Engineering Maths EMAT 30007, Hypothesis Testing (2)

Ksenia Shalonova

1 Likelihood ratio test for the exponential distribution:

(a) Download file DRUGTEST.xls from Blackboard. The file provides two samples: (1) clinical records that give the survival time in months from diagnosis of 30 sufferers from a certain disease and (2) survival times of 21 sufferers in a clinical trial of a new drug treatment. Assume that survival times are exponentially distributed. Perform a likelihood ratio test for whether the death rate is different for those getting the new drug.

- What is H_0 (corresponds to small model) and H_A (corresponds to big model)?
  Answer: H_0 :
- Find the **maximum possible value for the log-likelihood** of the **small model** M_s.

  $\hat{\lambda} = \frac{n}{\sum x_i} = \frac{51}{405.35} = 0.1258$ - MLE estimate
  $l(M_S) = l(\hat{\lambda}) = -156.72$ - log-likelihood

  One of the possible ways of getting the value is shown below.
  ```
  >> pd = fitdist(drud_and_no_drug,'Exponential');
  %NlogL specifies the negative log-likelihood for input data used to fit a distribution
  >> pd.NLogL
  ```

  You may need to convert row vector to column vector by using the following command:drud_and_no_drug=drud
  Notice: you have to swap the sign in you final answer.

- Find the **maximum possible value for the log-likelihood** of the **big model** M_B

  The log-likelihood function for M_B:
  $l(\lambda_{no\_drug}, \lambda_{drug}) = (30log\lambda_{no\_drug} - (\sum x_{no\_drug})\lambda_{no\_drug}) + (21log\lambda_{drug} - (\sum x_{drug})\lambda_{drug})$
  The maximum possible value for the log-likelihood:
  $l(M_B) = l(\lambda_{no\_drug}, \lambda_{drug}) = -154.77$

  ```
  >> pd = fitdist(no_drug,'Exponential');
  >> pd.NLogL
  >> pd = fitdist(drug,'Exponential');
  >> pd.NLogL
  ```
  Notice: you have to swap the sign again in you final answer.

- Perform **likelihood ratio test.** You can use MATLAB `lratiotest` function to get the test statistic χ^2 with the corresponding p-value. The the first two arguments are the loglikelihood maximums for two models and the third argument is the degrees of freedom for a chi-squared distribution.
  ```
  >> [h,pValue,stat] = lratiotest(-154.77,-156.72,1)
  ```
  Why is the is value 1 used for degrees of freedom? You may use a chi-squared distribution explicitly (instead of the function `lratiotest`) in order to get the same answer: $\chi^2 = 2(l(M_B) - l(M_S)) = 3.906$.
  What is the upper tail probability above 3.906?
- Interpret the result of the likelihood ratio test.

2 Testing the difference between two population means: independent samples

MATLAB ttest2(x,y) function returns a test decision for the null hypothesis that the data in vectors x and y comes from independent random samples from normal distributions with equal means and equal but unknown variances, using the two-sample t-test. The alternative hypothesis is that the data in x and y comes from populations with unequal means. The result h is 1 if the test rejects the null hypothesis at the 5% significance level (default parameter), and 0 otherwise.

(a) Download file SILICA.xls file from Blackboard. There was a study of the impact of calcium and gypsum on the flotation properties of silica in water. Fifty solutions of deionized water were prepared both with and without calcium/gypsum, and the level of flotation of silica was measured using a variable called 'zeta potential' (measured in millivolts). Carry out a test of hypothesis to compare the mean zeta potential values of the two types of solutions. Use $\alpha = 0.1$.

- Is it a two-tailed or one-tailed test? Write down H_0 and H_A.
- Can you conclude that the addition of calcium/gypsum to the solution impacts silica flotation level?

  ```
  >> [h,p,ci,stats] = ttest2(x1,x2, 'Alpha',0.01) %
  ```
- Now perform the same test using z-statistic and the rejection region (see lecture 4 notes). Do you get the same result? Why?

(b) Design engineers want to know if you may be more likely to purchase a vice product when your arm is flexed (when you carry a shopping bag) than when your arm is extended (when pushing a shopping cart). The researchers recruited 22 consumers. Half of the consumers were told to put their arm in a flex position and the other half werre told to put their arm in the extended position. Condumers were offered several choices between a vice and a virtue (e.g., movie ticket vs. shopping coupon) and a choice score (scale of 0 to 100) was determined for each. Higher scores indicated a greater preference for vice options. The average choice score for consumers with a flexed arm was 59, and the average for consumers with an extended arm was 43.

- In order to answer the question whether this experiment supports the researcher's theory, what will be H_0 and H_A? Is it a two-tailed or one-tailed test?
- Suppose the standard deviation of the choices scores for the flexed arm and extended arm are 4 and 2, respectively. Answer the question above by conducting hypothesis testing and use the significance level $\alpha = 0.05$.
- Now suppose the standard deviation of the choices scores for the flexed arm and extended arm are 10 and 15, respectively. Answer the question above by using the significance level $\alpha = 0.05$.

3 Testing the difference between two population means: matched pairs

MATLAB t̂test(x,y) function can be used for to test the null hypothesis that the pairwise difference between data vectors x and y has a mean equal to zero.

(a) Download file SHALLOW.xls from Blackboard. The table provides actual settlement values (in millimeters) for a sample of 13 structures built on a shallow foundation. These value were compared to settlement predictions made using a formula that accounts for dimension, rigidity etc. Test the hypothesis of no difference between the mean actual and mean predicted settlement values.

- State the null and alternative hypothesis.
- Interpret p-value (use $\alpha = 0.05$).

4 Proportions????