925.

Williams, Roberton, Hal Gordon, Dallas Burtraw, Jared Carbone, and Richard D. Morgenstern (2015) "The Initial Incidence of a Carbon Tax Across Income Groups," *National Tax Journal*, 68 (1), 195–214.

Appendices

A Optimal tax rules in the benchmark model

A.1 Implementability conditions

Let $\varphi \equiv \{\varphi_i\}$ be the market weights with $\varphi_i \geq 0$. Then, given aggregate levels c_t, h_t and Z_t, the individual levels can be found by solving the following static subproblem for each period t:

$$U(c_t, h_t, Z_t; \varphi) \equiv \max_{c_{i,t}, h_{i,t}} \sum_i \pi_i \varphi_i u(c_{i,t}, h_{i,t}, Z_t), \quad \text{s.t.} \sum_i \pi_i c_{i,t} = c_t, \quad \text{and} \quad \sum_i \pi_i e_i h_{i,t} = h_t. \tag{29}$$

The Lagrangian for this problem is

$$L = \sum_i \pi_i \varphi_i u(c_{i,t}, h_{i,t}, Z_t) + \theta_t^c \left(c_t - \sum_i \pi_i c_{i,t} \right) - \theta_t^h \left(h_t - \sum_i \pi_i e_i h_{i,t} \right),$$

where θ_t^c and θ_t^h are Lagrange multipliers. Applying the envelope theorem to problem (29), we get

$$U_{c,t} = \theta_t^c, \quad \text{and} \quad U_{h,t} = -\theta_t^h.$$

From the first order conditions of problem (29), we also have

$$\varphi_i u_{c,i,t} = \theta_t^c, \quad \text{and} \quad \varphi_i u_{h,i,t} = -e_i \theta_t^h.$$

It follows that

$$U_{c,t} = \varphi_i u_{c,i,t}, \tag{30}$$

$$U_{h,t} = \frac{\varphi_i u_{h,i,t}}{e_i}. \tag{31}$$

In any competitive equilibrium these optimality conditions must hold for every agent i. Hence, using (30), (31), and agents' first order conditions given by

$$\beta^t \frac{u_{c,i,t}}{u_{c,i,0}} = p_t, \quad \forall\, t \geq 0, \tag{32}$$

$$\frac{u_{h,i,t}}{u_{c,i,t}} = -(1 - \tau_{H,t}) e_i w_t, \quad \forall\, t \geq 0, \tag{33}$$

we obtain

$$\frac{U_{h,t}}{U_{c,t}} = \frac{u_{h,i,t}}{u_{c,i,t} e_i} = -w_t (1 - \tau_{H,t}), \tag{34}$$

and

$$\frac{U_{c,t}}{U_{c,0}} = \frac{u_{c,i,t}}{u_{c,i,0}} = \frac{p_t}{\beta^t}. \tag{35}$$

Given the relationships above we can derive the implementation condition which relies only on the aggregates c_t, h_t, and market weights φ. Let $c_{i,t}^m(c_t, h_t; \varphi)$ and $h_{i,t}^m(c_t, h_t; \varphi)$ be the arg max of problem (29). The budget constraint of agent i implies

$$\sum_{t=0}^{\infty} N_t p_t \left(c_{i,t}^m(c_t, h_t; \varphi) - (1 - \tau_{H,t}) w_t e_i h_{i,t}^m(c_t, h_t; \varphi) \right) \leq R_0 N_0 a_{i,0} + T,$$

which using (34) and (35) can be restated as

$$U_{c,0}\big(R_0N_0a_{i,0}+T\big)\geq\sum_{t=0}^{\infty}N_t\beta^t\Big(U_{c,t}c_{i,t}^m\big(c_t,h_t;\varphi\big)+U_{h,t}e_ih_{i,t}^m\big(c_t,h_t;\varphi\big)\Big),\quad\forall\ i. \tag{36}$$

A.2 Ramsey problem

A.2.1 Problem

Let $\lambda\equiv\{\lambda_i\}$ be the planner's welfare weight on type i, with $\sum_i\pi_i\lambda_i=1$. Define

$$\begin{aligned}W\left(c_t,h_t,Z_t;\varphi,\theta,\lambda\right)\equiv&\sum_i\pi_i\lambda_iu\left(c_{i,t}^m\left(c_t,h_t;\varphi\right),h_{i,t}^m\left(c_t,h_t;\varphi\right),Z_t\right)\\&+\sum_i\pi_i\theta_i\left[U_{c,t}c_{i,t}^m\left(c_t,h_t;\varphi\right)+U_{h,t}e_ih_{i,t}^m\left(c_t,h_t;\varphi\right)\right]\end{aligned}$$

where $\pi_i\theta_i$ is the Lagrange multiplier on the implementability constraint of agent i, and $\theta\equiv\{\theta_i\}$. The Ramsey problem can be written as

$$\max_{\substack{\{C_t,H_{1,t},H_{2,t},K_{1,t},K_{2,t},\\E_t,Z_t,\mu_t\}_{t=0}^{\infty},T,\varphi}}\sum_{t,i}N_t\beta^tW\left(c_t,h_t,Z_t;\varphi,\theta,\lambda\right)-U_{c,0}\sum_i\pi_i\theta_i\left(R_0N_0a_{i,0}+T\right)$$

subject to

$$\begin{aligned}N_tc_t+G_t+K_{t+1}+\Theta_t\left(\mu_t,E_t\right)&=\left(1-D\left(Z_t\right)\right)A_{1,t}F\left(K_{1,t},H_{1,t},E_t\right)+\left(1-\delta\right)K_t,\quad\forall\ t\geq0,\\E_t&=A_{2,t}G\left(K_{2,t},H_{2,t}\right),\quad\forall\ t\geq0,\\Z_t&=J\left(S_0,E_0^M,...,E_t^M,\eta_0,...,\eta_t\right),\quad\forall\ t\geq0,\\F_{K,t}G_{H,t}&=G_{K,t}F_{H,t},\quad\forall\ t\geq0,\\K_{1,t}+K_{2,t}&=K_t,\quad\forall\ t\geq0,\\H_{1,t}+H_{2,t}&=N_th_t,\quad\forall\ t\geq0,\end{aligned}$$

where $\beta^t\nu_{jt}$ for $j\in\{1,2,3\}$ are the Lagrange multipliers on the feasibility constraints in the order above. When using a functional form for households' utility below, it will also be convenient to add an additional constraint from the normalization of market weights. Because this constraint is a simple normalization, it has no impact on the resulting allocations.

In what follows, we assume that there is no initial wealth inequality, that is $a_{i,0}=a_{j,0}$ for every i and j. We relax this assumption in Appendix B.

A.2.2 First order conditions

The first order conditions are

$$[c_t] : W_c\left(c_t, h_t; \varphi, \theta, \lambda\right) - \nu_{1,t} = 0, \quad \forall\, t \geq 0, \tag{37}$$

$$[H_{1,t}] : W_h\left(c_t, h_t; \varphi, \theta, \lambda\right) + \nu_{1,t}\left(1 - D\left(Z_t\right)\right) A_{1,t} F_H\left(K_{1,t}, H_{1,t}, E_t\right) = 0, \quad \forall\, t \geq 0, \tag{38}$$

$$[H_{2,t}] : W_h\left(c_t, h_t; \varphi, \theta, \lambda\right) + \nu_{2,t} A_{2,t} G_H\left(K_{2,t}, H_{2,t}\right) = 0, \quad \forall\, t \geq 0, \tag{39}$$

$$[K_{1,t+1}] : -\nu_{1,t} + \left[\left(1 - D\left(Z_{t+1}\right)\right) A_{1,t+1} F_K\left(K_{1,t+1}, H_{1,t+1}, E_{t+1}\right) + (1-\delta)\right] \beta \nu_{1,t+1} = 0, \quad \forall\, t \geq 0, \tag{40}$$

$$[K_{2,t+1}] : -\nu_{1,t} + A_{2,t+1} G_K\left(K_{2,t+1}, H_{2,t+1}\right) \beta \nu_{2,t+1} + (1-\delta)\, \beta \nu_{1,t+1} = 0, \quad \forall\, t \geq 0, \tag{41}$$

$$\begin{aligned}[E_t] : &-\nu_{1,t}\left(\Theta_{E,t}\left(\mu_t, E_t\right) - \left(1 - D\left(Z_t\right)\right) A_{1,t} F_E\left(K_{1,t}, H_{1,t}, E_t\right)\right) - \nu_{2,t} \\ &- \sum_{j=0}^{\infty} \beta^j \nu_{3,t+j} J_{E_t^M, t+j}\left(1 - \mu_t\right) = 0, \quad \forall\, t \geq 0,\end{aligned} \tag{42}$$

$$[Z_t] : N_t W_Z\left(c_t, h_t, Z_t; \varphi, \theta, \lambda\right) - \nu_{1,t} D'\left(Z_t\right) A_{1,t} F\left(K_{1,t}, H_{1,t}, E_t\right) + \nu_{3,t} = 0, \quad \forall\, t \geq 0, \tag{43}$$

$$[\mu_t] : -\nu_{1,t} \Theta_{\mu,t}\left(\mu_t, E_t\right) + \sum_{j=0}^{\infty} \beta^j \nu_{3,t+j} J_{E_t^M, t+j} E_t = 0, \quad \forall\, t \geq 0, \tag{44}$$

$$[T] : \sum_i \pi_i \theta_i = 0, \tag{45}$$

and at $t = 0$,

$$\left[\tau_0^k\right] : U_c\left(c_0, h_0\right)\left(\left(1 - D\left(Z_0\right)\right) A_{1,0} F_K\left(K_{1,0}, H_{1,0}, E_0\right) - \delta\right) N_0 \sum_i \pi_i \theta_i a_{i,0} = 0 \tag{46}$$

$$[K_{1,0}] : \left[\left(1 - D\left(Z_0\right)\right) A_{1,0} F_K\left(K_{1,0}, H_{1,0}, E_0\right) + (1-\delta)\right] \nu_{1,0} - \kappa = 0 \tag{47}$$

$$[K_{2,0}] : A_{2,0} G_K\left(K_{2,0}, H_{2,0}\right) \nu_{2,0} + (1-\delta)\, \nu_{1,0} - \kappa = 0 \tag{48}$$

where κ is the Lagrange multiplier on the constraint $K_{1,0} + K_{2,0} = K_0$, and it follows that

$$\left(1 - D\left(Z_0\right)\right) A_{1,0} F_K\left(K_{1,0}, H_{1,0}, E_0\right) \nu_{1,0} = A_{2,0} G_K\left(K_{2,0}, H_{2,0}\right) \nu_{2,0},$$

which together with (38) and (39), implies that

$$\frac{F_K\left(K_{1,0}, H_{1,0}, E_0\right)}{F_H\left(K_{1,0}, H_{1,0}, E_0\right)} = \frac{G_K\left(K_{2,0}, H_{2,0}\right)}{G_H\left(K_{2,0}, H_{2,0}\right)}. \tag{49}$$

As in any other period, in $t = 0$ the requirement that the marginal rates of technical substitution are equated between sectors is satisfied at the second-best allocation. Therefore, in most of what follows we ignore the multiplier on this constraint.

A.3 Optimal taxes

A.3.1 Capital and Labor income taxes

From (37) and (38) we obtain

$$(1 - D(Z_t)) A_{1,t} F_{H,t} = -\frac{W_{h,t}}{W_{c,t}}, \quad \forall\, t \geq 0, \tag{50}$$

and using the intertemporal condition (40) we get

$$R^*_{t+1} \equiv 1 + r_{t+1} - \delta = \frac{1}{\beta} \frac{W_{c,t}}{W_{c,t+1}}, \quad \forall\, t \geq 0, \tag{51}$$

These two equations can be used to back out the optimal taxes on labor and capital income. Plugging (50) into (34) implies

$$\frac{U_{h,t}}{U_{c,t}} = \frac{W_{h,t}}{W_{c,t}} (1 - \tau_{H,t}),$$

which can be rearranged into

$$\tau_{H,t} = 1 - \frac{U_{h,t}}{U_{c,t}} \frac{W_{c,t}}{W_{h,t}}. \tag{52}$$

In any competitive equilibrium (35) holds, which together with $p_t = R_{t+1} p_{t+1}$ implies

$$\frac{U_{c,t+1}}{U_{c,t}} \beta R_{t+1} = 1.$$

Substituting this into (51), it follows that

$$\frac{R_{t+1}}{R^*_{t+1}} = \frac{W_{c,t+1}}{W_{c,t}} \frac{U_{c,t}}{U_{c,t+1}}. \tag{53}$$

A.3.2 Excise taxes of energy and emissions

From the abatement first-order condition (44) and the energy firm abatement decision (9) we have that

$$\tau_{E,t} = \frac{\Theta_{\mu,t}}{E_t} = \frac{1}{\nu_{1,t}} \sum_{j=0}^{\infty} \beta^j \nu_{3,t+j} J_{E^M_t, t+j}.$$

From the climate variable first-order condition (43) we have that

$$\nu_{3,t} = \nu_{1,t} D'_t A_{1,t} F(K_{1,t}, H_{1,t}, E_t) - N_t W_{Z,t},$$

hence the pollution tax is given by

$$\tau_{E,t} = \frac{1}{\nu_{1,t}} \sum_{j=0}^{\infty} \beta^j \left(\nu_{1,t+j} D'_{t+j} A_{1,t+j} F_{t+j} - N_{t+j} W_{Z,t+j} \right) J_{E^M_t, t+j}. \tag{54}$$

From the energy first-order condition (42) we have that

$$-\nu_{1,t} \Big(\Theta_{E,t} + (1 - \mu_t) \frac{\Theta_{\mu,t}}{E_t} - (1 - D(Z_t)) A_{1,t} F_{E,t} \Big) = \nu_{2,t}, \tag{55}$$

and combining the first-order conditions for sectoral labor supplies (38) and (39), it follows that

$$\frac{\nu_{2,t}}{\nu_{1,t}} = \frac{\left(1 - D\left(Z_t\right)\right) A_{1,t} F_{H,t}}{A_{2,t} G_{H,t}}.$$

From (4) and (8) we also have

$$\frac{\left(1 - D\left(Z_t\right)\right) A_{1,t} F_{H,t}}{A_{2,t} G_{H,t}} = p_{E,t} - \tau_{I,t} - \tau_{E,t}(1 - \mu_t) - \Theta_{E,t}.$$

Hence, using (5), (9), and (55) we have

$$-\Theta_{E,t} - (1 - \mu_t)\tau_{E,t} + p_{E,t} = p_{E,t} - \tau_{I,t} - \tau_{E,t}(1 - \mu_t) - \Theta_{E,t},$$

and therefore

$$\tau_{I,t} = 0. \tag{56}$$

A.4 Explicit formulas

A.4.1 Characterization of equilibrium

To obtain explicit formulas, it is convenient to normalize market weights as follows

$$\sum_j \pi_j \left(\varphi_j e_j^{\gamma(\sigma-1)}\right)^{\frac{1}{\sigma-(1-\sigma)\gamma}} = 1.$$

Using the period utility function defined in (23), the Lagrangian for the characterization problem defined by (15) is

$$L = \sum_i \pi_i \varphi_i \left[\frac{\left(c_{i,t}\left(1 - \varsigma h_{i,t}\right)^{\gamma}\right)^{1-\sigma}}{1-\sigma} + \frac{\left(1 + \alpha_0 Z_t^2\right)^{-(1-\sigma)}}{1-\sigma}\right] + \theta_t^c \left(c_t - \sum_i \pi_i c_{i,t}\right) - \theta_t^h \left(h_t - \sum_i \pi_i e_i h_{i,t}\right),$$

The first order conditions are

$$[c_{i,t}] : \varphi_i \left(c_{i,t}\left(1 - \varsigma h_{i,t}\right)^{\gamma}\right)^{1-\sigma} c_{i,t}^{-1} = \theta_t^c, \quad \forall\, t \geq 0, \tag{57}$$

$$[h_{i,t}] : \varphi_i \left(c_{i,t}\left(1 - \varsigma h_{i,t}\right)^{\gamma}\right)^{1-\sigma} \gamma\varsigma \left(1 - \varsigma h_{i,t}\right)^{-1} = e_i \theta_t^h, \quad \forall\, t \geq 0, \tag{58}$$

rearranging yields

$$c_{i,t} = \frac{\theta_t^h}{\theta_t^c} \frac{e_i\left(1 - \varsigma h_{i,t}\right)}{\gamma\varsigma},$$

so that

$$c_{i,t} = \left(\frac{\theta_t^c}{\varphi_i}\left(\frac{\theta_t^h}{\theta_t^c}\frac{e_i}{\gamma\varsigma}\right)^{\gamma(1-\sigma)}\right)^{-\frac{1}{\sigma-(1-\sigma)\gamma}}$$

$$1 - \varsigma h_{i,t} = \frac{\theta_t^c}{\theta_t^h}\frac{\gamma\varsigma}{e_i}\left(\frac{\theta_t^c}{\varphi_i}\left(\frac{\theta_t^h}{\theta_t^c}\frac{e_i}{\gamma\varsigma}\right)^{\gamma(1-\sigma)}\right)^{-\frac{1}{\sigma-(1-\sigma)\gamma}},$$

and summing across types (given that $c_t = \sum_i \pi_i c_{i,t}$, and $h_t = \sum_i \pi_i e_i h_{i,t}$)

$$c_t = \left(\theta_t^c \left(\frac{\theta_t^h}{\theta_t^c}\frac{1}{\gamma\varsigma}\right)^{\gamma(1-\sigma)}\right)^{-\frac{1}{\sigma-(1-\sigma)\gamma}} \sum_i \pi_i \left(\frac{e_i^{\gamma(1-\sigma)}}{\varphi_i}\right)^{-\frac{1}{\sigma-(1-\sigma)\gamma}} \tag{59}$$

$$1-\varsigma h_t = \frac{\theta_t^c}{\theta_t^h}\gamma\varsigma \left(\theta_t^c \left(\frac{\theta_t^h}{\theta_t^c}\frac{1}{\gamma\varsigma}\right)^{\gamma(1-\sigma)}\right)^{-\frac{1}{\sigma-(1-\sigma)\gamma}} \sum_i \pi_i \left(\frac{e_i}{\varphi_i}^{\gamma(1-\sigma)}\right)^{-\frac{1}{\sigma-(1-\sigma)\gamma}} \tag{60}$$

It follows that

$$c_{i,t}^m\left(c_t, h_t; \varphi\right) = \omega_i c_t, \tag{61}$$

$$1-\varsigma h_{i,t}^m\left(c_t, h_t; \varphi\right) = \frac{\omega_i}{e_i}\left(1-\varsigma h_t\right), \tag{62}$$

where

$$\omega_i = \frac{\left(\varphi_i \left(e_i\right)^{\gamma(\sigma-1)}\right)^{\frac{1}{\sigma-(1-\sigma)\gamma}}}{\sum_i \pi_i \left(\varphi_j e_j^{\gamma(\sigma-1)}\right)^{\frac{1}{\sigma-(1-\sigma)\gamma}}} = \left(\varphi_i \left(e_i\right)^{\gamma(\sigma-1)}\right)^{\frac{1}{\sigma-(1-\sigma)\gamma}}$$

Hence, we can write aggregate indirect utility $U\left(c_t, h_t, Z_t; \varphi\right)$ in terms of the aggregates c_t, h_t, and Z_t

$$U\left(c_t, h_t, Z_t, \varphi\right) = \sum_j \pi_j \varphi_j \left(\frac{\omega_j^{1+\gamma}}{e_j^{\gamma}}\right)^{1-\sigma} \frac{\left(c_t\left(1-\varsigma h_t\right)^{\gamma}\right)^{1-\sigma}}{1-\sigma} + \sum_i \pi_i \varphi_i \frac{\left(1+\alpha_0 Z_t^2\right)^{-(1-\sigma)}}{1-\sigma}, \tag{63}$$

$$= \frac{\left(c_t\left(1-\varsigma h_t\right)^{\gamma}\right)^{1-\sigma}}{1-\sigma} + \Gamma \frac{\left(1+\alpha_0 Z_t^2\right)^{-(1-\sigma)}}{1-\sigma}, \tag{64}$$

since from the normalization of market weights we have

$$\sum_j \pi_j \varphi_j \left(\frac{\omega_j^{1+\gamma}}{e_j^{\gamma}}\right)^{1-\sigma} = \sum_j \pi_j \left(\varphi_j e_j^{\gamma(\sigma-1)}\right)^{\frac{1}{\sigma-(1-\sigma)\gamma}} = 1,$$

and with $\Gamma \equiv \sum_i \pi_i \varphi_i$.

A.4.2 Explicit tax formulas

From (36), substituting the derivatives of U into the definition of $W\left(c_t, h_t, Z_t; \varphi, \theta, \lambda\right)$ we get

$$W\left(c_t, h_t, Z_t; \varphi, \theta, \lambda\right) = \sum_i \pi_i \lambda_i \left(\frac{\omega_i}{\varphi_i}\frac{\left(c_t\left(1-\varsigma h_t\right)^{\gamma}\right)^{1-\sigma}}{1-\sigma} + \frac{\left(1+\alpha_0 Z_t^2\right)^{-(1-\sigma)}}{1-\sigma}\right)$$
$$+ \sum_i \pi_i \theta_i \left[\left(c_t\left(1-\varsigma h_t\right)^{\gamma}\right)^{1-\sigma}\omega_i - \gamma\left(c_t\left(1-\varsigma h_t\right)^{\gamma}\right)^{1-\sigma}\left(1-\varsigma h_t\right)^{-1}\left(e_i - \omega_i\left(1-\varsigma h_t\right)\right)\right] \tag{65}$$

Collecting terms and simplifying we obtain

$$W\left(c_t, h_t, Z_t; \varphi, \theta, \lambda\right) = \Phi \frac{\left(c_t\left(1-\varsigma h_t\right)^{\gamma}\right)^{1-\sigma}}{1-\sigma} + \frac{\left(1+\alpha_0 Z_t^2\right)^{-(1-\sigma)}}{1-\sigma} + \Psi U_{h,t}. \tag{66}$$

where

$$\Phi \equiv \sum_i \pi_i \omega_i \left(\frac{\lambda_i}{\varphi_i} + (1-\sigma)(1+\gamma)\theta_i \right), \tag{67}$$

$$\Psi \equiv \sum_i \frac{\pi_i \theta_i e_i}{\varsigma}. \tag{68}$$

Substituting the derivatives into equation (52) we get

$$\tau_{H,t} = \frac{\Psi\varsigma(1-\varsigma h_t)^{-1}}{\Phi + \Psi\varsigma(1-\gamma(1-\sigma))(1-\varsigma h_t)^{-1}}, \tag{69}$$

substituting the derivatives into (53) yields

$$\frac{R_{t+1}}{R^*_{t+1}} = \frac{\Phi - \Psi\varsigma\gamma(1-\sigma)(1-\varsigma h_{t+1})^{-1}}{\Phi - \Psi\varsigma\gamma(1-\sigma)(1-\varsigma h_t)^{-1}}, \tag{70}$$

and substituting the derivatives into (54) we get

$$\tau_{E,t} = \frac{1}{\nu_{1,t}} \sum_{j=0}^{\infty} \beta^j \Big(\nu_{1,t+j} D'_{t+j} A_{1,t+j} F_{t+j} - N_{t+j} V_Z(Z_{t+j}) \Big) J_{E_t^M,t+j}, \tag{71}$$

with $\nu_{1,t}$ the multiplier of the resource constraint which we can express as

$$\nu_{1,t} = V_{c,t} + \sum_i \pi_i \theta_i MIC_{i,t}. \tag{72}$$

If we add—without loss of generality—the normalization of market weights as a constraint into the Ramsey problem, we obtain the following first order conditions with respect to market weights

$$\sum_t \beta^t N_t W_{\varphi_i,t} - \frac{\zeta}{\sigma - (1-\sigma)\gamma} \frac{\pi_i \omega_i}{\varphi_i} = 0, \quad \forall\ i.$$

From this equation we have that

$$\sum_{t=0}^{\infty} N_t \beta^t \frac{(c_t(1-\varsigma h_t)^\gamma)^{1-\sigma}}{1-\sigma} \frac{(1-\sigma)(1+\gamma)}{\sigma-(1-\sigma)\gamma} \frac{\pi_i\omega_i}{\varphi_i} \left(\frac{\lambda_i}{\varphi_i} + \theta_i \right) - \frac{\zeta}{\sigma-(1-\sigma)\gamma} \frac{\pi_i\omega_i}{\varphi_i} = 0, \quad \forall\ i,$$

and therefore

$$\frac{\lambda_i}{\varphi_i} + \theta_i = \frac{\zeta}{(1-\sigma)(1+\gamma)\sum_{t=0}^{\infty} N_t \beta^t \tilde{U}(c_t, h_t)}, \quad \forall\ i,$$

with

$$\tilde{U}(c_t, h_t) = \frac{(c_t(1-\varsigma h_t)^\gamma)^{1-\sigma}}{1-\sigma}.$$

Using the fact that

$$\sum_i \pi_i \theta_i = 0, \quad \sum_i \pi_i \omega_i = 1, \quad \text{and} \quad \sum_i \pi_i e_i = 1$$

it follows that

$$\sum_j \frac{\pi_j \lambda_j}{\varphi_j} = \frac{\zeta}{(1-\sigma)(1+\gamma)\sum_{t=0}^{\infty} N_t \beta^t \tilde{U}(c_t, h_t)},$$

and, therefore

$$\theta_i = \sum_j \frac{\pi_j \lambda_j}{\varphi_j} - \frac{\lambda_i}{\varphi_i}. \tag{73}$$

This allows us to rewrite

$$\begin{aligned}
\Phi &= \sum_i \pi_i \omega_i \left(\frac{\lambda_i}{\varphi_i} + (1-\sigma)(1+\gamma) \left(\sum_j \frac{\pi_j \lambda_j}{\varphi_j} - \frac{\lambda_i}{\varphi_i} \right) \right) \\
&= \sum_j \pi_j \frac{\lambda_j}{\varphi_j} + \big(1 - (1+\gamma)(1-\sigma)\big) \mathrm{cov}(\lambda_i/\varphi_i, \omega_i), \\
\Psi &= \frac{1}{\varsigma} \sum_j \pi_j \frac{\lambda_j}{\varphi_j} (1 - e_j) \\
&= - \frac{\mathrm{cov}(\lambda_i/\varphi_i, e_i)}{\varsigma},
\end{aligned}$$

where the last result is obtained using the normalization of productivity levels, $\sum_i \pi_i e_i = 1$. The implementability conditions can be rewritten as

$$\omega_i = \frac{U_{c,0} (R_0 N_0 a_{i,0} + T) + M e_i}{(1-\sigma)(1+\gamma) \sum_{t=0}^{\infty} N_t \beta^t \tilde{U}(c_t, h_t)}, \quad \forall\ i, \tag{74}$$

with

$$M \equiv \sum_{t=0}^{\infty} N_t \beta^t \gamma \left(c_t (1 - \varsigma h_t)^{\gamma} \right)^{1-\sigma} (1 - \varsigma h_t)^{-1}.$$

Since

$$\omega_i = \left(\varphi_i e_i^{\gamma(\sigma-1)} \right)^{\frac{1}{\sigma - (1-\sigma)\gamma}}$$

we can express market weights as

$$\varphi_i = \frac{\omega_i^{\sigma-(1-\sigma)\gamma}}{e_i^{\gamma(\sigma-1)}} = \frac{1}{e_i^{\gamma(\sigma-1)}} \left(\frac{U_{c,0} (R_0 N_0 a_{i,0} + T) + M e_i}{(1-\sigma)(1+\gamma) \sum_{t=0}^{\infty} N_t \beta^t \tilde{U}(c_t, h_t)} \right)^{\sigma-(1-\sigma)\gamma}$$

A.4.3 Comparison with first-best

First-best pollution tax To compare our second-best results with the first-best, we solve the same Ramsey problem except that we now allow for individualized lump-sum transfers. All first order conditions remain the same except for the one with respect to T given by (45): since we now have individualized instruments T_i, we obtain

$$\theta_i = 0, \quad \forall i, \tag{75}$$

hence for all t, $\sum_i \pi_i \theta_i MIC_{i,t} = 0$. From (73), this also implies that

$$\frac{\lambda_i}{\varphi_i} = \sum_j \frac{\pi_j \lambda_i}{\varphi_i}, \quad \forall i, \tag{76}$$

and as a consequence we have $\Psi = 0$, so that for all t, $\tau_{H,t} = 0$ and $\tau_{K,t} = 0$. Substituting for $\nu_{1,t}$ in (54), we can express the first-best tax as

$$\tau_{E,t}^{FB} = \sum_{j=0}^{\infty} \beta^j \left(\frac{V_{c,t+j}}{V_{c,t}} D'_{t+j} A_{1,t+j} F_{t+j} - \frac{N_{t+j} V_{Z,t+j}}{V_{c,t}} \right) J_{E_t^M,t+j} \ . \tag{77}$$

The first-best tax is equal to the social cost of the externality—*i.e.*, to the Pigouvian tax—evaluated at the first-best allocation.

The marginal cost of funds Let us now decompose the Pigouvian tax formula into a production damage component and a utility damage component:

$$\tau_{E,t}^{\text{Pigou,Y}} = \sum_{j=0}^{\infty} \beta^j \frac{V_{c,t+j}}{V_{c,t}} D'_{t+j} A_{1,t+j} F_{t+j} J_{E_t^M,t+j} \ ,$$

$$\tau_{E,t}^{\text{Pigou,U}} = (-1) \sum_{j=0}^{\infty} \beta^j \frac{N_{t+j} V_{Z,t+j}}{V_{c,t}} J_{E_t^M,t+j} \ .$$

If we define the marginal cost of funds as the ratio of the public to the private marginal utility of consumption,

$$\text{MCF}_t \equiv \frac{\nu_{1,t}}{V_{c,t}},$$

the share of marginal production damages occurring at time $t+s$ due to a marginal change in emissions at time t, as

$$\Delta_{t+s} \equiv \frac{\beta^j D'_{t+s} A_{1,t+s} F_{t+s} J_{E_t^M,t+s}}{\sum_{j=0}^{\infty} \beta^j D'_{t+j} A_{1,t+j} F_{t+j} J_{E_t^M,t+j}} \ ,$$

then the second-best tax given by (71) can be re-written as a function of the marginal cost of funds and the first-best tax rule evaluated at the second-best allocation

$$\tau_{E,t} = \sum_{j=0}^{\infty} \frac{\text{MCF}_{t+j}}{\text{MCF}_t} \Delta_{t+j} \ \tau_{E,t}^{\text{Pigou,Y}}\Big|_{\text{SB}} + \frac{\tau_{E,t}^{\text{Pigou,U}}\Big|_{\text{SB}}}{\text{MCF}_t} \ .$$

From (53), and using the fact that

$$\frac{V_{c,t+j}}{V_{c,t}} = \frac{U_{c,t+j}}{U_{c,t}}$$

we can also write the ratio of MCFs as

$$\frac{\text{MCF}_{t+j}}{\text{MCF}_t} = \prod_{k=1}^{j} \frac{R_{t+k}}{R^*_{t+k}} \ ,$$

from which we see that the ratio is equal to 1 if the capital tax is null for all future periods where current emissions generate production damages. Thus, as in Barrage (2019), the optimal tax on production damage is not distorted as long as, going forward, the capital income tax is optimally set to zero. Substituting for $\nu_{1,t}$ in the definition of the MCF, we also see that

$$\text{MCF}_t = 1 + \frac{\sum_i \pi_i \theta_i MIC_{i,t}}{V_{c,t}},$$

from which it appears that the MCF is equal to one when the implementation cost $-\sum_i \pi_i \theta_i MIC_{i,t}$ is null. In this situation, the second-best pollution tax corresponds to the Pigouvian tax formula evaluated at the second-best allocation.

The marginal implementation cost Using our functional form for U, we can show that

$$IC_{i,t} = \Big(c_t(1-\varsigma h_t)^{\gamma}\Big)^{(1-\sigma)} \left(\omega_i + \gamma\Big(\omega_i - \frac{e_i}{(1-\varsigma h_t)}\Big)\right), \tag{78}$$

from which we can write

$$MIC_{i,t} = (1-\sigma)\frac{IC_{i,t}}{C_t}. \tag{79}$$

Using the fact that $\sum_i \pi_i \theta_i = 0$, we can re-write the marginal implementation cost as

$$-\sum_i \pi_i \theta_i MIC_{i,t} = (\sigma - 1)\frac{\text{cov}(\theta_i, IC_{i,t})}{c_t}. \tag{80}$$

This term is equal to 0 when either $\sigma = 1$, or θ_i and $IC_{i,t}$ are uncorrelated.

Price effect To understand the role of σ, it is useful to go back to the origin of the the term $IC_i(c_t, h_t, \varphi)$. This term comes from households' budget constraint (2) in which we have substituted for the price and real wage using (34) and (35). From these equations, it appears that when making more resources available to households, the price goes down since

$$p_t = \beta^t \Big(\frac{c_t}{c_0}\Big)^{-\sigma} \Big(\frac{1-\varsigma h_t}{1-\varsigma h_0}\Big)^{\gamma(1-\sigma)}.$$

When $\sigma = 1$, the price effect exactly offsets the volume effect so that households' expenditures and nominal income remain unchanged, hence the planner does not need to change the value of the lump-sum transfer and the implementation cost remains constant.

Labor supply effect To determine the sign of the covariance term, we can examine the ratio of the period implemantation cost for two agents i and j such that $e_i > e_j$. From (78), we have

$$\frac{IC_{i,t}}{IC_{j,t}} = \frac{\omega_i + \gamma\Big(\omega_i - \frac{e_i}{(1-\varsigma h_t)}\Big)}{\omega_j + \gamma\Big(\omega_j - \frac{e_j}{(1-\varsigma h_t)}\Big)}.$$

Although the discounted sum of $IC_{i,t}$ is invariant across type, in period t this ratio may be below or above 1 depending on the value of the aggregate labor supply. In particular, we have

$$\frac{\partial \frac{IC_{i,t}}{IC_{j,t}}}{\partial h_t} = \frac{\varsigma\gamma(1+\gamma)\big(e_j\omega_i - e_i\omega_j\big)}{(1-\varsigma h_t)^2\Big(\omega_j(1+\gamma) - \frac{\gamma e_j}{(1-\varsigma h_t)}\Big)^2}. \tag{81}$$

From (74), we can also show that with homogeneous initial wealth (or full expropriation of initial wealth), when transfers plus initial assets are positive (as they are in our quantitative analysis) then

ω_i/e_i is strictly declining in e_i, hence for $e_i > e_j$, the derivative in (81) is negative. This result means that when h_t is high relative to its average value, the relative labor supply of highly productive households compared to less productive households is higher, hence more productive households need lower transfers to satisfy the planners' allocation at that period. If the more productive also have a lower marginal utility of consumption (hence a higher θ_i), then the covariance term in equation (80) is negative when aggregate labor supply is relatively high. Conversely, when transfers are negative, the derivative in (81) is positive and the covariance term is negative when the aggregate labor supply is low.

Differences from individual allocations We can express the aggregate utility defined using the planner's weights as follows

$$V(c_t, h_t, Z_t; \varphi, \lambda) = \frac{\sum_i \pi_i \lambda_i u(c_{i,t}, h_{i,t})}{\sum_i \pi_i \varphi_i u(c_{i,t}, h_{i,t})} \frac{\left(c_t \left(1 - \varsigma h_t\right)^{\gamma}\right)^{1-\sigma}}{1-\sigma} + \frac{\left(1 + \alpha_0 Z_t^2\right)^{-(1-\sigma)}}{1-\sigma},$$

hence the marginal utility of consumption from the planner's perspective is

$$V_{c,t} = \frac{\sum_i \pi_i \lambda_i u(c_{i,t}, h_{i,t})}{\sum_i \pi_i \varphi_i u(c_{i,t}, h_{i,t})} U_{c,t}.$$

From our characterization problem, we know that market weights are determined by the following expression

$$\varphi_i u_{c,i,t} = U_{c,t}, \quad \forall i,$$

from which we can rewrite

$$V_{c,t} = \sum_i \pi_i \lambda_i \frac{u_{c,i,t} c_{i,t}}{c_t} \tag{82}$$

Thus, between the first-best and the second-best case, the marginal utility of consumption will differ due to the path of aggregate consumption, as well as the distribution of individual allocations. Holding aggregate consumption constant, we see that an increase in the variance of $c_{i,t}$ has ambiguous effects. On the one hand, since $u(c, h)$ is concave in c, the average marginal utility is increasing with consumption inequalities. On the other hand, higher marginal utilities are weighted by lower consumption levels, hence increasing consumption dispersion reduces the relative weight given to high marginal utilities. The net effect depends on the curvature of the utility function. Substituting $u_{c,i,t}$ by its functional expression in (82), we have

$$V_{c,t} = \sum_i \pi_i \lambda_i \frac{\left(c_{i,t}(1 - \varsigma h_{i,t})^{\gamma}\right)^{1-\sigma}}{c_t}$$

and we see that when $\sigma = 1$, the two previous effects cancel each other and the distribution of individual allocations has no incidence on the marginal utility of consumption.

B Optimal tax rules with initial wealth inequality

In Appendix A.2.1, we describe the Ramsey problem with wealth inequality

B.1 First order conditions

For $t \geq 1$, the conditions are exactly the same as the ones derived above, in particular, we have that $\sum_i \pi_i \theta_i = 0$, which we use to simplify the equations below. The period-0 marginal rate of technical substitution constraint is no longer automatically satisfied, so let Γ_0 denote the Lagrange multiplier on this constraint. The first order conditions for period 0 are

$$[c_0] : W_{c,0} - \nu_{1,0} - U_{cc,0} \sum_i \pi_i \theta_i R_0 a_{i,0} = 0, \tag{83}$$

$$\begin{aligned}[H_{1,0}] : &W_{h,0} + \nu_{1,0} (1 - D_0) A_{1,0} F_{H,0} - U_{ch,0} \sum_i \pi_i \theta_i R_0 a_{i,0} \\ &- N_0 U_{c,0} \sum_i \pi_i \theta_i a_{i,0} (1 - \tau_{K,0}) (1 - D_0) A_{1,0} F_{KH,0} + \Gamma_0 (F_{HH,0} G_{K,0} - F_{KH,0} G_{H,0}) = 0,\end{aligned} \tag{84}$$

$$[H_{2,0}] : W_{H,0} + \nu_{2,0} A_{2,0} G_{H,0} - U_{ch,0} \sum_i \pi_i \theta_i R_0 a_{i,0} + \Gamma_0 (F_{H,0} G_{KH,0} - F_{K,0} G_{HH,0}) = 0, \tag{85}$$

$$\begin{aligned}[K_{1,0}] : &((1 - D_0) A_{1,0} F_{K,0} + (1 - \delta)) \nu_{1,0} - N_0 U_{c,0} \sum_i \pi_i \theta_i a_{i,0} (1 - \tau_{K,0}) (1 - D_0) A_{1,0} F_{KK,0} - \kappa \\ &+ \Gamma_0 (F_{HK,0} G_{K,0} - F_{KK,0} G_{H,0}) = 0,\end{aligned} \tag{86}$$

$$[K_{2,0}] : A_{2,0} G_{K,0} \nu_{2,0} + (1 - \delta) \nu_{1,0} - \kappa + \Gamma_0 (F_{H,0} G_{KK,0} - F_{K,0} G_{HK,0}) = 0, \tag{87}$$

$$\begin{aligned}[E_0] : &- \left(\mu_0 \Theta_0' - (1 - D_0) A_{1,0} F_{E,0}\right) \nu_{1,0} - \nu_{2,0} - \sum_{j=0}^{\infty} \beta^j \nu_{3,j} J_{E_0^M, j} (1 - \mu_0) \\ &- N_0 U_{c,0} \sum_i \pi_i \theta_i a_{i,0} (1 - \tau_{K,0}) (1 - D_0) A_{1,0} F_{KE,0} + \Gamma_0 (F_{HE,0} G_{K,0} - F_{KE,0} G_{H,0}) = 0,\end{aligned} \tag{88}$$

$$[Z_0] : N_0 W_{Z,0} - \nu_{1,0} D_0' A_{1,0} F_0 + \nu_{3,0} + N_0 U_{c,0} \sum_i \pi_i \theta_i a_{i,0} (1 - \tau_{K,0}) D_0' A_{1,0} F_{K,0} = 0. \tag{89}$$

B.2 Multiplier on period-0 marginal rate of technical substitution constraint

From (86) and (87), it follows that

$$\begin{aligned}\frac{\nu_{2,0}}{\nu_{1,0}} = &(1 - D_0) \frac{A_{1,0} F_{K,0}}{A_{2,0} G_{K,0}} - \frac{N_0 U_{c,0}}{\nu_{1,0}} \sum_i \pi_i \theta_i a_{i,0} (1 - \tau_{K,0}) (1 - D_0) \frac{A_{1,0} F_{KK,0}}{A_{2,0} G_{K,0}} \\ &+ \frac{\Gamma_0}{\nu_{1,0}} \frac{((F_{HK,0} G_{K,0} - F_{KK,0} G_{H,0}) - (F_{H,0} G_{KK,0} - F_{K,0} G_{HK,0}))}{A_{2,0} G_{K,0}}.\end{aligned}$$

From (84) and (85), it follows that

$$\begin{aligned}\frac{\nu_{2,0}}{\nu_{1,0}} = &(1 - D_0) \frac{A_{1,0} F_{H,0}}{A_{2,0} G_{H,0}} - \frac{N_0 U_{c,0}}{\nu_{1,0}} \sum_i \pi_i \theta_i a_{i,0} (1 - \tau_{K,0}) (1 - D_0) \frac{A_{1,0} F_{KH,0}}{A_{2,0} G_{H,0}} \\ &+ \frac{\Gamma_0}{\nu_{1,0}} \frac{((F_{HH,0} G_{K,0} - F_{KH,0} G_{H,0}) - (F_{H,0} G_{KH,0} - F_{K,0} G_{HH,0}))}{A_{2,0} G_{H,0}}.\end{aligned}$$

Hence, putting these two equations together, we obtain

$$\Gamma_0 = \frac{N_0 U_{c,0} \sum_i \pi_i \theta_i a_{i,0} (1-\tau_{K,0})(1-D_0) A_{1,0} (G_{K,0} F_{KH,0} - G_{H,0} F_{KK,0})}{\left\{ \begin{array}{c} G_{K,0} ((F_{HH,0} G_{K,0} - F_{KH,0} G_{H,0}) - (F_{H,0} G_{KH,0} - F_{K,0} G_{HH,0})) \\ -G_{H,0} ((F_{HK,0} G_{K,0} - F_{KK,0} G_{H,0}) - (F_{H,0} G_{KK,0} - F_{K,0} G_{HK,0})) \end{array} \right\}}.$$

B.3 Labor income taxes

From (84) and (83) we obtain

$$(1-D_0) A_{1,0} F_{H,0} = \frac{\left\{ \begin{array}{c} -W_{h,0} + U_{ch,0} \sum_i \pi_i \theta_i R_0 a_{i,0} \\ +N_0 U_{c,0} \sum_i \pi_i \theta_i a_{i,0} (1-\tau_{K,0})(1-D_0) A_{1,0} F_{KH,0} - \Gamma_0 (F_{HH,0} G_{K,0} - F_{KH,0} G_{H,0}) \end{array} \right\}}{W_{c,0} - U_{cc,0} \sum_i \pi_i \theta_i R_0 a_{i,0}} \tag{90}$$

Plugging (90) into (34) implies

$$\frac{U_{h,0}}{U_{c,0}} = \frac{\left\{ \begin{array}{c} W_{h,0} - U_{ch,0} \sum_i \pi_i \theta_i R_0 a_{i,0} \\ -N_0 U_{c,0} \sum_i \pi_i \theta_i a_{i,0} (1-\tau_{K,0})(1-D_0) A_{1,0} F_{KH,0} + \Gamma_0 (F_{HH,0} G_{K,0} - F_{KH,0} G_{H,0}) \end{array} \right\}}{W_{c,0} - U_{cc,0} \sum_i \pi_i \theta_i R_0 a_{i,0}} (1-\tau_{H,0}),$$

which can be rearranged into

$$\tau_{H,0} = 1 - \frac{U_{h,0}}{U_{c,0}} \frac{W_{c,0} - U_{cc,0} \sum_i \pi_i \theta_i R_0 a_{i,0}}{\left\{ \begin{array}{c} W_{h,0} - U_{ch,0} \sum_i \pi_i \theta_i R_0 a_{i,0} \\ -N_0 U_{c,0} \sum_i \pi_i \theta_i a_{i,0} (1-\tau_{K,0})(1-D_0) A_{1,0} F_{KH,0} + \Gamma_0 (F_{HH,0} G_{K,0} - F_{KH,0} G_{H,0}) \end{array} \right\}}. \tag{91}$$

B.4 Capital income taxes

From (40) and (83) we obtain

$$R_1^* \equiv 1 + r_1 - \delta = \frac{1}{\beta} \frac{W_{c,0} - U_{cc,0} \sum_i \pi_i \theta_i R_0 a_{i,0}}{W_{c,1}}. \tag{92}$$

In any competitive equilibrium (35) holds, which implies

$$\frac{U_{c,1}}{U_{c,0}} \beta R_1 = 1.$$

Substituting this into (51), it follows that

$$\frac{R_1}{R_1^*} = \frac{W_{c,1}}{W_{c,0} - U_{cc,0} \sum_i \pi_i \theta_i R_0 a_{i,0}} \frac{U_{c,0}}{U_{c,1}}. \tag{93}$$

B.5 Excise taxes of energy and emissions

From the abatement first-order condition (44) we have that

$$\Theta_0' = \frac{1}{\nu_{1,0}} \sum_{j=0}^{\infty} \beta^j \nu_{3,j} J_{E_0^M,j}.$$

From the climate variable first-order condition (89) we have that

$$\nu_{3,0} = \nu_{1,0} D_0' A_{1,0} F_0 - N_0 W_{Z,0} - N_0 U_{c,0} \sum_i \pi_i \theta_i a_{i,0} \left(1 - \tau_{K,0}\right) D_0' A_{1,0} F_{K,0}.$$

From the energy first-order condition (88) we have that

$$\left(1 - D_0\right) A_{1,0} F_{E,0} - \frac{\nu_{2,0}}{\nu_{1,0}} = \Delta_0' + N_0 \frac{U_{c,0}}{\nu_{1,0}} \sum_i \pi_i \theta_i a_{i,0} \left(1 - \tau_{K,0}\right) \left(1 - D_0\right) A_{1,0} F_{KE,0} - \frac{\Gamma_0}{\nu_{1,0}} \left(F_{HE,0} G_{K,0} - F_{KE,0} G_{H,0}\right).$$

Combining the first-order conditions for sectoral labor supplies (84) and (85), it follows that

$$\begin{aligned}
\frac{\nu_{2,0}}{\nu_{1,0}} = {} & \left(1 - D_0\right) \frac{A_{1,0} F_{H,0}}{A_{2,0} G_{H,0}} - \frac{N_0 U_{c,0}}{\nu_{1,0}} \sum_i \pi_i \theta_i a_{i,0} \left(1 - \tau_{K,0}\right) \left(1 - D_0\right) \frac{A_{1,0} F_{KH,0}}{A_{2,0} G_{H,0}} \\
& + \frac{\Gamma_0}{\nu_{1,0}} \frac{\left(\left(F_{HH,0} G_{K,0} - F_{KH,0} G_{H,0}\right) - \left(F_{H,0} G_{KH,0} - F_{K,0} G_{HH,0}\right)\right)}{A_{2,0} G_{H,0}},
\end{aligned}$$

and, therefore

$$\begin{aligned}
\left(1 - D_0\right) A_{1,0} F_{E,0} = {} & \Theta_0' + \left(1 - D_0\right) \frac{A_{1,0} F_{H,0}}{A_{2,0} G_{H,0}} + N_0 \frac{U_{c,0}}{\nu_{1,0}} \sum_i \pi_i \theta_i a_{i,0} \left(1 - \tau_{K,0}\right) \left(1 - D_0\right) \left(A_{1,0} F_{KE,0} - \frac{A_{1,0} F_{KH,0}}{A_{2,0} G_{H,0}}\right) \\
& + \frac{\Gamma_0}{\nu_{1,0}} \left(\frac{\left(\left(F_{HH,0} G_{K,0} - F_{KH,0} G_{H,0}\right) - \left(F_{H,0} G_{KH,0} - F_{K,0} G_{HH,0}\right)\right)}{A_{2,0} G_{H,0}} - \left(F_{HE,0} G_{K,0} - F_{KE,0} G_{H,0}\right)\right).
\end{aligned}$$

Then, from (9) we have that

$$\tau_{E,0} = \Theta_0' = \frac{1}{\nu_{1,0}} \sum_{j=0}^{\infty} \beta^j \nu_{3,j} J_{E_0^M,j}, \tag{94}$$

and from (4), (5), and (8) we have that

$$\left(1 - D_0\right) A_{1,0} F_{H,0} = \left(\left(1 - D_0\right) A_{1,0} F_{E,0} - \tau_{I,0} - \tau_{E,0}\right) A_{2,0} G_{H,0},$$

and therefore

$$\begin{aligned}
\tau_{I,0} = {} & N_0 \frac{U_{c,0}}{\nu_{1,0}} \sum_i \pi_i \theta_i a_{i,0} \left(1 - \tau_{K,0}\right) \left(1 - D_0\right) \left(A_{1,0} F_{KE,0} - \frac{A_{1,0} F_{KH,0}}{A_{2,0} G_{H,0}}\right) \\
& + \frac{\Gamma_0}{\nu_{1,0}} \left(\frac{\left(F_{HH,0} G_{K,0} - F_{KH,0} G_{H,0}\right) - \left(F_{H,0} G_{KH,0} - F_{K,0} G_{HH,0}\right)}{A_{2,0} G_{H,0}} - \left(F_{HE,0} G_{K,0} - F_{KE,0} G_{H,0}\right)\right).
\end{aligned}$$

Finally, using (43), (89) in (94) we get

$$\tau_{E,0} = \frac{1}{\nu_{1,0}} \sum_{j=0}^{\infty} \beta^j \left(\nu_{1,j} D_j' A_{1,j} F_j - N_j W_{Z,j}\right) J_{E_0^M,j} - N_0 \frac{U_{c,0}}{\nu_{1,0}} \sum_i \pi_i \theta_i a_{i,0} \left(1 - \tau_{K,0}\right) D_0' A_{1,0} F_{K,0} J_{E_0^M,0}.$$

C Optimal tax rules with Stone-Geary utility and heterogeneous preferences

The derivation of optimal tax rules in this extended version of the model closely follows the method applied to solve the benchmark model. This appendix highlights the differences with the benchmark presented in Appendix A.

C.1 Characterization of equilibrium

Let $\varphi \equiv \{\varphi_i\}$ be the market weights normalized so that

$$\sum_j \pi_j \left(\varphi_j e_j^{\gamma(\sigma-1)}\right)^{\frac{1}{1-(1+\epsilon+\gamma)(1-\sigma)}} = 1,$$

with $\varphi_i \geq 0$. Then, given aggregate levels c_t, d_t, h_t and Z_t, the individual levels can be found by solving the following static subproblem for each period t:

$$\begin{aligned} U\left(c_t, d_t, h_t, Z_t; \varphi\right) \equiv \max_{c_{i,t}, d_{i,t}, h_{i,t}} \sum_i \pi_i \varphi_i u_i\left(c_{i,t}, d_{i,t}, h_{i,t}, Z_t\right), \\ \text{s.t.} \ \sum_i \pi_i c_{i,t} = c_t, \quad \text{and} \quad \sum_i \pi_i d_{i,t} = d_t, \quad \text{and} \quad \sum_i \pi_i e_i h_{i,t} = h_t. \end{aligned} \tag{95}$$

Following the same steps as in Appendix A, we obtain the following solutions for this problem

$$c_{i,t}^m\left(c_t, d_t, h_t; \varphi\right) = \omega_i c_t, \tag{96}$$

$$d_{i,t}^m\left(c_t, d_t, h_t; \varphi\right) = \bar{d}_{i,t} + \omega_i\big(d_t - \bar{d}_t\big), \tag{97}$$

$$1 - \varsigma h_{i,t}^m\left(c_t, d_t, h_t; \varphi\right) = \frac{\omega_i}{e_i}(1 - \varsigma h_t), \tag{98}$$

with $\bar{d}_t = \sum_i \pi_i \bar{d}_{i,t}$, and where

$$\omega_i = \left(\varphi_i e_i^{\gamma(\sigma-1)}\right)^{\frac{1}{1-(1+\epsilon+\gamma)(1-\sigma)}} \tag{99}$$

which enables us to write the aggregate indirect utility in terms of the aggregates and market weights

$$U\left(c_t, d_t, h_t, Z_t\right) = \frac{\Big(c_t (d_t - \bar{d}_t)^{\epsilon} \left(1 - \varsigma h_t\right)^{\gamma}\Big)^{1-\sigma}}{1-\sigma} + \Gamma_\chi \frac{\big(1 + \alpha_0 Z_t^2\big)^{-(1-\sigma)}}{1-\sigma}, \tag{100}$$

with $\Gamma_\chi \equiv \sum_i \pi_i \varphi_i \chi_i$.

C.2 Implementability condition

From the first order conditions of problem (95) and applying the envelope theorem we have

$$U_{c,t} = \varphi_i u_{c,i,t}, \tag{101}$$

$$U_{d,t} = \varphi_i u_{d,i,t}, \tag{102}$$

$$U_{h,t} = \frac{\varphi_i u_{h,i,t}}{e_i}, \tag{103}$$

which together with the first order conditions of individual agents' problems give

$$\frac{U_{h,t}}{U_{c,t}} = \frac{u_{h,i,t}}{u_{c,i,t}e_{i,t}} = -w_t\left(1-\tau_{H,t}\right), \tag{104}$$

$$\frac{U_{d,t}}{U_{c,t}} = \frac{u_{d,i,t}}{u_{c,i,t}} = p_{E,t}+\tau_{D,t}, \tag{105}$$

and

$$\frac{U_{c,t}}{U_{c,0}} = \frac{u_{c,i,t}}{u_{c,i,0}} = \frac{p_t}{\beta^t}. \tag{106}$$

Using (104), (105), and (106) to substitute in households' budget constraint (28), we obtain the implementability conditions

$$U_{c,0}\left(R_0N_0a_{i,0}+T\right) \geq \sum_{t=0}^{\infty} N_t\beta^t\Big(U_{c,t}c^m_{i,t}\big(c_t,d_t,h_t;\varphi\big)+U_{d,t}d^m_{i,t}\left(c_t,d_t,h_t;\varphi\right)+U_{h,t}e_ih^m_{i,t}\left(c_t,d_t,h_t;\varphi\right)\Big),\ \ \forall i. \tag{107}$$

C.3 Ramsey problem

Let again $\lambda \equiv \{\lambda_i\}$ be the planner's welfare weight on type i, with $\sum_i \pi_i\lambda_i = 1$. Define the pseudo-utility function

$$\begin{aligned} W\big(c_t,d_t,h_t,Z_t;\varphi,\theta,\lambda\big) \equiv{}& \sum_i \pi_i\lambda_i u_i\big(c^m_{i,t}\big(c_t,d_t,h_t;\varphi\big),d^m_{i,t}\big(c_t,d_t,h_t;\varphi\big),h^m_{i,t}\big(c_t,d_t,h_t;\varphi\big),Z_t\big) \\ &+\sum_i \pi_i\theta_i\Big[U_{c,t}c^m_{i,t}\big(c_t,d_t,h_t;\varphi\big)+U_{d,t}d^m_{i,t}\big(c_t,d_t,h_t;\varphi\big)+U_{h,t}e_{i,t}h^m_{i,t}\big(c_t,d_t,h_t;\varphi\big)\Big], \end{aligned}$$

where $\pi_i\theta_i$ is the Lagrange multiplier on the implementability constraint of agent i, and $\theta \equiv \{\theta_i\}$. The new Ramsey problem can be written as

$$\max_{\substack{\{c_t,H_{1,t},H_{2,t},K_{1,t},K_{2,t},\\ d_t,E_{1,t},Z_t,\mu_t\}_{t=0}^{\infty},T,\varphi}} \sum_{t,i} N_t\beta^tW\left(c_t,d_t,h_t,Z_t;\varphi,\theta,\lambda\right) - U_{c,0}\sum_i \pi_i\theta_i\left(R_0N_0a_{i,0}+T\right),$$

subject to

$$\begin{aligned} N_tc_t+G_t+K_{t+1}+\Theta_t\left(\mu_t,E_t\right) &= \left(1-D\left(Z_t\right)\right)A_{1,t}F\left(K_{1,t},H_{1,t},E_{1,t}\right)+\left(1-\delta\right)K_t, \quad \forall\ t\geq 0,\\ E_t &= A_{2,t}G\left(K_{2,t},H_{2,t}\right), \quad \forall\ t\geq 0,\\ Z_t &= J\left(S_0,E^M_0,...,E^M_t,\eta_0,...,\eta_t\right), \quad \forall\ t\geq 0,\\ F_K(K_{1,t}H_{1,t},E_{1,t})G_H(K_{2,t}H_{2,t}) &= F_H(K_{1,t}H_{1,t},E_{1,t})G_K(K_{2,t}H_{2,t}), \quad \forall\ t\geq 0,\\ K_{1,t}+K_{2,t} &= K_t, \quad \forall\ t\geq 0,\\ H_{1,t}+H_{2,t} &= N_th_t, \quad \forall\ t\geq 0,\\ N_td_t+E_{1,t} &= E_t, \quad \forall\ t\geq 0,\\ \sum_j \pi_j\left(\varphi_je_j^{\gamma(\sigma-1)}\right)^{\frac{1}{1-(1+\epsilon+\gamma)(1-\sigma)}} &= 1, \end{aligned}$$

where $N_td_t+E_{1,t}=E_t$ is the only additional constraint compared to the benchmark problem.

C.4 Optimal taxes

Tax formulas From the first order conditions of the Ramsey problem, we can show that

$$\tau_{H,t} = 1 - \frac{U_{h,t}}{U_{c,t}} \frac{W_{c,t}}{W_{h,t}}, \tag{108}$$

$$\frac{R_{t+1}}{R^*_{t+1}} = \frac{W_{c,t+1}}{W_{c,t}} \frac{U_{c,t}}{U_{c,t+1}}, \tag{109}$$

$$\tau_{E,t} = \frac{1}{\nu_{1,t}} \sum_{j=0}^{\infty} \beta^j \left(\nu_{1,t+j} D'_{t+j} A_{1,t+j} F_{t+j} - N_{t+j} W_{Z,t+j} \right) J_{E^M_t, t+j}, \tag{110}$$

and

$$\tau_{I,t} = 0. \tag{111}$$

Using the first order conditions with respect to d_t, $E_{1,t}$ and c_t we have

$$W_{d,t} = W_{c,t}(1 - D(Z_t)) A_{1,t} F_{E,t},$$

which together with (105) and the final good firm's first order condition with respect to $E_{1,t}$ (given by (5) in the benchmark model) gives

$$\tau_{D,t} = \frac{U_{d,t}}{U_{c,t}} - \frac{W_{d,t}}{W_{c,t}}. \tag{112}$$

Using our functional form assumption, we can rewrite the pseudo-utility function as follows

$$W\left(c_t, d_t, h_t, Z_t; \varphi, \theta, \lambda\right) = \Phi \tilde{U}(c_t, d_t, h_t) + \frac{\sum_i \pi_i \lambda_i \chi_i}{\sum_i \pi_i \varphi_i \chi_i} \hat{U}(Z_t) + \Psi U_{h,t} + \Lambda_t U_{d,t}, \tag{113}$$

with

$$\tilde{U}(c_t, d_t, h_t) = \frac{(c_t (d_t - \bar{d}_t)^{\epsilon} (1 - \varsigma h_t)^{\gamma})^{1-\sigma}}{1-\sigma},$$
$$\hat{U}(Z_t) = \Gamma_\chi \frac{(1 + \alpha_0 Z_t^2)^{-(1-\sigma)}}{1-\sigma},$$

where

$$\Phi \equiv \sum_i \pi_i \omega_i \Big(\frac{\lambda_i}{\varphi_i} + \big(1-\sigma\big)\big(1+\epsilon+\gamma\big)\theta_i \Big), \tag{114}$$

$$\Psi \equiv \frac{1}{\varsigma} \sum_i \pi_i \theta_i e_i, \tag{115}$$

$$\Lambda_t \equiv \sum_i \pi_i \theta_i \bar{d}_{i,t}. \tag{116}$$

Substituting the derivatives into equations (108), (109), and (112), we get

$$\tau_{H,t} = 1 - \frac{\Phi + \Psi \frac{U_{ch,t}}{U_{c,t}} + \Lambda_t \frac{U_{cd,t}}{U_{c,t}}}{\Phi + \Psi \frac{U_{hh,t}}{U_{h,t}} + \Lambda_t \frac{U_{dh,t}}{U_{h,t}}} = \frac{\Psi \varsigma (1 - \varsigma h_t)^{-1}}{\Phi + \Psi \frac{\varsigma\left(1 - \gamma(1-\sigma)\right)}{(1 - \varsigma h_t)} + \Lambda_t \frac{\epsilon(1-\sigma)}{(d_t - \bar{d}_t)}}, \tag{117}$$

$$\frac{R_{t+1}}{R^*_{t+1}} = \frac{\Phi + \Lambda_{t+1}\frac{U_{cd,t+1}}{U_{c,t+1}} + \Psi\frac{U_{ch,t+1}}{U_{c,t+1}}}{\Phi + \Lambda_t\frac{U_{cd,t}}{U_{c,t}} + \Psi\frac{U_{ch,t}}{U_{c,t}}} = \frac{\Phi + \Lambda_{t+1}\frac{\epsilon(1-\sigma)}{(d_{t+1}-\bar{d}_{t+1})} - \Psi\frac{\varsigma\gamma(1-\sigma)}{(1-\varsigma h_{t+1})}}{\Phi + \Lambda_t\frac{\epsilon(1-\sigma)}{(d_t-\bar{d}_t)} - \Psi\frac{\varsigma\gamma(1-\sigma)}{(1-\varsigma h_t)}}, \tag{118}$$

and

$$\tau_{D,t} = \frac{\Lambda_t(d_t-\bar{d}_t)^{-1}U_{d,t}}{\Phi U_{c,t} + \Psi U_{hc,t} + \Lambda_t U_{dc,t}} = \frac{\Lambda_t\frac{\epsilon c_t}{(d_t-\bar{d}_t)^2}}{\Phi + \frac{\Psi\varsigma\gamma(\sigma-1)}{(1-\varsigma h_t)} - \frac{\Lambda_t\epsilon(\sigma-1)}{(d_t-\bar{d}_t)}}. \tag{119}$$

If we define

$$V\big(c_t, d_t, h_t, Z_t; \varphi, \theta, \lambda\big) \equiv \sum_i \pi_i\lambda_i u_i\big(c^m_{i,t}\big(c_t, d_t, h_t; \varphi\big), d^m_{i,t}\big(c_t, d_t, h_t; \varphi\big), h^m_{i,t}\big(c_t, d_t, h_t; \varphi\big), Z_t\big)$$

and

$$IC_i(c_t, d_t, h_t, \varphi) \equiv U_{c,t}c^m_{i,t}\big(c_t, d_t, h_t; \varphi\big) + U_{d,t}d^m_{i,t}\big(c_t, d_t, h_t; \varphi\big) + U_{h,t}e_{i,t}h^m_{i,t}\big(c_t, d_t, h_t; \varphi\big)$$

we can also express the optimal pollution tax as

$$\tau_{E,t} = \sum_{j=0}^{\infty}\beta^j\left(\frac{V_{c,t+j} + \sum_i\pi_i\theta_i MIC_{i,t+j}}{V_{c,t} + \sum_i\pi_i\theta_i MIC_{i,t}}D'_{t+j}A_{1,t+j}F_{t+j} - \frac{N_{t+j}V_{Z,t+j}}{V_{c,t} + \sum_i\pi_i\theta_i MIC_{i,t}}\right)J_{E^M_t,t+j}.$$

Comparison with the benchmark formula The previous expression is the same as the one found in our benchmark, and the optimal tax will again be equal to the social cost of pollution when the marginal implementation cost ($-\sum_i\pi_i\theta_i MIC_{i,t}$) is null, which is the case in the first-best.

Compared to our benchmark, the marginal implementation cost now includes an additional term from the derivative of U_d with respect to consumption. In particular, we again have

$$-\sum_i\pi_i\theta_i MIC_{i,t} = (\sigma-1)\frac{\text{cov}(\theta_i, IC_{i,t})}{c_t},$$

but now the ratio of the period implemantation cost for two agents i and j is

$$\frac{IC_{i,t}}{IC_{j,t}} = \frac{(1+\epsilon+\gamma)\omega_i + \frac{\epsilon\bar{d}_{i,t}}{(d_t-\bar{d}_t)} - \frac{\gamma e_i}{(1-\varsigma h_t)}}{(1+\epsilon+\gamma)\omega_j + \frac{\epsilon\bar{d}_{j,t}}{(d_t-\bar{d}_t)} - \frac{\gamma e_j}{(1-\varsigma h_t)}}.$$

Thus, the sign of the marginal implementation cost depends on a price effect through σ, and on an energy demand and labor supply effects from $\text{cov}(\theta_i, IC_{i,t})$. The covariance term is higher in periods when richer households (higher θ_i) work relatively less, or when they have higher energy needs relative to poor households compared to an average period.

The value of the optimal tax also depends on the marginal dis-utility from pollution ($V_{Z,t}$) which now accounts for the weights χ_i. In particular, we now have

$$\begin{aligned} V_{Z,t} &= -\sum_i\pi_i\lambda_i\chi_i 2\alpha_0 Z_t(1+\alpha_0 Z_t^2)^{\sigma-2} \\ &= -(1+\text{cov}(\lambda_i,\chi_i))2\alpha_0 Z_t(1+\alpha_0 Z_t^2)^{\sigma-2} \end{aligned}$$

where the last result is obtained using the normalization of the sums of λ_i and χ_i. Thus, when the planner has utilitarian preferences, for all i, $\lambda_i = 1$ and the distribution of χ_i has no impact on the aggregate marginal dis-utility form pollution. When the planner values more (resp. less) agents with higher marginal dis-utility from pollution, then the tax is set at a higher (resp. lower) level.

Note that we can again use the first order conditions with respect to market weights to obtain

$$\theta_i = \sum_j \frac{\pi_j \lambda_j}{\varphi_j} - \frac{\lambda_i}{\varphi_i}, \tag{120}$$

from which we can rewrite

$$\begin{aligned}
\Phi &= \sum_i \pi_i \omega_i \Big(\frac{\lambda_i}{\varphi_i} + \big(1-\sigma\big)\big(1+\epsilon+\gamma\big) \Big(\sum_j \frac{\pi_j \lambda_j}{\varphi_j} - \frac{\lambda_i}{\varphi_i} \Big) \Big) && (121) \\
&= \sum_j \pi_j \frac{\lambda_j}{\varphi_j} + \Big(1 - (1+\epsilon+\gamma)(1-\sigma) \Big) \mathrm{cov}(\lambda_i/\varphi_i, \omega_i), && (122) \\
\Psi &= \frac{1}{\varsigma} \sum_i \pi_i \Big(\sum_j \frac{\pi_j \lambda_j}{\varphi_j} - \frac{\lambda_i}{\varphi_i} \Big) e_i && (123) \\
&= -\frac{\mathrm{cov}(\lambda_i/\varphi_i, e_i)}{\varsigma}, && (124) \\
\Lambda_t &= \sum_i \pi_i \Big(\sum_j \frac{\pi_j \lambda_j}{\varphi_j} - \frac{\lambda_i}{\varphi_i} \Big) \bar{d}_{i,t} && (125) \\
&= -\mathrm{cov}(\lambda_i/\varphi_i, \bar{d}_{i,t}), && (126)
\end{aligned}$$

and obtain an expression for market weights

$$\varphi_i = \frac{1}{e_i^{\gamma(\sigma-1)}} \left(\frac{U_{c,0}(R_0 N_0 a_{i,0} + T) + \sum_t N_t \beta^t \Big(U_{h,t} \frac{e_i}{\varsigma} - U_{d,t} \bar{d}_{i,t} \Big)}{(1-\sigma)(1+\epsilon+\gamma) \sum_t N_t \beta^t \tilde{U}(c_t, d_t, h_t)} \right)^{1-(1+\epsilon+\gamma)(1-\sigma)}$$

D Calibration

D.1 Household heterogeneity

Productivity distribution We calibrate the ability distribution on the basis of hourly wage data that we obtain from the Survey of Consumer Finances (SCF). For each of the 6,015 households in the 2013 wave of the survey, we sum the hours worked on their main job and potential additional job(s) in a normal week. Annual labor supply of the respondent and their partner is then calculated by multiplying weekly hours worked by 52 minus the number of weeks they have spent unemployed during the past 12 months minus the number of weeks spend on holidays (which we assume equals 3 for each worker). The household hourly wage is then obtained as the household annual income from wages and salaries before taxes, divided by the household total annual labor supply (i.e., the sum of the respondent and their partner's labor supply). This number reflects how much households members were paid on average for each hour of work they supplied in the past year.

To obtain the hourly wage distribution, we make a few additional adjustments. We first drop all households with an hourly wage below \$1 or above \$1,000. We also restrict the sample to households who have worked at least 1 week over the past 12 months, who work at least 1 hour on a normal week, and with no member working above 100 hours. Finally we restrict the sample to households whose respondent is at least 18 years old, and at most 65 years old. Using this sub-sample, we divide households in ten groups of hourly wage deciles. These correspond to $I = 10$ groups with size $\pi_i = 0.10$. For each group, we compute the average hourly wage.

Asset distribution For each of the ten productivity groups, we divide again households in ten weighted deciles of net worth. For each sub-group, we compute the average net worth. This provides a table in which households are split in 100 groups of equal size, with for each of these groups the average hourly wage and the net worth.[28]

Because agents in our model are infinitely lived but hourly wage and asset holdings are positively correlated with age, we control for generational heterogeneity. To do so, we divide households in ten generations based on the age of the respondent, and compute the average hourly wage and net worth of each of the 100 groups within each generation. We then obtain the average hourly wage and net worth for each group as the average of that group over all generations. Table II below provides the results.

Table II: Distribution of households hourly wages and net worth by productivity deciles (rows) and net worth deciles (columns), controlling for generational differences.

	Net worth deciles										
Productivity deciles	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th	Hourly wage
1st	-4.59e+04	-7.00e+03	1.22e+03	7.45e+03	1.79e+04	3.25e+04	6.44e+04	1.12e+05	2.18e+05	1.10e+06	6.44e+00
2nd	-2.99e+04	-1.97e+03	4.89e+03	1.23e+04	2.50e+04	3.97e+04	6.46e+04	1.03e+05	1.83e+05	1.04e+06	1.11e+01
3rd	-4.13e+04	-6.00e+03	3.72e+03	1.29e+04	2.76e+04	4.47e+04	7.69e+04	1.09e+05	2.01e+05	7.19e+05	1.42e+01
4th	-4.56e+04	-2.65e+03	1.44e+04	3.31e+04	5.38e+04	7.48e+04	1.01e+05	1.50e+05	2.67e+05	7.64e+05	1.73e+01
5th	-4.94e+04	-2.15e+03	1.55e+04	3.58e+04	6.72e+04	9.53e+04	1.40e+05	2.07e+05	2.98e+05	1.10e+06	2.05e+01
6th	-3.82e+04	1.21e+04	3.94e+04	7.26e+04	1.14e+05	1.60e+05	2.13e+05	2.88e+05	4.60e+05	1.75e+06	2.41e+01
7th	-2.41e+04	3.79e+04	6.75e+04	1.03e+05	1.54e+05	2.06e+05	2.63e+05	3.58e+05	5.32e+05	1.23e+06	2.86e+01
8th	-2.93e+04	3.00e+04	7.10e+04	1.34e+05	2.11e+05	2.80e+05	3.90e+05	5.04e+05	6.94e+05	2.57e+06	3.48e+01
9th	4.38e+03	6.86e+04	1.44e+05	2.11e+05	3.07e+05	4.20e+05	5.53e+05	7.45e+05	1.08e+06	3.50e+06	4.47e+01
10th	-8.53e+04	1.40e+05	2.77e+05	4.43e+05	6.38e+05	8.55e+05	1.29e+06	2.14e+06	3.45e+06	1.00e+07	1.01e+02

Note: The rows correspond to productivity (*i.e.* hourly wage) decile groups. The last column corresponds to the average hourly wage for each productivity group. Columns 1 to 10 correspond to net worth decile groups within productivity groups. The number reported in these columns are the average net worth for each group. All groups are defined for a given generation, and values correspond to the weighted average across ten generation groups. Example: 1.10e+06 in the 1st row, 10th column, means that among the people that belong to the bottom 10% of the hourly wage distribution of their generation, the 10% wealthiest have an average net worth of \$1.10e+06.

[28] Kuhn and Ríos-Rull (2016) provide extensive descriptive statistics about income and wealth inequalities from the SCF. On the sub-sample of households from whom we compute the productivity distribution, we find that income is on average higher, wealth is on average lower, and the two variables are only slightly more correlated (correlation coefficient of 0.60 instead of 0.58).

Distribution of energy consumption Let us denote $X_{i,t}$ the expenditure share of energy for an household of type i at time t,

$$X_{i,t} \equiv \frac{d_{i,t}\big(p_{E,t} + \tau_{D,t}\big)}{d_{i,t}\big(p_{E,t} + \tau_{D,t}\big) + c_{i,t}}. \tag{127}$$

From the households' first order conditions we have

$$\frac{u_{d,i,t}}{u_{c,i,t}} = \frac{\epsilon c_{i,t}}{d_{i,t} - \bar{d_{i,t}}} = p_{E,t} + \tau_{D,t}, \tag{128}$$

with $u_{d,i,t}$, $u_{c,i,t}$ the marginal utility of energy and final good consumption of agent i at time t. Substituting the previous expression into (127) we get

$$X_{i,t} = \frac{d_{i,t}\big(\frac{\epsilon c_{i,t}}{d_{i,t} - \bar{d_{i,t}}}\big)}{d_{i,t}\big(\frac{\epsilon c_{i,t}}{d_{i,t} - \bar{d_{i,t}}}\big) + c_{i,t}}.$$

Rearranging the previous equation, we can express the necessity parameter of agent i in period t ($\bar{d}_{i,t}$) as a function of its observed consumption level ($d_{i,t}$), its observed energy consumption share ($X_{i,t}$), and the parameter of relative preference for energy (ϵ) common to all households

$$\bar{d}_{i,t} = d_{i,t}\left(1 - \epsilon \frac{(1 - X_{i,t})}{X_{i,t}}\right). \tag{129}$$

We obtain the initial distribution of households' energy expenditures and energy consumption shares from the Consumer Expenditure Surveys (CEX). To be consistent with the timing of DICE, we pool surveys from the 20 quarters between January 2011 and December 2015, for a total of 129,573 observations.

Energy expenditures (d_i) are obtained by summing expenditures on gasoline and motor oil, electricity, natural gas, fuel oil, and other fuels. The energy expenditure shares (X_i) are obtained by dividing energy expenditures by total expenditures. To determine hourly wages, we apply the same procedure as with the SCF. We first compute the household annual wage by summing the income received from salary or wages before taxes. We then compute the annual labor supply of the respondent and its partner: we multiply the number of hours usually worked per week by the number of weeks worked in the past twelve months, minus 3 weeks of imputed holidays. The household hourly wage is then the ratio of the household annual wage over annual hours. Just like with the SCF data, this number reflects how much households members were paid on average for each hour of work they supplied in the past year.[29]

In order to characterize the joint distribution of hourly wages and energy expenditure shares, we restrict our sample to working households, following the same sample definition as with the SCF. Using this sub-sample, we divide households in ten groups of hourly wage deciles. For each group, we compute the average hourly wage. For each of the ten groups, we divide again households in five weighted quintiles of energy expenditure share, and compute the average energy expenditure share. This provides a table in which households are split in 50 groups of equal size, with for each of these

[29] The bottom hourly wage is \$6.59 and the top hourly wage is \$110.12 (without generational adjustments).

groups the average hourly wage and the energy expenditure share.[30] Since energy consumption shares do not appear to be strongly determined by age among working households, we do not control for generational differences. However, we control for seasonality and yearly variations that could lead to overestimate consumption heterogeneity. We proceed in the same way as with generational controls: we group individuals based on their ranking relative to the people interviewed in the same month and same year. We then compute the average for each group over all time periods. The resulting distribution of initial energy shares $\{X_i\}_{i \in I}$ is presented in Table III.

Finally, in order to obtain the distribution of $\bar{d}_i$, we also need to determine ϵ. Relative to the data, the model gives us a degree of freedom, hence we assume ϵ is such that the group i with the lowest consumption share has $\bar{d}_i = 0$, which gives $\epsilon \simeq 0.0263$. [To be added: calibration going forward.]

Table III: Distribution of households energy expenditure shares by productivity deciles (rows) and expenditure share quintiles (columns), controlling for seasonality and time trend.

		Expenditure share quintiles					
		1st	2nd	3rd	4th	5th	Average
Productivity deciles	1st	2.69%	7.59%	11.42%	15.88%	24.39%	12.70%
	2nd	3.50%	8.07%	11.48%	15.26%	22.83%	12.51%
	3rd	4.13%	8.29%	11.31%	14.78%	21.79%	12.33%
	4th	4.09%	7.99%	10.86%	14.00%	20.46%	11.84%
	5th	4.09%	7.63%	10.33%	13.39%	19.45%	11.20%
	6th	3.93%	7.23%	9.75%	12.78%	18.86%	10.74%
	7th	3.83%	6.90%	9.25%	12.03%	17.89%	10.19%
	8th	3.47%	6.22%	8.44%	11.17%	16.96%	9.45%
	9th	3.04%	5.63%	7.76%	10.29%	16.05%	8.76%
	10th	2.56%	4.95%	7.01%	9.65%	15.60%	8.16%

Note: The rows correspond to productivity (*i.e.* hourly wage) decile groups. The column "Average" corresponds to the average energy expenditure share for each productivity group. Columns 1 to 5 correspond to energy expenditure share quintile groups within productivity decile groups. The numbers reported in these columns are the average energy expenditure shares for each group. All groups are defined for a given month and year, and values correspond to the weighted average across all periods. Example: 2.69% in the 1st row, 1st column, means that among the people that belong to the bottom 10% of the hourly wage distribution at the month × year they were interviewed, the 20% with lowest energy shares spend on average 2.69% of their total expenditures in energy. Sample: CEX from 2011 to 2015, only workers included.

[30] We chose to divide each decile group in quintiles instead of deciles in order to mitigate the impact of potential outliers.

D.2 Parameters choice

Baseline hours worked We also use the SCF 2013 to compute the initial labor supply that we impute to the model. To do so, we again restrict the sample to individuals between 18 and 65 years old. However, because our aim is not to compute hourly wages but to look at the average labor supply, we do not eliminate outliers based on their hourly wage or labor supply. In particular, we keep unemployed households for whom the hourly wage is not observed, as dropping them would lead to overestimate the average labor supply. For all households in the sample, we divide the annual labor supply by the number of working age individuals (individuals between 18 and 65). This yields an average of 1440 hours annually. Assuming a maximum labor supply capacity of 52 weeks per year and 100 hours per week per individual, this yields an average labor supply of 0.277 of the maximum capacity.

Table IV: Calibration summary: climate parameters.

Parameter	Description	Value
Carbon stocks		
S^{At}_{2015}	Initial carbon concentration in atmosphere (in GtC)	851
S^{Up}_{2015}	Initial carbon concentration in upper strata (in GtC)	460
S^{Lo}_{2015}	Initial carbon concentration in lower strata (in GtC)	1740
S^{At}_{eq}	Equilibrium carbon concentration in atmosphere (in GtC)	588
E^{land}_{2015}	Initial CO_2 emissions from land ($GtCO_2$ per year)	2.6
$g_{E^{\text{land}}}$	Decline rate of land emissions (per period)	0.115
Carbon cycle transition matrix		
$b_{1,1}$	Carbon cycle coefficient	0.88
$b_{2,1}$	Carbon cycle coefficient	0.047
$b_{3,1}$	Carbon cycle coefficient	0
$b_{1,2}$	Carbon cycle coefficient	0.12
$b_{2,2}$	Carbon cycle coefficient	0.94796
$b_{3,2}$	Carbon cycle coefficient	0.00075
$b_{1,3}$	Carbon cycle coefficient	0
$b_{2,3}$	Carbon cycle coefficient	0.005
$b_{3,3}$	Carbon cycle coefficient	0.99925
Radiative forcing		
κ	Forcings of equilibrium CO_2 doubling (Wm-2)	3.6813
$\mathcal{F}^{\text{Ex}}_{2015}$	Initial forcings of non-CO2 GHG (Wm-2)	0.5
$\mathcal{F}^{\text{Ex}}_{2100}$	2100 forcings of non-CO2 GHG (Wm-2)	1
$g_{\mathcal{F}^{\text{Ex}}}$	Rate of convergence of $\mathcal{F}$	1/17
Temperature		
T_{2015}	Initial atmospheric temperature change (C since 1900)	0.85
T^{Lo}_{2015}	Initial lower stratum temperature change (C since 1900)	0.0068
ζ_1	Climate model coefficient	0.1005
ζ_2	Climate model coefficient	1.1875
ζ_3	Climate model coefficient	0.088
ζ_4	Climate model coefficient	0.025

Note: All parameters are taken from DICE (2016).

Table V: Calibration summary: economic parameters.

Parameter	Description	Value	Source
Preferences			
β	Utility discount rate (per year)	1/(1.015)	DICE 2016
σ	Inverse of IES	1.45	DICE 2016
η^F	Frisch elasticity of labor supply	0.75	Chetty et al (2011)
ς	Labor dis-utility coefficient	1.885	To target η^F and h_{2015}
γ	Labor dis-utility exponent	0.709	To target η^F and h_{2015}
α_0	Relative preference for the environment	7.61e-05	Adapted from Barrage (2019)
Production damages			
a_1	Damage intercept	0	DICE 2016
a_2	Damage coefficient quadratic term	0.00175	DICE 2016 adjusted
a_3	Damage exponent	2	DICE 2016
Production first sector			
α	Return to scale on labor sector 1	0.3	DICE 2016
ν	Return to scale on energy sector 1	0.04	Golosov et al (2014)
δ	Depreciation rate on capital (per year)	0.1	DICE 2016
r_{2015}	Initial net rate of return on capital	0.023	To target steady state
Y_{2015}	Initial output (in trillions 2015 USD)	70.807	World Bank (2011-2015)
$hh_{1,2015}$	Initial share of labor in sector 1	0.976	To equate MPL across sectors
$kk_{1,2015}$	Initial share of capital in sector 2	0.926	To equate MPL across sectors
E_{2015}	Initial industrial emissions ($GtCO_2$ per year)	35.85	DICE 2016
h_{2015}	Initial labor supply per capita	0.277	Computed from SCF
$A_{1,2015}$	Initial TFP sector 1	141.9	To target Y_{2015}
Production second sector			
α_E	Return to scale on capital sector 2	0.403	Barrage (2019)
$A_{2,2015}$	Initial TFP sector 2	87.1	To target E_{2015}
Abatement costs			
$P^{\text{backstop}}_{2015}$	Backstop price in 2015 (in $/t$CO_2$)	550	DICE 2016
$g_{P^{\text{backstop}}}$	Decline rate backstop price (per period)	2.5%	DICE 2016
c_2	Exponent abatement cost function	2.6	DICE 2016
μ_{2015}	Initial abatement share	0.03	DICE 2016
Government			
G_t/Y_t	Government spending to GDP ratio	0.3030	IMF-GFS
B_{2015}	Initial public debt to GDP ratio	0.2220	IMF-GFS
$\tau_{H,2015}$	Initial tax rate on labor income	0.255	Trabandt & Uhlig (2012)
$\tau_{K,2015}$	Initial tax rate on capital income	0.411	Trabandt & Uhlig (2012)

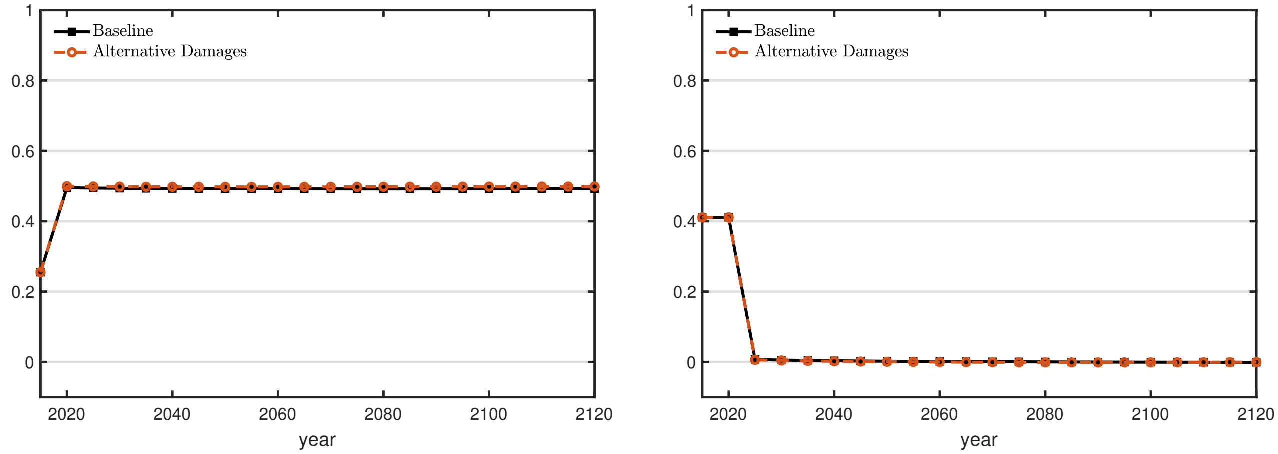
Calibration summary: economic parameters (continued).

Exogenous growth parameters			
$g_{A_{1,2015}}$	Initial TFP growth rate sector 1 (per period)	0.076	DICE 2016
$gg_{A_{1,t}}$	Decline rate TFP growth sector 1 (per year)	0.005	DICE 2016
$g_{A_{2,2015}}$	Initial TFP growth rate sector 2 (per period)	0.076	DICE 2016
$gg_{A_{2,t}}$	Decline rate TFP growth sector 2 (per year)	0.005	DICE 2016
N_{2015}	Initial population (in millions)	1,309	World bank (2015)
$N_{\max}$	Asymptotic population (in millions)	2,034	DICE 2016 US-adjusted
g_N	Rate of convergence of population	0.134	DICE 2016

Note: The adjustments relative to DICE 2016 for population and damages are described in the calibration section.

E Additional quantitative results

E.1 Alternative damages



(a) Optimal labor income Taxes

(b) Optimal capital income Taxes

Figure 6: Optimal Income Taxes, Alternative Damages.

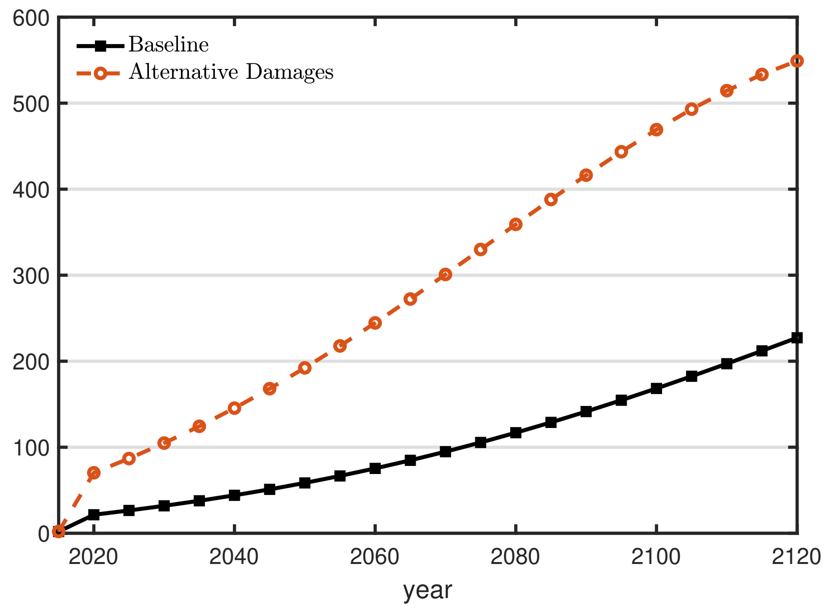


Figure 7: Optimal Carbon Taxes ($\$/tCO_2$), Alternative Damages.

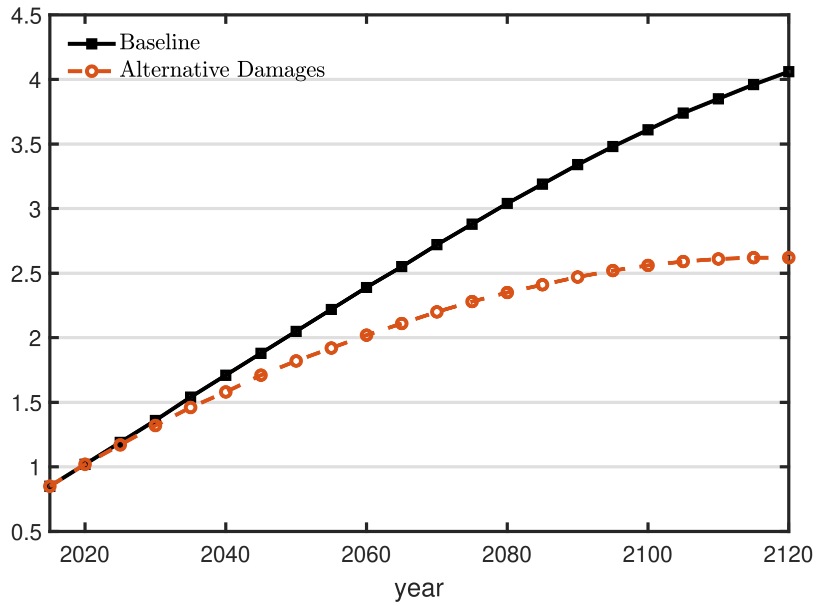


Figure 8: Temperature on the Optimal Path, Alternative Damages.

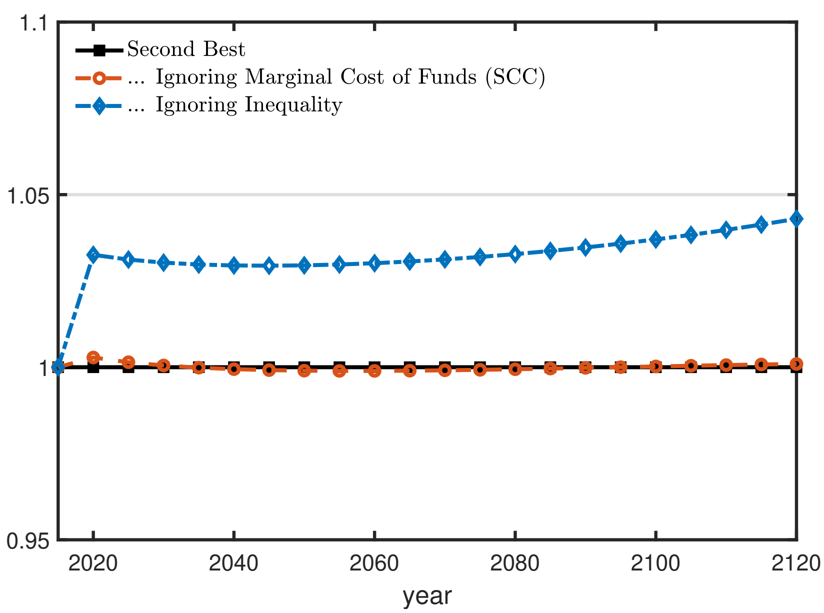
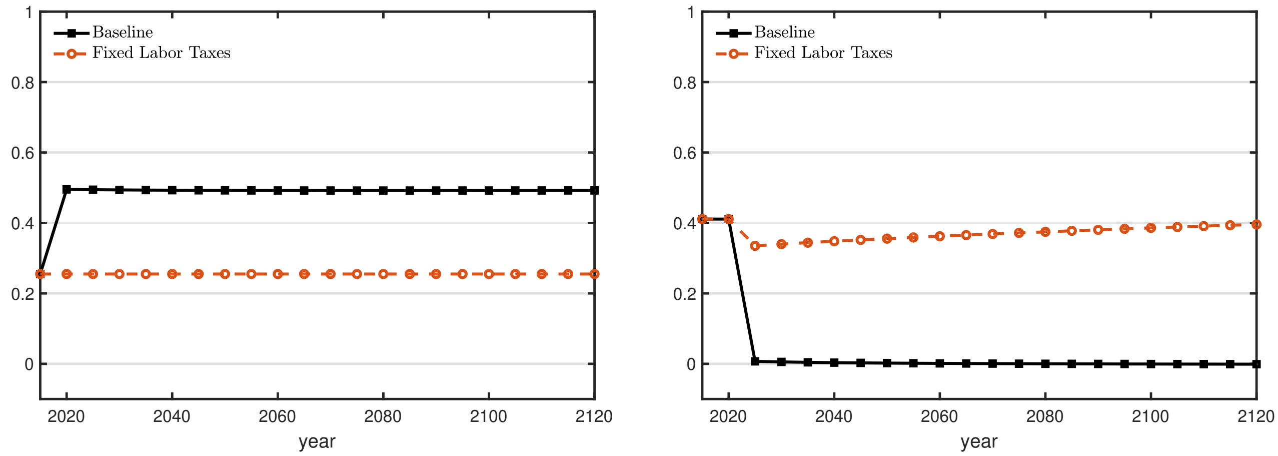


Figure 9: Carbon Tax Decomposition, Alternative Damages.

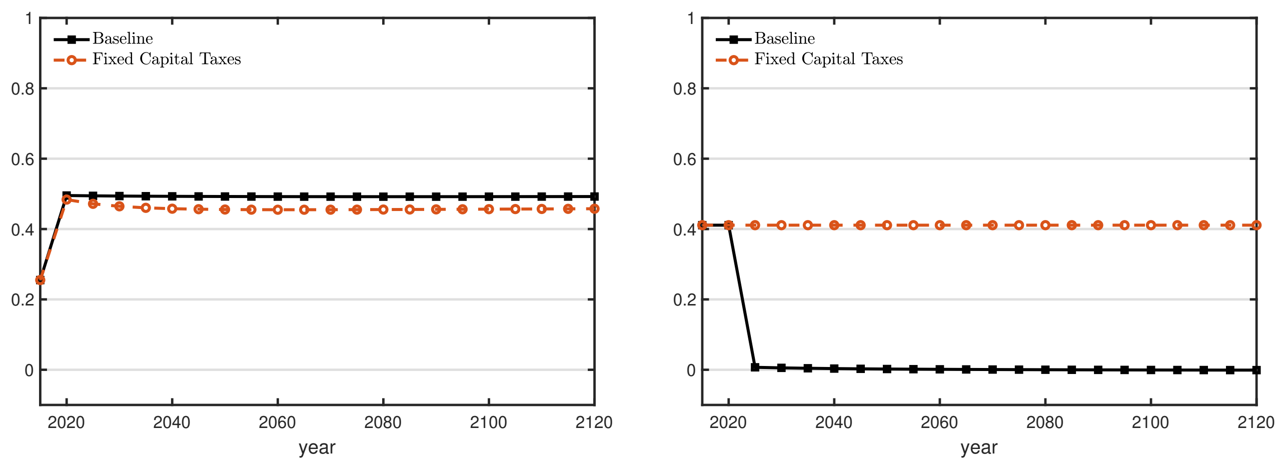
E.2 Third-best policies



(a) Optimal labor income Taxes

(b) Optimal capital income Taxes

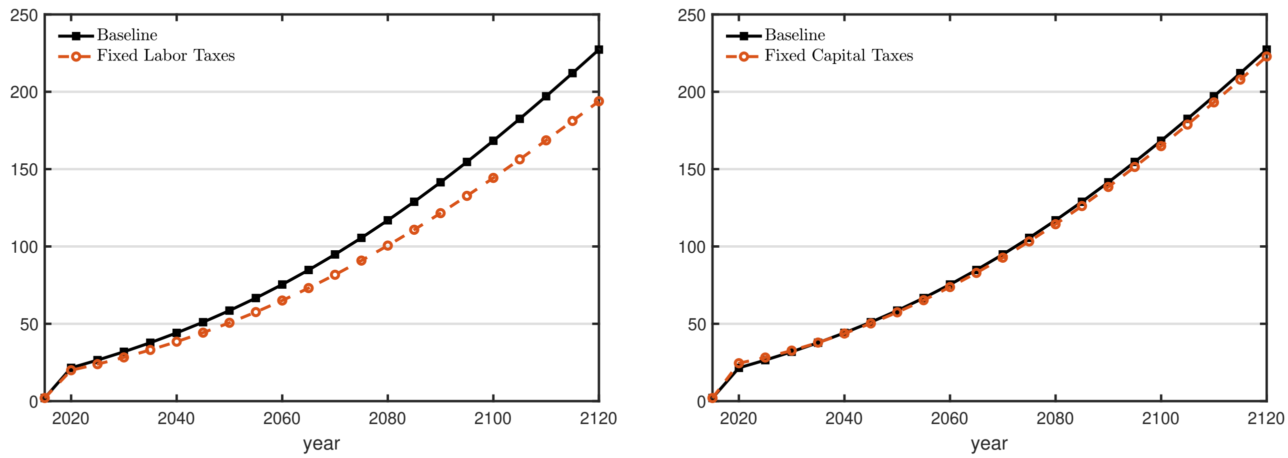
Figure 10: Optimal Income Taxes, Given Labor Tax.



(a) Optimal Labor-Income Taxes

(b) Optimal Capital-Income Taxes

Figure 11: Optimal Income Taxes, Given Capital Tax.



(a) Given labor income tax

(b) Given capital income tax

Figure 12: Optimal Carbon Taxes ($\$/tCO_2$), Given Income Taxes.

Table VI: Government Budget Adjustment, Given Capital-Income Taxes.

	Revenue Source			Revenue Use		
	Labor	Capital	Carbon	Gov. Cons.	Transfer	Interest
No Carbon Tax	31.5%	6.2%	0.0%	18.5%	2.1%	16.5%
Optimal Carbon Tax	30.8%	6.1%	1.1%	18.4%	2.2%	17.0%
Change	−0.7%	−0.2%	1.1%	−0.1%	0.0%	0.5%

Note: Numbers represent the present value of each component of the government budget constraint divided by the present value of GDP.

Table VII: Government Budget Adjustment, Given Labor-Income Taxes.

	Revenue Source			Revenue Use		
	Labor	Capital	Carbon	Gov. Cons.	Transfer	Interest
No Carbon Tax	17.2%	5.5%	0.0%	16.0%	5.0%	2.1%
Optimal Carbon Tax	17.0%	5.4%	0.8%	15.9%	5.7%	2.1%
Change	−0.2%	−0.1%	0.8%	−0.1%	0.7%	0.0%

Note: Numbers represent the present value of each component of the government budget constraint divided by the present value of GDP.

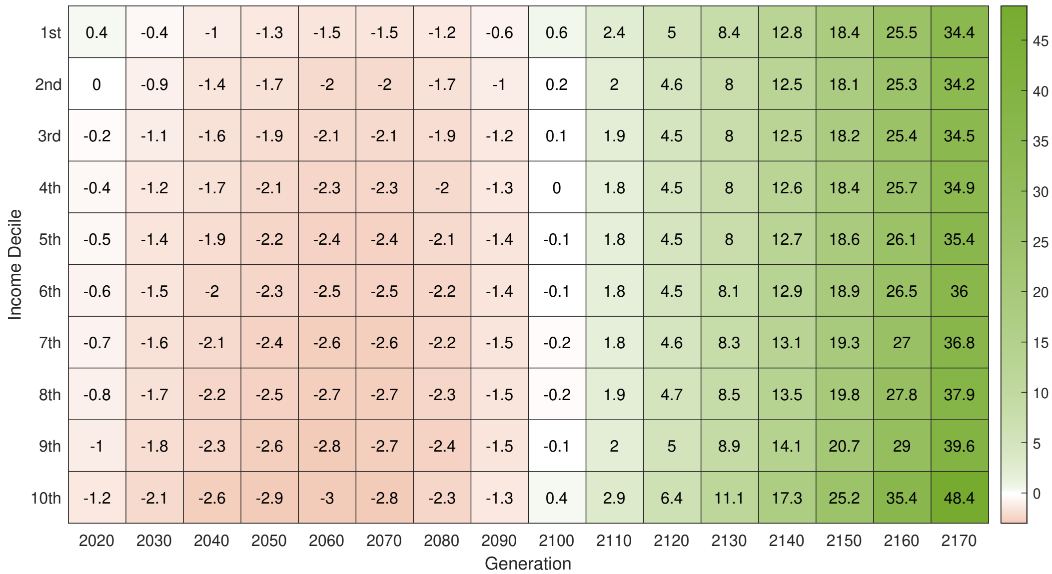


Figure 13: Period Welfare Gains (%), Given Capital-Income Taxes.

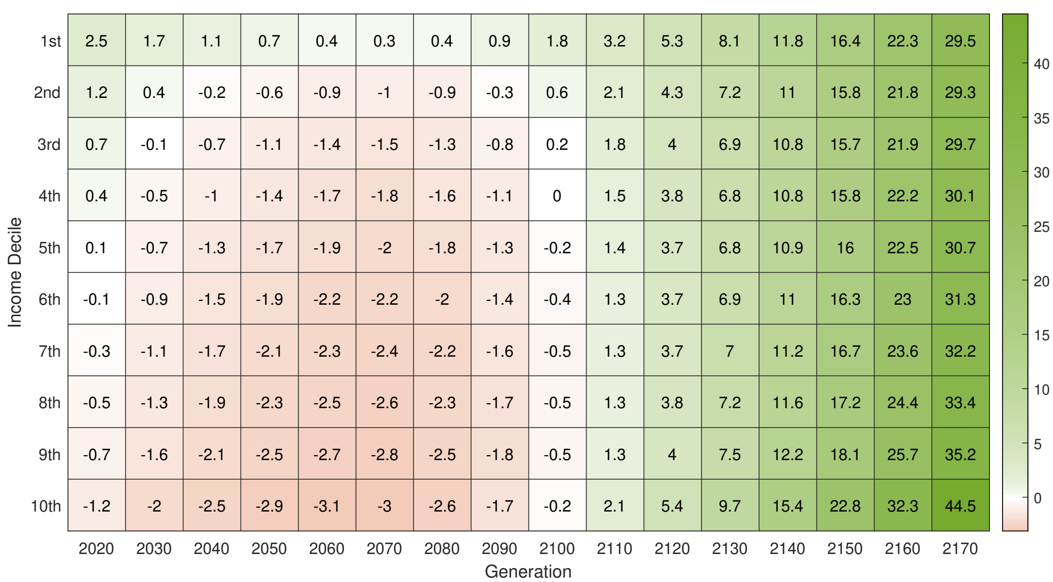
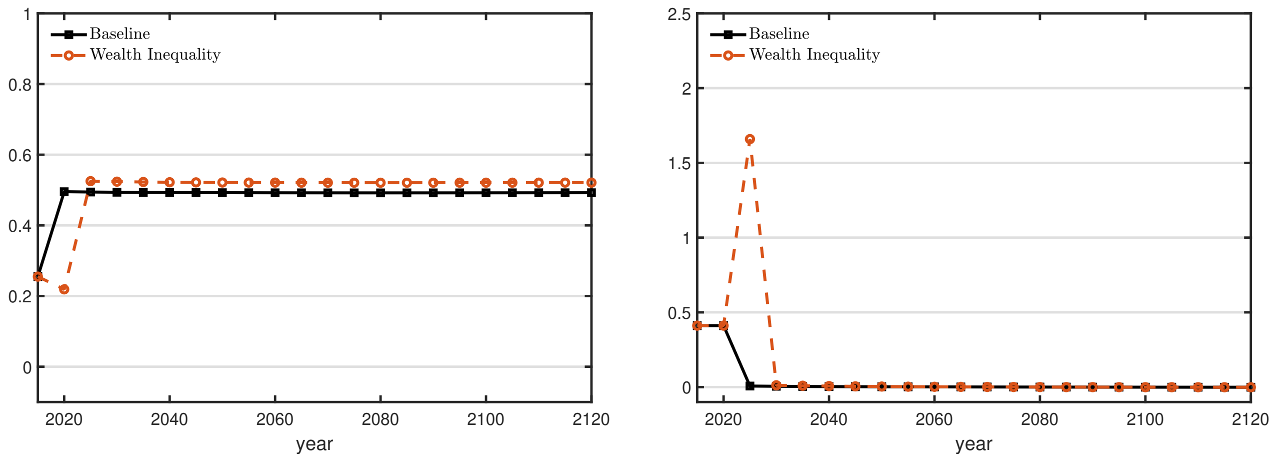


Figure 14: Period Welfare Gains (%), Given Labor-Income Taxes.

E.3 Initial wealth inequality



(a) Optimal Labor-Income Taxes (b) Optimal Capital-Income Taxes

Figure 15: Optimal Income Taxes, Initial Wealth Heterogeneity and Exogenous Initial Capital Tax.

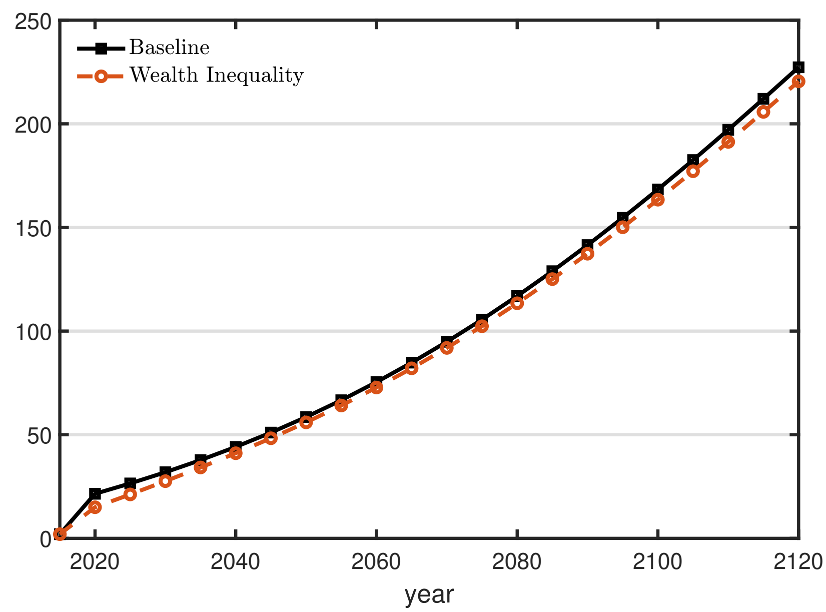


Figure 16: Optimal Carbon Taxes ($/t$CO_2$), Initial Wealth Heterogeneity and Exogenous Initial Capital Tax.

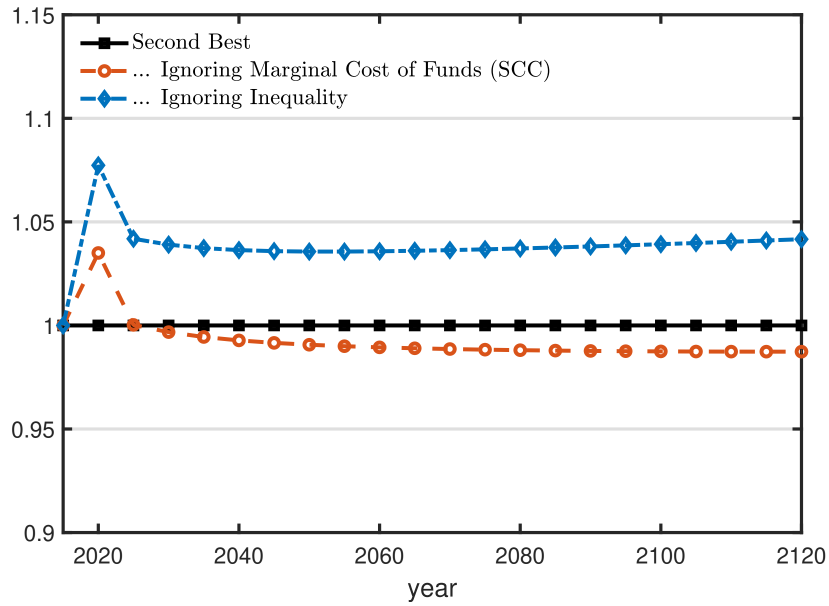


Figure 17: Carbon Tax Decomposition, Initial Wealth Heterogeneity and Exogenous Initial Capital Tax.

Table VIII: Government Budget Adjustment, Initial Wealth Heterogeneity and Exogenous Initial Capital Tax.

	Revenue Source			Revenue Use		
	Labor	Capital	Carbon	Gov. Cons.	Transfer	Interest
No Carbon Tax	34.2%	3.2%	0.0%	17.9%	18.1%	1.5%
Optimal Carbon Tax	33.6%	3.2%	0.9%	17.8%	18.5%	1.5%
Change	−0.6%	0.0%	0.9%	−0.1%	0.5%	0.0%

Note: Numbers represent the present value of each component of the government budget constraint divided by the present value of GDP.

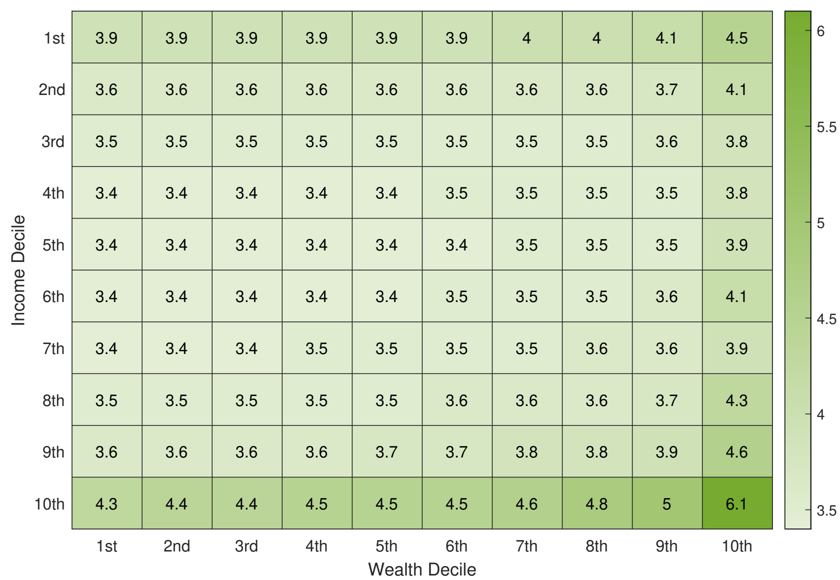


Figure 18: Welfare Gains (%), Initial Wealth Heterogeneity and Exogenous Initial Capital Tax.