Remote Sensing for Earth Observation and Surveillance – Homework #1

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In this report are discussed the proposed solutions to the problems related to advanced spaceborne radiometry, SAR interferometry basics and noise in images.

1 Advanced spaceborne radiometry

1) *Process the signal received by the first satellite to estimate its brightness temperature under the assumption of no RFI. Assess the accuracy of this measurement.*

The brightness temperature of the signal received by the first satellite can be obtained using

$$T_B = \frac{P}{KB}, \tag{1}$$

where P is the power of the received signal, K is the Boltzmann's constant, B is the bandwidth of the received signal. The latter is known since the sampling frequency is given ($B = f_s$).

To estimate T_B, the power of the signal must be estimated. Here the RFI radiation is neglected, and the signals from the ocean are assumed to be white in the spectral domain and as being circular normal. Thus, it is characterised by a 0-mean value ($\mathrm{E}[d_1(t)] = 0$). This means its power can be calculated easily

$$P_{d_1} = \mathrm{E}[|\mathrm{d_{1,i}}|^2]. \tag{2}$$

$$\hat{P}_{d_1} = \frac{1}{T_{\text{obs}}} \sum_i |d_{1,i}|^2 dt \tag{3}$$

With $dt = 1/f_s$. This gives an estimate of the brightness temperature. Regarding the accuracy of the measurement, in the case of a circular normal process x, $\sigma_x^2 = P_x$. In the case of this problem, the accuracy of the estimated power is then

$$\mathrm{var}(\hat{\mathrm{P}}_{\mathrm{d_1}}) = \frac{\mathrm{P}_{\mathrm{d_1}}^2}{\mathrm{B\,T_{obs}}} \longleftrightarrow \sigma_{\hat{\mathrm{P}}_{\mathrm{d_1}}} = \frac{\mathrm{P}_{\mathrm{d_1}}}{\sqrt{\mathrm{B\,T_{obs}}}} \tag{4}$$

From this, the estimation accuracy of the brightness temperature is derived. The results are found in Table 1.

2) *Compute the power spectrum of the signal processed at point 1, and check that the signal brightness temperature can be obtained by integrating the power spectrum.*

The power spectrum of the signal processed at point 1 can be obtained using the *dft* function, with a number of samples in frequency $N_f = 2^{\mathrm{ceil}(\log_2(\mathrm{N_t}))}$ where N_t is the number of samples in the time domain of the signal. The obtained Fourier transform D_1 leads to the calculation of the Power Spectral Density of the signal

$$S_{d_1} = dt\frac{|D_1|^2}{T_{obs}} \tag{5}$$

Table 1: Estimated quantities and their values.

Estimated Quantity	Estimated Value	Unit
$\hat{P}_{d_1}$	2.0673×10^{-14}	W
$\mathrm{var}(\hat{\mathrm{P}}_{\mathrm{d}_1})$	4.2738×10^{-33}	W^2
$\sigma_{\hat{P}_{d_1}}$	6.5375×10^{-17}	W
$\hat{T}_B$	149.7361	K
$\mathrm{var}(\hat{\mathrm{T}}_{\mathrm{B}})$	0.2242	K^2
$\sigma_{\hat{T}_B}$	0.4735	K

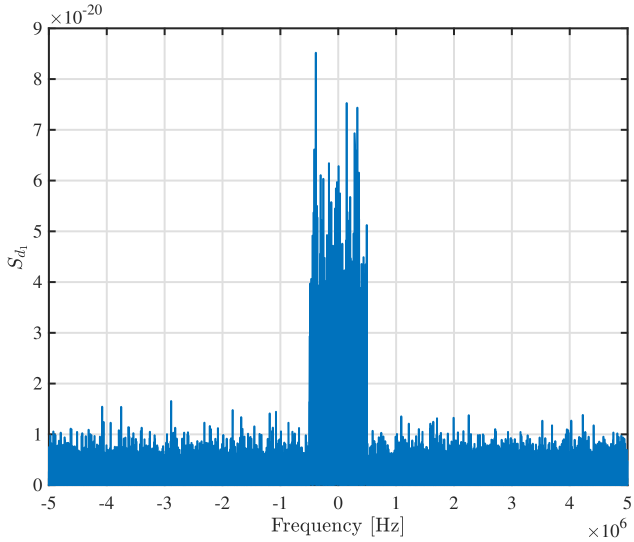


Figure 1: Power Spectrum Density of the signal received by the first satellite.

where the factor dt is a scale factor for dimensional consistency, and $T_{obs} = N_t\, dt$. The PSD is represented in Figure 1.

The power spectrum found above can be integrated to find T_B. This integration replaced by a sum over the samples and is exact as long as the sampling theorem is valid. The total power is thus

$$P_{d_1} = \sum_i S_{d_{1,i}}\, df = \sum_i S_{d_{1,i}} \frac{f_s}{N_f}, \tag{6}$$

and the brightness temperature is found: $T_B = 149.7361$ K. This is exactly the value found in the previous point.

3) *Compute the autocorrelation function of the signal processed at point 1, and check that the signal brigthness temperature can be obtained by taking the peak of the autocorrelation function.*

The autocorrelation can be obtained either using the Matlab functions or by simply using the *idft* function, since R_{d_1} is simply the inverse Fourier transform of the PSD of d_1. It is important to note that after using *idft*, a correction by a factor $N_f\, df$ has to be made. Indeed, this function divides the results by the number of frequency points and a scale factor for dimensional consistency is needed. R_{d_1} is plotted in Figure 2. As highlighted on the plot, the peak of the autocorrelation corresponds to the power of the signal. This is the exact same power as in the last two points, and thus it obviously leads to the same brightness temperature $T_B = 149.7361$ K.

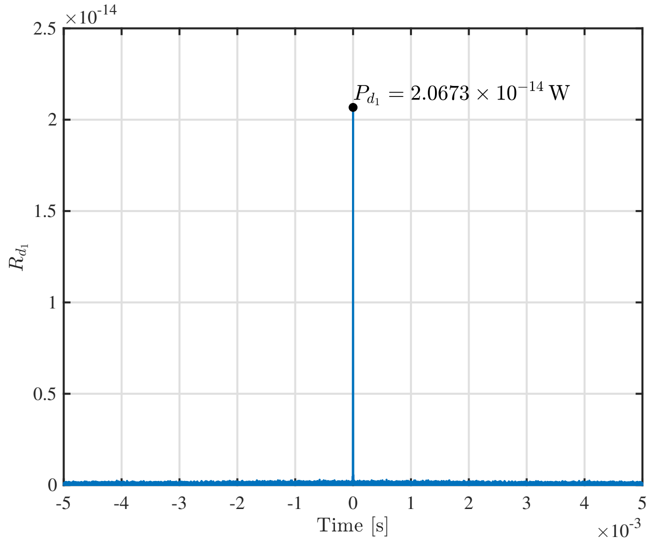


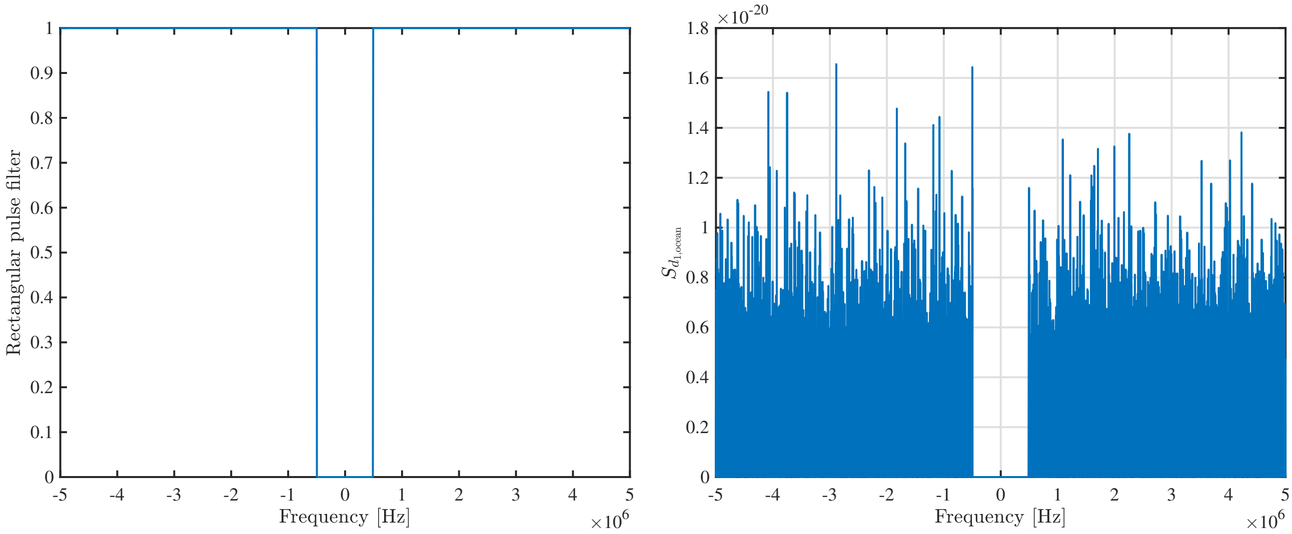
Figure 2: Autocorrelation function of the signal received by the first satellite.

4) *Observe the power spectrum computed at point 2. Can you spot an RFI? If so, propose and implement a procedure to measure the brightness temperature of the ocean while rejecting the RFI, and assess the accuracy of this measurement.*

By observing the power spectrum in Figure 1, the RFI is easily spotted. This peak in the middle of the frequency band is as a first approximation a tenth of the bandwidth. To assess more precisely the lower and upper bounds of the RFI bandwidth, the maximum of the spectrum outside a frequency range $[-f_s/20,\ f_s/20]$ is located. Then, the indexes of the vector of frequency for which the value of S_{d_1} is higher than this maximum are found. This gives the lower and upper bounds: $f_{\mathrm{RFI}} \in [-495071,\ 493622]$ Hz, leading to a bandwidth B_{RFI} of 988693 Hz.

The power of the signal emitted ocean can be retrieved by applying a filter in the bandwidth of RFI. As shown in Figure 11a, a rectangular pulse is chosen. It is important to note that some information will be lost by filtering the whole signal in that bandwidth. The filtered signal is shown in Figure 11b.

Then, by applying the same methodology as in point 2), the power and the brightness temperature are estimated. The difference here is the bandwidth. Indeed now $B_{\mathrm{o}} = B - B_{\mathrm{RFI}}$. The results are shown in Table 2. Of course some accuracy is lost by rejecting the signal of the ocean in the RFI bandwidth, because less bandwidth is used. However, the change in estimated temperature would not be significant and moreover, the variance and standard deviation would decrease since the whole bandwidth would be taking into account. A possibility to retrieve the signal of the ocean is to apply a threshold on the amplitude of the signal in B_{RFI}. This threshold is calculated as the ratio between the maximum of the whole signal and the maximum of the signal outside B_{RFI}. The figures are provided in the Annexes (section 4) for the sake of completeness. In the end, the estimated brightness temperature with this alternative approach is 101.7456 K, with an estimation accuracy $\sigma_{T_{B,\mathrm{o}}} = 0.3217$ K. As anticipated, the difference in the accuracy is very little (around 3.5% of relative error). However, attention must be paid in the signal



(a) Filter.

(b) Filtered signal.

Figure 3: Chosen filter and signal filtered for the ocean in the frequency domain.

in that bandwidth. Since it is a combination of the two signals, the first approach (using a notch filter) was preferred, even if some accuracy is lost.

Table 2: Estimated quantities and their values for the signal coming from the ocean.

Estimated Quantity	Estimated Value	Unit
$\hat{P}_\text{o}$	1.2450×10^{-14}	W
$\text{var}(\hat{\text{P}}_\text{o})$	1.7200×10^{-33}	W^2
$\sigma_{\hat{P}_\text{o}}$	4.1473×10^{-17}	W
$\hat{T}_{B,\text{o}}$	100.0658	K
$\text{var}(\hat{\text{T}}_{\text{B,o}})$	0.1111	K^2
$\sigma_{\hat{T}_{B,\text{o}}}$	0.3333	K

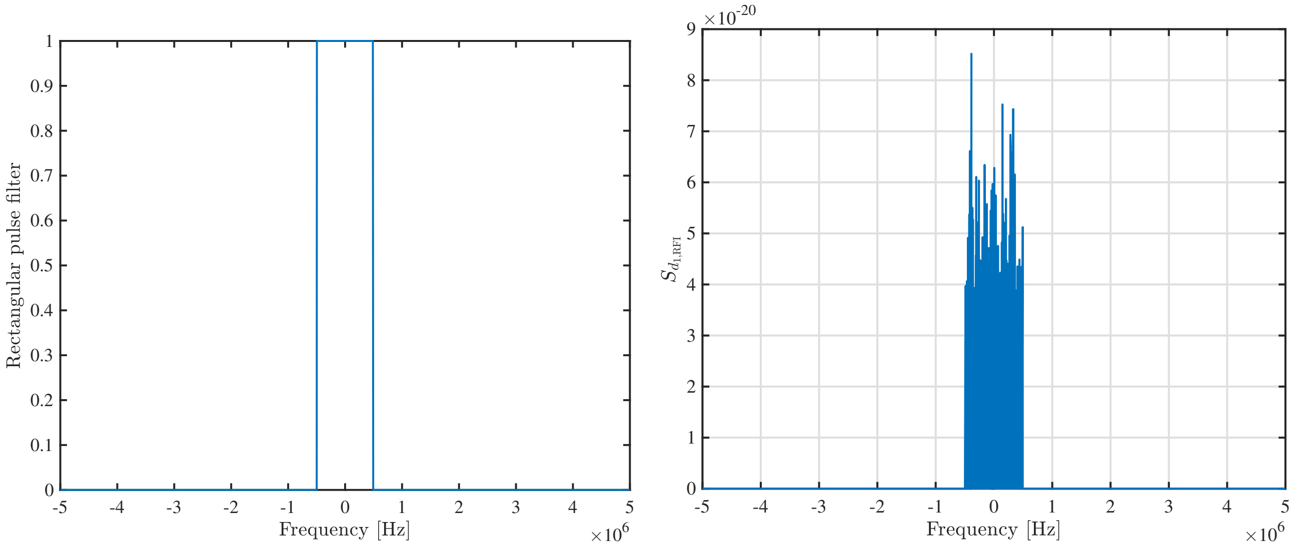
5) *Propose a procedure to estimate the brightness temperature of the RFI as well, and assess the accuracy of this measurement.*

In this case, another filter can be used to isolate the FRI (and as seen earlier some signal from the ocean in that bandwidth), as shown in Figure 4a. The filtered signal is also shown in Figure 4b. These figures are reported for the sake of clarity.

To estimate the power and the brightness temperature, the same methodology as seen in the previous point can be used. However, these two quantities can easily be obtained from the total power definition.

$$P_{d_1} = P_\text{o} + P_\text{RFI}, \quad \longleftrightarrow \quad K\,B\,T_B = K\,B_\text{o}\,T_{B,\text{o}} + K\,B_\text{RFI}\,T_{B,\text{RFI}}, \tag{7}$$

since P_o and P_RFI are uncorrelated. From this equation, $T_{B,\text{RFI}}$ is calculated. The results are shown in Table 3. In this case, the accuracy related to the brightness temperature is higher than in the previous cases. The estimated value of the temperature is also a lot higher.



(a) Filter. **(b)** Filtered signal.

Figure 4: Chosen filter and signal filtered for the RFI in the frequency domain.

Table 3: Estimated quantities and their values for the signal coming from the RFI.

Estimated Quantity	Estimated Value	Unit
$\hat{P}_{\mathrm{RFI}}$	8.2237×10^{-15}	W
$\mathrm{var}(\hat{\mathrm{P}}_{\mathrm{RFI}})$	6.8402×10^{-33}	W^2
$\sigma_{\hat{P}_{\mathrm{RFI}}}$	8.2705×10^{-17}	W
$\hat{T}_{B,\mathrm{RFI}}$	602.4492	K
$\mathrm{var}(\hat{\mathrm{T}}_{\mathrm{B,RFI}})$	36.7096	K^2
$\sigma_{\hat{T}_{B,\mathrm{RFI}}}$	6.0588	K

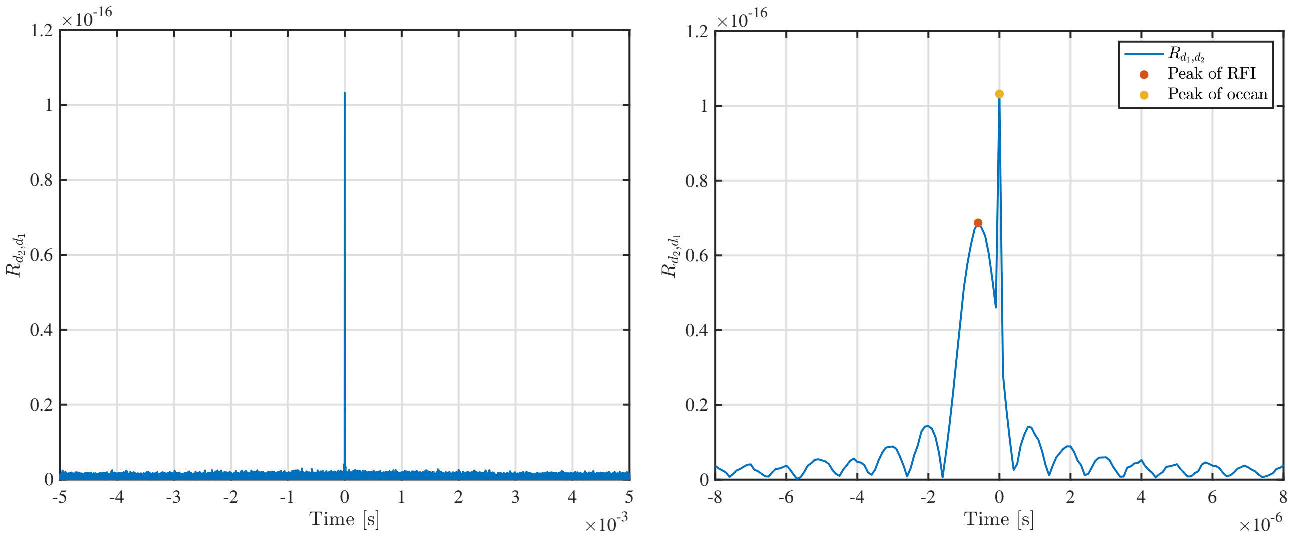
Note that, this time, using the alternative approach seen in the previous point, the obtained brightness temperature of the RFI differs by a large amount with respect to the one reported in Table 3. In this case, $\hat{T}_{B,\mathrm{RFI}} = 485.3934$ K and $\sigma_{\hat{T}_{B,\mathrm{RFI}}} = 4.8816$ K. The higher difference in the results related to RFI compared to the ocean is probably due to a higher variation of the amplitude of the spectrum in a much lower bandwidth. The introduction of the threshold (0.1943) lowers the signal in the bandwidth of the RFI by 0.1943 ($1 - 0.1943$ is exactly the ratio between the two calculated temperatures). A final note regarding the accuracy, also here it is improved.

6) *Let's now consider the signal received by the second satellite as well. Propose and implement a procedure to estimate the delays with which the radiation from the ocean and the RFI arrive at the second satellite w.r.t. the first one.*

   Estimating the delays with which the radiation from the ocean and the RFI arrive at the second satellite w.r.t. the first one is carried out by the use of the cross-correlation. Indeed, the signals received by the two satellites, in the time domain, are taken and the cross-correlation is shown in Figure 5. This can either be done with the function *conv2* of Matlab or by using the Fourier domain and going back to the time domain, taking care of the dimensional consistency.

$$S_{d_2,d_1} = S_{d_1} \cdot S_{d_2}^* \quad \longleftrightarrow \quad R_{d_1,d_2} = \mathcal{F}^{-1}\{S_{d_1,d_2}\}. \tag{8}$$

Two peaks are visible on Figure 5b. The one centered ($\tau = 0$ s) corresponds to the signal of the ocean. Indeed, by looking at the bandwidth of the ocean, it is large (around 90% of the full bandwidth), which explains the sharp peak (since the width is inversely proportional to the bandwidth). The other peak, delayed by $\tau = -6\,\mu$s, corresponds to the RFI, since its bandwidth is relatively small, explaining a wider peak. The attribution of the peaks to either the RFI or the ocean, from a physical point of view, may differ. It seems logical that the signal emitted by the ocean arrives with a certain delay to satellite two with respect to the first one. Moreover, since RFI is a noise (considered as a single point), it seems logical that noise produces 0 delay. Then, since the objective is to capture the thermal radiation produced by the ocean and estimate its brigthness temperature, it would make sense to have information on the ocean rather than the RFI.
In the end, the discussion regarding the bandwidth will be retained, since it is coherent from a mathematical point of view.



(a) Full plot. **(b)** Zoom.

Figure 5: Cross-correlation of the signals d_1 and d_2 (2 with respect to 1).

7) *What do you think the difference of the delays is due to? What information could you retrieve from this measurement?*

This difference of the delays can be due to different phenomena. First, the location of the source: the delay is due to the fact that the source is not emitting along a line perpendicular to the baseline and passing through the middle. The signals can also interfere (typically in this case the ocean and the RFI signals produce interference). Another phenomenon can be related to the passage of a signal passes through the atmosphere.
The difference of delays calculated in the previous point is $\Delta\tau = 6\mu$s. It is reasonable to assume a 2D-space (planar wave front). Indeed, as the baseline L is not enormous, this assumption holds. From $\Delta\tau$, the direction of arrival θ of the signal coming from the RFI can be retrieved, as described in Figure 6. Assuming a far field source, the two segments can be considered parallel and thus a rectangle triangle is formed. Knowing the difference of delays, the delta distance is known and thus the angle of arrival. Note that here the

speed is assumed equal to the speed of light, even though the actual speed is reduced by the presence of the atmosphere.

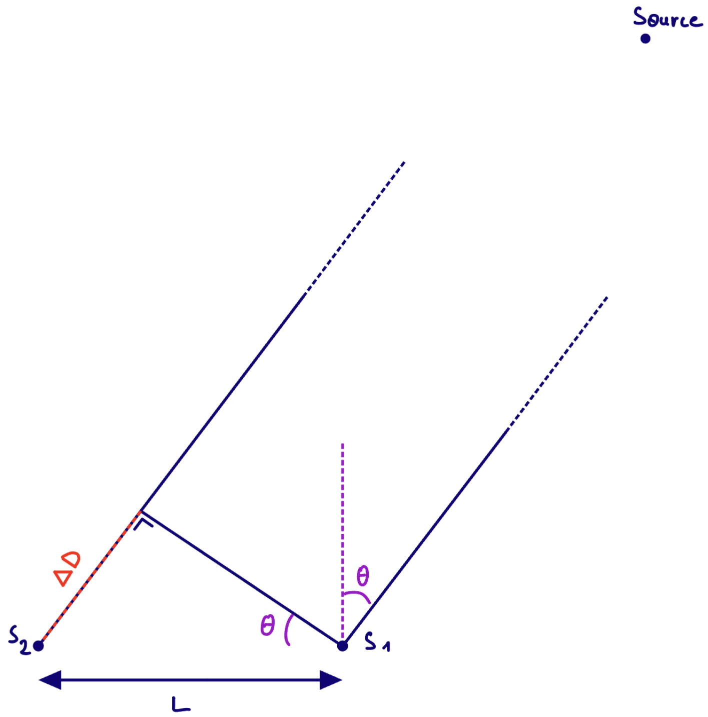


Figure 6: Scheme to determine the angle of arrival of a signal in the 2D-space.

$$\sin\theta = \frac{\Delta D}{L} = \frac{c\,\Delta\tau}{L} \tag{9}$$

This leads to $\theta = 5.1636°$.

2 SAR Interferometry basics

1) *Propose and implement a procedure to estimate elevation and deformation velocity for the selected targets, and assess estimation accuracy.*

   For each of the 1000 selected targets, the problem can be written in the following matrix form:

$$\boldsymbol{\varphi}_{10\text{x}1} = \mathbf{K}_{10\text{x}2}\mathbf{x}_{2\text{x}1} + \mathbf{w}_{10\text{x}1}, \tag{10}$$

   where $\mathbf{K} = [\mathbf{k_z}\ \mathbf{k_v}]$, and $\mathbf{x} = [z\, v]^T$.

   Firstly, this linear inverse problem is related parametric estimation. As the noise is characterised by a zero-mean, estimating elevation and deformation velocity can be carried out with a linear estimator: $\hat{\mathbf{x}} = \mathbf{B}\,\boldsymbol{\varphi}$. In this case, by looking at the phase noise covariance matrix, the noise is correlated. The optimal choice is thus the BLUE estimator since it is able to adapt to noise statistics. Indeed, the least-square estimator would lead to the same results as blue only in the case of uncorrelated noise (considered white noise if the variances are the same). In matrix from, the estimator is:

$$\mathbf{B} = (\mathbf{K}^H\mathbf{C}_\mathbf{w}^{-1}\mathbf{K})^{-1}\mathbf{K}^H\mathbf{C}_\mathbf{w}^{-1} \tag{11}$$

   The estimation accuracy is obtained as $\text{Var}(\hat{\mathbf{x}}) = \mathbf{B}\mathbf{C}_\mathbf{w}\mathbf{B}^H$, and the results are shown in Table 4. It is important to note that here the matrix $\mathbf{K}$ has size NxM with N>M, the case is overdetermined. Moreover, the estimation is not biased ($\mathbf{BK} = \mathbf{I}$) since BLUE is based on unbiasedness.

Table 4: Estimation accuracy of the BLUE estimator.

Estimated Quantity	Estimated Value	Unit
Var($\hat{z}$)	1.2321	m^2
$\sigma_{\hat{z}}$	1.1100	m
Var($\hat{v}$)	41.3935	$mm^2/year^2$
$\sigma_{\hat{v}}$	6.4338	mm/year

2) *Use the knowledge of the true target elevation and deformation velocity to assess estimation accuracy experimentally. Is it consistent with the assessment at point 1?*

   In order to assess estimation accuracy experimentally, it is straightforward to use the Mean Squared Error and the Root Mean Squared Error:

$$MSE = E[(\mathbf{x} - \hat{\mathbf{x}})^2] \quad \text{and} \quad \text{RMSE} = \sqrt{\text{MSE}}. \tag{12}$$

   Once again the results are shown in Table 5. It is can be seen with respect to Table 4, the results are coherent with the experimental data.

3) *Imagine that the two vectors k_z and k_v are parallel to one another, i.e.: $k_z = ak_v$, with a some constant. Would it be possible to solve the problem in this case?*

Table 5: Estimation accuracy of the BLUE estimator with respect the the experimental data.

Estimated Quantity	Estimated Value	Unit
MSE($\hat{z}$)	1.2278	m^2
RMSE($\hat{z}$)	1.1081	m
MSE($\hat{v}$)	42.1040	$mm^2/year^2$
MSE($\hat{v}$)	6.4888	mm/year

In the case $k_z = ak_v$, the determinant of $\mathbf{K}$ would be null, the matrix is then singular and $\mathbf{B}$ would be undefined. Consequently, the problem would not be solved with the BLUE estimator.

3 Noisy and blurred

1) *Propose and implement a procedure to estimate the unknown random misalignements and produce a corrected image.*

The first image is the reference one since no random misalignements are present, as seen in Figure 7. The procedure to estimate these unknown random misalignements of one image with respect to the first one is the following:

Figure 7: First image.

a. Grid definition: the spatial grid is first defined with the reference (0,0) chosen to be at the center of the image; The spatial sampling dx and dy are chosen equal to 1.

b. Spectral analysis: after the spatial grid is defined, the 2D Fourier transform of each image are performed (column-wise and then row-wise) with, for the same reasons as in section 1, $N_{f_{x,y}} = 2^{\mathrm{ceil}(\log_2(\mathrm{N_{x,y}}))}$. N_x and N_y are the dimensions of the image (number of columns and row respectively);

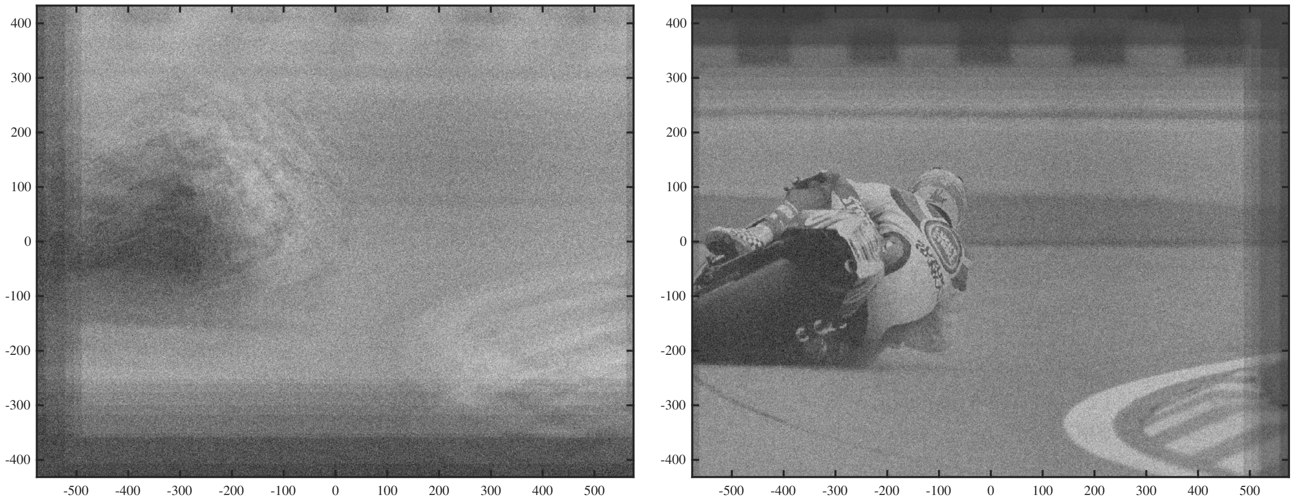
c. Interferometric analysis: the phase difference here is analysed using $S_i = S_2\, S_1^*$, where S_1 and S_2 are the FT of the images;

d. 2D cross-correlation: the displacement is estimated by taking the 2D $\mathcal{F}^{-1}$ of S_i. The peak of the cross-correlation (noted R) is found at a certain position (x_0, y_0). Practically, on Matlab, *[n,m] = find (R == max(R(:)))* is used to retrieve the two indexes. Then, the displacement is estimated

$$\Delta_x = n - N_x/2 \quad \text{and} \quad \Delta_y = n - N_y/2. \tag{13}$$

This procedure is carried out for the remaining images, the misalignment is corrected, and the averaging of the 10 pictures is executed. For the sake of comparison, the mean of the original images is shown in Figure 8a, and the result of the mean of the aligned images is shown in Figure 8b. Of course, the results are considerably better with the aligned images, which proves the efficiency of the procedure.

2) *Observe the 2D spectrum of the corrected image. Do you think it is possible to propose a procedure to further abate noise?*



(a) Mean of the original images. **(b)** Mean of the aligned images.

Figure 8: Results of the averaging of the 10 images, for the raw and the corrected data.

The 2D spectrum of the corrected image is represented in Figure 9. One bright spot is seen in the center (low-frequency region). This region is related to the mean value of the image. Also, symmetry is present around the center, which is expected for images with real values (this is the case here). The relatively uniform dark-grey background indicates that noise is not really present in the high frequencies. Still, the background in the high spatial frequencies is not dark. Consequently, to further abate noise, one should then use a low pass-filter in order to preserve low frequency content of the image, while reducing the noise at higher frequencies.

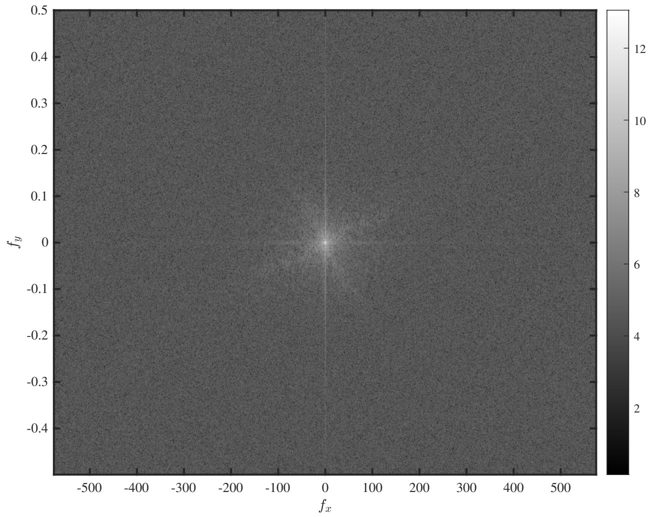


Figure 9: 2D spectrum of the corrected image.

As seen in Figure 10, a classic low-pass filter is able to further abate noise. This filter was constructed in the time domain with 2 filters (with the function *ones*, row-wise and column-wise), and applying it by convolution. One should finally note that a lot of filters exist, such as a Gaussian filter, or even a circular low-pass filter that could provide a better filtering of the image.

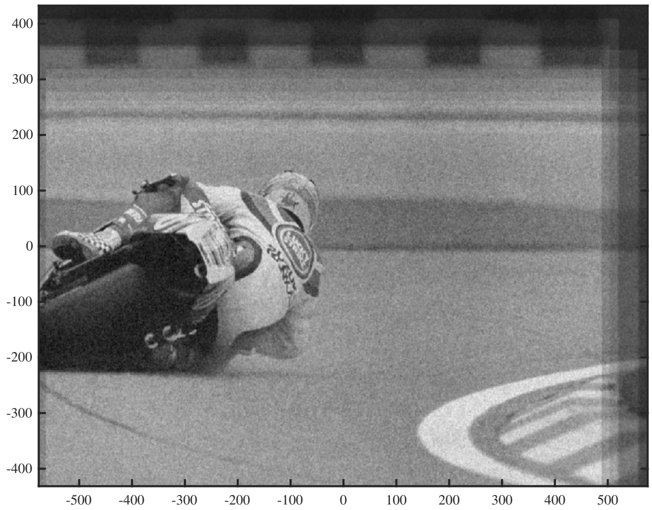
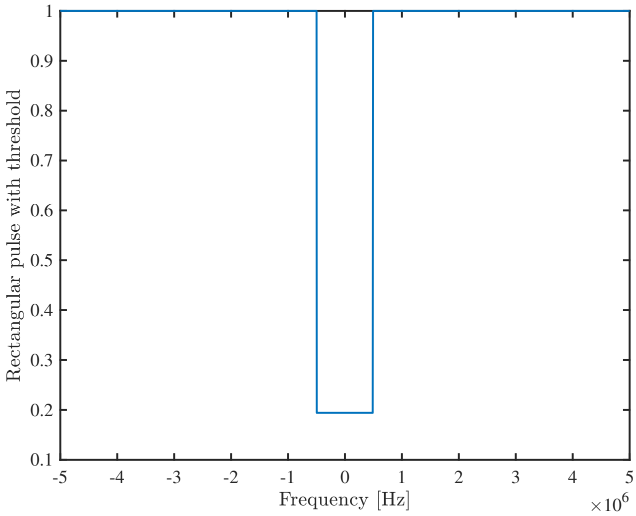
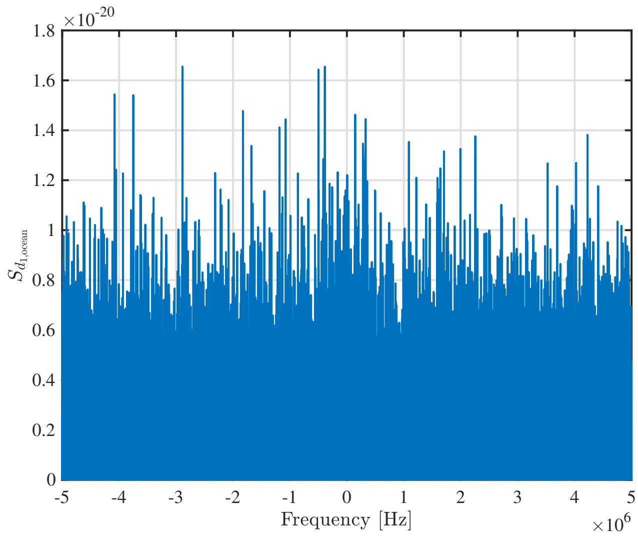


Figure 10: Low-pass filter applied to the corrected image.

4 Annexes



(a) Threshold.



(b) Obtained signal.

Figure 11: Threshold and obtained signal for the ocean in the frequency domain.