Tips:

1) Distinguish **Variance** and **Standard Deviation** in the question!!!
2) Is the **Standard Deviation** *known* when choosing test statistics???

A4 Q5. A digital output voltmeter has an input range of 0 to 30V and displays three significant figures XX.X. The manufacture claims an accuracy of +/- 2% of full scale. With a voltage reading of 5 V, what are the percent uncertainties of the reading due to accuracy and resolution?

Manufacturer Accuracy = ±2.0% of full scale

= ±0.02(30V)

= ±0.6V

% uncertainty of accuracy = ± (0.6V/5V)(100) = ±12%

The resolution of the device is 0.1 Volts. With a reading of 5V,

% uncertainty of resolution = (0.1V/5V)(100) = 2% (or ±1%)

A4 Q12. In using a temperature probe, he following uncertainties were determined:

Hysteresis +/- 0.1 C

Linearization error +/- 0.2% of the reading

Repeatability +/- 0.1 C

Resolution error +/- 0.1 C

Zero offset +/- 0.1 C

Determine the type of these errors (random or systematic) and the total uncertainty due to these effect for a temperature reading of 100 C.

12)

hysteresis	$\pm$ 0.1C	systematic
Lineariz.error	0.2% *of reading*	systematic
Resolution error	0.10C	random
zero off set	0.1C	systematic
repeatability	$\pm 0.1C$	random

$$B = (0.1^2 + [(.002)(100)]^2 + 0.1^2)^{1/2}$$
$$= 0.24C$$

Assuming that the random errors have been determined with samples>30,

$$P = (.10^2 + .1^2)^{1/2} = 0.14C$$

So total uncertainty

$$w = [B^2 + p^2]^{1/2} = [B^2 + (tS)^2]^{1/2}$$
$$w = [(0.24)^2 + (0.14)^2]^{1/2}$$
$$w = 0.28C$$

15. In measuring the power of a three phase electrical motor running a pump, the following data were obtained (based on one phase current measurement):

V (volt)	460	459	458	460	461	462	460	459
I (amp)	30.2	31.3	30.4	32.0	31.7	30.7	30.8	31.2
PF	0.78	0.80	0.79	0.82	0.81	0.77	0.78	0.80

For a three phase motor, the power P is related to the voltage V, current I and power factor PF by the formula

$$P = V \times I \times PF \times \sqrt{3}$$

a. Calculate the mean, the standard deviation and the random uncertainty of each parameter, V, I, PF and P at a 95% confidence level
b. Calculate the mean value of the power and the random uncertain of the man power at a 95% confidence level
c. If all the measured parameter have an accuracy of 1% of the reading of the measuring device, calculate the systematic uncertain in the power measurement at a 95% confidence level
d. Calculate the total uncertainty in the power measure at a 95% confidence level

15) **(a)**

$$\overline{V} = \frac{\sum V_i}{n} = 459.9 \qquad \overline{PF} = \frac{\sum PF_i}{n} = 0.79$$

$$S_V = [\frac{\sum (V_i - \overline{V})^2}{n-1}]^{1/2} = 1.25\,V \qquad S_{PF} = [\frac{\sum (PF_i - \overline{PF})^2}{n-1}]^{1/2} = 0.02$$

$$\bar{I} = \frac{\sum I_i}{n} = 31.0 \text{ amps} \qquad \bar{P} = \frac{\sum P_i}{n} = \frac{\sum V_i * I_i * PF_i \sqrt{3}}{n} = 19{,}629\, Watts$$

$$S_I = [\frac{\sum (I_i - \bar{I})^2}{n-1}]^{1/2} = 0.63\, amp \qquad S_P = [\frac{\sum (P_i - \bar{P})^2}{n-1}]^{1/2} = 787\, Watts$$

$P_V = tS_V = 2.95\,V$, $P_I = tS_I = 1.49$ amp, $P_{PF} = tS_{PF} = 0.05$, and $P_P = tS_P = 1{,}861\, Watts$

where "t" has been obtained from Student - t test Table 6.6 for

$\upsilon = n - 1 = 7$ for 95% confidence $(\frac{\alpha}{2} = 0.0025)$ to be 2.365

(b)

$$\bar{P} = \frac{\sum P_i}{n} = \frac{\sum V_i * I_i * PF_i \sqrt{3}}{n} = 19{,}629\, Watts$$

$$P_{\bar{P}} = t\frac{S_P}{\sqrt{n}} = 2.365 * \frac{787}{\sqrt{8}} = 658\, Watts \qquad 95\%\ confidence$$

(c) $B_V = B_{\bar{V}} = 1\%$ of $\overline{V} = 0.01*459.9 = 4.60$ V

$B_I = B_{\bar{I}} = 1\%$ of $\bar{I} = 0.01*31.0 = 0.31\, amp$

$B_{PF} = B_{\overline{PF}} = 1\%$ of $\overline{PF} = 0.01*0.79 = 0.79*10^{-2}$

Systematic uncertainty of power (applying Eq. 7.24 and utilizing Eq. 7.6 for simplification),

$$B_P = [\left(\frac{\partial P}{\partial V} B_V\right)^2 + \left(\frac{\partial P}{\partial I} B_I\right)^2 + \left(\frac{\partial P}{\partial PF} B_{PF}\right)^2]^{1/2}$$

a. $$\frac{B_P}{P} = [(\frac{B_V^2}{V})^2 + (\frac{B_I}{I})^2 + (\frac{B_{PF}}{PF})^2]^{1/2} = (0.01^2 + 0.01^2 + 0.01^2)^{1/2} = 0.017$$

$$B_P = 0.017 * 19629. = 334\, Watts$$

(d)

$$w_{\bar{P}} = [\ B_{\bar{P}}^2 + P_{\bar{P}}^2]^{1/2} = (334^2 + 658^2)^{1/2} = 738 \text{ Watts} \qquad \text{(With 95\% confidence level)}$$

most of the uncertainty is due to random effects.

A5

13. The yield of a chemical process is being studied. From previous experience with this process the standard deviation of yield is known to be 3. The past 5 days of plant operation have resulted in the following yields: 91.6, 88.75, 90.8, 89.95, and 91.3%. Use a significance level of 0.05.
    a. Is there evidence that the mean yield is not 90%? Use the P-value approach.
    b. What sample size would be required to detect a true mean yield of 85% with a probability of 0.95?

13) a) The parameter of interest is the true mean yield, μ.

H_0: $\mu = 90$

H_1: $\mu \neq 90$

Test statistic is $z_0 = \dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}}$ $\quad \bar{x} = 90.48$, $\sigma = 3$ $\quad z_0 = \dfrac{90.48 - 90}{3/\sqrt{5}} = 0.36$

P-value $= 2[1 - \Phi(0.36)] = 2[1 - 0.64058] = 0.7188$. Since the p-value > 0.05, we fail to reject H_0 and conclude the yield is not significantly different from 90%.

$$\boxed{\text{b) } n = \frac{(z_{\alpha/2} + z_\beta)^2 \sigma^2}{\delta^2} = \frac{(z_{0.025} + z_{0.05})^2 3^2}{(85 - 90)^2} = \frac{(1.96 + 1.65)^2 9}{(-5)^2} = 4.67}$$

$n \cong 5.$

14. The life in hours of a thermocouple used in a furnace is known to be approximately normally distributed with standard deviation σ = 20 hours. A random sample 15 thermocouples resulted in the following data: 553, 552, 567, 579, 550, 541, 537, 553, 552, 546, 538, 553, 581, 539, 529.

    a. Is there evidence to support the claim that mean life exceeds 540 hours? Use a fixed level test with $\alpha = 0.05$?

14) a) 1) The parameter of interest is the true mean life, μ.

2) H_0: $\mu = 540$

3) H_1: $\mu > 540$

4) $\alpha = 0.05$

5) $z_0 = \dfrac{\overline{x} - \mu}{\sigma / \sqrt{n}}$

6) $\boxed{\text{Reject } H_0 \text{ if } z_0 > z_\alpha \text{ where } z_{0.05} = 1.645}$

7) $\overline{x} = 551.33$, $\sigma = 20$

$$\boxed{z_0 = \frac{551.33 - 540}{20 / \sqrt{15}} = 2.19}$$

8) Since $\boxed{2.19 > 1.645}$, reject the null hypothesis and conclude there is sufficient evidence to support the claim the

life exceeds 540 hrs at $\alpha = 0.05$.

26. The dynamic modulus of concrete is obtained for two different concrete mixes. For the first mix, an $n = 33$ sample has a mean of 115.1 and a standard deviation of 0.47 psi. For the second mix, an $n = 31$ sample has a mean of 114.6 and a standard deviation of 0.38 psi. Test at a 0.05 significance level the null hypothesis of equality of mean dynamic modules versus the two sided alternative.

1. *Null hypothesis* $H_0 : \mu_1 - \mu_2 = 0$

   Alternative hypothesis $H_1 : \mu_1 - \mu_2 \neq 0$

2. *Level of significance:* $\alpha = 0.05$.

3. *Criterion:* The null hypothesis specifies $\delta = \mu_1 - \mu_0 = 0$. Since the samples are large, we use the large sample statistic where we estimate each population variance by the sample variance.

$$\boxed{Z = \frac{\overline{X} - \overline{Y} - \delta}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}}$$

   The alternative is two-sided so we reject the null hypothesis for $\boxed{Z > z_{.025}}$ or $\boxed{Z < -z_{.025}}$

4. *Calculations:* Since $n_1 = 33$, $n_2 = 31$, $\overline{x} = 115.1$, $\overline{y} = 114.6$, $s_1 = 0.47$, and $s_2 = 0.38$

$$\sqrt{\frac{.47^2}{33} + \frac{0.38^2}{31}} = 0.10655 \qquad z = \frac{115.1 - 114.6}{0.10655} = 4.69 > 1.96,$$

5. *Decision:* Because $4.69 > 1.96$, we reject the null hypothesis at the .05 level of significance. The P-value $2P[Z > 4.69]$ rounds to 0.00000 and gives extremely strong support for rejecting the null hypothesis.

28. Two machines are used for filling plastic bottles with a net volume of 16.0 ounces. The fill volume can be assumed normal, with standard deviation 0.020 and 0.025 for machines 1 and 2, respectively. A member of the quality engineering staff suspects that both machines fill to the same mean net volume, whether or not this volume is 16.0 ounces, A random sample of 10 bottles is taken from the output of each machine, as follows:

Machine 1		**Machine 2**	
16.03	16.01	16.02	16.03
16.04	15.96	15.97	16.04
16.05	15.98	15.96	16.02
16.05	16.02	16.01	16.01
16.02	15.99	15.99	16.00

a. Do you think the engineer is correct? Use the P-value approach.

b. If $\alpha = 0.05$, what is the power of the test in Part a) for a true difference in means of 0.04?

c. Find a 95% CI on the difference in means. Provide a practical interpretation of this interval.

d. Assuming equal sample sizes, what sample size should be used to ensure that b = 0.01 if the true difference in means is 0.04? Assume that $\alpha = 0.05$.

a) The parameter of interest is the difference in fill volume, $\mu_1 - \mu_2$ $\quad \delta = 0$

H_0: $\mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

H_1: $\mu_1 - \mu_2 \neq 0$ or $\mu_1 \neq \mu_2$

$$z_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$\bar{x}_1 = 16.015$ $\quad \bar{x}_2 = 16.005$

$\sigma_1 = 0.02$ $\quad \sigma_2 = 0.025$

$n_1 = 10$ $\quad n_2 = 10$

The test statistic is

$$z_0 = \frac{(16.015 - 16.005) - 0}{\sqrt{\frac{(0.02)^2}{10} + \frac{(0.025)^2}{10}}} = 0.99$$

P-value $= 2(1 - \Phi(0.99)) = 2(1 - 0.8389) = 0.3222$. Since p-value is greater than 0.05, we do not reject the null hypothesis.

b) Power = $1 - \beta$, where

$$\boxed{\beta = \Phi\left(z_{\alpha/2} - \frac{\Delta - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}\right) - \Phi\left(-z_{\alpha/2} - \frac{\Delta - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}\right)} = \Phi\left(1.96 - \frac{0.04}{\sqrt{\frac{(0.02)^2}{10} + \frac{(0.025)^2}{10}}}\right) - \Phi\left(-1.96 - \frac{0.04}{\sqrt{\frac{(0.02)^2}{10} + \frac{(0.025)^2}{10}}}\right)$$

$$= \Phi(1.96 - 3.95) - \Phi(-1.96 - 3.95) = \Phi(-1.99) - \Phi(-5.91) = 0.0233 - 0 = 0.0233$$

Power = 1 – 0.0233 = 0.977

c) $$(\bar{x}_1 - \bar{x}_2) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \le \mu_1 - \mu_2 \le (\bar{x}_1 - \bar{x}_2) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$(16.015 - 16.005) - 1.96\sqrt{\frac{(0.02)^2}{10} + \frac{(0.025)^2}{10}} \le \mu_1 - \mu_2 \le (16.015 - 16.005) + 1.96\sqrt{\frac{(0.02)^2}{10} + \frac{(0.025)^2}{10}}$$

$$-0.0098 \le \mu_1 - \mu_2 \le 0.0298$$

With 95% confidence, we believe the true difference in the mean fill volumes is between –0.0098 and 0.0298. Since 0 is contained in this interval, we can conclude there is no significant difference between the means.

d) Assume the sample sizes are to be equal, use $\alpha = 0.05$, $\beta = 0.01$, and $\Delta = 0.04$

$$n \cong \frac{(z_{\alpha/2} + z_\beta)^2 \boxed{(\sigma_1^2 + \sigma_2^2)}}{\delta^2} = \frac{(1.96 + 2.33)^2((0.02)^2 + (0.025)^2)}{(0.04)^2} = 2.08, \quad n = 11.79, \text{ use } n_1 = n_2 = 12$$

32. The melting points of two alloys used in formulating solder were investigated by melting 21 samples of each material. The sample mane and standard deviation of alloy 1 was $\bar{x}_1$=420.48, s_1= 2.34, and for alloy 2 they were $\bar{x}_2$=425, s_2= 2.5

a. Do the sample data support the claim that both alloys have the same melting point? Use a fixed-level test with $\alpha = 0.05$ and assume that both populations are normally distributed and have the same standard deviation.
b. Find the P-value for this test.

a) 1) The parameter of interest is the difference in mean melting point, $\mu_1 - \mu_2$

2) H_0: $\mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) H_1: $\mu_1 - \mu_2 \neq 0$ or $\mu_1 \neq \mu_2$

4) $\alpha = 0.05$

5) The test statistic is

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

6) $\boxed{\text{Reject the null hypothesis if } t_0 < -t_{\alpha/2,n_1+n_2-2}}$ where $-t_{0.025,40} = -2.021$ or $\boxed{t_0 > t_{\alpha/2,n_1+n_2-2}}$ where $t_{0.025,40} = 2.021$

7) $\bar{x}_1 = 420.48$ $\bar{x}_2 = 425$, $\Delta_0 = 0$ $s_p = \sqrt{\dfrac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}} = \sqrt{\dfrac{20(2.34)^2 + 20(2.5)^2}{40}} = 2.42$

$s_1 = 2.34$ $s_2 = 2.5$

$n_1 = 21$ $n_2 = 21$

$$t_0 = \frac{(420.48 - 425) - 0}{2.42\sqrt{\frac{1}{20} + \frac{1}{20}}} = -5.99$$

8) Since $-5.99 < -2.021$ reject the null hypothesis and conclude that the data do not support the claim that both alloys have the same melting point at $\alpha = 0.05$

b) P-value $= 2P(t < -5.424)$ P-value $< 0.001($