Case Study: Buckeye Power & Light Company

Executive Brief

Buckeye Power & Light Company (BP&L) is re-evaluating its coal procurement strategy ahead of their November procurement schedule. The company oversees three coal power plants, each with distinct quality and output requirements. BP&L supplies its three facilities through eleven vendors of varying size. All vendor prices and quality have been assessed based on their provided offer sheet, as well as past performance.

BP&L's coal procurement strategy has been formulated as a cost-minimizing linear program (LP), taking into account all quality and output requirements, as well as all vendor inventory available. The LP is then run through AMPL for calculations to arrive at the optimal strategy, along with sensitivity analysis for all parameters and constraints. The analysis additionally reveals certain vendors being particularly advantageous, which factor into later recommendations for renegotiations.

The solved LP includes guidance given for the optimal purchase strategy for BP&L, along with additional capacity available for the optimal basis. The optimal basis provided includes extra capacity up to around 175% for all three plants, with Steubenville up to nearly 220%. Thus, the proposed question of extra capacity between 15% and 20% is purely a risk analysis, as the entire range is within the optimal basis.

Additional cost-savings recommendations include renegotiating ash and moisture levels with particular vendors, reassessing internal ash and moisture requirements at specific plants, renegotiating price for some vendors that go unused with the optimal purchase strategy, and reaching out to the top three vendors for additional supply, if possible.

I. Introduction

Buckeye Power & Light Company (BP&L) is an energy utility based in southeastern Ohio supplying coal-based power through three steam-based plants. Each plant has separate quality requirements for the coal used, measured by Moisture, Ash, and Sulfur content. Each plant also has a separate minimum required energy to output each month, in BTUs.

BP&L's coal is supplied by eleven vendors, some of which provide coal to more than one plant. A combination of different vendors is used for each plant to meet all quality and output requirements. Currently, the utility carries an additional 20% capacity for unexpected spikes in demand. The purpose of the report is to analyze the options available to advise on the optimal amount of additional capacity to carry, as well as any other cost-saving measures.

II. Formulation

Given the objective of minimizing costs while meeting specific composition standards and other supply constraints, we have set up a linear program (LP) to optimize the solution. The decision variables will be in the form of *Xij*, representing the amount of coal to purchase (in tons) from each supplier; index *i* ranges from 1-11, in order as given in the Appendix; index *j* will be either *s*, *z*, or *a*, to reflect the destination plants of Steubenville, Zanesville, and Athens, respectively. For simplicity, some variables have been pruned if the vendor does not supply to a particular plant (such as *X1z*, as Willis Bros. does not serve Zanesville). Using the decision variables, the LP minimizes cost through the objective function, given by the sum of the amount of coal purchased from each vendor (*Xij*) times the price quoted by each vendor.

Decision Variables

x1a: Willis Bros. to Athens

x2a: MacMillan to Athens
x3a: K. Barnes to Athens
x4s: Foster & Hughes to Steubenville
x4z: Foster & Hughes to Zanesville
x4a: Foster & Hughes to Athens
x5s: Western to Steubenville
x5z: Western to Zanesville
x5a: Western to Athens
x6s: Pellham to Steubenville
x7s: McIntyre to Steubenville
x8s: Monongahela Consolidated to Steubenville
x8z: Monongahela Consolidated to Zanesville
x9z: Pope to Zanesville
x10z: Lyon Valley to Zanesville
x11z: Crescent Rock to Zanesville

Objective Function (Minimize)

30.8*x1a + 36.8*x2a + 34*x3a + 42*x4s + 41.6*x4z + 45.6*x4a + 43.92*x5s + 42.7*x5z + 41.48*x5a + 33.15*x6s + 32*x7s + 44.1*x8s + 45.36*x8z + 35*x9z + 33.12*x10z + 32.4*x11z

Constraints

The LP additionally considers all given constraints, some of which have been adjusted for analysis. For more direct sensitivity analysis, the BTU required at each plant has been adjusted back to 100%, rather than the 120% capacity as given. Additionally, the BTU requirements have been scaled by 1/2000 to change lbs. to tons, as all other units of measure for coal are given in tons:

BTUs for Steubenville (btuS): 12065*x4s + 12210*x5s + 11240*x6s + 11000*x7s + 12640*x8s >= 333333334
BTUs for Zanesville (btuZ): 12065*x4z + 12210*x5z + 12640*x8z + 12570*x9z + 11950*x10z + 12080*x11z >= 208333334
BTUs for Athens (btuA): 10980*x1a + 11590*x2a + 11550*x3a + 12065*x4a + 12210*x5a >= 250000000

Similarly, as the composition quality standards for Moisture, Ash, and Sulfur are calculated as a final percentage, the quality requirements have been scaled by the total weight purchased for each plant to give the proportion of each component as a percentage:

Moisture for Steubenville (moiS): 6.1*x4s + 6.2*x5s + 6.8*x6s + 6.3*x7s + 5.8*x8s <= 6*total weight to Steubenville (totS)

Moisture for Zanesville (moiZ): 6.1*x4z + 6.2*x5z + 5.8*x8z + 6.4*x9z + 6.8*x10z + 6.6*x11z <= 7*total weight to Zanesville (totZ)

Moisture for Athens: 6.2*x1a + 6*x2a + 6.4*x3a + 6.1*x4a + 6.2*x5a <= 7* total weight to Athens (totA)

Ash for Steubenville (ashS): 12*x4s + 14*x5s + 18*x6s + 17*x7s + 10*x8s <= 15*totS

Ash for Zanesville (ashZ): 12*x4z + 14*x5z + 10*x8z + 12*x9z + 13*x10z + 12080*x11z <= 11*totZ

Ash for Athens (ashA): 21*x1a + 20*x2a + 18*x3a + 12*x4a + 14*x5a <= 18*totA

Sulfur for Steubenville (sulS): 1*x4s + 0.9*x5s + 1.8*x6s + 2.2*x7s + 0.8*x8s <= 1*totS

Sulfur for Zanesville (sulZ): 1*x4z + 0.9*x5z + 0.8*x8z + 1*x9z + 0.9*x10z + 1.1*x11z <= 2*totZ

Sulfur for Athens (sulA): 1.2*x1a + 0.9*x2a + 1.1*x3a + 1*x4a + 0.9*x5a <= 1*totA

Finally, the individual inventory levels available at each supplier is given, to ensure that the purchase orders do not exceed actual supplies, given by weight in tons:

x1a <= 2500 Willis Bros.
x2a <= 9000 MacMillan
x3a <= 3000 K. Barnes
x4s + x4z + x4a <= 27000 Foster & Hughes
x5s + x5z + x5a <= 22500 Western
x6s <= 6000 Pellham
x7s <= 3000 McIntyre
x8s + x8z <= 30000 Monongahela Consolidated
x9z <= 3600 Pope
x10z <= 2700 Lyon Valley
x11z <= 2300 Crescent Rock

The complete LP and explicit formulation can be found in the Appendix.

III. Analysis

After formulating the problem, we now want to execute the LP and consider its output with a sensitivity analysis. To do that, we will run our formulated code using AMPL to solve the cost minimization problem subject to the given constraints. The constraints account for the BTU requirements for each facility, moisture levels, ash levels, sulfur levels, vendor inventory, and non-negativity.

Using the cplex, we obtain an objective value of 2,610,409.002. This number represents the lowest cost that satisfies the objective function with respect to the constraints implemented. Note: as the project case states, a plant must have enough BTUs every month to satisfy 100% of demand. Now, along with our objective value, we have our optimal solutions for how many tons of coal should be bought from each vendor for each of the plants available. The decision variables and values goes as follows:

For the **Athens** plant:

- Willis Bros. will supply 2500 tons at a price of $30.80/ton.
- MacMillan will supply 9000 tons at a price of $36.80/ton.
- K. Barnes will supply 3000 tons at a price of $34.00/ton.
- Western will supply 6846.03 tons at a price of $41.48/ton.

For the **Steubenville** plant:

- Foster & Hughes will supply 1712.83 tons at a price of $42.00/ton.
- Pellham will supply 4791.83 tons at a price of $33.15/ton.

- McIntyre will supply 190.314 tons at a price of $32.00/ton.

- Monongahela Consolidated will supply 20309.6 tons at a price of $44.10/ton.

For the **Zanesville** plant:

- Foster & Hughes will supply 690.4 tons at a price of $41.60/ton.
- Monongahela Consolidated will supply 9690.4 tons at a price of $45.36/ton.
- Pope will supply 3600 tons at a price of $35.00/ton.
- Lyon Valley will supply 2700 tons at a price of $33.12/ton.

After getting our optimal solution and optimal objective value, we want to use sensitivity analysis with our AMPL code to determine the shadow price, changes to the optimal solution, and to do an overall examination of the problem.

Upon using option solver cplex for sensitivity analysis, we display the following: ‘varname’, ‘var’, ‘var.rc’, ‘var.down’, ‘var.current’, and ‘var.up’. The prompts ‘varname’ and ‘var’ have previously told us the variable name and the optimal solution that goes with it. That is, the optimal amount of tons of coal that should be purchased from a vendor for a plant. The prompt ‘var.current’ tells us our current coefficients supporting the decision variables in the objective function. These coefficients happen to be the cost in dollars per ton of coal bought from each vendor to a specific plant. The ‘var.down’ and ‘var.up’ in the analysis lets us know the lowest and highest value of each coefficient that we can have in order for the solution to remain optimal at the same set of decision variable values. According to the output we came up with, the coefficient range goes as follows:

For decision variables:

- The price of coal from Willis Brothers (to Athens) can increase till $37.30/ton without changing the optimal basis.

- The price of coal from MacMillan (to Athens) can increase till $39.38/ton without changing the optimal basis.
- The price of coal from K. Barnes (to Athens) can increase till $39.23/ton without changing the optimal basis.
- The price of coal from Foster & Hughes (to Steubenville) can range between $40.67/ton and $43.23/ton without changing the optimal basis.
- The price of coal from Foster & Hughes (to Zanesville) can range between $37.40/ton and $43.61/ton without changing the optimal basis.
- The price of coal from Foster & Hughes (to Athens) must go below $40.98/ton before we change the optimal basis and start buying coal from Foster & Hughes for Athens.
- The price of coal from Western (to Steubenville) must go below $42.67/ton before we change the optimal basis and start buying coal from Western for Steubenville.
- The price of coal from Western (to Zanesville) must go below $38.63/ton before we change the optimal basis and start buying coal from Western for Zanesville.
- The price of coal from Western (to Athens) can range between $38.77/ton and $46.15/ton without changing the optimal basis.
- The price of coal from Pellham (to Steubenville) can range between $17.83/ton and $34.93/ton without changing the optimal basis.
- The price of coal from McIntyre (to Steubenville) can range between $29.32/ton and $36.53/ton without changing the optimal basis.
- The price of coal from Monongahela Consolidated (to Steubenville) can range between $31.28/ton and $45.90/ton without changing the optimal basis.

- The price of coal from Monongahela Consolidated (to Zanesville) can range between $41.79/ton and $58.18/ton without changing the optimal basis.
- The price of coal from Pope (to Zanesville) can increase till $43.41/ton without changing the optimal basis.
- The price of coal from Lyon Valley (to Zanesville) can increase till $39.44/ton without changing the optimal basis.
- The price of coal from Crescent Rock (to Zanesville) is infeasible to purchase at any positive price because of the quality of coal.

For energy constraints:

(1 Btu = 1 British Thermal Unit)

- The energy requirement in billions of Btus for the Athens plant can range from 627.556 (94.14%) to 1228.156 (184.22%) without changing the optimal basis (relative to the monthly requirement of 666.67).
- The energy requirement in billions of Btus for the Steubenville plant can range from 382.554 (91.81%) to 914.496 (219.48%) without changing the optimal basis (relative to the monthly requirement of 416.67).
- The energy requirement in billions of Btus for the Zanesville plant can range from 488.50 (97.70%) to 882.27 (176.45%) without changing the optimal basis (relative to the monthly requirement of 500).

For inventory/vendor supply constraints:

- The slack on the quantity of coal supplied by Willis Bros. is zero, which means that it is a binding constraint. The coal supplied by Willis Bros. can increase to 2785.6 tons without changing the optimal basis. The shadow price of -6.50 indicates that buying an additional ton of coal from Willis Bros. will decrease our total procurement cost by $6.50.

- The slack on the quantity of coal supplied by MacMillan is zero, which means that it is a binding constraint. The coal supplied by MacMillan can increase to 9325 tons without changing the optimal basis. The shadow price of -2.57 indicates that buying an additional ton of coal from MacMillan will decrease our total procurement cost by $2.57.
- The slack on the quantity of coal supplied by K. Barnes is zero, which means that it is a binding constraint. The coal supplied by K. Barnes can increase to 3497.94 tons without changing the optimal basis. The shadow price of -5.23 indicates that buying an additional ton of coal from K. Barnes will decrease our total procurement cost by $5.23.
- The slack on the quantity of coal supplied by Monongahela Consolidated is zero, which means that it is a binding constraint. The coal supplied by Monongahela Consolidated can increase to 31265.4 tons without changing the optimal basis. The shadow price of -1.80 indicates that buying an additional ton of coal from Monongahela Consolidated will decrease our total procurement cost by $1.80.
- The slack on the quantity of coal supplied by Pope is zero, which means that it is a binding constraint. The coal supplied by Pope can increase to 4276.58 tons without changing the optimal basis. The shadow price of -8.41 indicates that buying an additional ton of coal from Pope will decrease our total procurement cost by $8.41.
- The slack on the quantity of coal supplied by Lyon Valley is zero, which means that it is a binding constraint. The coal supplied by Lyon Valley X can increase to 3158.13 tons without changing the optimal basis. The shadow price of -6.32 indicates that buying an

additional ton of coal from Lyon Valley will decrease our total procurement cost by $6.32.

- The slack on the quantity of coal supplied by Foster & Hughes is positive, which means that is a non-binding constraint with a shadow price of zero. This quantity can decrease to 2403.23 tons without changing the optimal basis. The shadow price of zero indicates that buying an additional ton of coal from Foster & Hughes will not improve our objective value.
- The slack on the quantity of coal supplied by Western is positive, which means that is a non-binding constraint with a shadow price of zero. This quantity can decrease to 6846.03 tons without changing the optimal basis. The shadow price of zero indicates that buying an additional ton of coal from Western will not improve our objective value.
- The slack on the quantity of coal supplied by Pellham is positive, which means that is a non-binding constraint with a shadow price of zero. This quantity can decrease to 4791.93 tons without changing the optimal basis. The shadow price of zero indicates that buying an additional ton of coal from Pellham will not improve our objective value.
- The slack on the quantity of coal supplied by McIntyre is positive, which means that is a non-binding constraint with a shadow price of zero. This quantity can decrease to 190.31 tons without changing the optimal basis. The shadow price of zero indicates that buying an additional ton of coal from McIntyre will not improve our objective value.

<u>For moisture, ash, and sulfur constraints:</u>

- If we increase the moisture level for the Steubenville plant by one unit, the cost will go down by $3.14119.

- If we increase the ash level for the Zanesville plant by one unit, the cost will go down by $1.7463.
- If we increase the sulfur level for the Steubenville plant by one unit, the cost will go down by $4.69718.
- The moisture levels for the Zanesville plant and Athens plant are both zero; the ash levels for the Steubenville plant and Athens plant are both zero; the sulfur levels for the Zanesville plant and Athens plant are both zero. This implies that changes in the right hand sides of these constraints have no effect on the cost, and thus, the optimal objective value of the problem.

V. Solution/Recommendations

To decrease the cost of supplying coal to all three plants, it is recommended that the maximum levels of moisture and ash be reevaluated with the plant managers. Additionally, coal quality and price should be renegotiated with the vendors. Recommendation details below.

The Steubenville plant has a very strict maximum moisture level at 6%, which makes it very costly to supply the plant. Coal with lower levels of moisture is much more expensive. If possible, increasing the maximum moisture level to 7% at Steubenville would decrease costs significantly. This would also be in line with the two other plants that use a 7% maximum moisture level, and most vendors have coal with more than 6% moisture. If the current machinery at Steubenville can only process up to 6% moisture, then it is worth comparing the cost savings to any capital expenditures to raise the tolerable moisture level higher.

Monongahela Consolidated is the biggest supplier of coal, providing 30000 tons of coal in the current optimal state, at $44.10/ton. Monongahela Consolidated has the best quality of coal in terms of btu/lb., moisture, ash, and sulfur, which is why so much is purchased despite its high cost. Since so much coal is bought from them, even the smallest decrease in price per ton of coal would lead to a dramatic decrease in overall cost.

Western coal is great quality coal, second only to Monongahela Consolidated. The coal has high btu/lb. as well as low moisture, ash, and sulfur content. However, Western charges very high prices in comparison to its competitors, so in the current optimal state, only a small fraction of their supply is bought by the company. It would be ideal to renegotiate prices with Western down to a more reasonable price.

Crescent Rock has very good prices, but the ash level of the coal is too high (13%) to be practical to purchase for the Zanesville Plant, which requires 11% or less ash content. One possibility is to renegotiate the ash levels of coal from Crescent Rock. If they can decrease the ash level of their coal, then it would make sense for the company to purchase coal from them. Alternatively, if Zanesville increased their maximum ash content, then they could purchase the cheaper Crescent Rock coal. Zanesville's ash content maximum is much lower than the other plants, so similar cost comparisons should be made to consider capital expenditures if the current equipment cannot increase the maximum ash allowed.

From the sensitivity analysis explained in section "For inventory/vendor supply constraints" on page 12, there are possible major reductions in expense if certain vendors can increase their supply. Every extra ton produced Willis Bros., MacMillan, K. Barnes, Monongahela Consolidated, Pope, and Lyon Valley can reduce the cost per ton by $6.50, $2.57, $5.23, $1.80, $8.41, and $6.32, respectively.

Negotiating with all six suppliers may not be necessary. Instead, it would be ideal to focus on the top three suppliers: Willis Bros., Pope, and Lyon Valley. These vendors provide high quality products with reasonable prices. If they can sell more, then the company can use more of their coal and rely less on vendors like Foster & Hughes and Pelham. Foster & Hughes charges very high prices, so being able to replace that would bring significant cost savings. Pelham may be cheap, but has poor quality of coal, so for plants to meet quality requirements, the company would have to spend extra on premium coal with lower amounts of moisture, ash, and sulfur. The lack of these expenses lead to the dramatic decrease in cost per ton for Willis Bros., Pope, and Lyon Valley.

With these expenses gone, the company can cut back on costs while still meeting their requirements. Each plant will still meet the same btu/lb. and sulfur content requirements. Moisture, ash, supply, and cost will not necessarily remain the same as these are matters that can be changed and negotiated. Sensitivity analysis was done on these variables, but they are not applicable due to the flexibility of renegotiating their composition and cost. Implementing any of the suggested changes would improve the company's bottom line and minimize operating costs.

Appendix

AMPL code

```
# decision vars
# Xij, i indexed from 1 to 11 for each vendor on pg. 5 from top to bottom
#      j indexed to {s, z, a} for each destination
var x1a >= 0;   # Willis Bros.  to Athens
var x2a >= 0;   # MacMillan             to Athens
var x3a >= 0;   # K. Barnes             to Athens
var x4s >= 0;   # Foster & H     to Steubenville
var x4z >= 0;   #                       to Zanesville
var x4a >= 0;   #                       to Athens
var x5s >= 0;   # Western        to Steubenville
var x5z >= 0;   #                       to Zanesville
var x5a >= 0;   #                       to Athens
var x6s >= 0;   # Pellham        to Steubenville
var x7s >= 0;   # McIntyre       to Steubenville
var x8s >= 0;   # Monongahela C to Steubenville
var x8z >= 0;   #                       to Zanesville
var x9z >= 0;   # Pope                  to Zanesville
var x10z >= 0;  # Lyon Valley    to Zanesville
var x11z >= 0;  # Crescent Rock  to Zanesville

# Helper Vars, total weight for each plant and average cost for each plant
# not used in objective function, but for use in comparison/analysis later
var totS >= 0;
var totZ >= 0;
var totA >= 0;


# objective fn
minimize cost: 30.8*x1a + 36.8*x2a + 34*x3a + 42*x4s + 41.6*x4z + 45.6*x4a +
43.92*x5s + 42.7*x5z + 41.48*x5a + 33.15*x6s + 32*x7s + 44.1*x8s + 45.36*x8z + 35*x9z
+ 33.12*x10z + 32.4*x11z;

# constraints
subject to

# total tons purchased for each plant (helper var)
pltS: totS = x4s + x5s + x6s + x7s + x8s;
pltZ: totZ = x4z + x5z + x8z + x9z + x10z + x11z;
pltA: totA = x1a + x2a + x3a + x4a + x5a;

# BTU requirements for each facility (min)
# per Vicki's suggestion, adjusted BTU requirements back down to 100% (compared to
120% from case)
# assumes 2000 lbs per ton to convert BTU/lb to BTU/ton
# divides RHS (required BTU) by 2000 rather than each coefficient
# always rounded up to whole number (ceiling)
btuS: 12065*x4s + 12210*x5s + 11240*x6s + 11000*x7s + 12640*x8s >= 333333334;
             #800 billion / (1.2 * 2000)
btuZ: 12065*x4z + 12210*x5z + 12640*x8z + 12570*x9z + 11950*x10z + 12080*x11z >=
208333334; #500 billion / (1.2 * 2000)
```

```
btuA: 10980*x1a + 11590*x2a + 11550*x3a + 12065*x4a + 12210*x5a >= 250000000;
             #600 billion / (1.2 * 2000)

# Moisture levels (max)
# rescaled by x100
# mult RHS by total weight (rather than divide each LHS term by total weight for
proportion)
moiS: 6.1*x4s + 6.2*x5s + 6.8*x6s + 6.3*x7s + 5.8*x8s <= 6*totS;
moiZ: 6.1*x4z + 6.2*x5z + 5.8*x8z + 6.4*x9z + 6.8*x10z + 6.6*x11z <= 7*totZ;
moiA: 6.2*x1a + 6*x2a + 6.4*x3a + 6.1*x4a + 6.2*x5a <= 7*totA;

# Ash levels (max)
# rescaled by x100, mult RHS by total weight (same as Moisture)
ashS: 12*x4s + 14*x5s + 18*x6s + 17*x7s + 10*x8s <= 15*totS;
ashZ: 12*x4z + 14*x5z + 10*x8z + 12*x9z + 13*x10z + 12080*x11z <= 11*totZ;
ashA: 21*x1a + 20*x2a + 18*x3a + 12*x4a + 14*x5a <= 18*totA;

# Sulfur levels (max)
# rescaled by x100, mult RHS by total weight (same as Moisture)
sulS: 1*x4s + 0.9*x5s + 1.8*x6s + 2.2*x7s + 0.8*x8s <= 1*totS;
sulZ: 1*x4z + 0.9*x5z + 0.8*x8z + 1*x9z + 0.9*x10z + 1.1*x11z <= 2*totZ;
sulA: 1.2*x1a + 0.9*x2a + 1.1*x3a + 1*x4a + 0.9*x5a <= 1*totA;

# Inventory from each facility, in TONS (max)
inv1: x1a <= 2500;              # Willis Bros.
inv2: x2a <= 9000;              # MacMillan
inv3: x3a <= 3000;              # K. Barnes
inv4: x4s + x4z + x4a <= 27000; # Foster & H
inv5: x5s + x5z + x5a <= 22500; # Western
inv6: x6s <= 6000;              # Pellham
inv7: x7s <= 3000;              # McIntyre
inv8: x8s + x8z <= 30000;       # Monongahela C
inv9: x9z <= 3600;              # Pope
inv10: x10z <= 2700;            # Lyon Valley
inv11: x11z <= 2300;            # Crescent Rock
```

AMPL output