Lecture 1: The economic approach to the environment

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Table of Contents

A paradoxical field?

Many people see **economics** and the **environment** as two very distant – if not opposed – subjects. So what is environmental economics exactly?

A paradoxical field?

Many people see **economics** and the **environment** as two very distant – if not opposed – subjects. So what is environmental economics exactly?

Let's first define economics: one can find many definitions of economics as a field, such as:

- a relatively narrow one: "the way in which trade, industry, or money is organized, or the study of this" (Cambridge Dictionary);
- a relatively broad one: "the science which studies human behaviour as a relationship between (given) ends and scarce means which have alternative uses" (Lionel Robbins).

$\rightarrow$ As the study of the allocation of scarce resources, economics is clearly concerned with natural resources and the environment!

Environmental issues: what are we talking about?

Economists are interested in problems in which human behavior affects and/or is in return affected by the environment. This relates to a very broad set of environmental issues such as:

- biodiversity losses;
- land use;
- air pollution;
- water pollution;
- rising sea levels;
- extreme weather events;
- non-renewable resource depletion (e.g. oil, some minerals used for batteries);
- renewable resource depletion (e.g. fish stocks, forests);
- etc.

$\rightarrow$ As a global problem affecting all ecosystems, **climate change is closely linked to many of the above**. This is not a course focused exclusively on climate change, but we will often discuss it.

What's the difference?

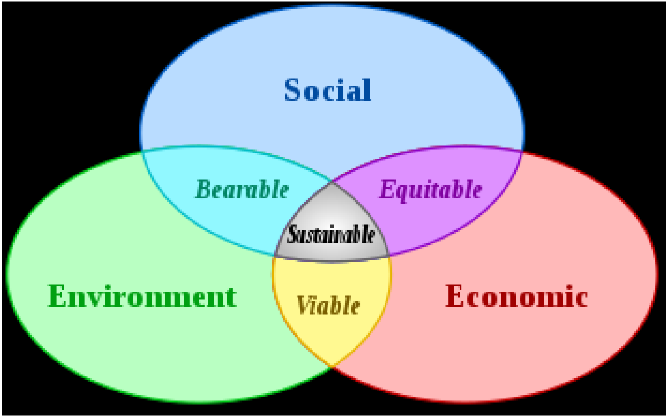
- **Environmental economics** starts from the framework of welfare economics. At the root of environmental economics there is the problem of environmental externalities. The goal is to understand their origin, their implications, and how to correct them to increase/maximize social welfare.

- **Ecological economics** is more trans-disciplinary. It does not build on the standard framework of welfare economics. Instead it starts from the study of ecosystems within which humans and the economy are embedded. Sustainability and fairness are key concerns, and there is much less focus on trade-offs.

- **Natural resources economics** is concerned with the allocation of natural resources, especially over time. At its origin it builds on the neoclassical growth framework. It is not necessarily concerned with externalities (although they are often involved in complex problems) but deals with the problem of scarcity.

This course covers environmental economics only. For those interested in ecological economics, I suggest reading *An Introduction to Ecological Economics* by Costanza et al.

Sustainability in environmental economics



Sustainability in ecological economics

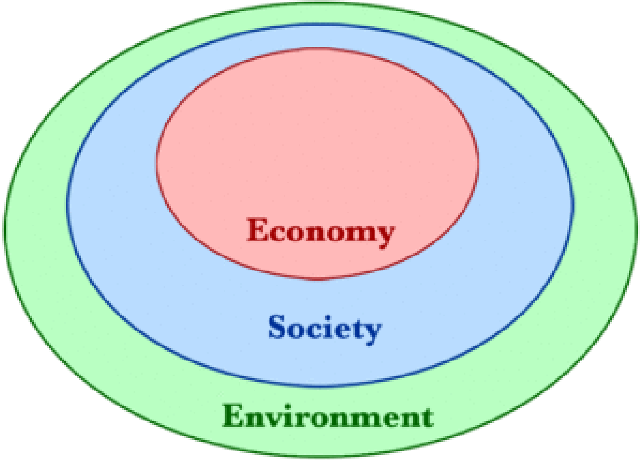


Table of Contents

The early thinkers

Sandmo (2015) "The Early History of Environmental Economics": presents the evolution of environmental concerns in economic thoughts.

- His history starts with Condorcet (1743-1794) who would have pioneered the idea of an environmental externality and the need for a government response. At the time, main concern is social justice, not efficiency: simple binary response to the problem (authorize or ban the polluting activity).
- Classical economists (XVIIIth – XIXth centuries): environment was not their chief concern, but: key contributions to *i)* the understanding of markets, which influenced the development of welfare economics (see Smith, Mill) and *ii)* growth theory, and its corollary, the question of sustainability (see Malthus, Ricardo, Jevons).
- The marginalist revolution (end of XIXth century): study competitive markets allocations and the conditions under which they maximize social utility (see Jevons, Menger, Walras). From this framework, a key assumption appears: there should be no externality (see Pareto, Marshall).

Pigou: externalities and public policies

Arthur Cecil Pigou (1877-1959): his great contribution is on the policy response to externalities. An externality arises when the social and the private marginal net products do not coincide, which may happen in two situations:

- when the social marginal benefit differs from the private marginal benefit;
- when the social marginal cost differs from the private marginal cost;

Since the environmental problem boils down to a discrepancy between the social and private marginal product, a tax policy can correct this gap:

"When competition rules and social and private net product at the margin diverge, it is theoretically possible to put matters right by the imposition of a tax or the grant of a subsidy." (Pigou [1920] 1952, 381)

$\rightarrow$ this is the origin of the so-called "Pigovian-tax", or "polluter pays principle": by setting a price on pollution, its impact on the polluted can be accounted for by the polluter, private and social interests can be aligned.

The interaction of tools and problems in economics

After Pigou, economics has for long remained silent about environmental issues. It's only from the 1960's that environmental economics started to progressively emerge as a field. Why?

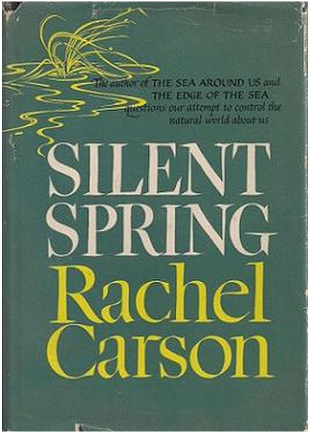
According to Sandmo (2015):

- the growth of environmental problems, which became more and more pressing issues;
- the increase in standards of living which led to give a higher relative importance to environmental issues;
- for a long time, economists have mainly focused on the core topics of the discipline, and the environment was not considered as one. The discipline has then expended to cover much more topics (e.g. education, health);
- economists took time to realize that they had the tools to think about environmental problems. Koopmans 1957, 170: *"The solution of important problems may be delayed because the requisite tools are not perceived. Or the availability of certain tools may lead to an awareness of problems, important or not, that can be solved with their help"*.

1952: The Great Smog of London

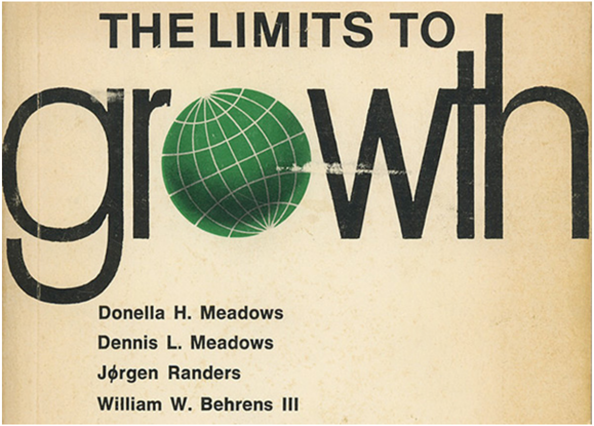
Between December 5 and 9, 1952, special weather conditions (no wind and temperature inversion) lead to high emissions of air pollutants (mostly from coal) trapped under a low layer. Estimates of the time pointed to 100,000 people injured and 4,000 deaths. Recent estimates suggest 12,000 deaths. Critical episode that raised awareness of the consequences of human activities on pollution and health.

1962: publication of *Silent Spring*



The book warns about the effect of pesticides on the environment, impacting animals and the whole food supply. It led to new agricultural policies and was key in the development of the environmental movement and the US Environmental Protection Agency (EPA).

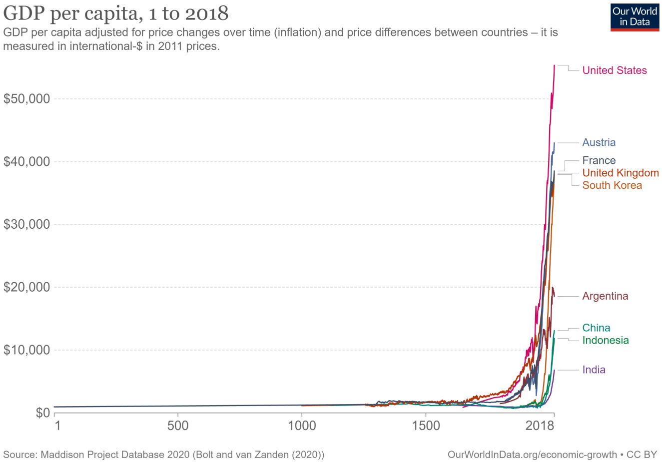
1972: publication of *The Limits to Growth*



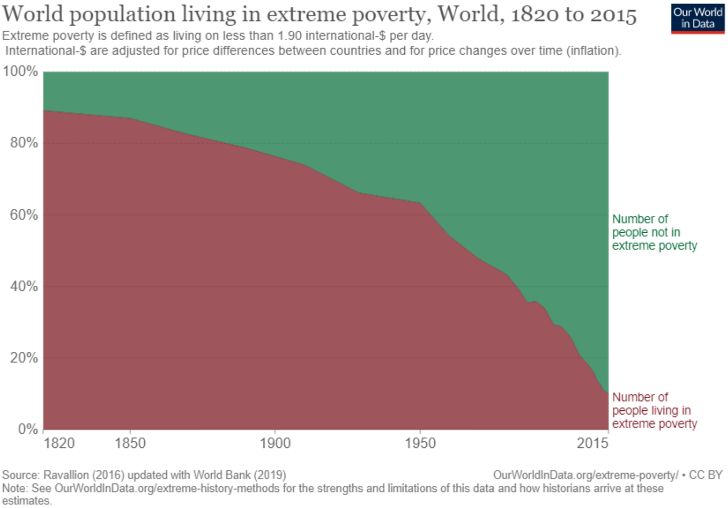
Controversial report from the Club of Rome. From computer simulations, conclude that resource depletion would limit economic growth. The publication coincided with the first oil shock of 1973, raising further attention about its conclusions.

Table of Contents

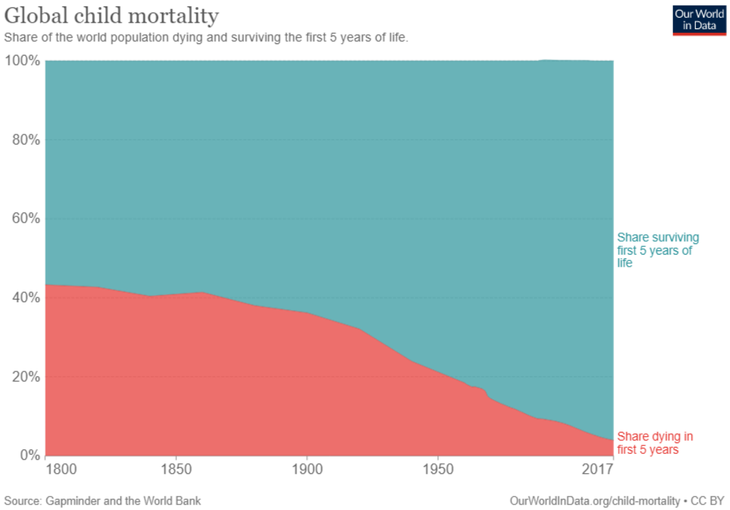
GDP over the long run



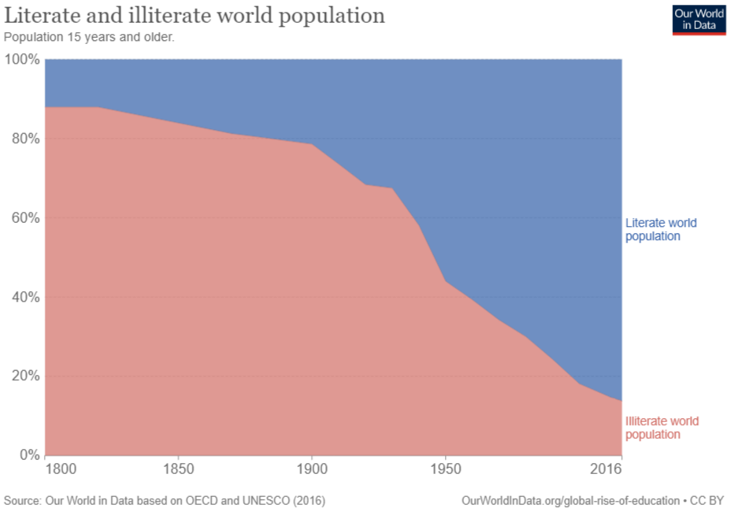
Economic growth took off in the late XVII$^{\text{th}}$ century: unprecedented phenomenon.



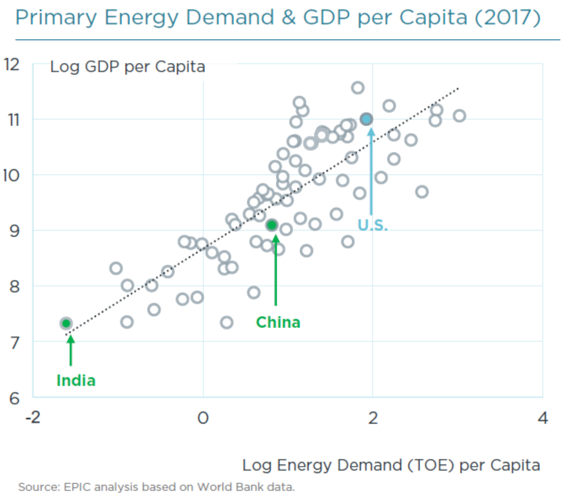
At the same time, many people lifted out of extreme poverty (...)



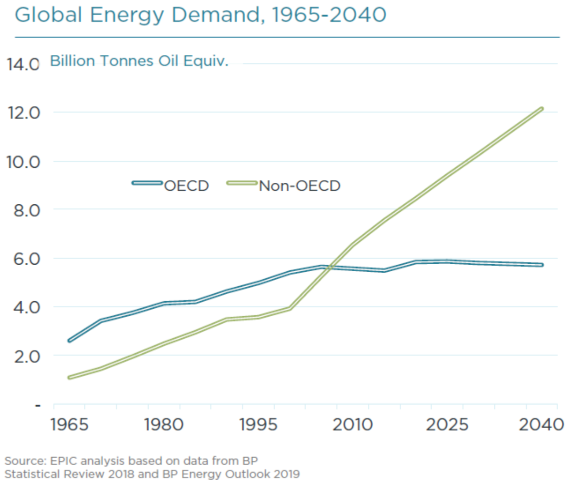
(...) which also led to better health conditions (...)



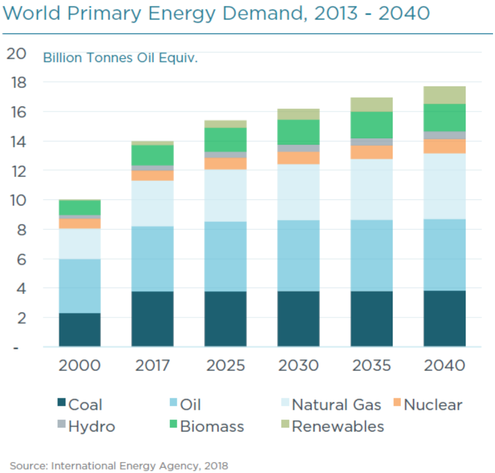
(...) and other benefits such as higher levels of education.



Economic growth has gone hand in hand with energy demand. So far, weak evidence of absolute "decoupling" (...)

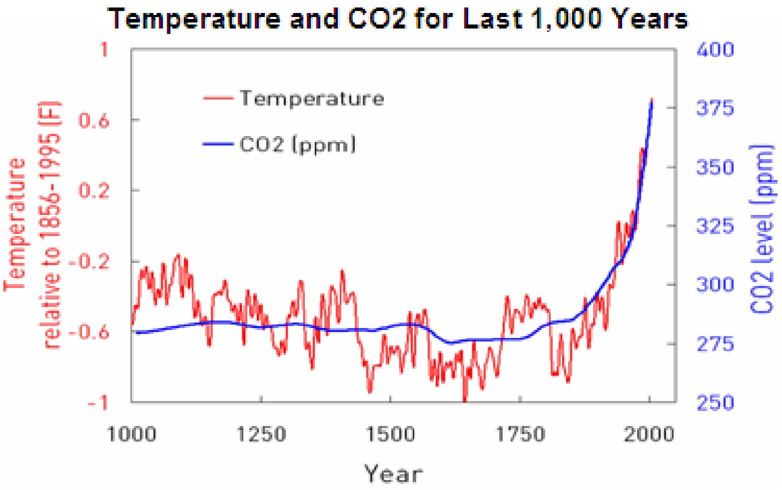


(...) and so we can forecast even greater energy demand in the coming decades.



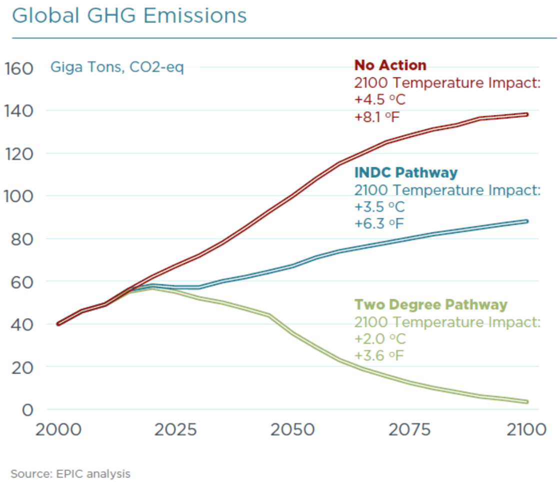
Forecasts also suggest that this demand will in part be met by more fossil fuels. Of course, the likelihood of this scenario largely depends on which policies are implemented in between.

Trends in CO_2 concentration and global average temperature



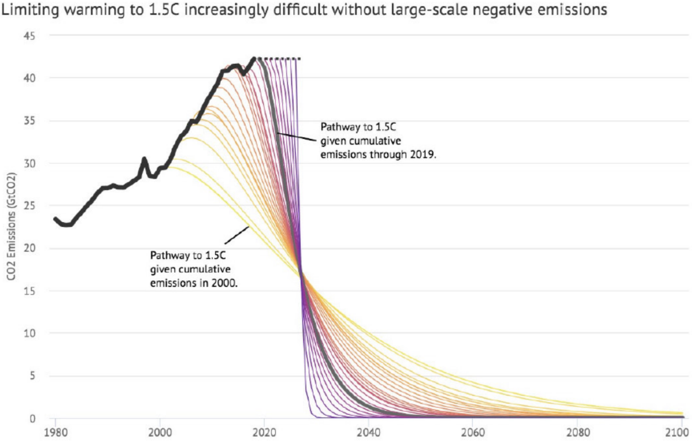
Source: Environmental Defense Fund

Climate change: what is to be expected



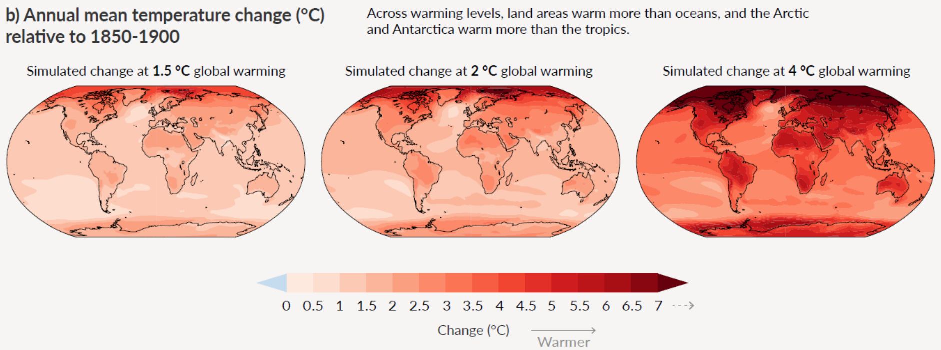
Different future scenarios are possible. Note: these figures mask great uncertainty, especially as emissions increase.

Climate change: pathways to +1.5°C limit warming



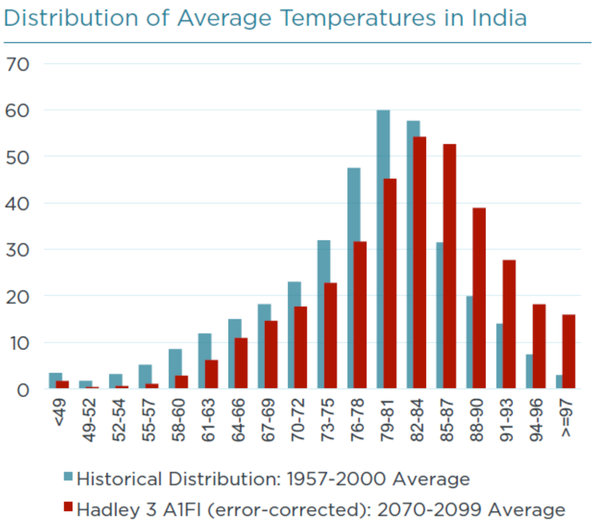
The +1.5°C target is extremely unlikely to be achieved without negative emissions.

Average temperature increase masks spatial heterogeneity (...)

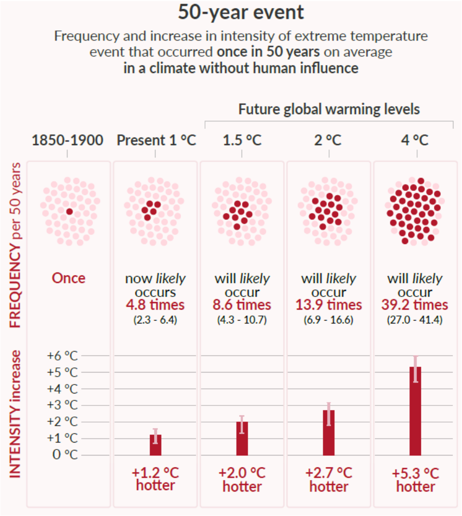


Source: IPCC, AR6

(...) as well as time heterogeneity

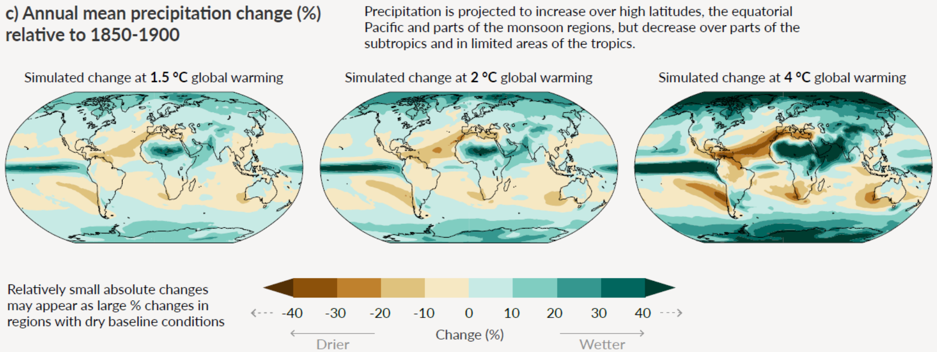


Expected frequency of extreme temperatures



Source: IPCC, AR6

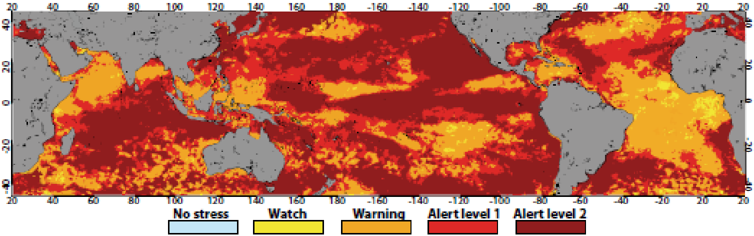
Beyond temperatures, precipitations are greatly affected as well



Source: IPCC, AR6

Current state of oceans' warming

Figure SPM.3. Map showing the maximum heat stress experienced during the 2014–2017 global coral bleaching event

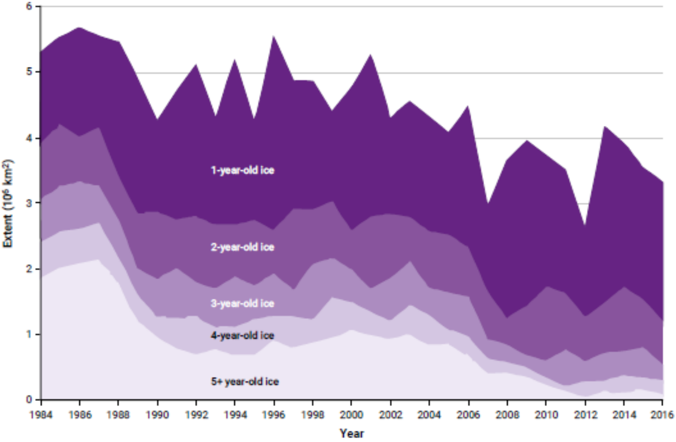


Source: National Oceanic and Atmospheric Administration (2017).

Note: Alert level 2 heat stress indicates widespread coral bleaching and significant mortality; alert level 1 heat stress indicates significant coral bleaching; lower levels of stress may also have caused some bleaching.

Short-term evolution of Arctic ice by age

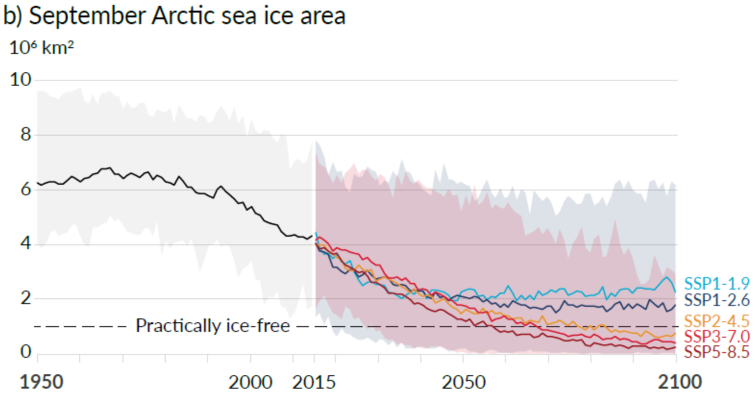
Figure SPM.6. Reduction in the extent of Arctic sea ice by age



Source: United States National Snow and Ice Data Center (2017).

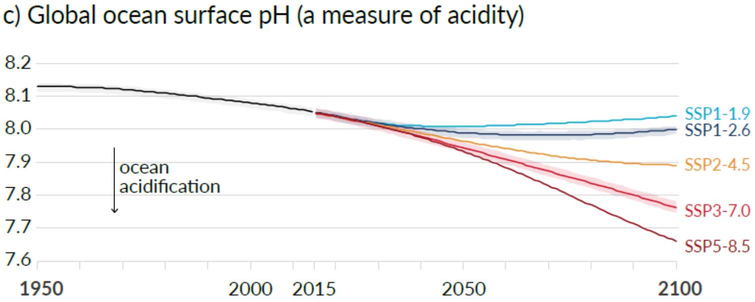
Note: A few decades ago, a large proportion of Arctic sea ice survived the summer melt. In 1984, more than a third of sea ice was older than five years. Figure SPM.6 shows the sharp reduction in sea ice of that age since then.

Expected evolution of Arctic ice



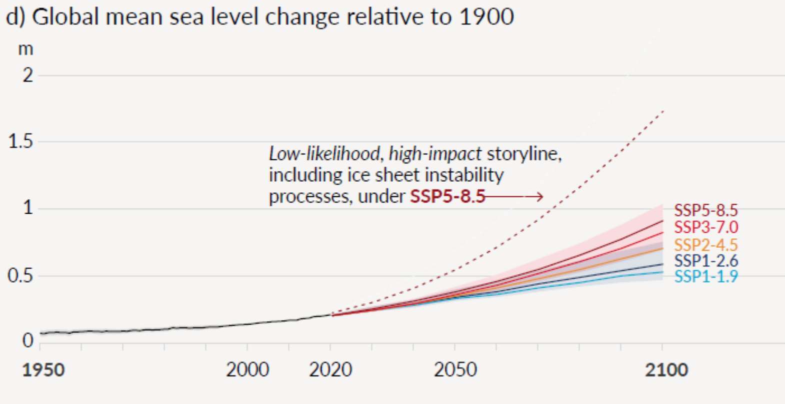
Source: IPCC, AR6

Expected acidification of oceans



Source: IPCC, AR6

Sea level rise



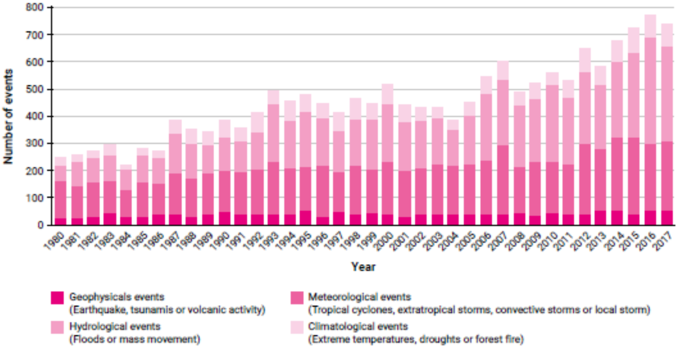
Source: IPCC, AR6

Very slow process, that will continue well beyond the XXIth century. By 2300, great uncertainty: "sea level rise greater than 15m cannot be ruled out with high emissions" (IPCC, AR6).

Some consequences of +7 meters

Short-term evolution of environmental disasters

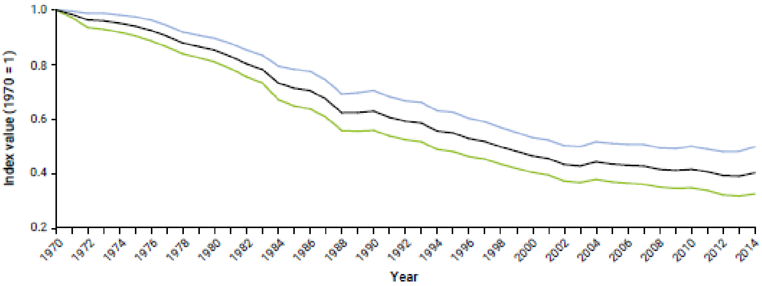
Figure SPM.7. Trends in numbers of loss-related natural events



Source: Munich Re (2017).

Short-term evolution of biodiversity

Figure SPM.2. Global Living Planet Index

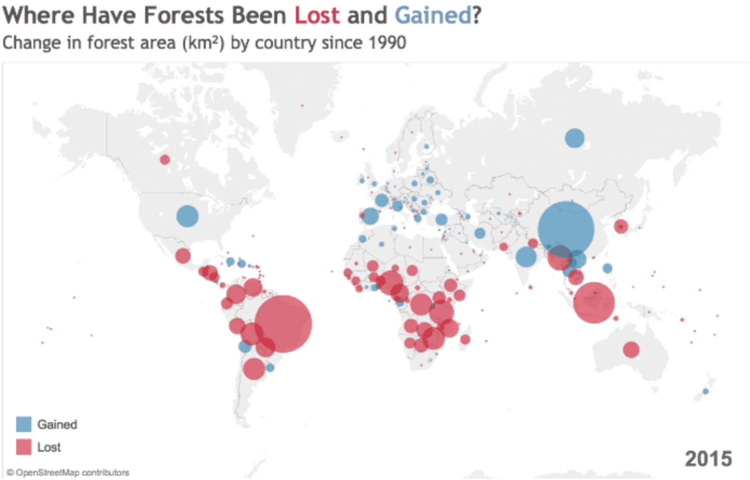


Source: World Wide Fund for Nature and Zoological Society of London (2018).

Note: The centre line shows the index values, indicating a 60 per cent decline between 1970 and 2014, and the upper and lower lines represent the 95 per cent confidence limits surrounding the trend. This is the average change in population size of 4,005 vertebrate species, based on data from 16,704 times series from terrestrial, freshwater and marine habitats.

Multiple causes: deforestation, land use, agricultural practices (c.f. *Silent Spring*), climate change, etc.

Deforestation around the world



Source: World bank data. Note: new forests do not substitute for the old-growth forests that have much richer ecosystems.

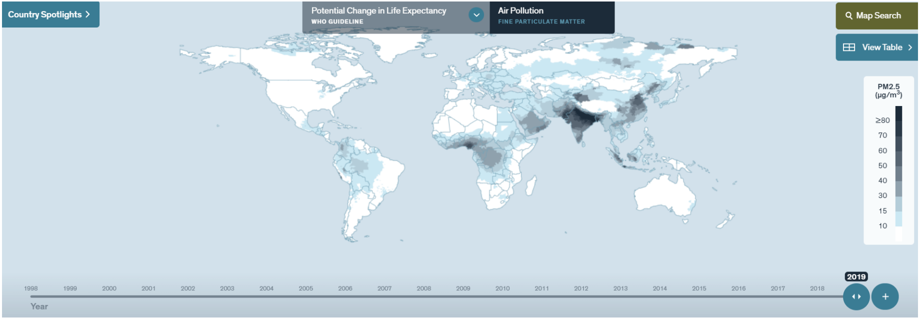
Water scarcity and women labor

Figure SPM.5. Summary of global progress in providing basic drinking water services and the disproportionate impact on women in sub-Saharan countries who still lack access to basic drinking water services

Source: UNICEF and WHO (2012); WHO and UNICEF (2017).

→ Economic development has lifted out of poverty billions of people in the past decades. Still, in several regions of the world living conditions remain difficult and may be further impacted by environmental degradation, such as droughts and reduced agricultural yields. Tight link between environmental and livig conditions.

Heterogeneous concentration of air pollutants



Compared to CO_2, relatively short-lived and more local pollutants:

- makes it easier for policies;
- but clearly not a second order issue!

Assessing the impact of air pollution on mortality

How to estimate the causal impact of long-term exposure to air pollution? Many confounding factors!

Ebenstein et al (2017): exploit quasi-experimental variation in particulate matters generated by China's Huai River Policy.

- cities north of the river get free or heavily subsidized coal for indoor heating during the winter;
- cities south of the river don't.

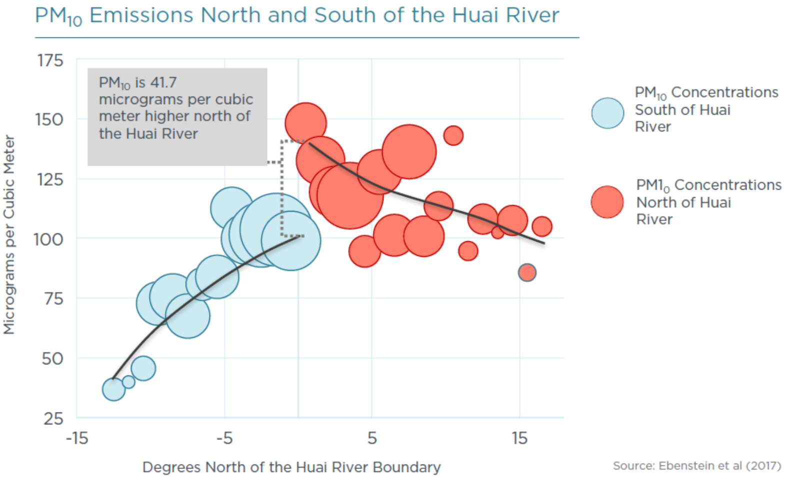
$\rightarrow$ Regression discontinuity design based on distance from the Huai River. Formally:

$$PM_j = \alpha_0 + \alpha_1 N_j + f(L_j) + N_j f(L_j) + X_j \gamma + u_j \quad (1)$$

$$Y_j = \beta_0 + \beta_1 \hat{PM}_j + f(L_j) + N_j f(L_j) + X_j \psi + \epsilon_j \quad (2)$$

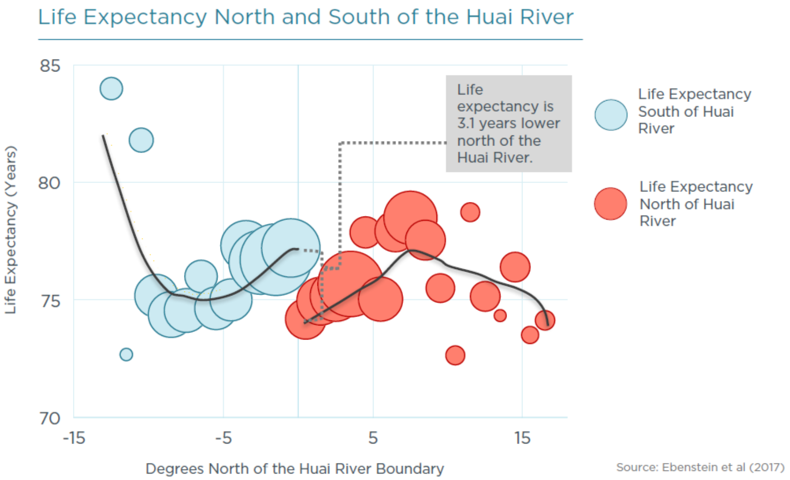
where Y captures the mortality rate, N is a north/south of the river dummy, and L captures the distance to the river.

Huai River policy: effect on PM concentration



Pollution is 40% higher on the north side of the river.

Huai River policy: effect on life expectancy



Life expectancy is about 3 years lower north of the River

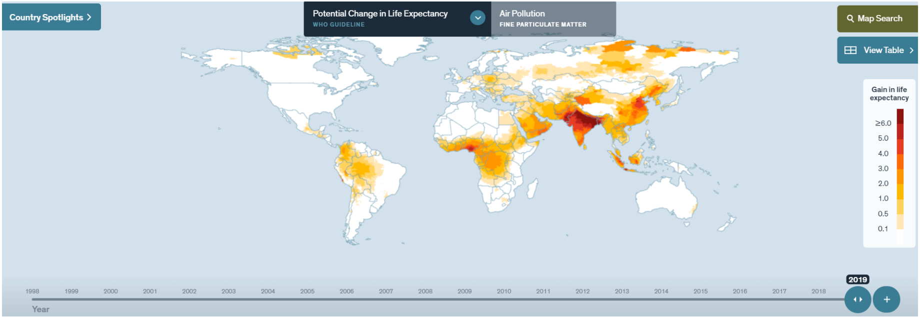
Huai River policy: econometric results

Table 2. RD estimates of the impact of the Huai River Policy

Outcome	[1]	[2]	[3]
Pollution and life expectancy			
PM_{10}	27.4*** (9.5)	31.8*** (9.1)	41.7*** (12.9)
Life expectancy at birth, y	−2.4** (1.0)	−2.2* (1.1)	−3.1*** (0.9)
Cause-specific mortality (per 100,000, log)			
Cardiorespiratory	0.30** (0.14)	0.22* (0.13)	0.37*** (0.11)
Noncardiorespiratory	0.06 (0.10)	0.08 (0.09)	0.13 (0.08)
RD type	Polynomial	Polynomial	LLR
Polynomial function	Third	Linear	
Sample	All	5°	

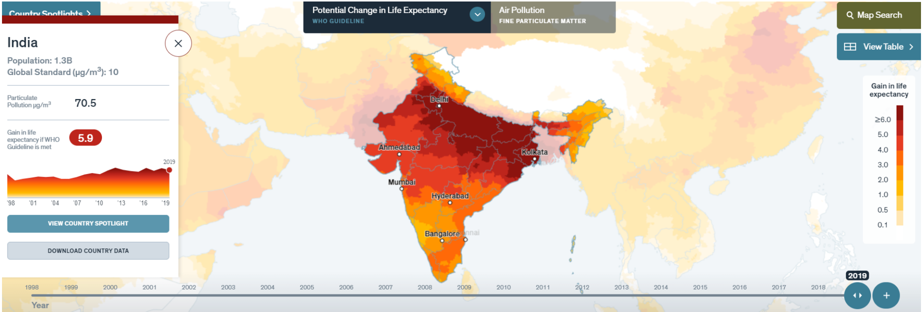
Column [1] reports OLS estimates of the coefficient on a north of the Huai River dummy after controlling for a polynomial in distance from the Huai River interacted with a north dummy using the full sample (n = 154) and the control variables from *SI Appendix*, Table S1. Column [2] reports this estimate for the restricted sample (n = 79) of DSP locations within 5° of the Huai River. Column [3] presents estimates from local linear regression (LLR), with triangular kernel and bandwidth selected by the method proposed by Imbens and Kalyanaraman (14).
*Significant at 10%.
**Significant at 5%.
***Significant at 1%.

The effect of air pollution on life expectancy – World



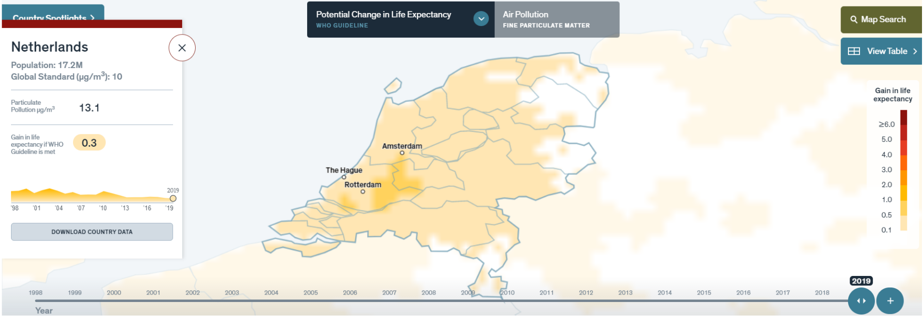
Very heterogeneous impacts: reflect heterogeneous levels of development and population density.

The effect of air pollution on life expectancy – India



Very high and heterogeneous impacts, with an upward trend.

The effect of air pollution on life expectancy – Netherlands



Great progress since the times of the Great smog in developed economies. Still, not a minor issue.

Air pollution and other mortality causes

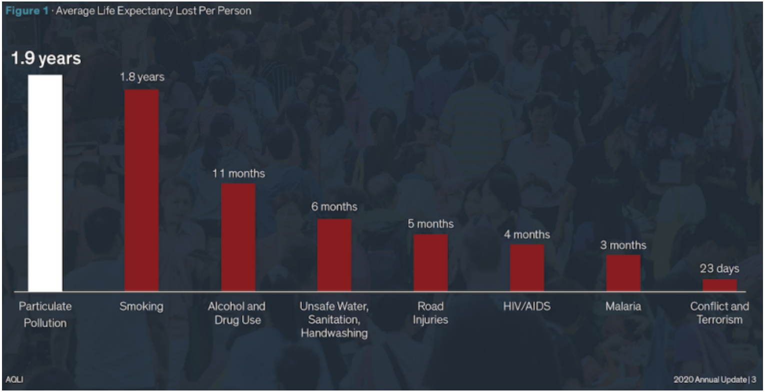


Table of Contents

Economic efficiency: definition

Welfare economics is interested in the allocation of resources and how it relates to social well-being. One of the critical concepts in welfare economics is *economic efficiency*.

Simple definition from Perman et al: *"An allocation of resources is said to be efficient if it is not possible to make one or more persons better off without making at least one other person worse off."*

For an allocation to be efficient in a production economy, three conditions must be met:

1. efficiency in consumption;
   - MRS must be equalized
2. efficiency in production;
   - MRT must be equalized
3. product-mix efficiency.
   - MRS must be equal to MRT

Multiple efficient allocations

An efficient allocation is by no mean unique: in the figure below, all allocations on the CC curve are efficient $\rightarrow$ for all these allocations, it is not possible to increase the utility of A or B without hurting the other.

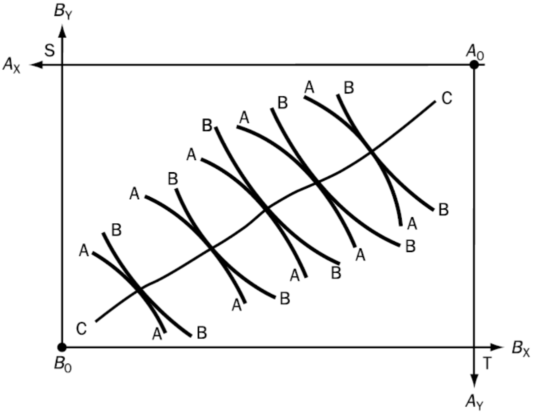


Figure 5.4 The set of allocations for consumption efficiency

Source: Perman et al.

The social welfare function and optimal allocations

Social welfare function: it is a function that ranks allocations according to their level of social desirability.

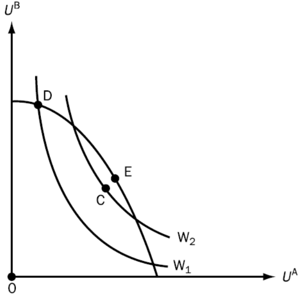


Figure 5.7 Welfare and efficiency

Source: Perman et al.

Note: in the previous figure, allocation C is preferred to D, yet D is efficient while C is not. E is efficient and is preferred to both C and D. $\rightarrow$ Improving economic efficieny will not always lead to a more desirable social outcome.

Effiency, optimality, and the benefits of a reform

In practice, how to decide whether a reform is desirable?

- Option 1: compare the outcome of the SWF with the *ex ante* and *ex post* allocations and implement the reform *iff* social welfare increases.
  - Problem: ideally, the social planner knows the SWF. In practice, institutions reflect imperfectly social preferences and there is no straightforward rule to aggregate them.
- Option 2: implement a reform *iff* it is Pareto improving.
  - Problem: situations in which Pareto improvements are possible are very scarce.
- Option 3: implement a reform *iff* the winners can compensate the losers and still be better off (Kaldor-Hicks improvement).
  - Problem: if these transfers are theoretically possible but do not happen, tensions may emerge between winners and losers.

$\rightarrow$ Judging about the desirability of a policy change is always very difficult and requires an important part of subjectivity. When in addition there is uncertainty about the resulting allocations (i.e. always), it gets even more complex.

First welfare theorem

Assumptions: Let's consider a competitive economy characterized by the following assumptions:

- markets are complete;
- markets are perfectly competitive;
- agents have perfect information;
- all resources are subject to private property rights;
- there are no externalities;
- there are no public goods;
- utility and production functions are "well-behaved".
- consumers maximize their utility, firms maximize their profits.

Theorem (not a formal statement): *any equilibrium allocation is Pareto efficient.*

First welfare theorem: caveats

The first welfare theorem is a very strong result that sets a useful benchmark to understand markets' properties. **Two caveats** are in order:

1. **The previous assumptions are extremely strong**: they depict an idealized economy. In reality, they are never fully met. Thus, we can hardly expect market equilibria to actually produce efficient allocations.
2. **Efficiency is not optimality**: under the previous assumptions, a competitive equilibrium will be efficient. This says nothing about its optimality or social desirability.

$\rightarrow$ Two critical roles for economic policies: (1) restoring efficiency, (2) redistributing resources to get closer to optimality.

Environmental economists are generally concerned with the first objective, i.e. correcting market failures due to environmental externalities. In theory, the second problem can be dealt with separately (see Second welfare theorem). In practice however, the question of redistribution can rarely be addressed independently (see lectures 4 and 5).

Environmental policies to correct externalities

- When thinking about an environmental problem, it is often useful to start from a simplistic benchmark and assume that the externality is the only market failure.
- From there, one can think about the policy instruments that could be used to deal with it.
- Then, one needs to consider the externality and potential policies in a more realistic context where multiple market failures exist and potentially interact.
- In both cases, the objective of environmental economists will be two-fold:
  - quantify the relative importance of the externality;
  - determine how to correct it, given the constraints imposed by the economic environment.

→ These two objectives will be the subject of lectures 2 and 3 to 6 respectively.

Table of Contents

Road-map

The course will be divided into six lectures covering the following themes:

- Lecture 1 – The economic approach to the environment
- Lecture 2 – Determining regulation targets
- Lecture 3 – Environmental policies: the social planner, the state, and governments
- Lecture 4 – Environmental policies: efficiency and equity
- Lecture 5 – Environmental policies: public support
- Lecture 6 – Environmental policies: trade, firms, and technologies

We will cover the two main questions of environmental economics: by how much should we reduce pollution from economic activities? (lecture 2) And how to attain these pollution targets, i.e. which policies are preferable and what are their obstacles? (lectures 3 to 6)

Assessment

You will receive three grades:

- two from take-home exercises:
  - the problem set of tutorial 4 will be graded;
  - one critical review of an article (due date: week 6).
- one from the final exam:
  - writing a policy report. Specific guidelines will be provided later in the course.

For tutorial 4: you can work in groups, as long as the exercise is handed in individually.

Table of Contents

A simple economy

Assumptions: to keep things simple, let's assume the economy is made of:

- two final goods X and Y;
- produced from two inputs K and L;
- and consumed by two individuals A and B.

We can write the utility functions of individual A and B as:

$$U^A(X^A, Y^A) \quad ; \quad U^B(X^B, Y^B)$$

and the production function of goods X and Y as:

$$X(K^X, L^X) \quad ; \quad Y(Y^K, Y^L)$$

(everything we will show can easily be extended to an arbitrarily large number of individuals, goods, and inputs)

Efficient allocation: the problem

To study efficient allocations, we apply the Pareto principle: we maximize the utility of a given individual subject to other individuals' utility being higher than a given threshold. In our case, this gives:

$$\max_{X^A, Y^A} U^A(X^A, Y^A) \tag{3}$$

subject to:

$$U^B(X^B, Y^B) \geq \bar{U} \tag{4}$$

$$X_1(K_1^X, L_1^X) + X_2(K_2^X, L_2^X) \geq X^A + X^B \tag{5}$$

$$Y_1(K_1^Y, L_1^Y) + Y_2(K_2^Y, L_2^Y) \geq Y^A + Y^B \tag{6}$$

$$K \geq K_1^X + K_2^X + K_1^Y + K_2^Y \tag{7}$$

$$L \geq L_1^X + L_2^X + L_1^Y + L_2^Y \tag{8}$$

Efficient allocation: the problem

The previous problem can be solved using the following Lagrangian:

$$
\begin{aligned}
\mathcal{L} = U^A(X^A, Y^A) + \lambda_1(U^B(X^B, Y^B) - \bar{U}) \\
+\lambda_2(X_1(K_1^X, L_1^X) + X_2(K_2^X, L_2^X) - X^A - X^B) \\
+\lambda_3(Y_1(K_1^Y, L_1^Y) + Y_2(K_2^Y, L_2^Y) - Y^A - Y^B) \\
+\lambda_4(K - K_1^X - K_2^X - K_1^Y - K_2^Y) \\
+\lambda_5(L - L_1^X - L_2^X - L_1^Y - L_2^Y)
\end{aligned}
$$

Efficiency in consumption

First order conditions:

$$\frac{\partial \mathcal{L}}{\partial X^A} = \frac{\partial U^A}{\partial X^A} - \lambda_2 = 0 \tag{9}$$

$$\frac{\partial \mathcal{L}}{\partial Y^A} = \frac{\partial U^A}{\partial Y^A} - \lambda_3 = 0 \tag{10}$$

$$\frac{\partial \mathcal{L}}{\partial X^B} = \lambda_1 \frac{\partial U^B}{\partial X^B} - \lambda_2 = 0 \tag{11}$$

$$\frac{\partial \mathcal{L}}{\partial Y^B} = \frac{\partial U^B}{\partial Y^B} - \lambda_3 = 0 \tag{12}$$

If we rearrange these conditions we obtain:

$$\frac{\frac{\partial U^A}{\partial X^A}}{\frac{\partial U^A}{\partial Y^A}} = \frac{\lambda_2}{\lambda_3} = \frac{\frac{\partial U^B}{\partial X^B}}{\frac{\partial U^B}{\partial Y^B}} \tag{13}$$

$\rightarrow$ Consumption efficiency requires that marginal rates of utility substitution equalize.

Efficiency in consumption

Graphically:

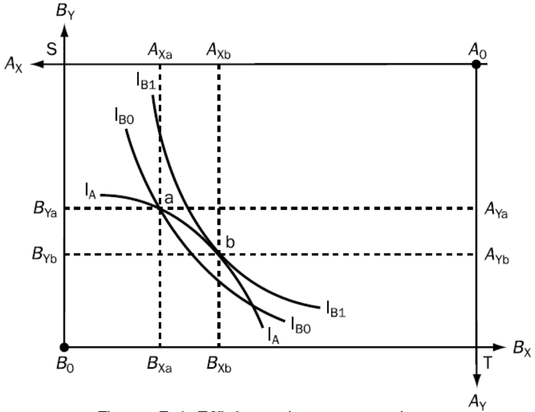


Figure 5.1 Efficiency in consumption

Source: Perman et al.

Efficiency in consumption

Graphically:

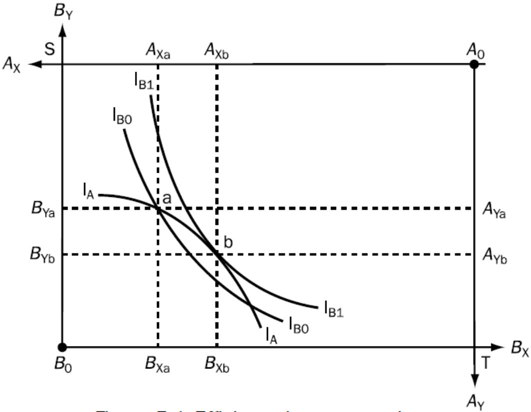


Figure 5.1 Efficiency in consumption

Source: Perman et al.

$\rightarrow$ a is not an efficient allocation because B can be made better off without hurting A. b is efficient because we cannot make A or B better off without hurting the other. The two indifferent curves have identical slopes at b.

Efficiency in production

Similarily, one can take the first order conditions w.r.t. K_1^X, K_2^X, L_1^X, L_2^X, K_1^Y, K_2^Y, L_1^Y, L_2^Y and show that production efficiency requires that marginal rates of technical substitution equalize (see Perman et al, appendix of chapter 5):

$$\frac{\frac{\partial X_1}{\partial L_1^X}}{\frac{\partial X_1}{\partial K_1^X}} = \frac{\frac{\partial X_2}{\partial L_2^X}}{\frac{\partial X_2}{\partial K_2^X}} = \frac{\frac{\partial Y_1}{\partial L_1^Y}}{\frac{\partial Y_1}{\partial K_1^Y}} = \frac{\frac{\partial Y_2}{\partial L_2^Y}}{\frac{\partial Y_2}{\partial K_2^Y}} \tag{14}$$

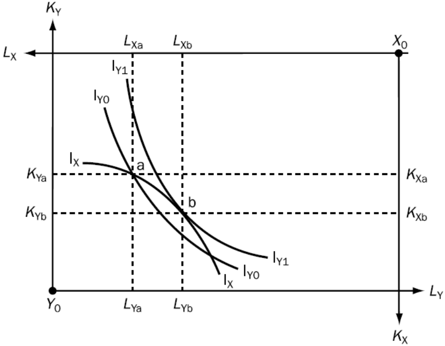


Figure 5.2 Efficiency in production

Source: Perman et al.

Product-mix efficiency

Finally, rearranging the FOCs one can also show that for an allocation to be efficient, the marginal rates of transformation must equalize the marginal rate of utility substitution:

$$\frac{\frac{\partial Y}{\partial K}}{\frac{\partial X}{\partial K}} = \frac{\frac{\partial Y}{\partial L}}{\frac{\partial X}{\partial L}} = \frac{\frac{\partial U^A}{\partial X^A}}{\frac{\partial U^A}{\partial Y^A}} = \frac{\frac{\partial U^B}{\partial X^B}}{\frac{\partial U^B}{\partial Y^B}} \tag{15}$$

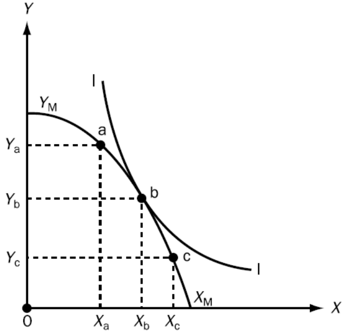


Figure 5.3 Product-mix efficiency

Source: Perman et al.

The social welfare function and optimal allocations

Social welfare function: it is a function that ranks allocations according to their level of social desirability. In our case, a social welfare function takes the form $W = W(U^A, U^B)$ and is non-decreasing in both arguments. At the optimum we must have:

$$\frac{\frac{\partial W}{\partial U_A}}{\frac{\partial W}{\partial U_B}} = \frac{\frac{\partial U_B}{\partial X_B}}{\frac{\partial U_A}{\partial X_A}} = \frac{\frac{\partial U_B}{\partial Y_B}}{\frac{\partial U_A}{\partial Y_A}}$$

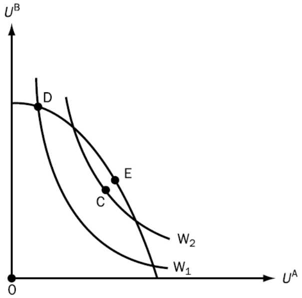


Figure 5.7 Welfare and efficiency

Source: Perman et al.

Externalities: definition and classification

One particular case of market failure is the existence of externalities, i.e. situations in which individuals produce by their actions external effects that affect the well-being of other individuals, without any counterpart. They can be classified as follows:

Table 5.6 Externality classification

Arising in	Affecting	Utility/production function
Consumption	Consumption	$U^{A}(X^{A}, Y^{A}, X^{B})$
Consumption	Production	$X(K^{X}, L^{X}, Y^{A})$
Consumption	Consumption and production	$U^{A}(X^{A}, Y^{A}, X^{B})$ and $Y(K^{Y}, L^{Y}, X^{B})$
Production	Consumption	$U^{A}(X^{A}, Y^{A}, X)$
Production	Production	$X(K^{X}, L^{X}, Y)$
Production	Consumption and production	$U^{A}(X^{A}, Y^{A}, Y)$ and $X(K^{X}, L^{X}, Y)$

Source: Perman et al.

Note: although other market failures may be relevant to the study of environmental issues, externalities is the most common and will be our main focus. We will later see examples where externalities interact with other market failures (imperfect competition, asymmetric information, non-maximizing agents, etc.).

Externalities: a formal example

Let's consider an endowment economy – that is a simplified version of our production economy – with two consumers A and B and two goods X and Y. Assuming no externality, the utility of A and B can be expressed as:

$$U^A(X^A, Y^A) \quad ; \quad U^B(X^B, Y^B)$$

Let's now assume that when A consumes good X, it also affects the welfare of B. Then, their respective utility functions write:

$$U^A(X^A, Y^A) \quad ; \quad U^B(X^B, Y^B, {\color{red}X^A})$$

$\rightarrow X^A$ now enters the utility of B, but X^A is still chosen by A only.

Efficient allocation with a consumption externality

The objective is to maximize $U^A(X^A, Y^A)$, subject to:

- $U^B(X^B, Y^B, X^A) \geq \bar{U}$
- $\bar{X} \geq X^A + X^B$
- $\bar{Y} \geq Y^A + Y^B$

We write the following Lagrangian:

$$\begin{aligned}\mathcal{L} = U^A(X^A, Y^A) + \lambda(U^B(X^B, Y^B, X^A) - \bar{U}) \\ +\mu(\bar{X} - X^A - X^B) + \delta(\bar{Y} - Y^A - Y^B)\end{aligned}$$

Efficient allocation with a consumption externality

The first order conditions give:

$$\frac{\partial L}{\partial X^A} = \frac{\partial U^A}{\partial X^A} + \lambda \frac{\partial U^B}{\partial X^A} - \mu = 0 \tag{16}$$

$$\frac{\partial L}{\partial X^B} = \lambda \frac{\partial U^B}{\partial X^B} - \mu = 0 \tag{17}$$

$$\frac{\partial L}{\partial Y^A} = \frac{\partial U^A}{\partial Y^A} - \delta = 0 \tag{18}$$

$$\frac{\partial L}{\partial Y^B} = \lambda \frac{\partial U^B}{\partial Y^B} - \delta = 0 \tag{19}$$

(+ Kuhn-Tucker conditions)

Efficient allocation with a consumption externality

Substituting, we get:

$$\frac{\partial U^B}{\partial X^B}\frac{\partial Y^B}{\partial U^B} = \frac{\partial U^A}{\partial X^A}\frac{\partial Y^A}{\partial U^A} + \lambda\frac{\partial U^B}{\partial X^A}\frac{\partial Y^A}{\partial U^A} \tag{20}$$

Equation 20 above is necessary to obtain an efficient allocation in this economy. The red term corresponds to the externality created by the consumption of X^A on consumer B. The social impact of this consumption is therefore different than its private impact.

Market equilibrium allocation with a consumption externality

Objective : agent $i = A, B$ maximizes U^i under its budget constraint, that is, $w_i \geq pX^i + Y^i$ where p is the price of X and Y is the numéraire. The two Lagrangians write:

$$\mathcal{L}^A = U^A(X^A, Y^A) + \lambda(w^A - pX^A - Y^A) \tag{21}$$

$$\mathcal{L}^B = U^B(X^B, Y^B, X^A) + \lambda(w^B - pX^B - Y^B) \tag{22}$$

Since A only decides on X^A and Y^A and B only decides on X^B and Y^B, for $i = A, B$ the FOCs give:

$$\frac{\partial \mathcal{L}^i}{\partial X^i} = \frac{\partial U^i}{\partial X^i} - \lambda p = 0 \tag{23}$$

$$\frac{\partial \mathcal{L}^i}{\partial Y^i} = \frac{\partial U^i}{\partial Y^i} - \lambda = 0 \tag{24}$$

(+ Kuhn-Tucker conditions) which leads to:

$$p = \frac{\partial U^B}{\partial X^B}\frac{\partial Y^B}{\partial U^B} = \frac{\partial U^A}{\partial X^A}\frac{\partial Y^A}{\partial U^A} \neq \frac{\partial U^A}{\partial X^A}\frac{\partial Y^A}{\partial U^A} + \lambda\frac{\partial U^B}{\partial X^A}\frac{\partial Y^A}{\partial U^A} \tag{25}$$

$\rightarrow$ the competitive equilibrium (on the left of $\neq$) ignores the externality, and therefore leads to an inefficient allocation.