Wealth Inequality, Network Topology and Financial Crisis

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Abstract

A theoretical financial network model is proposed for understanding the relationship between wealth inequality and financial crises, or instability. Financial assets link individual asset and liability holders to form a static economic network. The total number of financial assets an individual owns represents their *in-degree* and the distribution of those assets is imposed by an *in-degree distribution*—equivalent to the wealth distribution. A network's topology varies with the level of wealth inequality and total wealth, and together they determine network contagion in the event of a random negative income shock to an individual. Simulations demonstrate that increasing wealth inequality, imposed exogenously through the degree distribution's skewness parameter, makes a wealthy network less stable—as measured by the share of individuals failing financially. Aggregate wealth also has an inverted U-shaped effect on the model's network stability. The simulation results suggest a unique structural role for the distribution of accumulated assets in macro-financial stability.

Keywords: Wealth inequality, financial crisis, growth and fluctuations, financial network, degree distribution.

JEL-Classification: D31, G01, L14, N10

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1 Introduction

The share of wealth held by top percentiles in the US provocatively peaks before both the Great Crash and the more recent global financial crisis.[1] This correlation raises important questions: Is inequality a destabilizing economic force? Or, is it, along with financial fragility, a symptom of deeper economic perturbations? The objective of this paper is to understand the relationship between wealth inequality and macroeconomic instability as manifested through the financial sector. The approach can be summarized thusly: consider two identical economies that are observed at a point in time and distinguished only by their distributions of wealth. Which is more unstable in the event of a negative income shock? To answer, an interpersonal financial network model is constructed using elements of graph theory. The model is then repeatedly simulated, generating predictions about the endogenous role of wealth distributions on financial stability. One goal is to demonstrate how the wealth distribution can alter the configuration of the financial economy into more or less stable arrangements.

The proposed theoretical model provides a direct channel from top wealth inequality to the vulnerability of a financial network in the event of a shock. There exists one type of financial asset that represents an individual's claim on some future cash flow. That cash flow is assumed to be generated by another individual's labor income, so that an individual owning a financial asset is naturally linked to the individual whose income generates the cash flow. The flows across these links are what Hyman Minsky called a "complex system of money in/money out transactions,"[2] and Kregel (2014) makes the point that only a "slight disturbance" in money flows is necessary to cause instability and "widespread financial distress." This paper investigates how wealth inequality, by changing the organization of financial links, determines the level of distress.

Axel Leijonhufvud once described a network economy as a "web of contracts and understandings" between agents. Wealth, as a collection of financial assets, by definition creates financial links in a network economy. Network nodes represent households or individuals and the total number of financial assets an individual owns represents their *in-degree*. The distribution of those assets is imposed by an *in-degree distribution*—equivalent to the wealth distribution. As the distribution of

[1] See Kopczuk & Saez (2004). Piketty & Saez (2014), Saez & Zucman (2014), and Kopczuk (2015).
[2] See (Minsky, 1986a, p. 69).

wealth changes the distribution of links in the network also changes—thereby altering the topology of the interpersonal financial network. Since labor income is assumed to be homogenous, changing the degree distribution changes the wealth distribution. It is shown in simulations that the model network economy is more unstable (measured by the share of individuals whose net worth drops below a predetermined threshold) in the event of a random negative shock when it (a) exhibits high wealth inequality, and (b) is sufficiently wealthy in aggregate. Additionally, an inverted U-shaped relationship is found between aggregate network wealth and instability.

Contagion occurs when the random negative income shock decreases one individual's net worth to the point of financial failure, which prompts failure costs that wipe out collateral wealth. Importantly, an individual node's net worth is assumed to collateralize their financial liabilities, much like an asset-backed security. The network structure implies one individual's net worth is linked to, and dependent on, the net worth of others. Therefore decreases in net worth spread.

The network model embeds several features of Minsky's Financial Instability Hypothesis—a framework that generates endogenous instability in a financial economy of connected banks and firms rather than individuals.[3] The following key tenets are incorporated: interrelated balance sheets of individuals, where one's asset is always another's liability; assets/liabilities represent commitments to future cash flows; a collapse in asset values stifling future cash flows; and a growing financial economy increasing the scale of contagion.

Of course, the model is a gross simplification of a financial capitalist economy (e.g. assuming a static network, with one type of financial asset serviced by (uniform) labor income cash flows and individual net worth acting as collateral). But by stripping away layers of financial intermediaries, it becomes possible to expose the latent financial relationships between individual creditors and debtors and to understand how the interpersonal distribution of wealth in the economy may impact its overall stability. Though the setup also ignores network formation dynamics, or consumption and saving decisions by individuals, it provides a tractable model that can be simulated and whose results are generalizable. The only known network models of individual wealth inequality reside in a small statistical mechanics literature,[4] demonstrating a power law degree distribution of wealth

[3]See Minsky (1975) and Minsky (1986b) for longer expositions, or Minsky (1992) for a brief summary.
[4]See Lee & Kim (2007), Kim et al. (2008), and Coelho et al. (2005)

often results from network formation dynamics. None consider contagion or network instability as this paper does.

Financial network models are frequently used, however, to model financial crises amongst banks. Allen & Gale (2000) were one of the first to show in a simple bank network model that the configuration of financial links mattered for contagion—complete networks were more stable than incomplete ones. More recently, Acemoglu et al. (2015) stress network structure as the determining factor in contagion, but they largely look at the magnitude and frequency of negative shocks to the network in order to analyze its stability, not network topology. And Glasserman & Young (2015) abandon topology measures altogether in favor of bank-specific sufficient statistics to evaluate bank network contagion risks. They conclude that factors beyond pure spillovers, such as confidence in counterparties and bankruptcy costs (included in the model below), are responsible for substantial economic losses from contagion.[5]

A more nuanced finding in the financial network model literature is the nonmonotonic effect of connectivity on contagion. Increases in interdependence initially increase contagion and spillovers, but after a certain threshold, the increased linkages create a more robust financial system (Nier et al. (2007), Elliott et al. (2014a)). Gai & Kapadia (2010) characterize this nonmonotonicity as a "robust-yet-fragile" trait of financial markets, something the below model also captures.

The below model is derived from the framework of Elliott et al. (2014a). However, nodes represent individuals rather than banks or countries, and financial links do not exist outside of the network. Their emphasis is also on the levels of financialization in the network, both intensive and extensive margins, rather than the skewness of financial assets as in this paper.

While the finance network literature has not considered the role of inequality, the income inequality literature has considered financial crises—with mixed results. In a qualitative survey of 84 crises across 21 countries over the past century, Morelli & Atkinson (2015) examine both the levels of and changes in income inequality preceding a crisis episode. They conclude that the impact of either is ambiguous. Rajan (2011) argues increasing US income inequality was but one critical "fault line" in the crisis because it prompted policies that ultimately relaxed credit to un-

[5] Both Acemoglu et al. (2015) and Glasserman & Young (2015) are derived from Eisenberg & Noe (2001), a network model of equilibrium clearing payments among banks used to measure contagion.

sustainable levels. Testing the Rajan hypothesis, Bordo & Meissner (2012) regress changes in real credit growth on lagged changes in top income shares. They find no effect amongst a panel of 14 countries between 1870 and 2010 and thus conclude no link between inequality and crisis exists.[6] A dynamic stochastic general equilibrium model by Kumhof et al. (2015)[7] is conceived, like this paper's model, around assets linking households—in their case the top 5% to the bottom 95%. The authors demonstrate that a sequence of increasing income inequality, rising household debt of the bottom 95%, and increasing financial assets of the top 5% causes higher leverage and thus a higher probability of crisis. Stiglitz (2012) instead stresses a Keynesian mechanism: the marginal propensity to consume. Increasing income inequality decreases aggregate demand because wealthy households consume less willingly at the margin than poor households. The policy reaction is also loose credit.

In the income inequality literature cited above, the most common mechanism linking inequality to instability is household debt, measured as the debt-income ratio. Mason & Jayadev (2014) show, however, that a set of so-called "Fisher dynamics" (i.e. interest rate change, inflation, and income growth) account for most, if not all, of the increase in US household leverage since 1980—the same structural lever modeled by Kumhof et al. (2015). In other words, increasing household debt-income ratios do not necessarily imply newly issued debt, and new debt is the critical vehicle of the inequality-household debt-instability story. This paper argues instead that an economy's specific financial network configuration as determined by the wealth distribution is the critical determinant.

The rest of the paper is organized as follows: Section 2 derives the theoretical financial network model, presents its mechanics, and introduces concepts of instability. Financial network parameter estimates are shared in Section 3, to motivate model calibration. Section 4 describes the method to simulate random static networks and also presents results, including the finding that increasing wealth inequality is destabilizing in wealthy networks. We conclude in Section 5 and suggest extensions for further research.

[6] Gu & Huang (2014), criticizing the econometric methods of Bordo & Meissner (2012), argue that income inequality, in Anglo-Saxon countries, *does* determine credit growth—and therefore leads to financial crisis.

[7] Often cited as an earlier IMF Working paper, Kumhof & Ranciere (2010).

2 Financial Network Model

In this section we introduce the wealth inequality network model, building off of Elliott et al. (2014a). Our model notably disregards financial intermediaries and instead relies on the latent financial links between asset and liability holders to form an interpersonal financial network economy. This enables a more tractable model between the economy's wealth distribution, how it translates to the network topology, and overall financial (in)stability.

2.1 Setup and Financial Assets

We consider a static financial network composed of nodes $i = 1, \ldots, n \in N$ where each node represents wealth owning individuals or households. (The terms node or individual are used interchangeably throughout.) We exclude firms, banks, and other types of organizations to simplify our model and to argue that variations in the distribution of wealth between individuals have network consequences.[8]

Links, or edges, connect two nodes and represent a financial claim between them. A financial asset is simply a claim on future cash flows. The links between all nodes in our network can be represented by an $n \times n$ matrix $\mathbf{G}$, called an *adjacency matrix*, with binary entries where $G_{ij} = 1$ if node i has some financial claim on node j and 0 otherwise. Claims are directional, implying an asset position for i and a liability for j and a direction of future cash flows from j to i. Matrix $\mathbf{G}$ is thus composed of creditors (rows) making financial claims on debtors (columns). Though individuals are along both dimensions of the matrix, financial claims need not be reciprocated—and $\mathbf{G}$ need not be symmetric. Our network can be summarized as an unweighted directed graph $\mathcal{G}(N, G)$ whose edges indicate the existence, and paths, of financial flows between individuals.

Assume there exists only one type of financial asset held by individuals and households, a type of asset-backed security. Each security is a claim on future labor income cash flows generated by the liability holder, with their net worth serving as collateral.[9] A node i owns d_i financial assets, where $d_i = \sum_j G_{ij}$ is called the node's *in-degree*. This also represents the total number

[8] To be sure, many individuals rely on opaque institutions and organizations to hide private wealth. See Zucman (2014) and Zucman (2015) for a detailed analysis on hidden private wealth.

[9] Node net worth is discussed in detail in Section 2.3.

of individuals i holds claims against (a row sum in $\mathbf{G}$). A financial asset-owning node may also back the value of an asset themselves, a function of their own valuation. Let d_j^{out} represent the total number of financial liabilities node j is collateralizing, where $d_j^{out} = \sum_i G_{ij}$ (a column sum in $\mathbf{G}$). Called the *out-degree*, d_j^{out} represents the number of financial outflows from individual j to claimant nodes. Financial assets are distributed according to some probability distribution $f(d_i)$ called the *degree distribution*.[10] Only some fraction $c \in (0,1)$ of each individual's overall net worth is collateralized and can be claimed by, and owed to, other individuals holding financial assets within the network.

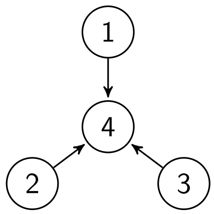
We define a matrix $\mathbf{C}$ to describe the relative ownership claims on each node's value in the network. This *cross-holdings matrix* has, for $i \neq j$, element

$$C_{ij} = \begin{cases} c\frac{G_{ij}}{d_j^{out}} & \text{if } d_j^{out} > 0 \\ 0 & \text{else.} \end{cases} \tag{1}$$

Our unweighted adjacency matrix $\mathbf{G}$ has become a weighted matrix $\mathbf{C}$ of financial claims between nodes, where element C_{ij} is the share of individual j's future cash flows, backed by their net worth, claimed by i. The total number of asset holders d_j^{out} holding assets backed by individual's j's wealth are each entitled to an equal portion of future cash flows. Node cash flows not claimed by other individuals $(1-c)$ are saved. (Savings do not accumulate as our model is static.) The savings of each node are summarized in a diagonal matrix $\hat{\mathbf{C}}$, with element $\hat{C}_{jj} = 1 - \sum_i C_{ij}$. From this definition we can also rewrite the total sum of claims made on individual j as $\sum_i C_{ij} = 1 - \hat{C}_{jj}$.

To illustrate, consider the network in Figure 1a, where $n = 4$ and $c = 0.5$. The corresponding adjacency and cross-holdings matrices are in Figure 1b. Notice, from $\mathbf{G}$'s bottom row, that node 4 has financial assets which are claims on the cash flows of nodes 1, 2 and 3, but has no cash flow obligations itself, and thus an in-degree $d_4 = 3$ but an out-degree $d_4^{out} = 0$. Because $c = 0.5$, 50 percent of nodes 1, 2, and 3's future incomes flow to node 4, seen in the last row of $\mathbf{C}$.

[10] Note that $\sum d_i = \sum d_j^{out}$ so that total assets equal total liabilities and the economy's balance sheet balances.



(a) Graph of network

$$\mathbf{G} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}, \ \mathbf{C} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ .5 & .5 & .5 & 0 \end{bmatrix}$$

(b) Corresponding adjacency and cross-holdings matrices.

Figure 1: Example of a four-node network

2.1.1 Microfoundations

Consider one possible microfoundation for our network model thus far. Suppose our static network is an endowment economy, whereby all nodes are endowed with a single type of financial asset—like our asset-backed security. The endowments are randomly distributed between nodes according to some probability distribution $f(d_i)$, where d_i is the total number of financial assets node i owns. Which nodes then back each of the d_i securities node i owns is randomly determined.

A richer form of heterogeneity could allow for variation amongst nodes rather than endowments. Let ρ represent a symmetrically distributed stochastic discount factor where $\rho \in \{\rho_l, \rho_\mu, \rho_h\}$.[11] If a node is assigned the lower discount factor ρ_l, then the node must borrow to consume single good y as they have a preference for consumption. In this circumstance $d_l < d_l^{out}$ and the individual is a net debtor. If a node receives the higher discount factor ρ_h it is a lender with a preference for accumulating assets. In this event $d_h > d_h^{out}$ and the individual is a net creditor. Should the node receive the mean discount factor ρ_μ, then $d_\mu = d_\mu^{out}$.

Another possibility is to consider an economy of entrepreneurs. Each node is endowed with some productive asset and an intermediary good, drawn from a distribution. The intermediary good may be consumed, but nodes prefer to consume a final consumption good that requires the interaction of at least two intermediary goods. Credit, fixed in aggregate, is extended between entrepreneurs to produce the final consumption good, which may be used to repay liabilities.

[11] In Krusell & Smith (1998), a dynamic general equilibrium model using stochastic discount factors generates a Pareto wealth distribution in the tails that closely fits the empirical estimates of Wolff (1994).

2.2 Real Assets

In addition to the financial asset, there exist $k = \{1, \ldots, m\} \in M$ real, or physical, assets in the network. Think of productive assets like land or human capital. A matrix $\mathbf{D}$, describing the pattern of real asset claims and analogous to the cross-holdings matrix $\mathbf{C}$, is composed of elements D_{ik}, which denote individual i's share of real asset k . We can now describe the *gross value* of individual i's total assets V_i as the sum of their real asset claims (each at their respective prevailing market price, p_k) and financial asset claims (backed by the liability holder's own gross value).

$$V_i = \sum_k D_{ik} p_k + \sum_j C_{ij} V_j. \tag{2}$$

Written in matrix notation, we have $\mathbf{V} = \mathbf{Dp} + \mathbf{CV}$, and solving for the gross value of each individual in the network yields a vector of values $\mathbf{V}$, where

$$\mathbf{V} = (\mathbf{I} - \mathbf{C})^{-1} \mathbf{Dp}. \tag{3}$$

Note, however, that the gross value of individual i's total assets V_i double counts real asset claims D_{ik}. They appear not only in the first term of Equation (2) but also in the second term as a component of other individuals' own valuations V_j. Therefore, in the next section, we derive a measurement of node net worth.

We simplify our model by assuming there exists one type of real asset, human capital, with $m = n$ different units. Because the only real asset in this economy is human capital, it cannot be owned by anyone else, though others may have claim to the future cash flows generated by it.[12] In other words, each node in the network is endowed with one unit of labor that is inelastically supplied. Output is generated by a linear production function with labor or human capital as the only argument, $y = l$ where $l \equiv D_{ii}$. Because human capital, or labor, is owned entirely by the individual endowed with it, we let $\mathbf{D} = \mathbf{I}_n$. Human capital prices are homogeneous and normalized to one, such that $p_k = 1 \ \forall \ k$. Total output in this static economy is equal to the sum of real assets

[12] Allowing $\mathbf{D}$ to represent human capital takes into consideration a common critique of Piketty (2014), best articulated by Blume & Durlauf (2015), that aggregating financial and physical assets at prevailing market prices crucially ignores the important contemporary role human capital plays in generating cash flows.

made into commodities, or total human capital:

$$Y = \sum_{i=1}^{n} y = \operatorname{Tr} \mathbf{D} = m = n. \tag{4}$$

Because real assets are homogeneous in our network, the model is designed to specifically study how the distribution of financial assets $f(d_i)$ impacts the network's overall stability.

2.3 Net Worth

A node's net worth is defined as total assets (real and financial) less liabilities.[13] To derive an expression, we sum real assets and ownership claims on other individuals' wealth (inflows) and subtract claims on one's own wealth (outflows):

$$v_i = \sum_{k} D_{ik} p_k + \sum_{j \neq i} C_{ij} V_j - \left(\sum_{j \neq i} C_{ji} \right) V_i. \tag{5}$$

Note that the first two terms are simply individual i's gross value, or the sum of real and financial assets defined in Equation (2). From this we subtract total liabilities. In matrix form we have,

$$\mathbf{v} = \mathbf{Dp} + \mathbf{CV} - (\mathbf{I} - \hat{\mathbf{C}})\mathbf{V} = \mathbf{Dp} + [\mathbf{C} - (\mathbf{I} - \hat{\mathbf{C}})]\mathbf{V}$$

where $\mathbf{I} - \hat{\mathbf{C}}$ is a diagonal matrix representing weighted total obligations in the network and $\mathbf{C}$ represents weighted total claims. Substituting the gross value from Equation (3) for $\mathbf{V}$ and rearranging leads to a unique interpretation of net worth.

$$\begin{aligned}
\mathbf{v} &= \mathbf{Dp} + [\mathbf{C} - (\mathbf{I} - \hat{\mathbf{C}})]\mathbf{V} \\
&= \mathbf{Dp} + [\mathbf{C} - (\mathbf{I} - \hat{\mathbf{C}})][(\mathbf{I} - \mathbf{C})^{-1}\mathbf{Dp}] \\
&= ((\mathbf{I} - \mathbf{C}) + \mathbf{C} - \mathbf{I} + \hat{\mathbf{C}})(\mathbf{I} - \mathbf{C})^{-1}\mathbf{Dp} \\
\mathbf{v} &= \hat{\mathbf{C}}(\mathbf{I} - \mathbf{C})^{-1}\mathbf{Dp} \\
\mathbf{v} &= \mathbf{ADp}
\end{aligned} \tag{6}$$

[13] See, for example, Davies & Shorrocks (1999) and Davies et al. (2007). In Elliott et al. (2014a) this is also called a node's *market value*, since their model's nodes represent firms or banks.

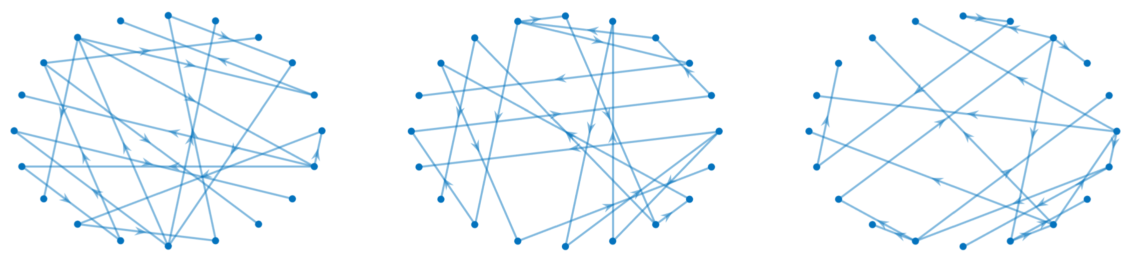
Net worth can now be derived from the overall claims between all nodes in the network (matrix $\mathbf{A}$) made on the underlying real assets (matrix $\mathbf{D}$ at price $\mathbf{p}$) of the economy. Since each real asset represents a node's human capital, net worth is simply derived from the cumulative claims on future output generated by another's human capital.

The utility of introducing matrix $\mathbf{A} = \hat{\mathbf{C}}(\mathbf{I} - \mathbf{C})^{-\mathbf{1}}$, called the *dependency matrix*, is that it summarizes the total claims between all nodes, i.e. the sum of direct and indirect dependencies between individuals in the network.[14] It is possible for element A_{ij} to be nonzero even if the corresponding element in the cross-holdings matrix, C_{ij}, is zero—an indication of indirect claims by i on j via other nodes in the network but no direct claims. The dependency matrix $\mathbf{A}$ is not unlike Leontief's input-output matrix, Elliott et al. (2014a) posit, in its ability to summarize the interconnections of a network economy. It is instructive to examine the differences between direct holdings (from cross-holdings matrix $\mathbf{C}$) and total direct and indirect holdings (from dependency matrix $\mathbf{A}$) in the examples in Section 2.6.

The dependency matrix also simplifies the accounting considerably. Claims on individual real assets, rather than both financial assets and liabilities, become a sufficient statistic to determine an individual's overall net worth when calculating the impacts of a shock as they reverberate through the network. In fact, all wealth is derived from human capital.

Given that we can calculate the net worth of an individual v_i using Equation (2.3), why not directly model the wealth distribution with $f(v_i)$? Using $f(v_i)$, rather than $f(d_i)$, to model wealth inequality obscures the critical role that interconnectedness plays in the financial network. It is precisely the connecting structure of the network that determines whether or not a shock causes contagion. In order to have a tractable link structure in our adjacency matrix the random network's inequality must be derived from the degree distribution, $f(d_i)$. Finally, the degree distribution of the network characterizes the same magnitude of wealth inequality given by the distribution of individual net worths, without loss of generality.

[14] Matrix $\mathbf{A}$ has all nonnegative elements and is also column-stochastic, thus each of its columns sums to 1 ($\sum_i A_{ij} = 1$).



(a) $\gamma = 2.35$ **(b)** $\gamma = 1.65$ **(c)** $\gamma = 1.025$

Figure 2: Random network graphs ($n = 20$)

2.4 Wealth Inequality

The wealth distribution of the network can be decomposed into its real and financial components. We have already assumed real assets, in the form of human capital, are fixed and equal for all individuals. Financial assets, then, entirely determine the wealth distribution, defined by the degree distribution of financial assets $f(d_i)$. Wealthier individuals have more positive financial claims and links to other individuals in the network than less wealthy individuals. A deterministic degree distribution, for example, captures perfect equality of financial wealth. Let a Pareto distribution describe the degree distribution of an unequal society where the probability of someone having d_i financial assets is given by $p(d_i) = a d_i^{-\gamma}$, with $\gamma > 0$.[15]

The aggregate financial wealth of the entire network is equal to total number of financial claims $\sum d_i$. Because our network is static and the number of individuals n remains fixed, increasing the number of assets in the network increases total financial wealth. This is akin to the economy growing through increased credit, or financialization at the extensive margin.

The graphs in Figure 2 illustrate how a random network's structure changes with financial wealth inequality via the Pareto parameter γ. Each network with $n = 20$ and expected in-degree $E[d_i] = 1$ is generated randomly for a specified γ. The highest Pareto parameter ($\gamma = 2.35$) corresponds to the lowest inequality among the three graphs. Its financial claims are more evenly spread out compared to the most unequal random network graph ($\gamma = 1.025$).[16]

[15] Our network model simulation results are robust to allowing wealth inequality to be determined by the out-degree distribution $f(d_i^{out})$ rather than the in-degree distribution $f(d_i)$.

[16] Each graph is generated thusly: draw a random Pareto distribution of financial claims d_i, truncated at the top

2.5 Shocks, Financial Failure, and Contagion

Though our model is static, contagion is evaluated dynamically. We therefore introduce a time subscript to specify periods in relation to the initial shock in period $t = 0$.

Recall that, initial real asset prices are set to 1 so that $\mathbf{p}$ is a vector of ones. A random exogenous income shock at time $t = 0$ impacts one individual's net worth through their real asset price, such that $p_i = 1$ becomes $\tilde{p}_i = \lambda p_i = \lambda \ \forall \ t \geqslant 1$, where $\lambda \in [0, 1)$. The magnitude of the negative real asset price shock $\tilde{p}_i$ is decreasing in λ. All other individuals in the network experience no real asset price shock at $t = 0$, and thus the vector of real asset prices after the initial shock $\tilde{\mathbf{p}}$ contains a value λ in the i^{th} row and 1 everywhere else.[17] Because there exists a uniform risk $(\frac{1}{n})$ of shock, no risk premia are priced into financial assets.

The negative shock to an individual's financial wealth, transmitted through an exogenous price drop in their human capital, could represent the loss of a job or earning capacity. If, as a result of this income shock, the individual's wealth $v_{i,t}$ should fall below some threshold $\underline{v}_i$ they experience *financial failure*. Network instability is defined by the accumulation of many individuals failing financially. Financial failure triggers bankruptcy costs β_i. They are not to be taken literally (net worth remains positive), but instead as representative of increased financial burdens faced when an individual's net worth is depressed by some relative amount. Such burdens could include direct costs like attorney and accounting fees as well as indirect costs such as lost income, increased future borrowing costs, loss of collateral or counterparty confidence. We denote this as $\beta_i(\tilde{\mathbf{p}})I_{v_{i,t}<\underline{v}_i}$, where I is an indicator function taking a value of 1 if $v_{i,t} < \underline{v}_i$ and 0 otherwise.

We assume that $\underline{v}_i = \theta v_i > 0$, with $\theta \in (0, 1)$ remaining constant throughout the dynamic contagion process. Parameter θ describes individual financial fragility. A high θ implies a more easily breached valuation threshold and likelier financial failure in the event of a shock, whereas a low value means more robust personal finances. The failure threshold $\underline{v}_i$ remains positive because financial duress and accompanying cash flow strains need not imply negative net worth in our model, only a financial setback such that creditors are not repaid and penalties imposed.[18]

to ensure $E[d_i] = 1$ across distributions; randomly link financial claims d_i to other nodes to create adjacency matrix $\mathbf{G}$; plot directed graph $\mathbf{G}$.

[17] One could consider the notation $\tilde{\mathbf{p}}_i$ to indicate that individual i experiences the negative shock.

[18] If $\underline{v}_i \leqslant 0$, it would imply individual human capital value is 0 or negative, an unrealistic scenario.

Let $\mathbf{b}_{t-1}$ represent a vector of failure costs with element $b_{i,t-1} = \beta_i(\tilde{\mathbf{p}}) I_{v_{i,t-1} < \underline{v}_i}$. By definition, $\beta_i = 0 \ \forall \ i$ at $t = 1$ because no individuals have failed yet. The first iteration of calculating new node valuations occurs at $t = 1$,[19] so equation (2.3) is rewritten to incorporate failure costs:

$$\mathbf{v_t} = \hat{\mathbf{C}}(\mathbf{I} - \mathbf{C})^{-\mathbf{1}}(\mathbf{D}\tilde{\mathbf{p}} - \mathbf{b_{t-1}}) = \mathbf{A}(\mathbf{D}\tilde{\mathbf{p}} - \mathbf{b_{t-1}}) \text{ for } t = 1, \ldots, T. \tag{7}$$

The dependency matrix $\mathbf{A}$ not only describes the share of an individual's wealth owed to claimants, but also the failure costs absorbed by those same claimants in the event of an individual's financial failure. This is the source of contagion. When an individual fails financially their remaining net worth (collateral) is wiped out due to failure costs—we assume $\beta_{i,t} = v_{i,t}$ in our calibration in Section 4.2. The failure costs spread according to the dependency matrix $\mathbf{A}$.

Consider the following example of dynamic contagion (illustrated in Figure 3). Suppose some individual j is the first financial failure as a consequence of receiving human capital price shock $\tilde{p}_j = \lambda$ at time $t = 0$, such that $v_{j,1} < \underline{v}_j = \theta v_j$ in the first re-evaluation at $t = 1$. This prompts failure costs $\beta_{j,1}$ which deplete collateral wealth and are partially absorbed by, for example, individual i's dependency on j as represented by a nonzero value for element A_{ij} in the dependency matrix. Such codependence implies i's value decreases by the amount $A_{ij}\beta_{j,1}$ in period $t = 2$. Should i's value $v_{i,2}$ fall below $\underline{v}_i$, it would incur its own failure cost $\beta_{i,2}$ and as a consequence alter the values, in period $t = 3$, of all individuals i is financially connected to (directly or indirectly) through the dependency matrix $\mathbf{A}$.

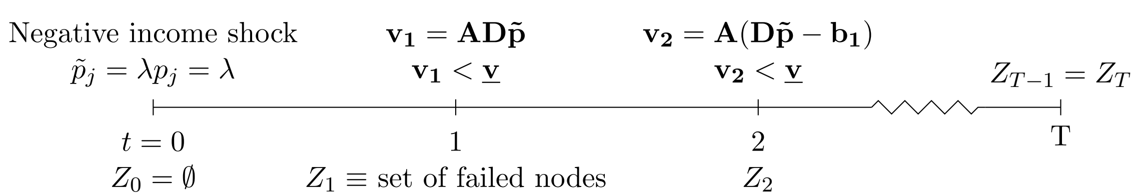


Figure 3: Timeline of Network Contagion

A static financial network gives way to a dynamic process of cascading failures. The instability is initiated by a decrease in one individual's earning capacity and wealth, hindering their ability to

[19] An algorithm in Appendix Section A.1 describes the process of iteratively calculating node valuations in the event of a negative shock in order to determine the number of total financial failures resulting from the initial shock.

service financial debts and thus provide cash flows for the financial claims creditors have on their output.[20] This cessation of cash outflows to creditors decreases each creditor's wealth, setting off progressive failures as any decline in a creditor's wealth below their own failure threshold would cause additional failure costs.

Any shock to individual net worth could initiate contagion. Because our network's wealth is ultimately derived from human capital, it is the intuitive income source to receive the shock. The model also emphasizes the role of network topology on instability by shocking only individual labor income rather than all labor income, which could cause instability because of the scope of the shock and not necessarily network structure.

A simple algorithm to identify the set of nodes failing is outlined in Section A.1 of the Appendix. Each iteration of the algorithm represents one period t and appends another group of individuals, Z_t, who fail as a direct result of the preceding $t-1$ group's failures. Contagion stops when no new individuals in the network fail.

2.5.1 From Contagion to Crisis

We define the level of network instability as proportional to the share of individuals in the network who have failed financially $S = \frac{|Z_T|}{n}$. A financial crisis could be said to manifest when a sufficient share of the network fails financially, though we are agnostic about a specific threshold. Network failure in our model is driven by drops in the value of initially real, but then financial, assets. This is congruent with empirical definitions of financial crisis, which may specify the magnitude asset values must drop.

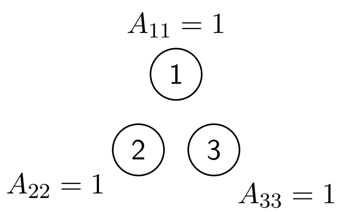
2.6 Example

We present one example of a simple network with $n = 3$ nodes and increasing numbers of financial assets to help elucidate concepts from the model. The example is illustrative of the network and

[20]This appropriates Minsky's position on financial instability: "the behavior and particularly the stability of the economy change as the relation of payment commitments to the funds available for payments changes and the complexity of financial arrangements evolve." (Minsky, 1986a, p. 197)

matrix structures, not contagion effects.[21] Throughout, we assume $\mathbf{D} = \mathbf{I}_3$ and $p_k = 1 \ \forall\ k$.

First, consider an unconnected network. No edges linking any nodes exist (Figure 4a). In a network with no financial claims, each individual keeps all future cash flows and their net worth depends only on their human capital—which is homogeneous. When a shock occurs, only the wealth of the individual experiencing the shock declines, but every other node is isolated. No contagion can occur. (Adjacency matrix $\mathbf{G}$ is omitted from Figure 4b because, like $\mathbf{C}$, it is all zeros.)



(a) Graph of network

$$\mathbf{C} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ \mathbf{A} = \hat{\mathbf{C}}(\mathbf{I} - \mathbf{C})^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

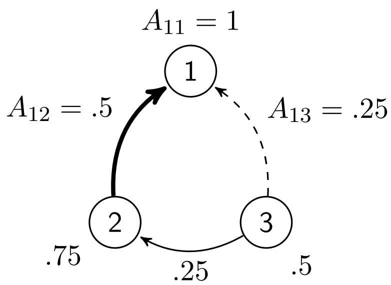
(b) Corresponding cross-holdings and dependency matrices

Figure 4: Three-node network with no financial assets

Now, suppose two financial assets are introduced into our network (see Figure 5). The total share of a node's net worth that may be claimed by other nodes, c, is 0.5. All elements in the diagonal savings matrix $\hat{\mathbf{C}}$ will be 1, unless financial claims are made on a node's value and it equals 0.5. The network's two financial assets represent two claims: node 1 has a claim on node 2's future cash flows and node 2 has a claim on node 3's. Therefore $d_1 = d_2 = 1 = d_2^{out} = d_3^{out}$ while $d_3 = 0 = d_1^{out}$. According to Equation (1), $C_{12} = 0.5$. Node 1, therefore, has claim to half of node 2's cashflows while node 2 retains the other half. The same relationship holds between nodes 2 and 3, where $C_{2,3} = 0.5$. Importantly, nodes 1 and 3 are indirectly connected through node 2 even though no direct link exists. (Note the dashed edge in Figure 5a.) Hence $A_{13} = 0.25 > C_{13} = 0$, because node 2 claims half of node 3's net worth, and node 1 claims half of 2's. Node 1 also has the highest net worth ($A_{11} = 1.75$) of which 0.75 is derived from the other two nodes. Node 2 has a net worth of 0.75, of which 0.25 is derived from node 3, and node 3 has no financial assets and thus a net worth of only 0.5 (equal to its own savings). A shock to node 1 would have no effect on the other nodes since no other nodes have financial claims on node 1 or are dependent on node 1's net worth. Only if nodes 2 or 3 were shocked could multiple nodes fail (nodes 1 and 2) since others

[21] Because this is the smallest possible network that can display a variety of link structures, a shock to any connected node may or may not immediately cause failure for all nodes.

are dependent on them.



(a) Graph of network

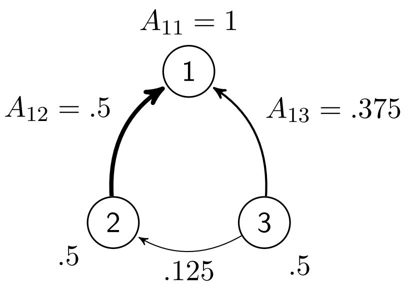
$$\mathbf{C} = \begin{bmatrix} 0 & .5 & 0 \\ 0 & 0 & .5 \\ 0 & 0 & 0 \end{bmatrix}, \ \mathbf{A} = \hat{\mathbf{C}}(\mathbf{I} - \mathbf{C})^{-\mathbf{1}} = \begin{bmatrix} 1 & .5 & .25 \\ 0 & .5 & .25 \\ 0 & 0 & .5 \end{bmatrix}$$

(b) Corresponding cross-holdings and dependency matrices

Figure 5: Three-node network with two financial assets

Next, we introduce another asset into the network giving a total of three financial assets in the network. (See Figure 6.) Node 1 gains an explicit financial claim on node 3. The in-degree of each node is now $d_1 = 2, d_2 = 1, d_3 = 0$. Of the 0.5 share of node 3's value that is securitized within the network, half goes to node 2 and the other half to node 1. But because node 1 has a claim on node 2's value, it also indirectly receives cash flows from node 3 via node 2 as well. Thus its indirect total cash inflows from node 3 are greater than its direct cash flows, or $A_{13} = 0.375 > C_{13} = 0.25$. In this graph with three financial assets, contagion depends on which individual is initially shocked. For example, if $\lambda = 0$ and node 1 were shocked (such that $\tilde{p}_1 = 0$), then only node 1 would fail financially. No other nodes depend on its value so its failure would not disrupt the net worth of others. If, on the other hand, node 3 were shocked (for the same λ) then because its value backs the financial assets held by the other nodes it would cause all three nodes to fail.

Finally, suppose all nodes are linked such that $d_i = d_i^{out} = n - 1 \ \forall \ i$ (Figure 7). The network has absorbed the maximum possible number of financial assets $(n^2 - n)$ and represents a *complete graph*—a special case of a *regular graph* where all nodes have equal degree. Each node has equal net worth: 0.6 from oneself and 0.2 from each of the other two nodes. Since everyone is connected in both directions, any shock will precipitate contagion.

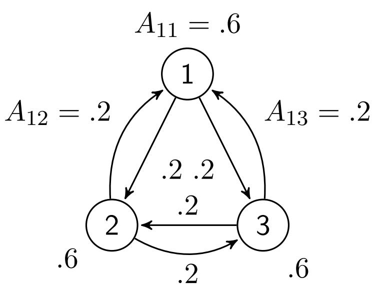


(a) Weighted network graph

$$\mathbf{C} = \begin{bmatrix} 0 & .5 & .25 \\ 0 & 0 & .25 \\ 0 & 0 & 0 \end{bmatrix}, \mathbf{A} = \begin{bmatrix} 1 & .5 & .375 \\ 0 & .5 & .125 \\ 0 & 0 & .5 \end{bmatrix}$$

(b) Corresponding cross-holdings and dependency matrices

Figure 6: Three-node network with three financial assets



(a) Weighted network graph

$$\mathbf{C} = \begin{bmatrix} 0 & .25 & .25 \\ .25 & 0 & .25 \\ .25 & .25 & 0 \end{bmatrix}, \mathbf{A} = \begin{bmatrix} .6 & .2 & .2 \\ .2 & .6 & .2 \\ .2 & .2 & .6 \end{bmatrix}$$

(b) Corresponding cross-holdings and dependency matrices

Figure 7: Three-node network with maximum $(n-1)$ financial assets

3 Empirics on Financial Networks

To motivate the choice of a Pareto distribution to model inequality of financial assets (and thus financial connections), in this section we first describe several empirical findings from the financial network literature on the connectivity of financial institutions through interbank lending as well as the distribution of those connections.[22] Then we present estimates fitting various datasets of individual wealth to Pareto (power-law) distributions along with their goodness of fit and tests against alternative distributions.

[22]We must rely on existing research as only aggregate lending data are publicly available. Fedwire Funds Service, a large value transfer service operate by the Federal Reserve—though not unique to federal funds lending—provides bank-level data of the US federal funds market.

3.1 Interbanking Networks

In a seminal work, Furfine (1999) developed an algorithm to parse transactions data of the federal funds market for bilateral overnight lending.[23] Summarizing interbank lending market concentration during the first quarter of 1998, Furfine finds that the top 1% of financial institutions in the federal funds market account for two thirds of all assets. They also represent 86 percent of federal funds sold and 97 percent of federal funds bought. These levels of financial market concentration are within the range of parameter estimates we test in our simulations in the next section.

Empirical estimates of various financial network structural parameters from Blasques et al. (2015) are based on data from Dutch interbank markets between 2008 and 2011.[24] Amongst the top 50 lending banks, the authors estimate a mean in-/out- degree of 1.04, with standard deviations of 1.6 and 1.84, respectively. On average, banks lend to or borrow from an average of 1.04 different banks. At the same time, they find very positively skewed in-/out- degree distributions, supporting the Pareto distribution we impose on our model.

Bech & Atalay (2010) describe the topology of the federal funds market in the US between 1997 and 2006—also using Fedwire data and the Furfine (1999) algorithm. In 2006, banks had an average in-/out- degree of 3.3 ± 0.1 for overnight interbank lending.[25] Among many other parameters describing the topology of the federal funds market, they estimate the out-degree distribution for banks on a representative day in their sample period, concluding that a power-law distribution provides the best fit with a parameter estimate of 1.76 ± 0.02. Their results lend support to our model's degree distribution parameterization, described in Section 4.2.

The aforementioned papers only consider unsecured overnight interbank lending. Bargigli et al. (2015) study both secured and unsecured lending for varying maturities, reflecting our own model more closely—which posits financial assets are secured by an individual borrower's labor income and

[23] All subsequent papers cited in this section rely on the Furfine (1999) algorithm, or adaptations of it, to generate their interbank lending data from the broader Fedwire data. An important caveat of the resulting Furfine interbank lending data are their dependence on transactions occurring through Federal Reserve balance sheets, but not the banks' own lending.

[24] Unlike the Fedwire data in Furfine (1999), the authors use TARGET2 interbank lending data (the Eurosystem equivalent of Fedwire) which specifies individual borrowing and lending institutions for indicated bilateral credit payments. The Dutch interbank lending data has also been cross-validated against Italian and Spanish interbank lending data to minimize type I errors.

[25] The authors define a directed link as going from lender to borrower. Thus their definition of a bank's out-degree corresponds to our own definition of an individual's in-degree (cash flows directed in towards the asset holder).

hence a longer maturity. The authors estimate the the in-/out- degree distributions of the Italian Interbank Network (IIN) between 2008 and 2012, and for 2012 they report power-law parameters on interval [1.8, 3.5]. A similar parameterization is applied in our model's Pareto degree distribution of individual financial assets. Their expected degrees of networks with long-term maturities are also within our range of mean degree values.

Though we abstract from financial intermediaries writ large, our interpersonal financial network framework emphasizes the latent interconnectedness of parties in a financial economy. Estimates on existing networks are therefore helpful guides for reasonable calibration.

3.2 Financial Distributions

The Pareto distribution, or power law, is typically used to estimate top shares.[26] Thus our model more accurately describes a network of top financial asset holders where we assume financial assets are Pareto distributed. According to the Survey of Consumer Finances (SCF), between 1989 and 2007 US households in the top 1% of households by net worth typically owned one third of all wealth, around 29 percent of all assets, and also nearly one third of all financial assets. The top 10% held nearly two thirds of all wealth and assets, and over 70 percent of all financial assets. The bottom 50%, however, never held more than 3 percent of financial assets or 6.7 percent of all assets (which almost entirely consisted of real estate). We argue that since top wealth holders describe the majority of financial assets, their network topology is a sufficient determinant of overall financial instability.

Given that the power-law relationship $p(x) = \Pr(X = x) = Cx^{-\gamma}$ implies $\ln p(x) = \mathit{constant} + \gamma \ln x$, the approximate linear relationship on a log-log plot suggests its absolute slope is a reasonable estimate of the parameter γ. Since Pareto (1896), power-law distributions have been traditionally estimated thusly: construct a histogram representing the frequency distribution of the variable x; plot on a log-log scale; finally, if approximately linear, estimate its slope to find the scaling parameter.[27]

[26] See Kennickell (2009) for estimates of the nonparametric wealth distribution in the US using Survey of Consumer Finances data and Vermeulen (2014) for a detailed discussion on estimating top tails in wealth distributions.

[27] In Pareto (1896), $\hat{\gamma}$ was approximately 1.5—and conjectured to be fixed.

For numerous reasons outlined in Clauset et al. (2009), the above estimation method is problematic. Instead the authors propose a maximum likelihood estimation method whereby the scaling parameter γ is estimated conditional on a correct estimate of the lower bound value for power-law behavior x_{min}—as chosen by Kolmogorov-Smirnov statistics. Following the methodology of Clauset et al. (2009) and applying it to the 1989 and 2010 Survey of Consumer Finances, we find a wide range of plausible power-law fittings for US household data on total net worth, financial assets, and total debt.[28] We repeat the exercise for comparable variables using three international datasets from the Luxembourg Wealth Study (LWS): the UK in 2007, and Australia and Italy in 2010.

Results vary by country (Table 1). The US data are the least representative of a Pareto, or power-law, distribution. Though parameter estimates are easily fitted to the data, hypothesis testing rejects a statistically significant goodness of fit between generated data and fitted data.[29] The Pareto distribution fits US financial asset data from 1989 best, though only 60 percent of comparisons between generated and fitted data fail to reject the null that they come from the same Pareto distribution. In all other sets of US data we reject the null the majority of the time. However, we also reject any alternative distributions (the exponential and lognormal, both with and without cutoff values) as good fits of the US data.[30] Fitted Pareto parameters range from 1.450 (US net worth in 2010), indicating high inequality, to 2.208 (US financial assets in 1989), indicating much lower inequality.

Data for the UK, Australia and Italy consistently fit a Pareto distribution, across all household variables. In at least 87 percent of the comparisons between generated Pareto distributions and fitted Pareto distributions we cannot reject a difference between the two. Alternative distributions are also unanimously rejected as possible models. Though the Pareto is a uniformly good fit of the LWS data, the scaling parameter estimates are much higher than for the US data, with a minimum of only 2.224 (AUS financial assets in 2010) and a maximum of 3.571 (AUS liabilities in 2010). One reason why may be that each data series is measured in local currency units, so

[28] Estimation programs are available online at http://tuvalu.santafe.edu/∼aaronc/powerlaws/.

[29] Generated data come from 2,500 randomly generated Pareto distributions simulated from our fitted parameter estimates.

[30] Using the R package `poweRlaw` , we also test against alternative poisson distributions for the US data. Using a Vuong test, also outlined in Clauset et al. (2009), we prefer a Pareto distribution against a poisson in all cases.

larger cutoff values of the power-law behaving region suggest a less skewed distribution (and higher scaling parameter) above it. Another reason could be over-sampling high-earning households in the SCF survey population.

A Pareto distribution estimates top wealth inequality in the tail of the distribution, thus our interpersonal financial network is representative of top financial asset holders and their influence on stability. Along with the empirical literature on interbank networks, our estimates of Pareto parameters for 15 different wealth series suggest that our range of calibrated γ values [1.025, 2.375], for the simulation in the proceeding section, are reasonable.

4 Simulation

4.1 Setup

In a static random network the number of nodes is fixed and links are established following some probabilistic rule. Let d_i be drawn independently from the Pareto distribution $p(d_i) = ad_i^{-\gamma}$, where γ is the Pareto, or power-law, parameter and a is a normalizing constant.[31] For example, suppose a random draw from the degree distribution yields an in-degree for individual i of 10. Ten financial assets are owned by i, each backed by the net worths of 10 different individuals. As a creditor, i is represented by a row in the adjacency matrix $\mathbf{G}$. Those 10 financial claims are randomly distributed to debtors, represented along columns, in $\mathbf{G}$, so long as $G_{ii} = 0$. In other words, the Pareto draw tells us the row sum of G_i, which is then randomly distributed in columns along row i.

If the expected in-degree of our network is $d = E[d_i]$, then, because our network size is fixed at $n = 100$, any increase in aggregate wealth $\sum d_i$ directly increases d.

One characteristic of the Pareto distribution is that its scaling parameter γ decreases in the distribution's skewness. Therefore, it is a natural inequality measure for a Pareto distribution while also directly related to top percentile shares: if a random variable is Pareto distributed, then the share going to the top q percent of the population is equal to $S(q) = (\frac{100}{q})^{\frac{1-\gamma}{\gamma}}$. The Gini index

[31] Assuming a Pareto distribution only amongst top wealth holders is trivial since our simulated network includes $n = 100$ nodes and thus no distribution within the top 1%.

can also be directly derived from the Pareto shape parameter with $GINI = (2\gamma - 1)^{-1}$ when $\gamma > \frac{1}{2}$. Each relationship illustrates that wealth inequality is decreasing in γ.

To understand the effects of the network's aggregate financial assets $\sum d_i$ and wealth inequality $\frac{1}{\gamma}$ on its stability, we generate a random network, shock one individual randomly, and then evaluate (according to the algorithm in Section A.1 of the Appendix) the total percentage of nodes in the network that have failed financially S—our measure of the network's instability. Each simulation is repeated 1,000 times for each set of parameter values with the share of failing nodes S averaged across iterations. Each iteration generates a unique graph $\mathcal{G}(N, G)$ with a network structure that conforms to an exogenously imposed financial wealth distribution and level of total wealth.[32] We follow the below procedure, adapted from Elliott et al. (2014a):

Step 1 Generate a static, directed random network $\mathbf{G}$ with parameter d_i represented by a truncated Pareto probability distribution. (The distribution is truncated to isolate the effect of γ for a given d. At each level of γ a maximum in-degree is set so that d remains constant.)

Step 2 Derive the cross-holdings matrix $\mathbf{C}$ from $\mathbf{G}$ using Equation (1).

Step 3 Calculate individuals' starting values $v_i \; \forall \; i \in n$, given an initial real asset price of $p_k = 1$, and determine failure threshold values $\underline{v}_i = \theta v_i$ for some $\theta \in (0, 1)$.

Step 4 Randomly choose an individual j to experience a negative income shock and decrease its real asset price to $\tilde{p}_j = \lambda p_j$.

Step 5 Assume all other real asset prices remain at 1 and calculate the number of nodes failing according to the algorithm in Section A.1 of the Appendix.

The set of all nodes Z_T who have failed financially, calculated at the algorithm's terminal step, yields the share of nodes in the network who have failed S. Results are reported with a graph of S against the wealth inequality parameter γ for varying levels of aggregate wealth d.

[32] Additional simulations (not reported) considered $n = 500$ and $n = 1,000$ and gave indistinguishable results. For computational ease, all simulation results are generated with $n = 100$.

4.2 Calibration

We briefly summarize the choice of exogenous parameters in our simulated static random network models. (See Table 2.)

The share of an individual's net worth that can be securitized (c) characterizes the percentage of future income flows claimed by creditors in the model. An analogous, and available, macroeconomic variable measuring the burden of liabilities is the debt service ratio (DSR), the share of an individual's income repaying debt. Aggregate estimates from Drehmann & Juselius (2012) require making several assumptions concerning average credit maturity, lending rates, and total outstanding credit. Across a panel of both advanced and developing economies, the aggregate debt-service ratio for households ranges from 5.1 percent in Italy in 2010 to 20.3 percent for Denmark.[33]

Because nodes in our model represent individuals or households who also produce, make financing decisions, and determine the output of the economy, we also consider the DSR of private non-financial firms and corporations. Looking at year 2010 again, Italy has a private non-financial firm aggregate DSR of 12.9 percent and Denmark's equals 29.5 percent. For non-financial corporations the rates are even higher in 2010: 40.6 percent in Italy and 55.5 percent in Denmark.

The Federal Reserve produces two similar aggregate DSR estimates for the US: household debt service payments and household financial obligations, both as shares of personal disposable income.[34] Financial obligations include rent payments on tenant-occupied property, auto lease payments, homeowners' insurance, and property tax payments. Thus its ratio is larger, peaking at 18.1 percent in the fourth quarter of 2007 while the DSR was only 13.1 percent in the same period. The BIS data for private non-financial firms and also corporations in the US in 2010 are 15.8 percent and 39.4 percent, respectively.

Heterogeneity of debt burdens may skew aggregate estimates, thus we examine their distributions. In 1989 and 2010 in the US, for example, top wealth holders have a greater DSR than middle portions of the wealth distribution but lower than the household average. (See Figures A.2.1 and

[33] Data are available online at http://www.bis.org/statistics/dsr.htm

[34] Data are available from FRED online.
Household debt service payments series: https://research.stlouisfed.org/fred2/series/TDSP
Household financial obligations series: https://research.stlouisfed.org/fred2/series/FODSP

A.2.2 in Section A.2 of the Appendix.) Generally, BIS household aggregate estimates are lower than averages calculated from household survey data for overlapping years.[35]

We consider $c \in [0.05, 0.5]$, which captures the full range of DSR estimates. Letting variable c take a value of 0.3 in our baseline scenario models an economy with reasonable household cash flow obligations. Higher c values are more akin to firms and corporations than individuals, but more congruent with the units of analysis in the network literature (Section 3).

In the event of a financial failure, such that $v_i < \underline{v}_i = \theta v_i$, an individual incurs bankruptcy costs or some increased economic burden as a consequence of their depressed net worth. We follow Elliott et al. (2014a) and let θ take on a range of values in [0.8, 0.98]. This provides a wide enough spectrum such that individuals are either very robust to valuation changes or incredibly sensitive.

Since the advent of the US Bankruptcy Act in 1978, the majority of consumer bankruptcy cases are filed under Chapter 7 protection, a form of bankruptcy in which assets (above some exemption threshold) are liquidated to pay off creditors of secure debt but the debtor's future income streams are untouched. For example, in 2014 approximately two thirds of all consumer bankruptcy petitions filed in US courts were under Chapter 7.[36] Our model assumes that, as in Chapter 7, financially failing individuals liquidate their remaining asset position to cover their failure costs. Because failure costs equal the value of the individual's wealth after failure in period t, or $\beta_i = v_{i,t}$, a failed individual's wealth drops to 0 after incurring bankruptcy costs and collateral wealth is lost.

Recall that, in our model, an income shock lowers an individual's human capital price, so that $\tilde{p}_i = \lambda p_i = \lambda$. A negative shock may decrease an individual's labor-earning capacity by varying amounts, depending on an individual's level of savings, the number of wage earners in a household, support systems of friends and family and other financial coping mechanisms. The human capital price decline could be very large if, for example, it was caused by some physical injury preventing a wage earner from earning any labor income through their human capital. In such an instance λ would be small. On the other hand, the income shock may be very small if earning capacity is not greatly inhibited, and so λ is large. The range of λ values tested is in $[0, 0.9]$. Whenever $\lambda < \theta$ a

[35] The distribution of household DSRs is calculated using Survey of Consumer Finance (SCF) data for the US and Luxembourg Income Study (LIS) data for France and other countries (not shown).

[36] Data are available online at http://www.uscourts.gov/statistics-reports/bapcpa-report-2014.

failure, and contagion, can occur.

Concerning historical US wealth inequality measurements, Wolff (1992) finds the top 1% of individuals approximately own as little as 19 percent of household wealth (excluding retirement wealth) in 1976 and as much as 38 percent in 1922. These translate to Pareto parameter values of 1.56 and 1.27, assuming top wealth shares are described by a power law.[37] In 1962, the first iteration of the Federal Reserve's household survey, the Survey of Consumer Finances (SCF)[38], found a Gini coefficient of 0.72 in wealth with a corresponding top 1% wealth share of 32 percent. In its second iteration in 1983, the SCF found a Gini coefficient for wealth of 0.74 (top wealth share of 31 percent). Using more recent SCF waves, Kennickell (2009) decomposes the wealth distribution. In 1989 the top 1% owned 28.3 percent of financial assets and in 2007 it owned 31.5 percent. Assuming a power law describes top wealth shares for the US in those years, the equivalent Pareto parameters are 1.38 in 1989 and 1.33 in 2007.

Our values for the Pareto parameter γ are in the interval [1.025, 2.375], which corresponds to a range of Gini coefficients from 0.9524 to 0.2667. The corresponding range of top 1% shares is from 89.4 percent to 6.95 percent. The parameter space is credible and within the range of empirical estimates of wealth, asset, and liability inequalities estimated in Section 3 and in the literature.[39]

Changes in γ also change the mean $d = E[d_i]$ of the in-degree distribution $f(d_i)$. Therefore, we must truncate the Pareto distribution in order to hold d constant as γ varies. In this manner we may isolate the distribution effect from the aggregate wealth effect. With $n = 100$, possible d values are restricted to the interval [1, 2]. For example, suppose $\gamma = 2.375$ (minimal inequality). The maximum possible d_i is 99 (it is not feasible to have $d_i \geqslant n$). When $\max\{d_i\} = 99$ and $\gamma = 2.375$, then $d = 2$ and represents an upper bound on expected in-degree values under our Pareto distribution. For each level of γ we adjust the maximum d_i accordingly.

Our baseline model calibration is the fullowing: $c = 0.3, \theta = 0.92, \beta_{i,t} = \underline{v}_{i,t}, \lambda = 0, \gamma =$

[37]Solve for γ in $S(0.01) = 100^{\frac{1-\gamma}{\gamma}}$.

[38]The earliest Federal Reserve Board wealth survey was called the Survey of Financial Characteristics of Consumers.

[39]See Vermeulen (2014), Table 8, for Pareto parameter estimates which merge Forbes billionaire data with national surveys, such as the SCF. In his broad survey of power laws in economics, Gabaix (2009) finds 1.5 to be the median estimate found for top wealth.

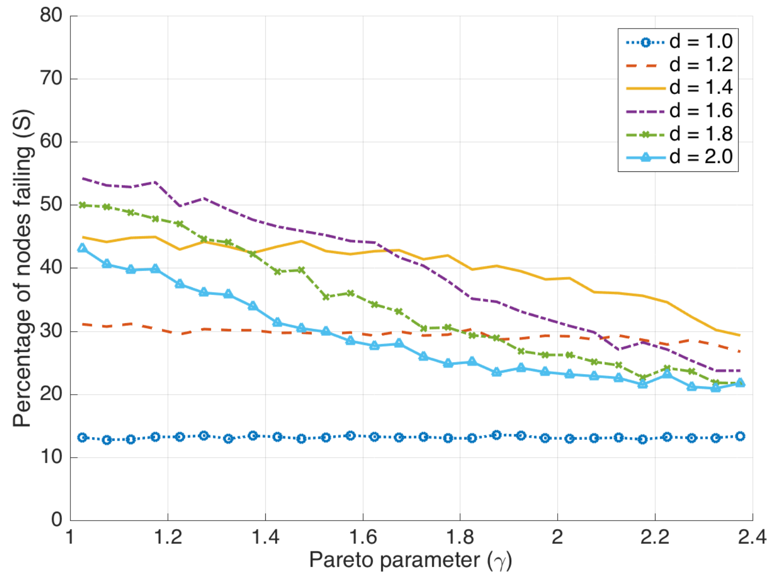


Figure 8: Baseline model

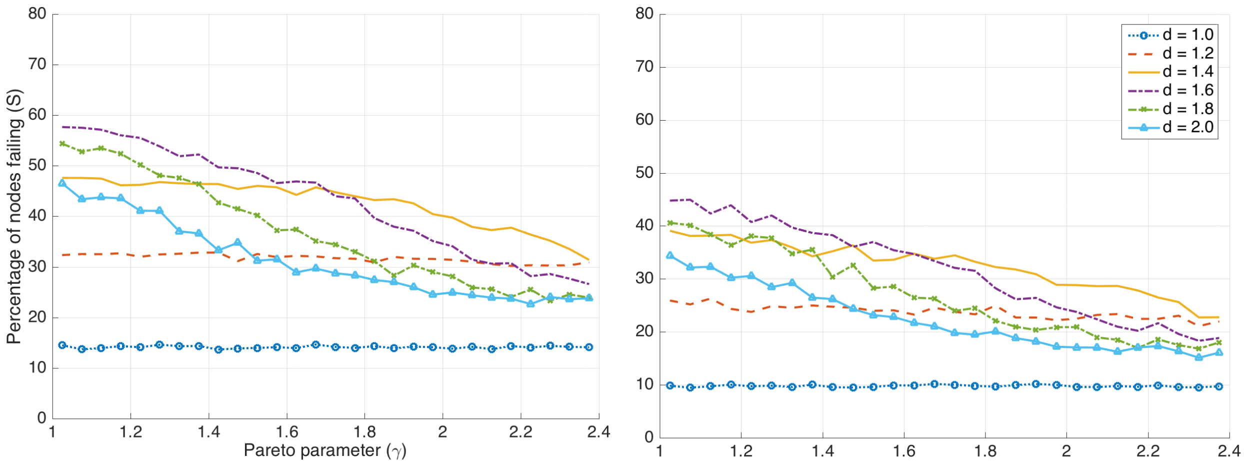
Notes: Aggregate wealth is increasing in expected in-degree d. Calibrated with $c = 0.3$, $\theta = 0.92$, and $\lambda = 0$. As γ increases wealth inequality decreases. The domain of $\gamma = [1.025, 2.375]$ corresponds to Gini coefficients of [0.952, 0.267] and top 1% wealth shares of [0.894, 0.070]. Percentage of financial failures is average of 1,000 iterations.

$[1.025, 2.375]$ and $d = [1, 2]$. The full range of calibrations is summarized in Table 2.

4.3 Results

Results from our baseline simulation (Figure 8) illustrate that as γ decreases (inequality rises), the share S of individuals in the economy failing financially increases, but only when the network is sufficiently wealthy. Wealth inequality, in other words, is destabilizing only when the economy attains a minimum level of wealth. In our baseline model this approximately occurs when $d = 1.4$. At or above this level of financial wealth, increasing wealth inequality causes greater financial contagion and therefore a greater likelihood of financial crisis. The positive contribution of inequality on instability is most pronounced when our network's wealth has an expected in-degree of 1.6, but remains significant for higher levels of wealth as well.

Unlike wealth inequality, the effect of increasing aggregate wealth on stability is notably non-monotonic. Initial increases in aggregate wealth (from $d = 1.0$ to 1.2) increase the share of financial failures but are immune to any effects from inequality. At moderate levels of aggregate wealth ($d = 1.4$) instability is higher still, but now inequality begins to have a destabilizing impact as it



(a) Poorest node ($\min\{v_i\}$) shocked.

(b) Richest node ($\max\{v_i\}$) shocked.

Figure 9: TARGETED SHOCKS

NOTES:Aggregate wealth is increasing in expected in-degree d. Calibrated with $c = 0.3$, $\theta = 0.92$, and $\lambda = 0$. As γ increases wealth inequality decreases. The domain of $\gamma = [1.025, 2.375]$ corresponds to Gini coefficients of [0.952, 0.267] and top 1% wealth shares of [0.894, 0.070]. Percentage of financial failures is average of 1,000 simulations.

goes up (γ decreases). The strongest effect of wealth inequality occurs in a moderately wealthy network ($d = 1.6$), when moving from very low wealth inequality to very high inequality roughly doubles the size of the contagion—from around 25 to over 50 percent of the network failing. Finally, at the highest levels of aggregate wealth ($d \geqslant 1.8$), inequality remains positively and significantly related to contagion, however the level, or the share of the network failing financially, is smaller than at moderate levels of wealth—nearly 20 percentage points less at some levels of inequality.

We emphasize two results revealed in our simulations:

1. wealth inequality positively increases network instability for moderate to wealthy networks;
2. and aggregate wealth has an inverted U-shaped relationship with instability—initially increasing, but then decreasing.

The network economy is therefore most unstable, or vulnerable to negative shocks, when it is both wealthier (higher d) and unequal (low γ). The model also reveals an important interaction between an economy's level of wealth inequality and total aggregate wealth, reflecting the "robust-yet-fragile" nonmonotonicity found in other network models.[40]

Financial contagion occurs independent of the node subjected to the random income shock.

[40] Gai & Kapadia (2010), Nier et al. (2007), and Elliott et al. (2014a).

Figure 9 depicts two scenarios of identically calibrated networks. In the left panel, the poorest node in the network receives the negative income shock and in the right the richest node is shocked. The level of contagion, while significant and nearly identical to our baseline model in both, is different as is the likelihood of financial crisis. When the poorest node ($\min\{v_i\}$) receives the income shock, a greater share of the network fails for both a given level of inequality and aggregate wealth than when the richest node ($\max\{v_i\}$) is shocked. This makes sense because poorer nodes have more liabilities, and thus their failure spreads to a greater number of nodes than when the richest node is shocked. The stronger effect on instability of shocking the poorest node is more muted at lower aggregate wealth levels. When the richest node is shocked, networks are more robust by over ten percentage points for the wealthiest networks ($d \geqslant 1.6$) and approximately five percentage points for the least wealthy networks ($d = 1$).

The overall pattern of our baseline model, observed when a random node is shocked, however, persists: increasing wealth inequality (decreasing γ) causes a greater share of individuals to fail in networks of at least moderate wealth while increasing the aggregate wealth (increasing d) of the network is initially destabilizing but then stabilizing.

4.3.1 Regular Graphs

To emphasize the importance of both aggregate wealth and the financial wealth distribution on network stability, we study random regular graph simulations. Regular graphs feature equal in-degrees and thus represent perfect financial asset equality in our model. The only parameters changing are c, the percentage of future cash flows owed by an individual to other claimants, and d, the in-degree of all individuals. No longer restricted by the degree distribution parameter γ, d can take on a broader set of values. Results are presented in Figure 10.

As d increases the aggregate wealth of the network increases. When $c > 0.15$, there exists a stark pattern: the share of nodes failing increases sharply when aggregate wealth is low and increasing, but quickly drops again as aggregate wealth increases beyond some level. (The particular level depends on c.) Increasing c extends the in-degree values for which instability is high, before dropping to zero in even wealthier networks. Like our models in Figures 8 and 9, the equal network displays

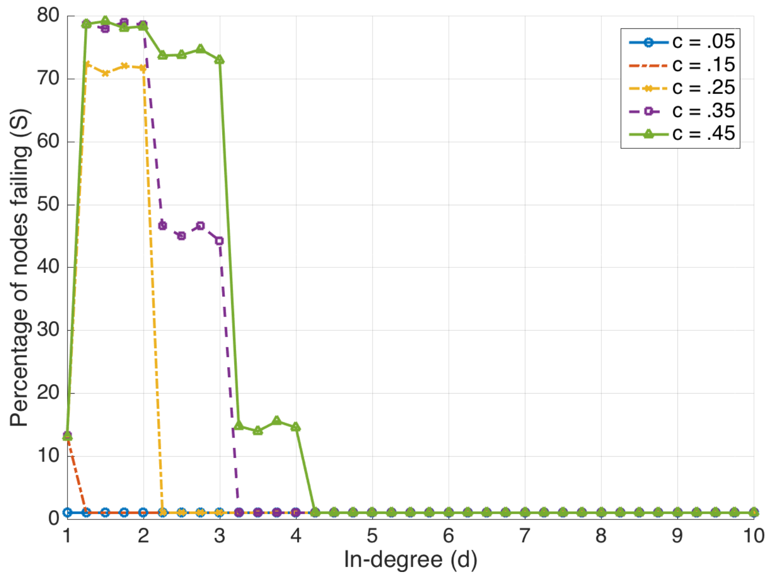


Figure 10: Regular (equal) network

Notes: Regular network contains fixed in-degree d_i for each node, hence there exists perfect wealth equality. Aggregate wealth is increasing in expected in-degree d. Calibrated with $\theta = 0.92$, and $\lambda = 0$. Percentage of financial failures is average of 1,000 iterations.

increasing instability as aggregate wealth increases from low to moderate levels, but decreasing instability as wealth increases further. Decreasing c or financialization at the intensive margin, however, also significantly lowers instability.[41]

Simulation results for the full range of calibrated parameter values described in Table 2 are presented in the Appendix, Section A.3. The model is particularly sensitive to the c parameter—as evidenced by the regular network simulations in Figure 10—as well as the θ parameter, , the measure of an individual's personal robustness under financial stress, or the economy's ability to absorb depleted cash flows on asset claims. The parameter λ, inversely proportional to the size of the income shock $\tilde{p}_i = \lambda p_i$, is nearly indiscriminate in its effect on contagion (see Figure A.3.3). So long as the condition $\lambda < \theta$ holds, an initial negative income shock will always cause at least one financial failure which will catalyze contagion within the network.

[41] The step-function-like behavior of the regular network results are due to the fact that individuals must have integer values of $d_i = d$. A rounding function in the program simply rounds up to the next integer.

5 Conclusion

Keynes once described the relationship between debtors and creditors as forming "the ultimate foundation of capitalism."[42] This paper's central goal was to examine the relationship between inequality and financial crisis by reducing the financial economy into a network of creditors and debtors who were linked through financial assets. While the income inequality literature has posited that inequality's association with debt may drive financial crises, studying the distribution of the asset side of the balance sheet helps illuminate a topographical relationship between inequality and financial crisis. In this vein, Jayadev (2013) concludes his summary of the inequality-crisis literature that "wealth/net worth may be the more critical variable, especially when financial crises are driven by asset bubbles."

The financial network model presented is a radically simplified interpretation of a financial economy, one that eliminates intermediaries and instead relies on the latent financial pathways that link individual asset and liability holders. Implicit financial links between individuals are made explicit in a directed network graph. A link indicates the presence of a financial asset, and the direction of cash flows, between individuals. It follows that changing the distribution of financial assets changes the distribution of links in the network. This model of wealth inequality and financial instability, derived from the framework of Elliott et al. (2014a), demonstrates with simulations that changes to the network topology have two main effects: first, increasing top wealth inequality, conditional on a network's overall wealth, increases instability; and second, aggregate network wealth should have an increasing and then decreasing effect on instability—measured by the share of nodes in the network who are determined to have failed financially.

An intuitive metaphor for understanding how the network attributes wealth inequality and aggregate wealth work in tandem to determine financial stability is to consider a Jenga tower, the block-building game. If each block represents a financial asset or link, then a small tower is relatively stable regardless of the distribution of the blocks. As the Jenga tower grows, however, the distribution of blocks becomes critical to its stability.

One implication of these findings is that future increases in wealth inequality (as predicted by

[42](Keynes, 1920, p. 236).

Piketty) in the US and other financially advanced economies may increase macroeconomic instability, meaning a greater likelihood of financial crisis in the event of some negative income shock. The consequences for moral hazard, systemic risk, and too-big-to-fail, among other regulatory concerns, could be great. Another broader implication is the incitement to reduce inequality for cogent economic—not simply moral—reasons. Rising inequality will always have broad welfare effects, but macroeconomic health may also be at stake.

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Tables

Table 1: Empirical Pareto Estimates

		US 1989	US 2010	UK 2007	Australia 2010	Italy 2010
net worth	$\hat{\gamma}$	1.475	1.450	2.809	2.729	2.904
	$\hat{x}_{min}$	146,468	206,670	940,162	978,558	495,000
Hypothesis testing	PL	reject	reject	fail (98)	fail (92)	fail (98)
	Alt.	reject	reject	reject	reject	reject
financial assets	$\hat{\gamma}$	2.208	1.493	3.254	2.224	2.382
	$\hat{x}_{min}$	5,102,103	184,330	788,000	495,660	59,777
	PL	fail (60)	reject	fail (98)	fail (87)	fail (98)
	Alt.	reject	reject	reject	reject	reject
liabilities	$\hat{\gamma}$	1.988	2.036	3.086	3.571	3.393
	$\hat{x}_{min}$	158,376	217,700	147,000	554,457	109,900
	PL	fail (16)	fail (6)	fail (93)	fail (98)	fail (94)
	Alt.	reject	reject	reject	reject	reject

Sources: US: Survey of Consumer Finances (SCF); UK, Australia, Italy: Luxembourg Wealth Study (LWS)

Notes: Australian, Italian, UK and US data are all in local currency units. SCF (US only) financial asset data are the total market value of financial investments and products, deposit accounts, cash and other financial assets owned by household members, including pension assets as well as life insurance. LWS (GBR, AUS, ITA) financial asset data exclude pension assets and other long-term savings. Net worth data are total assets minus total liabilities, except Italy 2010, where disposable net worth is measured. Hypothesis testing: (PL) null hypothesis of fitted power-law distribution and generated power-law distribution (using estimated parameters) being the same, by Kolmogorov-Smirnov statistic; and (Alt.) null hypothesis of fitted alternative distribution and generated alternative distribution (using estimated parameters) being the same, by Kolmogorov-Smirnov statistic. Alternative distributions tested are an exponential distribution and log normal distribution, both with and without cutoff values ($\hat{x}_{min}$). If we fail to reject a null, the percentage of 2,500 simulated fittings of generated and fitted data which fail to reject null is reported in parentheses.

Table 2: Parameter calibration for static random network simulations

Variable	Values	Source(s)
c	$[0.05, 0.5]$	Author's estimates (Section A.2), Drehmann & Juselius (2012), BIS, FRB St. Louis
θ	$[0.8, 0.98]$	Elliott et al. (2014a)
β_i	$\underline{v}_i$	UScourts.gov (Federal Caseload Statistics)
λ	[0, 0.9]	
γ	[1.025, 2.375]	Author's estimates (Section 3.2), Elliott et al. (2014b)
d	[1, 2]	Blasques et al. (2015), Elliott et al. (2014b)

Appendix

A.1 Failure Algorithm

This algorithm is used the determine the ordering of individuals who fail financially in the event of an initial income shock. It finds what Elliott et al. (2014a) refer to as the *best-case* equilibrium, i.e. there exist the fewest number of failures and highest values $v_{i,t}$ possible.

The initial financial shock occurs at period $t = 0$, changing real asset price values to $\tilde{\mathbf{p}}$. Let Z_t represent the set of financially failed individuals at period t, where $Z_0 = \emptyset$. Then for periods $t \geqslant 1$:

Step 1 Let $\mathbf{b}_{t-1}$ be a vector of failure costs with element $b_{i,t-1} = \beta_i$ if $i \in Z_{t-1}$ and 0 otherwise. By definition, $\beta_i = 0 \; \forall \; i$ at $t = 1$.

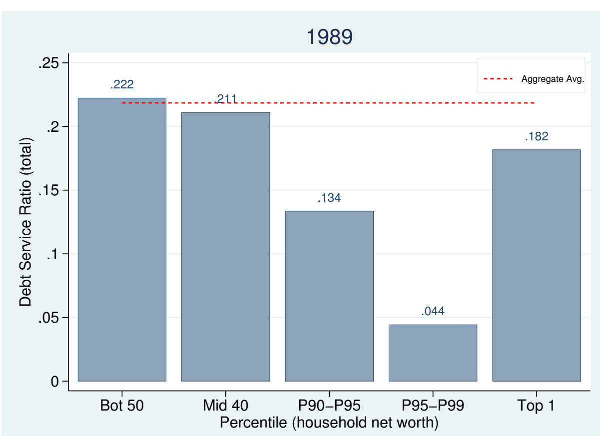
Step 2 Let Z_t be the set of all j where $v_{j,t} < 0$ and:

$$\mathbf{v_t} = \mathbf{A}(\mathbf{D}\tilde{\mathbf{p}} - \mathbf{b_{t-1}}) - \underline{\mathbf{v}}.$$

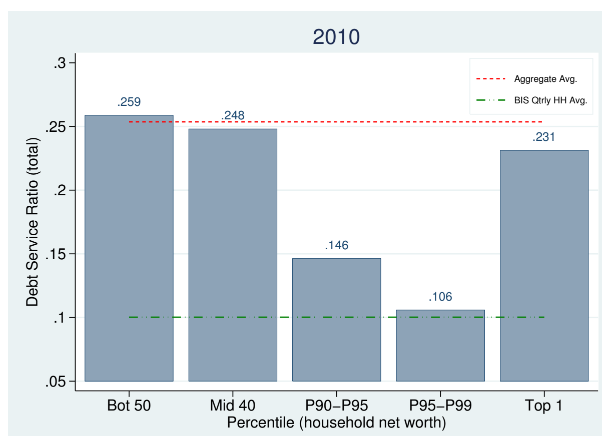
Step 3 Stop iterations if $Z_t = Z_{t-1}$, otherwise return to Step 1.

The resulting set Z_T, at terminal period T, is the corresponding set of individuals who have failed financially. An important feature is that the individuals added each period $(Z_t - Z_{t-1})$ are those individuals whose financial failures were catalyzed by the preceding set of cumulative failures. For example, Z_1 is the first group of individuals to fail and Z_2 includes the group of individuals who fail in the second period as a direct result of the individuals failing during period $t = 1$.

A.2 Distributions of Household Debt Service Ratio (DSR)

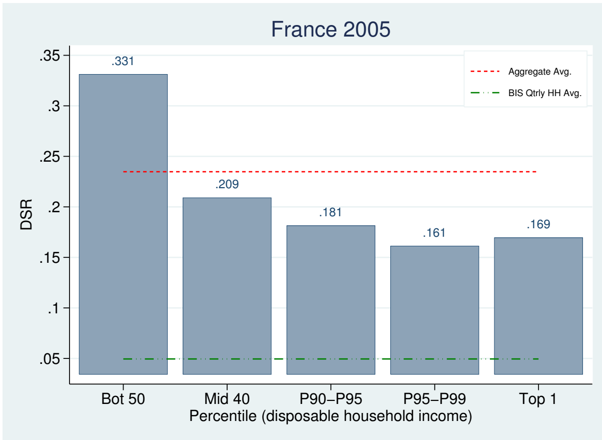


Source: Survey of Consumer Finances (SCF)



Source: Survey of Consumer Finances (SCF)

Figure A.2.1: US: 1989, 2010



SOURCE: Luxembourg Income Study (LIS)

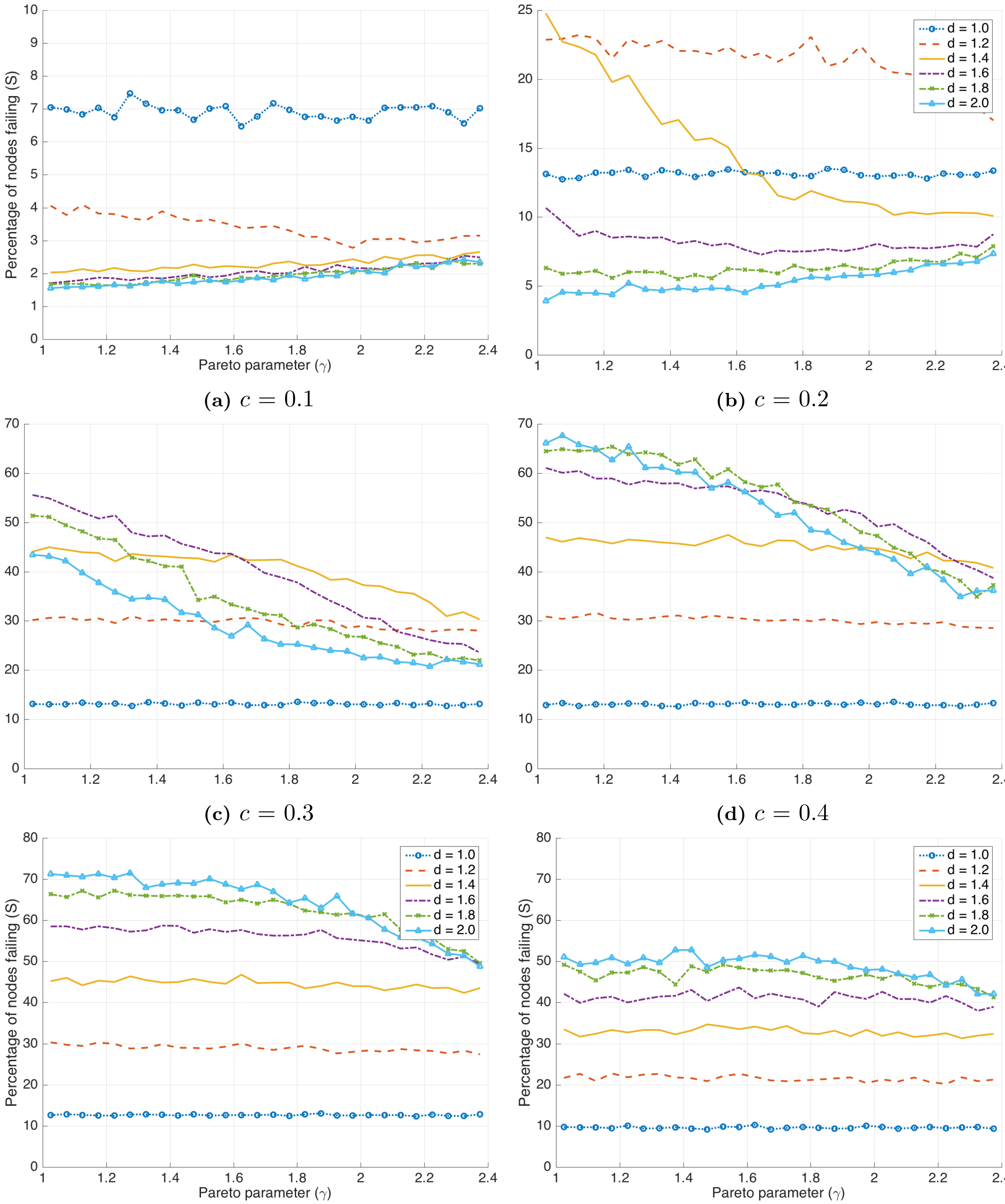
Figure A.2.2: FRANCE: 2005

A.3 Additional Parameterizations

A.3.1 Changes in parameter c

The parameter c determines the share of each node's value that can be securitized and claimed by other nodes. It measures the share of a node's cash flows that are sent to creditor nodes, an approximation of the level of financialization in the network at the intensive margin. Simulation results of the static random network for values of $c = \{0.1, 0.2, 0.3, 0.4, 0.6, 0.8\}$, and $\theta = 0.92$ and $\lambda = 0$, are shown in Figure A.3.1.

When $c = 0.1$, less than seven percent of the nodes fail under all levels of wealth inequality and aggregate wealth. Only as the share of individual wealth that can be claimed increases ($c \geqslant 0.2$) does the positive effect of wealth inequality on S begin to assert itself at moderate levels of wealth ($d \in [1.4, 1.6]$). As financialization at the intensive margin, c, continues increasing instability at the highest levels of aggregate wealth keeps increasing until we reach a maximum amount of contagion at approximately $c = 0.6$. (See Figure A.3.1.) Hereafter, instability declines as c increases. Thus the inverted U-shaped effect of financialization (at the extensive margin) on instability observed from increasing network wealth (d) appears to also take shape when financialization at the intensive margin (c) is also increased—though the downward sloping portion occurs at values of c that are well beyond any reasonably estimated debt servicing burden, commercial or private. These results broadly echo those of Drehmann & Juselius (2012) who show debt service burdens positively predict economic downturns.



(a) $c = 0.1$ **(b)** $c = 0.2$

(c) $c = 0.3$ **(d)** $c = 0.4$

(e) $c = 0.6$ **(f)** $c = 0.8$

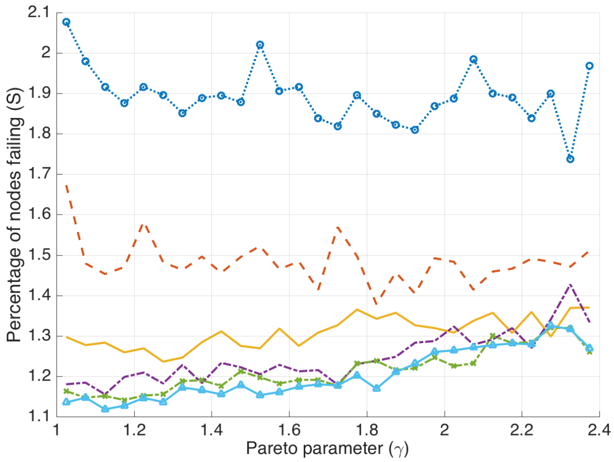
Figure A.3.1: Changes in parameter c

Notes: Pareto distributed in-degree d. Aggregate wealth is increasing in expected in-degree d. $\theta = 0.92$ and $\lambda = 0$. As γ increases wealth inequality decreases. The domain of $\gamma = [1.025, 2.375]$ corresponds to Gini coefficients of [0.952, 0.267] and top 1% wealth shares of [0.894, 0.070]. Percentage of financial failures is average of 1,000 simulations.

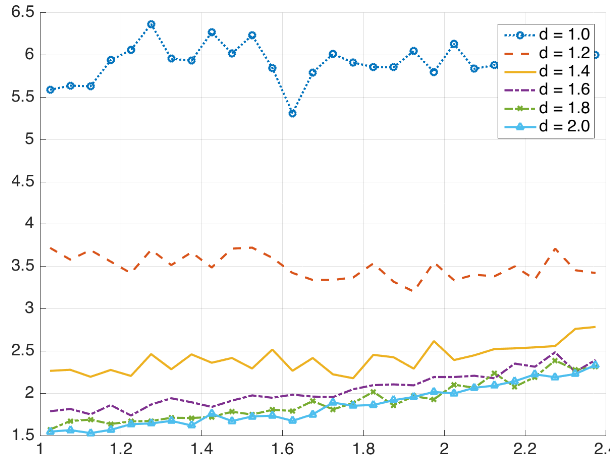
A.3.2 Changes in parameter θ

The parameter θ determines the financial robustness of an individual node in the event of an income shock. Since financial failure is predicated on $v_i < \underline{v}_i$ and $\underline{v}_i = \theta v_i$, the smaller θ is the more financially robust an individual is. An individual's financial fragility is increasing in θ. Simulation results of the static random network for values of $\theta = \{0.8, 0.84, 0.88, 0.92, 0.94, 0.98\}$, and $c = 0.3$ and $\lambda = 0$, are shown in Figure A.3.2.

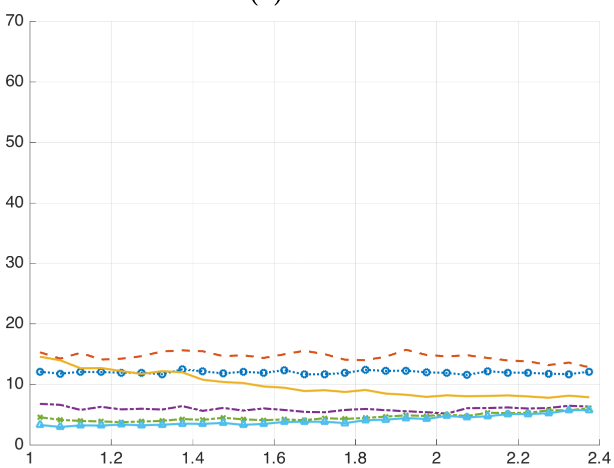
As θ increases an individual is more likely to breach $\underline{v}_i$ in the event that they personally experience an income shock or absorb failures indirectly through the dependency matrix $\mathbf{A}$. When θ is smallest (0.8), individuals are especially robust to any shock and the share of failing nodes is very low ($S < 2\%$). See Figure A.3.2. When θ increases (0.88) individual financial vulnerability increases, but contagion is still very low and unaffected by inequality. When θ is high ($\geqslant 0.92$), only a slight disturbance can tip an individual into financial failure and contagion spreads easily. The impact of wealth inequality on contagion is also strongly felt, but, again, is dependent on the network's aggregate wealth level.



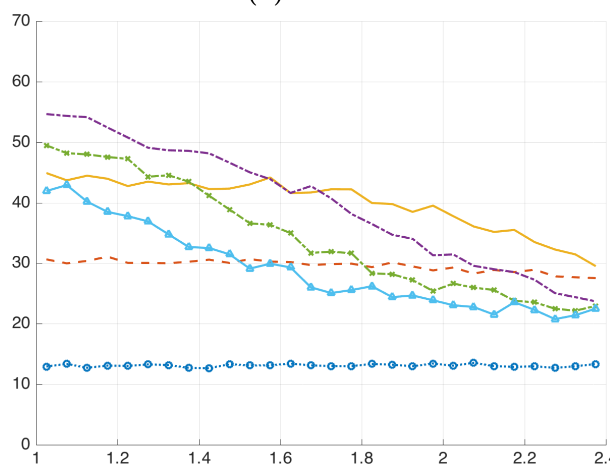
(a) $\theta = 0.8$



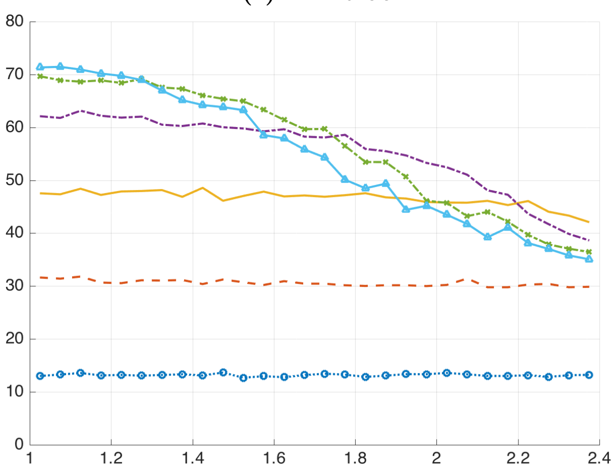
(b) $\theta = 0.84$



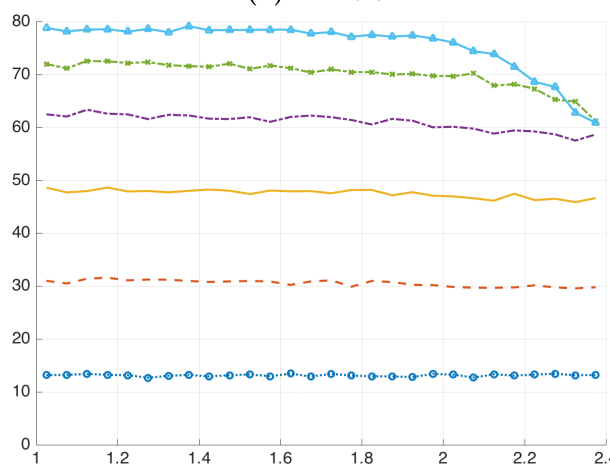
(c) $\theta = 0.88$



(d) $\theta = 0.92$



(e) $\theta = 0.94$



(f) $\theta = 0.98$

Figure A.3.2: Changes in parameter θ

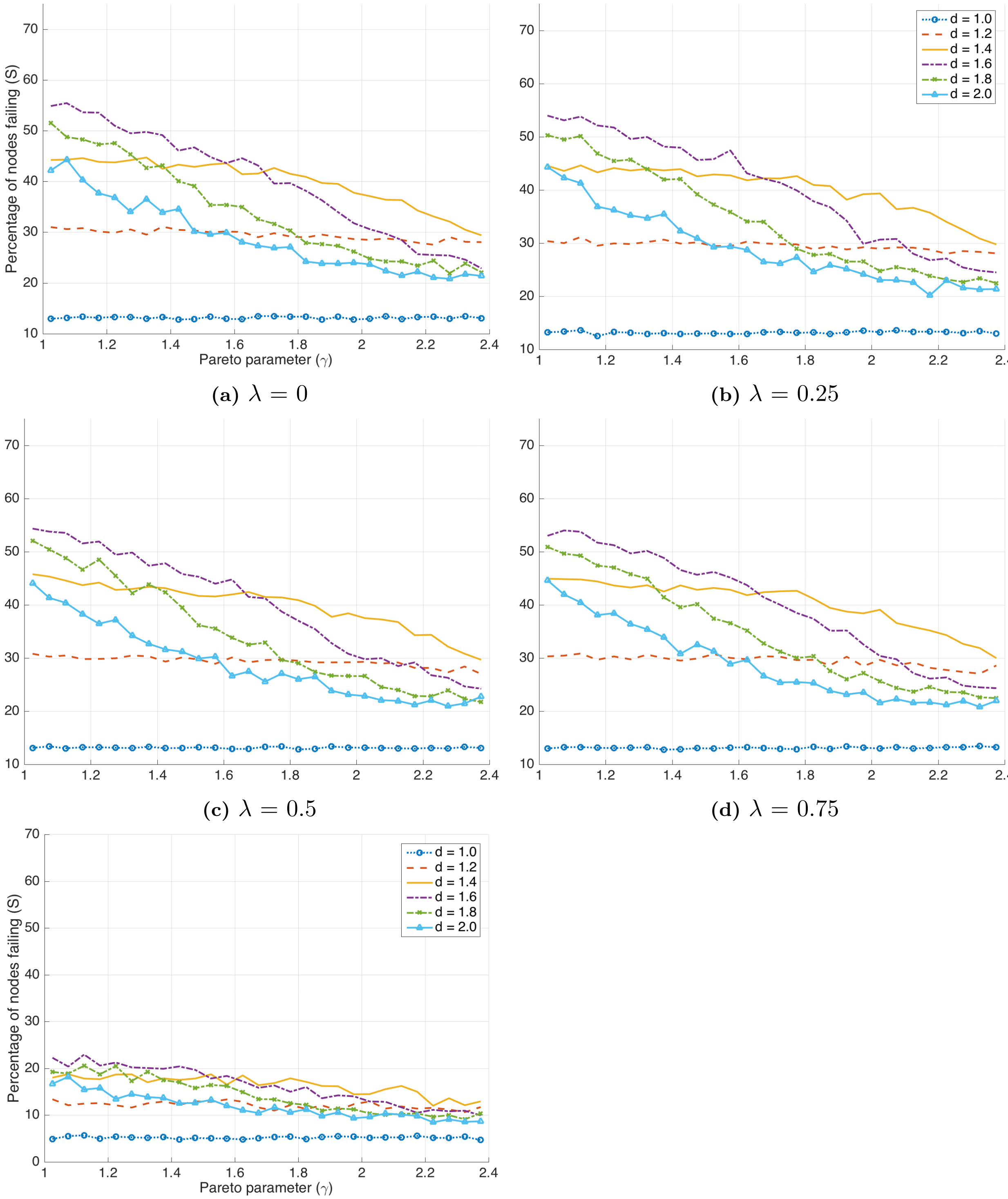
Notes: Pareto distributed in-degree d. Aggregate wealth is increasing in expected in-degree d. $c = 0.3$ and $\lambda = 0$. As γ increases wealth inequality decreases. The domain of $\gamma = [1.025, 2.375]$ corresponds to Gini coefficients of [0.952, 0.267] and top 1% wealth shares of [0.894, 0.070]. Percentage of financial failures is average of 1,000 simulations.

A.3.3 Changes in parameter λ

The parameter λ determines the magnitude of the random income shock imposed on a single node. An income shock decreases the market price of the node's real asset to $\tilde{p}_k = \lambda p_k$, where $p_k = 1$ and $\lambda \in [0, 1)$. Therefore as the magnitude of the income shock is decreasing in λ. Simulation results of the static random network for values of $\lambda = \{0, 0.25, 0.5, 0.75, 0.9\}$, and $c = 0.3$ and $\theta = 0.92$, are shown in Figure A.3.3.

Only when λ is very close to θ in value (0.9) is there any significant decrease in contagion. If $\lambda < \theta$, no matter the size of the shock the overall pattern of our simulation results holds: increasing inequality causes an increase in the percentage of nodes failing, conditional on a certain level of aggregate wealth; and increasing the aggregate wealth of the network, first increases then decreases network stability.[43]

[43] We test one counterfactual simulation in which a random individual receives a positive income shock, setting $\lambda = 2$. Because contagion is a property of net worth decreasing below some threshold value, we expect increases in net worth to have no effect on contagion. As our model would predict, the network is perfectly stable and no financial failures occur at any level of aggregate wealth. Contagion is conditional upon some negative shock.



(a) $\lambda = 0$

(b) $\lambda = 0.25$

(c) $\lambda = 0.5$

(d) $\lambda = 0.75$

(e) $\lambda = 0.9$

Figure A.3.3: Changes in parameter λ.

Notes: Pareto distributed in-degree d. Aggregate wealth is increasing in expected in-degree d. $c = 0.3$ and $\theta = 0.92$. As γ increases wealth inequality decreases. The domain of $\gamma = [1.025, 2.375]$ corresponds to Gini coefficients of [0.952, 0.267] and top 1% wealth shares of [0.894, 0.070]. Percentage of financial failures is average of 1,000 simulations.