Torsion Solution Procedure:

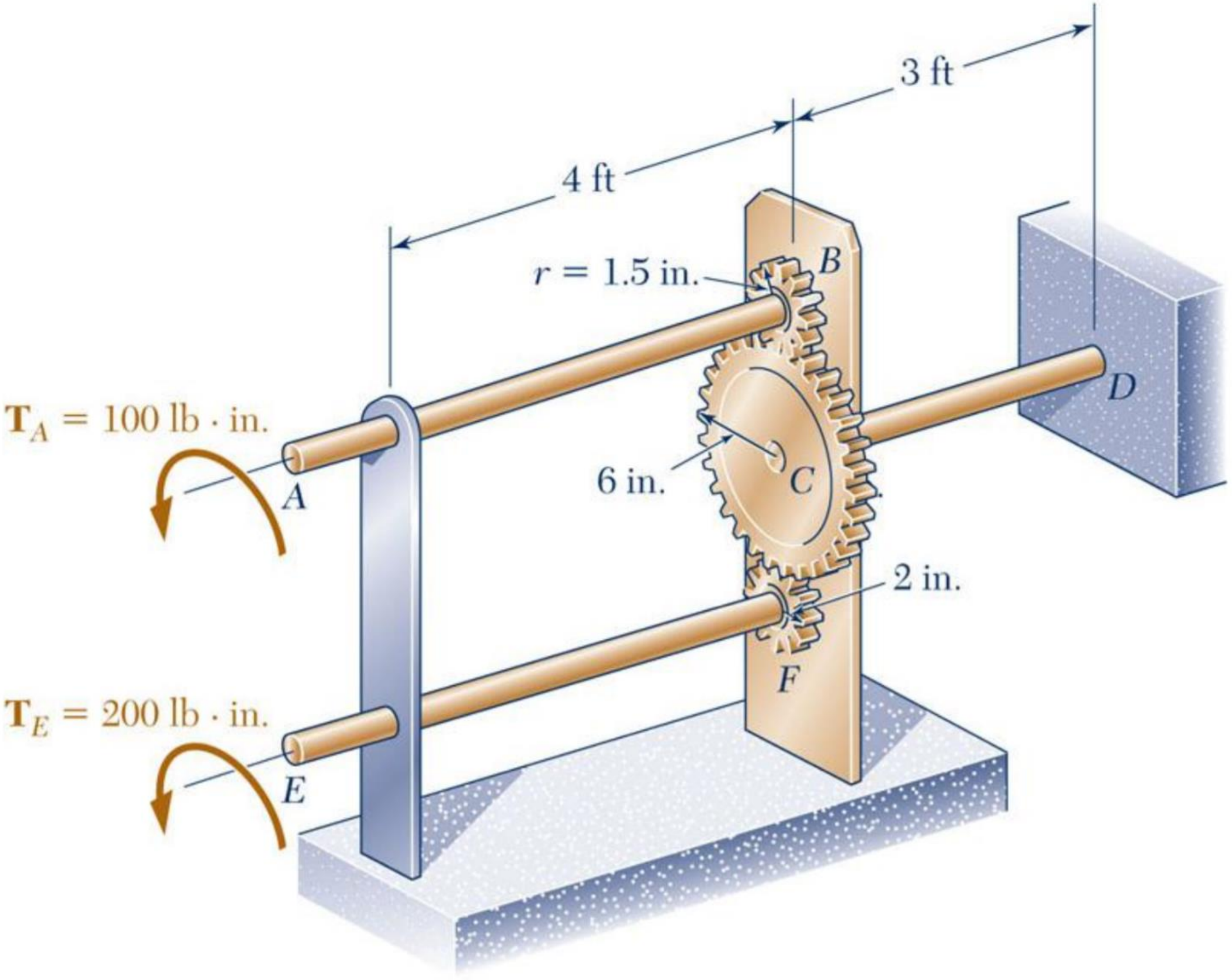
1) FBD of each wheel
2) Equilibrium Equations
3) Actual target values

Naming conventions

- Torque in shaft: T_{AB}
- Torque on wheel: T_A
- Total Angle of twist of a wheel: ϕ_B
- Respective Angle of twist: $\phi_{B/A}$

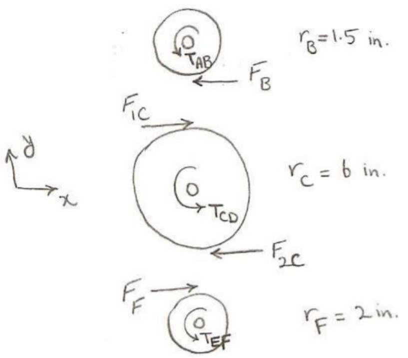
5- Three solid shafts, each of $\frac{3}{4}$ in. diameter, are connected by the gears shown. Knowing that $G = 11.2\times10^6$ psi, determine *(a)* the angle through which end A of shaft AB rotates, *(b)* the angle through which end E of shaft EF rotates.

(Answer: (a) φ_A = 24.5° ↺ (b) φ_E = 19.37° ↺)



Solution

$\boxed{\text{FBD}}$ for gears B, C & F



$\boxed{\text{Equilib. Eq's}}$

Gear B: $\Sigma M_z = 0 \longrightarrow T_{AB} = r_B \times F_B$

$\longrightarrow F_B = \dfrac{100}{1.5} = 66.7$ lb.in.

Gear F: $\Sigma M_z = 0 \longrightarrow T_{EF} = r_F \times F_F$

$\longrightarrow F_F = \dfrac{200}{2} = 100$ lb.in.

action & reaction:

$F_{1C} = F_B = 66.67$ lb.in. ; $F_{2C} = F_F = 100$ lb.in.

Gear C: $\Sigma M_z = 0 \longrightarrow T_{CD} = F_{1C} \times r_C + F_{2C} \times r_C = (100 + 66.67) \times 6 = 1000$ lb.in.

$\boxed{\text{Angle of twist}}$

in shaft CD: $\Phi_C = \dfrac{T_{CD} \cdot L_{CD}}{G \cdot J_{CD}} \longrightarrow \Phi_C = \dfrac{1000 \times 3 \times 12}{11.2 \times 10^6 \times 0.031063} = 0.10348$ rad

$J_{AB} = J_{CD} = J_{EF} = \dfrac{\pi}{32} \times \left(\dfrac{3}{4}\right)^4 = 0.031063$

For gears in contact we know: $r_B \cdot \Phi_B = r_C \cdot \Phi_C = r_F \cdot \Phi_F \longrightarrow \begin{cases} \Phi_B = \dfrac{r_C}{r_B} \Phi_C = 0.41391 \text{ rad} \\ \Phi_F = \dfrac{r_C}{r_F} \Phi_C = 0.31043 \text{ rad} \end{cases}$

$\Phi_A = \Phi_B + \Phi_{A/B}$

$\Phi_{A/B} = \dfrac{T_{AB} \times L_{AB}}{G \cdot J} = \dfrac{100 \times 4 \times 12}{11.2 \times 10^6 \times 0.031063} = 0.01380$ rad

$\longrightarrow \Phi_A = 0.41391 + 0.01380 = 0.4277$ rad $= 24.5°$

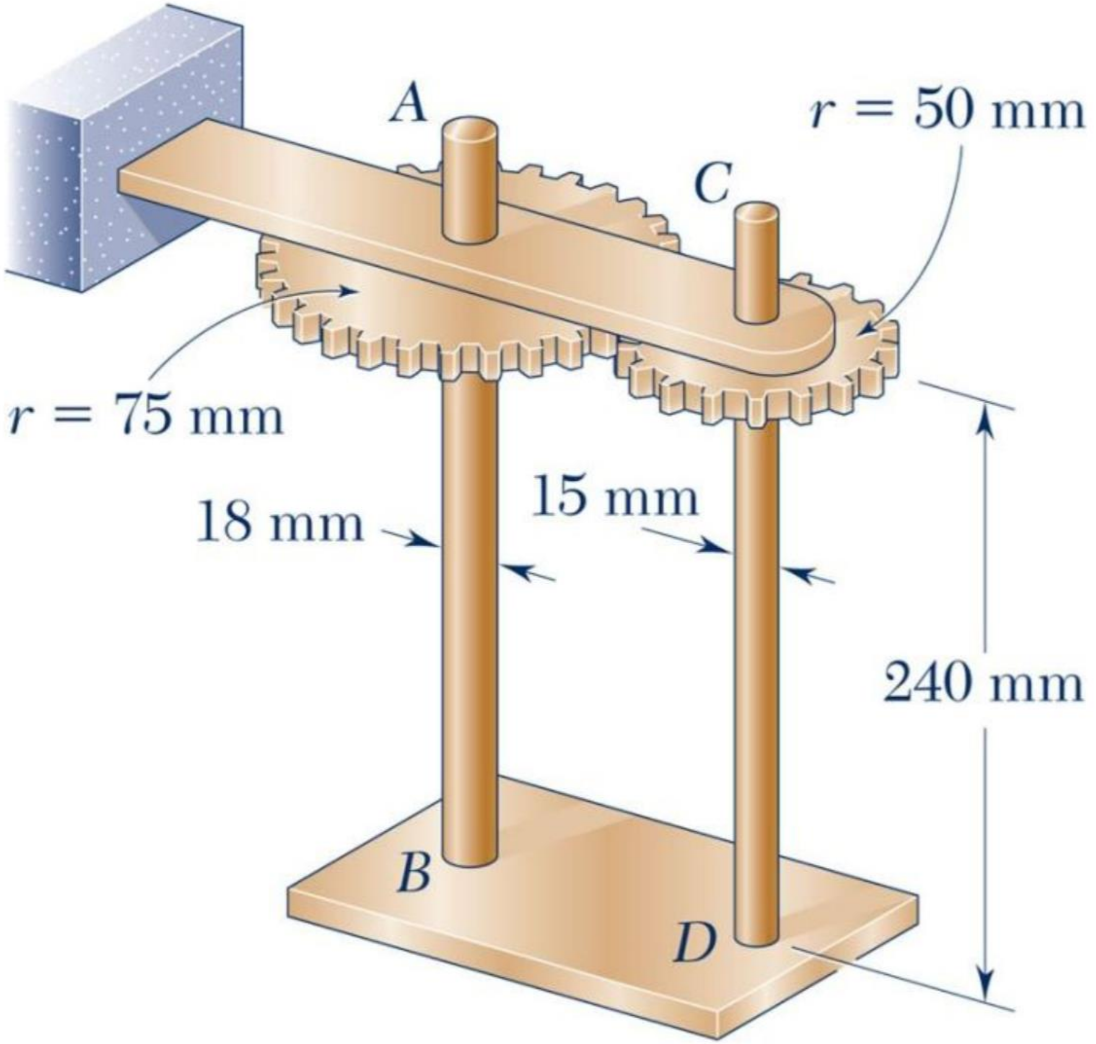
$\Phi_E = \Phi_F + \Phi_{E/F}$

$\Phi_{E/F} = \dfrac{T_{EF} \cdot L_{EF}}{G \cdot J} = \dfrac{200 \times 4 \times 12}{11.2 \times 10^6 \times 0.031063} = 0.027594$ rad

$\longrightarrow \Phi_E = 0.31043 + 0.027594 = 0.338024$ rad $= 19.37°$

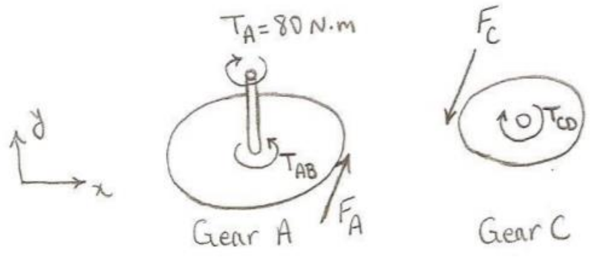
7- At a time when rotation is prevented at the lower end of each shaft a 80-N.m torque is applied to end A of shaft AB. Knowing that G = 77.2 GPa for both shafts, determine *(a)* the maximum shearing stress in shaft CD, *(b)* the angle of rotation at A.

(Answer: (a) τ_{CD} = 41.9 MPa (b) φ_A = 0.663°)



Solution

FBD for gears A, C



Equilib. Eqs

Gear A: $T_A = T_{AB} + r_A \times F_A$

$\rightarrow T_{AB} + 0.075 \times F_A = 80 \quad \cdots (1)$

Gear C: $T_{CD} = r_C \times F_C$

$\rightarrow T_{CD} = 0.05 \times F_C \quad \cdots (2)$

F_A and F_C are action and reaction, $F_A = F_C \quad \cdots (3)$

we have $\underline{3}$ equations (1,2,3) and $\underline{4}$ unknowns (T_{AB}, F_A, T_{CD}, F_C). So, the problem is indeterminant and we need one more equation, which is Compatibility of rotations.

we know that: $r_A \Phi_A = r_C \Phi_C \rightarrow r_A \times \dfrac{T_{AB} \cdot \cancel{L_{AB}}}{\cancel{G} \cdot J_{AB}} = r_C \times \dfrac{T_{CD} \cdot \cancel{L_{CD}}}{\cancel{G} \cdot J_{CD}}$

$$J_{AB} = \frac{\pi d_{AB}^4}{32} = \frac{\pi \times 18^4}{32} = 10306 \text{ mm}^4$$

$$J_{CD} = \frac{\pi d_{CD}^4}{32} = \frac{\pi \times 15^4}{32} = 4970 \text{ mm}^4$$

$$\rightarrow 75 \times \frac{T_{AB}}{10306} = 50 \times \frac{T_{CD}}{4970}$$

$$\rightarrow T_{AB} = 1.382\, T_{CD} \quad \cdots (4)$$

$$\xrightarrow[(1),(2),(3),(4)]{\text{Solving}} \begin{cases} T_{AB} = 38.368 \text{ N.m} \\ T_{CD} = 27.755 \text{ N.m} \end{cases}$$

(a) Shear stress shaft CD: $\tau_{CD} = \dfrac{T_{CD} \cdot r_{CD}}{J_{CD}} = \dfrac{27.755 \times 10^3 \times 7.5}{4970} = 41.88 \text{ MPa}$

(b) $\Phi_A = \dfrac{T_{AB}\, L}{G \cdot J_{AB}} = \dfrac{38.368 \times 10^3 \times 240}{77.2 \times 10^3 \times 10306} = 0.01157 \text{ rad} = 0.663^\circ$

General Procedure for Bending:

- Find Centroid
- Find **Moment of Inertia I**
- Find Stresses
- Stress Distribution Diagram

Composite Beam Bending:

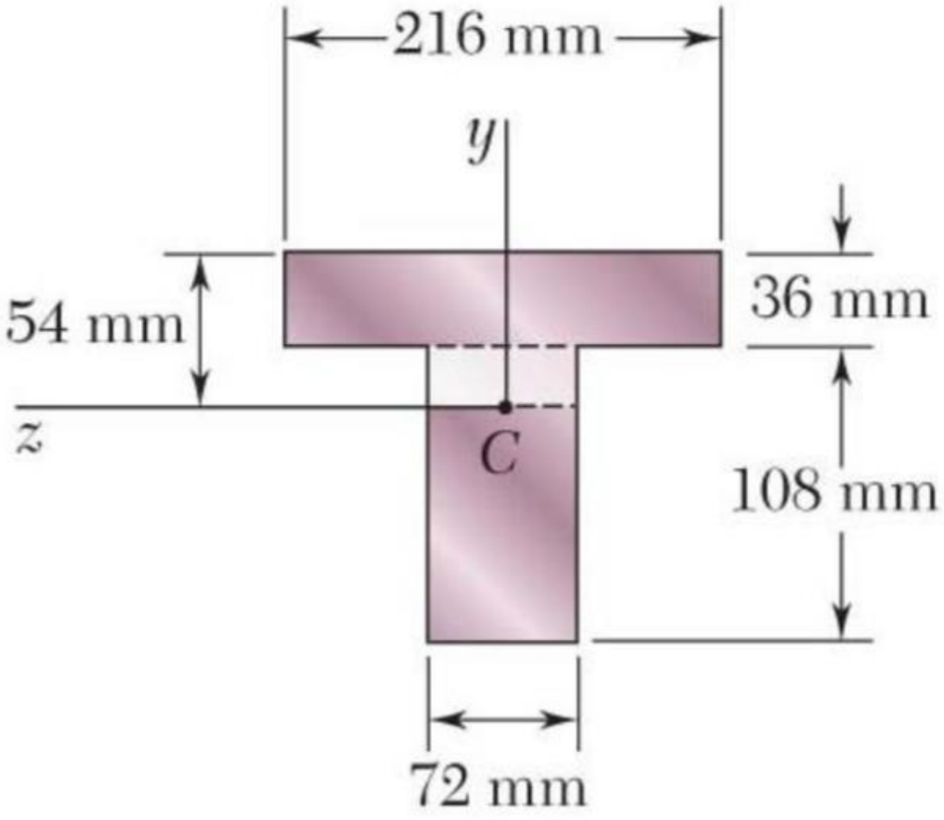
- Use a table to track A, nA, $\bar{y}$, and $nA\bar{y}$

Eccentric Loading:

- Sometimes the ‘eccentric nature’ is not obvious, thus memorizing formula is not useful
- Essentially, its just a combination of axial loading + bending.
- Solve the bending *without* worrying about the axial loading (e.g. finding centroid the normal way)

3- Knowing that a beam of the cross-section shown is bent about a horizontal axis and that the bending moment is 6 kN.m, determine the total force acting on the top flange.

(Answer: F = 58.8 kN)



Find I

$$I_{total} = I_{flange} + I_{web}$$

$$I_{flange} = \frac{1}{12} \times 216 \times 36^3 + 216 \times 36 \times (18+18)^2 = 1.092 \times 10^7 \ mm^4$$

$$I_{web} = \frac{1}{12} \times 72 \times 108^3 + 108 \times 72 \times 36^2 = 1.764 \times 10^7 \ mm^4$$

$$I_{total} = 2.856 \times 10^7 \ mm^4$$

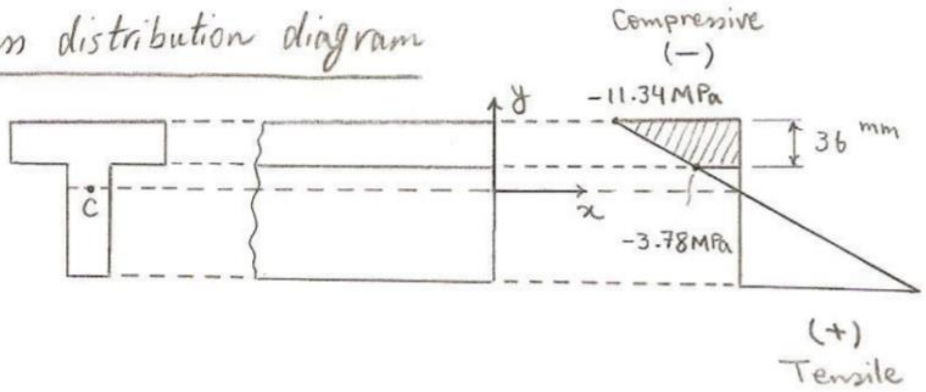
Find stress

$$\sigma_{top} = -\frac{M y_{top}}{I_{total}} = -\frac{6 \times 10^6 \times 54}{2.856 \times 10^7} = -11.34 \ MPa$$

Since it's not clear in which direction the moment is acting, the σ and force could be positive or negative. Here it's assumed that bending is acting in positive z-direction.

$$\sigma_{flange\text{-}bottom} = -\frac{M y_{Fl.\text{-}Bot.}}{I_{total}} = -\frac{6 \times 10^6 \times 18}{2.856 \times 10^7} = -3.78 \ MPa$$

Stress distribution diagram



Total Force

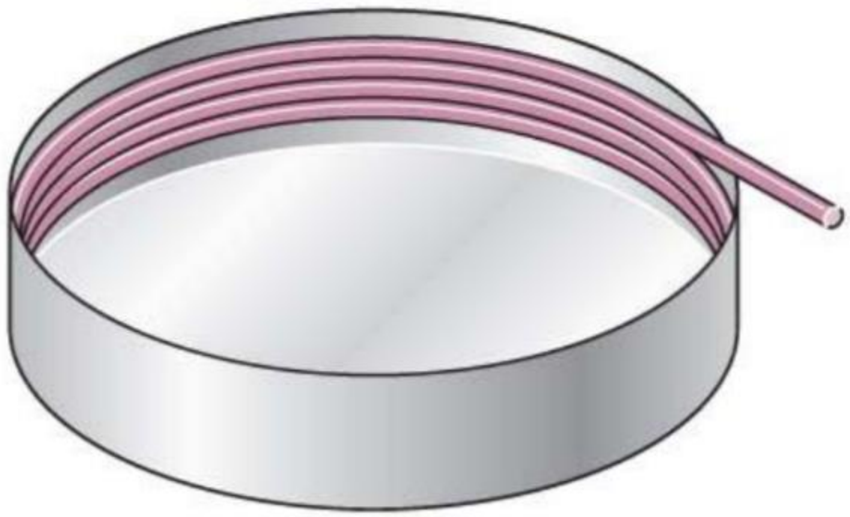
$$F = \int \sigma \, dA = \text{Area}_{\text{hatched portion}} \times \text{Flange width}$$

$$= \frac{(-11.34 - 3.78)}{2} \times 36 \times 216$$

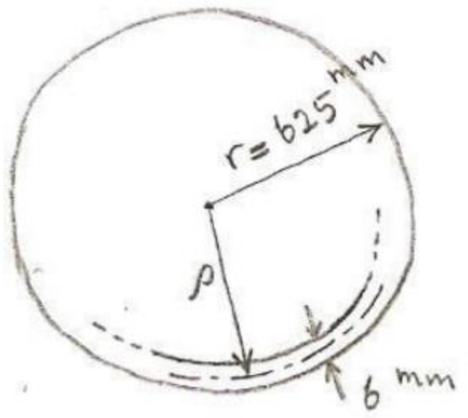
$$\rightarrow F = -58787 \ N = -58.8 \ kN$$

5- Straight rods of 6-mm diameter and 30-m length are stored by coiling the rods inside a drum of 1.25-m inside diameter. Assuming that the yield strength is not exceeded, determine *(a)* the maximum stress in a coiled rod, *(b)* the corresponding bending moment in the rod. Use E = 200 GPa.

(Answer: (a) σ = 965 MPa (b) M = 20.5 N.m)



(a) $\sigma_{max} = E \cdot \varepsilon_{max} = E \times \left(-\frac{y_{max}}{\rho}\right)$



y_{max} = rod's radius = 3 mm

$\rho = \frac{1.25 \times 10^3}{2} - 3 = 622$ mm

$\longrightarrow \sigma_{max} = 200 \times 10^3 \times \left(-\frac{3}{622}\right) = -964.6$ MPa

(b) $\sigma_{max} = -\frac{Mc}{I} = -964.6$ MPa

$I = \frac{\pi d^4}{64} = 63.62$ mm^4

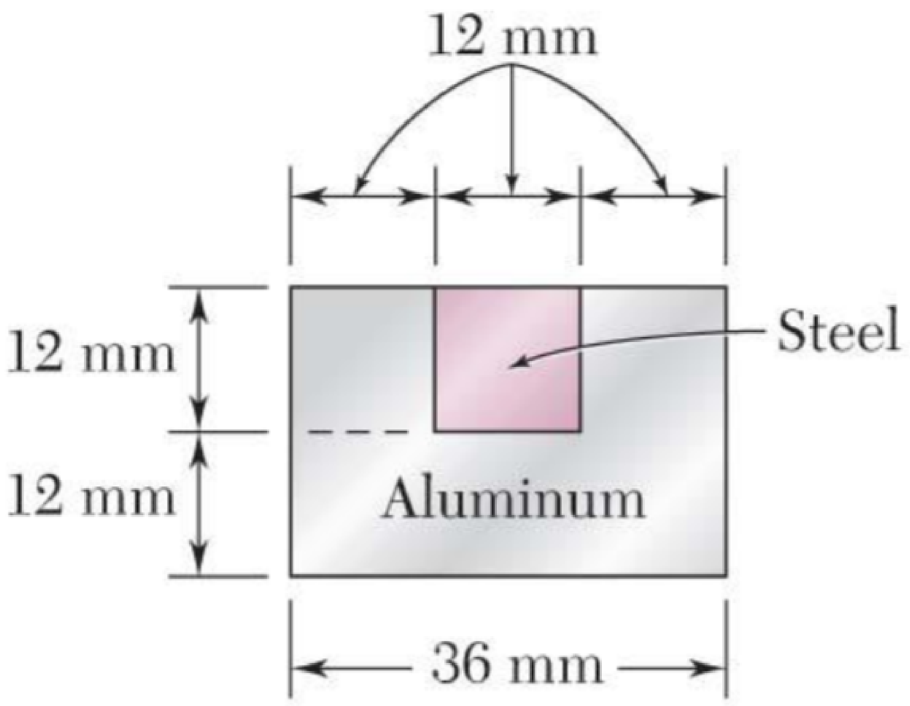
$c = 3$ mm

$\longrightarrow M = \frac{964.6 \times 63.62}{3} = 20456$ N.mm

$\longrightarrow M = 20.5$ N.m

7- A steel bar (E_S = 210 GPa) and an aluminum bar (E_a = 70 GPa) are bonded together to form the composite bar shown. Determine the maximum stress in (a) the aluminum, (b) the steel, when the bar is bent about a horizontal axis, with $\boldsymbol{M}$ = 200 N.m.

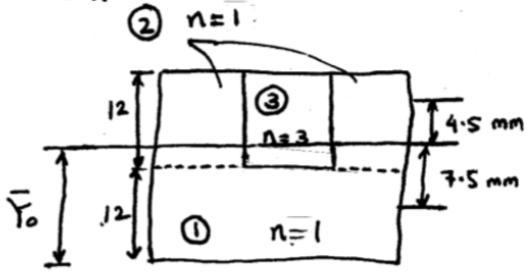
(Answer: (a) σ_a= 51.2 MPa ; (b) σ_s = -119.5 MPa)



we need to select a reference material first. Here, we select aluminum as the reference. Therefore: For aluminum ; $n=1$

For steel ; $n = \frac{E_s}{E_a} = \frac{210}{70} = 3$

	A (mm²)	nA (mm²)	$\bar{y}_o$ (mm)	$nA\bar{y}_o$ (mm³)
①	432	432	6	2592
②	288	288	18	5184
③	144	432	18	7776
	Σ	1152		15552



$$\bar{Y}_o = \frac{\Sigma nA\bar{y}_o}{\Sigma nA} = \frac{15552}{1152} = 13.5 \text{ mm}$$ (the neutral axis lies 13.5 mm above the bottom)

Now, we calculate I_1, I_2 & I_3:

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2 = \frac{1}{12}(36)(12)^3 + (432)(7.5)^2 = 29.484\times10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{1}{12}(24)(12)^3 + (288)(14.5)^2 = 9.288\times10^3 \text{ mm}^4$$

$$I_3 = \frac{n_3}{12} b_3 h_3^3 + n_3 A_3 d_3^2 = \frac{3}{12}(12)(12)^3 + (432)(4.5)^2 = 13.932\times10^3 \text{ mm}^4$$

$$\therefore \quad I = I_1 + I_2 + I_3 = 52.704\times10^3 \text{ mm}^4 = 52.704\times10^{-9} \text{ m}^4$$

$$M = 60 \text{ N.m} \quad , \quad \sigma = -\frac{nMy}{I}$$

a) Aluminum: $n=1$, $y = -13.5\text{mm} = -0.0135\text{m}$

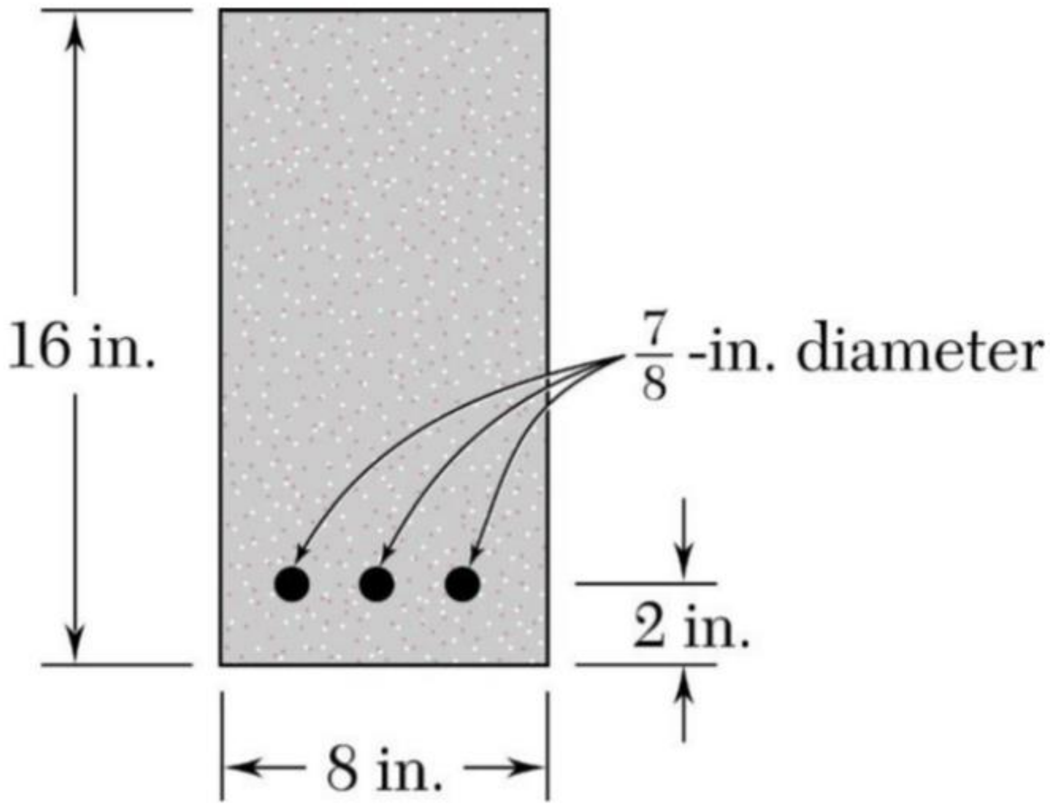
$$\therefore \sigma_a = -\frac{(1)(200)(-0.0135)}{52.704\times10^{-9}} = 51.2\times10^6 \text{ Pa} \Rightarrow \sigma_a = 51.2 \text{ MPa}$$

b) Steel: $n = 3$, $y = 10.5 \text{ mm} = 0.0105 \text{ m}$

$$\therefore \sigma_s = -\frac{(3)(200)(0.0105)}{52.704\times10^{-9}} = -119.5\times10^6 \text{ Pa} \Rightarrow \sigma_s = -119.5 \text{ MPa}$$

8- A concrete beam is reinforced by three steel rods placed as shown. The modulus of elasticity is 3×10^6 psi for the concrete and 30×10^6 psi for the steel. Using an allowable stress of 1350 psi for the concrete and 20 ksi for the steel, determine the largest allowable positive bending moment in the beam.

(Answer: M = 389 kip.in = 32.4 kip.ft)



Solutions

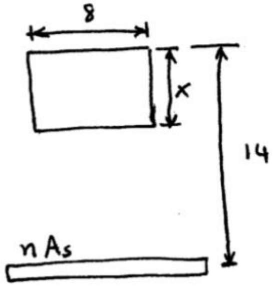
To obtain the transformed section of a reinforced concrete beam, we replace the total cross-sectional area A_s of the steel bars by an equivalent area nA_s where $n = E_s/E_s$.

$$\therefore \quad n = \frac{30\times10^6}{3\times10^6} = 10 \qquad A_s = 3\left(\frac{\pi}{4}d^2\right) = 3\left(\frac{\pi}{4}\right)\left(\frac{7}{8}\right)^2 = 1.804 \text{ in}^2$$

$$\Rightarrow \quad nA_s = 18.04 \text{ in}^2$$

next, we locate the neutral axis:

$$8(x)\frac{x}{2} - (18.04)(14-x) = 0 \qquad 4x^2 + 18.04x - 252.56 = 0$$



Solve for x:

$$x = \frac{-18.04 + \sqrt{18.04^2 + (4)(4)(252.56)}}{(2)(4)}$$

$$= 6.005 \text{ in}$$

$$\therefore \quad 14 - x = 7.995 \text{ in}$$

To find the moment of inertia I:

$$I = \frac{1}{3}(8)x^3 + nA_s(14-x)^2 = \frac{1}{3}(8)(6.005)^3 + (18.04)(7.995)^2$$

$$= 1730.4 \text{ in}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \qquad \therefore \quad M = \frac{\sigma I}{ny}$$

For the concrete: $\quad n = 1 \qquad |y| = 6.005 \text{ in} \qquad |\sigma| = 1350 \text{ psi}$

$$\therefore \quad M = \frac{(1350)(1730.5)}{(1)(6.005)} = 389\times10^3 \text{ lb.in} = 389 \text{ kip.in}$$

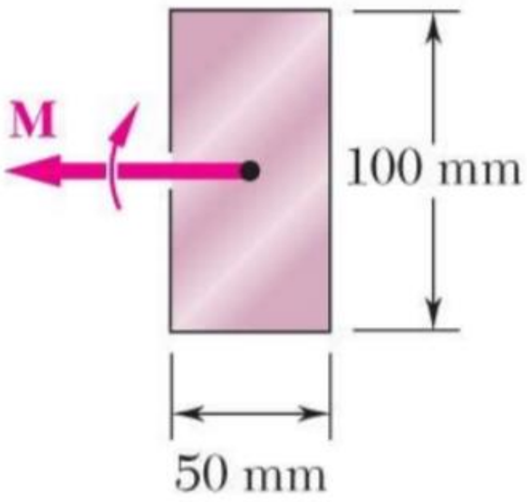
steel: $\quad n = 10, \qquad |y| = 7.995 \qquad \sigma = 20\times10^3 \text{ psi}$

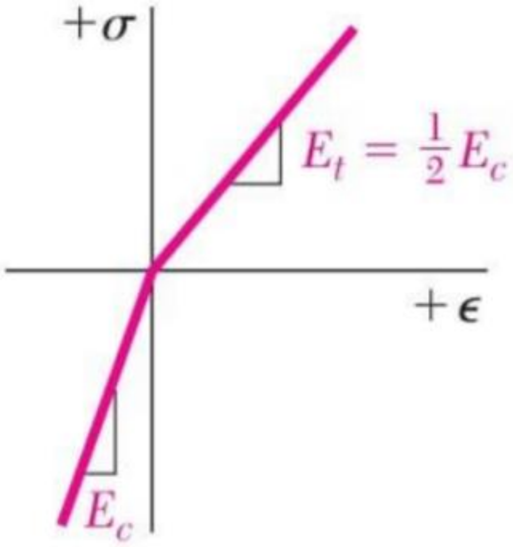
$$M = \frac{(20\times10^3)(1730.5)}{(10)(7.995)} = 433\times10^3 \text{ lb.in} = 433 \text{ kip.in}$$

choose the smallest value: $\quad M = 389 \text{ kip.in} = 32.4 \text{ kip.ft}$

10- The rectangular beam shown is made of a plastic for which the value of the modulus of elasticity in tension is one-half of its value in compression. For a bending moment M = 600 N.m, determine the maximum (a) tensile stress, (b) compressive stress.

(Answer: (a) σ_T= 6.15 MPa ; (b) σ_C = -8.69 MPa)



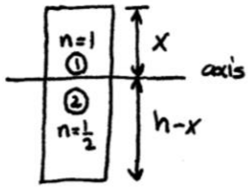


Taking the compression side as the reference $\Rightarrow$ $n = 1$ (for compression side)

$\Rightarrow$ For the tension side $n = \frac{E_t}{E_c} = \frac{1}{2}$

The location of the neutral axis

$$n_1 b x \frac{x}{2} - n_2 b (h-x) \frac{h-x}{2} = 0$$



$$\frac{bx^2}{2} - \frac{b}{4}(h-x)^2 = 0$$

$$x^2 = \frac{1}{2}(h-x)^2 \Rightarrow x = \frac{1}{\sqrt{2}}(h-x)$$

$$x = \frac{1}{\sqrt{2}+1} h = 0.41421 h = 41.421 \text{ mm}$$

$$h - x = 58.579 \text{ mm}$$

Now, we calculate the moment of inertia:

$$I_1 = n_1 \tfrac{1}{3} b x^3 = (1)(\tfrac{1}{3})(50)(41.421)^3 = 1.1844 \times 10^6 \text{ mm}^4$$

$$I_2 = n_2 \tfrac{1}{3} b (h-x)^3 = (\tfrac{1}{2})(\tfrac{1}{3})(50)(58.579)^3 = 1.675 \times 10^6 \text{ mm}^4$$

$$I = I_1 + I_2 = 2.8595 \times 10^6 \text{ mm}^4 = 2.8595 \times 10^{-6} \text{ m}^4$$

a) tensile stress: $n = \frac{1}{2}$ $y = -58.579 \text{ mm} = -0.058579 \text{ m}$

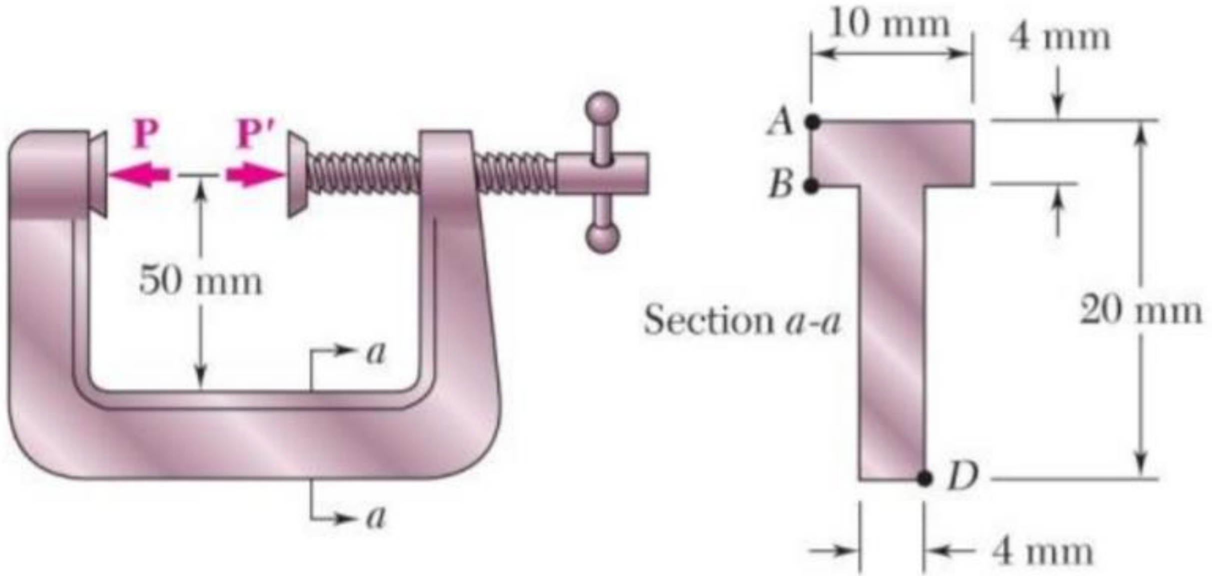
$$\sigma = -\frac{nMy}{I} = -\frac{(0.5)(600)(-0.058579)}{2.8595 \times 10^{-6}} = 6.15 \times 10^6 \text{ Pa} \quad \sigma_T = 6.15 \text{ MPa}$$

b) compressive stress: $n = 1$ $y = 41.421 \text{ mm} = 0.04142 \text{ m}$

$$\sigma = -\frac{nMy}{I} = -\frac{(1)(600)(0.041421)}{2.8595 \times 10^{-6}} = -8.69 \times 10^6 \text{ Pa} \quad \sigma_c = -8.69 \text{ MPa}$$

11- Knowing that the clamp shown has been tightened on wooden planks being glued together until P = 400 N, determine in section *a-a* (a) the stress at point *A*, (b) the stress at point *D*, (c) the location of the neutral axis.

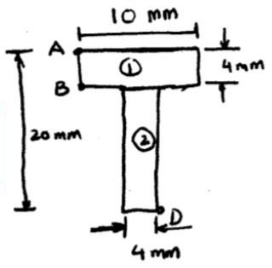
(Answer: (a) σ_A= 52.7 MPa (b) σ_B = -67.2 MPa, (c) neutral axis lies at 11.20 mm above *D*)



First we need to locate the neutral axis

	$A\ (mm)^2$	$\bar{y}_o\ (mm)$	$A\bar{y}_o\ (mm^3)$
1	40	18	720
2	64	8	512
	104		1232

$$\boxed{\bar{Y}_o = \frac{1232}{104} = 11.846\ mm}$$



section a-a

∴ The centroid lies 11.846 mm above point D.

Eccentricity : $e = (50 + 20 - 11.846) = -58.154\ mm$

Therefore : Bending couple $M = Pe = (400)(-58.154 \times 10^{-3}) = -23.262\ N.m$

$A = 104\ mm^2 = 104 \times 10^{-6}\ m^2$

Now we calculate the moment of inertia

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(10)(4)^3 + (40)(6.154)^2 = 1.568 \times 10^3\ mm^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12}(4)(16)^3 + (64)(3.846)^2 = 2.312 \times 10^3\ mm^4$$

$$I = I_1 + I_2 = 3.8802 \times 10^3\ mm^4 = 3.8802 \times 10^{-9}\ m^4$$

a) The stress at point A $\quad y = 20 - 11.846 = 8.154\ mm = 8.154 \times 10^{-3}\ m$

$$\sigma_A = \frac{P}{A} - \frac{My}{I} = \frac{400}{104 \times 10^{-6}} - \frac{(-23.262)(8.154 \times 10^{-3})}{3.8802 \times 10^{-9}} = 52.7 \times 10^6\ Pa$$

$\sigma_A = 52.7\ MPa$

b) Stress at point D $\quad y = -11.846\ mm = -11.846 \times 10^{-3}\ m$

$$\sigma_B = \frac{P}{A} + \frac{My}{I} = \frac{400}{104 \times 10^{-6}} - \frac{(-23.262)(-11.846 \times 10^{-3})}{3.8802 \times 10^{-9}} = -67.2 \times 10^6\ Pa$$

$\sigma_B = -67.2\ MPa$

c) Location of the neutral axis : $\quad \sigma = 0$

$$\boxed{\sigma = \frac{P}{A} - \frac{My}{I} = 0}$$

$$\Rightarrow y = \frac{I}{Ae} = \frac{3.8802 \times 10^{-9}}{(104 \times 10^{-6})(-58.154 \times 10^{-3})} = -0.642 \times 10^{-6}\ m = -0.642\ mm$$

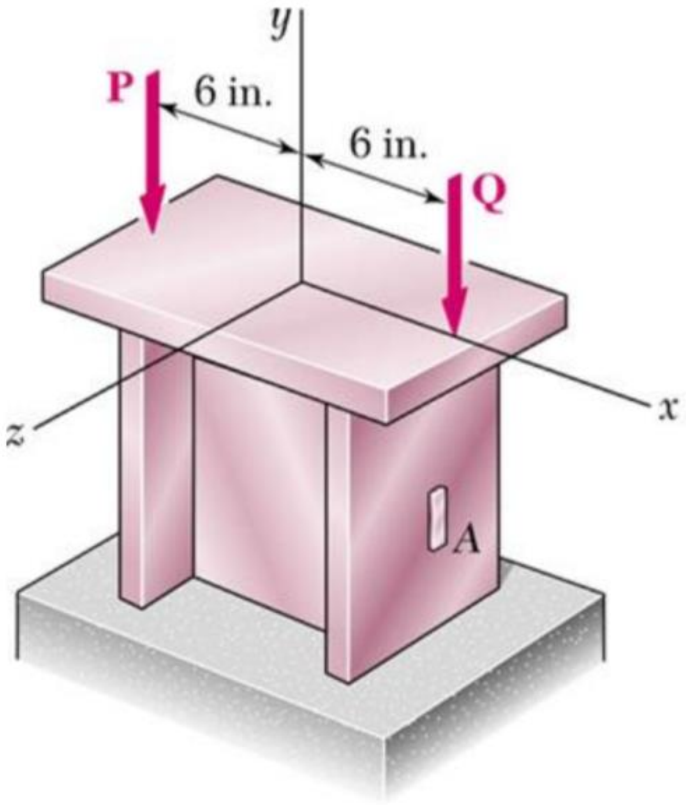
the neutral axis lies $11.846 - 0.642 = 11.205\ mm$ $\quad$ 11.20 mm above D

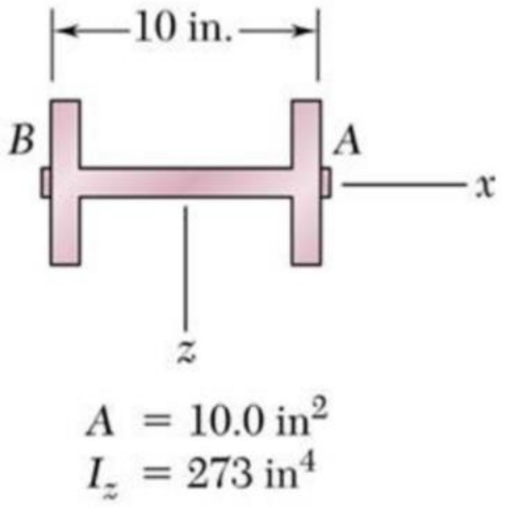
12-* A short length of a rolled-steel column supports a rigid plate on which two loads **P** and **Q** are applied as shown. The strains at two points A and B on the centerline of the outer faces of the flanges have been measured and found to be

$$\varepsilon_A = -400 \times 10^{-6} \text{ in/in} \quad \varepsilon_B = -300 \times 10^{-6} \text{ in/in}$$

Knowing that E = 29 x 10^6 psi, determine the magnitude of each load.

(Answer: P = 44.2 x 10^3 lb = 44.2 kips, Q = 57.3 x 10^3 kips = 57.3 kips)





First, we find the stresses at A and B from strain gages :

$$\sigma_A = E\varepsilon_A = (29\times10^6)(-400\times10^{-6}) = -11.6\times10^3 \text{ psi}$$

$$\sigma_B = E\varepsilon_B = (29\times10^6)(-300\times10^{-6}) = -8.7\times10^3 \text{ psi}$$

Now, the centric force : $F = P + Q$

the Bending couple : $M = 6P - 6Q$; $c = 5$ in

$$\therefore \quad \sigma_A = -\frac{F}{A} + \frac{Mc}{I} = -\frac{P+Q}{10} + \frac{(6P-6Q)(5)}{273}$$

$$-11.6\times10^3 = +0.00989P - 0.20989Q \quad \text{---- (1)}$$

$$\sigma_B = -\frac{F}{A} - \frac{Mc}{I} = -\frac{P+Q}{10} - \frac{(6P-6Q)(5)}{273}$$

$$-8.7\times10^3 = -0.20989P + 0.00989Q \quad \text{---- (2)}$$

Solving (1) & (2) gives:

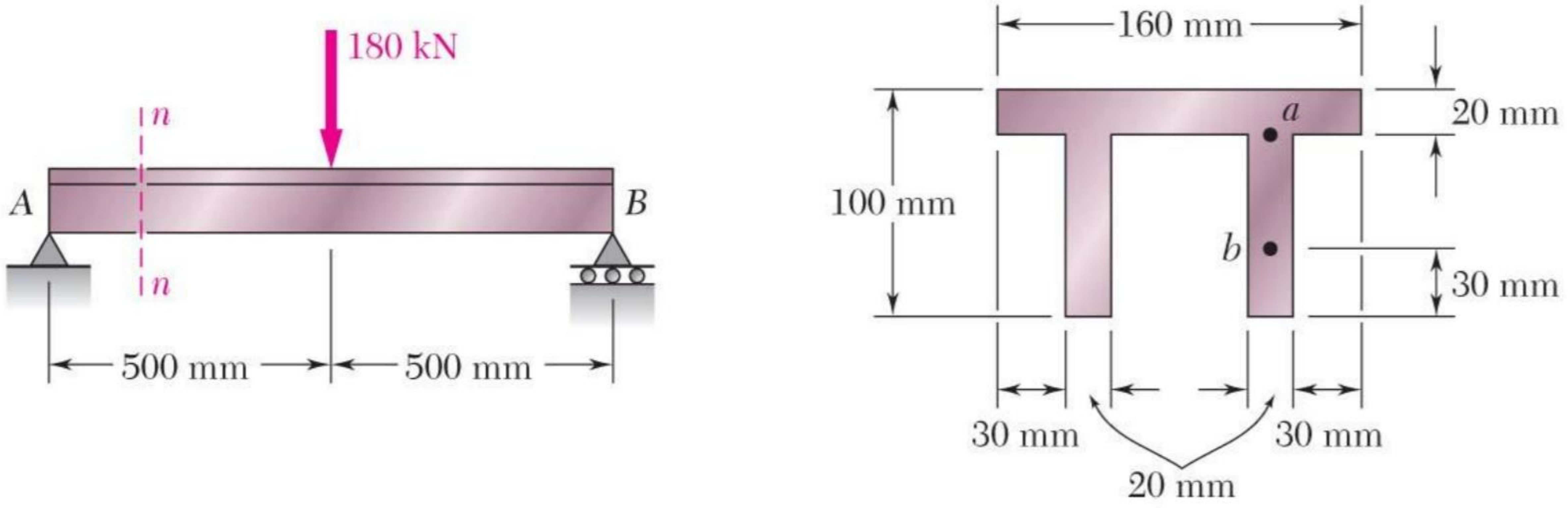
$$P = 44.2\times10^3 \text{ lb} = 44.2 \text{ kips}$$

$$Q = 57.3\times10^3 \text{ lb} = 57.3 \text{ kips}$$

Shear Flow:

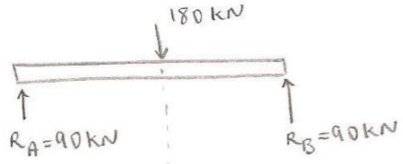
2- For the beam and loading shown, consider section *n-n* and determine the shearing stress at *(a)* point *a*, *(b)* point *b*.

(Answer: (a) τ_a = 31.0 MPa (b) τ_b = 23.2 MPa)

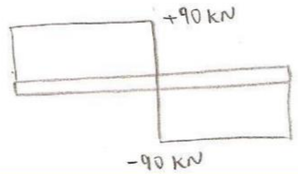


Solution

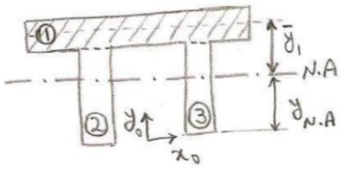
FBD for the beam



Shear diagram



Transrverse shear stress Equation : $\tau = \dfrac{VQ}{It}$ $\longrightarrow$ $V_{n-n} = 90\,kN = 9\times10^4\,N$



(a) $V = 9\times10^4\,N$

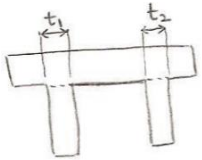
$Q_a = \int y\,dA = \bar{y}_1 \cdot A_1$ $\longrightarrow$ for finding $\bar{y}_1$, we first need to locate N.A

$$y_{N.A} = \frac{A_1 y_1 + 2A_2 y_2}{A_1 + 2A_2} = \frac{160\times20\times90 + 2\times20\times80\times40}{160\times20 + 2\times20\times80} = 65\ mm$$

$\longrightarrow \bar{y}_1 = 90 - 65 = 25\ mm$ $\longrightarrow Q_a = \bar{y}_1 \times A_1 = 25\times160\times20 = 8\times10^4\ mm^3$

$I = I_1 + 2I_2$ $\quad I_1 = \frac{1}{12}bh^3 + Ad^2 = \frac{1}{12}\times160\times20^3 + 160\times20\times25^2 = 2.107\times10^6\ mm^4$

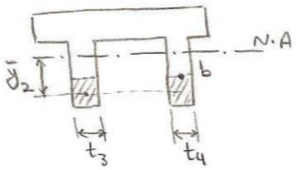
$I_2 = \frac{1}{12}bh^3 + Ad^2 = \frac{1}{12}\times20\times80^3 + 20\times80\times(65-40)^2 = 1.853\times10^6\ mm^4$



$\longrightarrow I = I_1 + 2I_2 = 5.813\times10^6\ mm^4$

$t_a = t_1 + t_2 = 40\ mm$ $\longrightarrow \tau_a = \dfrac{V.Q_a}{I.t_a} = \dfrac{9\times10^4\times8\times10^4}{5.813\times10^6\times40} = 30.96\ mm$

(b) $Q_b = \bar{y}_2 \cdot A_{hatched} = (65-15)\times2\times20\times30 = 6\times10^6\ mm^3$



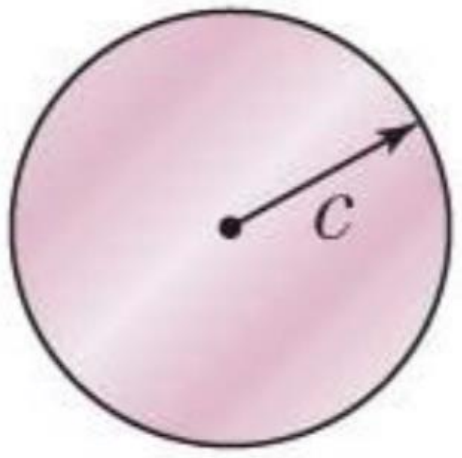
$t_b = t_3 + t_4 = 40\ mm$ $\longrightarrow \tau_b = \dfrac{V.Q_b}{I.t_b} = \dfrac{9\times10^4\times6\times10^6}{5.813\times10^6\times40}$

$\longrightarrow \tau_b = 23.22\ MPa$

4- A beam having the cross section shown is subjected to a vertical shear V. Determine *(a)* the horizontal line along which the shearing stress is maximum, *(b)* the constant k in the following expression for the maximum shearing stress

$$\tau_{max} = k\frac{V}{A}$$

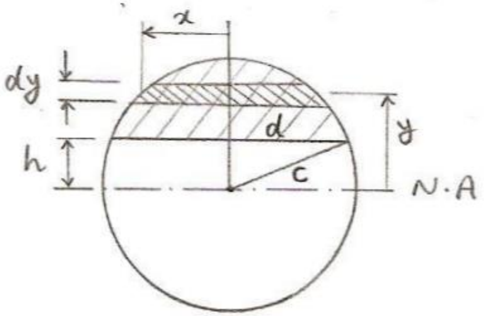
where A is the cross-sectional area of the beam.



(Answer: (a) t=2c (b) k= 1.333)

(a) $\tau = \frac{VQ}{It}$ As mentioned in the solution to the problem 3, V and I are constant throughout the cross section. So, we only need to take the changes in Q and t into account.

Considering an arbitrary section of the cross section, with a distance h to center (N.A),



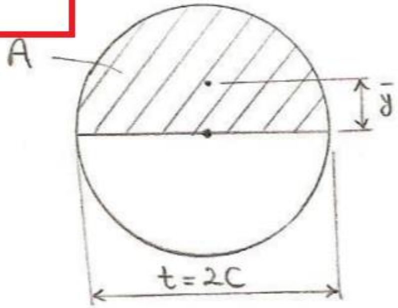
$$\left.\begin{array}{l} Q = \int y\,dA \\ dA = 2\times x\,dy = 2\times\sqrt{c^2-y^2}\,dy \end{array}\right\} \longrightarrow Q = 2\int_{y=h}^{c} y\sqrt{c^2-y^2}\,dy$$

let $(c^2-y^2) = t^2 \longrightarrow -2y\,dy = 2t\,dt$

$$\longrightarrow Q = 2\int y\sqrt{c^2-y^2}\,dy = -2\int t^2\,dt = -\tfrac{2}{3}t^3 = -\tfrac{2}{3}(c^2-y^2)^{3/2}\Big|_{y=h}^{c} = \tfrac{2}{3}(c^2-h^2)^{3/2}$$

$$\left.\begin{array}{l} t = 2d \\ h^2+d^2 = c^2 \end{array}\right\} \longrightarrow t = 2\sqrt{c^2-h^2} \longrightarrow \tau = \frac{VQ}{It} = \frac{V}{I}\,\frac{\frac{2}{3}(c^2-h^2)^{3/2}}{2(c^2-h^2)^{1/2}} = \frac{1}{3}\frac{V}{I}(c^2-h^2)$$

$$\frac{d\tau}{dh} = 0 \longrightarrow \frac{1}{3}\frac{V}{I}(-2h) = 0 \longrightarrow h = 0 \longrightarrow \tau_{max} \text{ occurs at center}$$

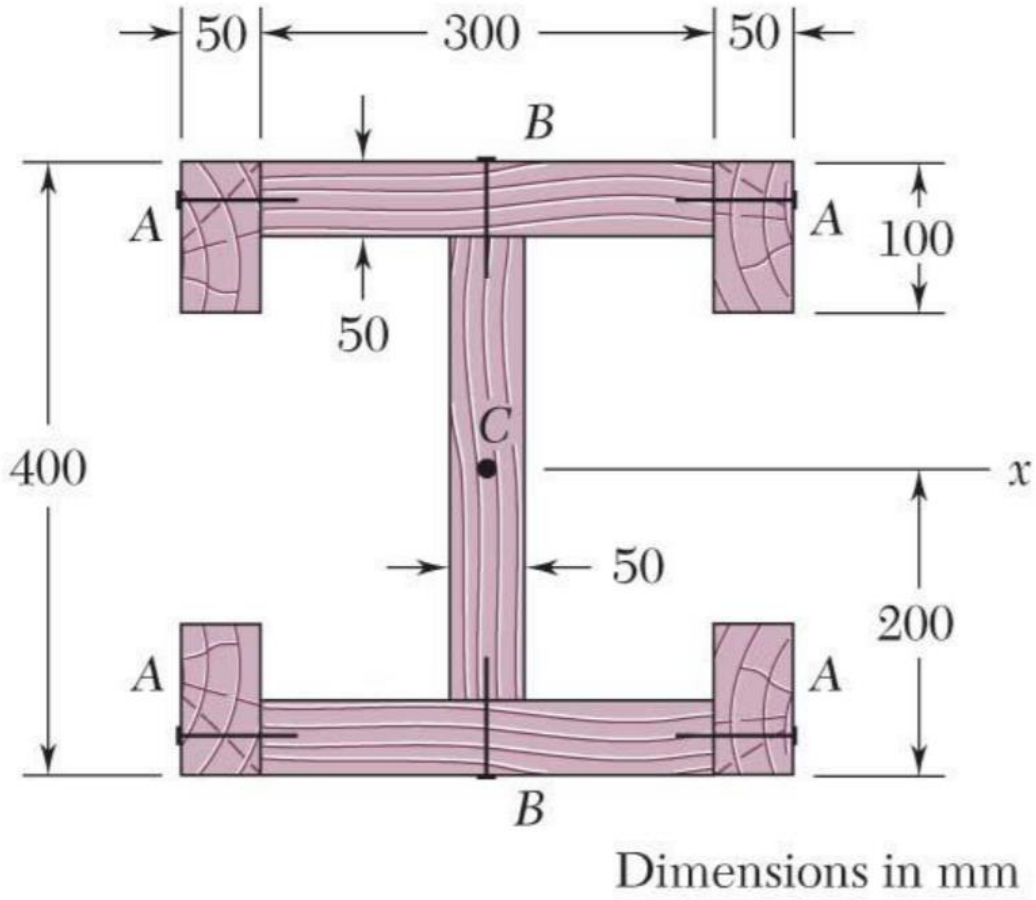


(b) $\tau = \frac{VQ}{It}$ for a semi-circle: $\bar{y} = \frac{4c}{3\pi}$; $A = \frac{\pi}{2}c^2$

$$\longrightarrow Q = \bar{y}\cdot A = \tfrac{2}{3}c^3 \quad \& \quad t = 2c \longrightarrow \tau = \frac{V}{\frac{\pi}{4}c^4}\times\frac{\frac{2}{3}c^3}{2c} = \frac{4}{3}\frac{V}{\pi c^2} \longrightarrow k = \frac{4}{3}$$

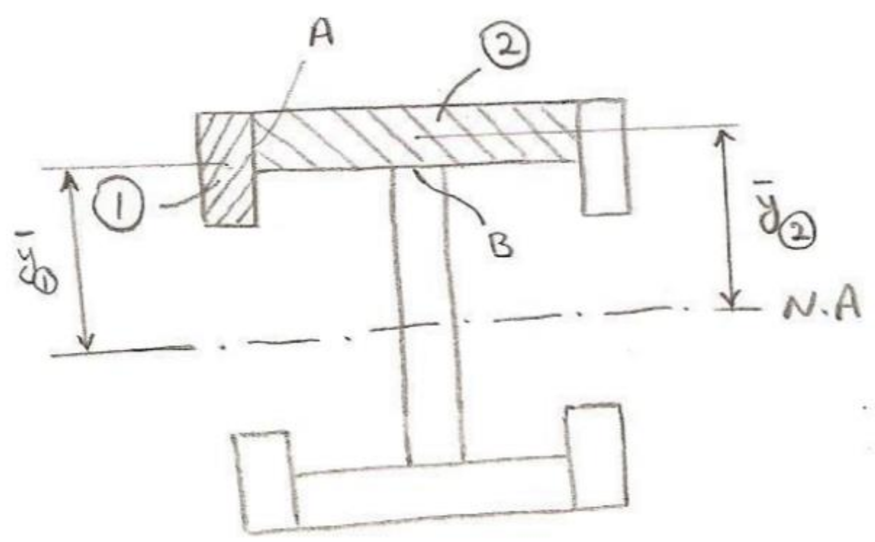
5- The built-up wooden beam shown is subjected to a vertical shear of 8 kN. Knowing that the nails are spaced longitudinally every 60 mm at A and every 25 mm at B, determine the shearing force in the nails *(a)* at A, *(b)* at B. (Given: $I_x = 1.504 \times 10^9\ mm^4$.)

(Answer: (a) F_A = 239 N (b) F_B = 549 N)



shear flow equation: $q = \frac{VQ}{I}$

$$\rightarrow \begin{cases} q_A = \dfrac{VQ_A}{I} \\ q_B = \dfrac{VQ_B}{I} \end{cases}$$



$$Q_A = Q_1 = \bar{y}_1 \cdot A_1 = (200-50) \times 50 \times 100 = 7.5 \times 10^5 \ mm^3$$

$$Q_B = Q_2 + 2Q_1$$

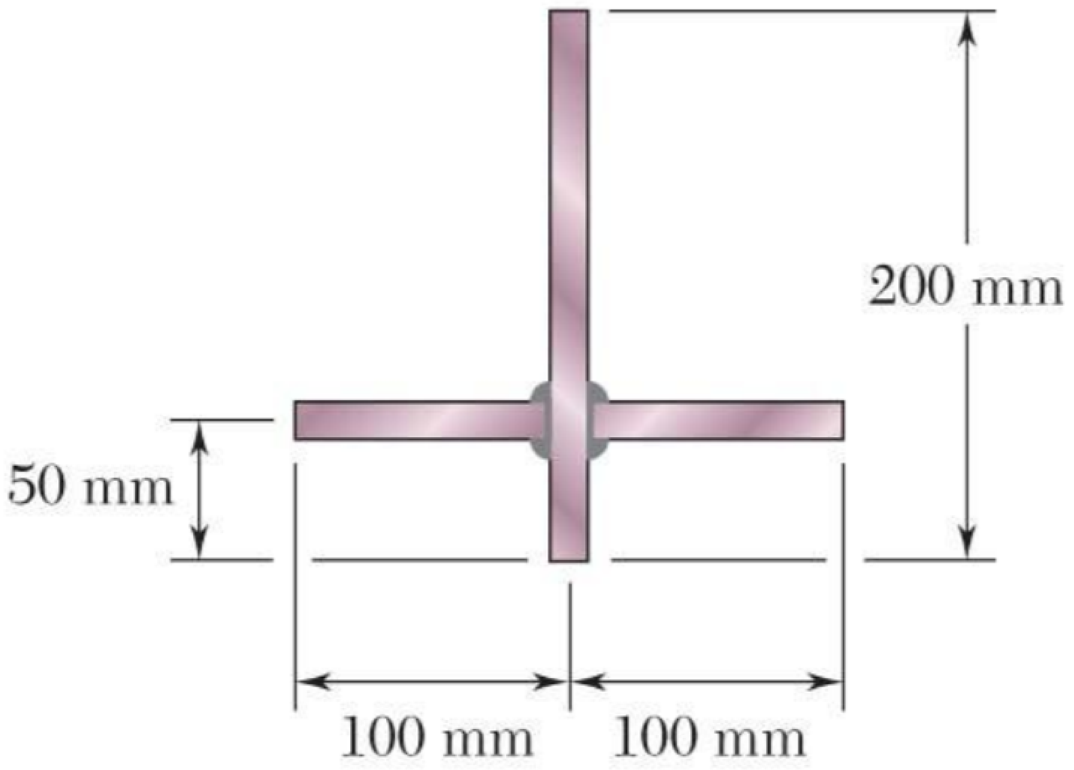
$$Q_2 = \bar{y}_2 \cdot A_2 = (200-25) \times 300 \times 50 = 2.625 \times 10^6 \ mm^3 \longrightarrow Q_B = 4.125 \times 10^6 \ mm^3$$

$$\rightarrow \begin{cases} q_A = \dfrac{8 \times 10^3 \times 7.5 \times 10^5}{1.504 \times 10^9} = 3.99 \ N/mm \\ q_B = \dfrac{8 \times 10^3 \times 4.125 \times 10^6}{1.504 \times 10^9} = 21.94 \ N/mm \end{cases}$$

(a) $F_A = q_A \cdot S_A = 3.99 \times 60 = 239.4 \ N$

6- Three plates each 12 mm thick, are welded together to form the section shown. For a vertical shear of 100 kN, determine the shear flow through the welded surfaces and sketch the shear flow in the cross section.

(Answer: q_m = 848 kN/m)

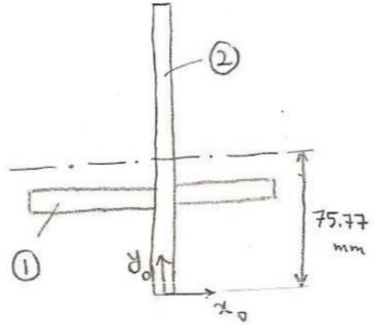


$$q = \frac{VQ}{I} \qquad V = 100 \text{ kN} = 10^5 \text{ N}$$

For finding Q and I we first need to locate neutral axis,

$$y_{N.A} = \frac{2 \times y_{①} \times A_{①} + y_{②} \times A_{②}}{2A_{①} + A_{②}} = \frac{2 \times 50 \times (100-6) \times 12 + 100 \times 200 \times 12}{2 \times 94 \times 12 + 200 \times 12}$$

$$\rightarrow y_{N.A} = 75.77 \text{ mm}$$



$$I = 2I_{①} + I_{②}$$

$$I_{①} = \frac{1}{12}bh^3 + Ad^2 = \frac{1}{12} \times 94 \times 12^3 + 94 \times 12 \times 25.77^2 = 7.626 \times 10^5 \text{ mm}^4$$

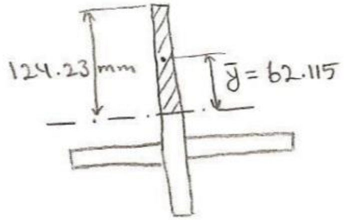
$$I_{②} = \frac{1}{12}bh^3 + Ad^2 = \frac{1}{12} \times 200 \times (100 - 75.77)^2 = 9.410 \times 10^6 \text{ mm}^4$$

$$\rightarrow I = 1.0934 \times 10^7 \text{ mm}^4$$

$$Q_{weld} = \bar{y}_{①} \times A_{①} = 25.77 \times (12 \times 94) = 2.907 \times 10^4 \text{ mm}^3$$

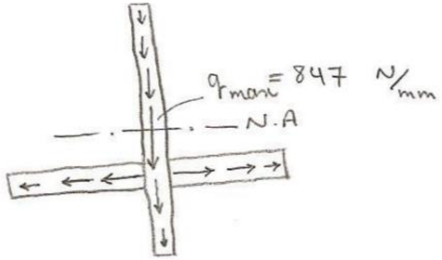
$$q_{weld} = \frac{VQ}{I} = \frac{10^5 \times 2.907 \times 10^4}{1.0934 \times 10^7} = 265.87 \text{ N/mm}$$

To draw the shear flow diagram, we need to find the q_{max} which happens on the neutral axis,



$$q_{max} = \frac{VQ}{I} \qquad Q_{max} = \bar{y} \cdot A = 62.115 \times (124.23 \times 12) = 9.26 \times 10^4 \text{ mm}^3$$

$$\rightarrow q_{max} = \frac{10^5 \times 9.26 \times 10^4}{1.0934 \times 10^7} = 847 \text{ N/mm}$$



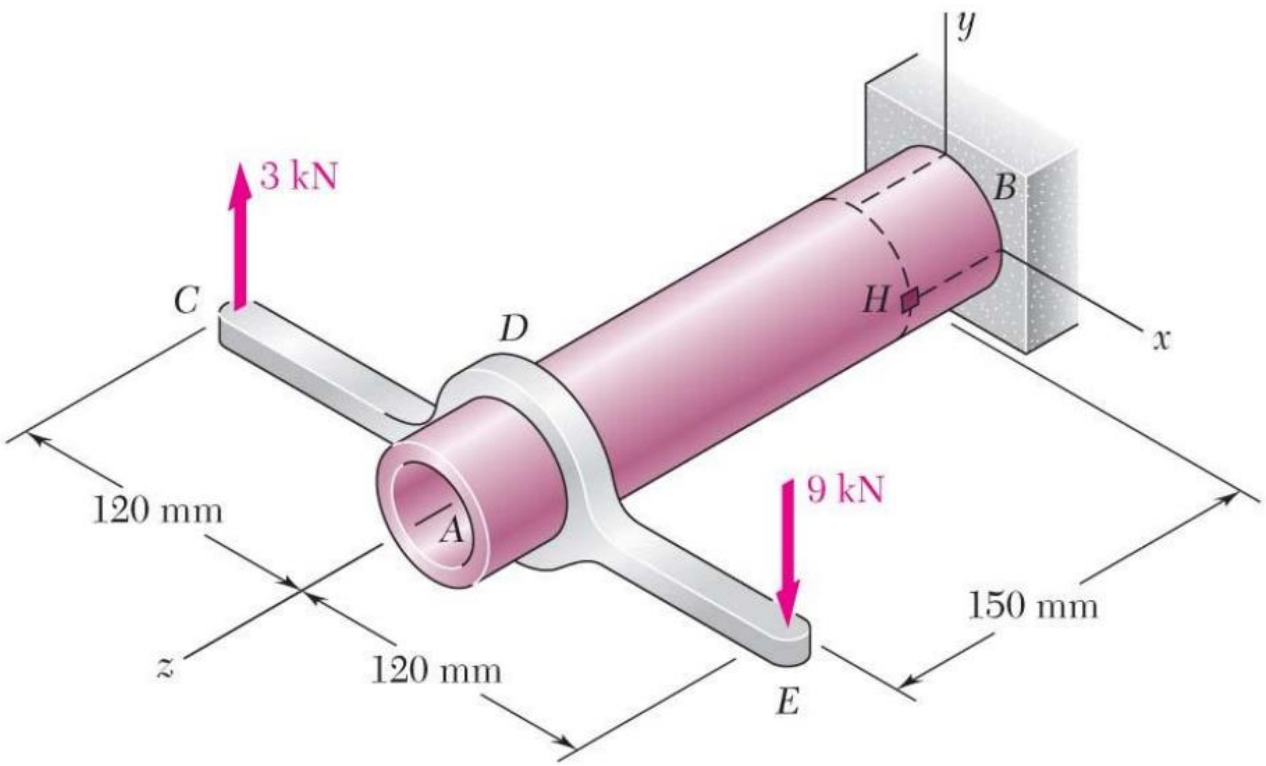
Combined Loading and Yield:

1) **Critical Section**: section with max internal load of the structure
2) **Equivalent Internal Forces** of the critical section (at centroid)
3) **Critical Point H**: max stress point
4) **Stress Transformation**: max shear and stress
5) **Yield Criterion**, **Factor of Safety**

Tips: Do *not* mess up **radius** and **diameter**; pipe inner diameter needs to subtract **2** x thickness

The steel pipe *AB* has a 72-mm outer diameter and a 5-mm wall thickness. Knowing that the arm *CDE* is rigidly attached to the pipe, determine the principal stresses, principal planes, and the maximum shear stress at point *H*.

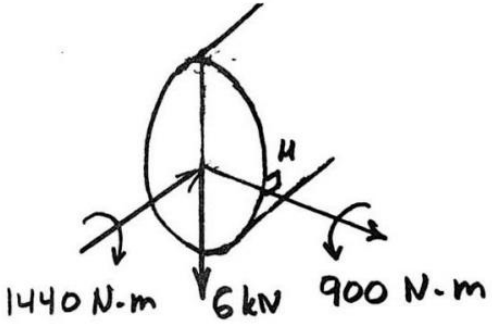
(Answer: σ_a=55Mpa, σ_b=-55MPa, -45º, 45º, τ_{max}=55MPa**)**



Replace the forces at C and E by an equivalent force-couple system at D.

$$F_D = 9 - 3 = 6 \text{ kN} \downarrow$$

$$T_D = (9\times10^3)(120\times10^{-3}) + (3\times10^3)(120\times10^{-3}) = 1440 \text{ N}\cdot\text{m}$$



At the section containing point H,

$P = 0$, $V = 6$ kN, $T = 1440$ N·m

$$M = (6\times10^3)(150\times10^{-3}) = 900 \text{ N}\cdot\text{m}$$

Section properties: $d_o = 72$ mm $\quad c_o = \frac{1}{2}d_o = 36$ mm $\quad c_i = c_o - t = 31$ mm

$$A = \pi(c_o^2 - c_i^2) = 1.0524\times10^3 \text{ mm}^2 = 1.0524\times10^{-3} \text{ m}^2$$

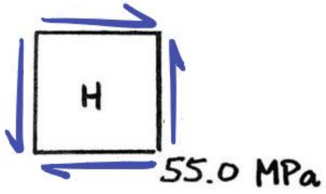
$$I = \frac{\pi}{4}(c_o^4 - c_i^4) = 593.84\times10^{-3} \text{ mm}^4 = 593.84\times10^{-9} \text{ m}^4$$

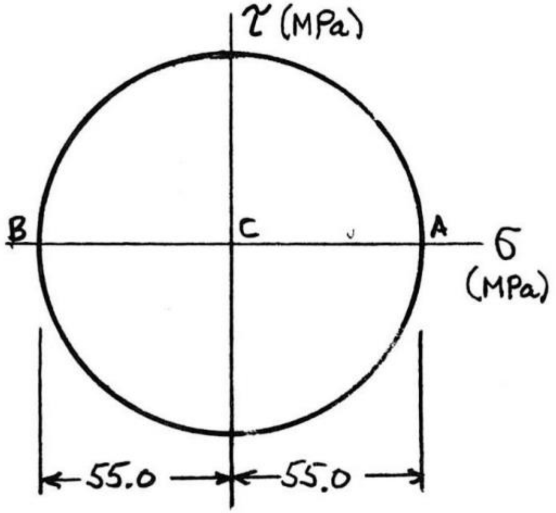
$$J = 2I = 1.1877\times10^{-6} \text{ m}^4$$

For half-pipe, $Q = \frac{2}{3}(c_o^3 - c_i^3) = 11.243\times10^3 \text{ mm}^3 = 11.243\times10^{-6} \text{ m}^3$

At point H. $\boxed{\text{Point H lies on the neutral axis of bending. } \sigma_H = 0.}$

$$\boxed{\tau_H = \frac{Tc}{J} + \frac{VQ}{It}} = \frac{(1440)(36\times10^{-3})}{1.1877\times10^{-6}} + \frac{(6\times10^3)(11.243\times10^{-6})}{(593.84\times10^{-9})(10\times10^{-3})} = 55.0 \text{ MPa}$$





Use Mohr's circle.

$\sigma_c = 0$

$R = 55.0$ MPa

$\sigma_a = \sigma_c + R = 55.0$ MPa

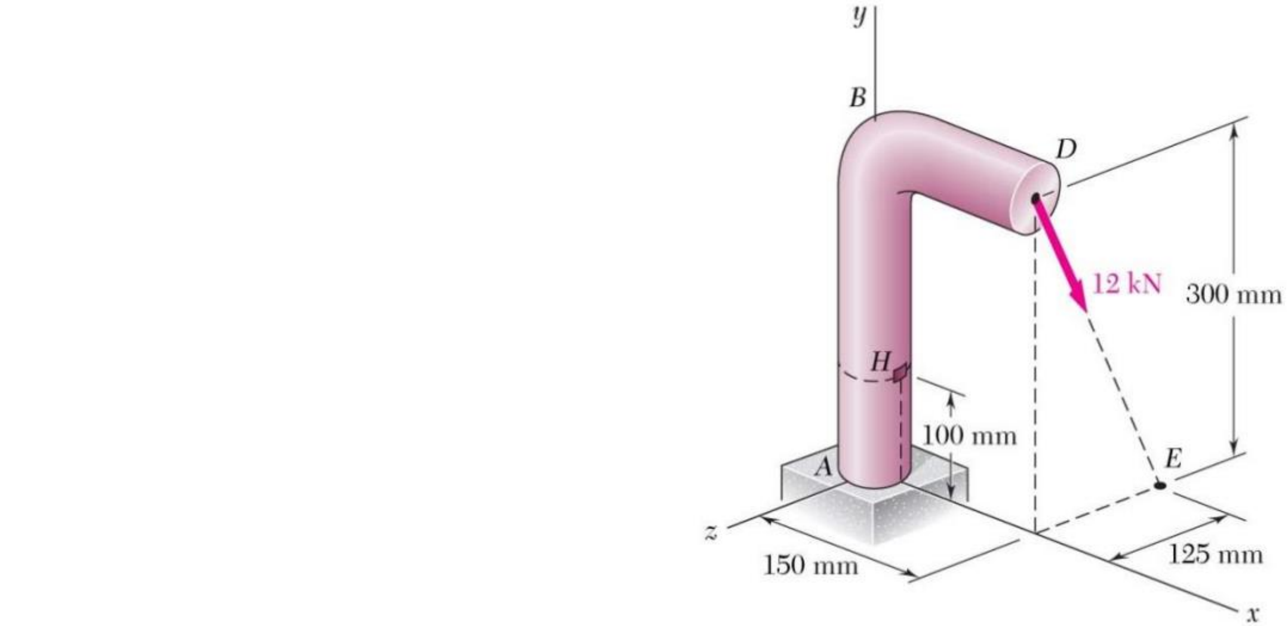
$\sigma_b = \sigma_c - R = -55.0$ MPa

$\theta_a = -45°$, $\theta_b = +45°$

$\tau_{max} = R = 55.0$ MPa

A 12-kN force is applied as shown to the 60-mm-diameter cast iron post *ABD*. At point *H*, determine (a) the principal stresses and principal planes, (b) the maximum shearing stress.

(Answer: σ_a=4Mpa, σ_b=-86.3MPa, 12.1°, 102.1°, τ_{max}=45.1MPa **)**



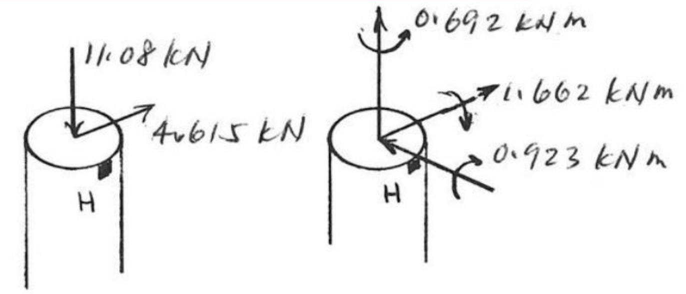
At point D $\quad F_x = 0$

$$F_y = -\left(\frac{300}{325}\right)(12) = -11.08 \text{ kN}$$

$$F_z = -\left(\frac{125}{325}\right)(12) = -4.615 \text{ kN}$$

Moment of equivalent force-couple system at C, the centroid of the section containing point H

$$\boxed{\vec{M} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0.15 & 0.2 & 0 \\ 0 & -11.08 & -4.615 \end{vmatrix}} = 0.923\,\vec{i} + 0.692\,\vec{j} - 1.662\,\vec{k} \text{ kN.mm}$$



Section properties

$d = 60$ mm $\quad c = \frac{1}{2}d = 30$ mm

$A = \pi c^2 = 2827 \text{ mm}^2$

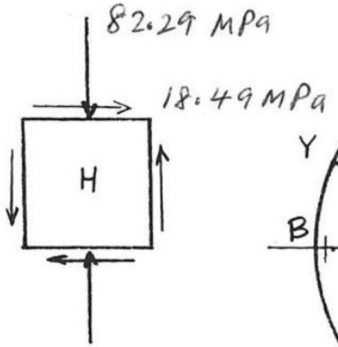
$I = \frac{\pi}{4}c^4 = 0.6362 \times 10^6 \text{ mm}^4$

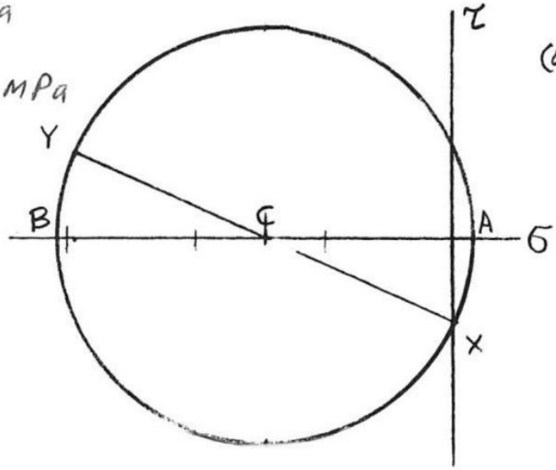
$J = 2I = 1.2724 \times 10^6 \text{ mm}^4$

For a semicircle $\quad Q = \frac{2}{3}c^3 = 18 \times 10^3 \text{ mm}^3$

At point H $\quad \sigma_H = -\frac{P}{A} - \frac{Mc}{I} = -\frac{(11.08 \times 10^3)}{(2827)} - \frac{(1.662 \times 10^6)(30)}{(0.6362 \times 10^6)} = -82.29 \text{ MPa}$

$$\tau_H = \frac{Tc}{J} + \frac{VQ}{It} = \frac{(0.692 \times 10^6)(30)}{1.2724 \times 10^6} + \frac{(4.615 \times 10^3)(18 \times 10^3)}{(0.6362 \times 10^6)(60)} = 18.49 \text{ MPa}$$





(a) $\quad \sigma_c = \frac{\sigma_H}{2} = -41.145 \text{ MPa}$

$R = \sqrt{\left(\frac{\sigma_H}{2}\right)^2 + \tau_H^2} = 45.109 \text{ MPa}$

$\sigma_a = \sigma_c + R = 4 \text{ MPa}$

$\sigma_b = \sigma_c - R = -86.3 \text{ MPa}$

$\tan 2\theta_p = \frac{2\tau_H}{|\sigma_H|} = 0.4494$

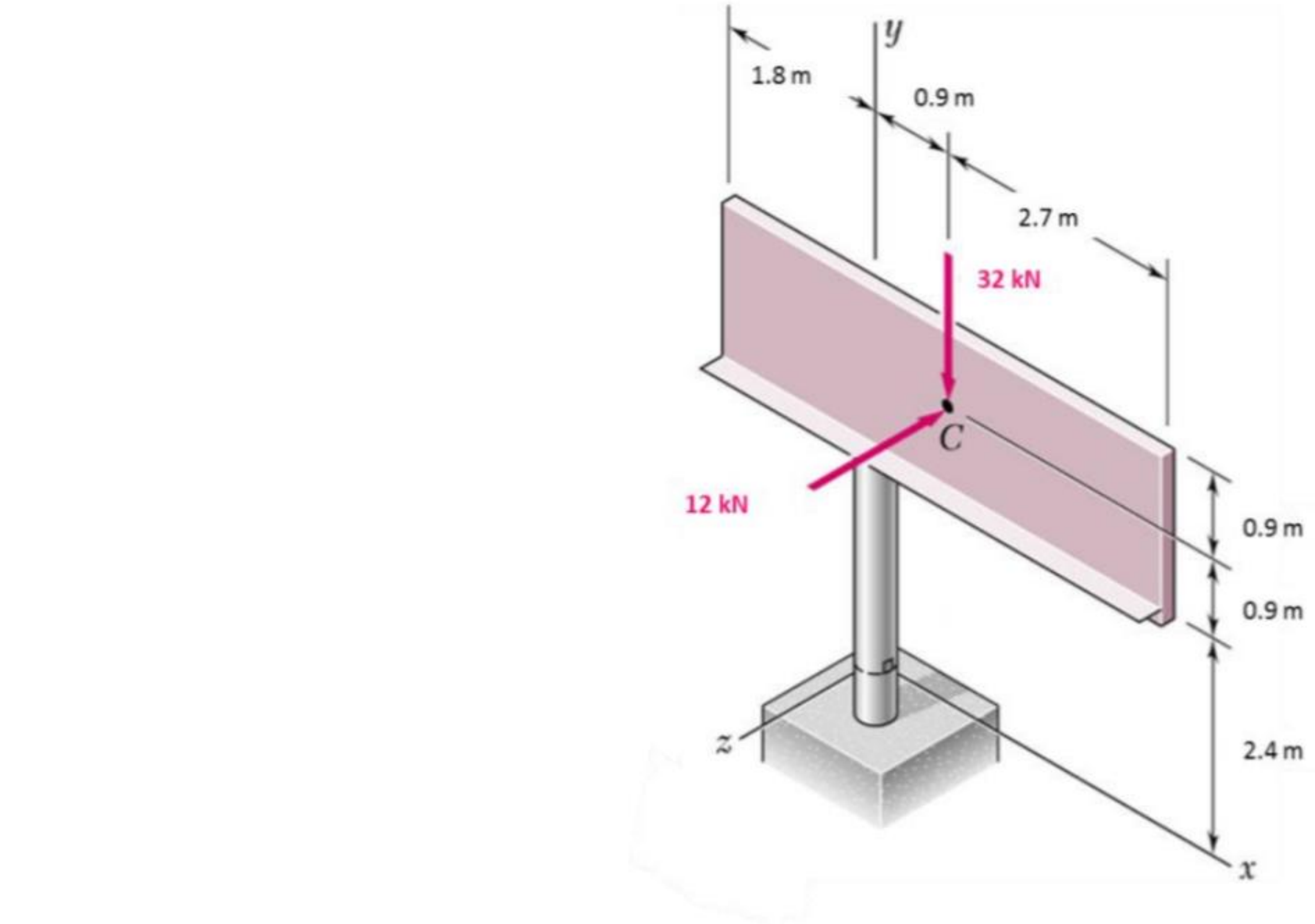
$\theta_a = 12.1°, \quad \theta_b = 102.1°$

(b) $\quad \tau_{max} = R = 45.1 \text{ MPa}$

1- The billboard shown weights **32 kN** and is supported by a structural steel tube that has a **380-mm** outer diameter and a **12-mm** wall thickness. At a time when the resultant of the wind pressure is **12 kN** located at the center *C* of the billboard, determine whether the structural tube is safe or not? Structural steel yield strength is **250 MPa**.

.

(Answer: Yes; Tresca S.F.=5.78; Mises S.F.=5.82)



1. Find Critical Section:

By inspection, and due to maximum moment arm the critical section is at the fixed support

2. Find Equivalent internal forces at the centroid of critical Section

Applied Force Vector $\vec{F} = -32\,\vec{j} - 12\,\vec{k}$ KN

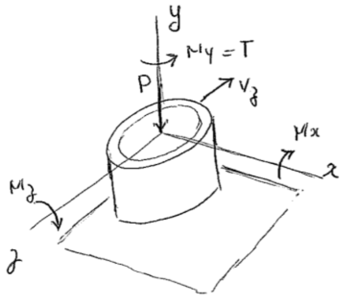
Moment arm (origin: (0,0,0) and c: (0.9, 3.3, 0) m)

$\vec{r} = 0.9\,\vec{i} + 3.3\,\vec{j}$ m

$\vec{M} = \vec{r} \times \vec{F} = (0.9\,\vec{i} + 3.3\,\vec{j}) \times (-32\,\vec{j} - 12\,\vec{k})$

$= -39.6\,\vec{i} + 10.8\,\vec{j} - 28.8\,\vec{k}$ KN.m

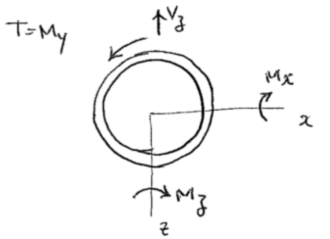
$M_x = -39.6$ KN.m $M_y = 10.8$ KN.m $M_z = -28.8$ KN.m



$P = -32$ KN

$V_z = -12$ KN

Top View



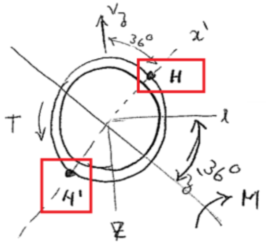
5.

3. Critical point

To find max normal stress due to bending combine Mx and Mz

$M = \sqrt{M_z^2 + M_x^2} = 48.97$ KN.m

$\tan^{-1} \frac{M_z}{M_x} = 36.03^\circ$ from x-axis

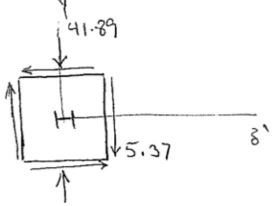


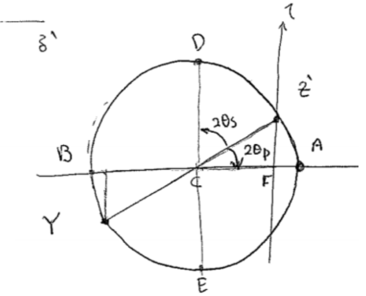
Note that dominant loading is bending. Point H is the critical point

σ_H : Max Normal stress

τ_H : Max due to torsion + contr. from Shear

State of Stress at H





Principal stresses / Max Shear

z' : (0 , 5.37) CW

Y : (−41.89 , −5.37) CCW

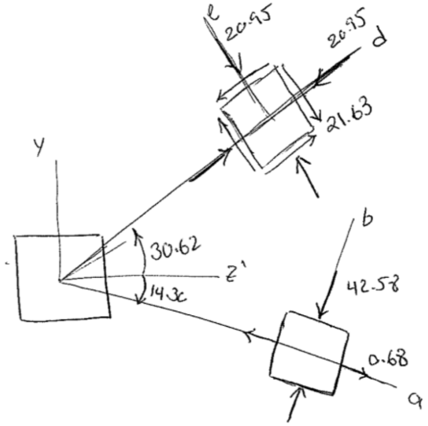
$\tan 2\theta_p = \frac{2\tau_{zy}}{\sigma_{z'} - \sigma_y} = \frac{2(5.37)}{41.89} \Rightarrow \theta_p : 14.38^\circ$

$\sigma_{av} = -20.95$ MPa

$R = 21.63$

$\sigma_a = \sigma_{av} + R = 0.68$

$\sigma_b = \sigma_{av} - R = -42.58$ MPa



$\sigma_1 = 0.68$

$\sigma_2 = 0$ MPa

$\sigma_3 = -42.58$

$\tau_{max} = R = 21.63$

6/7 Yield Criterion / Factor of Safety

Principal Stresses are

$\sigma_1 = 0.68$ MPa

$\sigma_2 = 0$ MPa

$\sigma_3 = -42.58$ MPa

A. Max Shear or Tresca

$\boxed{\tau_{max} < \frac{\sigma_Y}{2}}$ $\tau_{max} = 21.63 < 125$

$S.F. = \boxed{\frac{125}{21.63}} = 5.78$

B. Max distorsion energy or Mises σ_a, σ_b Non-zero Principal stresses

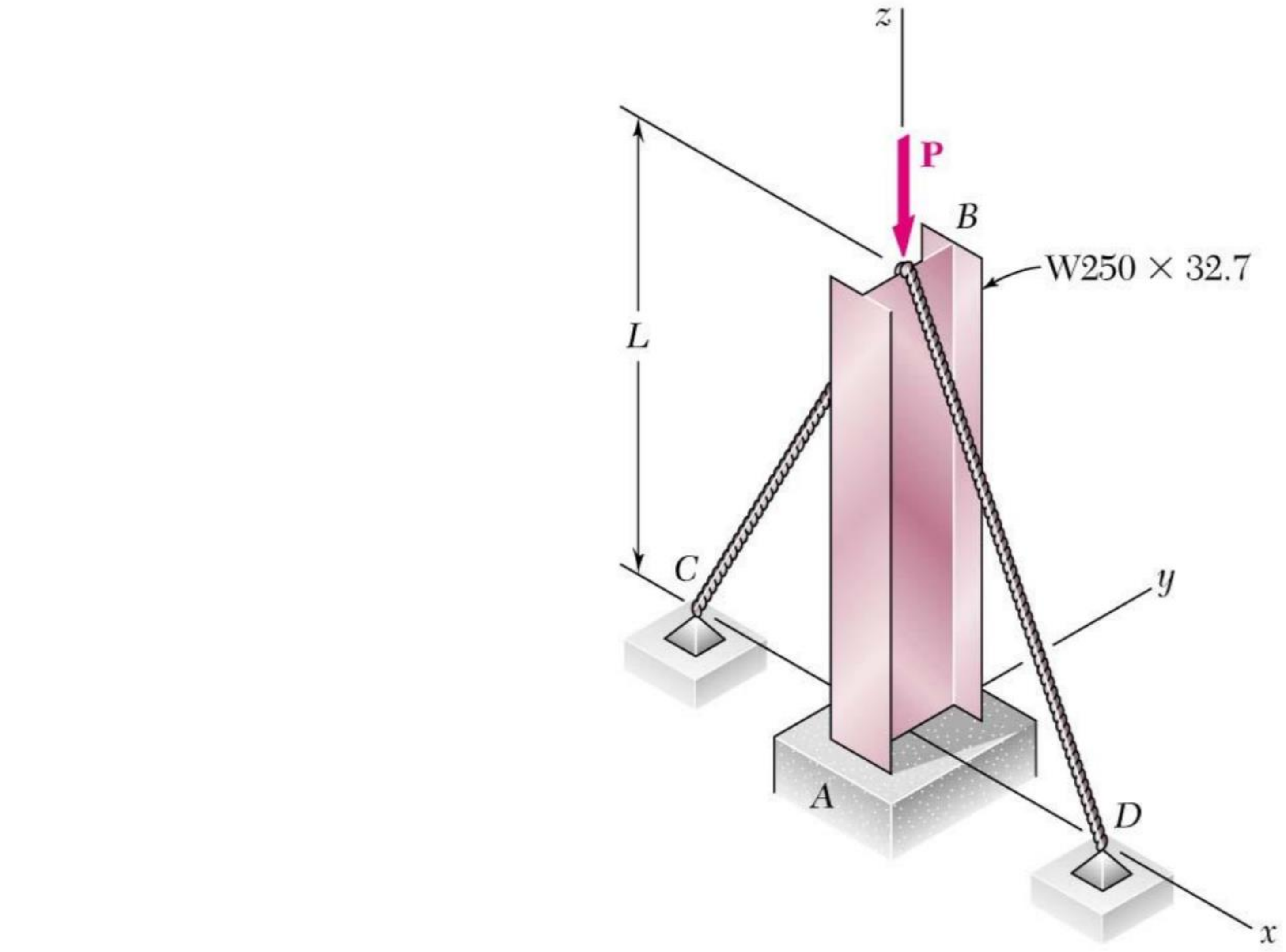
$\sigma_{eq} = \sqrt{\sigma_a^2 - \sigma_a\sigma_b + \sigma_b^2} = 42.92$ MPa

$S.F. = \frac{\sigma_Y}{\sigma_{eq}} = \frac{250}{42.92} = 5.82$

Buckling:

4- Column AB carries a centric load P of magnitude 60kN. Cables BC and BD are taut and prevent motion of point B in the xz plane. Using Euler's formula and a factor of safety of 2.2, and neglecting the tension in the cables, determine the maximum allowable length L. Use E=200GPa.

(Answer: $L = 12.01m$)



W250X32.7: $I_x = 48.9\times10^6\ mm^4$, $I_y = 4.73\times10^6\ mm^4$

$P = 60\ kN.$

$$\boxed{P_{cr} = (F.S.)P = (2.2)(60) = 132\ kN}$$

$\boxed{\underline{\text{Buckling in xz-plane}}.\quad L_e = 0.7L}$

$$P_{cr} = \frac{\pi^2 E I_y}{(0.7L)^2} \qquad L = \frac{\pi}{0.7}\sqrt{\frac{EI_y}{P_{cr}}}$$

$$L = \frac{\pi}{0.7}\sqrt{\frac{(200\times10^9)(4.73\times10^{-6})}{132000}} = 12.01\ m.$$

$\boxed{\underline{\text{Buckling in yz-plane}}.\quad L_e = 2L}$

$$P_{cr} = \frac{\pi^2 E I_x}{(2L)^2} \qquad L = \frac{\pi}{2}\sqrt{\frac{EI_x}{P_{cr}}} = \frac{\pi}{2}\sqrt{\frac{(200\times10^9)(48.9\times10^{-6})}{132000}} = 13.52\ m$$

Smaller value for L governs. $L = 12.01\ m$