Tutorial 1 – An introduction to environmental economics

Thomas Douenne – University of Amsterdam

Let's consider an economy composed of H different types of individuals. We asssume a unit size population with n_h individuals of type h (i.e. $\sum_{h=1}^{H} n_h = 1$). Individuals differ in two ways. First, with respect to their productivity: we assume that individuals of type h exogenously supply l_h units of effective labor, with unit wage normalized to 1. Second, with respect to their preferences. Individual preferences are defined over two consumption goods, x and y, and aggregate pollution, E. x is a clean good and does not pollute. y is a dirty good and generates pollution. In particular, aggregate pollution is given by:

$$E = \sum_{h=1}^{H} n_h e_h \tag{1}$$

with $e_h = \phi y_h$ where ϕ is the pollution intensity of the dirty good. We assume that the dirty good is a necessity: individuals of type h need to consume a minimum amount of $\bar{y}_h$ and experience utility from that good only when they consume more than this level. In particular, we assume that preferences can be represented by the following utility function:

$$u^h(x_h, y_h, E) = x_h + \gamma \ln(y_h - \bar{y}_h) - \chi E \tag{2}$$

In addition of individual consumers, we also suppose that in both sectors (clean and dirty) there is one representative competitive firm producing from a linear technology such that producing one unit of good x (resp. y) requires a_x (resp. a_y) units of effective labor.

Decentralized problem

We first consider the behavior of private agents under a laissez-faire scenario.

1. Write down the profit function of both firms. Keeping in mind that both sectors are competitive, show that in equilibrium the firms will sell at prices $p_x = a_x$ and $p_y = a_y$.

2. Write down agent h problem and the associated Lagrangian. Show that the first order conditions give:

$$\frac{\frac{\partial u^h}{\partial x_h}}{\frac{\partial u^h}{\partial y_h}} = \frac{p_x}{p_y} \tag{3}$$

3. Using equation (2), show that the equilibrium demands for x_h and y_h are given by:

$$x_h = \frac{l_h - p_y \bar{y}_h}{p_x} - \gamma \tag{4}$$

$$y_h = \bar{y}_h + \gamma \frac{p_x}{p_y} \tag{5}$$

4. Show that the aggregate level of pollution in the decentralized equilibrium is:

$$E^{eq} = \phi\Big(\gamma \frac{a_x}{a_y} + \sum_h n_h \bar{y}_h\Big) \tag{6}$$

Central planner's problem

Let's now consider a central planner deciding on the optimal allocation. We assume that the planner gives equal weight to the utility of each individual.

5. Write down the planner's problem — *i.e.* its objective function, the feasibility constraint, and the pollution constraint — and express it as a Lagrangian.

6. Show that for any individual of type h the first order conditions give:

$$\frac{\partial u^h}{\partial y_h} = \frac{a_y}{a_x}\frac{\partial u^h}{\partial x_h} - \phi \sum_{k=1}^{H} n_k \frac{\partial u^k}{\partial E}. \tag{7}$$

7. For any individual of type h, show that the consumption of dirty good at the optimum (y_h^*) is given by:

$$y_h^* = \bar{y}_h + \gamma \frac{a_x}{a_y + \phi \chi a_x} \tag{8}$$

8. Explain why we cannot derive such expression for x_h^*.

9. Compute E^*, the optimal level of pollution, and compare it to E^{eq}.

Government's problem

Let's now assume that a government cannot choose all allocations, but can tax the polluting good, so that its final price is $q_y = p_y + t_y$. Also assume that the government can redistribute the tax revenue through a uniform lump-sum transfer T to all agents.

10. Write down the government budget constraint.

11. Show that in equilibrium, the firms sell at prices $q_x = a_x$ and $q_y = a_y + t_y$.

12. Show that the equilibrium demands for x_h and y_h are given by:

$$x_h = \frac{l_h + T - q_y \bar{y}_h}{q_x} - \gamma \tag{9}$$

$$y_h = \bar{y}_h + \gamma \frac{q_x}{q_y} \tag{10}$$

13. Compute the level of tax t_y necessary for the government to reach the optimal level of pollution.

14. We denote σ_h the effort rate on the pollution tax, defined as the ratio between an household contribution to this tax over its income. Compute the effort rate of this tax. If $\bar{y}_h$ is the same for all types h, how does the effort rate evolves with income?

15. Consider the case where $\bar{y}_h$ is a function of income, so that we can write $\bar{y}_h = \bar{y}(l_h)$. Under which condition over this function is the tax progressive?