TNM087 – Image Processing and Analysis
Lab 3 – Filtering in the frequency domain

TASK 1 Preparation

The preparation task consists of a number of problems that should be solved using Matlab. Your answers (in Swedish or English) should be written in the document *Lab_3.1_Preparation_Answers.docx*, where you also insert the required images. To save the images you can use the MATLAB functions `imwrite` or `imsave`. Make sure to save the images in an uncompressed format, such as **.tif** or **.png**.
Don't scale the images when inserting them in the word document. Before submitting the answer document on Lisam, first save the document as **.pdf**!

For the preparation tasks you do not need to submit your m-file. However, it is strongly recommended that you save your experiments in an m-file, in case you need to go back and correct anything later. Sometimes, you can also re-use your code in later tasks.

There are a number of MATLAB functions that are useful regarding Fourier transform of an image, for example, $fft2$, $ifft2$, $fftshift$, $ifftshift$, abs and $angle$. See the lecture notes for Chapter 4 (**FÖ 4 and 5 in the course**) on **page 74** to learn how they work.

1) 2D Fourier spectrum

In this part of the task, you are going to study the Fourier spectrum of a number of test images. Our main goal in this part is to make some associations between specific component of an image and its Fourier transform. As discussed in the lecture notes for Chapter 4 (**FÖ 4 and 5 in the course**) on **page 73**, the dc term of the Fourier transform usually dominates the values of the spectrum. Therefore, to bring out the details, the **log transformation** of the spectrum is used. We recommend that you write a simple MATLAB code to find and display the log transformation of the spectrum of an image in order to do the following experiments (you do not need to submit this m-file). In the lecture notes for Chapter 4 (**FÖ 4 and 5 in the course**) on **page 74**, you can find exactly how such a MATLAB function should look like.

Read the image $characterTestPattern.tif$ into MATLAB and scale it on $[0, 1]$. Let us call this image cTP.

View the test image cTP and note especially different horizontal, vertical and diagonal lines (bars) in this test image.

Problem 1) Find the **log transformation** of the Fourier spectrum of cTP and call it $Spec1$. Insert your result in the answer document.

Observe specially the effect of vertical, horizontal and diagonal bars in cTP on its spectrum $Spec1$.

Now, shift the test image by the following MATLAB command, that circularly shifts cTP, 100 pixels in the x direction and -200 pixels in the y direction.

$$cTP\_shift = circshift(cTP, [100, -200]);$$

View cTP_shift to see how this circular shift affected the test image.

Problem 2) Find the log transformation of the Fourier spectrum of cTP_shift and call it $Spec2$. Insert your result in the answer document.

Problem 3) Are there any differences between $Spec2$ and $Spec1$? How does shift affect the spectrum of the Fourier transform?

Now, rotate the test image by the following MATLAB command, that rotates cTP 15° counterclockwise.

$$cTP\_rot = imrotate(cTP, 15, 'bicubic');$$

View cTP_rot to see how this rotation affected the test image.

Problem 4) Find the log transformation of the Fourier spectrum of cTP_rot and call it $Spec3$. Insert your result in the answer document.

Problem 5) Are there any differences between $Spec3$ and $Spec1$? How does rotation in the spatial domain affect the Fourier spectrum? (Ignore some distortions caused by the added black area around the image after rotation (cTP_rot))

Read the image $characterTestPattern\_2.tif$ into MATLAB and scale it on $[0, 1]$. Let us call this image $cTP2$. View the test image $cTP2$ and specially notice that this image is almost identical with cTP, only the vertical bars under the large **a** are eliminated.

Problem 6) Find the log transformation of the Fourier spectrum of $cTP2$ and call it $Spec4$. Insert your result in the answer document.

Problem 7) Compare $Spec4$ and $Spec1$ and explain how the elimination of vertical bars affected the spectrum. **HINT:** Look specially at the **horizontal** axis of the spectrum.

Problem 8) Explain what would happen to the spectrum if the horizontal bars were eliminated from cTP?

Read the image $characterTestPattern\_3.tif$ into MATLAB and scale it on $[0, 1]$. Let us call this image $cTP3$. View the test image $cTP3$ and specially notice that this image is almost identical with cTP, only the diagonal bars at the bottom-left corner are eliminated.

Problem 9) Find the log transformation of the Fourier spectrum of $cTP3$ and call it $Spec5$. Insert your result in the answer document.

Problem 10) Compare $Spec5$ and $Spec1$ and explain how the elimination of diagonal bars affected the spectrum. **HINT:** Look specially at the diagonal axes of the spectrum.

2) Period and Frequency

In this part of this task you are going to study the relationship between the period in the spatial domain and the frequency in the Fourier domain.

Read the image $verticalbars\_2.tif$ into MATLAB and call it $v2$. This image is binary of logical type, meaning that it contains either 0 or 1. Thus, don't divide it by 255 in MATLAB. The index 2 in this file name indicates the period of the vertical bars, which is $P = 2$. View $v2$ and notice that each black/white stripe (bar) is 1 pixel wide. One-pixel-wide stripes (giving the period of $P = 2$) are the narrowest possible in a digital image. Therefore, the period will be the smallest possible period ($P = 2$) which implies the highest possible frequency. This means that the peaks will appear at the furthest point on each side of the center on the horizontal axis having the frequency of $f = \frac{1}{P} = \frac{1}{2} = 0.5$ cycles/pixel.

Find the **log transformation** of the Fourier spectrum of $v2$ and call it $Spec6$. You don't need to insert your result. Have a close look at $Spec6$; there should be three dominant peaks in the spectrum. One of them is the dc-term in the center of the spectrum. The other two should appear at the furthest point on each side of the horizontal axis. **Since this image is of even size, you only see one of these two at the left.**

HINT: See also assignment 10 in "lektion 3".

Problem 11) Where would these three dominant peaks appear if $v2$ is transposed, i.e. if the vertical bars become horizontal?

Read the image $verticalbars\_4.tif$ into MATLAB and call it $v4$. The image is binary of logical type. The index 4 in this file name indicates the period of the vertical bars, which is $P = 4$.

Problem 12) What is the frequency of these stripes? Where would the three dominant peaks in the spectrum for this image appear?

As you might have noticed, these images are 300×300 pixels. If we now increase the period to $P = 300$, the image will only consist of one white and one black bar (each 150 pixels wide).

Problem 13) What is the frequency of these stripes? Where would the three most dominant peaks in the spectrum for this image appear?

3) The importance of the spectrum and the phase angle

In this part of this task you are going to study the importance of the spectrum and the phase angle by exchanging the spectra and the phase angles of two different images.

The MATLAB functions $abs(F)$ and $angle(F)$ can be used to obtain the spectrum and the phase angle of the Fourier transform image F.

If you want to combine the spectrum $Spec$ and the phase angle Ang to obtain the image in the spatial domain you can use the following command:

$$real(ifft2(Spec.* \exp(i * Ang)))$$

Make sure that you understand how the above command works.

Read the two Einstein images $Einstein1.jpg$ and $Einstein2.jpg$ into MATLAB, scale them on $[0,1]$, and call them $E1$ and $E2$, respectively.

Problem 14) Take the Fourier spectrum of $E1$ and the Fourier phase angle of $E2$ and combine them to get an image in the spatial domain. Call your result $E1\_E2$ and insert it in the answer document.

Problem 15) Take the Fourier spectrum of $E2$ and the Fourier phase angle of $E1$ and combine them to get an image in the spatial domain. Call your result $E2\_E1$ and insert it in the answer document.

Problem 16) Is the spectrum or the phase angle that has more effect on the structure of an image based on your visual analysis of the above results?