Optimal Climate Policy with Incomplete Markets*

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Abstract

This paper examines dynamic, optimal climate policy in a fiscal climate-economy model with incomplete markets. Initially, in a simple two-period model, we isolate several mechanisms through which inequality, risk, and borrowing constraints affect optimal climate policy. We then introduce a new fiscal climate-economy framework with incomplete markets, which we calibrate to reflect critical aspects of the U.S. economy, particularly household inequality and risk, carbon emissions, and abatement technologies. Within this framework, we solve a Ramsey problem for a government that has access to multiple instruments, including carbon taxes and government debt. In all scenarios, optimal carbon taxes grow at a faster rate than GDP until the economy reaches carbon neutrality. Moreover, we show that granting the planner the flexibility to determine the timing of debt and lump-sum transfers significantly affects redistribution, insurance, aggregate variables, and welfare. Yet, surprisingly, this flexibility has minimal influence on the optimal trajectory of climate policy.

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1 Introduction

In the face of the escalating climate crisis, determining the optimal approach for governments to levy carbon taxes amidst household inequality and risk becomes both policy-relevant and urgent. What are the implications of carbon taxation for redistribution, insurance provision, and overall welfare in this critical context? This paper introduces a fiscal climate-economy framework incorporating incomplete markets to address these pressing questions. We further explore and emphasize the strategic role of fiscal policy in combating the consequences of climate change.

Our analysis is conducted in two stages. First, we analytically explore the effects of inequality, risk, and borrowing constraints on optimal climate policy using a simple two-period model. This step allows us to transparently identify the relevant trade-offs. Then, we develop a quantitative, macroeconomic, and novel framework that combines a climate-economy model featuring fiscal policy and heterogeneity with the standard incomplete markets (SIM) model incorporating uninsurable idiosyncratic risk.[1] We proceed to analyze a Ramsey problem in this context, characterizing an optimal, dynamic fiscal policy over the transition.

Through this approach, we derive new insights on optimal climate policy design and the trade-offs policymakers face. Theoretically, we find that risk, inequality, and the presence of borrowing constraints raise the costs of climate policy in terms of foregone consumption, but also raise the benefits by substituting away from private consumption, which is risky and unequally distributed, toward a cleaner environment, which benefits all households. The effect on optimal carbon taxation is therefore ambiguous. Quantitatively, when we allow the government to optimize government debt, we find that it chooses to provide very large lump-sum transfers in earlier periods to provide redistribution and relax households' borrowing constraints. While this has significant impacts on macroeconomic aggregates such as labor supply and capital stock, the optimal carbon tax path is, surprisingly, hardly affected. In all cases, optimal carbon taxes increase rapidly over time, at a faster rate than GDP, until the economy reaches carbon neutrality.

In our two-period model, we examine a basic endowment economy where the government can set the level of climate policy, conceptualized as a consumption tax, for analytical simplicity. While immediately reducing private consumption, this policy offers dual benefits: it funds a public good and generates future endowment gains. These two forms of benefits are the counterparts of the utility and production benefits of climate change mitigation in our infinite-horizon model. We solve a Ramsey problem in this economy and analytically investigate the impact of inequality, idiosyncratic risk, and borrowing constraints on the optimal policy rule.

This simplified model reveals several mechanisms by which inequality and risk influence the optimal climate policy. In scenarios where the climate policy solely produces public good benefits, the presence of inequality and risk modifies the optimal policy via the opportunity cost associated with reduced private consumption. On the one hand, increased inequality among households across dif-

[1] The climate-economy model builds upon Douenne, Hummel, and Pedroni (2023). The standard incomplete markets model was originally developed by Bewley (1986), Imrohoruglu (1989), Huggett (1993), and Aiyagari (1994).

ferent states or over time due to binding borrowing constraints elevates the average marginal utility of private consumption, which calls for a less stringent climate policy. This represents an income effect: as private consumption becomes scarcer, its value increases. On the other hand, the existence of inequality—among households, states of the world, and periods—enhances the value of climate policy as it provides a way to substitute unequal/risky private consumption for equal/safe public good consumption, serving as an indirect means of redistribution/insurance. This constitutes a substitution effect.

When the utility is CRRA, we show that the first effect dominates if and only if the intertemporal elasticity of substitution (IES) is below 1. In that case, it is optimal to pursue a less ambitious environmental policy. Similarly, when climate policies lead to future endowment gains, the same mechanisms apply, driven by the opportunity cost of reduced private consumption. Moreover, inequality, risk, and borrowing constraints also influence the direct benefits of the policy. As before, endowment gains are more valued as consumption becomes "scarcer" with inequality and risk, yet they are less valued as they disproportionately advantage richer/luckier households with a lower marginal utility of consumption. We find that with CRRA utility and holding other mechanisms constant, inequality necessitates a more stringent climate policy if it decreases over time. In such cases, the policy acts to augment the relative size of the less unequal future endowments, thereby mitigating consumption inequality. However, binding borrowing constraints diminish the policy's appeal by making transferring resources to the future less attractive, as consumption is scarcer in the present. Likewise, the precautionary savings prompted by risk diminish the policy's advantages by rendering agents wealthier in the future, thus decreasing the value of future endowment gains.

The insights derived from the two-period model are valuable, yet they can only take us this far. To advance our understanding, we turn to a quantitative framework. Our infinite-horizon economy features a population of households with preferences over consumption, leisure, and climate. Household labor productivity is subject to idiosyncratic risk against which they cannot insure: they can only invest in a risk-free asset subject to a borrowing constraint. The final consumption-investment good is produced using capital, labor, and energy. Energy, in turn, is produced in a second sector using capital and labor. Energy production generates CO_2 emissions that firms can mitigate by paying abatement costs. Non-abated CO_2 emissions accumulate and cause climate change, affecting households through production and direct utility damages. Since the decision of firms to engage in abatement activities depends on the cost of CO_2 emissions to them, the government can mitigate climate change by setting a tax on emissions. In addition, the government has access to proportional taxes on capital, labor, energy production, lump-sum transfers, and debt. It uses these instruments to finance expenditures. Population and total factor productivities in both sectors grow at exogenous rates, which implies the economy is on the equilibrium balanced growth path. Within this framework, we define a Ramsey problem: choosing time-varying policy instruments to maximize welfare over transition toward a final, endogenously determined balanced growth path.

We calibrate our model to the data, setting its parameters based on the U.S. economy. To align with global scales, we adjust the U.S. economy's emissions and GDP to match those of the world, modifying the population to maintain the U.S. GDP per capita. By doing this instead of adding emissions from the rest of the world exogenously, we ensure that the U.S. planner accounts for the negative effects of its emissions abroad, as officially intended by the U.S. administration (see Section 4). In calibrating the climate component, we draw on recent advancements in integrated assessment modeling meant to better capture the impulse response of temperatures to emissions. Specifically, we use the climate model of Dietz and Venmans (2019) and calibrate it based on IPCC (2021) and Friedlingstein et al. (2022), with additional parameters taken from Barrage and Nordhaus (2023). The calibration of the macroeconomic aspects of our model is guided by the goal of replicating three sets of statistics: macroeconomic variables, inequality metrics, and measures of idiosyncratic risk. This quantitative approach ensures that our model accurately reflects the macroeconomic and environmental realities captured in the data, crucially highlighting the interconnection between these dimensions. We rely on established climate models and standard macro statistics used in macro public finance to facilitate a comparison of our findings with existing studies and current policy proposals.

Our analysis begins with a calibrated economy, serving as the foundation for quantitatively exploring optimal fiscal and climate policies. To tackle the optimal policy problem within our framework, we employ numerical techniques developed in Dyrda and Pedroni (2023), approximating the trajectories of fiscal instruments over time using orthogonal polynomials. We adopt a utilitarian welfare criterion and decompose the welfare gains from different policies into four distinct components: level, insurance, redistribution, and climate.

In an initial examination, we consider four scenarios regarding the trajectory of optimal carbon taxes based on whether the government can utilize (i) revenues from carbon taxes and (ii) government debt. The first scenario illuminates the non-environmental benefits derived from carbon taxation, while the second assesses the significance of deviations from Ricardian equivalence for optimal climate policy. Throughout this analysis, we assume that taxes on capital and labor remain constant at their current levels, a constraint we will relax in subsequent experiments.

Under our baseline scenario, where the government optimizes over the path of carbon taxes and adjusts lump-sum transfers to maintain a constant debt-to-output ratio, the carbon tax initiates at $45/t$CO_2$ and increases until it reaches the backstop price, at around $465/t$CO_2$ in 2145, marking the transition to a carbon-neutral economy. This trajectory is preceded by a gradual increase in emissions until 2050, primarily driven by GDP growth, after which emissions steadily decline towards zero as the proportion of abated emissions rises. The emissions profile remains relatively stable throughout the 21st century, leading to a nearly linear increase in atmospheric temperature, projected to stabilize at +2.6°C around 2140.

We compare our baseline scenario with alternative cases where the government allocates carbon tax revenues towards wasteful spending instead of transfers and scenarios allowing for flexible adjustments in the debt-to-output ratio. The critical insight from this analysis is the significance of the government's ability to utilize carbon tax proceeds for insurance and redistribution in shaping optimal climate policy rather than the capacity to adjust government debt levels freely.

In scenarios where carbon tax revenues are directed towards wasteful spending, regardless of whether the government can freely adjust debt, the start of the climate transition is delayed by 25 years compared to the baseline. Intriguingly, once initiated, the transition to carbon neutrality occurs much more swiftly, with the economy achieving this state by 2080 — 60 years ahead of the baseline schedule. This reduces cumulative emissions and a lower long-term temperature increase (2.3°C compared to 2.6°C in the baseline scenario). However, it is worth noting that the rise in temperature and the associated damages manifest earlier in these scenarios, leading to higher cumulative damages.

Our quantitative findings indicate that the ability to adjust debt-to-GDP ratios and the timing of lump-sum transfers during the transition to carbon neutrality exert only a limited impact on the trajectories of optimal carbon taxes and the evolution of atmospheric temperature. This observation holds irrespective of whether carbon tax revenues are allocated towards adjusting the level of transfers or financing wasteful spending. In economies with flexible debt-to-GDP ratios, the achievement of carbon neutrality, marked by hitting the backstop prices, occurs approximately ten years earlier or later (depending on fiscal closure assumptions) compared to economies with fixed debt-to-GDP ratios. However, the paths of optimal carbon taxes and the progression of atmospheric temperature show minimal differences between economies with and without the ability to adjust debt.

Instead of significantly altering climate policy, the planner utilizes flexibility in debt to advance lump-sum transfers by up to 25 percent of GDP. This strategy enhances economic efficiency by moving agents away from borrowing constraints, improving insurance and redistribution for households, thereby elevating overall welfare. Despite minor variations in climate policy, the allocation of resources differs substantially between economies with divergent debt policies. Front-loading lump-sum transfers necessitate the issuance of a significant amount of debt during the transition to carbon neutrality, which, in turn, crowds out private capital by more than 20 percent compared to scenarios with a constant debt-to-output ratio. A generous transfer policy also disincentivizes households from working, contributing to a further GDP reduction relative to fixed debt scenarios. Despite the macroeconomic slowdown, scenarios with a flexible debt policy result in higher welfare than their fixed debt counterparts. A crucial caveat to these findings is that, under our calibration, the economy features interest rates that exceed the output growth rate (i.e., $r - g > 0$) throughout the transition to carbon neutrality. The implications under an alternative calibration, where (i.e., $r - g < 0$), remain to be explored. Similarly, the full-blown Ramsey problem results, where we allow all the fiscal instruments to vary over time, are still pending.

Related literature. Our paper contributes to the literature analyzing fiscal policy in the presence of environmental externalities. In particular, it contributes to the literature studying optimal carbon taxation in second-best economies and carbon taxation in the presence of inequality and risk.

In an important paper, Golosov, Hassler, Krusell, and Tsyvinski (2014) explore a dynamic stochastic general-equilibrium (DSGE) model to understand the externality effects of fossil fuel usage on climate change. They derive a straightforward formula for the marginal externality damage of emissions that is proportional to current GDP. This formula simplifies the calculation of the optimal carbon tax, suggesting it should be slightly higher than commonly cited estimates, based on parameters from recent natural science research. A separate literature studies optimal environmental taxes in the presence of distortionary taxation in static representative-agent frameworks (e.g., Sandmo, 1975; Bovenberg and de Mooij, 1994; Bovenberg and van der Ploeg, 1994; Bovenberg and Goulder, 1996). Barrage (2020) brings these two strands of literature together by studying optimal climate policy in a dynamic general-equilibrium model with distortionary taxation. The main takeaway is that, in the presence of tax distortions, the optimal tax on carbon is lower than the social cost of carbon (SCC).

Another stream of papers study optimal pollution taxation with distortionary taxes and heterogeneous agents (e.g., Kaplow, 2012; Jacobs and de Mooij, 2015; Jacobs and van der Ploeg, 2019). When tax distortions arise as an optimal response to inequality, it is generally no longer optimal to deviate from the Pigouvian principle; i.e., tax distortions do not justify taxing carbon below its social cost. Douenne, Hummel, and Pedroni (2023) generalize this result to a dynamic environment: in a climate-economy model based on Barrage (2020), they theoretically show that optimal carbon taxes are on average Pigouvian, and they find that temporary deviations from the SCC are quantitatively negligible. They also find that inequalities reduce the SCC by making consumption relatively more valuable, even though the effect does not appear to be quantitatively large. Since this literature focuses on complete market economies to obtain theoretical results, little is known about the effect of uninsurable idiosyncratic risk on optimal climate policy. One notable exception is Belfiori and Macera (2023) who study climate policy with inequality and incomplete markets, though with a different focus: they consider a planner solving a constrained efficiency problem à la Davila, Hong, Krusell, and Ríos-Rull (2012) and study the optimal level of carbon capture across regions, abstracting from other fiscal instruments.

We contribute to this literature by quantifying optimal carbon taxes in an economy featuring uninsurable idiosyncratic income risk. In this setting, the planner has an additional mandate of providing insurance to households via adjustments in taxes, particularly through capital taxes and transfers. This leads to further distortions in the economy which, depending on the instruments available to the planner, may also affect the optimal path of climate policies. We provide a comprehensive quantitative analysis of these effects under different policy scenarios, various sources of household heterogeneity, and alternative calibrations of our model.

An abundant literature also studies the distributional effects of carbon taxation (for recent examples, see van der Ploeg et al., 2022; Känzig, 2023). Close to our work, several papers have recently studied this question within heterogeneous agents incomplete markets models. Fried, Novan, and Peterman (2018) study the distributional effects of exogenous carbon tax reforms in an OLG economy with heterogeneous households exposed to idiosyncratic productivity shocks. In a follow-up paper, Fried, Novan, and Peterman (2021) search for the optimal revenue-recycling of an exogenous carbon tax reform. To manage the complexity of this problem, they abstract from the climate-economy interaction and focus on the steady state. In contrast, our approach allows us to study a setting in which the planner chooses policy instruments to maximize social welfare, enabling us to examine the optimal carbon tax and its interplay with concerns for redistribution and insurance. Our method is also well-suited to analyze the transition, and our climate model permits a comprehensive welfare analysis of climate policies. Another recent contribution to this literature is Benmir and Roman (2022) who study the distributional effects of implementing the net-zero target in the U.S. based on a HANK framework. Their economy accounts for multiple frictions, but they only consider exogenous policies.

We also contribute to the macro public finance literature, which focuses on taxation and optimal policy design in economies with incomplete markets. Early seminal works in this area, exploring fiscal policy under incomplete markets, include studies by Conesa and Krueger (1999), Domeij and Heathcote (2004), Heathcote (2005), and Conesa, Kitao, and Krueger (2009). Research on optimal policies in this field has typically focused on analyzing once-and-for-all policy changes, with Boar and Midrigan (2022) and Kina, Slavík, and Yazici (2023) being among the most recent examples. Only recently have studies started to examine dynamic, optimal policies, as seen in Krueger and Ludwig (2016), Itskhoki and Moll (2019), and Dyrda and Pedroni (2023). We contribute to this strand of research by studying the optimal, time-varying carbon taxation in an economy blending together a state-of-the-art macroeconomic framework with a cutting-edge climate model.

The remainder of the paper is structured as follows. Section 2 presents a simple two-period model to illustrate the main mechanisms at play. Section 3 lays out the infinite-horizon model, the planning problem, and the solution method. Section 4 details our calibration. Section 5 presents our main quantitative results. Finally, Section 6 concludes.

2 Two-period model

To illustrate the mechanisms driving the results of our infinite-horizon model, we first present a simple two-agent two-period example. To keep things as simple as possible, we consider a partial equilibrium endowment economy, and we abstract from modeling the environmental externality. Our simple model is based on the idea that the cost of carbon taxation is forgone consumption, and the benefits are the provision of a public good (from direct utility benefits) and higher future endowments (from the mitigation of production damages).

2.1 Model features

The model features 2 periods $t \in \{1, 2\}$, 2 types of households $i \in \{L, H\}$, two states of the world $j \in \{l, h\}$, and a government. L corresponds to the (poor) agent with a lower expected endowment, and H corresponds to the (rich) agent with a higher expected endowment. In the absence of aggregate risk, agents receive perfectly negatively correlated shocks. We denote by l the state of the world where the poor agent receives a positive shock, and by h the state of the world where the rich agent receives a positive shock. We also assume that the expected value of the shocks is zero for both agents.

Preferences Households value both private and public consumption, respectively denoted c and G. We assume that their preferences over these two goods are additively separable, i.e. that their utility can be written as

$$U(c, G) = u(c) + V(G),$$

with $u_c > 0$, $u_{cc} < 0$, $V_G \geq 0$, and $V_{GG} \leq 0$. When useful, we will refer to the following CRRA utility function,

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma},$$

where the parameter σ corresponds to the inverse of the IES. When not specified, the results are provided for a more general time-additive utility function.

Endowments There is no production: in the first period, a proportion p_i of households of type i receives an endowment $\omega_{1,i}$, with $\omega_{1,H} \geq \omega_{1,L}$. In the second period, households of type i receive $g(\tau) \times \omega^j_{2,i}$ with $\mathbb{E}_j\left[\omega^j_{2,H}\right] \geq \mathbb{E}_j\left[\omega^j_{2,L}\right]$, where $\mathbb{E}_j$ denotes the expectation with respect to the shocks, and where $g(\tau)$ scales the size of the aggregate endowment in the second period as a function of the policy. Households can save their endowment in the first period, and their savings a_i is remunerated at a fixed rate r. The period discount rate is denoted β, and for simplicity, we assume that $\beta(1+r) = 1$.

Government The government has access to a policy instrument τ that has three effects. First, this instrument is modeled as a consumption tax that applies uniformly to both agents and both periods, and therefore reduces private consumption. Second, the revenue from this tax finances the public good G. Third, a higher tax leads to higher endowments in the second period through the function $g(\tau)$, with $g(0) = 1$, $g_\tau(\tau) \geq 0$, and $g_{\tau\tau}(\tau) \leq 0$. This approach provides a reduced-form version of a more sophisticated model where a carbon tax leads to costly pollution abatement, thereby reducing consumption but leading to higher future production through the mitigation of production damages and higher utility through a cleaner environment.

2.2 Optimal policy without borrowing contraints

In this section, we consider the case where households are allowed to borrow in period 1. We study the implications of borrowing constraints in Section 2.3.

Definition *A competitive equilibrium is* (a_L, a_H, τ, G) *such that*

(*i*) *for* $i \in \{L, H\}$, $j \in \{l, h\}$, a_i *solves*

$$\max_{a_i} u(c_{1,i}) + \beta \mathbb{E}_j[u(c_{2,i})^j] + V(G),$$

subject to

$$\begin{aligned}(1+\tau)c_{1,i} &= \omega_{1,i} - a_i,\\ (1+\tau)c^j_{2,i} &= (1+r)a_i + g(\tau)\omega^j_{2,i},\end{aligned}$$

(*ii*) *and the government budget constraint holds, i.e.*

$$G = \tau\Big(C_1 + \frac{C_2}{1+r}\Big),$$

with $C_1 = p_L c^j_{1,L} + p_H c^j_{1,H}$ *and* $C_2 = p_L c^j_{2,L} + p_H c^j_{2,H}$.

The Ramsey problem is to choose a_L, a_H, τ, and G to maximize welfare subject to equilibrium conditions. To better highlight the numerous mechanisms driving the optimal policy, we separately consider the case where the policy serves to finance a public good (Section 2.2.1), and where it serves to generate future endowment gains (Section 2.2.2).

2.2.1 Public good provision (utility damages)

When the policy only serves to finance a public good, the optimal policy corresponds to the following Samuelson rule:

$$V_G(G) = \frac{1}{C}\mathbb{E}_i\Big[c_{1,i}u_c(c_{1,i}) + \beta\mathbb{E}_j\big[c^j_{2,i}u_c(c^j_{2,i})\big]\Big], \tag{1}$$

where $C \equiv C_1 + \frac{C_2}{1+r}$ denotes aggregate consumption, and $\mathbb{E}_i$ denotes the cross-sectional expectation. Propositions 1 and 2 state the effect of risk and inequality on the optimal policy (see their proofs in Appendix A).

Proposition 1 (***Public good provision with inequality***) *When there is no risk but households are ex-ante unequal, the optimal public good policy is given by*

$$V_G(G) = \mathbb{E}_i\big[u_c(c_i)\big] + \frac{\text{cov}_i(c_i, u_c(c_i))}{c}, \tag{2}$$

where c_i denotes agent i's (constant) per period consumption, and $c \equiv \frac{C}{1+\frac{1}{1+r}}$ denotes the aggregate per period consumption. The optimal provision of public goods is affected by inequality in two opposite ways: i) inequality decreases public good provision by increasing households average marginal utility of private consumption, and ii) it increases it because the public good indirectly provides redistribution. When utility is CRRA, the net effect of inequality on the level of public good is negative if $\sigma > 1$, positive if $\sigma < 1$, and null if $\sigma \to 1$.

Proposition 2 (***Public good provision with risk***) *When households are ex-ante identical but face idiosyncratic endowment risk in the second period, the optimal public good policy is given by*

$$V_G(G) = \mathbb{E}_j\big[u_c(c^j_2)\big] + \beta\frac{\text{cov}_j(c^j_2, u_c(c^j_2))}{C}. \tag{3}$$

The optimal provision of public good is affected by risk in three ways: i) risk decreases public good provision by increasing households expected marginal utility of private consumption, and ii) it increases it because the public good indirectly provides insurance. In addition, iii) risk also induces precautionary savings that mitigate these two mechanisms by increasing the expected value and reducing the uncertainty of future consumption. When utility is CRRA, the net effect of risk on the level of public good is negative if $\sigma > 1$, positive if $\sigma < 1$, and null if $\sigma \to 1$.

Propositions 1 and 2 are reminiscent of Proposition 3 in Douenne et al. (2023). Inequality and risk affect the optimal provision of public good through the distributions of consumption and—by implication—of the marginal utility of consumption. The underlying mechanisms can be understood as an income and a substitution effect. On the one hand, as inequality and risk increase, the average marginal utility of private consumption increases because the benefits of consuming more increase more for poor/unlucky households than it decreases for rich/lucky households: this is an income effect, consumption is in a sense more scarce, and therefore more desirable. On the other hand, when the private good is more unequal or more risky, substituting it with the equal and safe public good is more desirable: this is a substitution effect, financing the public good is a way to provide redistribution and insurance.

In addition, risk also affects the optimal provision of public good through its effect on household decisions: precautionary savings (which occur assuming $u_{ccc} \geq 0$) mitigate the drop in future consumption and reduce future consumption risk, which attenuates the previous two mechanisms.

2.2.2 Endowment gains (production damages)

When the policy only serves to generate future endowment gains, the optimal policy is given by the following formula:

$$g_\tau(\tau) = \frac{\mathbb{E}_i\Big[c_{1,i}u_c(c_{1,i}) + \beta\mathbb{E}_j\big[c^j_{2,i}u_c(c^j_{2,i})\big]\Big]}{\beta\mathbb{E}_i\Big[\mathbb{E}_j\big[\omega^j_{2,i}u_c(c^j_{2,i})\big]\Big]}. \tag{4}$$

Thus, while the costs of the policy given by the numerator of (4) are the same as in the public good provision case (see equation (1)), the benefits are not. In particular, risk and inequality further affect the optimal policy through the denominator of (4). Propositions 3 and 4 characterize the additional channels through which inequality and risk affect the optimal policy in this case (see their proofs in Appendix A).

Proposition 3 (***Endowment gains with inequality***) *When there is no risk but households are ex-ante unequal, the optimal mitigation policy is given by*

$$g_\tau(\tau) = \frac{C \times \mathbb{E}_i\big[u_c(c_i)\big] + (1 + \frac{1}{1+r}) \times \mathrm{cov}_i(c_i, u_c(c_i))}{\beta\Big(\omega_2\mathbb{E}_i\big[u_c(c_i)\big] + \mathrm{cov}_i(\omega_{2,i}, u_c(c_i))\Big)}. \tag{5}$$

Inequality affects both the opportunity cost and the benefits of the policy. The effect of inequality on the opportunity cost of the policy is given by Proposition 1. The effect of inequality on the benefits of the policy is determined by the denominator

$$\mathcal{D} \equiv \beta\Big(\omega_2\mathbb{E}_i\big[u_c(c_i)\big] + \mathrm{cov}_i(\omega_{2,i}, u_c(c_i))\Big). \tag{6}$$

Holding the opportunity cost of the policy constant, inequality affects the policy: i) positively by increasing households average marginal utility of private consumption, and ii) negatively as rich households with low marginal utility of consumption experience a larger fraction of endowment gains. In addition, iii) when utility is CRRA, the benefits—and the level—of the policy are higher if and only if endowment inequality is lower in the second than in the first period.

As in Proposition 1, mechanisms *i*) and *ii*) can be understood as an income and a substitution effect. The intuition behind mechanism *iii*) is that the cost of the consumption tax is proportional to households' consumption, while the benefits are proportional to their endowment in period 2: if inequality is decreasing, poor households benefit proportionally more than rich households. In other words, when inequality is decreasing over time, taxing consumption to increase future endowments is an indirect way to provide redistribution by increasing the relative share of future (less unequal) income.

One reason why inequality can make reallocating endowments to later periods more valuable is that poor households can borrow, and benefit as of period 1 from future endowment gains. In section 2.3, we examine what happens when borrowing is not possible for poor households.

Proposition 4 **(*Endowment gains with risk*)** *When households are ex-ante identical but face idiosyncratic endowment risk in the second period, the optimal mitigation policy is given by*

$$g_\tau(\tau) = \frac{C \times \mathbb{E}_j\left[u_c(c_2^j)\right] + \text{cov}_j(c_2^j, u_c(c_2^j))}{\beta\Big(\omega_2 \mathbb{E}_j\left[u_c(c_2^j)\right] + \text{cov}_j(\omega_2^j, u_c(c_2^j))\Big)}. \tag{7}$$

Risk affects both the opportunity cost and the benefits of the policy. The effect of risk on the opportunity cost of the policy is given by Proposition 2. The effect of risk on the benefits of the policy is determined by the denominator,

$$\mathcal{D} \equiv \beta\Big(\omega_2 \mathbb{E}_j\left[u_c(c_2^j)\right] + \text{cov}_j(\omega_2^j, u_c(c_2^j))\Big). \tag{8}$$

Holding the opportunity cost of the policy constant, risk affects the policy: i) positively by increasing households expected marginal utility of private consumption, and ii) negatively as lucky households with low marginal utility of consumption experience a larger fraction of endowment gains. In addition, iii) risk induces precautionary savings that reduce the benefits—and the level—of the policy by reducing the utility value of future endowment gains.

The net effect of risk on equation (8) is ambiguous. Its sign depends on the curvature of the utility function, but also on the extent to which endowment shocks are self-insured and on the relative size of period 2's endowment (i.e., to what extent shocks to $\omega_{2,i}$ are passed on to $c_{2,i}$). All these forces affect the trade-off between the income and substitution effects from mechanisms *i*) and *ii*).

2.3 Optimal policy with borrowing contraints

We now assume that households' savings decisions are subject to a borrowing constraint,

$$a_i \geq 0, \quad \forall i.$$

Let's consider the case where there is no uncertainty but agents are ex ante heterogeneous. In particular, let's assume that for all values of τ, $\omega_{1,L} < g(\tau)\omega_{2,L}$, and $\omega_{1,H} > g(\tau)\omega_{2,H}$, so that only type L is constrained.

Propositions 5 and 6 state the effect of the borrowing constraint on the optimal policy when the policy is used for public good provision and when it is used for endowment gains (see their proofs in Appendix A).

Proposition 5 (***Public good provision with inequality and borrowing constraints***) *When there is no risk but households are ex-ante unequal and poor households are borrowing constrained, the optimal public good policy is given by*

$$V_G(G) = \mathbb{E}_i\big[u_c(c_{2,i})\big] + \frac{\mathrm{cov}_i(c_i, u_c(c_{2,i}))}{c} + \frac{1}{C}\mathbb{E}_i\big[c_{1,i}\big(u_c(c_{1,i}) - u_c(c_{2,i})\big)\big], \tag{9}$$

where the last term illustrates that the borrowing constraint adds inequality between periods for a given household. When utility is CRRA, the effect of a borrowing constraint on the level of public good is negative if $\sigma > 1$, positive if $\sigma < 1$, and null if $\sigma \to 1$.

Thus, the presence of a borrowing constraint prevents poor households from smoothing their consumption. This leads to further consumption inequality, as consumption is now also unequal for a given household between periods. The way this additional form of inequality affects the optimal public good policy is similar to the mechanisms presented in Proposition 1.

Proposition 6 (***Endowment gains with inequality and borrowing constraints***) *When there is no risk but households are ex-ante unequal and poor households are borrowing constrained, the optimal mitigation policy is given by*

$$g_\tau(\tau) = \frac{\mathbb{E}_i\big[u_c(c_{2,i})\big] + \frac{\mathrm{cov}_i(c_i, u_c(c_{2,i}))}{c} + \frac{1}{C}\mathbb{E}_i\big[c_{1,i}\big(u_c(c_{1,i}) - u_c(c_{2,i})\big)\big]}{\beta\mathbb{E}_i\Big[\omega_{2,i}u_c(c_i) + \omega_{2,i}\big(u_c(c_{2,i}) - u_c(c_i)\big)\Big]}. \tag{10}$$

The presence of a borrowing constraint affects both the opportunity cost and the benefits of the policy. The effect of the borrowing constraint on the opportunity cost of the policy is given by Proposition 5. The effect of the borrowing constraint on the benefits of the policy is determined by the denominator

$$\mathcal{D} \equiv \beta\Big(\omega_2\mathbb{E}_i\big[u_c(c_i)\big] + \mathrm{cov}_i(\omega_{2,i}, u_c(c_i))\Big) + \beta\mathbb{E}_i\Big[\omega_{2,i}\big(u_c(c_{2,i}) - u_c(c_i)\big)\Big],$$

where the second term captures the effect of the borrowing constraint and is negative. Thus, tightening the borrowing constraint leads to a lower policy level through this channel.

The intuition behind the last mechanism is that when poor households are constrained in their borrowing, they are richer in the future than in the present. Thus, their willingness to pay for future consumption improvements is reduced. In the context of climate change, if poor households expect to be richer in the future but cannot smooth their consumption, then climate mitigation efforts are less valuable to them because those efforts transfer resources in periods where those resources are less valued.

To sum up, inequality, risk, and the presence of borrowing constraints have ambiguous effects on the optimal climate policy as they affect it through multiple channels that can play in opposite directions. In the next section, we present an infinite-horizon model to study these questions quantitatively.

3 Infinite-horizon model

In this section, we set up the infinite-horizon model where households face uninsurable idiosyncratic income risk and where the government aims to provide redistribution, insurance, and to address climate change. The presentation follows Dyrda and Pedroni (2023) to which we add an energy sector, climate change, and carbon taxation as in Barrage (2020) and Douenne et al. (2023).

3.1 Households

Time is discrete and the horizon is infinite. Population grows at an exogenous rate of n_t. Households have preferences over consumption c_t, labor h_t, and the climate Z_t,

$$\mathbb{E}_0\left[\sum_t \beta^t u(c_t, h_t, Z_t)\right]. \tag{11}$$

Households' labor productivity is denoted by $e \in E$ with $E \equiv \{e_1, ..., e_L\}$, and follows a Markov process with transition matrix Γ. The only asset available to households is a risk-free one, denoted a, which can take values in $A \equiv [\underline{a}, \infty)$. Thus, households are indexed by $(a, e) \in S$, with $S \equiv A \times E$.

Given a sequence of taxes on labor and capital income, $\{\tau_t^h, \tau_t^k\}_{t=0}^{\infty}$, transfers $\{T_t\}_{t=0}^{\infty}$, and prices $\{r_t, w_t, R_t\}_{t=0}^{\infty}$, with $R_t \equiv 1 + (1 - \tau_t^k)(r_t - \delta)$, in each period t each household chooses how much to consume, $c_t(a, e)$, work $h_t(a, e)$, and save, $a_{t+1}(a, e)$, to solve

$$v_t(a, e) = \max_{c_t, h_t, a_{t+1}} u\big(c_t(a, e), h_t(a, e), Z_t\big) \; + \beta \sum_{e_{t+1} \in E} v_{t+1}\big(a_{t+1}(a, e), e_{t+1}\big)\Gamma_{e, e_t+1} \tag{12}$$

subject to

$$(1 + \tau^c)c_t(a, e) + a_{t+1}(a, e) = (1 - \tau_t^h)w_t e h_t(a, e) + R_t a_t + T_t, \tag{13}$$

$$a_{t+1}(a, e) \geq \underline{a}, \tag{14}$$

where τ^c is an exogenous and constant consumption tax.[2]

We denote aggregate consumption and hours worked with capital letters,

$$C_t = \int_S c_t(a, e) d\lambda_t, \quad H_t = \int_S h_t(a, e) d\lambda_t, \tag{15}$$

with $\{\lambda_t\}_{t=0}^{\infty}$ a sequence of probability measures defined over the Borel sets $\mathcal{S}$ of the space S, where the initial measure λ_0 is given.

3.2 Firms

There are two production sectors, each represented by a representative firm.

[2] As in Dyrda and Pedroni (2023), we add this as a parameter for calibration purposes, but this is not an instrument for the planner.

3.2.1 Final good sector

The first sector produces the final consumption-investment good, Y_t, using a constant-returns-to-scale technology, $F(.)$, with capital, labor, and energy inputs, denoted $K_{1,t}$, $H_{1,t}$, and E_t. Final good production is subject to climate damages $D(Z_t)$, such that

$$Y_{1,t} = \big(1 - D(Z_t)\big) A_{1,t} F(K_{1,t}, H_{1,t}, E_t), \tag{16}$$

with $A_{1,t}$ the total factor productivity of the final good sector. The representative final good firm chooses capital, labor, and energy given the respective real factor prices r_t, w_t, and p_t^e, and makes zero profit. The first-order conditions are

$$r_t = \big(1 - D(Z_t)\big) A_{1,t} F_{K,t}, \tag{17}$$

$$w_t = \big(1 - D(Z_t)\big) A_{1,t} F_{H,t}, \tag{18}$$

$$p_t^e = \big(1 - D(Z_t)\big) A_{1,t} F_{E,t}. \tag{19}$$

3.2.2 Energy sector

The second sector produces energy, E_t, using a constant-returns-to-scale technology, $G(.)$, with capital and labor inputs, $K_{2,t}$ and $H_{2,t}$, both assumed to be fully mobile across sectors. Energy production is given by

$$E_t = A_{2,t} G\big(K_{2,t}, H_{2,t}\big), \tag{20}$$

with $A_{2,t}$ the total factor productivity of the energy sector. Energy production is polluting, with industrial CO_2 emissions given by

$$E_t^M = (1 - \mu_t) E_t, \tag{21}$$

where μ_t represents the fraction of energy coming from clean technologies. The cost of emission abatement is given by $\Theta_t(\mu_t) E_t$, with $\Theta_{\mu,t}, \Theta_{\mu\mu,t} > 0$, and $\Theta_t(0) = 0$. The representative firm's profits in the energy sector are given by

$$\mathcal{P}_t = (p_t^e - \tau_t^i) E_t - \tau_t^e (1 - \mu_t) E_t - w_t H_{2,t} - r_t K_{2,t} - \Theta_t(\mu_t, E_t), \tag{22}$$

with τ_t^i the excise intermediate-goods tax on total energy production, E_t, and τ_t^e the excise tax on carbon emissions, E_t^M. Since the abatement cost function is linear in E_t, profits in the energy sector are null. The representative energy firm chooses capital, labor, and abatement such that

$$r_t = \big(p_t^e - \tau_t^i - \tau_t^e(1 - \mu_t) - \Theta_{E,t}\big) A_{2,t} G_{K,t}, \tag{23}$$

$$w_t = \big(p_t^e - \tau_t^i - \tau_t^e(1 - \mu_t) - \Theta_{E,t}\big) A_{2,t} G_{H,t}, \tag{24}$$

$$\tau_t^e = \frac{\Theta_{\mu,t}}{E_t}. \tag{25}$$

3.3 Government

The government has access to proportional income taxes on capital, τ_t^k, and labor, τ_t^h, taxes on total energy production, τ_t^i, on carbon emissions, τ_t^e, as well as a fixed consumption tax, τ^c. Each period it uses these instruments to finance an exogenous stream of expenses, G_t, and lump-sum transfers, T_t. The government can also issue debt, B_{t+1}, whose sequence must remain bounded. The governments inter-temporal budget constraint is

$$G_t + T_t + R_t B_t = \tau^c C_t + \tau_t^h w_t H_t + \tau_t^k (r_t - \delta) K_t + \tau_t^i E_t + \tau_t^e E_t^M + B_{t+1}. \tag{26}$$

3.4 Climate

We use the climate model of Dietz and Venmans (2019) in order to capture two key features of climate dynamics: the temperature response to emissions is almost immediate and permanent, and temperature is almost linear in cumulative emissions.[3] This model therefore correctly approximates the impulse response of temperature to emissions, which is essential for a proper quantitative assessment of the effect of climate policies on welfare. Formally, global mean surface temperature change relative to pre-industrial levels, Z_t, follows the law of motion

$$Z_{t+1} = Z_t + \epsilon \big(\zeta \mathcal{E}_t - Z_t \big), \tag{27}$$

with ζ the transient climate response to cumulative carbon emissions (TCRE, see Dietz and Venmans, 2019), ϵ a parameter for the speed of adjustment of temperature to an emission pulse, and $\mathcal{E}_t$ the cumulative emissions that evolve as follows:

$$\mathcal{E}_{t+1} = \mathcal{E}_t + E_t^M + E_t^{\text{ex}}, \tag{28}$$

where E_t^{ex} represents exogenous land emissions.

3.5 Competitive equilibrium

A competitive equilibrium and a balanced growth path are defined in our environment as follows.

Definition 1 *Given K_0, B_0, an initial distribution λ_0, and a policy $\pi \equiv \{\tau_t^h, \tau_t^k, \tau_t^i, \tau_t^e, T_t\}_{t=0}^{\infty}$, a* ***competitive equilibrium*** *is a sequence of value functions $\{v_t\}_{t=0}^{\infty}$, an allocation $X \equiv \{c_t, h_t, a_{t+1}, Z_t, E_t, \mu_t, K_{1,t}, K_{2,t}, K_{t+1}, H_{1,t}, H_{2,t}, H_t, B_{t+1}\}_{t=0}^{\infty}$, a price system $P \equiv \{R_t, w_t, r_t, p_t^e\}_{t=0}^{\infty}$, and a sequence of distributions $\{\lambda_t\}_{t=0}^{\infty}$, such that for all t:*

1. *the allocations solve the consumers' and the firms' problems given prices and policies;*

[3] For a discussion of how these properties feature in other climate-economy models, see Mattauch et al. (2020) and Dietz et al. (2021). For further references on the temperature response to emissions over time, see Joos et al. (2013), Ricke and Caldeira (2014) and references therein. For further references on the linear relationship between cumulative emissions and temperatures, see Matthews et al. (2009), Gillett et al. (2013), or the summaries provided in IPCC (2021).

2. *the sequence of probability measures* $\{\lambda_t\}_{t=1}^{\infty}$ *satisfies*

$$\lambda_{t+1}(\mathcal{S}) = \int_S Q_t((a,e),\mathcal{S})d\lambda_t, \quad \forall \mathcal{S} \text{ in the Borel } \sigma\text{-algebra of } \mathcal{S}, \tag{29}$$

where Q_t *is the transition probability measure;*

3. *the government budget constraint* (26) *is satisfied in every period, and debt is bounded;*

4. *temperature change satisfies equation* (27) *in every period, and;*

5. *markets clear, i.e., the following equations are satisfied:*

$$H_t = H_{1,t} + H_{2,t}, \tag{30}$$

$$K_t = K_{1,t} + K_{2,t}, \tag{31}$$

$$C_t + G_t + K_{t+1} + \Theta_t(\mu_t, E_t) = \big(1 - D(Z_t)\big) A_{1,t} F(K_{1,t}, H_{1,t}, E_t) + (1-\delta)K_t, \tag{32}$$

$$E_t = A_{2,t} G(K_{2,t}, H_{2,t}), \tag{33}$$

$$H_t = \int_S e h_t(a,e) d\lambda_t, \tag{34}$$

$$K_t + B_t = \int_S a d\lambda_t. \tag{35}$$

The economy is on a ***balanced growth path*** *if all the aggregate variables grow at a constant rate and the economy satisfies competitive equilibrium conditions.*

3.6 Ramsey problem

We assume that the government announces and commits to a sequence of policies at time zero.

Definition 2 *Given* K_0, B_0, *and* λ_0, *for every policy* π, ***equilibrium allocation rules*** $X(\pi)$ *and* ***equilibrium price rules*** $P(\pi)$ *are such that* $\{\pi, X(\pi), P(\pi)\}$ *together with the corresponding* $\{v_t\}_{t=0}^{\infty}$ *and* $\{\lambda_t\}_{t=1}^{\infty}$ *constitute a competitive equilibrium. Given a welfare function* $\mathcal{W}(\pi)$, *the* ***Ramsey problem*** *is to* $\max_{\pi\in\Pi} \mathcal{W}(\pi)$ *subject to* $X(\pi)$ *and* $P(\pi)$ *being equilibrium allocation and price rules, and* Π *is the set of policies* $\pi \equiv \{\tau_t^h, \tau_t^k, \tau_t^i, \tau_t^e, T_t\}_{t=0}^{\infty}$ *for which an equilibrium exists.*

If we assume that the planner has utilitarian preferences, then the planner's objective is given by

$$\mathcal{W}(\pi) = \int_S \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \tilde{\beta}^t u\big(c_t(a_0, e_0|\pi), h_t(a_0, e_0|\pi), Z_t(\pi)\big) \right] d\lambda_0, \tag{36}$$

where $\tilde{\beta} \geq \beta$ represents the discount rate of the planner. Specifically, we follow Farhi and Werning (2007) and allow the planner to value the future more than households in order to reconcile the impatience of individual decision-makers with the more ethical approach to intertemporal welfare of the social planner.[4]

[4]This disagreement over the discount rate can be microfounded by an OLG economy where individuals attach some altruistic weight to their offspring, which the planner accounts for in addition to valuing future generations directly (see Bernheim, 1989; Farhi and Werning, 2007). For applications of this modeling to climate change economics, see e.g. Belfiori (2017), Barrage (2018), and van der Ploeg and Rezai (2021).

3.7 Solution method

When markets are complete, the optimal fiscal system can be characterized analytically using the method introduced by Werning (2007). We make use of this approach to characterize the Pigouvian tax formula, i.e. the first-best tax rule that can then be evaluated at any equilibrium allocation (see Appendix B). To compute optimal policy when markets are incomplete, we use numerical methods.

Our solution method builds on Dyrda and Pedroni (2023). To convert an infinite-dimensional Ramsey problem, defined above, into a finite-dimensional one we assume the existence of a Ramsey balanced growth path—in the long run, all optimal fiscal instruments, including government debt, grow at a constant rate and the economy settles in a new balanced growth path. To lower the dimensionality of the problem we approximate the paths of fiscal instruments in the time domain using a combination of orthogonal polynomials as follows:

$$x_t = \left(\sum_{i=0}^{m_{x0}} \alpha_i^x P_i(t)\right) \exp\left(-\lambda^x t\right) + \left(1 - \exp\left(-\lambda^x t\right)\right) \left(\sum_{j=0}^{m_{xF}} \beta_j^x P_j(t)\right), \quad t \leq t_F, \tag{37}$$

where x_t can be any of the fiscal instruments $\{\tau_t^h, \tau_t^k, \tau_t^i, \tau_t^e, T_t\}$; $\{P_i(t)\}_{i=0}^{m_{x0}}$ and $\{P_j(t)\}_{j=0}^{m_{xF}}$ are families of Chebyshev polynomials; $\{\alpha_i^x\}_{i=0}^{m_{x0}}$ and $\left\{\beta_j^x\right\}_{j=0}^{m_{xF}}$ are weights on the consecutive elements of the family; λ^x controls the convergence rate of the fiscal instrument; and t_F is the period after which the instrument becomes constant. The orders of the polynomial approximations are given by m_{x0} and m_{xF} for the short-run and long-run dynamics. With the approximation at hand, we optimize the parameters to maximize the objective function (e.g. welfare) over the transition between the balanced growth paths.

4 Mapping the Model to the Data

This section describes the details of our calibration. All parameters are summarized in Table V of Appendix C.

Our calibration is based on the U.S. economy. We follow Douenne et al. (2023) and scale up the U.S. so that its emissions and GDP are those of the world, with the population being adjusted to preserve the U.S. GDP per capita. A close alternative would be to simply consider the U.S. economy and assume that emissions from the rest of the world evolve proportionally to domestic ones. The caveat of this alternative approach is that it would lead the U.S. to ignore the negative impact of its emissions abroad and consider only the U.S. SCC.[5] Our approach is intended to ensure that policies reflect the global SCC, which is consistent with the stated objectives of the U.S. administration which claims that "It is essential that agencies capture the full costs of greenhouse gas emissions as accurately as possible, including by taking global damages into account." (Executive Order 13990 of Jan 20, 2021).

[5] It is the approach adopted by Benmir and Roman (2022) who impose an emission cap but do not study the SCC. A third approach, adopted by Barrage (2020), is to directly calibrate the model to the world economy. In this latter case, one needs to assume that there exists a global planner choosing a carbon tax as well as global income taxes and transfers.

4.1 Climate model

We calibrate the climate model of Dietz and Venmans (2019) based on IPCC (2021). We set the initial cumulative carbon emissions to $\mathcal{E}_{2020} = 2390\text{GtCO}_2$ and the initial temperature change to $Z_{2020} = 1.07$°C. We take the report's best estimate for the TCRE, at $\zeta = 0.00045$°C/GtCO$_2$. For the speed of adjustment of temperature to an emission pulse, we follow Dietz and Venmans (2019) and set $\epsilon = 0.5$.

We calibrate initial industrial and land emissions from the Global Carbon Project (Friedlingstein et al., 2022), at $E^M_{2020} = 36.33\text{GtCO}_2$/year and $E^{\text{ex}}_{2020} = 3.96\text{GtCO}_2$/year. Note that these values represent net emissions, i.e. after abatement. Land use emissions being exogenous, we set their path following DICE 2023 (Barrage and Nordhaus, 2023) and assume that gross emissions exogenously decline by 10% every five years and are abated at the same rate as industrial emissions.

4.2 Damages

We model production damages following Dietz and Venmans (2019), i.e.

$$D(Z_t) = 1 - \exp(-\frac{\alpha_1}{2} Z^2). \tag{38}$$

This exponential-quadratic specification leads to a damage curve similar to DICE 2023, although damages are higher in their calibration: with a baseline parameter $\alpha_1 = 0.01$, damages amount to 2% of output at 2°C warming, and 7.7% at 4°C warming (against 1.4% and 5.5% in DICE 2023). We base our calibration on Dietz and Venmans (2019)'s central value of $\alpha_1 = 0.01$, but we adjust this parameter to split damages between production and utility. Following Barrage (2020), we assume that 74% of damages at 2.5°C warming come from output losses, and 26% come from direct utility impacts. This leads to $\alpha_1 = 0.00737$, and it enables us to determine the parameters associated with utility damages (α_z, see below).

4.3 Households

In our model, the primary unit of analysis is a *household*, as opposed to an individual. Consequently, we measure all pertinent statistics in the data at the household level, employing the equivalence scales suggested by the US Census. Subsequently, in the context of the household problem (12), we interpret consumption, hours, and asset positions on a per-capita basis within each household. To make progress on a quantitative front, we aim to discipline preference parameters as well as the labor productivity process that the households face. We do so by targeting three sets of statistics: (i) macroeconomic variables, (ii) inequality statistics, and (iii) measures of idiosyncratic risk. We discuss our strategy in what follows.

Preferences. Households have preferences over consumption, labor, and climate in the model. We impose the following utility function:

$$u\left(c_t, h_t, Z_t\right) = \frac{\left(c^{\gamma}(1-\varsigma h)^{1-\gamma}\right)^{1-\sigma} + \left(1+\alpha_z\left(Z^2\right)\right)^{\sigma-1}}{1-\sigma} \tag{39}$$

We discipline preference parameters $\{\beta, \gamma, \sigma, \varsigma, \alpha_z\}$ as follows. First, we match a capital-output ratio of 2.6 computed from NIPA for the period 2009-2019.[6] Second, we target the intertemporal elasticity of substitution (IES) of 1/1.5; a number well within the range of estimates used in the quantitative macro literature. To discipline the labor supply margin we target the average hours worked in the entire population 0.25 and we impose that the average Frisch elasticity equals 1.0.[7] Since household-level Frisch elasticities depend on the household's labor supply, we measure the intensive-margin average Frisch elasticity with the unweighted average of household-level Frisch elasticities for employed households, that is

$$\Psi \equiv \int_{h(a,e) \geq \underline{h}} \left(\gamma + (1-\gamma)\frac{1}{\sigma}\right) \frac{1-h(a,e)}{h(a,e)} \, d\lambda_0(a,e). \tag{40}$$

Finally, the parameter α_z is set to ensure that 26% of damages are directly associated with the utility.

Labor productivity. In our model, we represent the stochastic process governing household labor productivity as a combination of two components: a persistent component, denoted as e_P, governed by a Markov matrix Γ_P, and a transitory component, e_T, defined by a probability vector P_T.[8] This process includes four persistent and six transitory productivity levels. By normalizing the average productivity to one, we are left with 26 free parameters within the labor income process.

These parameters are carefully calibrated, guided by a set of specific targets. These targets are derived from the partitioning of the population, as well as considerations of inequality and risk. The following discussion delves into these aspects, elucidating how they inform and shape the parameterization of our model.

Population. We align the partitioning of the household population in both the model and the data. Utilizing the Survey of Consumer Finances (SCF) as the data source, we categorize the population into four distinct groups: workers, business owners, retirees, and non-working households. This classification is designed to be both mutually exclusive and comprehensive.

A household is categorized as a business owner if either the head or the spouse is actively involved in business ownership, and the household's total labor income is surpassed by both its business and capital income. Retiree households are identified based on two criteria: first, both the head and the spouse must have declared retirement prior to the survey year; second, the household should not fall under the business owner category. Non-working households are those that do not qualify as business owners or retirees and have no labor income. Conversely, any household that does not fit into the aforementioned categories is classified as a worker. To streamline our analysis, we further consolidate retirees and non-working households into a single category termed 'Inactive Households', which

[6] Capital is defined as nonresidential and residential private fixed assets and purchases of consumer durables. For more details, see Appendix D.1.

[7] To obtain the average hours worked we use the Current Population Survey (CPS) and compute average annual hours worked for the entire working-age population independent of their employment status, which is 1269. Assuming that the households can work at most 100 hours per capita per week for 52 weeks in a year, we get $1269/(52 \times 100) = 0.25$.

[8] In the model's notation, $\Gamma = \Gamma_P \otimes \text{diag}(P_T)$, and $e = e_P + e_T e_P^{\eta}$. For example, if $\eta = 0$, the transitory shocks are additive, whereas if $\eta = 1$, they are multiplicative.

simplifies the demographic segmentation.

We map this categorization to the model as follows. We reserve one persistent productivity state to account for business owners, which serves as a shortcut to represent the role of entrepreneurial income (see also Dyrda and Pedroni (2023) for a similar approach). Second, we classify all households with hours worked below the threshold $\underline{h}$ as inactive households in the model. We ensure that the model matches the shares in population, earnings, income, and wealth (Table I) for these two groups. Then, residually, we classify all other households as workers.

Table I: Population Partitions: Model vs. Data

	Shares			
	Population	**Earnings**	**Income**	**Wealth**
	Workers			
Data	67.2	82.7	69.1	44.9
Model	70.9	86.3	78.7	47.0
	Business Owners			
Data	5.8	13.7	16.1	33.0
Model	6.6	13.7	14.8	31.2
	Inactive Households			
Data	27.0	3.6	14.8	22.2
Model	22.5	0.0	6.5	21.8

Notes: Data comes from 2019 wave of the SCF. Details about the definitions of subgroups of the population can be found in Appendix D.3.

Inequality and Income Risk. We focus on several key metrics related to inequality: the share of wealth, earnings, and hours owned by each quintile, the Gini coefficient, and the proportion held by the bottom and top 5% of the distribution. For wealth and earnings data, we rely on the Survey of Consumer Finances (SCF), and for hours distribution, we utilize the Current Population Survey (CPS). The efficacy of our model in meeting these specific targets is detailed in Table II. Additionally, to capture the joint distribution dynamics of earnings and wealth, we target the cross-sectional correlation between these two variables. Our approach to modeling income risk is informed by the labor income process characteristics documented in Pruitt and Turner (2020a). Leveraging their insights, we calculate and target the variance, Kelly skewness, and Moors kurtosis of labor income growth rates. In computing these labor-income moments within the model, we exclude households in the entrepreneurial state and focus on active households by conditioning on employment status.

Table II: Benchmark Model Economy: Target Statistics and Model Counterparts

(1) Macroeconomic aggregates

	Target	Model
Intertemporal elasticity of substitution	0.66	0.66
Capital to output	2.57	2.54
Average Frisch elasticity (Ψ)	1.0	1.0
Average hours worked	0.24	0.25
Transfer to output (%)	14.7	14.7
Debt to output (%)	104.5	104.5
Fraction of hhs with negative net worth (%)	10.8	11.5
Correlation between earnings and wealth	0.51	0.43

(2) Cross-sectional distributions

	Bottom (%)	Quintiles					Top (%)	Gini
	0–5	1st	2nd	3rd	4th	5th	95–100	
Wealth								
Data	−0.5	−0.5	0.8	3.4	8.9	87.4	65.0	0.85
Model	−0.2	0.1	1.7	3.6	6.7	88.1	70.0	0.85
Earnings								
Data	−0.1	−0.1	3.5	10.8	20.6	65.2	35.3	0.65
Model	0.0	0.1	3.6	12.0	17.7	66.6	37.5	0.65
Hours								
Data	0.0	2.7	13.8	19.2	27.9	36.4	11.1	0.34
Model	0.0	0.4	11.4	26.1	28.3	33.9	8.9	0.35

(3) Statistical properties of labor income

	Target	Model
Variance of 1-year growth rate	2.33	2.32
Kelly skewness of 1-year growth rate	−0.12	−0.13
Moors kurtosis of 1-year growth rate	2.65	2.65

4.4 Production

We assume that both sectors feature a Cobb-Douglas technology, and we parametrize the production functions as in Douenne et al. (2023), i.e.

$$F(K_{1,t}, H_{1,t}, E_t) = K_{1,t}^{\alpha} H_{1,t}^{1-\alpha-\nu} E_t^{\nu}, \tag{41}$$

$$G(K_{2,t}, H_{2,t}) = K_{2,t}^{\alpha_E} H_{2,t}^{1-\alpha_E}, \tag{42}$$

with $\alpha = 0.3$, $\nu = 0.04$ (from Golosov et al., 2014), and $\alpha_E = 0.597$ (from Barrage, 2020). We calibrate initial total factor productivities to match global GDP (from the World Bank) and aggregate industrial emissions (from Friedlingstein et al., 2022), and we borrow their growth rates from DICE 2023. Finally, we adapt the abatement cost function from DICE 2023, so that

$$\Theta(\mu_t)E_t = P_t^{\text{back}} \frac{\mu_t^{c_2}}{c_2} E_t, \tag{43}$$

with $c_2 = 2.6$, and P_t^{back} the backstop price that starts at 696.2\$/tCO_2 in 2020 and declines by 1% per year until 2050 and by 0.1% per year thereafter.

4.5 Fiscal Policy

We calibrate consumption, labor, and capital taxes by extending the analysis and measurements presented in Trabandt and Uhlig (2011) up to 2019 (detailed in Appendix D.4), setting these taxes to their average levels between 2015 and 2019. This approach results in an initial capital tax, τ_t^k, of 33.6 percent, an initial labor income tax, τ_t^l, of 27.7 percent, and an initial consumption tax, τ_t^c, of 4.2 percent. The initial tax on total energy production, τ_t^i, is set at 0.0 percent, and the initial carbon emission tax, also denoted as τ_t^e, is set at 0.6 percent. We ensure that the government's debt-to-GDP ratio aligns with the 2019 level of 104.5 percent in the initial balanced growth path. Our model's lump-sum transfer is mapped to personal transfer receipts in the National Income and Product Accounts (NIPA), encompassing social security, Medicare, Medicaid, and unemployment insurance payments. This mapping is justified as we model retired and unemployed households as unproductive, and in our framework, lump-sum transfers represent a baseline income for those not working. Consequently, we set the lump-sum transfer to GDP ratio at 14.7 percent, as detailed in Appendix D.1.

5 Main results

In this section with present quantitative results on the optimal path of climate policy (Section 5.1) and its welfare implications (Section 5.2).

5.1 Optimal climate policy

Scenarios Below we consider four scenarios for the path of optimal carbon taxes assuming the government can or cannot use (i) the revenues from carbon taxes and (ii) government debt. The first gives a sense of the importance of the non-environmental gains from carbon taxes. The second sheds light on how important inequality, risk, and a failure of Ricardian equivalence is for optimal climate policy. Throughout we maintain the assumption that capital and labor taxes are fixed at their current levels, an assumption that we will relax in future experiments.

Baseline In the baseline scenario, depicted by the black curve in all figures, the government optimizes carbon taxes subject to the constraint that the lump-sum transfer adjusts each period so that the debt-to-output ratio remains constant. Figure 1 plots the path of optimal carbon taxes in this scenario. The carbon tax starts at \$45/tCO_2 and increases relatively fast, at a higher rate than the growth of GDP, in contrast to Golosov et al. (2014). When the backstop price is reached, at around \$465/tCO_2 in 2145, the economy becomes carbon neutral. This is preceded by a slow increase of emissions until 2050—driven primarily by an increase in GDP (see Figure 6 in Appendix E)—after which emissions slowly converge towards zero as the share of abated emissions increases (see Figure 2a). The relatively flat profile of emissions over the 21st century results in a close to linear increase in atmospheric temperature, that converges to +2.6°C around 2140 (see Figure 2b).

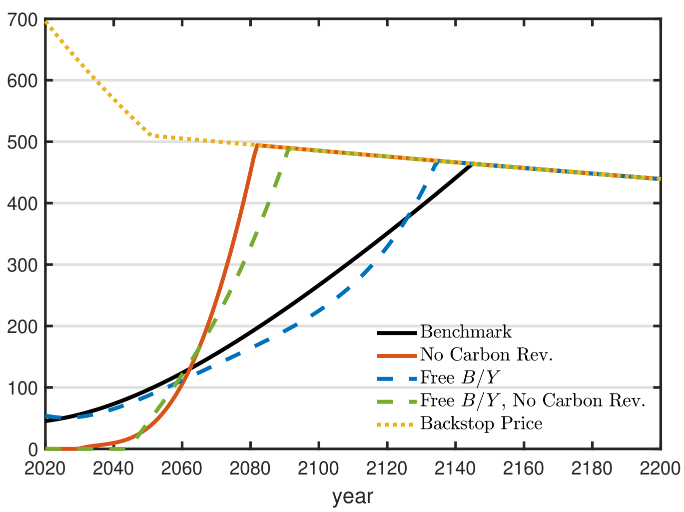
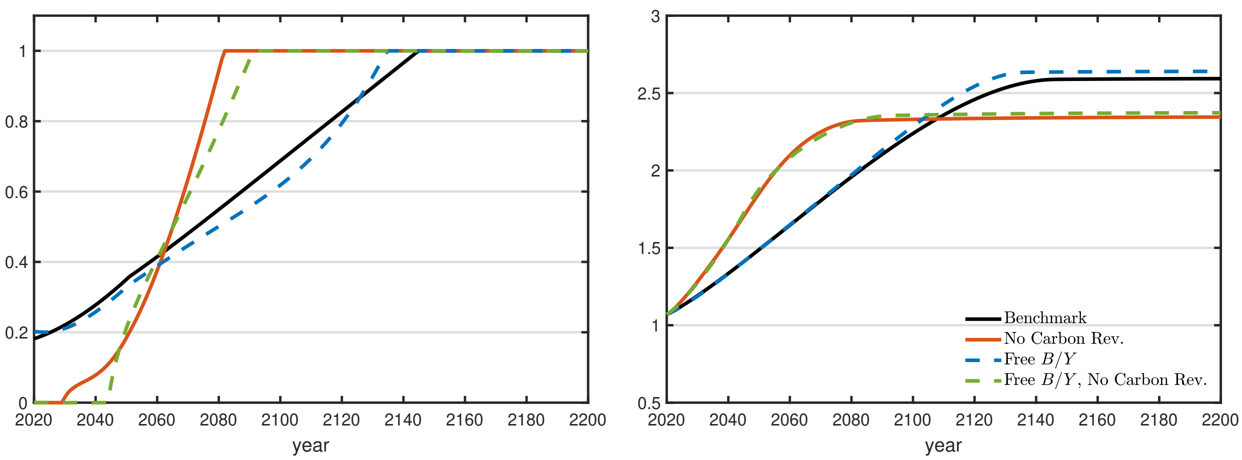


Figure 1: Optimal Carbon Taxes and Backstop Price (in \$/tCO_2).

Notes: This figure plots the paths of optimal carbon taxes in the baseline where all instruments are fixed and the carbon tax revenue is redistributed lump-sum (black), when debt/GDP is allowed to change (blue), and when the revenue is thrown away (red). All emissions are abated (i.e., $\mu = 1$) when the carbon tax attains the level of the backstop price.

Effect of revenue use The second scenario (red curves) maintains a constant debt-to-output ratio, but in this case, the revenues from carbon taxes are used to finance an increase in wasteful government spending. While the revenue generated by carbon taxation is not very large as a share of GDP—see the difference in transfers between red and black curves on Figure 3—throwing it away has a very large effect on the optimal path of climate policy. Indeed, the inability to compensate higher energy prices with an increase in transfers shifts all the benefits of climate policy to the future. Figure 1 shows that in this case, the planner finds it optimal to delay the climate transition: the carbon tax remains very close to 0 for about 25 years. As illustrated in our two-period model, the effect of discounting on optimal climate policy is further exacerbated by borrowing constraints that make current households poorer than future households, a mechanism that is alleviated by the increase in transfers in the baseline. Interestingly, while the climate transition is postponed, the overall ambition is not reduced when the government cannot use the revenues from carbon taxation. After staying close to zero for about 25 years, carbon taxes increase very rapidly to limit the rapid increase in temperatures, and hit the backstop price already around 2080. The economy thus reaches carbon neutrality sooner compared to the case where the government can use the revenues from carbon taxation to provide insurance and redistribution. The result is a lower level of cumulative emissions and lower long-run temperature increases (2.3°C compared to 2.6°C in the baseline), though the temperature increase and corresponding damages occur earlier (see Figure 2b).[9]



(a) Share of industrial emissions abated (in %).

(b) Atmospheric Temperature Change (in °C).

Figure 2: Abatement and Temperature Change.

Effect of debt In the final two scenarios (depicted by the blue and green dashed curves), the government can optimize over the path of both carbon taxes and lump-sum transfers, thus allowing the debt-to-output ratio to vary over time. The third scenario (blue) assumes the government can use the proceeds from carbon taxes, whereas in the fourth scenario (green) these revenues are again used to finance an increase in wasteful government spending. Comparing these scenarios with the two previous cases where the debt-to-output ratio was fixed, we find that allowing the government to choose the path of lump-sum transfers leads to lower optimal carbon taxes, though the effect is much smaller compared to the effect of throwing away the revenues from carbon taxation.

The finding that optimal climate policy is hardly affected by adjustments in the debt-to-output ratio is rather striking, since the economies with and without optimal adjustments to lump-sum transfers and government debt look vastly different. Figure 3 illustrates this point. When the government can

[9] Despite lower long run temperatures, the higher costs of carbon taxation in this scenario imply that the discounted sum of climate damages is higher than in the baseline.

jointly optimize over carbon taxes and lump-sum transfers (hence, is not restricted to maintain a constant debt-to-output ratio), then it becomes optimal to issue large amounts of debt in order to finance substantial transfers. This policy is a way for the planner to provide redistribution and relax households' borrowing constraints, as in Dyrda and Pedroni (2023). The large transfers lead to a sizable reduction in labor supply through wealth effects, as can be seen from Figure 7a in Appendix E. In addition to labor supply, the capital stock is also much smaller when the government can freely adjust government debt, see Figure 7b in Appendix E. This result is driven by a combination of factors: lower output due to a reduction in labor supply, higher government debt crowding out private capital, and reduced precautionary savings due to substantially larger transfers.

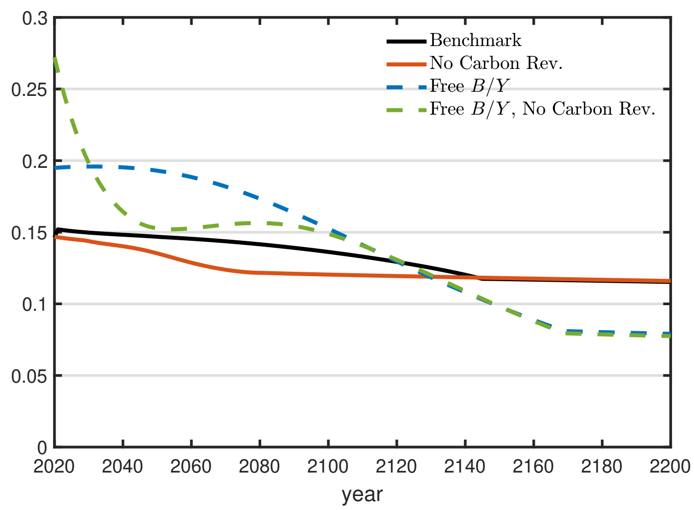


Figure 3: Lump-sum Transfers to GDP Ratio.

Thus, the failure of Ricardian equivalence has only modest effect on optimal climate policy. Put differently, carbon taxation cannot be used as a substitute for debt in raising large amounts of revenues with the aim of relaxing households' borrowing constraints.

Comparison with Pigouvian taxes Figure 4 below plots the ratio of optimal carbon taxes over the Pigouvian taxes, defined as the first-best tax rules evaluated at the equilibrium allocations (see Appendix B). In the scenarios where the government redistributes the carbon tax revenue lump-sum, optimal carbon taxes are consistently below their Pigouvian counterparts, with average ratios slightly below 70% over the 21st century. When the government uses the carbon tax revenue to finance wasteful spending, the optimal tax starts at close to 0% of the Pigouvian level, to increase to about 50% above at the end of the century. Thus, when other fiscal instruments are fixed at sub-optimal levels, the optimal carbon tax may deviate from the Pigouvian rate. In a model with complete markets, Douenne et al. (2023) show that when income taxes are exogenously set below their optimal value, it is optimal to tax carbon below its social cost. In future experiments, we will investigate whether a similar logic holds with incomplete markets.

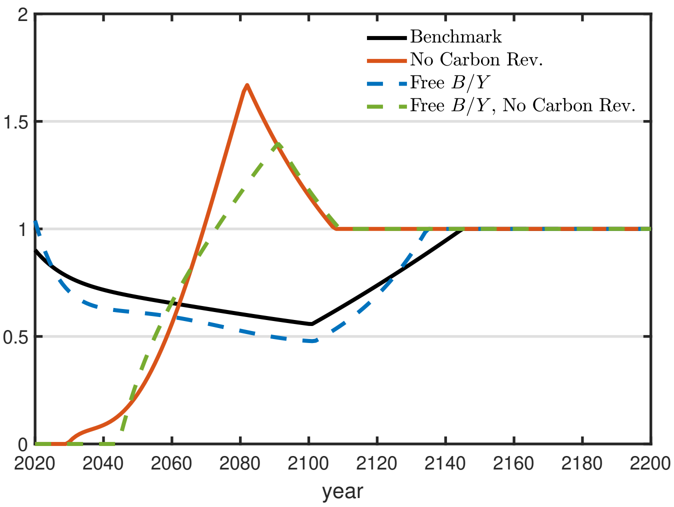


Figure 4: Optimal Carbon Taxes relative to Pigouvian Taxes.

Notes: This figure plots the ratio of optimal carbon taxes over Pigouvian taxes (defined as the first-best tax rule evaluated at the equilibrium allocation) in the baseline where all instruments are fixed and the carbon tax revenue is redistributed lump-sum (black), when debt/GDP is allowed to change (blue), and when the revenue is thrown away (red).

5.2 Welfare

To investigate the welfare implications of the policy scenarios studied above, we compare them to an economy similar to our benchmark—i.e., with income taxes and the debt-to-output ratio fixed at their current levels—where the planner does not optimize the carbon tax but instead sets it at the Pigouvian level and either redistributes the proceeds lump-sum or uses them to finance an increase in wasteful spending. Table III shows that all our scenarios lead to welfare gains relative to this counterfactual. The total welfare gains (Δ) are naturally larger when the government is not restricted to maintain a constant debt-to-output ratio (columns 3 and 4), by about 2.5 percentage points in consumption equivalents. While the ability to use debt has much smaller implications for the optimal path of climate policy than the ability to use the carbon tax revenue, it has significantly larger effects on welfare.

Table III additionally provides a decomposition of welfare gains. Following Dyrda and Pedroni (2023), we decompose the total gains into components associated to the level of aggregate variables (Δ_L), insurance (Δ_I), and redistribution (Δ_R). Additionally, we compute gains associated with the change in climate damages impacting utility directly (Δ_C). Formally, we have

$$1 + \Delta = (1 + \Delta_L)(1 + \Delta_I)(1 + \Delta_R)(1 + \Delta_C). \tag{44}$$

In all scenarios, the bulk of the welfare gain is driven by the level component, which indicates an overall improvement in efficiency. In addition, in all four scenarios optimal carbon taxes have distributional benefits as well. Interestingly, however, compared to a setting where taxes are set at their sub-optimal Pigouvian levels, households face additional risk when carbon taxes are optimized, leading to a welfare loss from insurance, except in the case where the government can issue debt and uses the proceeds from carbon taxes to finance larger transfers. Finally, we find negligible welfare losses from the mitigation

Table III: Welfare Decomposition.

	Scenarios			
	Benchmark	No Carbon Rev.	Free B/Y	Free B/Y, No Carbon Rev.
Δ	0.17%	0.78%	2.66%	3.43%
Δ_L	0.13%	0.68%	1.69%	2.36%
Δ_I	-0.06%	-0.30%	0.09%	-0.17%
Δ_R	0.09%	0.39%	0.85%	1.20%
Δ_C	-0.01%	-0.01%	-0.01%	-0.01%

Notes: Total welfare gains (Δ) and their decomposition in components associated to levels (Δ_L), insurance (Δ_I), redistribution (Δ_R), and climate utility (Δ_C). Production gains from climate change mitigation are capture by the "level" component. Welfare gains are computed in terms of consumption equivalent, with the benchmark being a scenario where the planner sets carbon taxes at the Pigouvian level and uses the revenues to finance lump-sum transfers (Columns 1 and 3) or wasteful spending (Columns 2 and 4).

of climate utility damages relative to the Pigouvian counterfactual. It should be noted that these gains (Δ_C) do not account for the production benefits of mitigation, which are captured by the level gains (Δ_L), and represent the largest share of climate impacts at low levels of warming.[10]

6 Conclusion

This paper studies the optimal path of carbon taxes in a climate-economy model with distortionary taxes where households are unequal and face uninsurable risk. In a simple two-period model, we highlight several mechanisms through which inequality, risk, and borrowing constraints affect optimal climate policy. We then introduce a new fiscal climate-economy model in the spirit of Barrage (2020) with incomplete markets, which we calibrate to match features of the U.S. economy with emissions scaled up to the global level. We then use the model to study quantitatively policy scenarios where the government chooses the optimal carbon tax with different assumptions on the revenue use and ability to issue debt. We find that when the government uses the carbon tax revenue to finance wasteful spending as opposed to lump-sum transfers, the climate transition is significantly delayed but eventually catches up. When the government has the ability to use government debt, the path of optimal carbon taxes is, surprisingly, hardly affected. This is true despite the fact that debt policy has important implications for the amount of optimal redistribution and insurance, and the behavior of macroeconomic aggregates. In the future, we wish to solve the full Ramsey problem where the government also optimizes over capital and labor taxes, and explore the implications of household heterogeneity in energy spending shares for optimal climate policy.

[10] Our calibration of utility versus production damages, based on Barrage (2020), leads to more convexity in utility damages that only account for a very small portion of total damages at low temperatures.

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Appendices

A Derivations two-period model

A.1 Derivations general case

The Lagrangian of the planner's problem is

$$\begin{aligned}\mathcal{L} = &\sum_i p_i \Big(\sum_j \pi_j \big(u(c_{1,i}) + \beta u(c_{2,i}^j) \big) \Big) + V(G) + \sum_i p_i \lambda_i \Big(u_c(c_{1,i}) - \beta(1+r) \sum_j \pi_j u_c(c_{2,i}^j) \Big) \\ &+ \nu \Big[\tau \Big(\sum_i p_i \Big(c_{1,i} + \frac{\sum_j \pi_j c_{2,i}^j}{1+r} \Big) \Big) - G \Big].\end{aligned}$$

The first-order conditions yield

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial \tau} = &\sum_i p_i \frac{\partial c_{1,i}}{\partial \tau} \Big(u_c(c_{1,i}) + \lambda_i u_{cc}(c_{1,i}) + \nu\tau \Big) + \sum_i p_i \Bigg(\sum_j \pi_j \Bigg(\frac{\partial c_{2,i}^j}{\partial \tau} \Big(\beta u_c(c_{2,i}^j) - \lambda_i \beta (1+r) u_{cc}(c_{2,i}^j) + \frac{\nu\tau}{(1+r)} \Big) \Bigg) \Bigg) \\ &+ \nu \sum_i p_i \Big(c_{1,i} + \frac{\sum_j \pi_j c_{2,i}^j}{1+r} \Big) = 0, \\ \frac{\partial \mathcal{L}}{\partial G} = &V_G(G) - \nu = 0,\end{aligned}$$

and $\forall i$,

$$\frac{\partial \mathcal{L}}{\partial a_i} = p_i \frac{\partial c_{1,i}}{\partial a_i} \Big(u_c(c_{1,i}) + \lambda_i u_{cc}(c_{1,i}) + \nu\tau \Big) + p_i \sum_j \pi_j \frac{\partial c_{2,i}^j}{\partial a_i} \Big(\beta u_c(c_{2,i}^j) - \lambda_i \beta(1+r) u_{cc}(c_{2,i}^j) + \frac{\nu\tau}{(1+r)} \Big) = 0,$$

with the following expressions for the partial derivatives, for $i \in \{L, H\}$ and $j \in \{l, h\}$:

$$\frac{\partial c_{1,i}}{\partial \tau} = -\frac{c_{1,i}}{(1+\tau)}, \qquad \frac{\partial c_{2,i}^j}{\partial \tau} = \frac{g_\tau(\tau)\omega_{2,i}^j - c_{2,i}^j}{(1+\tau)}, \qquad \frac{\partial c_{1,i}}{\partial a_i} = \frac{-1}{1+\tau}, \qquad \frac{\partial c_{2,i}^j}{\partial a_i} = \frac{1+r}{1+\tau}.$$

Using the partial derivatives, the FOCs w.r.t. a_i give

$$u_c(c_{1,i}) + \lambda_i u_{cc}(c_{1,i}) + \nu\tau = \sum_j \pi_j (1+r) \Big(\beta u_c(c_{2,i}^j) - \lambda_i \beta(1+r) u_{cc}(c_{2,i}^j) + \frac{\nu\tau}{(1+r)} \Big),$$

which, using the Euler equations, simplifies to

$$\lambda_i u_{cc}(c_{1,i}) = -\lambda_i \beta (1+r)^2 \mathbb{E}_j \big[u_{cc}(c_{2,i}^j) \big]. \tag{45}$$

Assuming that $u_{cc}(c)$ has constant sign, this implies that $\forall i$,

$$\lambda_i = 0.$$

We can make use of this result and the partial derivatives to simplify the FOC w.r.t. τ:

$$\sum_i p_i \frac{\partial c_{1,i}}{\partial \tau} \Big(u_c(c_{1,i}) + \nu\tau \Big) + \sum_i p_i \Bigg(\sum_j \pi_j \Bigg(\frac{\partial c_{2,i}^j}{\partial \tau} \Big(\beta u_c(c_{2,i}^j) + \frac{\nu\tau}{(1+r)} \Big) \Bigg) \Bigg) + \nu \sum_i p_i \Big(c_{1,i} + \frac{\sum_j \pi_j c_{2,i}^j}{1+r} \Big) = 0.$$

Simplifying further, we have

$$\sum_i p_i \Big(c_{1,i} u_c(c_{1,i}) + \beta \sum_j \pi_j \Big(c^j_{2,i} u_c(c^j_{2,i}) \Big) \Big) =$$
$$\sum_i p_i \Big(\sum_j \pi_j \Big(g_\tau(\tau) \omega^j_{2,i} \Big) \Big(\beta u_c(c^j_{2,i}) + \frac{\nu\tau}{(1+r)} \Big) \Big) + \nu \sum_i p_i \Big(c_{1,i} + \frac{\sum_j \pi_j c^j_{2,i}}{1+r} \Big),$$

or equivalently,

$$\mathbb{E}_i \Big[c_{1,i} u_c(c_{1,i}) + \beta \mathbb{E}_j \big[c^j_{2,i} u_c(c^j_{2,i}) \big] \Big] = V_G(G) C + g_\tau(\tau) \beta \mathbb{E}_i \Big[\mathbb{E}_j \big[\omega^j_{2,i} u_c(c^j_{2,i}) \big] \Big] + g_\tau(\tau) \beta \tau V_G(G) \omega_2,$$

with

$$C = \mathbb{E}_i \Big[c_{1,i} + \frac{\mathbb{E}_j \big[c^j_{2,i} \big]}{1+r} \Big], \quad \text{and} \quad \omega_2 = \mathbb{E}_i \Big[\mathbb{E}_j \big[\omega^j_{2,i} \big] \Big].$$

A.2 Derivations public good provision

We consider the case where $g_\tau(.) = 0$ and $g(\tau) = 1$. The optimal policy is given by the following Samuelson rule,

$$V_G(G) = \frac{1}{C} \Big(\mathbb{E}_i \Big[c_{1,i} u_c(c_{1,i}) + \beta \mathbb{E}_j \big[c^j_{2,i} u_c(c^j_{2,i}) \big] \Big] \Big).$$

A.2.1 Risk

In the absence of ex ante inequality, we have $\forall i$, $c_{1,i} = c_1$, $\mathbb{E}_j[c^j_{2,i}] = \mathbb{E}_j[c^j_2]$, thus

$$V_G(G) = \frac{1}{C} \Big(c_1 u_c(c_1) + \beta \mathbb{E}_j \big[c^j_2 u_c(c^j_2) \big] \Big). \tag{46}$$

Decomposing the expectation term, we have

$$V_G(G) = \frac{1}{C} \Big(c_1 u_c(c_1) + \beta \Big(\mathbb{E}_j \big[c^j_2 \big] \mathbb{E}_j \big[u_c(c^j_2) \big] + \text{cov}_j(c^j_2, u_c(c^j_2)) \Big) \Big).$$

Using the Euler equation, $\beta(1+r) = 1$, and the fact that $C = c_1 + \frac{\mathbb{E}_j[c^j_2]}{1+r}$, we obtain

$$V_G(G) = \mathbb{E}_j \big[u_c(c^j_2) \big] + \beta \frac{\text{cov}_j(c^j_2, u_c(c^j_2))}{C}. \tag{47}$$

When utility is CRRA, $c u_c(c) = c^{1-\sigma}$, hence from equation (46), when $\sigma \to 1$, consumption risk has no effect on $V_G(G)$. From Jensen's inequality, it also follows from this equation that higher risk leads to more (resp. less) public good provision if $\sigma < 1$ (resp. > 1).

A.2.2 Inequality

In the absence of risk, when households are ex-ante unequal, we have $u_c(c_{1,i}) = u_c(c_{2,i}) = u_c(c_i)$, hence $c_{1,i} = c_{2,i} = c_i$, and

$$V_G(G) = \frac{1}{c} \Big(\mathbb{E}_i \big[c_i u_c(c_i) \big] \Big). \tag{48}$$

where

$$c \equiv \frac{C}{1+\frac{1}{1+r}}$$

denotes average per period consumption. We can again decompose the expectation term to get

$$\begin{aligned} V_G(G) &= \frac{1}{c}\Big(\mathbb{E}_i\big[c_i\big]\big[u_c(c_i)\big] + \text{cov}_i(c_i, u_c(c_i))\Big) \\ &= \mathbb{E}_i\big[u_c(c_i)\big] + \frac{\text{cov}_i(c_i, u_c(c_i))}{c}. \end{aligned} \tag{49}$$

The main difference with equation (47) is that, while risk leads to heterogeneous consumption in the second period, inequality in endowment—if known ex ante — is smoothed over time and therefore affects consumption in both periods.

Again, when utility is CRRA, $cu_c(c) = c^{1-\sigma}$, hence from equation (48), when $\sigma \to 1$, consumption inequality has no effect on $V_G(G)$. From Jensen's inequality, it also follows from this equation that higher inequality leads to more (resp. less) public good provision if $\sigma < 1$ (resp. > 1).

A.3 Derivation endowment gains

We consider the case where $V_G(.) = 0$. The optimal mitigation policy is given by the following rule:

$$g_\tau(\tau) = \frac{\mathbb{E}_i\Big[c_{1,i}u_c(c_{1,i}) + \beta\mathbb{E}_j\big[c_{2,i}^j u_c(c_{2,i}^j)\big]\Big]}{\beta\mathbb{E}_i\Big[\mathbb{E}_j\big[\omega_{2,i}^j u_c(c_{2,i}^j)\big]\Big]}.$$

A.3.1 Risk

In the absence of ex ante inequality, we have $\forall i$, $c_{1,i} = c_1$, $\mathbb{E}_j[c_{2,i}^j] = \mathbb{E}_j[c_2^j]$, and $\mathbb{E}_j[\omega_{2,i}^j] = \mathbb{E}_j[\omega_2^j]$, thus following similar steps as above, we have

$$g_\tau(\tau) = \frac{C \times \mathbb{E}_j\big[u_c(c_2^j)\big] + \text{cov}_j(c_2^j, u_c(c_2^j))}{\beta\mathbb{E}_j\big[\omega_2^j u_c(c_2^j)\big]}, \tag{50}$$

or, after decomposing the expectation of the denominator,

$$g_\tau(\tau) = \frac{C \times \mathbb{E}_j\big[u_c(c_2^j)\big] + \text{cov}_j(c_2^j, u_c(c_2^j))}{\beta\Big(\omega_2\mathbb{E}_j\big[u_c(c_2^j)\big] + \text{cov}_j(\omega_2^j, u_c(c_2^j))\Big)}. \tag{51}$$

From equation (51) and using Jensen's inequality, we see that risk affects the policy positively by increasing the expected value of the marginal utility of consumption. Because consumption in period 1 is identical across types, endowments shocks in period 2 are entirely passed on to consumption in period 2, hence higher risk leads to a higher covariance between ω_2^j and $u_c(c_2^j)$, which affects the policy negatively. In addition, from equation (50), it is straightforward to see that precautionary savings—that for any j leads to an increase in c_2^j and thus a decrease in $u_c(c_2^j)$—negatively affect the policy's benefits captured by the denominator, and therefore reduce the optimal policy level through this channel.

A.3.2 Inequality

In the absence of risk, when households are ex ante unequal, we have $u_c(c_{1,i}) = u_c(c_{2,i}) = u_c(c_i)$, hence $c_{1,i} = c_{2,i} = c_i$, and

$$g_\tau(\tau) = \frac{C \times \mathbb{E}_i\big[u_c(c_i)\big] + (1 + \frac{1}{1+r}) \times \text{cov}_i(c_i, u_c(c_i))}{\beta\Big(\omega_2 \mathbb{E}_i\big[u_c(c_i)\big] + \text{cov}_i(\omega_{2,i}, u_c(c_i))\Big)}.$$

Let's define household total endowment as

$$\omega_i \equiv \omega_{1,i} + \frac{g(\tau)\omega_{2,i}}{1+r},$$

and let $\kappa(\tau)$ denote the share of aggregate endowment received in period 2, i.e.,

$$\kappa(\tau) \equiv \frac{g(\tau)\sum_i p_i \omega_{2,i}}{\sum_i p_i \omega_i}.$$

Using this notation, let's express household i's endowment in period 2 as

$$\omega_{2,i} = \tilde{\kappa}(\tau)\omega_i + \Delta_i,$$

with $\tilde{\kappa}(\tau) = \kappa(\tau)/g(\tau)$. The term Δ_i, which is such that $\sum_i p_i \Delta_i = 0$, is positive for households of type i if these households receive a higher share of their endowment in period 2 compared to other households. Thus, when inequality is growing (resp. declining) over time, $\Delta_H > 0$ (resp. $\Delta_L > 0$).

When utility is CRRA, household consumption can be expressed as $c_i = \alpha\omega_i$, with

$$\alpha = \frac{(1+r)}{(1+\tau)(2+r)},$$

hence we have

$$\begin{aligned}
\mathcal{D} &= \beta \mathbb{E}_i\Big[\omega_{2,i} u_c(c_{2,i})\Big] \\
&= \beta \sum_i p_i \frac{\omega_{2,i}}{(\alpha\omega_i)^\sigma} \\
&= \frac{\beta}{\alpha^\sigma} \sum_i p_i \Big(\tilde{\kappa}(\tau)\omega_i^{1-\sigma} + \Delta_i \omega_i^{-\sigma}\Big).
\end{aligned}$$

The first term in brackets is reminiscent of the mechanisms studied above: unequal endowments lead to unequal consumption, which affects the valuation of endowment gains through the average of the marginal utility of consumption, given by $(\alpha\omega_i)^{-\sigma}$, weighted by consumption gains, given by ω_i. Again, when $\sigma \to 1$, these two effects cancel out.

In addition, the second term in brackets measures the impact of the timing of inequality. In particular, when rich households get a lower share of their endowment in the second period, i.e., when inequality decreases over time with $\Delta_H < 0$,

$$\sum_i p_i \Delta_i \omega_i^{-\sigma} > 0. \tag{52}$$

A.4 Derivation borrowing constraint

A.4.1 Optimal policy

Assuming endowments are such that only household L is constrained in the first period, the Lagrangian of this problem is

$$
\begin{aligned}
\mathcal{L} =& p_L\big(u(c_{1,L}) + \beta u(c_{2,L})\big) + p_H\big(u(c_{1,H}) + \beta u(c_{2,H})\big) + V(G) \\
&+ p_H \lambda_H \big(u_c(c_{1,H}) - \beta(1+r)u_c(c_{2,H})\big) \\
&+ \nu\bigg(\tau\Big(p_L c_{1,L} + p_H c_{1,H} + \frac{p_L c_{2,L} + p_H c_{2,H}}{1+r}\Big) - G\bigg).
\end{aligned}
$$

The first-order conditions yield

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \tau} =& p_L \frac{\partial c_{1,L}}{\partial \tau}\Big(u_c(c_{1,L}) + \nu\tau\Big) + p_L \frac{\partial c_{2,L}}{\partial \tau}\Big(\beta u_c(c_{2,L}) + \frac{\nu\tau}{(1+r)}\Big) \\
&+ p_H \frac{\partial c_{1,H}}{\partial \tau}\Big(u_c(c_{1,H}) + \lambda_H u_{cc}(c_{1,H}) + \nu\tau\Big) + p_H \frac{\partial c_{2,H}}{\partial \tau}\Big(\beta u_c(c_{2,H}) - \lambda_H \beta(1+r)u_{cc}(c_{2,H}) + \frac{\nu\tau}{(1+r)}\Big) \\
&+ \nu\Big(p_L c_{1,L} + p_H c_{1,H} + \frac{p_L c_{2,L} + p_H c_{2,H}}{1+r}\Big) = 0, \\
\frac{\partial \mathcal{L}}{\partial G} =& V_G(G) - \nu = 0, \\
\frac{\partial \mathcal{L}}{\partial a_H} =& p_H \frac{\partial c_{1,H}}{\partial a_H}\Big(u_c(c_{1,H}) + \lambda_H u_{cc}(c_{1,H}) + \nu\tau\Big) + p_H \frac{\partial c_{2,H}}{\partial a_H}\Big(\beta u_c(c_{2,H}) - \lambda_H \beta(1+r)u_{cc}(c_{2,H}) + \frac{\nu\tau}{(1+r)}\Big) = 0,
\end{aligned}
$$

with the following expressions for the partial derivatives, for $i \in \{L, H\}$:

$$
\frac{\partial c_{1,i}}{\partial \tau} = -\frac{c_{1,i}}{(1+\tau)}, \qquad \frac{\partial c_{2,i}}{\partial \tau} = \frac{g'(\tau)\omega_{2,i} - c_{2,i}}{(1+\tau)}, \qquad \frac{\partial c_{1,H}}{\partial a_H} = \frac{-1}{1+\tau}, \qquad \frac{\partial c_{2,H}}{\partial a_H} = \frac{1+r}{1+\tau}.
$$

Using the partial derivatives, the FOC w.r.t. a_H gives

$$
u_c(c_{1,H}) + \lambda_H u_{cc}(c_{1,H}) = (1+r)\Big(\beta u_c(c_{2,H}) - \lambda_H \beta(1+r)u_{cc}(c_{2,H})\Big).
$$

Using the Euler equations, we have

$$
\lambda_H u_{cc}(c_{1,H}) = -\lambda_H \beta(1+r)^2 u_{cc}(c_{2,H}),
$$

which, assuming that $u_{cc}(c)$ has constant sign, implies that

$$
\lambda_H = 0.
$$

We can make use of this result and the partial derivatives to simplify the FOC w.r.t. τ, and following the same steps as in the benchmark model, we obtain the same formula (abstracting from risk),

$$
\begin{aligned}
V_G(G)&\Big(p_L c_{1,L} + p_H c_{1,H} + \frac{p_L c_{2,L} + p_H c_{2,H}}{1+r}\Big) + g_\tau(\tau)\Big(p_L \omega_{2,L}\beta u_c(c_{2,L}) + p_H \omega_{2,H}\beta u_c(c_{2,H})\Big) \\
&+ g_\tau(\tau)V_G(G)\Big(p_L \omega_{2,L}\frac{\tau}{1+r} + p_H \omega_{2,H}\frac{\tau}{1+r}\Big) \\
&= p_L c_{1,L} u_c(c_{1,L}) + p_L c_{2,L}\beta u_c(c_{2,L}) + p_H c_{1,H} u_c(c_{1,H}) + p_H c_{2,H}\beta u_c(c_{2,H}), \qquad (53)
\end{aligned}
$$

or, more concisely,

$$\mathbb{E}_i\Big[c_{1,i}u_c(c_{1,i}) + \beta c_{2,i}u_c(c_{2,i})\Big] = V_G(G)C + g_\tau(\tau)\beta\mathbb{E}_i\Big[\omega_{2,i}u_c(c_{2,i})\Big] + \tau V_G(G)\frac{g_\tau(\tau)\omega_2}{(1+r)},$$

Thus, the optimal policy is determined by the same trade-off between the direct cost from reduced consumption and the benefits from public good provision, increase in consumption through higher future endowments, and increase in public good provision through a higher fiscal base. Although the formula is the same in the presence of a binding borrowing constraint, the allocations differ.

A.4.2 Public good provision

We consider the case where $g_\tau(\tau) = 0$ and $g(\tau) = 1$. When households of type L are borrowing constrained, the optimal policy is given by the following Samuelson rule,

$$\begin{aligned} V_G(G) &= \frac{1}{C}\mathbb{E}_i\Big[c_{1,i}u_c(c_{1,i}) + \beta c_{2,i}u_c(c_{2,i})\Big] \\ &= \frac{1}{C}\mathbb{E}_i\Big[c_{1,i}\big(u_c(c_{1,i}) - u_c(c_{2,i})\big) + \big(c_{1,i} + \beta c_{2,i}\big)u_c(c_{2,i})\Big] \end{aligned} \tag{54}$$

Using $\beta(1+r) = 1$, $c_{1,i} + \frac{c_{2,i}}{1+r} = C_i$, and $\mathbb{E}_i[C_i] = C$, we obtain

$$V_G(G) = \mathbb{E}_i\big[u_c(c_{2,i})\big] + \frac{\text{cov}_i(c_i, u_c(c_{2,i}))}{c} + \frac{1}{C}\mathbb{E}_i\big[c_{1,i}\big(u_c(c_{1,i}) - u_c(c_{2,i})\big)\big].$$

From equation (54), when utility is CRRA we have

$$V_G(G) = \frac{1}{C}\mathbb{E}_i\big[c_{1,i}^{1-\sigma} + \beta c_{2,i}^{1-\sigma}\big],$$

hence, when $\sigma \to 1$, the presence of inequality and a binding borrowing constraint have no effect on the optimal provision of public good. Similarly to before, the effect of consumption inequality on public good provision can be signed using Jensen's inequality (with inequality calling for less public good when $\sigma > 1$, and more public good when $\sigma < 1$). To further study the impact of the borrowing constraint, let's consider that households of type L can borrow a given amount a_L in the first period, and have to repay $(1+r)a_L$ in the second period. We consider the case where a_L is small enough that the household's borrowing constraint is still binding, i.e. $c_{1,L} + a_L < c_{2,L} - (1+r)a_L$. We have

$$V_G(G) = \frac{1}{C}\bigg(\Big(1 + \frac{1}{1+r}\Big)c_H^{1-\sigma} + (c_{1,L} + a_L)^{1-\sigma} + \beta(c_{2,L} - (1+r)a_L)^{1-\sigma}\bigg).$$

Taking the derivative of $V_G(G)$ w.r.t. a_L, we have,

$$\begin{aligned} \frac{\partial V_G(G)}{\partial a_L} &= \frac{1}{C}\Big((1-\sigma)(c_{1,L} + a_L)^{-\sigma} - \beta(1+r)(1-\sigma)(c_{2,L} - (1+r)a_L)^{-\sigma}\Big) \\ &= \frac{1-\sigma}{C}\Big((c_{1,L} + a_L)^{-\sigma} - (c_{2,L} - (1+r)a_L)^{-\sigma}\Big). \end{aligned}$$

Since $\sigma > 0$ by assumption, it follows that:

$$\begin{cases} \frac{\partial V_G(G)}{\partial a_L} & > 0, \quad \text{if } \sigma < 1, \\ \frac{\partial V_G(G)}{\partial a_L} & = 0, \quad \text{if } \sigma \to 1, \\ \frac{\partial V_G(G)}{\partial a_L} & < 0, \quad \text{if } \sigma > 1. \end{cases}$$

Thus, relaxing the borrowing constraint (i.e., increasing a_L) increases $V_G(G)$—and thus reduces public good provision—if and only if $\sigma > 1$.

A.4.3 Endowment gains

We consider the case where $V_G(.) = 0$. The optimal mitigation policy is given by the following rule:

$$g_\tau(\tau) = \frac{\mathbb{E}_i\left[u_c(c_{2,i})\right] + \frac{\text{cov}_i(c_i, u_c(c_{2,i}))}{c} + \frac{1}{C}\mathbb{E}_i\left[c_{1,i}\left(u_c(c_{1,i}) - u_c(c_{2,i})\right)\right]}{\beta\mathbb{E}_i\Big[\omega_{2,i}u_c(c^*_{2,i}) + \omega_{2,i}\big(u_c(c_{2,i}) - u_c(c^*_{2,i})\big)\Big]},$$

with $c^*_{2,i}$ household i's consumption if there was no borrowing constraint. Thus, if we denote by $\mathcal{D}$ the denominator of the previous equation, we have

$$\mathcal{D} = \beta\Big(\omega_2\mathbb{E}_i\left[u_c(c^*_{2,i})\right] + \text{cov}_i(\omega_{2,i}, u_c(c^*_{i,2}))\Big) + \beta\mathbb{E}_i\Big[\omega_{2,i}\big(u_c(c_{2,i}) - u_c(c^*_{2,i})\big)\Big],$$

or, since absent a borrowing constraint consumption is equalized across periods,

$$\mathcal{D} = \beta\Big(\omega_2\mathbb{E}_i\left[u_c(c_i)\right] + \text{cov}_i(\omega_{2,i}, u_c(c_i))\Big) + \beta\mathbb{E}_i\Big[\omega_{2,i}\big(u_c(c_{2,i}) - u_c(c_i)\big)\Big],$$

The presence of the borrowing constraint affects $\mathcal{D}$ through the additional term,

$$\beta\mathbb{E}_i\Big[\omega_{2,i}\big(u_c(c_{2,i}) - u_c(c_i)\big)\Big],$$

which, since only households of type L are subject to the constraint, can also be written as

$$\beta p_L\omega_{2,L}\big(u_c(c_{2,L}) - u_c(c_L)\big).$$

Given that poor households are unable to borrow, they consume more in period 2 than in period 1, hence this additional term is negative.

B Pigouvian tax with incomplete markets

Let us define the Pigouvian tax as the first-best tax formula evaluated at the equilibrium allocation. To clearly distinguish inequality from risk, let us denote the planner's period welfare function as

$$V(c_t, h_t, Z_t; \pi, \lambda) \equiv \sum_i \lambda_i \sum_{s^t} \pi_{i,t}(s^t|s_0) u(c_t^i(s^t), h_t^i(s^t), Z_t). \tag{55}$$

Following the approach of Douenne et al. (2023), we can show that in the first-best, i.e. when the planner has access to individualized lump-sum transfers and households can trade state-contingent contracts, the optimal carbon tax is

$$\tau_t^{e,FB} = \sum_{j=0}^{\infty} \beta^j \left(\frac{V_{c,t+j}}{V_{c,t}} D'_{t+j} A_{1,t+j} F_{t+j} - \frac{V_{Z,t+j}}{V_{c,t}} \right) J_{E_t^M, t+j},$$

where $J_{E_t^M,t+j}$ denotes the marginal impact of CO_2 emissions in period t on the climate in period $t+j$, $D'_{t+j} A_{1,t+j} F_{t+j}$ denotes the marginal impact of climate on production, and V_c, V_Z denote the aggregate marginal utility from consumption and climate from the perspective of the planner. When utility is additively separable in the climate variable, inequality and risk do not affect the value of V_Z.

To understand their impact on V_c, let us decompose individuals' consumption and labor at a given history as the product of an individual-history component and an aggregate component,

$$c_t^i(s^t) \equiv \omega_t^{c,i}(s^t) c_t, \tag{56}$$

$$h_t^i(s^t) \equiv \omega_t^{h,i}(s^t) h_t. \tag{57}$$

From this decomposition, we can express V_c, the aggregate marginal utility of consumption from the perspective of the planner, as

$$\begin{aligned} V_c(c_t, h_t, Z_t; \pi, \lambda) = \sum_i \lambda_i \sum_{s^t} \pi_{i,t}(s^t|s_0) \bigg(& u_c(c_t^i(s^t), h_t^i(s^t), Z_t) \left(\omega_t^{c,i}(s^t) + \frac{\partial \omega_t^{c,i}(s^t)}{\partial c_t} c_t \right) \\ & + u_h(c_t^i(s^t), h_t^i(s^t), Z_t) \frac{\partial \omega_t^{h,i}(s^t)}{\partial c_t} h_t \bigg). \end{aligned} \tag{58}$$

Thus, the marginal utility of consumption from the planner's perspective is the sum of three terms. The first term is the expected value of households' marginal utility of consumption weighted by their share of aggregate consumption, that we denote by

$$\tilde{V}_c(c_t, h_t, Z_t; \pi, \lambda) \equiv \sum_i \lambda_i \sum_{s^t} \pi_{i,t}(s^t|s_0) u_c(c_t^i(s^t), h_t^i(s^t), Z_t) \omega_t^{c,i}(s^t). \tag{59}$$

The second and third terms are consumption and labor reallocation components that depend on current allocations as well as on the planners' preferences and constraints,

$$\vartheta^c(c_t, h_t, Z_t; \pi, \lambda) \equiv \sum_i \lambda_i \sum_{s^t} \pi_{i,t}(s^t|s_0) u_c(c_t^i(s^t), h_t^i(s^t), Z_t) \frac{\partial \omega_t^{c,i}(s^t)}{\partial c_t} c_t, \tag{60}$$

$$\vartheta^h(c_t, h_t, Z_t; \pi, \lambda) \equiv \sum_i \lambda_i \sum_{s^t} \pi_{i,t}(s^t|s_0) u_h(c_t^i(s^t), h_t^i(s^t), Z_t) \frac{\partial \omega_t^{h,i}(s^t)}{\partial c_t} h_t. \tag{61}$$

In the first-best, and more generally with complete markets, we have $\vartheta^c(c_t, h_t, Z_t; \pi, \lambda) = \vartheta^h(c_t, h_t, Z_t; \pi, \lambda) = 0$, so that $V_c = \tilde{V}_c$. Thus, we can express the Pigouvian tax, i.e. the first-best tax formula evaluated at the equilibrium allocation, as

$$\tau_t^{e,Pigou} = \sum_{j=0}^{\infty} \beta^j \left(\frac{\tilde{V}_{c,t+j}}{\tilde{V}_{c,t}} D'_{t+j} A_{1,t+j} F_{t+j} - \frac{V_{Z,t+j}}{\tilde{V}_{c,t}} \right) J_{E_t^M, t+j},$$

with

$$\tilde{V}_c(c_t, h_t, Z_t; \pi, \lambda) = \mathbb{E}_i \left[\mathbb{E}^i_{s^t} \left[u_c(c_t^i(s^t), h_t^i(s^t), Z_t) \omega_t^i(s^t) \right] \right], \tag{62}$$

where $\mathbb{E}_i$ denotes the cross-sectional expectation and $\mathbb{E}^i_{s^t}$ the expectation over histories for an individual i.[11] Importantly, this tax depends only on the path of equilibrium allocations, and does not require to determine how a marginal increase in aggregate consumption would be redistributed along this equilibrium path, or how it would affect the allocation of aggregate labor, i.e. it does not require to know ϑ^c and ϑ^h. While ϑ^c and ϑ^h matter for the optimal carbon tax, they depend on mechanisms that only occur in second-best environments and are therefore not part of the Pigouvian tax (just like the marginal cost of funds in second-best complete markets economies, see Barrage, 2020; Douenne et al., 2023).

[11] If agents are identical at $t = 0$, then the two expectation terms are redundant. If we start at $t = 0$ from a distribution such that the probability distributions of histories are heterogeneous, then the two expectation terms are not redundant anymore.

C Calibration

Table IV: Calibrated Model Parameters

Description	Parameter	Value
Preferences and technology		
Consumption share	γ	0.74
Preference curvature	σ	1.69
Discount factor	β	0.995
Weight on leisure	ς	1.979
Weight on damages in utility	α_Z	2.16×10^{-4}
Borrowing constraint	$\underline{a}$	-0.080
Ratio of TFPs	A_2/A_1	6.831
Fiscal policy		
Government expenditure	G	0.069
Transfers	T	0.088
Labor productivity process		
Productivity process curvature	η	1.12

Persistent shock

$$\Gamma_P = \begin{bmatrix} 0.994 & 0.002 & 0.004 & 3\mathrm{E}{-5} \\ 0.019 & 0.979 & 0.001 & 9\mathrm{E}{-5} \\ 0.023 & 0.000 & 0.977 & 5\mathrm{E}{-5} \\ 0.000 & 0.000 & 0.012 & 0.987 \end{bmatrix} \quad e_P = \begin{bmatrix} 0.185 \\ 0.305 \\ 0.537 \\ 27.223 \end{bmatrix}$$

Transitory shock

$$P_T = \begin{bmatrix} 0.357 \\ 0.002 \\ 0.467 \\ 0.004 \\ 0.025 \\ 0.176 \end{bmatrix} \quad e_T = \begin{bmatrix} 0.07 \\ 0.09 \\ 3.12 \\ 3.16 \\ 7.80 \\ 9.51 \end{bmatrix}$$

Table V: Exogenously Imposed Parameters

Parameter	Description	Value	Source
Production first sector			
a_1	Damage coefficient	0.01	Dietz and Venmans (2019)
α	Return to scale on labor sector 1	0.3	DICE 2023
ν	Return to scale on energy sector 1	0.04	Golosov et al (2014)
δ	Depreciation rate on capital (per year)	0.1	DICE 2023
Y_{2020}	Initial output (in trillions 2023 USD)	83.476	World Bank (2016-2020)
Production second sector			
α_E	Return to scale on capital sector 2	0.597	Barrage (2020)
E_{2020}	Init. gross indus. emissions ($GtCO_2$ per year)	38.23	Friedlingstein et al (2022)
Climate			
S_{2020}	Initial cumulative carbon emissions (in GtCO2)	2390	IPCC (2021)
T_{2020}	Initial atmos. temp. change (C since 1900)	1.07	IPCC (2021)
ϵ	Initial pulse-adjustment timescale	0.5	Dietz and Venmans (2019)
ζ	Trans. clim. resp. to cum. emissions (TCRE)	0.00045	IPCC (2021)
E^{land}_{2020}	Init. gross CO_2 emis. land ($GtCO_2$ per year)	4.17	Friedlingstein et al (2022)
$g_{E^{\text{land}}}$	Ex. decline rate of gross land emissions (per period)	0.1	DICE 2023
Abatement costs			
P^{back}_{2020}	Backstop price in 2020 (in $/tCO_2$)	696.2	DICE 2023
$g^{P^{\text{back}}}_{2020}$	Decline rate backstop price 2020-2050 (per year)	1%	DICE 2023
$g^{P^{\text{back}}}_{2050}$	Decline rate backstop price after 2050 (per year)	0.1%	DICE 2023
c_2	Exponent abatement cost function	2.6	DICE 2023
μ_{2020}	Initial abatement share	0.0513	DICE 2023
Exogenous growth parameters			
$g_{A_{1,2020}}$	Initial TFP growth rate sector 1 (per period)	0.082	DICE 2023
$gg_{A_{1,t}}$	Decline rate TFP growth sector 1 (per year)	0.0072	DICE 2023
$g_{A_{2,2020}}$	Initial TFP growth rate sector 2 (per period)	0.082	DICE 2023
$gg_{A_{2,t}}$	Decline rate TFP growth sector 2 (per year)	0.0072	DICE 2023
N_{2020}	Initial population (in millions)	1,368	World Bank US-adjusted
$N_{\max}$	Asymptotic population (in millions)	1,910	DICE 2023 US-adjusted
g_N	Rate of convergence of population	0.145	DICE 2023
Fiscal Policy			
τ^k	Capital income tax (%)	33.6*	Appendix D.4
τ^h	Labor income tax (%)	27.7*	Appendix D.4
τ^c	Consumption tax (%)	4.2*	Appendix D.4
τ^i_t	Energy tax (%)	0.0	Appendix D.4
τ^e_t	Initial carbon emission tax (%)	0.6	Appendix D.4

D Data

In what follows we describe our procedure to obtain macroeconomic data and cross-sectional moments at the household level. We use the Current Population Survey (CPS) to construct the cross-sectional moments for hours, the Survey of the Consumer Finances (SCF) to construct the cross-sectional moments for wealth, earnings and income, and the Consumption Expenditure Survey (CEX) to construct the cross-sectional moments for consumption. Finally, we discuss the computation of our targets for the statistical properties of the labor income process based on the data provided by Pruitt and Turner (2020b) from the Internal Revenue Services.

D.1 National Income and Product Accounts (NIPA)

Following Aiyagari and McGrattan (1998) we define physical capital as the sum of nonresidential and residential private fixed assets and purchases of consumer durables. Therefore, our definition excludes government's fixed assets. We compute the average capital-output ratio, following the outlined definition, for period the 2009-2019 using two tables provided by the U.S. Bureau of Economic Analysis: Table 1.1. Current-Cost Net Stock of Fixed Assets and Consumer Durable Goods for the capital series and Table 1.1.5. Gross Domestic Product for the GDP series. We obtain the ration of 2.57.

We define the investment-output ratio in a way consistent with the capital-output ratio, that is, we compute investment as the sum of nonresidential and residential private fixed assets and purchases of consumer durables and exclude the government's investment. We compute the average investment-output ratio, following the outlined definition, for 2009-2019 period using two tables provided by the U.S. Bureau of Economic Analysis: Table 1.5. Investment in Fixed Assets and Consumer Durable Goods for the capital series and Table 1.1.5. Gross Domestic Product for the GDP series.

The third statistic we discipline using data from the NIPA tables is the transfers-to-output ratio. We define transfers in the data as personal current transfer receipts, which include social security transfers, medicare, medicaid, unemployment benefits, and veteran benefits. We choose this for two reasons: First, we include retired and unemployed households in our inequality moments. Second, lump-sum transfers in the model can be interpreted as a basic income in the case of not working. We compute the transfers to output ratio, following the outlined definition, for 2009-2019 period using two tables provided by the U.S. Bureau of Economic Analysis: Table 2.1. Personal Income and Its Disposition for the construction of the transfers and Table 1.1.5. Gross Domestic Product for the GDP series. We obtain the average ratio of 0.147.

D.2 Equivalence Scale

We construct cross-sectional statistics at the household level. To account for the distribution of household sizes we use an equivalence scale. This way we take into the consideration the number of people living in the household and how these people share resources and take advantage of economies of scale. We use an equivalence adjustment based on a three-parameter scale that reflects and follows the procedure of the U.S. Census Bureau:[12]

1. On average, children consume less than adults.
2. As family size increases, expenses do not increase at the same rate.
3. The increase in expenses is larger for a first child of a single-parent family than the first child of a two-adult family.

The three-parameter scale is calculated in the following way:

- One and two adults: scale = (number of adults)$^{0.5}$
- Single parents: scale = (number of adults + $(0.8 \times$ first child$)$ + $0.5 \times$ other children)$^{0.7}$
- All other families: scale = (number of adults + $0.5 \times$ number of children)$^{0.7}$

We apply the same equivalence measures to the SCF and the CEX.

D.3 The Survey of Consumer Finances (SCF).

D.3.1 Partition of the Population

We partition the groups of households in the SCF into four categories: workers, business owners, retirees, and non-wrking households. The partition is mutually exclusive and exhaustive. The following table summarizes the shares for each of the household type in the 2019 SCF sample.

	Workers	Business Owners	Retirees	Non-working
2019 Share (%)	67.19	5.84	9.02	17.95

Business Owners. Business owner households are defined as (1) one of the head or the spouse of the household is an active business owner, and (2) total household labor income is less than both the total household business income and the total household capital income.

Retirees. A household is defined as a retiree household if (1) both the head and the spouse of the household declared a retirement year prior to the survey year, and (2) the household is not a business owner household.

Non-working. A household is non-working if (1) the household is not a business owner household, (2) the household is not a retiree household, and (3) the household earns no labor income.

[12] See the link here: https://www.census.gov/topics/income-poverty/income-inequality/about/metrics/equivalence.html

Workers. All households that do not fall into the above three categories are classified as workers.

D.4 Time Series for Tax Rates

In this section we provide a description of the procedure we use to obtain average, effective tax rates for the United States by updating and extending the approach by Trabandt and Uhlig (2011). There are four rates computed: the average effective personal income tax rate, the average effective consumption tax rate, the average effective capital tax rate, and the average effective labor income tax rate. There are three main sources of data: the OECD database, the AMECO database, and the BEA statistics.

Variable Names and Associated Dataset. There are a total of two tables (T11000 from section 1 and T60200 from section 6) used from BEA, two tables (*simplied non-financial accounts table* and *revenue statistics for tax revenue table*) from OECD, and two variables (*private final consumption expenditure* and *total final consumption expenditure of general government*) from AMECO. In particular, the T11000 table is downloaded from Section 1 and T60200 from Section 6. We extract *"Gross wages and salaries"* and *"Net Operating Surplus"*, corresponding to line 3 and line 9 from table T11000. We extract variable for compensation of employees from *"Government"* that includes the federal and state amount from table T60200. As a result of a modification in industry classification, the table layout undergoes changes over time, resulting in the existence of four Excel sheets, with no fixed line number assigned to this variable. For reference purposes, we will utilize line 76 for this variable, as it corresponds to the line number in the statistics for the period from 1948 to 1987.

For the OECD data, the data catalogue webpage provides a search function, which allows us to locate the tables of interest. The simplified non-financial accounts table is downloaded for the USA, transaction sector Households and non-profit stitutions serving households (*SS14_S15*), in the national currency unit. The variables (with the associated variable code) used are: Consumption of fixed capital (SK1R), Received property income (SD4R), Paid property income (SD4P), and Gross operating surplus and mixed income (SB2G_B3G).

Similarly, the revenue statistics for the tax revenue table are downloaded for the USA, sector Total, in the national currency unit. The variables (along with their associated variable codes) used are: Taxes on financial and capital transaction (4400), General taxes (5110), Excises (5121), Taxes on individual income, profits and capital gain (1100), Taxes on corporate income (1200), Social security contributions (2000), Taxes from Employers (2200), Taxes on payroll and workforce (3000), and Recurrent taxes on immovable property (4100).

The annual macro-economic database of the European Commission's Directorate General for Economic and Financial Affairs (AMECO) is accessed from Ameco Online. The variables are acquired via the search function in the Ameco Online platform by choosing the USA as the country and the national currency as the unit. We will refer to the variable for private final consumption expenditure as *PFCE* and total final consumption expenditure of general government as *GFCE*.

Every variable is encoded in national current currencies in millions of dollars. The following equations are used to calculate the effective tax rates, utilizing variable codes for ease of reference. *Lines* correspond to tables from BEA, *codes* refer to tables from OECD, and *variable names* pertain to variables from AMECO or those further calculated in the text. For each year, the effective tax rates are determined using the following equations. After obtaining the tax rates for each year, we calculate the average of the tax rates from 1995 to 2019.

Personal Income Tax Rate (PITR) The effective personal income tax rate is calculated by

$$\frac{1100}{\text{line 4} + (\text{OSPUE + PEI})} \tag{63}$$

with *OSPUE* + *PEI* calculated as

$$\text{OSPUE + PEI} = SB2G\_B3G + SD4R - SD4P - 1 \times SK1R$$

We follow the practice by Tranbandt and Uhlig (reference to be added) to set the indicator to 1, i.e., we subtract the consumption of fixed capital from the operating surplus and mixed income.

Consumption Tax Rate The effective consumption tax rate is calculated by

$$\frac{5110 + 5121}{PFCE + GFCE - \text{line 76} - 5110 - 5121} \tag{64}$$

Labor Income Tax Rate The effective labor income tax rate is calculated by

$$\frac{\text{PITR} + \text{line 4} + 2000 + 3000}{\text{line 4} + 2200} \tag{65}$$

Capital Tax Rate The effective capital tax rate is calculated by

$$\frac{\text{PITR} \times (\text{OSPUE + PEI}) + 4400 + 4100 + 1200}{\text{line 11}} \tag{66}$$

E Additional Figures

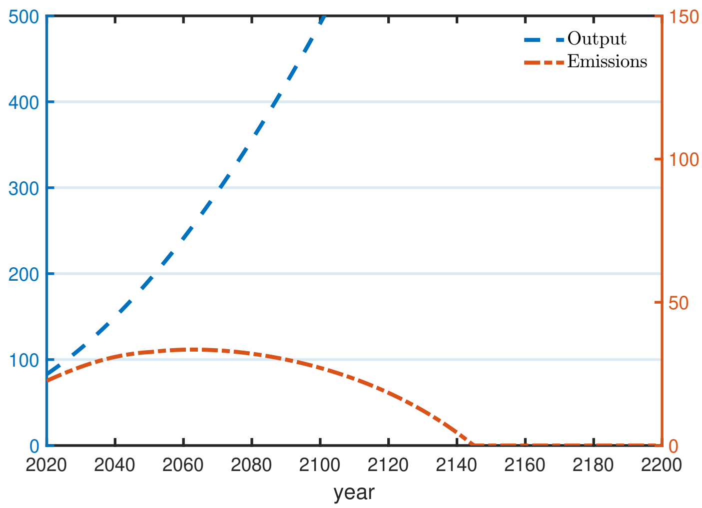


Figure 5: Output (in trillions of \$), and Emissions (in $GtCO_2$) in the Baseline.

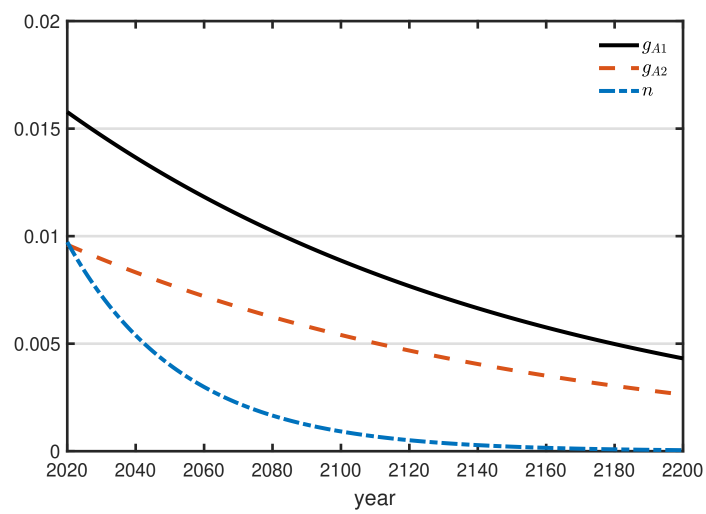
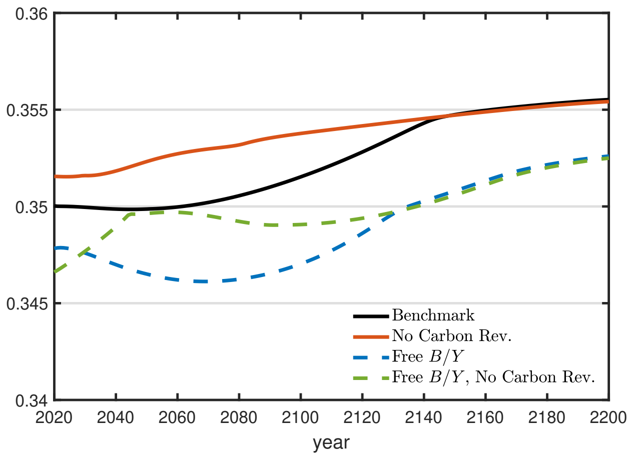
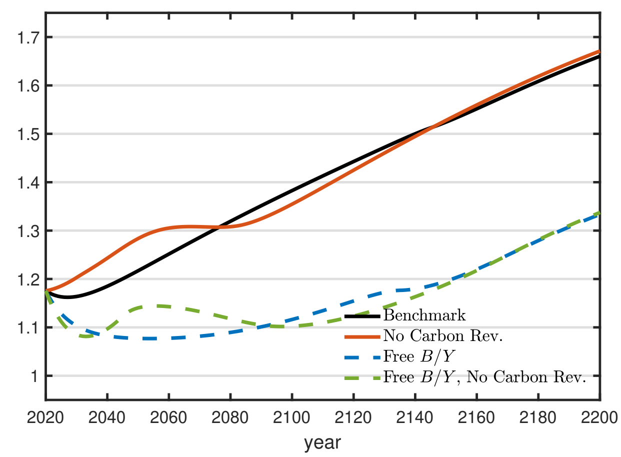


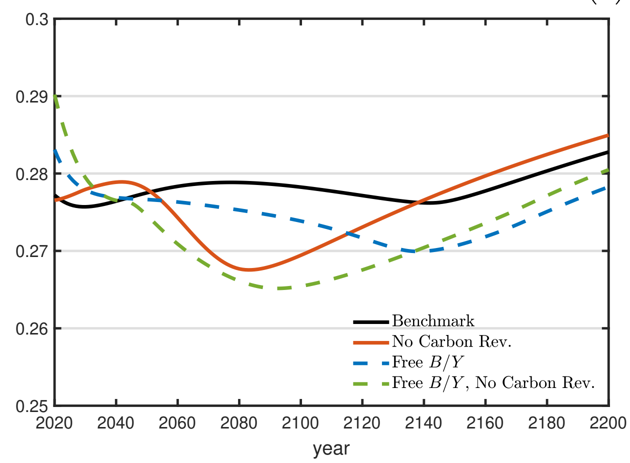
Figure 6: Exogenous Growth Rates



(a) Labor



(b) Capital



(c) Aggregate Consumption

Figure 7: Allocations