Un test pour un problème 3D (corrigé)

On pose $\Lambda = \{(x,y) : -1 < x,y < 1\} \subset \mathbb{R}^2$, $\Omega = \Lambda \times (0,h) \subset \mathbb{R}^3$, $\partial\Omega_s = \Lambda \times \{h\}$, $\partial\Omega_i = \Lambda \times \{0\}$, $\partial\Omega_{lat} = \partial\Lambda \times (0,h)$.

- Résoudre $-\Delta \boldsymbol{u} + \boldsymbol{\nabla} p = \boldsymbol{f}$, $\operatorname{div}(\boldsymbol{u}) = 0$ dans Ω avec
- $\boldsymbol{u} = 0$ sur $\partial\Omega_{lat}$,
- $u_z = 0$,$\frac{\partial u_x}{\partial z} = \frac{\partial u_y}{\partial z} = 0$ sur $\partial\Omega_s \cup \partial\Omega_i$.

Avec

- $f_x = \left(\begin{array}{c} -[8x^2\left(y^2-1\right)^2\left(\frac{1}{4}h^4 - \frac{3}{2}hz^2 + z^3\right) - \left(x^2-1\right)^2\left(y^2-1\right)^2(3h-6z) + \\ 8y^2\left(x^2-1\right)^2\left(\frac{1}{4}h^3 - \frac{3}{2}hz^2 + z^3\right) + \\ 4\left(x^2-1\right)\left(y^2-1\right)^2\left(\frac{1}{4}h^3 - \frac{3}{2}hz^2 + z^3\right) + \\ 4\left(x^2-1\right)^2\left(y^2-1\right)\left(\frac{1}{4}h^3 - \frac{3}{2}hz^2 + z^3\right)] + \\ 3x^2y^4z^5 \end{array} \right)$

- $f_y = \left(\begin{array}{c} -[8x^2\left(y^2-1\right)^2\left(\frac{1}{4}h^4 - \frac{3}{2}hz^2 + z^3\right) - \left(x^2-1\right)^2\left(y^2-1\right)^2(3h-6z) + \\ 8y^2\left(x^2-1\right)^2\left(\frac{1}{4}h^3 - \frac{3}{2}hz^2 + z^3\right) + \\ 4\left(x^2-1\right)\left(y^2-1\right)^2\left(\frac{1}{4}h^3 - \frac{3}{2}hz^2 + z^3\right) + \\ 4\left(x^2-1\right)^2\left(y^2-1\right)\left(\frac{1}{4}h^3 - \frac{3}{2}hz^2 + z^3\right)] + \\ 4x^3y^3z^5 \end{array} \right)$

- $f_z = \left(\begin{array}{c} \left(\frac{1}{4}h^3z - \frac{1}{2}hz^3 + \frac{1}{4}z^4\right)\left(24y\left(x^2-1\right)^2 + 32xy^2\left(x^2-1\right) + 16x\left(x^2-1\right)\left(y^2-1\right)\right) + \\ \left(\frac{1}{4}h^3z - \frac{1}{2}hz^3 + \frac{1}{4}z^4\right)\left(24x\left(y^2-1\right)^2 + 32x^2y\left(y^2-1\right) + 16y\left(x^2-1\right)\left(y^2-1\right)\right) + \\ \left(3z^2 - 3hz\right)\left(4x\left(x^2-1\right)\left(y^2-1\right)^2 + 4y\left(x^2-1\right)^2\left(y^2-1\right)\right) + \\ 5x^3y^4z^4 \end{array} \right)$