Engineering Maths EMAT 30007, Hypothesis Testing (2)

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1 Likelihood ratio test for the exponential distribution:

(a) Download file DRUGTEST.xls from Blackboard. The file provides two samples: (1) clinical records that give the survival time in months from diagnosis of 30 sufferers from a certain disease and (2) survival times of 21 sufferers in a clinical trial of a new drug treatment. Assume that survival times are exponentially distributed. Perform a likelihood ratio test for whether the death rate is different for those getting the new drug.

- What is H_0 (corresponds to small model) and H_A (corresponds to big model)?

- Find the **maximum possible value for the log-likelihood** of the **small model** M_s.

  $\hat{\lambda} = \frac{n}{\sum x_i} = \frac{51}{405.35} = 0.1258$ - MLE estimate
  $l(M_S) = l(\hat{\lambda}) = -156.72$ - log-likelihood

  One of the possible ways of getting the value is shown below.

  ```
  >> pd = fitdist(drud_and_no_drug,'Exponential');
  %NlogL specifies the negative log-likelihood for input data used to fit a distribution
  >> pd.NLogL
  ```

  You may need to convert row vector to column vector by using the following command:drud_and_no_drug=drud
  Notice: you have to swap the sign in you final answer.

- Find the **maximum possible value for the log-likelihood** of the **big model** M_B

  The log-likelihood function for M_B:
  $l(\lambda_{no\_drug}, \lambda_{drug}) = (30log\lambda_{no\_drug} - (\sum x_{no\_drug})\lambda_{no\_drug}) + (21log\lambda_{drug} - (\sum x_{drug})\lambda_{drug})$
  The maximum possible value for the log-likelihood:
  $l(M_B) = l(\lambda_{no\_drug}, \lambda_{drug}) = -154.77$

  ```
  >> pd_1 = fitdist(no_drug,'Exponential');
  >> pd_1.NLogL
  >> pd_2 = fitdist(drug,'Exponential');
  >> pd_2.NLogL
  ```

  Notice: you have to swap the sign again in you final answer.

- Perform **likelihood ratio test.** You can use MATLAB `lratiotest` function to get the test statistic χ^2 with the corresponding p-value. The the first two arguments are the loglikelihood maximums for two models and the third argument is the degrees of freedom for a chi-squared distribution.

  ```
  >> [h,pValue,stat] = lratiotest(-154.77,-156.72,1)
  ```

  Why is the is value 1 used for degrees of freedom? You may use a chi-squared distribution explicitly (instead of the function `lratiotest`) in order to get the same answer: $\chi^2 = 2(l(M_B) - l(M_S)) = 3.906$.
  What is the upper tail probability above 3.906?

- Interpret the result of the likelihood ratio test.

2 Testing the difference between two population means: independent samples

MATLAB ttest2(x,y) function returns a test decision for the null hypothesis that the data in vectors x and y comes from independent random samples from normal distributions with equal means and equal but unknown variances, using the two-sample t-test. The alternative hypothesis is that the data in x and y comes from populations with unequal means. The result h is 1 if the test rejects the null hypothesis at the 5% significance level (default parameter), and 0 otherwise.

(a) Download file SILICA.xls file from Blackboard. There was a study of the impact of calcium and gypsum on the flotation properties of silica in water. Fifty solutions of deionized water were prepared both with and without calcium/gypsum, and the level of flotation of silica was measured using a variable called 'zeta potential' (measured in millivolts). Carry out a test of hypothesis to compare the mean zeta potential values of the two types of solutions. Use $\alpha = 0.1$.

- Is it a two-tailed or one-tailed test? Write down H_0 and H_A.
- Can you conclude that the addition of calcium/gypsum to the solution impacts silica flotation level?

  ```
  >> [h,p,ci,stats] = ttest2(x1,x2, 'Alpha',0.01) %
  ```
- Now perform the same test using z-statistic and the rejection region (see lecture 4 notes). Do you get the same result? Why?

(b) Design engineers want to know if you may be more likely to purchase a vice product when your arm is flexed (when you carry a shopping bag) than when your arm is extended (when pushing a shopping cart). The researchers recruited 22 consumers. Half of the consumers were told to put their arm in a flex position and the other half werre told to put their arm in the extended position. Condumers were offered several choices between a vice and a virtue (e.g., movie ticket vs. shopping coupon) and a choice score (scale of 0 to 100) was determined for each. Higher scores indicated a greater preference for vice options. The average choice score for consumers with a flexed arm was 59, and the average for consumers with an extended arm was 43.

- In order to answer the question whether this experiment supports the researcher's theory, what will be H_0 and H_A? Is it a two-tailed or one-tailed test?
- Suppose the standard deviation of the choices scores for the flexed arm and extended arm are 4 and 2, respectively. Answer the question above by conducting hypothesis testing and use the significance level $\alpha = 0.05$.
- Now suppose the standard deviation of the choices scores for the flexed arm and extended arm are 10 and 15, respectively. Answer the question above by using the significance level $\alpha = 0.05$.

3 Testing the difference between two population means: matched pairs

MATLAB ttest(x,y) function can be used for to test the null hypothesis that the pairwise difference between data vectors x and y has a mean equal to zero.

(a) Download file SHALLOW.xls from Blackboard. The table provides actual settlement values (in millimeters) for a sample of 13 structures built on a shallow foundation. These value were compared to settlement predictions made using a formula that accounts for dimension, rigidity etc. Test the hypothesis of no difference between the mean actual and mean predicted settlement values.

- State the null and alternative hypothesis.
- Interpret p-value (use $\alpha = 0.05$).

4 Testing the difference between two population proportions

(a) When you sign in to your Facebook account, you are granted access to more than1 million relying party (RP) websites. RP websites were categorized as server-flow or client-flow websites. Of the 40 server-flow sites studied, 20 were found to be vulnerable to impersonation attacks. Of he 54 clien-flow sites, 41 were found to be vulnerable to impersonation attacks. Do these results indicate that a client-flow website is more likely to be vulnerable to an attack than a server-flow website? Test using $\alpha = 0.01$.