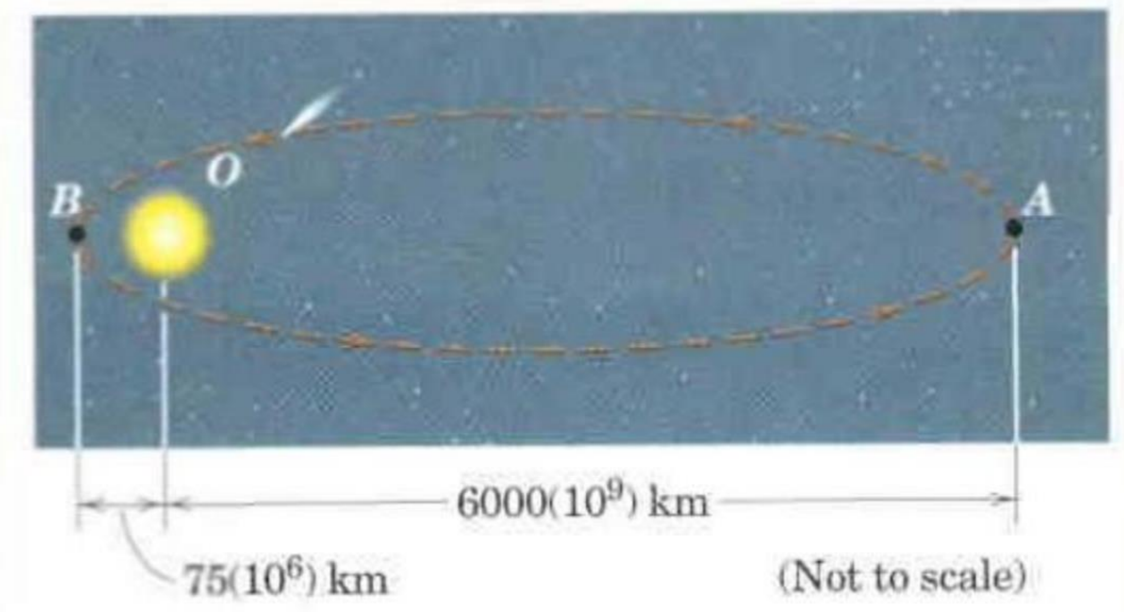
Dynamics Problems

Thomas G

Tips

- Be bold
- Be clever
- Be cautious
- Assumptions for something seemingly trivial may be a deadly trap!

25. Angular Impulse and Momentum - A comet is in the highly eccentric orbit shown in the figure. Its speed at the most distant point A, which is at the outer edge of the solar system, is v_A= 740 m/s. Determine its speed at the point B of closest approach to the sun.



The only considerable force acting on the comet, the gravitational force exerted on it by the sun, is central (points to the sun center O), angular momentum about O is conserved.

conservation of angular momentum about O:

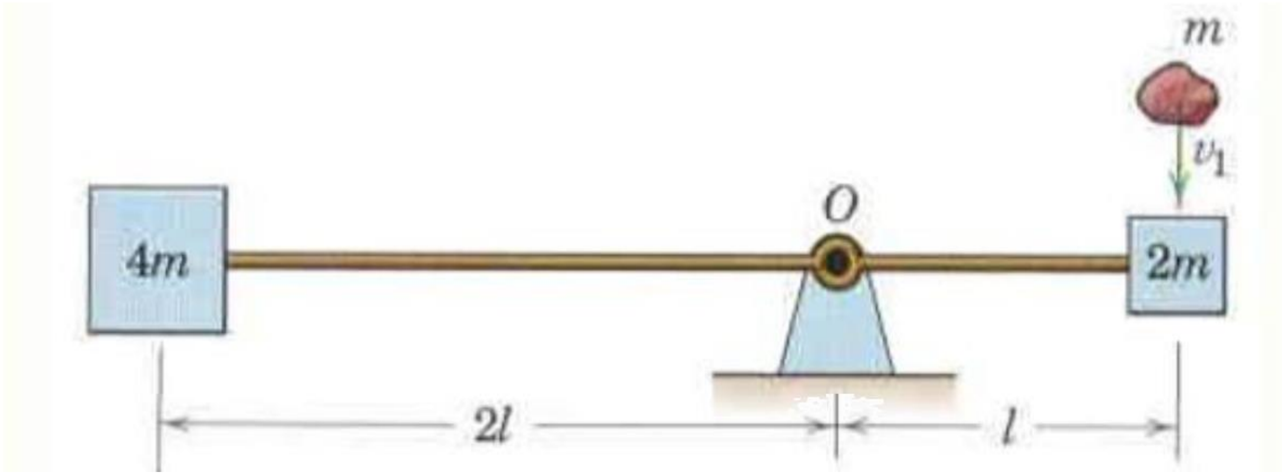
$$(H_o)_A = (H_o)_B$$

$$m r_A V_A = m r_B V_B$$

$$V_B = \frac{r_A V_A}{r_B} = \frac{6000(10^6)\,740}{75(10^6)}$$

$$\boxed{V_B = 59\,200 \ \frac{m}{s}}$$

26. Angular Impulse and Momentum - The assembly of the light rod and two end masses is at rest when it is struck by the falling object travelling with speed of v_1 as shown. The object adheres to and travels with the right-hand end mass. Determine the angular velocity of the assembly just after impact. The pivot at O is frictionless, and all three masses may be assumed to be particles.



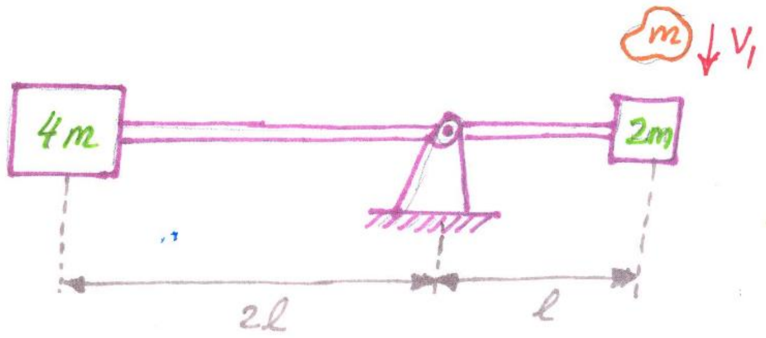
If the $\boxed{\text{angular impulses associated with the weights}}$ during the collision process is $\boxed{\text{ignored}}$, then the angular momentum of the system about O is conserved, during impact:

Conservation of angular momentum about O:

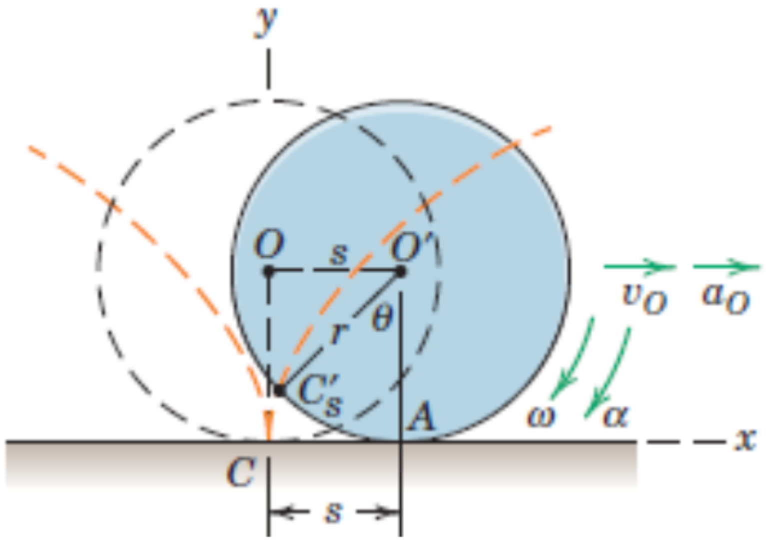
$$(H_0)_1 = (H_0)_2 \qquad mv_1 l = (m+2m)(\dot{\theta}_2)l + 4m(2l\dot{\theta}_2)2l$$

$$\dot{\theta}_2 = \frac{v_1}{19l} \qquad \text{CW (clockwise)}$$

Note: Each angular momentum term is written in the form mvd, and the final transverse velocities are expressed as radial distances times angular velocity $\dot{\theta}_2$.



3. Absolute Motion - A wheel of radius r rolls on a flat surface without slipping. Determine the angular motion of the wheel in terms of the linear motion of its center O. Also determine the acceleration of a point on the rim of the wheel as the point comes into contact with the surface on which the wheel rolls.



The origin of fixed coordinate x-y is taken arbitrarily but conveniently at the point of contact between C on the rim of the wheel and the ground. When point C has moved along its cycloidal path to C', $\boxed{\text{its new coordinates \& their time derivates}}$ become:

Absolute Position Coordinate**

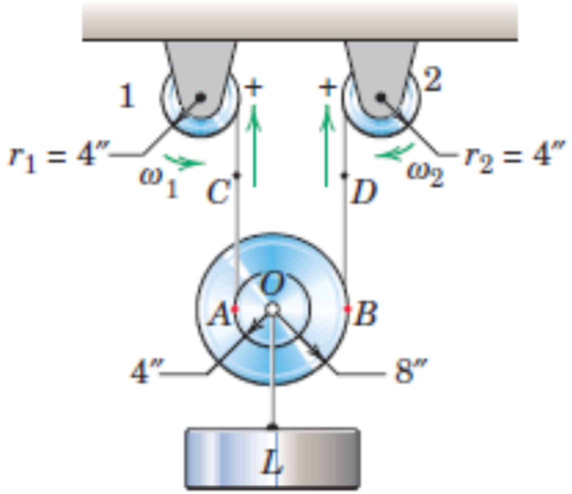
$$\begin{cases} x = S - r\sin\theta = r(\theta - \sin\theta) \\ \dot{x} = r\dot{\theta}(1-\cos\theta) = V_0(1-\cos\theta) \\ \ddot{x} = \dot{V}_0(1-\cos\theta) + V_0\dot{\theta}\sin\theta \end{cases} \qquad \begin{cases} y = r - r\cos\theta = r(1-\cos\theta) \\ \dot{y} = r\dot{\theta}\sin\theta = V_0\sin\theta \\ \ddot{y} = \dot{V}_0\sin\theta + V_0\dot{\theta}\cos\theta \end{cases}$$

For desired instant of contact, $\theta = 0$, $\ddot{x} = 0$, $\ddot{y} = r\omega^2$

4. Absolute Motion - The load L is being hoisted by the pulley and cable arrangement shown. Each cable is wrapped securely around its respective pulley so it does not slip. The two pulleys to which L is attached are fastened together to form a single rigid body. Calculate the velocity and acceleration of the load L and the corresponding angular velocity ω and angular acceleration α of the double pulley under the following conditions:

Case (a) Pulley 1: $\omega_1 = \dot{\omega}_1 = 0$ (pulley at rest) Pulley 2: $\omega_2 = 2$ rad/sec, $\alpha_2 = \dot{\omega}_2 = -3$ rad/sec^2

Case (b) Pulley 1: $\omega_1 = 1$ rad/sec, $\alpha_1 = \dot{\omega}_1 = 4$ rad/sec^2 Pulley 2: $\omega_2 = 2$ rad/sec, $\alpha_2 = \dot{\omega}_2 = -2$ rad/sec^2



Solution

Case (a)

$$dS_B = \overline{AB}\, d\theta \qquad dS_O = \overline{AO}\, d\theta$$

$$V_B = \overline{AB}\, \omega \qquad V_O = \overline{AO}\, \omega$$

$$(a_B)_t = \overline{AB}\, \alpha \qquad a_O = \overline{AO}\, \alpha$$

$V_D = r_2 \omega_2 = 4(2) = 8$ in/s

$a_D = r_2 \alpha_2 = 4(-3) = -12$ in/s²

$\omega = V_B / \overline{AB} = V_D / \overline{AB} = \frac{8}{12} = \frac{2}{3}$ rad/s (CCW)

$\alpha = (a_B)_t / \overline{AB} = a_D / \overline{AB} = \frac{-12}{12} = -1$ rad/s² (CW)

The corresponding motion of O and the load L is:

$V_O = \overline{AO}\, \omega = 4\left(\frac{2}{3}\right) = \frac{8}{3}$ in/s

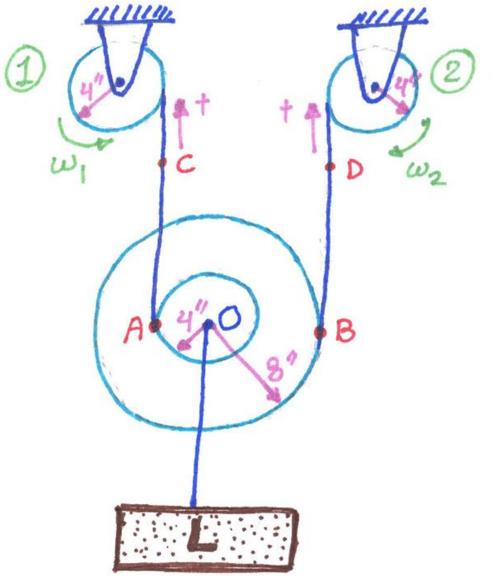
$a_O = \overline{AO}\, \alpha = 4(-1) = -4$ in/s²

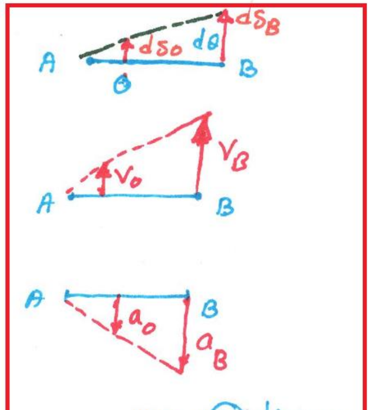
Case (b) $dS_B - dS_A = \overline{AB}\, d\theta$, $V_B - V_A = \overline{AB}\, \omega$, $(a_B)_t - (a_A)_t = \overline{AB}\, \alpha$

$dS_O - dS_A = \overline{AO}\, d\theta$, $V_O - V_A = \overline{AO}\, \omega$, $a_O - (a_A)_t = \overline{AO}\, \alpha$

$V_C = r_1 \omega_1 = 4(1) = 4$ in/s , $V_D = r_2 \omega_2 = 4(2) = 8$ in/s²

$a_C = r_1 \alpha_1 = 4(4) = 16$ in/s , $a_D = r_2 \alpha_2 = 4(-2) = -8$ in/s²

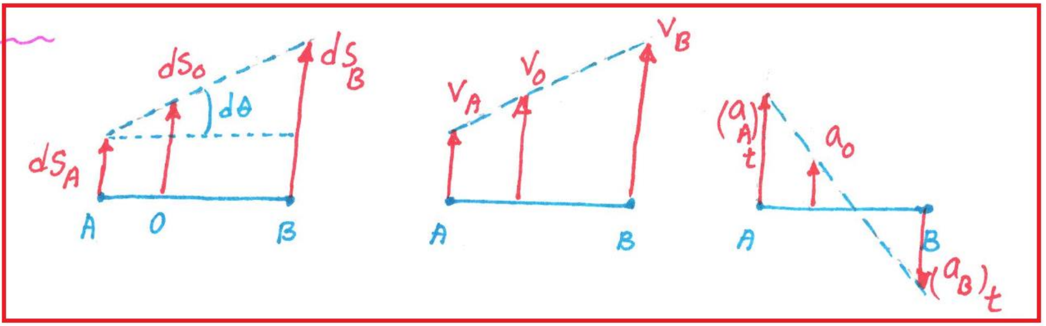




case (a) diagram

we have for the angular motion of the double pulley:

Solution



$$\omega = \frac{V_B - V_A}{\overline{AB}} = \frac{V_D - V_C}{\overline{AB}} = \frac{8-4}{12} = \frac{1}{3} \text{ rad/s (CCW)}$$

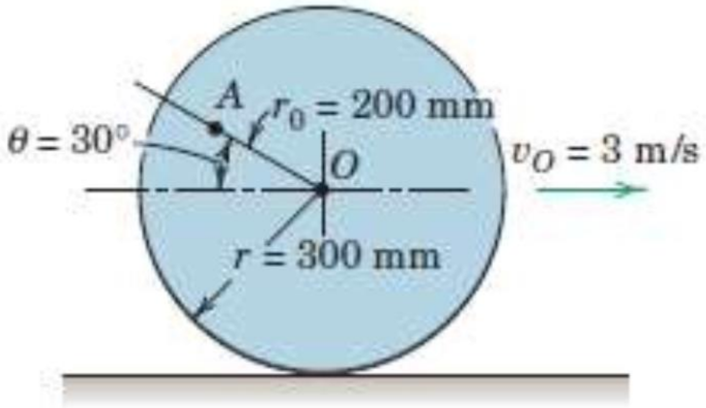
$$\alpha = \frac{(a_B)_t - (a_A)_t}{\overline{AB}} = \frac{a_D - a_C}{\overline{AB}} = \frac{-8-16}{12} = -2 \text{ rad/s}^2 \text{ (CW)}$$

The corresponding motion of O and the load L is:

$$V_O = V_A + \overline{AO}\, \omega = V_C + \overline{AO}\, \omega = 4 + 4\left(\frac{1}{3}\right) = \frac{16}{3} \text{ in/s}$$

$$a_O = (a_A)_t + \overline{AO}\, \alpha = a_C + \overline{AO}\, \alpha = 16 + 4(-2) = 8 \text{ in/s}^2$$

1. Instantaneous center of zero velocity - The wheel rolls to the right without slipping, with its center O having a velocity $v_O = 3$ m/s. Locate the instantaneous center of zero velocity and use it to find the velocity of point A for the position indicated.

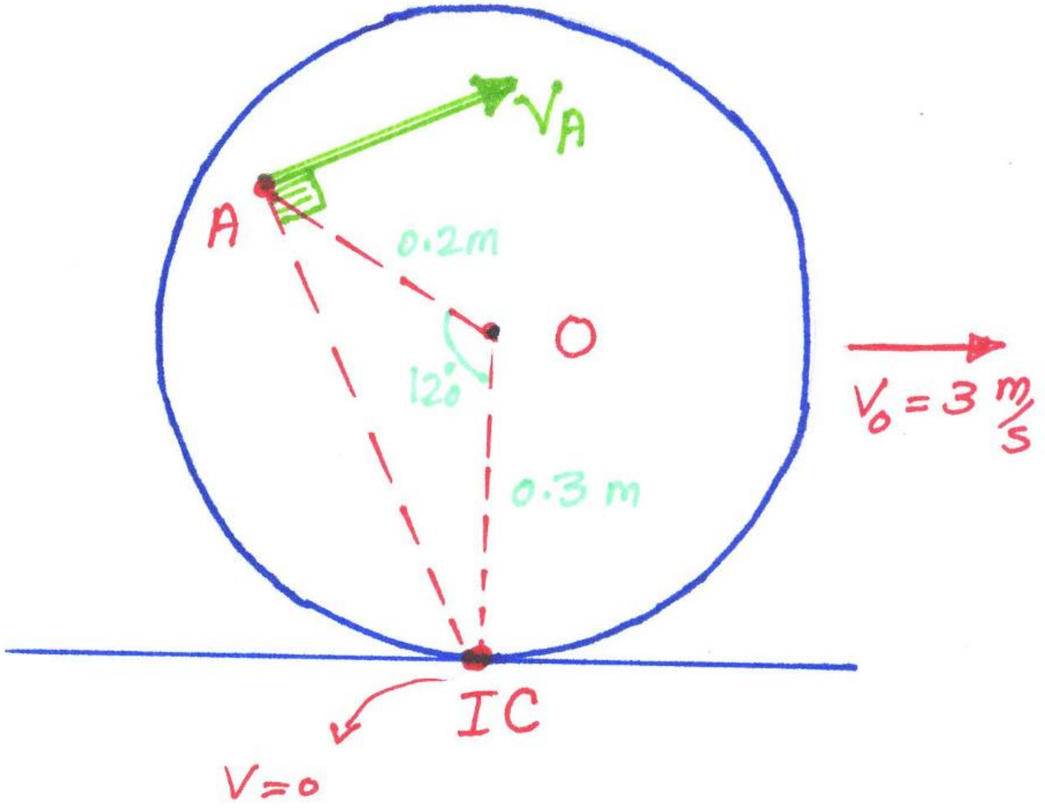


Solution

Rolls without slipping.

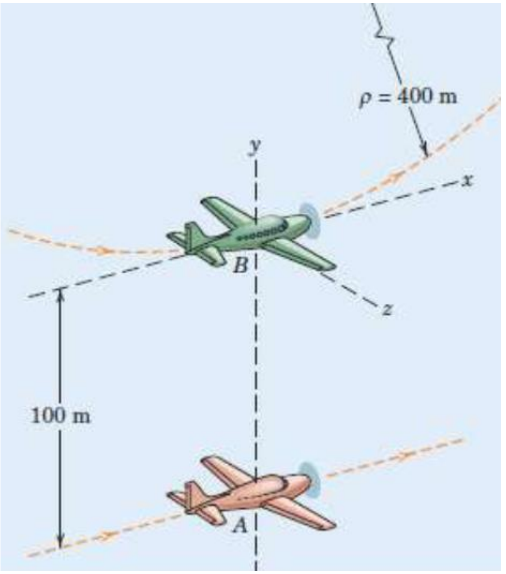
$V_0 = 3\ \frac{m}{s}$, $V_A = ?$

- The point on the rim of the wheel in contact with the ground has no velocity if the wheel is not slipping, then it is instantaneous center of zero velocity.



The angular velocity of the wheel becomes:

9. Motion relative to rotating axes - Aircraft B has a constant speed of 150 m/s as it passes the bottom of a circular loop of 400-m radius. Aircraft A flying horizontally in the plane of the loop passes 100 m directly below B at a constant speed of 100 m/s. (a) Determine the instantaneous velocity and acceleration which A appears to have to the pilot of B, who is fixed to his rotating aircraft. (b) Compare your results for part (a) with the case of erroneously treating the pilot of aircraft B as nonrotating.



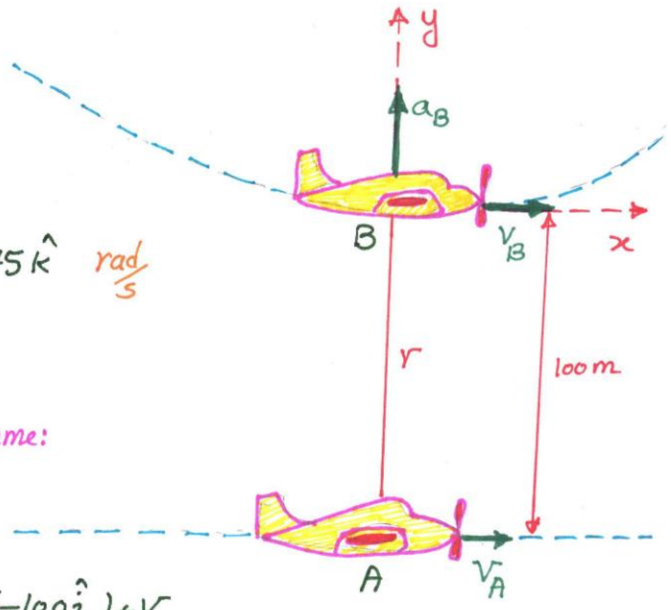
Solution

$\vec{V}_A = 100\,\hat{i}$ m/s

$\vec{V}_B = 150\,\hat{i}$ m/s

$\vec{\omega} = \frac{V_B}{\rho}\,\hat{k} = \frac{150}{400}\,\hat{k} = 0.375\,\hat{k}$ rad/s

$\vec{r} = \vec{r}_{A/B} = -100\,\hat{j}$ m



Velocity equation - rotating frame:

$$\vec{V}_A = \vec{V}_B + \boxed{\vec{\omega}\times\vec{r}} + \vec{V}_{rel}$$

$$100\,\hat{i} = 150\,\hat{i} + 0.375\,\hat{k} \times (-100\hat{j}) + V_{rel}$$

$$\boxed{\rightarrow \vec{V}_{rel} = -87.5\,\hat{i} \text{ m/s}}$$

Acceleration Equation - rotating frame: $\vec{a}_A = \vec{a}_B + \dot{\vec{\omega}}\times\vec{r} + \vec{\omega}\times(\vec{\omega}\times\vec{r}) + 2\vec{\omega}\times\vec{V}_{rel} + \vec{a}_{rel}$

$$0 = 56.2\hat{j} + 0\times(-100\hat{j}) + 0.375\hat{k}\times[0.375\,k \times(-100\hat{j})]$$

$$+\ 2[0.375\,\hat{k}\times(-87.5\hat{i})] + \vec{a}_{rel} \rightarrow \vec{a}_{rel} = -4.69\,\hat{k} \text{ m/s}^2$$

For motion relative to translating frames:

$V_{A/B} = V_A - V_B = 100\,\hat{i} - 150\,\hat{i} = -50\hat{i}$ m/s

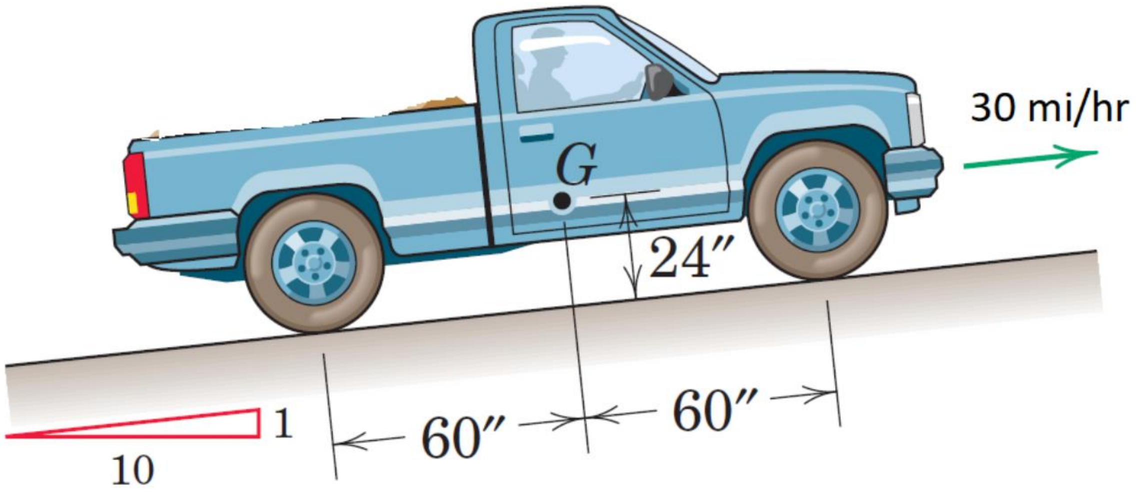
$a_{A/B} = a_A - a_B = 0 - 56.2\,\hat{j} = -56.2\,\hat{j}$ m/s²

Note: $V_{rel} \neq V_{A/B}$ & $a_{rel} \neq a_{A/B}$

The rotation of pilot B makes a difference in what he/she observes

6-1

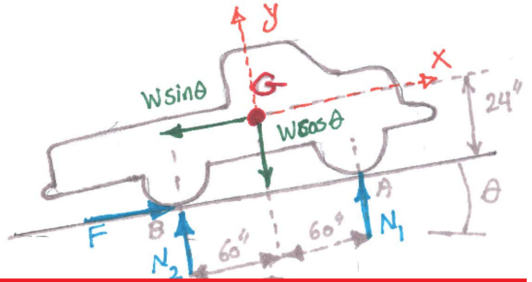
The pickup truck weighs 3220 lb and reaches a speed of 30 mi / hr from rest in a distance of 200 ft up the 10-percent incline with constant acceleration. Calculate the normal force under each pair of wheels and the friction force under the rear driving wheels. The effective coefficient of friction between the tires and the road is known to be at least 0.80.



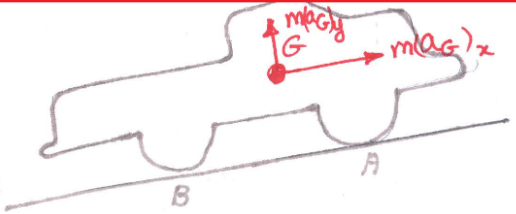
Solution

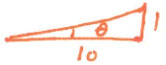
① Draw FBD and Kinetic diagram: The FBD of complete truck shows the normal forces N_1 and N_2, the friction force F in the direction to oppose the slipping of the driving wheels, and the weight W represented by its two components.

FBD



Kinetic Diagram





$\theta = \tan^{-1}\left(\frac{1}{10}\right) = 5.71°$

② Kinematic analysis → to find acceleration of the truck from velocity

$v_{Gx}^2 = 2a_{Gx}S \rightarrow a_{Gx} = \frac{v^2}{2S} = \frac{(44)^2}{2(200)} = 4.84 \ ft/s^2$ $\quad$ 30 mile/hr = 44 ft/s

The truck's center of mass has rectilinear translation with above acceleration

③ Apply 3 equations of motion

$\Sigma F_x = m a_{Gx} \rightarrow F - 320 = \left(\frac{3220}{32.2}\right)(4.84) \rightarrow \boxed{F = 804 \ lb}$

$\Sigma F_y = m a_{Gy} \rightarrow N_1 + N_2 - 3200 = 0$ ①

$\Sigma M_G = I_G \alpha = 0 \rightarrow 60 N_1 + 804(24) - N_2(60) = 0$ ②

Solving Eq. ① and ② simulatneously gives:

$\boxed{N_1 = 1441 \ lb}$, $\boxed{N_2 = 1763 \ lb}$

The friction force of 804 lb, requires minimum $\frac{F}{N_2} = 0.46 = \frac{804}{1763}$ as coefficient of friction. Since, our coefficient of friction is at least 0.80, the surfaces are rough enough to support the calcalated value of F so that our result is correct.

Alternative Solution

From kinetic diagram we see that N_1 and N_2 can be obtained independently of one another by writing seperate moment equations about A and B.

$$\boxed{\begin{aligned} \Sigma M_A &= m a_G d \\ \Sigma M_B &= m a_G d \end{aligned}}$$

$120 N_2 - 60(3200) - 24(320) = 484(24)$ ①

$3200(60) - 320(24) - 120 N_1 = 484(24)$ ②

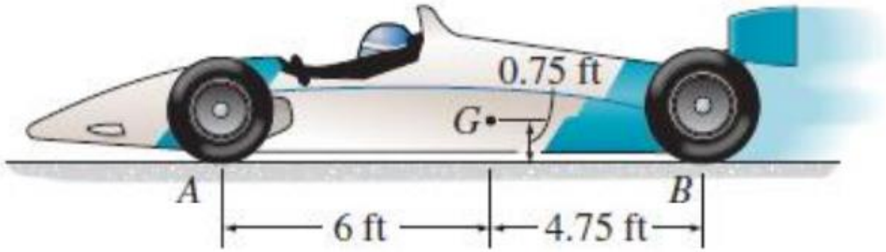
① $N_2 = 1763 \ lb$

② $N_1 = 1441 \ lb$

*17–36.

Determine the maximum acceleration that can be achieved by the car without having the front wheels A leave the track or the rear drive wheels B slip on the track. The coefficient of static friction is $\mu_s = 0.9$. The car's mass center is at G, and the front wheels are free to roll. Neglect the mass of all the wheels.

The weight of car is W=1550 lb.



Equations of Motion:

$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad F_B = \frac{1550}{32.2}a \tag{1}$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_A + N_B - 1550 = 0 \tag{2}$$

$$\circlearrowleft + \Sigma M_G = 0; \qquad N_B(4.75) - F_B(0.75) - N_A(6) = 0 \tag{3}$$

If we $\boxed{\text{assume that the front wheels are about to leave the track, } N_A = 0.}$ Substituting this value into Eqs. (2) and (3) and solving Eqs. (1), (2), (3),

$$N_B = 1550 \text{ lb} \qquad F_B = 9816.67 \text{ lb} \qquad a = 203.93 \text{ ft/s}^2$$

Since $F_B > (F_B)_{max} = \mu_s N_B = 0.9(1550)$ lb $= 1395$ lb, the rear wheels will slip. Thus, the solution must be reworked so that the rear wheels are about to slip.

$$F_B = \mu_s N_B = 0.9 N_B \tag{4}$$

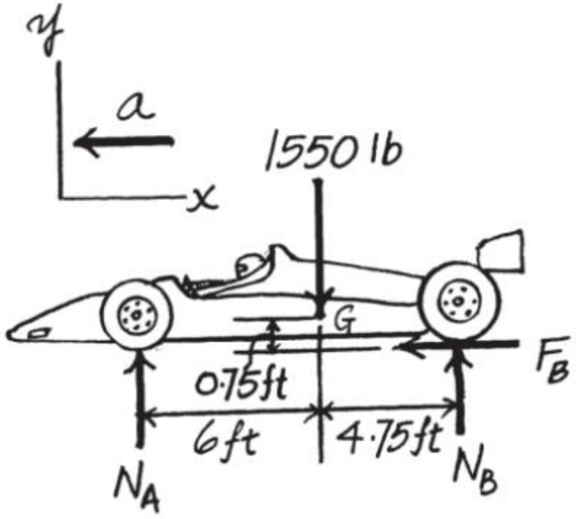
Solving Eqs. (1), (2), (3), and (4) yields

$$N_A = 626.92 \text{ lb} \qquad N_B = 923.08 \text{ lb}$$

$$a = 17.26 \text{ ft/s}^2 = 17.3 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

1) Friction *Only* Occur on **Driving Wheel**

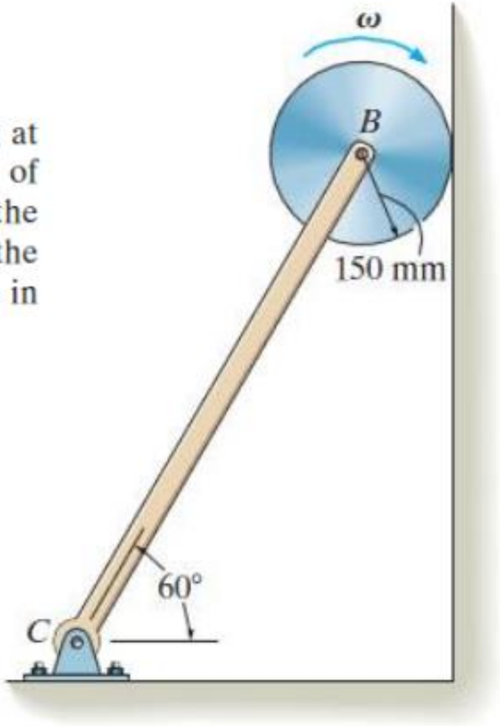
2) Leave the track means **N=0**



(a)

*17–72.

The disk has a mass of 20 kg and is originally spinning at the end of the strut with an angular velocity of $\omega = 60$ rad/s. If it is then placed against the wall, where the coefficient of kinetic friction is $\mu_k = 0.3$, determine the time required for the motion to stop. What is the force in strut BC during this time?



$$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad F_{CB} \sin 30° - N_A = 0$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad F_{CB} \cos 30° - 20(9.81) + 0.3N_A = 0$$

$$\circlearrowleft + \Sigma M_B = \boxed{I_B\,\alpha;} \qquad 0.3N_A(0.15) = \boxed{\left[\frac{1}{2}(20)(0.15)^2\right]\alpha}$$

$$N_A = 96.6\text{ N}$$

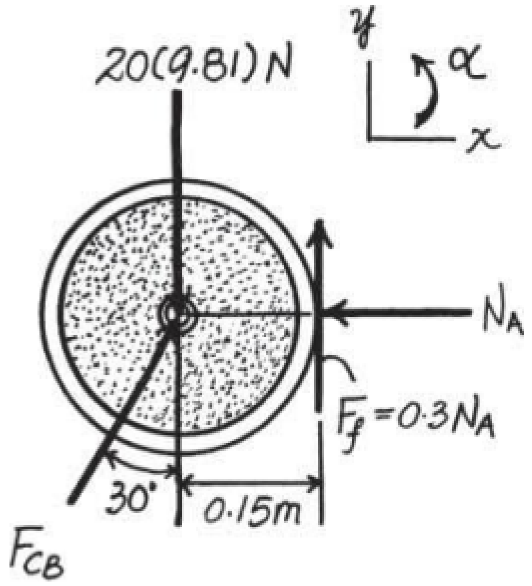
$$F_{CB} = 193\text{ N}$$

$$\alpha = 19.3\text{ rad/s}^2$$

$$\circlearrowleft + \qquad \omega = \omega_0 + \alpha_c t$$

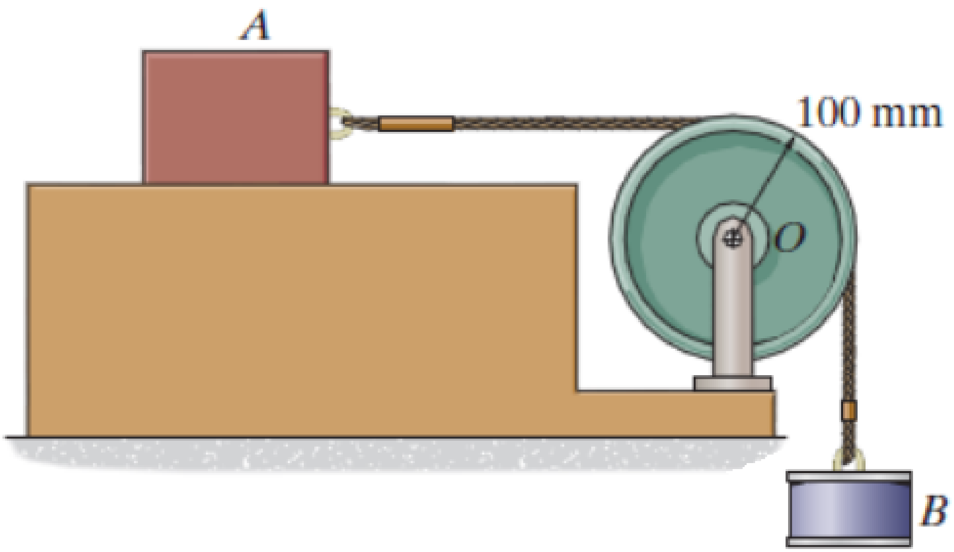
$$0 = 60 + (-19.3)\,t$$

$$t = 3.11\text{ s}$$



*17–88.

The 15-kg block A and 20-kg cylinder B are connected by a light cord that passes over a 5-kg pulley (disk). If the system is released from rest, determine the cylinder's velocity after its has traveled downwards 2 m. The coefficient of kinetic friction between the block and the horizontal plane is $\mu_k = 0.3$. Assume the cord does not slip over the pulley.



Equations of Motion: Since the block is in motion, $F_A = \mu_k N_A = 0.3N_A$. Referring to the free-body diagram of block A shown in Fig. a,

$$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_A - 15(9.81) = 0 \qquad N_A = 147.15 \text{ N}$$

$$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \quad T_1 - 0.3(147.15) = 15a \qquad \textbf{(1)}$$

Referring to the free-body diagram of the cylinder, Fig. b,

$$+\uparrow \Sigma F_y = m(a_G)_y; \quad T_2 - 20(9.81) = -20a \qquad \textbf{(2)}$$

Since the pulley rotates about a fixed axis passing through point O, $\alpha = \frac{a}{r} = \frac{a}{0.1} = 10a$. The mass moment of inertia of the pulley about O is $I_O = \frac{1}{2}mr^2 = \frac{1}{2}(5)(0.1^2) = 0.025 \text{ kg}\cdot\text{m}^2$. Writing the moment equation of motion about point O using the free-body diagram of the pulley shown in Fig. c,

$$+\Sigma M_O = I_O\alpha; \qquad T_1(0.1) - T_2(0.1) = -0.025(10a) \qquad \textbf{(3)}$$

Solving Eqs. (1) through (3) yields

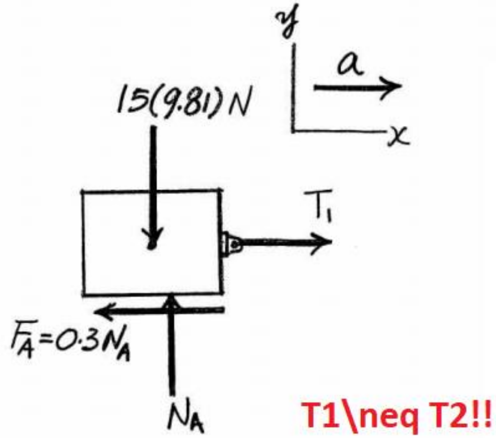
$$a = 4.0548 \text{ m/s}^2 \qquad T_2 = 115.104 \text{ N} \qquad T_1 = 104.967 \text{ N}$$

Kinematics: Since the acceleration is constant,

$$(+\downarrow) \qquad v^2 = v_0^2 + 2a_c(s - s_0)$$

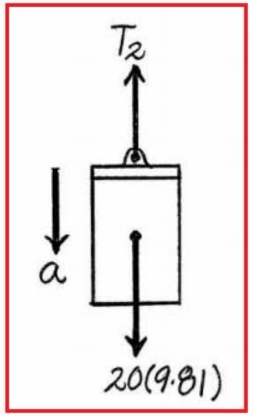
$$v^2 = 0^2 + 2(4.0548)(2 - 0)$$

$$v = 4.027 \text{ m/s} = 4.03 \text{ m/s} \downarrow \qquad \textbf{Ans.}$$

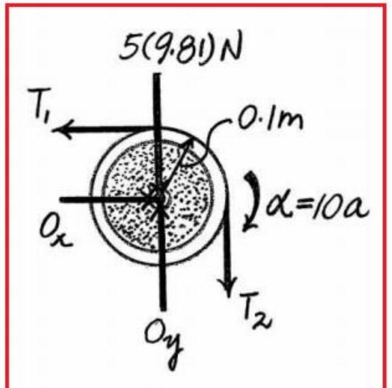


T1\neq T2!!

(a)



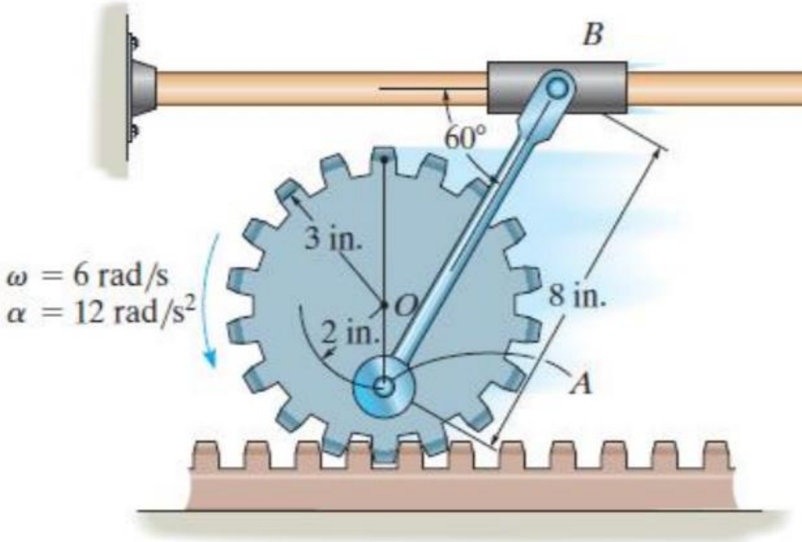
(b)



(c)

*16–124.

At a given instant, the gear has the angular motion shown. Determine the accelerations of points A and B on the link and the link's angular acceleration at this instant.



For the gear

$$v_A = \omega r_{A/IC} = 6(1) = 6 \text{ in./s}$$

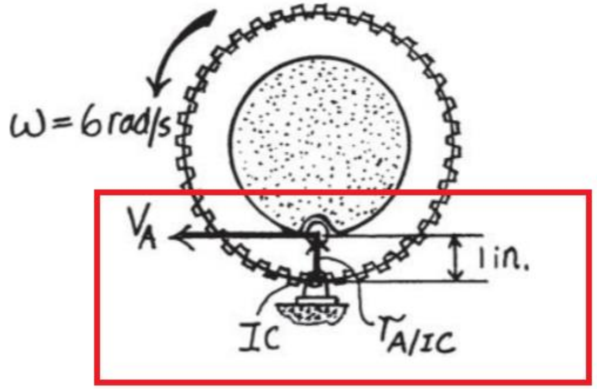
$\mathbf{a}_O = -12(3)\mathbf{i} = \{-36\mathbf{i}\} \text{ in./s}^2 \qquad \mathbf{r}_{A/O} = \{-2\mathbf{j}\} \text{ in.} \qquad \alpha = \{12\mathbf{k}\} \text{ rad/s}^2$

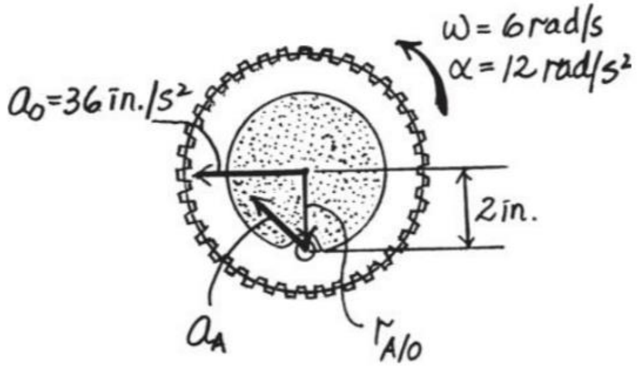
$$\mathbf{a}_A = \mathbf{a}_0 + \alpha \times \mathbf{r}_{A/O} - \omega^2 \mathbf{r}_{A/O}$$

$$= -36\mathbf{i} + (12\mathbf{k}) \times (-2\mathbf{j}) - (6)^2(-2\mathbf{j})$$

$$= \{-12\mathbf{i} + 72\mathbf{j}\} \text{ in./s}^2$$

$$a_A = \sqrt{(-12)^2 + 72^2} = 73.0 \text{ in./s}^2 \qquad \theta = \tan^{-1}\left(\frac{72}{12}\right) = 80.5^\circ$$





For link AB The IC is at ∞, so $\omega_{AB} = 0$, i.e.,

$$\omega_{AB} = \frac{v_A}{r_{A/IC}} = \frac{6}{\infty} = 0$$

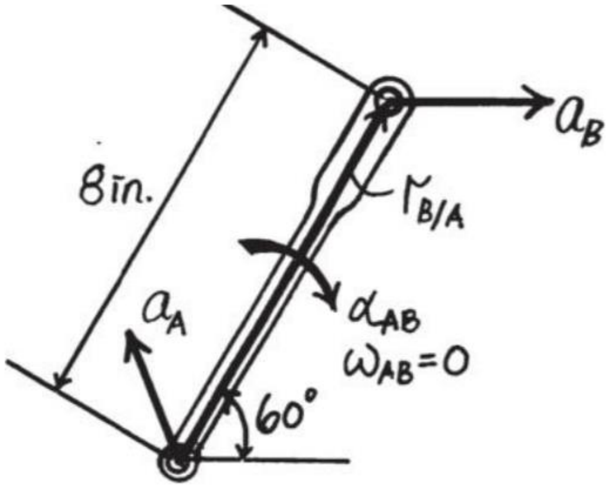
$\mathbf{a}_B = a_B\mathbf{i} \qquad \alpha_{AB} = -\alpha_{AB}\,\mathbf{k} \qquad \mathbf{r}_{B/A} = \{8 \cos 60^\circ\mathbf{i} + 8 \sin 60^\circ\mathbf{j}\} \text{ in.}$

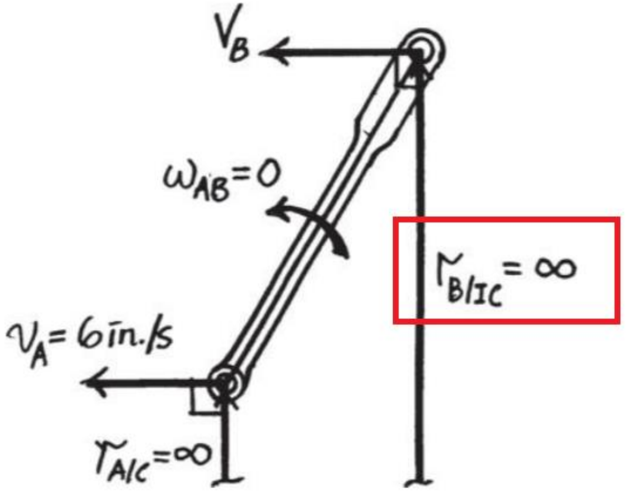
$$\mathbf{a}_B = \mathbf{a}_A + \alpha_{AB} \times \mathbf{r}_{B/A} - \omega^2 \mathbf{r}_{B/A}$$

$$a_B\,\mathbf{i} = (-12\mathbf{i} + 72\mathbf{j}) + (-\alpha_{AB}\,\mathbf{k}) \times (8 \cos 60^\circ\mathbf{i} + 8 \sin 60^\circ\mathbf{j}) - \mathbf{0}$$

$(\overset{+}{\rightarrow}) \qquad a_B = -12 + 8 \sin 60^\circ(18) = 113 \text{ in./s}^2 \rightarrow$

$(+\uparrow) \qquad 0 = 72 - 8 \cos 60^\circ \alpha_{AB} \qquad \alpha_{AB} = 18 \text{ rad/s}^2$





***15–8.**

If the jets exert a vertical thrust of $T = (500t^{3/2})$N, where t is in seconds, determine the man's speed when $t = 3$ s. The total mass of the man and the jet suit is 100 kg. Neglect the loss of mass due to the fuel consumed during the lift which begins from rest on the ground.

SOLUTION

Free-Body Diagram: The thrust **T** must overcome the weight of the man and jet before they move. Considering the equilibrium of the free-body diagram of the man and jet shown in Fig. *a*,

$$\boxed{+\uparrow \Sigma F_y = 0; \qquad 500t^{3/2} - 100(9.81) = 0 \qquad t = 1.567 \text{ s}}$$

Principle of Impulse and Momentum: Only the impulse generated by thrust **T** after $t = 1.567$ s contributes to the motion. Referring to Fig. *a*,

$$(+\uparrow) \qquad m(v_1)_y + \Sigma \int_{t_1}^{t_2} F_y dt = m(v_2)_y$$

$$100(0) + \int_{1.567\text{ s}}^{3\text{ s}} 500t^{3/2} dt - 100(9.81)(3 - 1.567) = 100v$$

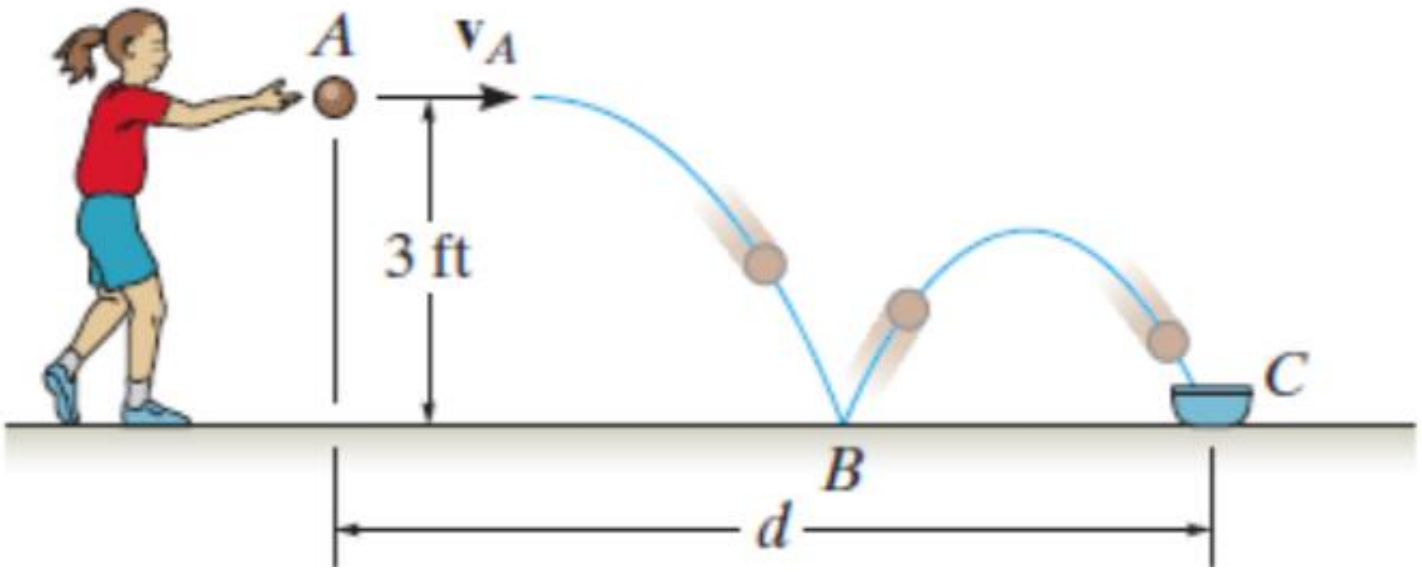
$$\left(200t^{5/2}\right)\Big|_{1.567\text{ s}}^{3\text{ s}} - 1405.55 = 100v$$

$$v = 11.0 \text{ m/s}$$

(a)

15–66.

If the girl throws the ball with a horizontal velocity of 8 ft/s, determine the distance d so that the ball bounces once on the smooth surface and then lands in the cup at C. Take $e = 0.8$.



SOLUTION

$(+\downarrow)\quad v^2 = v_0^2 + 2a_c(s - s_0)$

$(v_1)_y^2 = 0 + 2(32.2)(3)$

$(v_1)_y = 13.90\downarrow$

$(+\downarrow)\quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$3 = 0 + 0 + \frac{1}{2}(32.2)(t_{AB})^2$

$t_{AB} = 0.43167\text{ s}$

$(+\downarrow)\quad \boxed{e = \frac{(v_2)_y}{(v_1)_y}}$

$0.8 = \frac{(v_2)_y}{13.90}$

$(v_2)_y = 11.1197\uparrow$

$(+\downarrow)\quad v = v_0 + a_c t$

$11.1197 = -11.1197 + 32.2(t_{BC})$

$t_{BC} = 0.6907\text{ s}$

Total time is $t_{AC} = 1.1224\text{ s}$

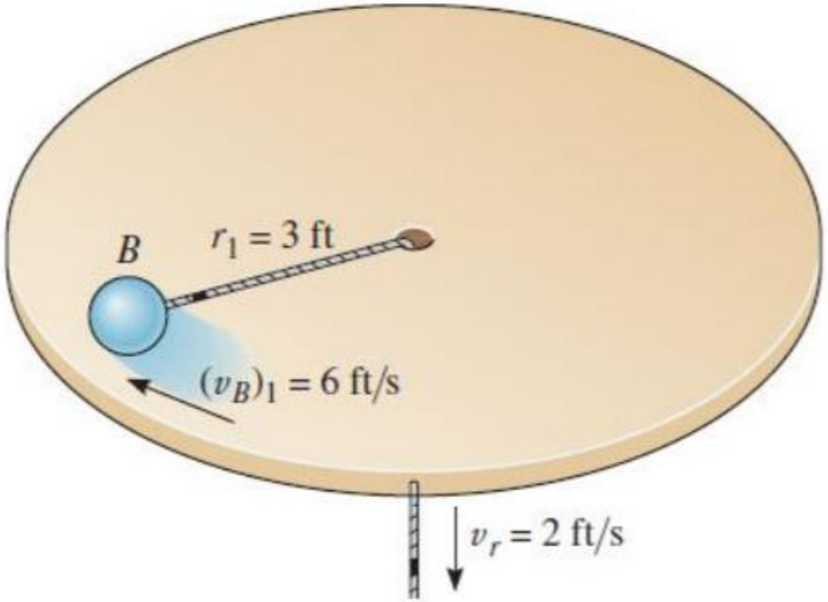
$d = v_A(t_{AC})$

$d = 8(1.1224)$

$d = 8.98\text{ ft}$

***15–104.**

A 4-lb ball B is traveling around in a circle of radius $r_1 = 3$ ft with a speed $(v_B)_1 = 6$ ft/s. If the attached cord is pulled down through the hole with a constant speed $v_r = 2$ ft/s, determine how much time is required for the ball to reach a speed of 12 ft/s. How far r_2 is the ball from the hole when this occurs? Neglect friction and the size of the ball.



SOLUTION

$$v = \sqrt{(v_\theta)^2 + (2)^2}$$

$$12 = \sqrt{(v_\theta)^2 + (2)^2}$$

$$v_\theta = 11.832 \text{ ft/s}$$

$$H_1 = H_2$$

$$\frac{4}{32.2}(6)(3) = \frac{4}{32.2}(11.832)(r_2)$$

$$r_2 = 1.5213 = 1.52 \text{ ft}$$

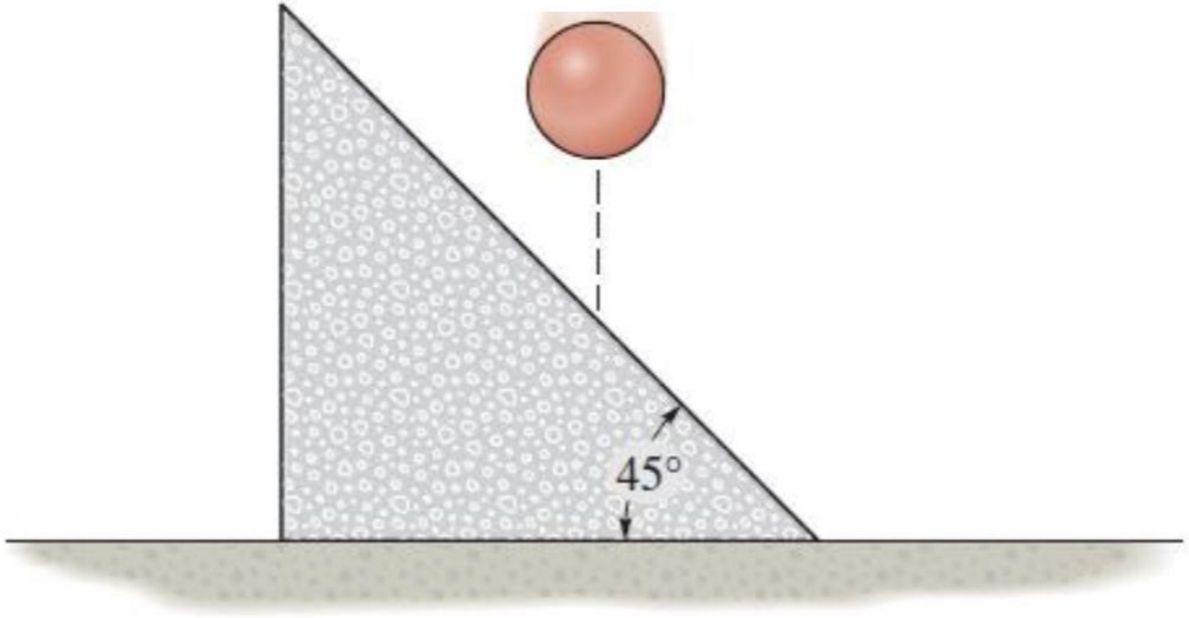
$$\Delta r = v_r t$$

$$(3 - 1.5213) = 2t$$

$$t = 0.739 \text{ s}$$

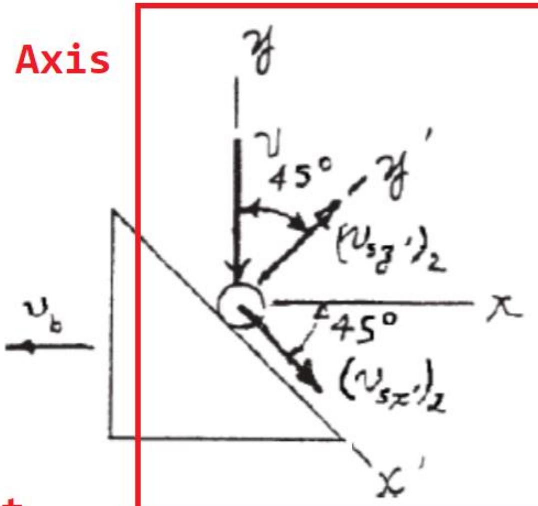
15–79.

The sphere of mass m falls and strikes the triangular block with a vertical velocity v. If the block rests on a smooth surface and has a mass $3m$, determine its velocity just after the collision. The coefficient of restitution is e.



SOLUTION

Adjust Reference Axis



Conservation of "x′" Momentum:

$$m\,(v)_1 = m\,(v)_2$$

$$(\searrow +) \qquad m(\,v \sin 45°) = m(v_{sx'})_2$$

$$(v_{sx'})_2 = \frac{\sqrt{2}}{2}\,v$$

Coefficient of Restitution (y′):

Only on Line of Impact

$$\boxed{e = \frac{(v_b)_2 - \left(v_{s_{y'}}\right)_2}{\left(v_{s_{y'}}\right)_1 - (v_b)_1}}$$

$$(+\swarrow) \qquad e = \frac{v_b \cos 45° - \left[-\left(v_{sy'}\right)_2\right]}{v \cos 45° - 0}$$

$$\left(v_{sy'}\right)_2 = \frac{\sqrt{2}}{2}\,(ev - v_b)$$

Conservation of "x" Momentum:

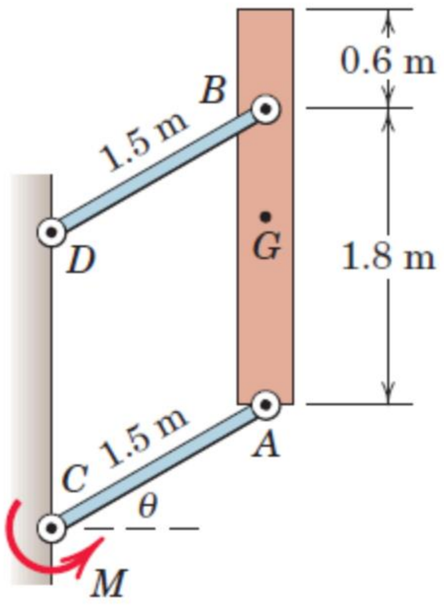
$$0 = m_s\,(v_s)_x + m_b\,v_b$$

$$(\overset{+}{\leftarrow}) \qquad 0 + 0 = 3mv_b - m\left(v_{sy'}\right)_2 \cos 45° - m(v_{sx'})_2 \cos 45°$$

$$3v_b - \frac{\sqrt{2}}{2}(ev - v_b)\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\,v\frac{\sqrt{2}}{2} = 0 \qquad v_b = \left(\frac{1+e}{7}\right)v$$

6-2

The vertical bar AB has a mass of 150 kg with center of mass G midway between the ends. The bar is elevated from rest at $\theta = 0$ by means of the parallel links of negligible mass, with a constant couple M = 5 kN·m applied to the lower link at C. Determine the angular acceleration α of the links as a function of and find the force B in the link DB at the instant when $\theta = 30°$.

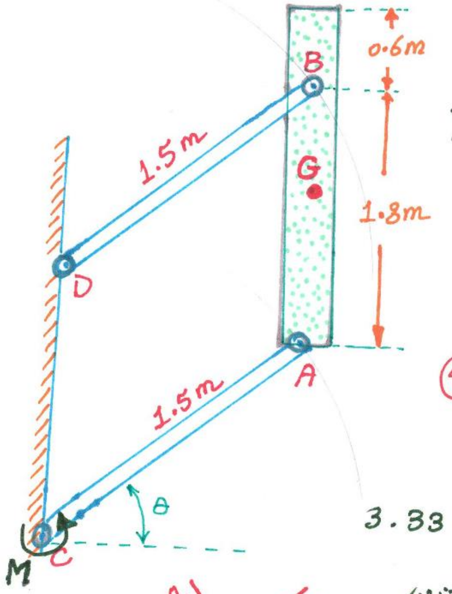


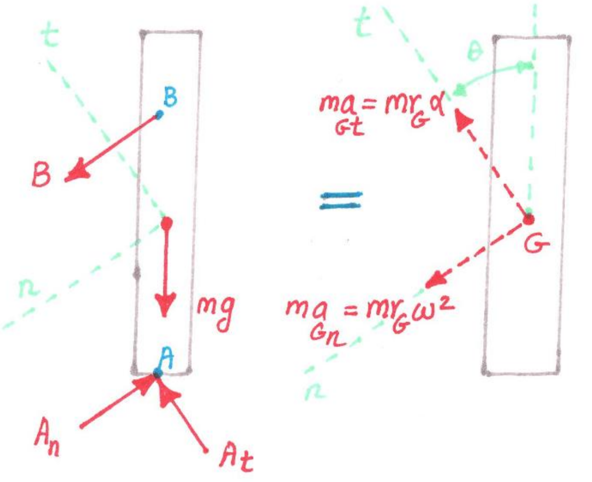
① Type of motion of the bar:

Curvilinear translation

② Appropriate frame of reference: with circular motion of the mass center G, we choose n–t coordinates.

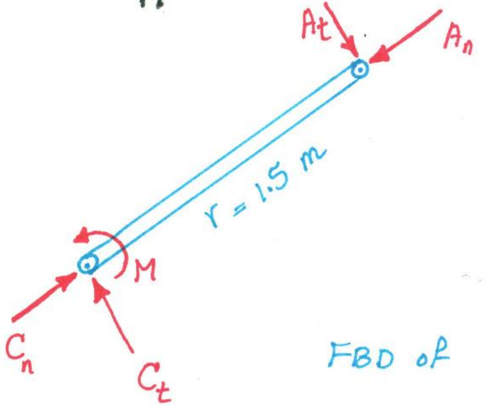
③ FBD and kinetic diagram:





FBD of bar AB

Kinetic diagram of bar AB



FBD of link AC

with negligible mass of the links, the tangential component A_t of the force at A can be obtained from FBD of AC where $\Sigma M_C \approx 0$

and $A_t = \dfrac{M}{\overline{AC}} = \dfrac{5}{1.5} = 3.33$ KN

The force at B is along the link.

The A_n and B forces depend on the resultant force in n-direction, and hence on $mr\omega^2$ at $\theta = 30°$.

④ Equations of motion

$\Sigma F_t = m\,a_{Gt}$ $\qquad A_t - mg\cos\theta = mr\alpha$

$3.33 - 0.15(9.81)\cos\theta = 0.15(1.5\alpha)$ $\qquad \alpha = 14.81 - 6.54\cos\theta$ rad/s²

with α known as function of θ, the angular velocity ω of the links is obtained from $\omega\,d\omega = \alpha\,d\theta$ equation:

$$\int_0^{\omega} \omega\,d\omega = \int_0^{\theta} (14.81 - 6.54\cos\theta)\,d\theta$$

$$\omega^2 = 29.6\,\theta - 13.08\sin\theta$$

Evaluate at $\theta = 30°$ → $\begin{cases} (\omega^2) = 8.97 & (\text{rad/s})^2 \\ \alpha = 9.15 & \text{rad/s}^2 \end{cases}$ and:

$$\begin{cases} mr\omega^2 = 0.15(15)(8.97) = 2.02 & \text{KN} \\ mr\alpha = 0.15(1.5)(9.15) = 2.06 & \text{KN} \end{cases}$$

If the moment about point A is calculated, the forces of A_n, A_t & weight are eliminated and force B can directly be found:

$\Sigma M_A = m a_G d$ $\qquad 1.8\cos 30°\, B = 2.02(1.2)\cos 30° + 2.06(0.6)$

$B = 2.14$ kN