Remote Sensing for Earth Observation and Surveillance – Homework #2

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In this report are discussed the proposed solutions to the problems related to Interferometric SAR (InSAR) systems and SAR signal processing. Note that some information is taken from graphs in the statement of this homework. These graphs are not included in this report.

1 LoS & AT deformation

1) *For distributed targets, it can be shown that the two images obtained by processing the same acquisition under different squint angles are completely uncorrelated with one another. Can you provide an (intuitive) explanation?*

   In the case of distributed targets, the signal is random, for interference reasons. However, interference also depends on the acquisition geometry. Since in this case, there are two different squint angles ($\pm\psi$), the interference patterns are completely different. Therefore, the two images are completely uncorrelated with one another.

2) *Discuss whether the APS depends on the squint angle.*

   In this case, only the troposphere is considered, because this is the water vapor that is slowing down the wave. The height of the troposphere (only several km) is very small compared to the altitude of the typical orbits (in LEO from 450 to 700-800 km). For small squint angles, the APS can be assumed being independent of ψ. This is because the difference induced by the APS between the two points will be negligible, since these two points are very close to each other. However, for big squint angles, that difference could become more important. In the case of this design, the squint angle will be small, and thus the APS is assumed dependent only on the distance with respect to the reference point.

3) *Determine the maximum tolerable phase noise to fulfill the accuracy requirement on both components of the deformation as a function of the squint angle. Tip: don't forget the APS.*

   The two images being completely uncorrelated, two phase noises must be computed, one related to the AT and one related to the LoS:

$$\sigma_{dr} = \sqrt{(\sigma^2_{pn,r} - \sigma^2_{\text{APS}})\left(\frac{4\pi}{\lambda}\right)^2} \tag{1}$$

$$\sigma_{dx} = \sqrt{\sigma^2_{pn,x}\left(\frac{8\pi\sin\psi}{\lambda}\right)^2} \tag{2}$$

   Since, in point 2), it was concluded that the APS does not depend on small squint angles, σ_{APS} is equal in $-\psi$ and ψ. Consequently, for σ_{dx}, when taking the difference between $\Delta\varphi$ for the two angles, and computing the accuracy, the contribution of σ_{APS} disappears. The measurement is intended to be valid w.r.t. a non-moving reference point at a maximum distance of 1 km from the investigated area, with an accuracy (1σ) required to be 1 cm about the LoS component and 2 cm about the AT component. Using this information, $\sigma_{\text{APS}} = 1$ mm is found, while σ_{dr} and σ_{dx} are simply 1 cm and 2 cm.

Manipulating the equations, $\sigma_{pn,r}$ and $\sigma_{pn,x}$ are found. Note that a squint angle of 2° is chosen.

$$\sigma_{pn,r} = 30.47° \tag{3}$$
$$\sigma_{pn,x} = 4.27° \tag{4}$$

These two values mean that, as long as the phase noise for the full resolution case is less than 30.47°, and the sub-aperture case, is less than 4.27°, the InSAR system is working well. It is important to note that these values are obtained without margins. They will be added in the next point.

4) *Assuming a Signal-to-Noise Ratio (SNR) equal to 10 dB, determine how many looks are required to contain phase noise within the limit derived at point 3. Tip: don't forget temporal decorrelation.*

The number of uncorrelated looks (or pixels) is obtained by using the Cramér-Rao Bound (CRB):

$$\sigma_{pn}^2 \geq \frac{1-|\gamma|^2}{2\,L_{eq}|\gamma|^2} \tag{5}$$

where $|\gamma = |\gamma_{\text{SNR}}||\gamma_{\text{temporal}}| = \frac{SNR}{SNR{=}1}0.7 = 0.6363$. Here, it is assumed that the exact same orbit is repeated, and thus γ_{terrain} is neglected.

This allows to find two different L_{eq} for the two components of the deformation.

$$L_{eq,r} \geq 3 \tag{6}$$
$$L_{eq,x} \geq 132 \tag{7}$$

By applying a 50% margin, $L_{eq,r} \geq 5$ and $L_{eq,x} \geq 198$. It is important to remember that the number of uncorrelated looks for the sub-aperture case depends on ψ. By reducing it even more, the number of uncorrelated looks required will increase drastically, as seen in Figure 1. This is due to the value of $\sin\psi$ being closer and closer to 0, decreasing $\sigma_{pn,x}$. The choice of this specific ψ will be addressed in the following point.

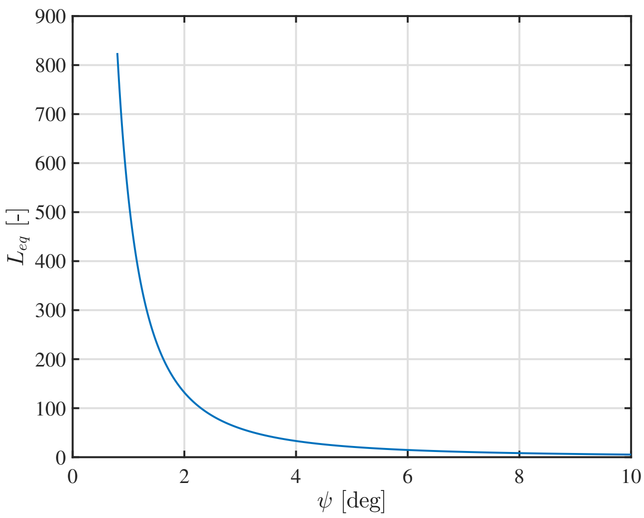


Figure 1: Evolution of the minimum number of uncorrelated looks required for different values of the squint angle ψ (no margins included).

5) & 6) *Identify possible values of along-track and ground range resolution that are compatible with your assessment at points 3 and 4. Tip: consider that images obtained by processing subapertures come at a lower resolution.*
Derive antenna length, transmitted bandwidth, and incidence angle.

The following results are obtained by assuming the different parameters in Table 1.

Table 1: Parameters assumed for the design.

Parameter	Value	Unit
L_{ant}	2.5	m
B	50	MHz
θ	20	deg
H_{orbit}	500	km

This allows to compute the AT resolution (full aperture case)

$$\Delta x_{\text{A}_\text{s},\text{tot}} = L_{\text{ant}}/2 = 1.25\,\text{m}. \tag{8}$$

Then, the total aperture is computed

$$A_s = \frac{\lambda R}{2\Delta x_{\text{A}_\text{s},\text{tot}}} = 50.04\,\text{km} \tag{9}$$

with $R = \frac{H}{\cos\theta}$.
Then, since the squint angle is known ($\psi = 2°$), the sub-aperture is calculated as follows (see Figure 2):

$$A_{sub} = 2(\frac{A_s}{2} - D) = 2(\frac{A_s}{2} - \tan\psi R) = 12.88\,\text{km}. \tag{10}$$

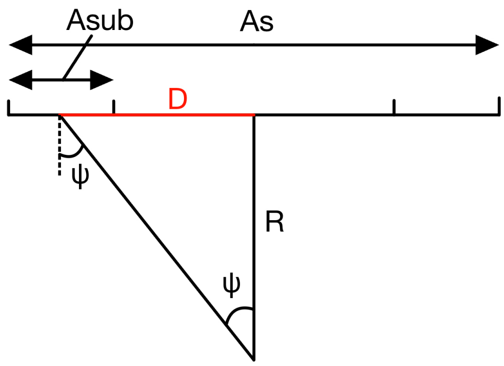


Figure 2: Scheme of the full and sub-apertures geometry.

This small squint angle, together with the antenna length and the chosen orbit, provide a feasible aperture. If L_{ant} increases, ψ has to be lower, to obtain a feasible length. Regarding the AT resolution of the sub-aperture case, the resolution is then:

$$\Delta x_{A_{\text{sub}}} = \frac{\lambda R}{2A_{\text{sub}}\cos\psi} = 4.86\,\text{m}. \tag{11}$$

Considering now the LoS and ground range resolutions, they are obviously equal for both cases

$$\Delta r = \frac{c}{2\,B} = 3\,\mathrm{m}, \tag{12}$$
$$\Delta y = \frac{\Delta r}{\sin\theta} = 8.77\,\mathrm{m}. \tag{13}$$

This allows to compute the number of uncorrelated looks for both cases (full and sub-apertures)

$$L_{eq} = \frac{100}{\Delta y}\frac{100}{\Delta x_{\mathrm{A_s,tot}}} = 913\,\mathrm{pixels} \quad \text{(Full aperture)} \tag{14}$$
$$L_{eq} = \frac{100}{\Delta y}\frac{100}{\Delta x_{\mathrm{A_{sub}}}} = 235\,\mathrm{pixels} \quad \text{(Sub aperture)} \tag{15}$$

Of course, the number of uncorrelated pixels in the case of the sub-aperture case is lower, since the resolution is lower. (Note that the maximum resolution for the sub-aperture is when $A_{sub} = A_s/2$, here $A_{sub} \approx A_s/4$). Of course the difference of L_{eq} r with respect to the CRB is bigger than for x. Nevertheless, these results are compatible with the results found in the previous points. The two L_{eq} being higher than the ones found in point 4, this leads to a lower amount of phase noise than in point 3, guaranteeing that the system is working. For the design, L_{eq} computed for the sub-aperture will be kept, as it is the most restrictive.

7) *Propose a basic design of the SAR system by deriving the following parameters: orbital height, Pulse Repetition Frequency (PRF), chirp duration, height of the physical antenna, extent of the ground swath. Note: you have some degrees of freedom. Yet, the values of the parameters have to be consistent with one another.*

The orbital height was already defined in Table 1 (H = 500 km). The height of the physical antenna is chosen according to typical values for SAR antennas between 0.6 m and 4 m. Also this value is the result of a tradeoff as it will be explained at the end of this section.

$$H_{\mathrm{ant}} = 2\,\mathrm{m} \tag{16}$$

Now, in order to derive the PRI, attention must be paid to the condition for unambiguous SAR imaging: the spatial sampling dx has to be lower than the maximum along-track resolution allowed by the antenna beamdwidth. Consequently, $dx = 0.95 * \Delta x_{\mathrm{A_s,tot}}$ is selected.

The Pulse Repetition Interval (PRI) and the Pulse Repetition Frequency (PRF) are thus

$$PRI = dx/v_{\mathrm{sat}} = 0.1560\,\mathrm{ms} \tag{17}$$
$$PRF = 1/PRI = 6410.7\,\mathrm{Hz} \tag{18}$$

with v_{sat} the velocity of a circular orbit at altitude H. The choice of $\Delta x_{\mathrm{A_s,tot}}$ would allow to find a PRI compatible with the full and the sub-aperture case. Indeed, since the resolution is coarser with the sub-aperture, $PRI_{\mathrm{max,sub}}$ will be higher than $PRI_{\mathrm{max,full}}$. Any PRI matching the condition on the full aperture case will also match the condition on $PRI_{\mathrm{max,sub}}$, and it is the case here.

The transmitted chirp duration (corresponding to the pulse length/duration) is then found, taking into account a duty cycle of 15% (found from the SAOCOM mission):

$$T_g = dc\,PRI = 0.0234\,\text{ms} \tag{19}$$

Finally, the ground swath extent can be computed, using the elevation beamwidth

$$\Delta\theta = \frac{\lambda}{H_{\text{ant}}} = 6.74° \tag{20}$$

Knowing $\Delta\theta$, the ground swath extent is computed

$$Swath = H\left(\tan\theta_{\text{max}} - \tan\theta_{\text{min}}\right) = 66.68\,\text{km} \tag{21}$$

with $\theta_{\text{max}} = \theta + \frac{\Delta\theta}{2}$ and $\theta_{\text{min}} = \theta - \frac{\Delta\theta}{2}$, as represented in Figure 3.

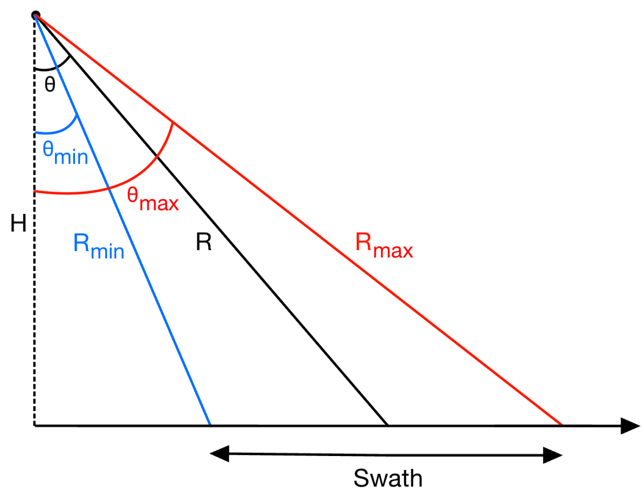


Figure 3: Scheme of the swath extent.

It is important to check the consistency of the results. In that regard, the maximum and minimum delays are defined:

$$d_{\text{max}} = \frac{2\,R_{\text{max}}}{c} \tag{22}$$

$$d_{\text{min}} = \frac{2\,R_{\text{min}}}{c} \tag{23}$$

with $R_{\text{max}} = H/\cos\theta_{\text{max}}$ and $R_{\text{min}} = H/\cos\theta_{\text{min}}$. And indeed, the following inequality is verified:

$$PRI > d_{\text{max}} - d_{\text{min}} = 0.1524\,\text{ms}. \tag{24}$$

This ensures to reject range ambiguities. The tradeoff between the PRI being valid for both sub and full aperture cases, while satisfying the condition on the delays is typical for SAR systems. This last condition is linked to $\Delta\theta$ and thus the swath, while the condition on PRI_{max} is linked to the AT resolution. This means that it is not possible to achieve at the same time an arbitrary large swath, and an arbitrary fine along track resolution.

In summary, Table 2 shows the parameters of the SAR system.

Table 2: Summary of the SAR design parameters.

Parameter	Value	Unit
L_{ant}	2.5	m
H_{ant}	2	m
B	50	MHz
θ	20	deg
H_{orbit}	500	km
$\Delta x_{\text{A}_\text{s},\text{tot}}$	1.25	m
A_s	50.04	km
A_{sub}	12.88	km
$\Delta x_{A_{\text{sub}}}$	4.86	m
Δr	3	m
L_{eq}	235	[-]
PRF	6140.8	Hz
T_g	0.0234	ms
Swath	66.68	km

8) *With reference to the parameters derived at the previous points, evaluate the required power in transmission and draw the graph of the resulting NESZ along the ground swath. For this point, assume a Noise Figure $F = 6$ dB, an antenna efficiency $\xi = 0.7$.*

In this section, the transmitted power and the NESZ will be computed for the full resolution case. First, assuming a planar antenna, the gain is computed:

$$G = \frac{4\pi}{\lambda^2} A_{\text{ant}}\, \xi = 29\,\text{dB} \tag{25}$$

with $A_{\text{ant}} = L_{\text{ant}} * H_{\text{ant}}$. Also, $A_{\text{eff}} = G\frac{\lambda^2}{4\pi}$. Note that the formula $G = \frac{4\pi}{\Delta\theta\,\Delta\psi}\,\xi$ leads to the same result in the case of the full resolution since $\Delta\psi = \frac{\lambda}{L_{\text{ant}}}$. For the sub-aperture case $\Delta\psi = \frac{\lambda}{2\,A_{\text{sub}}\,\cos\psi}$.

Then, the effective number of pusles is found as:

$$N_\tau = \frac{A_\text{s}}{v_{\text{sat}}\,PRI}. \tag{26}$$

Regarding N_0, it is assumed that all temperatures are equal to $T_{\text{ref}} = 290$ K, leading to

$$N_0 = K_b\,F\,T_{\text{ref}} \tag{27}$$

Atmospheric losses are also considered (therefore contributing to a higher power at it will be seen), assuming a loss of 0.02 dB/km: $L_{\text{atm}} = 0.02\,R$. Regarding the backscattering coefficient, it is taken at $\theta = 20°$, for Medium Rough Soil at L-Band, HH polarization ($\sigma^0 = -8$ dB). The Radar Cross Section is thus

$$\text{RCS} = \Delta_r\,\Delta x_{\text{A}_\text{s},\text{tot}}\,\frac{\sigma^0}{\sin\theta}. \tag{28}$$

These quantities allow to compute the transmitted power:

$$P_{tx} = \frac{(4\pi R^2)^2\,L_{\text{atm}}\,N_0\,\text{SNR}}{G\,A_{\text{eff}}\,RCS\,N_\tau\,T_g\,f_{\text{ant}}^2}, \tag{29}$$

with f_{ant} taken as $f_{\text{ant}}(\psi = 0, \theta) = \left[\text{sinc}(\frac{20° - \theta}{\Delta\theta})\right]^2$. Since here the case when $\theta = 20°$ is considered, $f_{\text{ant}} = 1$.

This leads to a transmitted power

$$P_{\text{Tx}} = 4.91\,\text{kW}. \tag{30}$$

Regarding the NESZ, it is calculated as follows:

$$\text{NESZ} = \frac{\sin\theta}{\Delta x \Delta r} \frac{N_0(4\pi R^2)^2}{N_\tau T_g P_{\text{Tx}}} \frac{1}{G_{ant} A_{ant} f_{ant}^2(\theta)} \tag{31}$$

It depends on θ and thus on R, as it is depicted in Figure 4. Assuming the transmitted power found here above, the NESZ can be plotted along the swath (from θ_{min}, R_{min} to θ_{max}, R_{max}. The NESZ corresponding to $\theta = 20°$ is -30.19 dB. Comparing these values to the SAOCOM mission with a peak power of 6.7 kW and a requirement on the NESZ $<$ -28 dB for some modes, it seems acceptable.

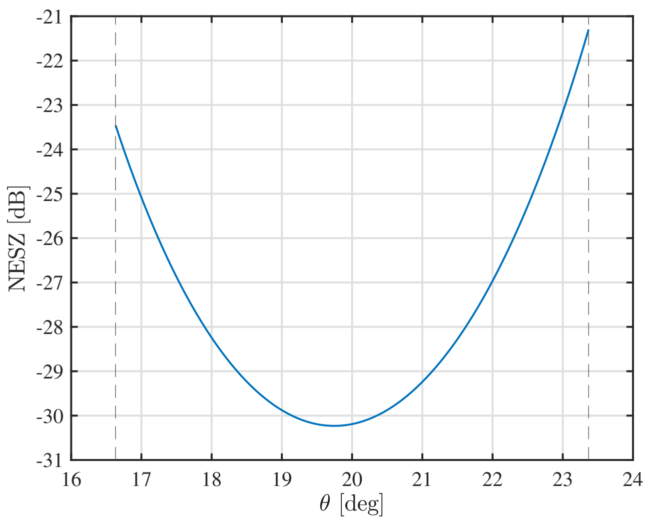


Figure 4: Evolution of the NESZ along the swath for the 0-Doppler case.

Note that to calculate the transmitted power for the sub-aperture case, N_τ, RCS, T_g and the gain would change (due to the change in AT resolution and, of course, the reduced synthetic aperture). Moreover, regarding the antenna pattern, another constant factor $\text{sinc}\left(\frac{\psi}{\Delta\psi}\right)$ should be added.

9) *Consider now a further coherence loss induced by terrain decorrelation, and discuss the requirement about the orbital tube to ensure that your design is valid. Note: the orbital tube is the allowed displacement of the orbit from one pass to another.*

Up to point 8), coherence loss induced by terrain decorrelation was neglected. By considering it now, the terrain decorrelation is expressed as:

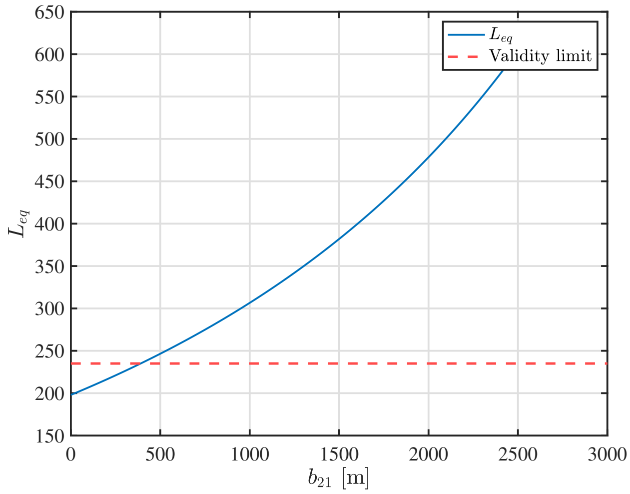
$$|\gamma_{\text{terrain}}| = 1 - \frac{|b_{21}|}{\frac{B}{f_0} R_1 \tan\theta_1} \tag{32}$$

with the index 1 refers to the case when $\theta = 20°$ and the index 2 refers to the position after one orbit.

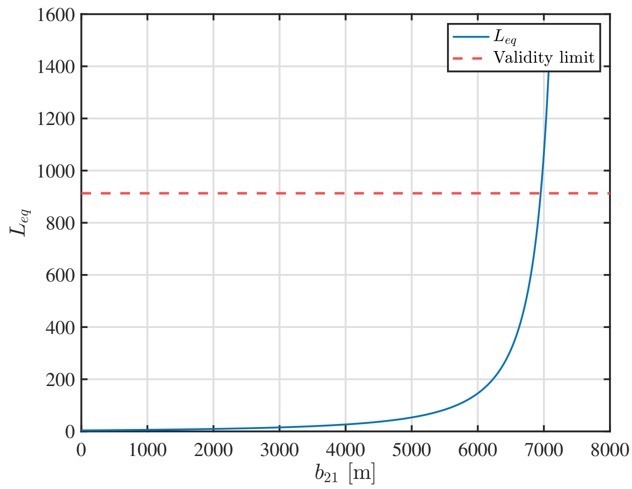
In order to establish the requirement about the orbital tube, this parameter must be known. The methodology to determine the maximum allowed displacement of the orbit from one pass to another is the following: the value of the normal baseline is gradually increased. For each value of b_{21} the InSAR coherence is calculated:

$$|\gamma| = |\gamma_{\text{SNR}}|\,|\gamma_{\text{temporal}}|\,|\gamma_{\text{terrain}}|. \tag{33}$$

This value is used in the Cramér-Rao bound to determine the new value of L_{eq} (number of uncorrelated looks) required for a valid design. The result is shown in Figure 5a, and it is seen that for the sub-aperture case, the limit on the normal baseline is $b_{21} = 392.19$ m. For the full aperture case, of course, the maximum value of b_{21} increases quite significantly, due to the higher difference between the designed number of uncorrelated looks and the required ones. It is seen in Figure 5b that $b_{21,\text{max}} = 6952.36$ m. Therefore, the limit of b_{21} for the sub-aperture case is kept to determine the requirements on the orbital tube.



(a) Sub-aperture case.



(b) Full aperture case.

Figure 5: Evolution of the theoretical L_{eq} (50% margin included) as a function of the normal baseline.

As seen in Figure 6, the displacement Δ_y of the orbit is found knowing the normal baseline, the orbit height and both angles of incidence. The orbit height is assumed to remain constant from one pass to another. θ_1 is known, as previously said, and θ_2 is found easily using the following equation:

$$\theta_2 - \theta_1 = \tan^{-1}\frac{b_{21}}{R_1} \tag{34}$$

Then, geometrically, y_1 and y_2 are found, leading to Δ_y:

$$\Delta_y = y_2 - y_1 = H\left(\tan\theta_2 - \tan\theta_1\right) = 417.47\,\text{m}. \tag{35}$$

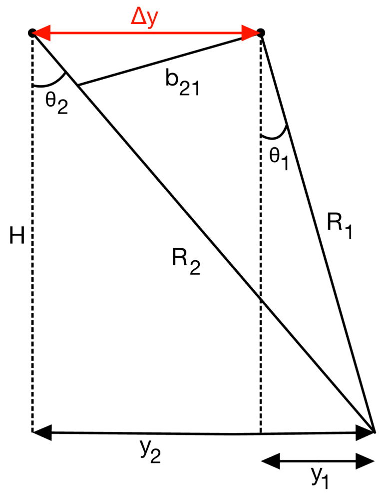


Figure 6: Scheme of the displacement of the orbit from one pass to another (orbital tube).

This value is obviously close to the normal baseline, since b_{21} is very small compared to the altitude of the orbit. Considering some margins the requirement on the orbital tube is then: Δ_y should remain below 400 meter to ensure that the design is still valid.

10) *Assume now to use ascending and descending orbits to measure all three components of the deformation vector, and calculate the accuracy of each component in the North, East, Up reference system.*

Firstly, it is assumed that the orbit is North-South such that the along-track component is the North component, and the two Line of Sight components are perfectly in the plane East-Up, as seen in Figure 7.

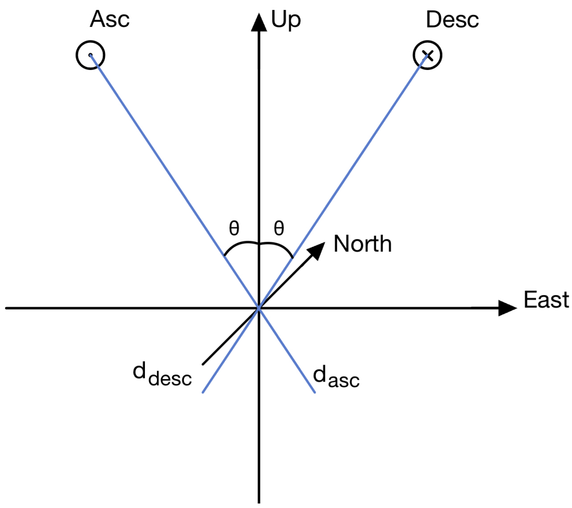


Figure 7: Scheme of the deformation vector in the North, East, UP reference system.

Following a similar procedure seen to retrieve the along-track component of the deformation in the statement of the assignment, the two displacement vectors d_{asc} and d_{desc} can be written as

$$\begin{cases} d_{\text{asc}} & = -d_{\text{U}} \cos\theta + d_{\text{E}} \sin\theta \\ d_{\text{desc}} & = -d_{\text{U}} \cos\theta - d_{\text{E}} \sin\theta \end{cases}, \tag{36}$$

while the along-track component is simply:

$$d_x = d_{\text{N}}. \tag{37}$$

Focusing on Equation 36, it can be written in matrix form:

$$\begin{bmatrix} d_{\text{asc}} \\ d_{\text{desc}} \end{bmatrix} = \begin{bmatrix} -\cos(\theta) & \sin(\theta) \\ -\cos(\theta) & -\sin(\theta) \end{bmatrix} \cdot \begin{bmatrix} d_{\text{U}} \\ d_{\text{E}} \end{bmatrix} \tag{38}$$

Inverting the equation, the following rotation matrix between d_{asc}, d_{desc} and d_{U}, d_{E} is found and thus the two displacements*:

$$\begin{bmatrix} d_{\text{U}} \\ d_{\text{E}} \end{bmatrix} = \begin{bmatrix} \frac{-1}{2\cos(\theta)} & \frac{-1}{2\cos(\theta)} \\ \frac{1}{2\sin(\theta)} & \frac{-1}{2\sin(\theta)} \end{bmatrix} \cdot \begin{bmatrix} d_{\text{asc}} \\ d_{\text{desc}} \end{bmatrix} = \begin{bmatrix} \frac{-d_{\text{asc}} - d_{\text{desc}}}{2\cos(\theta)} \\ \frac{d_{\text{asc}} - d_{\text{desc}}}{2\sin(\theta)} \end{bmatrix} \tag{39}$$

The accuracy in each direction can then be determined by the use of this rotation matrix, and knowing that the along-track component is along the North direction. As a reminder from the requirements, the accuracy on the LoS component of the deformation vector is $\sigma_{dr} = 1$ cm, and the AT component is $\sigma_{dx} = 2$ cm.

$$\begin{cases} \sigma_{\text{N}} & = \sigma_{dx} = 2\,\text{cm} \\ \sigma_{\text{U}} & = \sqrt{\left(\frac{-1}{2\cos(\theta)}\right)^2 + \left(\frac{-1}{2\cos(\theta)}\right)^2}\,\sigma_{dr} = \frac{\sqrt{2}}{2\,|\cos\theta|}\,\sigma_{dr} = 0.75\,\text{cm} \\ \sigma_{\text{E}} & = \sqrt{\left(\frac{1}{2\sin(\theta)}\right)^2 + \left(\frac{-1}{2\sin(\theta)}\right)^2}\,\sigma_{dr} = \frac{\sqrt{2}}{2\,|\sin\theta|}\,\sigma_{dr} = 2.07\,\text{cm} \end{cases} \tag{40}$$

*This relation is validated by A. Pepe and F. Calò (2017) in their paper "A Review of Interferometric Synthetic Aperture RADAR (InSAR) Multi-Track Approaches for the Retrieval of Earth's Surface Displacements", and also by another paper of A. Pepe (2017) named Generation of Earth's Surface Three-Dimensional (3-D) Displacement Time-Series by Multiple-Platform SAR Data.

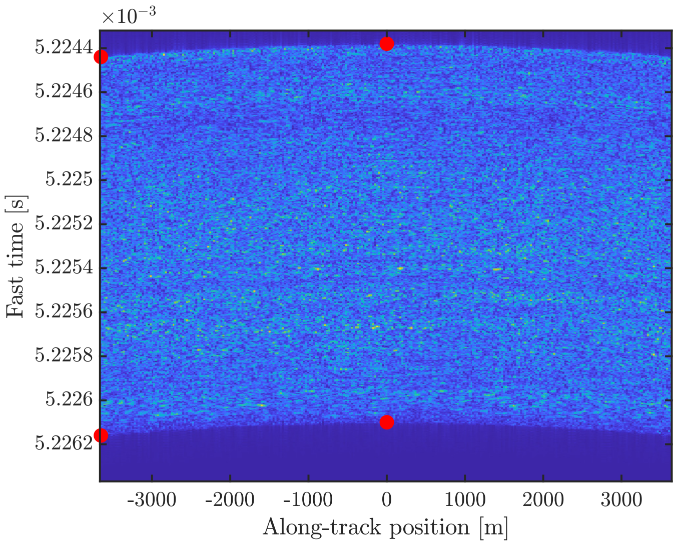
2 SAR signal processing

1) *Assess the maximum range migration in meters by taking a look at the magnitude of range compressed data.*

In Figure 8b is represented the range compressed data. To assess the maximum range migration, the difference in the fast time between the two red points has to be evaluated. It can be measured taking the upper points, or the lower points. In any cases, the results will be identical. The difference in fast time between the two points at the bottom gives a $\Delta_{t,\text{fast}} = 6e{-}8$ s. In order to obtain the range migration, the speed of light (c = 299792458 m/s) is taken into account. Note that here half of the speed of light is taken into account, ensuring a one-way distance from the SAR to the target. When considering the two other points, an identical value of found, as previously anticipated.

$$\Delta_{r,\max} = \Delta_{t,\text{fast}}\, c/2 = 9\,\text{m}. \tag{41}$$

(a) Image chosen.



(b) Magnitude of the range-compressed data, represented with respect to the AT position and the fast time.

Figure 8: Chosen filter and signal filtered for the ocean in the frequency domain.

2) *Focus the data onto the coordinate grid defined by the vectors x_{ax} and y_{ax} using Time Domain Back Projection. The image plane can be assumed to be at a height z = 0. Tip: there's no universal convention about the sign of the phase of the data. If you have problems, try taking the complex conjugate of the data. Important: use the value of speed of light found in D.c.*

After defining the grid based on x_{ax} and y_{ax}, a loop over the slow time (N_τ). For each position (for each time τ), the distance from the antenna to any point of the grid ($R_{n,xy}$) is computed. Then, an interpolation is executed (using *interp1*), taking the RC signal acquired by antenna. After that, a phase rotation is carried out. Finally, a sum over τ is done. Note that the focus is applied at a constant height $z = 0$ m and the complex conjugate of the RC data is used for the computations. Mathematically, with S_{RC} being the results of the interpolation, one has

$$I(x,y) = \int S_{RC}(t = \frac{2R_n(\tau;x,y)}{c};\tau)\ \exp{(j\frac{4\pi}{\lambda}R_n(\tau;x,y))}d\tau \tag{42}$$

The result is shown in Figure 9. Note that here, no weights are applied since the resolution is unknown yet.

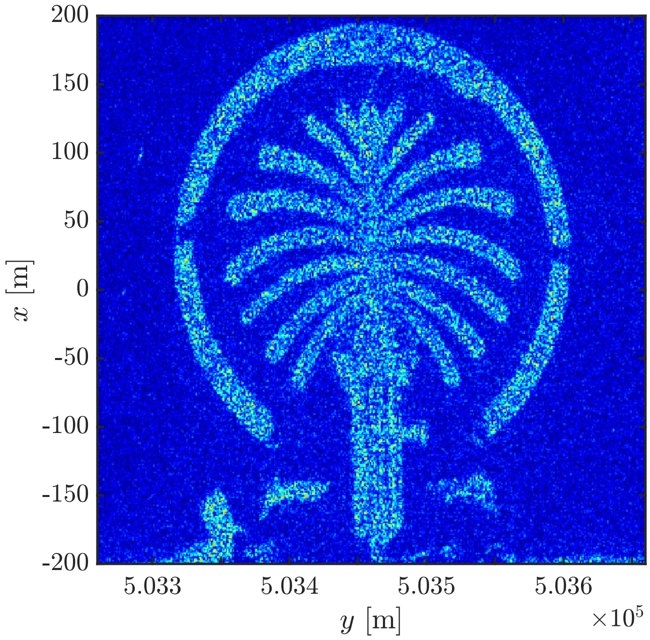


Figure 9: Focusing of the image by Time Domain Back Projection.

3) *Evaluate the 2D Fourier Transform of the focused SAR image and represent it in dB. Can you tell the resolution of the SAR image based on a (naked-eye) evaluation of the 2D spatial bandwidth?*

Figure 10 shows the 2D Fourier Transform of the focused SAR image by TDBP, in dB. It has been computed using a *Fast Fourier Transform* along the columns and the rows. Based on a naked-eye evaluation, the dimensions of the red square are $0.6\,\text{m} \times 0.6\,\text{m}$. Then, by taking the spatial bandwidth $B = 0.3\ \text{m}^{-1}$, the resolution for both the x and y-directions is

$$\Delta x = \Delta y = 1/B = 3.3334\,\text{m}. \tag{43}$$

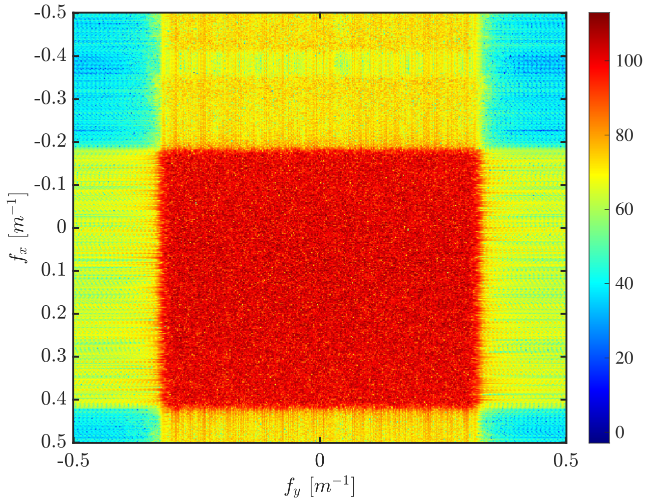
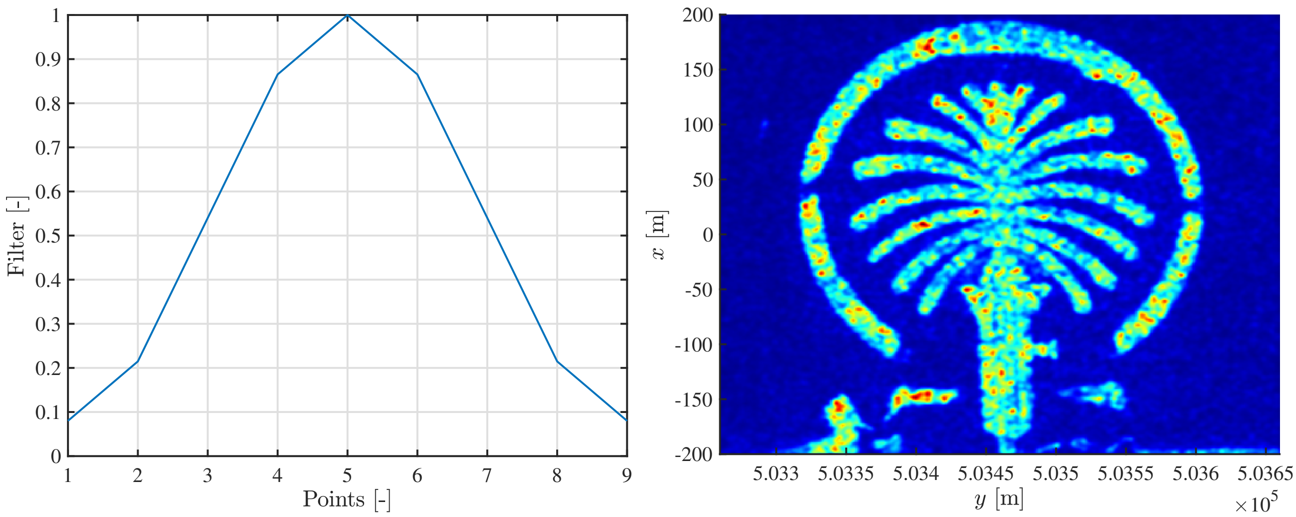


Figure 10: 2D Fourier Transform of the focused image by Time Domain Back Projection, in dB.

4) *Implement a despeckling procedure and comment on the result*

   Based on Figure 9, speckles are observed. In that regard, a despeckling procedure can be implemented. For example, a Hamming filter can be used as seen in Figure 11a. The procedure is simply convoluting twice the image with the same filter in the two dimensions. The result is shown in Figure 11b. The objective of this filter is to reduce the "granularity" effect seen in Figure 9. Of course, the image is now more blurred, due to the low-pass filter. This is also why the background is darker in Figure 11b than in Figure 9.



(a) Hamming filter (9). **(b)** Hamming Filter applied on the image.

Figure 11: Focused range compressed data after elimination of the leakage, with two methods.

5) *Now type D = Generate rc data(MyPicture,1) to generate a new data affected by a direct return from the transmitter, referred to as leakage. The data is to be modelled as: $d_{leak}(t,\tau) = d_{clean}(t,\tau) + A \cdot g(t)$, with A a (very high) constant and $g(t)$ a waveform that describes the impact of the direct signal from the transmitter. The waveform $g(t)$ is unknown, but it is assumed to be the same for all values of slow time τ. Devise and*

implement a procedure to restore the clean range compressed data by eliminating $A \cdot g(t)$, refocus the data and comment on the results.

Since the waveform is assumed to be the same for all values of slow time τ and A is a very high constant, one way of restoring the clean range compressed data is to average the data affected by leakage. The average is carried out using the *mean* over the dimension 2 of the matrix (over the slow time). This gives as a result one vector of dimension $N_t \times 1$. Then, what is left is simply subtracting to the RC data with leakage this vector. Finally, the resulting data is focused, and the result is shown in Figure 13a. Note that one could have thought of simply subtracting one column randomly picked. The random choice is justified by the assumption of $A \cdot g\ (t) = \text{cst}\ \forall\, \tau$. However, as it can be seen in Figure 13b, it does not lead to a result as satisfying as in Figure 13a.

Note also that here, since the resolution is known, weights can be intergated to the TDBP algorithm. They are in the form:

$$W_n(\psi(\tau; x, y)) = rectpuls\left(\frac{\psi(\tau; x, y)}{\Delta\psi}\right) \tag{44}$$

with $\Delta\psi = \frac{\lambda}{L_x} = \frac{\lambda}{2\,\Delta x}$, and Δx found in point 3. The images in Figure 13 are focused with these weights, i.e.

$$I(x, y) = \int W_n(\psi(\tau; x, y))\, S_{RC}(t = \frac{2R_n(\tau; x, y)}{c}; \tau)\ \exp{(j\frac{4\pi}{\lambda}R_n(\tau; x, y))} d\tau \tag{45}$$

As it can be seen, the result is not significantly different compared to Figure 13a, only brighter spots appear in some portions of the image.

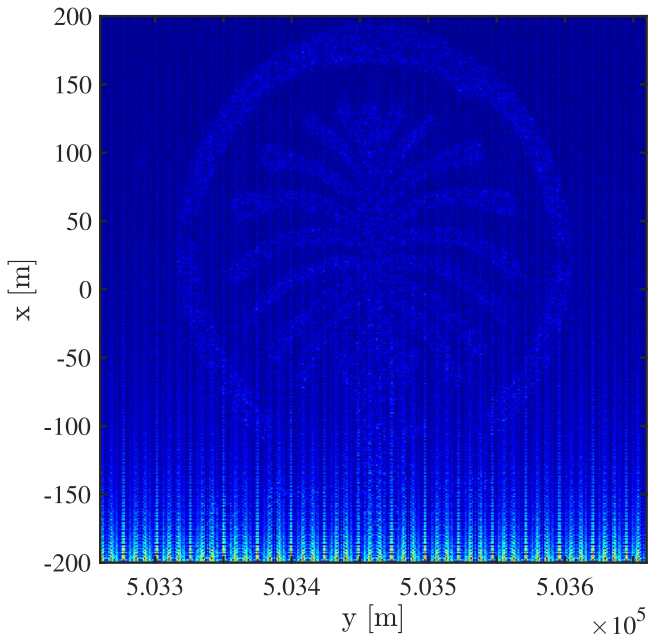
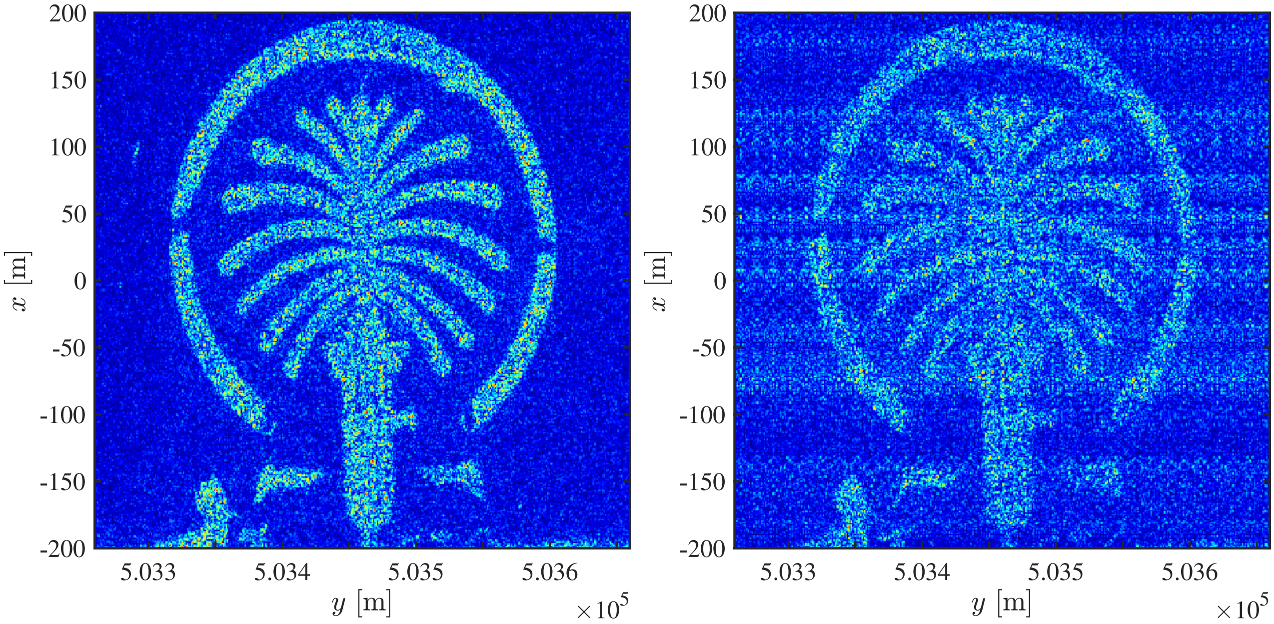


Figure 12: Focused range compressed data before elimination of the leakage.



(a) Estimate of the leakage by average.

(b) Estimate of the leakage with one column only.

Figure 13: Focused range compressed data after elimination of the leakage, with two methods, with the TDBP algorithm with weights.