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Show that $m(a+bx) = a + b \times m(x)$ (Part 1)

$m(Y) = \frac{1}{N}\sum_{i=1}^{N} y_i$ and $y_i = a + bx_i$ Using this logic ...

$$m(a+bx) = \frac{1}{N}\sum_{i=1}^{N}(a+bx_i)$$

$$m(a+bx) = \frac{1}{N}\sum_{i=1}^{N} a+bx_i = \frac{a}{N}\sum_{i=1}^{N} 1 + \frac{b}{N}\sum_{i=1}^{N} x_i = a + \frac{b}{N}\sum_{i=1}^{N} x_i$$

$$a + b\underbrace{\left(\frac{1}{N}\sum_{i=1}^{N} x_i\right)}_{= m(X)}$$

$$\boxed{\therefore a + b \times m(x) = m(a+bx) \quad \checkmark \text{ linear}}$$

Show that $cov(x, x) = s^2$ (Part 2)

$$cov(x,x) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))(x_i - m(x))$$

$$= \frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))^2 = s^2$$

$$\boxed{\therefore cov(x,x) = s^2 \quad \checkmark}$$

Show that $cov(X, a+bY) = b\,cov(X,Y)$ (Part 3)

$$cov(X, a+by) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))[(a+by_i) - (a+b\,m(Y))]$$

$$= \frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))\,b(y_i - m(Y))$$

$$= b\left(\frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))(y_i - m(Y))\right)$$ *given function

$$\boxed{cov(x, a+by) = b\,cov(x,y) \quad \checkmark}$$

Show that $cov(a+bx, a+by) = b^2 cov(X,Y)$ (part 4)

$$cov(a+bx, a+by) = \frac{1}{N}\sum_{i=1}^{N} [(a+bx_i) - (a+b\,m(x))][(a+by_i) - (a+bm(x))]$$

$$= \frac{1}{N}\sum_{i=1}^{N} b(x_i - m(x))\, b(y_i - m(y))$$

$$= b^2 \underbrace{\left(\frac{1}{N}\sum_{i=1}^{N} (x_i - m(x))(y_i - m(y))\right)}_{\text{Given}}$$

$$\boxed{cov(a+bx, a+by) = b^2 cov(x,v) \quad ✓}$$

Part 5

For $b>0$, the data is only increasing and order is preserved. This means that for the median value of x, the median value of a+bx will be equal to the median value of a+b med(x). $\boxed{\text{So yes, } med(a+bx) = a+b\,med(x) \; ✓}$

Proof: $X = \{2, 4, 6, 8, 10\}$ where $a=2$, $b=2$

$a+bx = \{6, 10, 14, 18, 22\}$

$med(a+bx) = 14$

$a+b\,med(x) = 2+2(6) = 14$

} Values match ✓

For IQR: $IQR = Q_{.75} - Q_{.25} \xrightarrow[\text{transformation}]{\text{linear}} (a+bQ_{.75}) - (a+bQ_{.25}) = b(Q_{.75} - Q_{.25})$

$\therefore IQR(a+bx) = b \cdot IQR(x)$

$$\boxed{\therefore IQR(a+bx) \neq a+b \cdot IQR(x) \text{ as } a \text{ is canceled}}$$

Part 6

Example: $x = \{2, 4, 6, 8, 10\}$

$m(x) = \frac{2+4+6+8+10}{5} = 6 \rightarrow (m(x))^2 = 36$ and $\sqrt{m(x)} = \sqrt{6} = 2.45$

$x^2 = \{4, 16, 36, 64, 100\}$

$m(x^2) = \frac{4+16+36+64+100}{5} = 44$

$$\boxed{\begin{array}{c} 44 \neq 36 \\ m(x^2) \neq (m(x))^2 \end{array}}$$

$\sqrt{x} = \{\sqrt{2}, 2, \sqrt{6}, 2\sqrt{2}, \sqrt{10}\}$

$m(\sqrt{x}) = \frac{\sqrt{2}+2+\sqrt{6}+2\sqrt{2}+\sqrt{10}}{5} = 2.37$

$$\boxed{\begin{array}{c} 2.37 \neq 2.45 \\ m(\sqrt{x}) \neq \sqrt{m(x)} \end{array}}$$