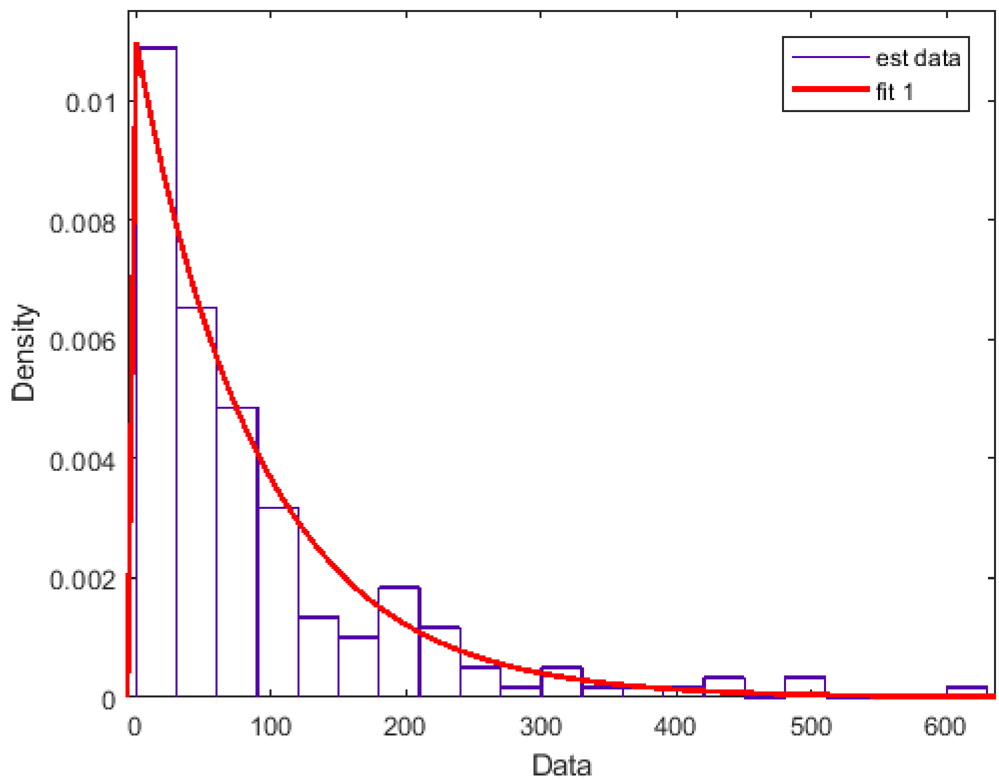
Engineering Maths EMAT 30007, Parameter Estimation(2)
Ksenia Shalonova

1 Evaluating parameters in exponential distribution using MLE (Maximum Likelihood Estimate):

(a) To estimate the *m*ean of the exponential distribution in MATLAB given sample data you can use either `expfit` command. Download data from AIRCRAFT_COND.xlsx from Blackboard. The command `expfit` uses MLE for parameter estimation.

```
>>est=xlsread('aircraft_cond.xlsx');% read xlsx file
>>est=est(:); % convert matrix to vector
>>id1 = find(isnan(est)); % find empty elements in the vector
>>est([id1]) = [] % delete empty elements - now vector contains 199 non-empty values

>>[muhat,muci] = expfit(est) % point estimate for mean (muhat) and confidence interval (muci)
>>[muhat,muci] = expfit(est, 0.01) % estimates with 99% confidence interval
```




```
Distribution:    Exponential
Log likelihood:  -1096.49
Domain:          0 <= y < Inf
Mean:            90.9196
Variance:        8266.37

Parameter  Estimate  Std. Err.
mu         90.9196   6.44512
```

(b) To estimate the *m*ean of the exponential distribution in MATLAB given sample data you can use a more general command `mle`. You should get exactly the same results as with `expfit` command.

```
>>[phat,pci] = mle('exp',est)
```

(c) **IMPORTANT** Do you get the same estimate and confidence interval as in the lecture? What parameter was estimated in the lecture?

(d) How can we get the point estimate for $\bar{\lambda}$ given the point estimate for its inverse $\bar{x}$? This task seems to be rather straight forward.

(e) How to estimate the confidence interval for λ. The first step is to work out the `standard error`. In the lecture we used the asymptotic formula for the standard error of maximum likelihood estimators.

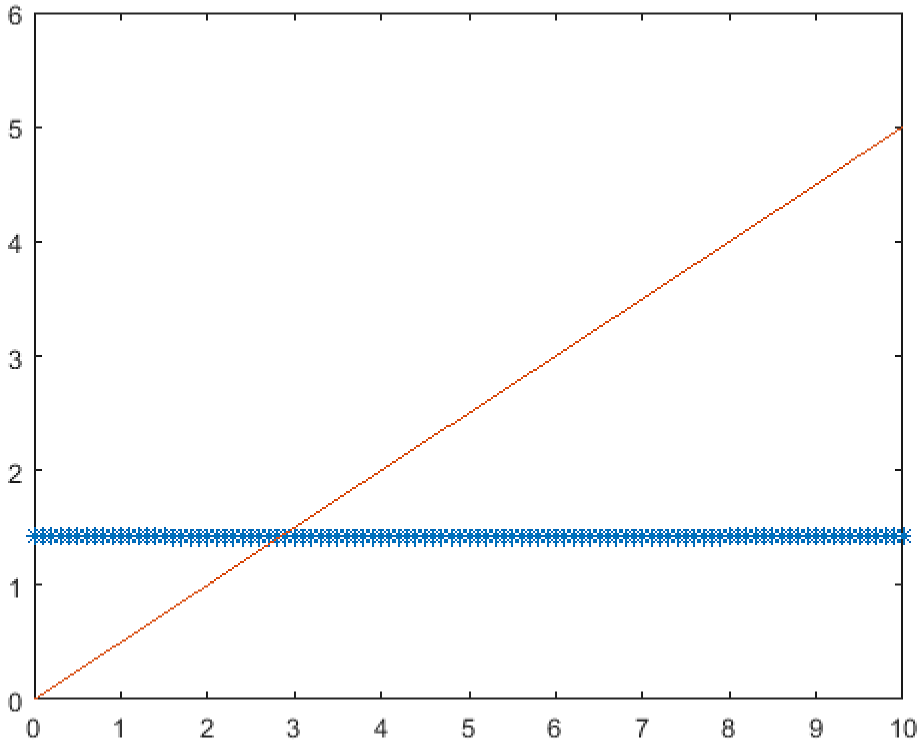
(f) An approximate 95% confidence interval for λ can be found using the asymptotic normality of the maximum likelihood estimator.

2 Hazard rate in Weibull distribution and Exponential distribution:

(a) Manufactured items usually fail at some time after they start to be used, and biological entities also have limited lifetimes. The `survival function` (probability of surviving until a particular time) is $R(t) = 1-F(t)$. The `hazard rate` function (failure rate) is worked out by the formula: $h(t) = \frac{f(t)}{1-F(t)}$ or $h(t) = \frac{f(t)}{R(t)}$ where $f(t)$ and $F(t)$ are `pdf` and `cdf` of the distribution. The hazard function describes how an items ages where t affects its risk of failure. This constant hazard function in the exponential distribution corresponds to the the Poisson processes without memory, i.e. the chance of failing does not depend on what happened before and and how long the item has already survived.

```
>>t = 0:0.1:10;
>>h_e = exppdf(t,0.7)./(1-expcdf(t,0.7));
>>h_w = wblpdf(t,2,2)./(1-wblcdf(t,2,2));
>>plot(t,h_e,'*');
>>hold on
>>plot(t,h_w,'-');
```

The graph shows the hazard rate for exponential (asterisks) and Weibull (solid line) distributions. The Weibull hazard rate increases with age, whereas the exponential hazard rate stays the same. What was α (or shape) parameter in the Weibull distribution?



(b) Now visualise on the graph the hazard rate for Weibull distribution by changing its shape parameter: (1) $\alpha = 0.5$ and (2) $\alpha = 1$. What happens with the hazard function? *Note that in MatLab the shape parameter in Weibull distribution is the last parameter.*

(c) **Weibull distribution: hydraulic pumps**. Download file "HYDPUMP.xlsx" from Blackboard. In order to evaluate the performance of hydraulic pumps, 20 pumps were tested and the number of pumps still running at the end of each week were recorded for a period of 6 weeks. Assume that the lifetime of pumps follow a Weibull distribution.

Work out:

- 95% confidence interval for the shape parameter
- 95% confidence interval for the scale parameter
- use the estimates to find the probability that the pump fail before 3 weeks $(1 - R(t))$
- estimate the reliability of the pump ($R(t)$ function) at $t = 4$ weeks
- find `hazard rate` and plot it on the graph
- compute `hazard rate` at $t = 5$ and interpret its value

(d) **Weibull distribution: bearings**. Download file "BEARINGS.xlsx" from Blackboard. The Weibull distribution can be applied to the life lengths of a sample of $n = 138$ roller bearings. The Excel table gives the number of bearings still in operation at the end of each 100-hour period until all bearings failed.

Work out:

- 99% confidence interval for the shape parameter
- 99% confidence interval for the scale parameter
- estimate the reliability of the bearings at $t = 300$ hours
- estimate the probability that a roller bearing will fail before 300 hours
- find `hazard rate` and plot it on the graph

(e) **Weibull distribution: computer memory chips**. Download file "MEMCHIP.xlsx" from Blackboard. Assume that the lifetime (in years) of a memory chip has a Weibull failure time distribution. Fifty chips were placed on test and the number of survivors was recorded at the end of each year, for a period of 8 years (.xlsx file). Work out:

- 95% confidence interval for the shape parameter
- 95% confidence interval for the scale parameter
- use the estimates to find the probability that the chip fail before 5 years
- estimate the reliability of the memory chip at $t = 7$ years
- find `hazard rate` and plot it on the graph
- compute `hazard rate` at $t = 4$ and interpret its value

(f) Suppose the failure time distribution for a product can be approximated using a normal distribution with $\mu = 3$ and $\sigma = 1.5$.

- find `hazard rate` and plot it on the graph

(g) Can you make up data that will give a decreasing `hazard rate`?