EMAT30007 Applied Statistics

Lecture 11: revision and further models

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Revision 1

Simple linear regression and multiple linear regression: you will NOT need to perform any calculations, but rather understand and interpret MATLAB outputs

a) Recall the structure of simple and multiple linear regression models (LMs) and be able to use and interpret (1) alternative representations (such as algebraic notation, matrix notation and random variable notation) and (2) residual plots.
b) Understand the assumptions underlying both linear regression and multiple linear regression models.
c) Estimate parameters (MLE) and perform hypothesis testing (AIC, BIC, likelihood, R2).
d) Model selection.

Model building: you will NOT need to perform any calculations, but rather understand and interpret MATLAB outputs

a) Understand how qualitative and quantitative predictors can be combined in models.
b) Be able to interpret non-linear relationships between variables using polynomials of predictors.

Revision 2

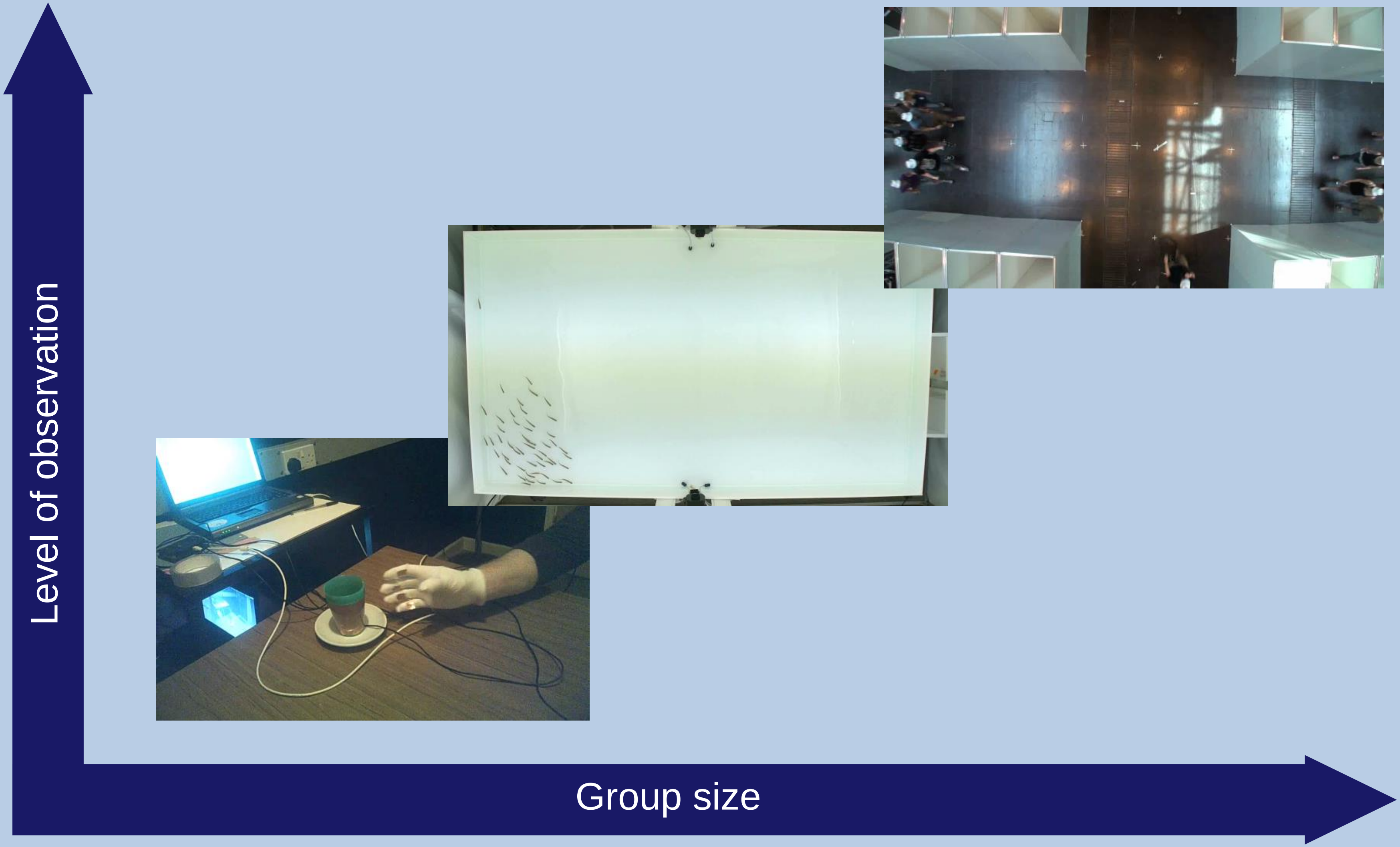
Experimental design and ANOVA: you will NOT need to perform any calculations, but rather understand and interpret MATLAB outputs

a) Understand four major steps in the statistical experimental design.
b) Be able to interpret one-way Anova and two-way Anova results.

Generalised linear models (Logistic regression): you will NOT need to perform any calculations, but rather understand and interpret MATLAB outputs

a) Understand the assumptions underlying generalised linear models (GLMs) including interpretation of two types of residuals.
b) Estimate parameters (MLE) and perform hypothesis testing (AIC, BIC, likelihood, deviance).

Scales of collective movements

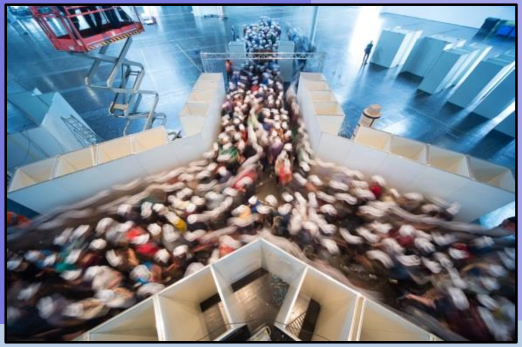


Collective movements

Animals + humans:

- spread of diseases
- conservation
- …

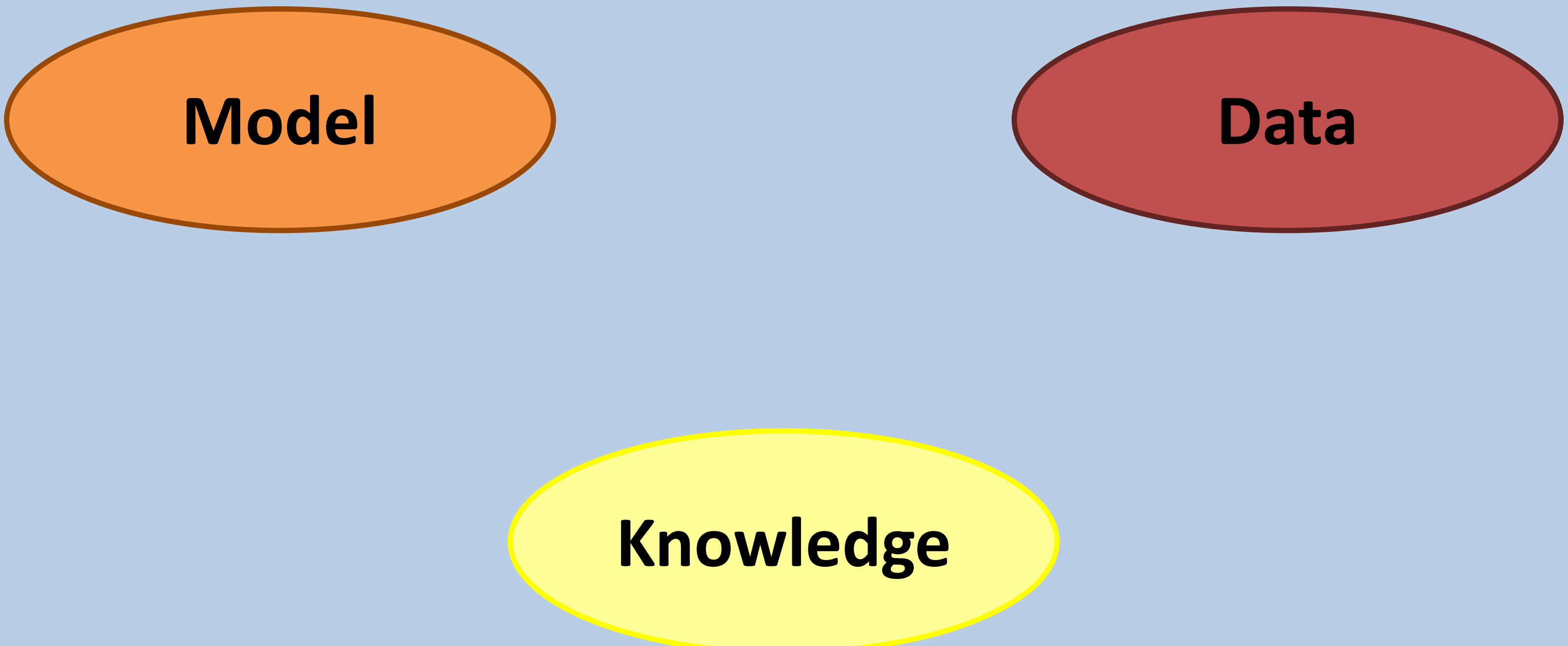
Why is studying social movement important?



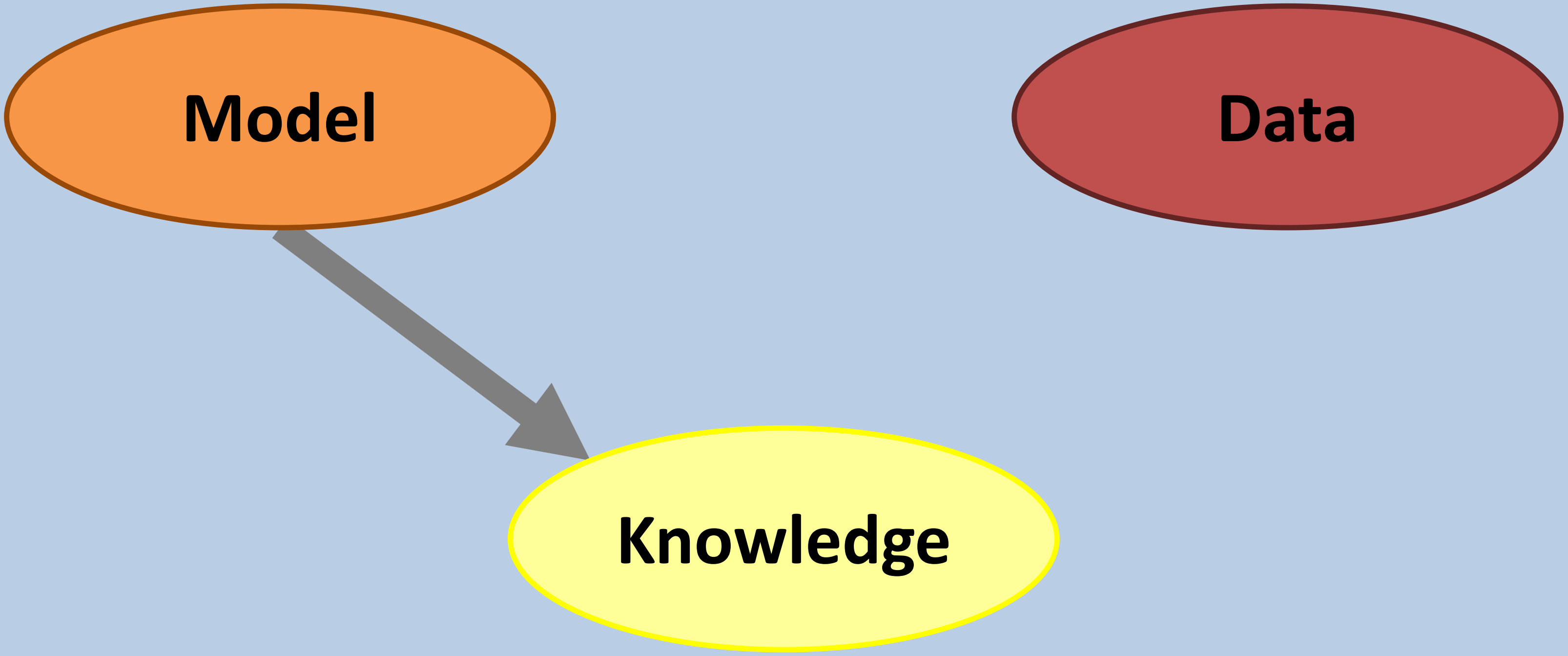
Humans:

- behavioural diagnostics
- building design
- …

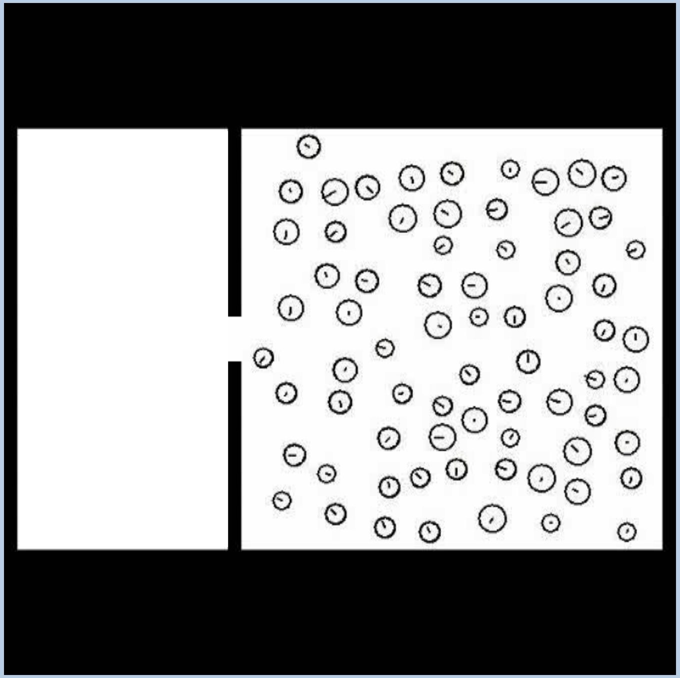
Studying collective movement



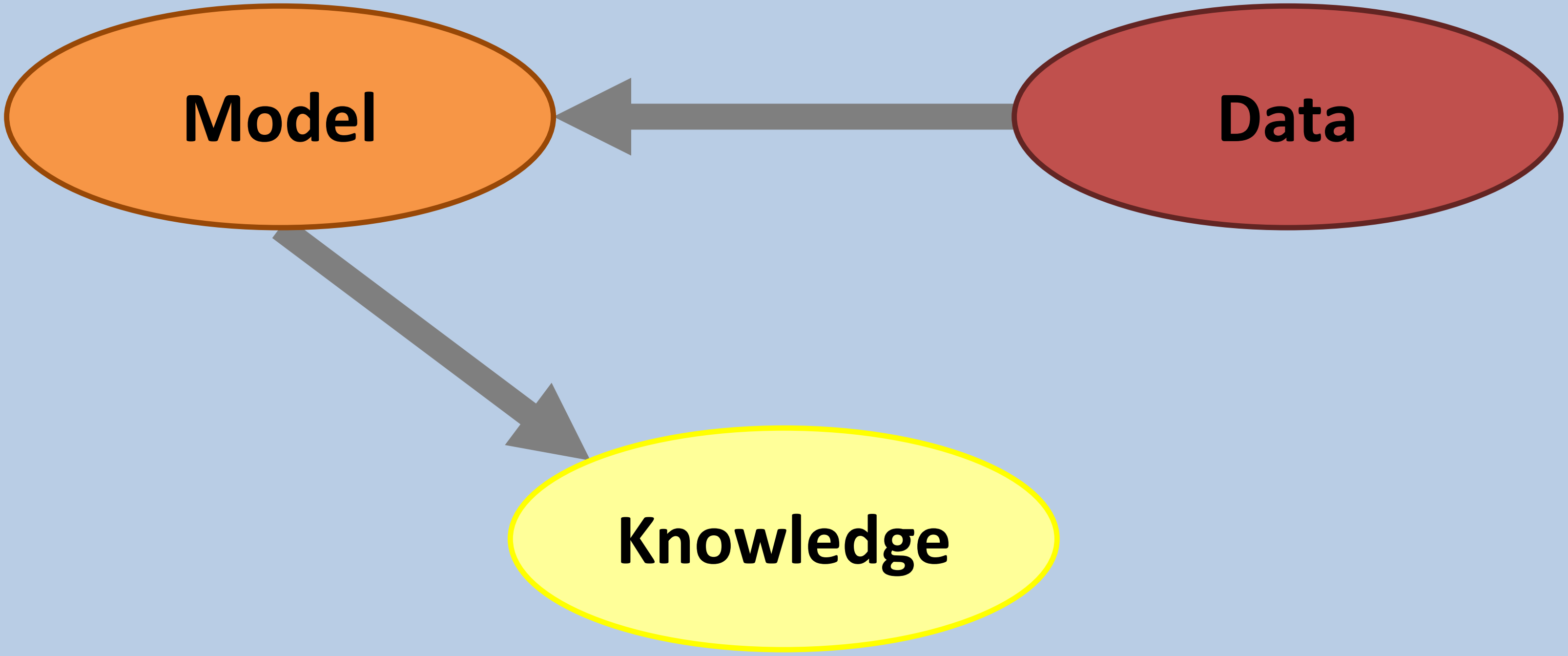
Studying collective movement



Models for collective movement



Studying collective movement



Context – pedestrian crowds

- Route choice
- Evacuation timing
- Risk taking
- Helping behaviour

Decision-making in evacuations

Observational data and experiments with real crowds are difficult to obtain.

Decision-making in evacuations

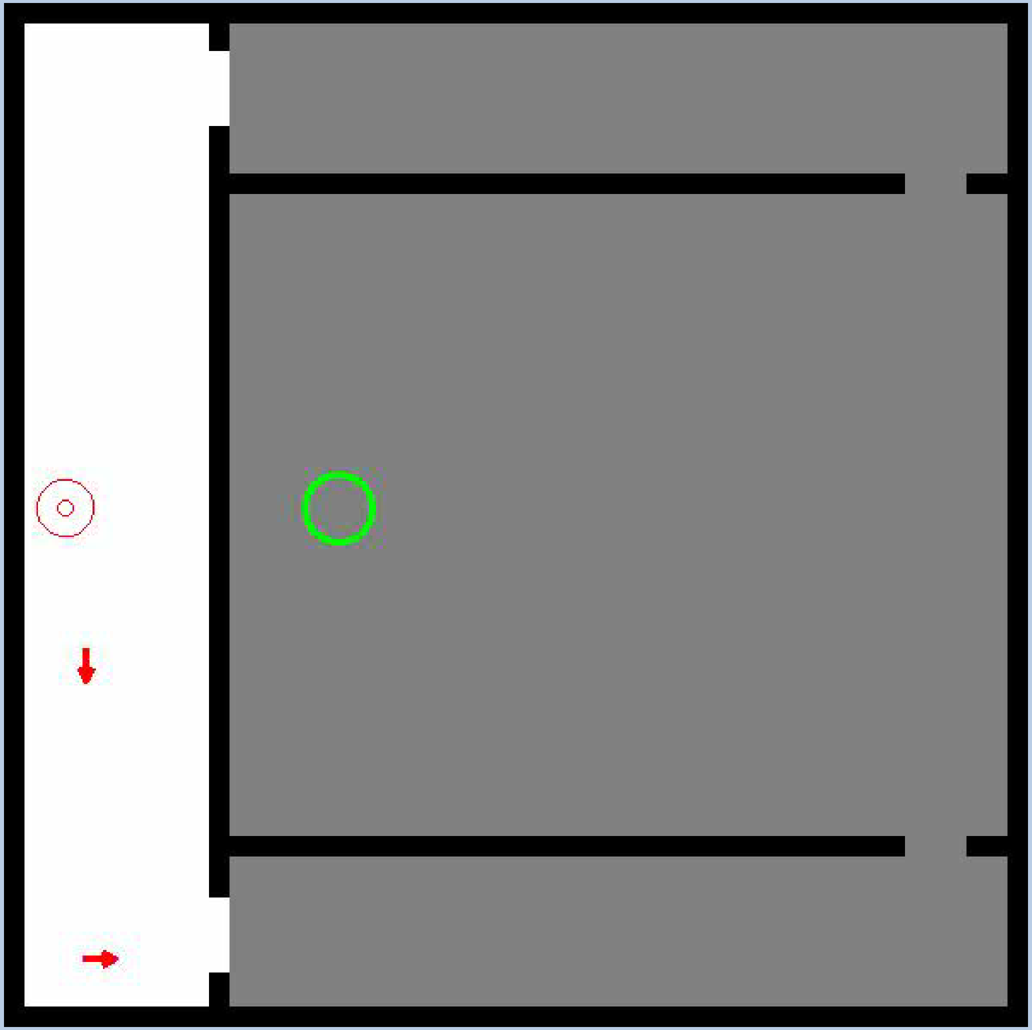
Observational data and experiments with real crowds are difficult to obtain.

Instead, use a *virtual environment* to investigate this.

Here, give one example, study:

- route choice
- following others
- ability to change decisions
- the effect of motivation/stress

Experiments

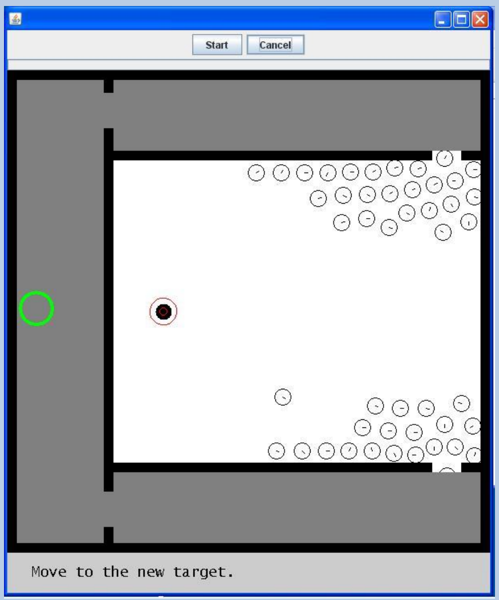


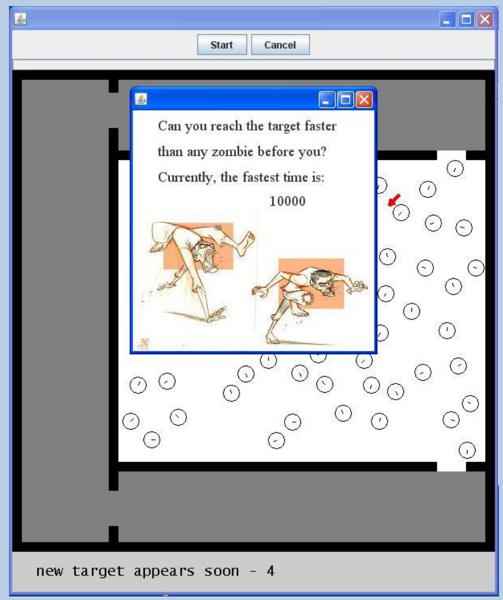
Treatments

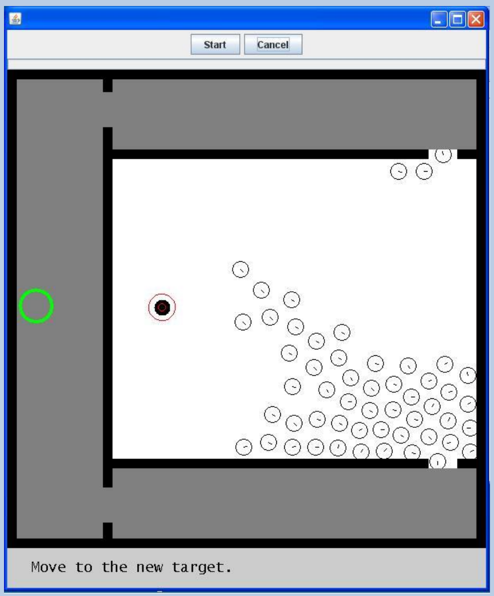
Control

Motivation (M)

Asymmetric (A)

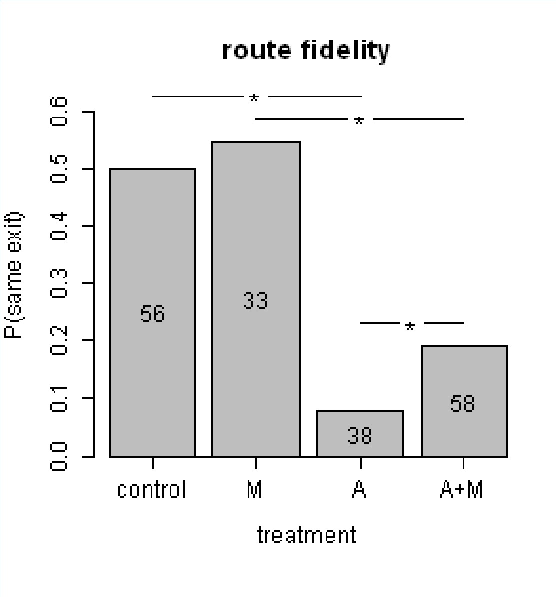






Four treatments: control, M, A, A+M

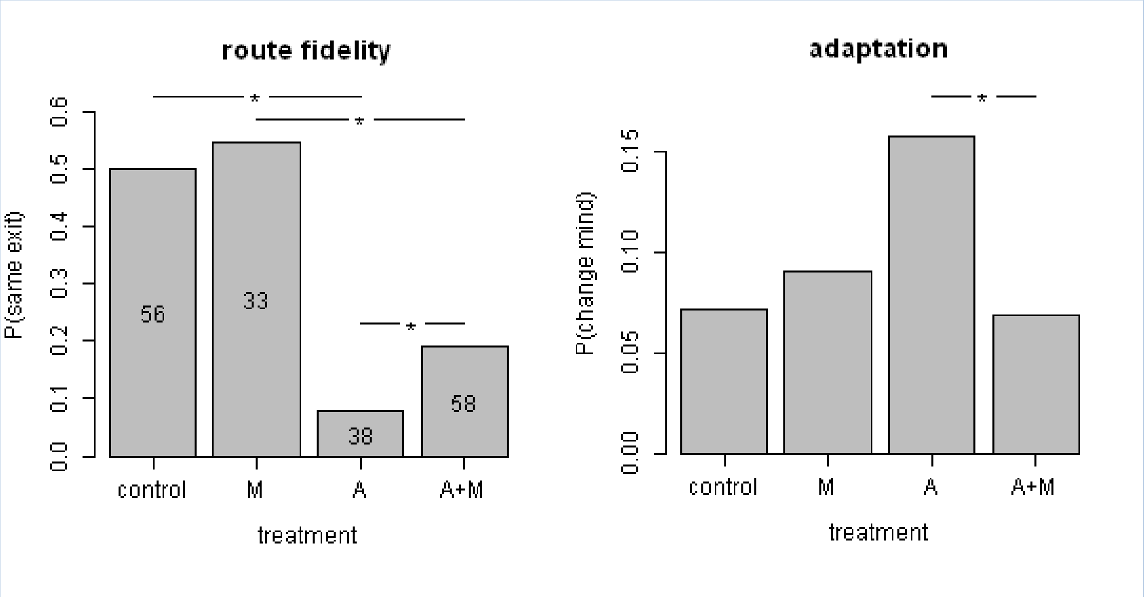
Results



185 participants

* P<0.1

Results



185 participants

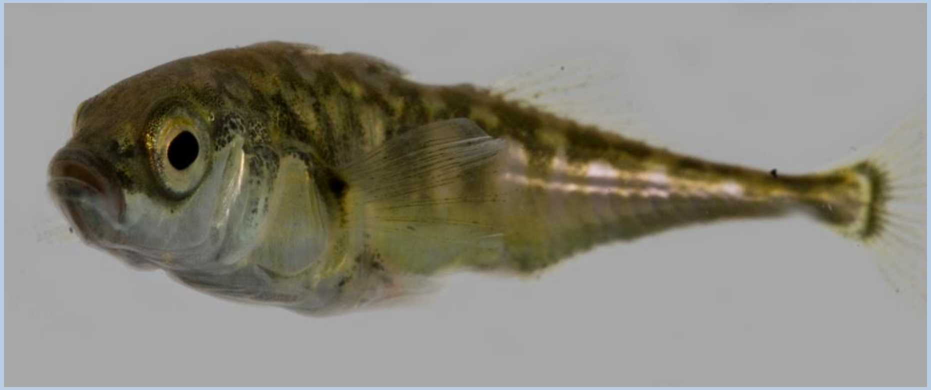
* P<0.1

Summary

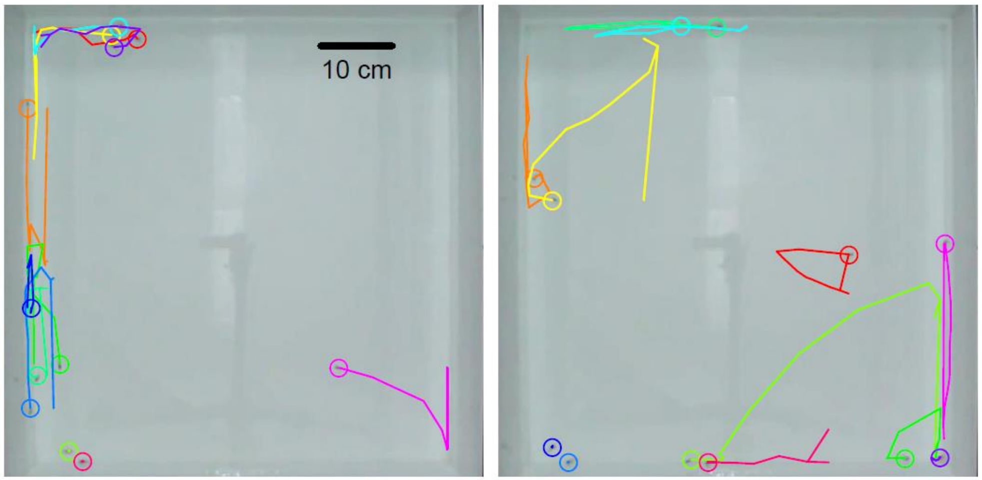
Motivation affects decision-making

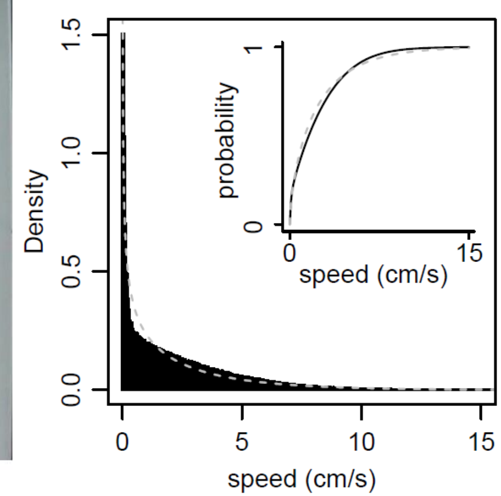
- Other experiments:
  - Reactions to combinations of different sources of information
  - Helping behaviour
  - Following instructions
  - Risk-taking

Intermittent social movement in shoaling guppies and sticklebacks

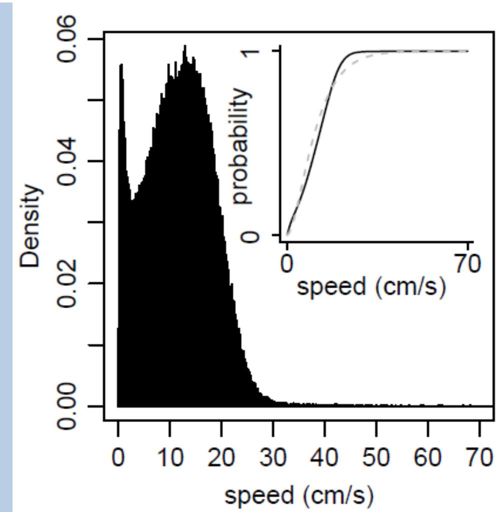


Movement of fish shoals

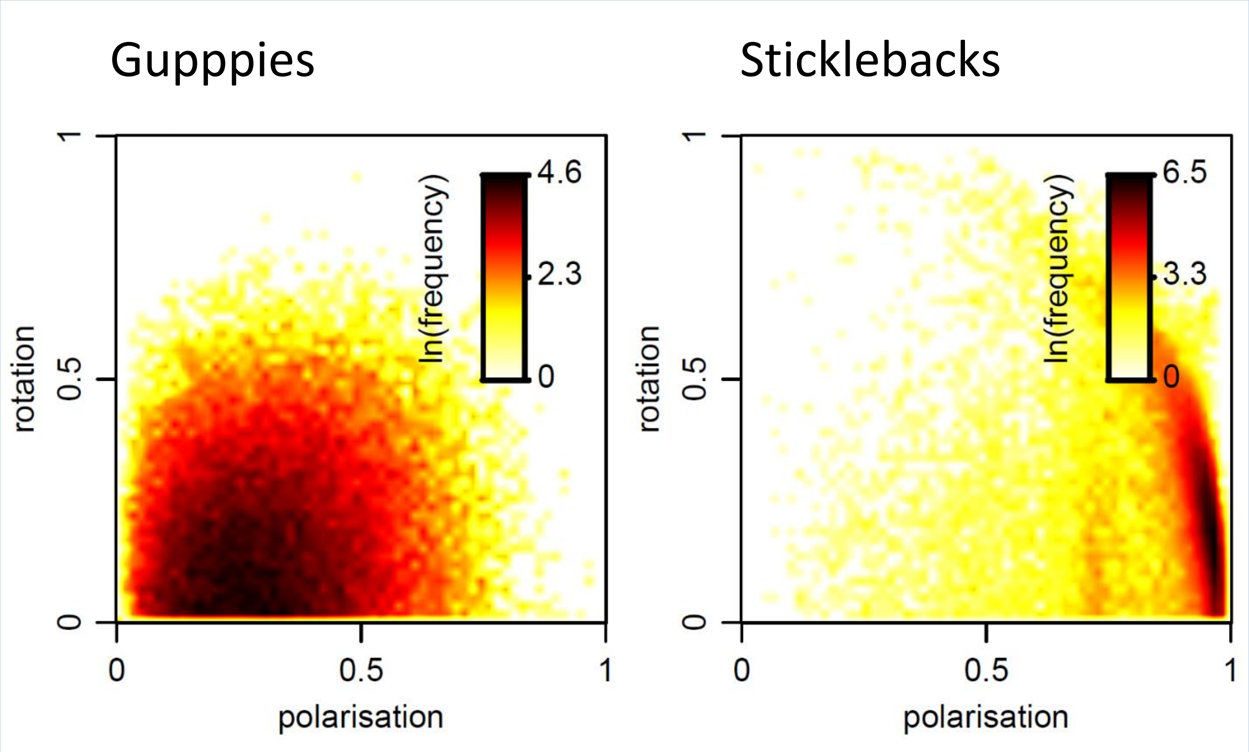




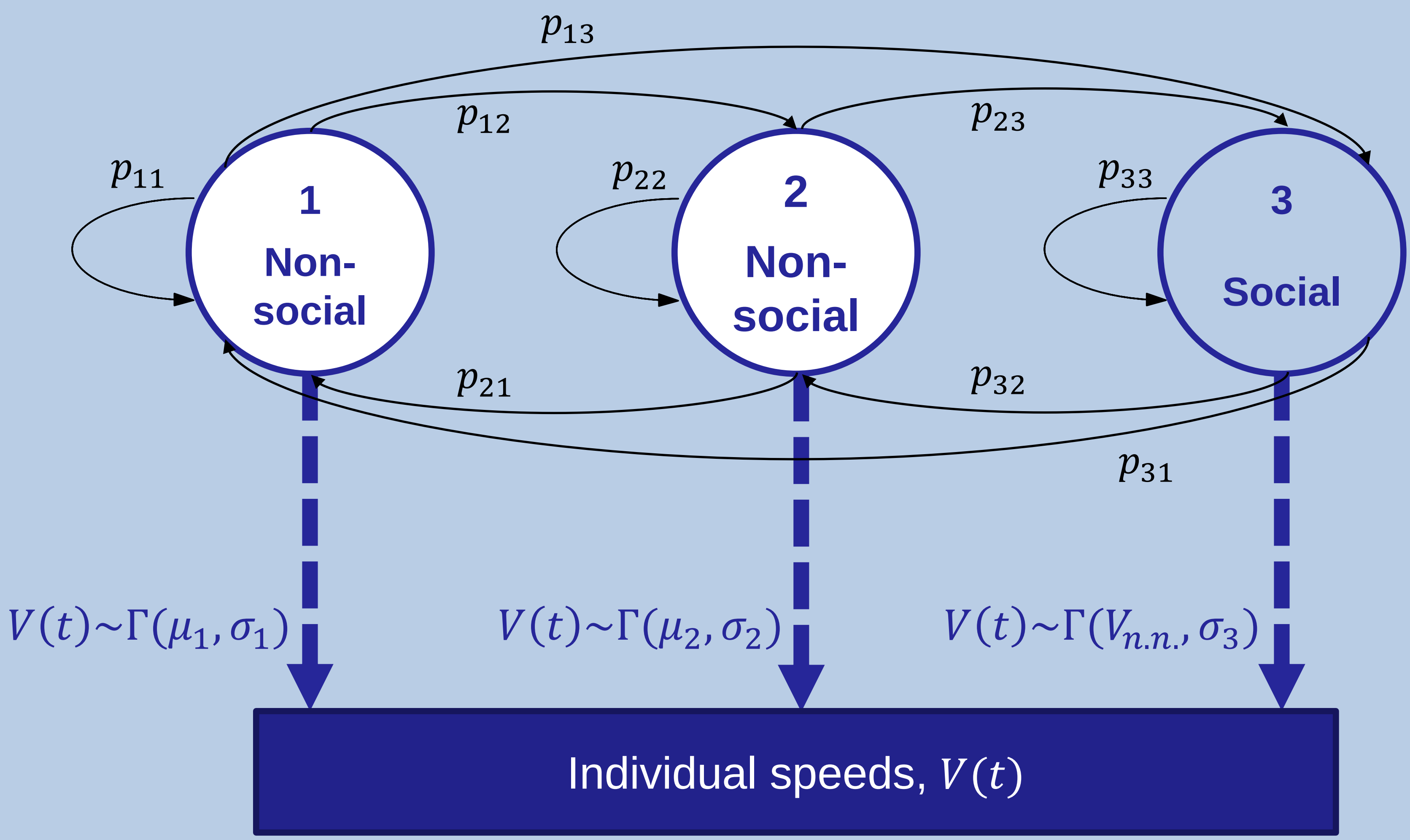
- Shoals of 12 guppies or 8 sticklebacks
- Obtain trajectories
- Square (guppies) and circular (sticklebacks) tank



Shoal-level movement



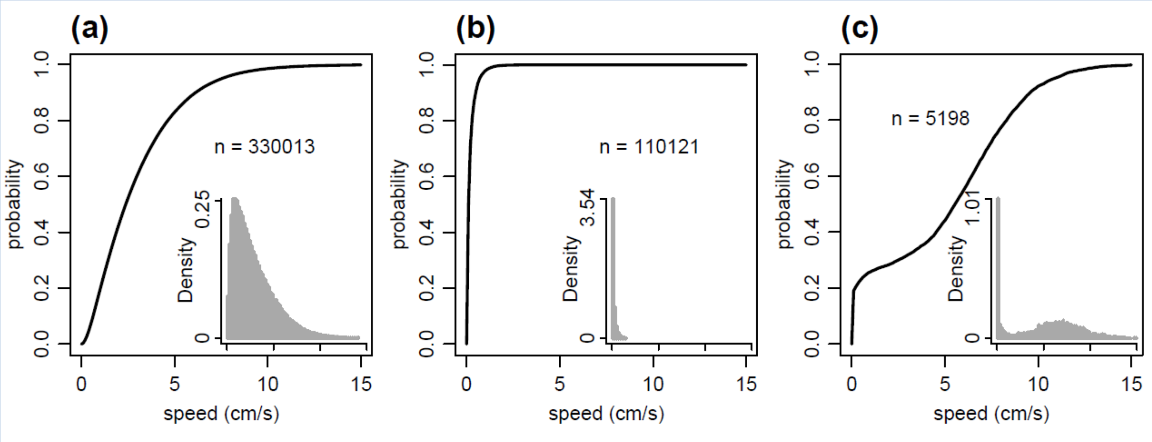
Behaviour characterisation using HMMs



Evidence for intermittent social behaviour

AIC of social model is better than expected by chance for both species ($P \ll 0.01$).

Gupppies



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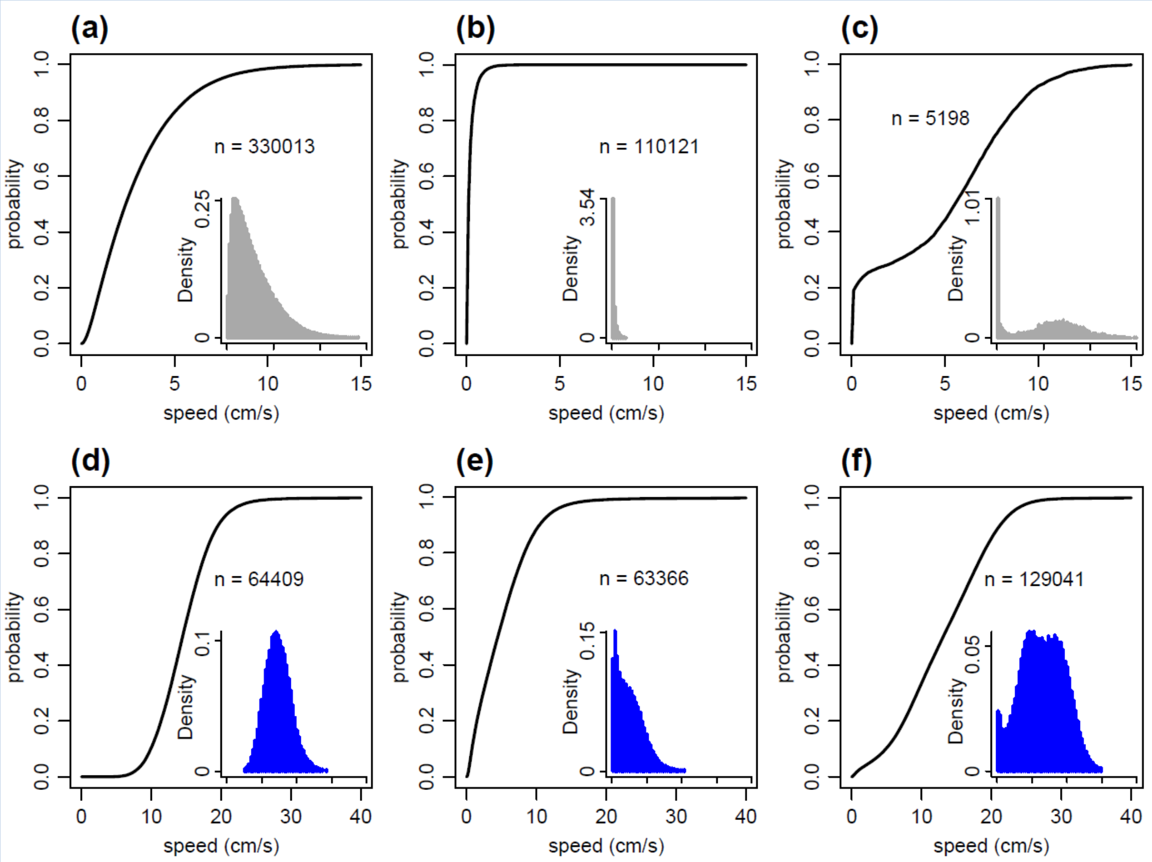
Gupppies

Sticklebacks



Fishy discussion

Intermittent social movement in fish shoals

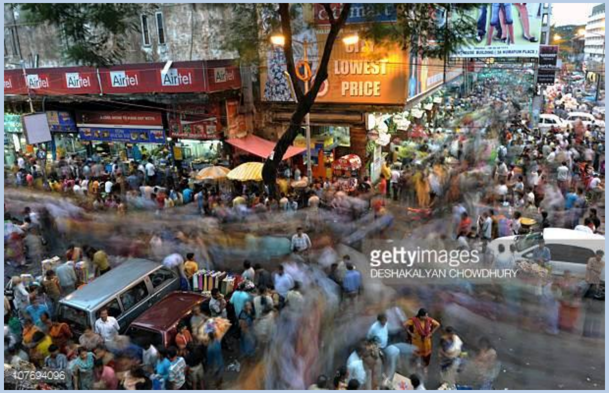
Microscopic analysis of interactions

Direct contrast of hypotheses for behaviour

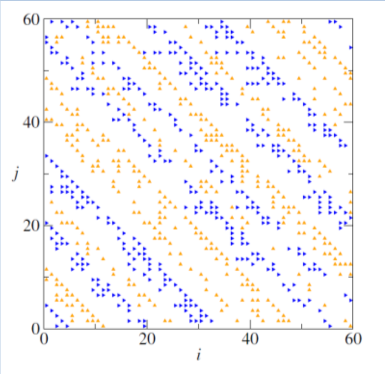
Models are simple, but versatile

Detailed investigation of social movement?

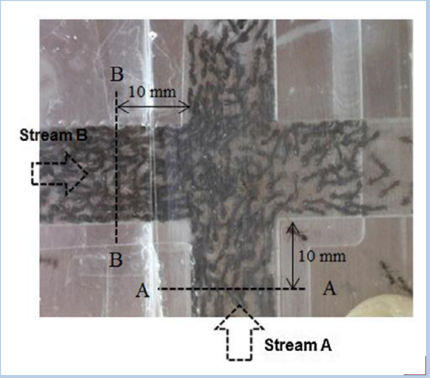
Interactions at the infrastructure level: intersecting pedestrian streams

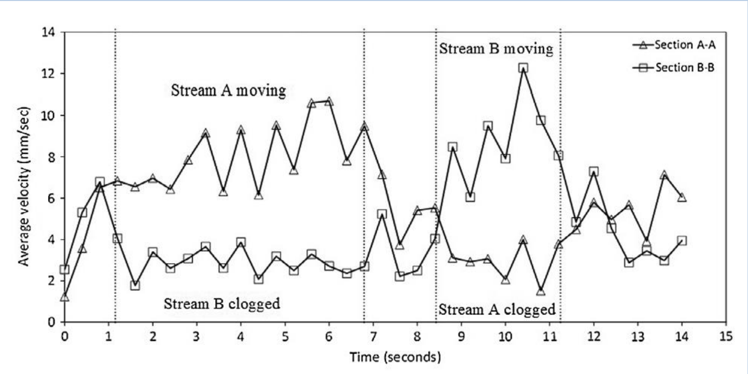


Previous Work

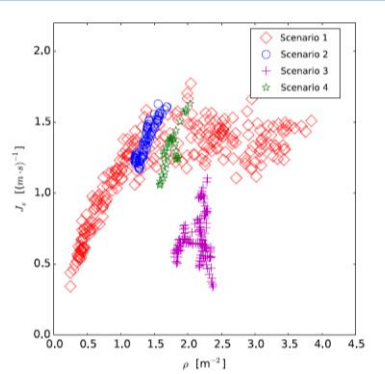


Cividini, Appert-Rolland, et al. (2013)

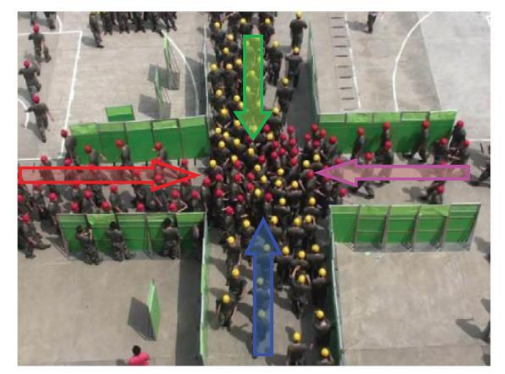


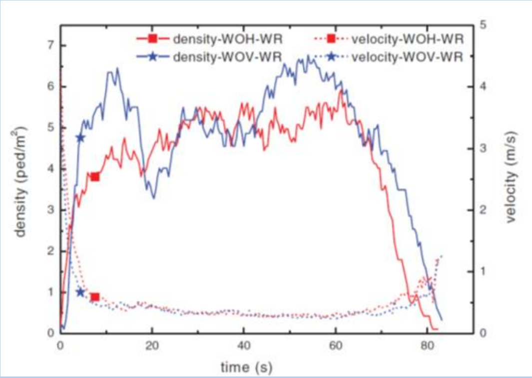


Dias, Sarvi, et al. (2013)



Zhang, Seyfried (2014)





Lian et al. (2015)

My Approach

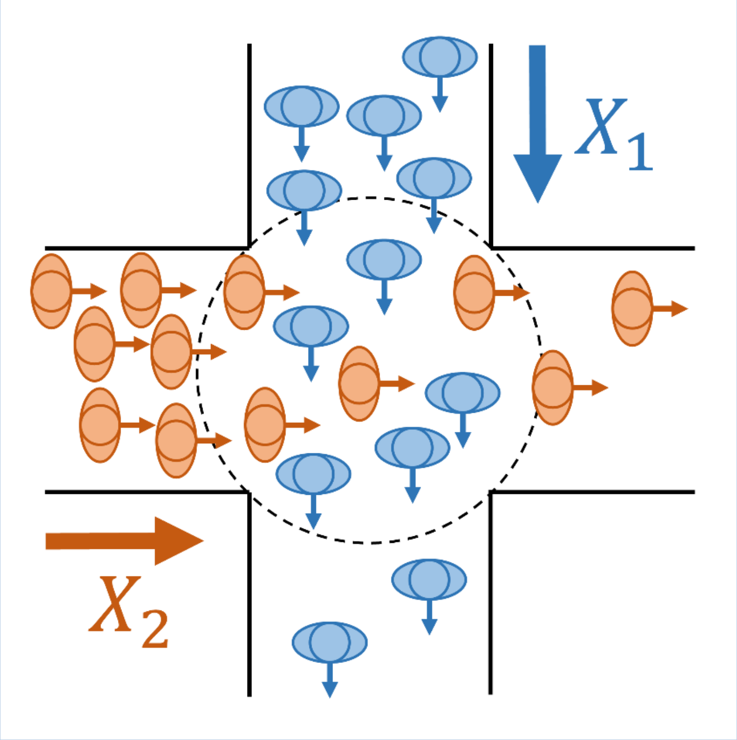
… to test for interactions at the infrastructure level:

- Different scenarios we might reasonably expect
  - Stable states
  - Densities
  - Dynamics

  → ***macroscopic models***

- Which model is the most plausible, given empirical evidence

Stream Populations

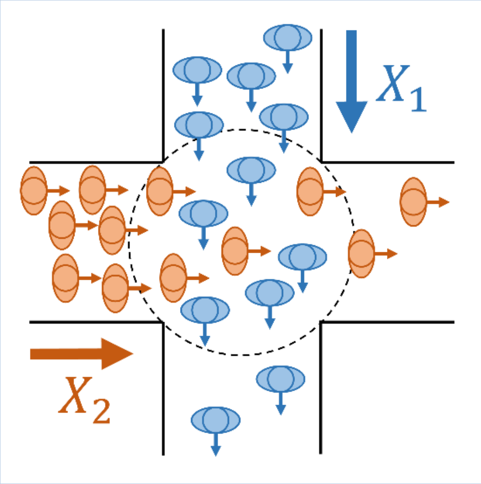


Here $X_1 = 5$ and $X_2 = 4$.

Modelling Stream Populations

$$\frac{dX_1}{dt} = f_{in}(X_1, X_2) - f_{out}(X_1, X_2)$$

$$\frac{dX_2}{dt} = f_{in}(X_2, X_1) - f_{out}(X_2, X_1)$$



Assumptions:

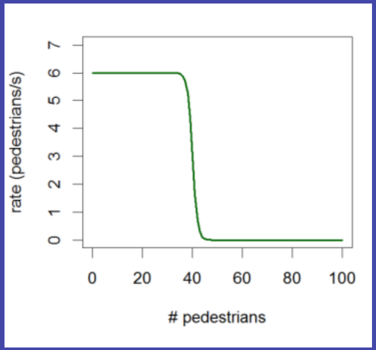
- Streams are symmetric
- Density inside intersection drives dynamics

Modelling Stream Populations

$$\frac{dX_1}{dt} = f_{in}(X_1, X_2) - f_{out}(X_1, X_2)$$

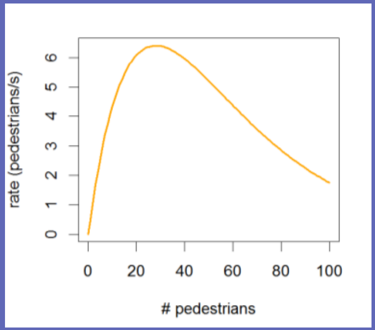
INFLOW

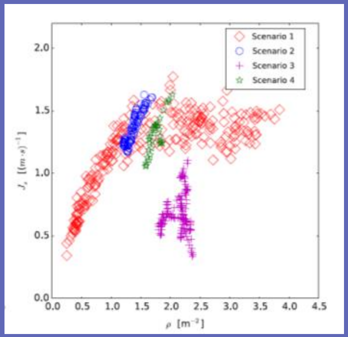
$$f_{in}(X_1, X_2) = \frac{\alpha}{1 + e^{(X_1 - \gamma)}}$$



OUTFLOW

$$f_{out}(X_1, X_2) = \mu X_1 e^{-\varepsilon X_1}$$





Zhang, Seyfried (2014)

Different Models

Null Model (no interaction), model 1:

$$f_{in}(X_1, X_2) = \frac{\alpha}{1 + e^{(X_1 - \gamma)}} \qquad f_{out}(X_1, X_2) = \mu X_1 e^{-\varepsilon X_1}$$

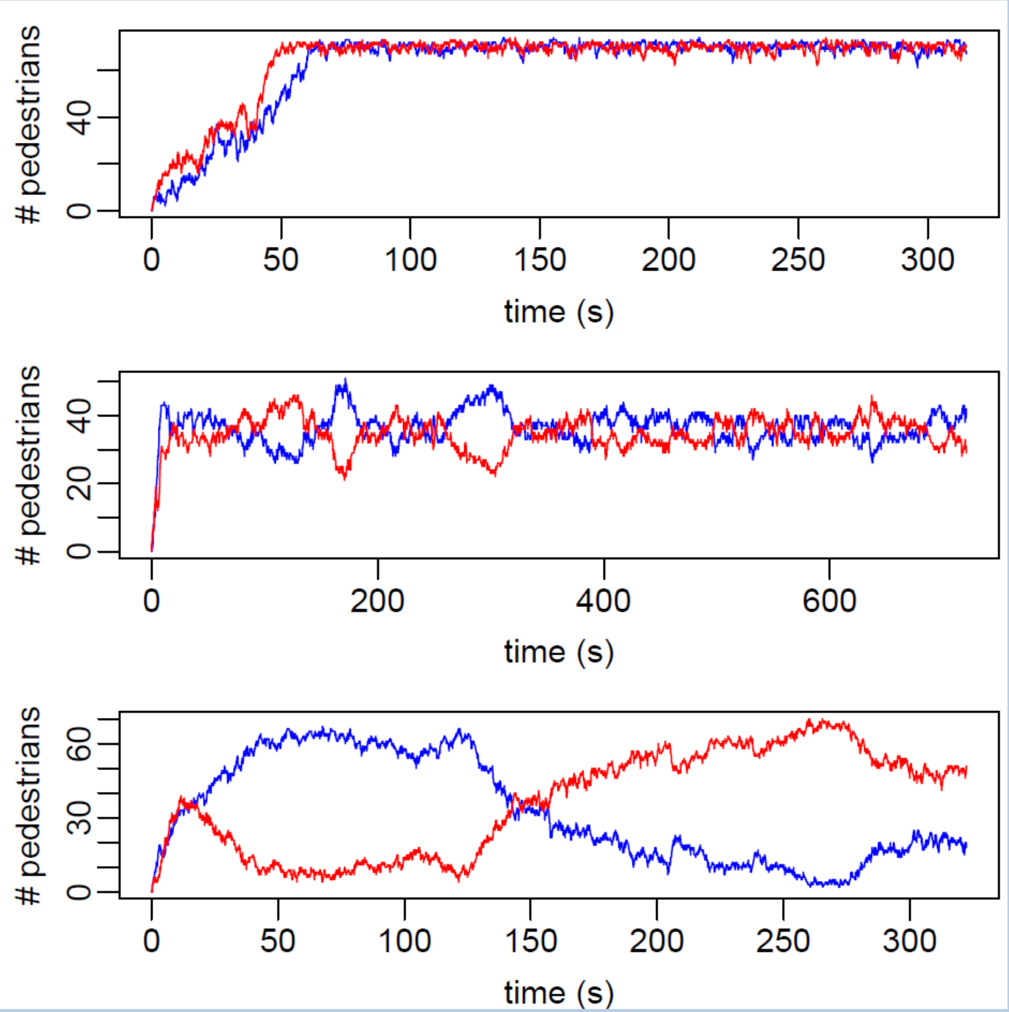
Linear density interaction, model 2:

$$f_{in}(X_1, X_2) = \frac{\alpha}{1 + e^{(X_1 + X_2 - \gamma)}} \qquad f_{out}(X_1, X_2) = \mu X_1 e^{-\varepsilon [X_1 + X_2]}$$

Non-linear density interaction, model 3:

$$f_{in}(X_1, X_2) = \frac{\alpha}{1 + e^{(X_1 + X_2 - \gamma)}} \qquad f_{out}(X_1, X_2) = \mu X_1 e^{-\varepsilon X_1 - \delta \sqrt{X_1 X_2}}$$

Simulations - Pedestrian Numbers



Model 1

$\alpha = 7;\ \gamma = 70;$
$\mu = 0.039;\ \varepsilon = 0.036.$

Model 2

$\alpha = 8;\ \gamma = 70;$
$\mu = 0.039;\ \varepsilon = 0.036.$

Model 3

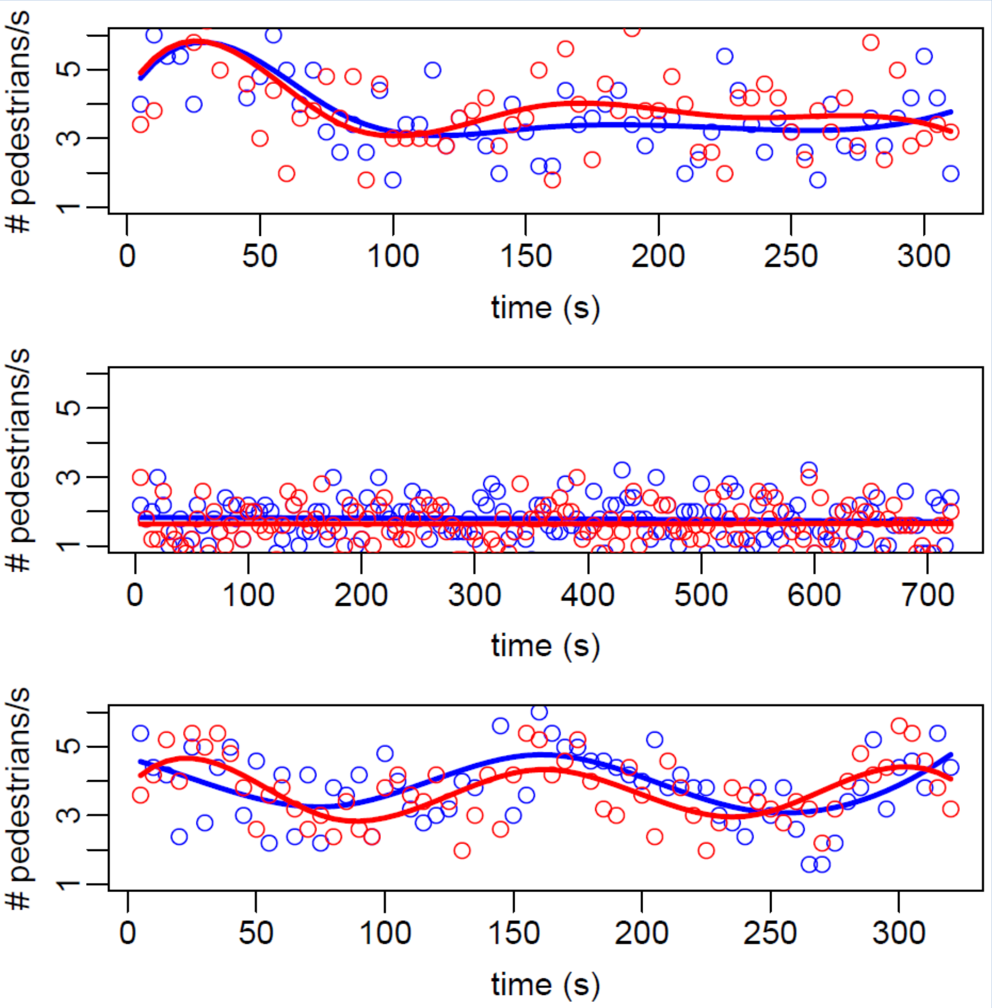
$\alpha = 8;\ \gamma = 70;$
$\mu = 0.039;$
$\varepsilon = 0.036;\ \delta = 0.01.$

Simulations - Outflows

Model 1

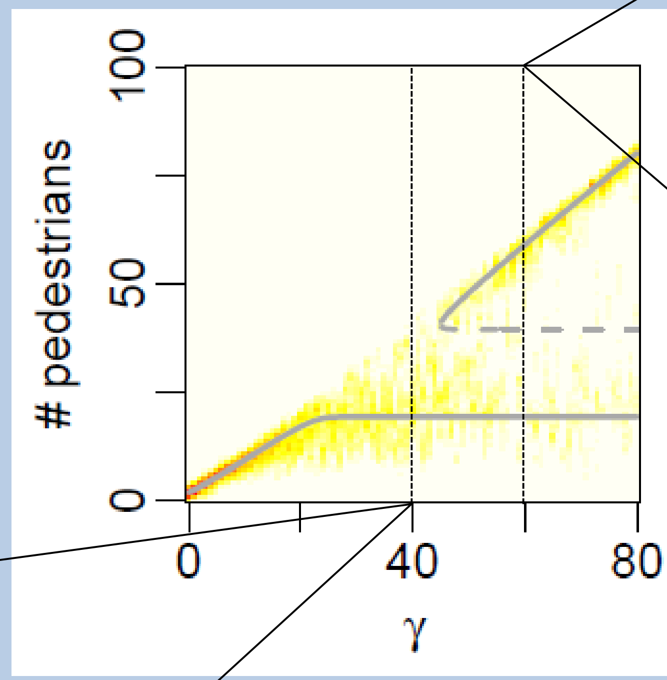
Model 2

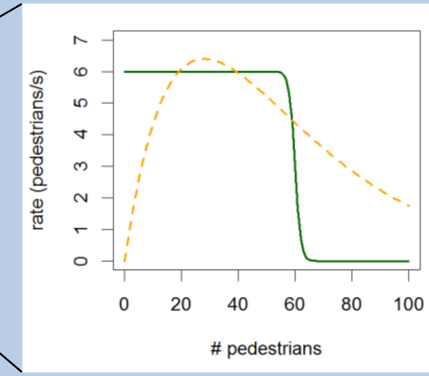
Model 3

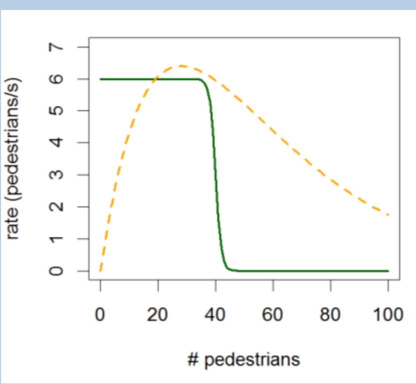


Stability Analysis 1

Find ***fixed points***:







Bifurcation diagram for varying γ (model 1);

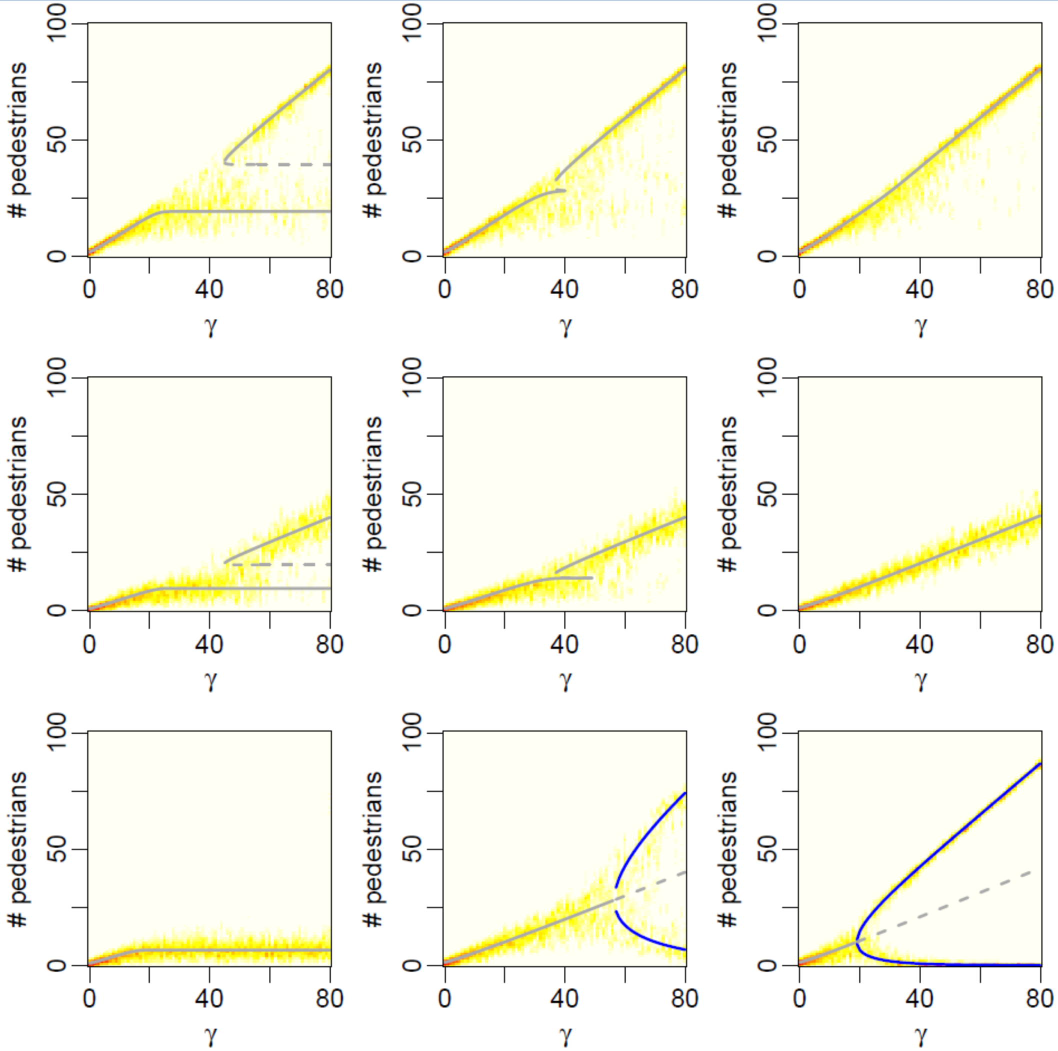
$$f_{in} = \frac{\alpha}{1 + e^{(X_1 - \gamma)}}$$

Stability Analysis 2

Model 1

Model 2

Model 3



Model Fitting using ABC

Exploring the parameter space comprehensively is difficult, so models need to be fitted to data.

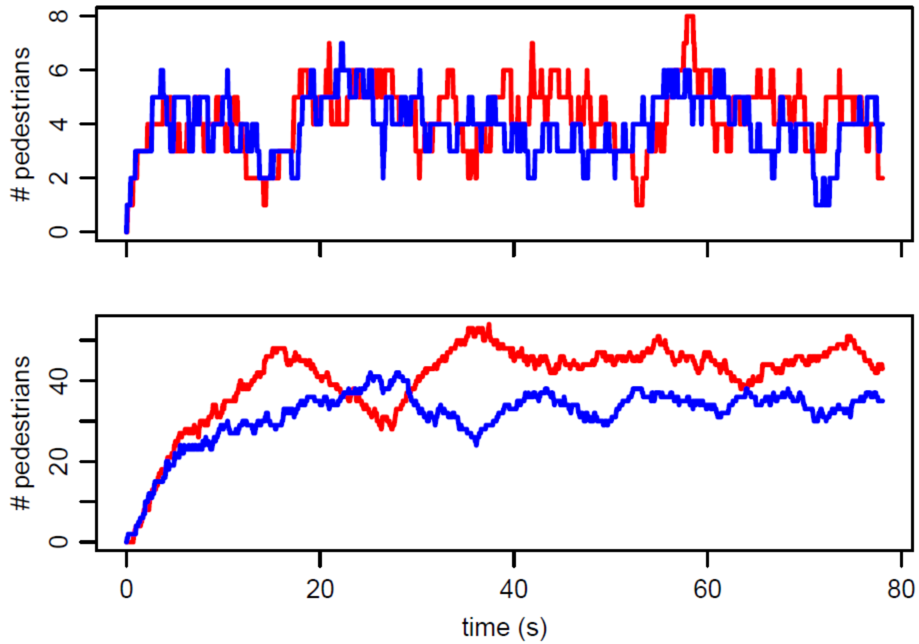
Pedestrian data

Courtesy BaSiGo Project

Pedestrian data

Inflow gate 1m wide

Inflow gate 4m wide



Model Fitting using ABC

Exploring the parameter space comprehensively is difficult, so models need to be fitted to data.

Discreteness and multi-dimensionality of data (# pedestrians and time) mean likelihood functions of models are not readily available.

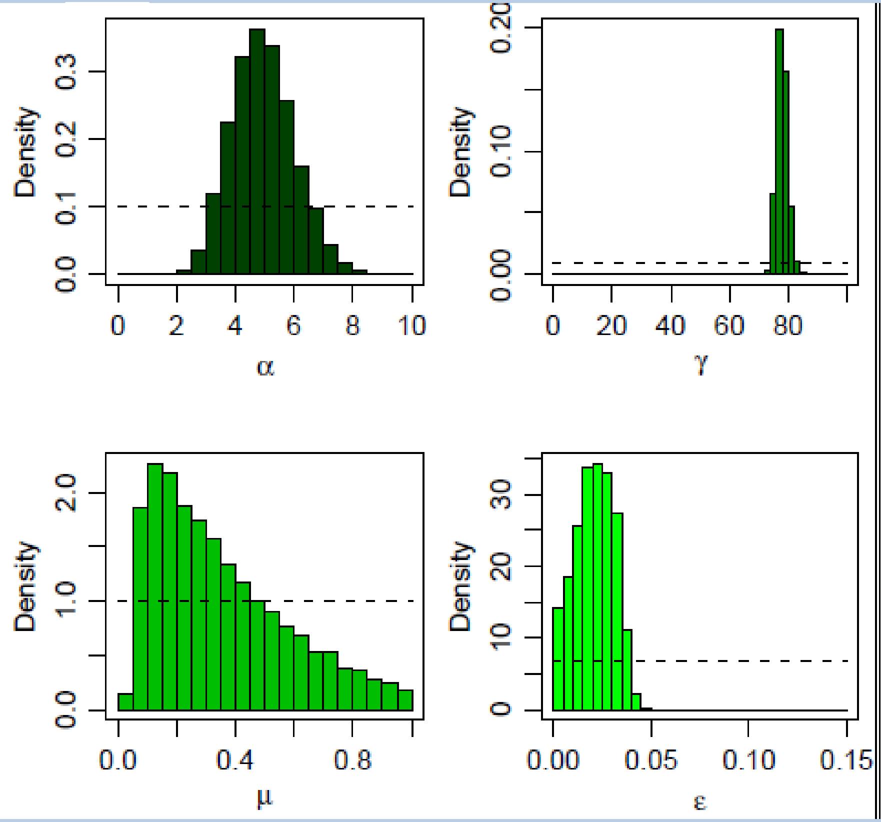
Use ***Approximate Bayesian Computation***:

1) Sample parameters from prior distribution
2) Simulate from model using these parameter values
3) If *difference* between simulation and data is below a threshold, accept priors, otherwise reject
4) Return to (1)

… do this LOTS of times to get posterior distribution of parameters.

Model Fitting using ABC

Doing this for a given data set (courtesy Holl, Chraibi, BaSiGo project, FZ Jülich):



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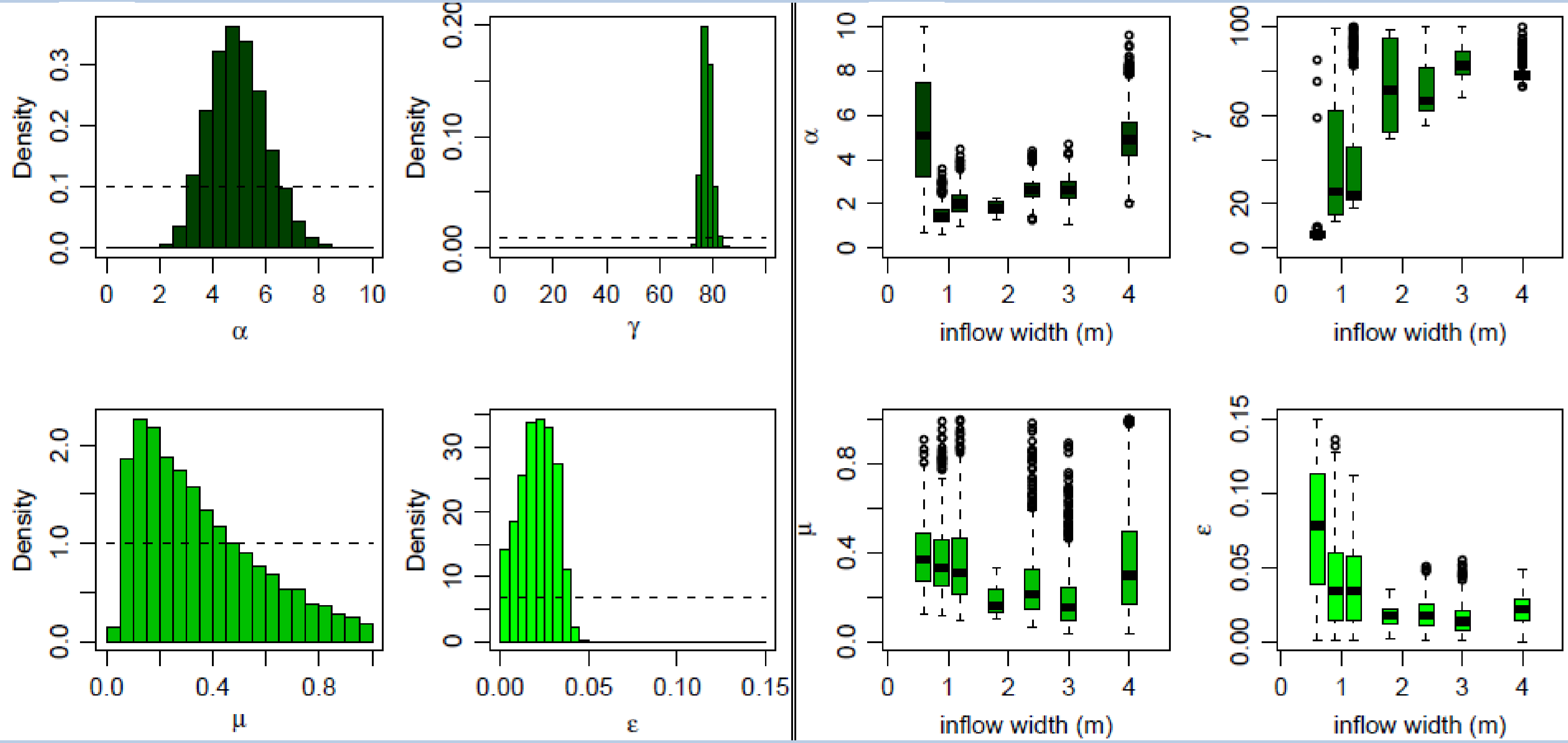
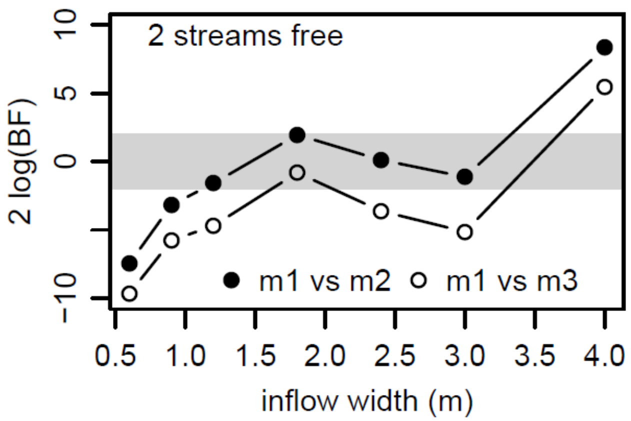


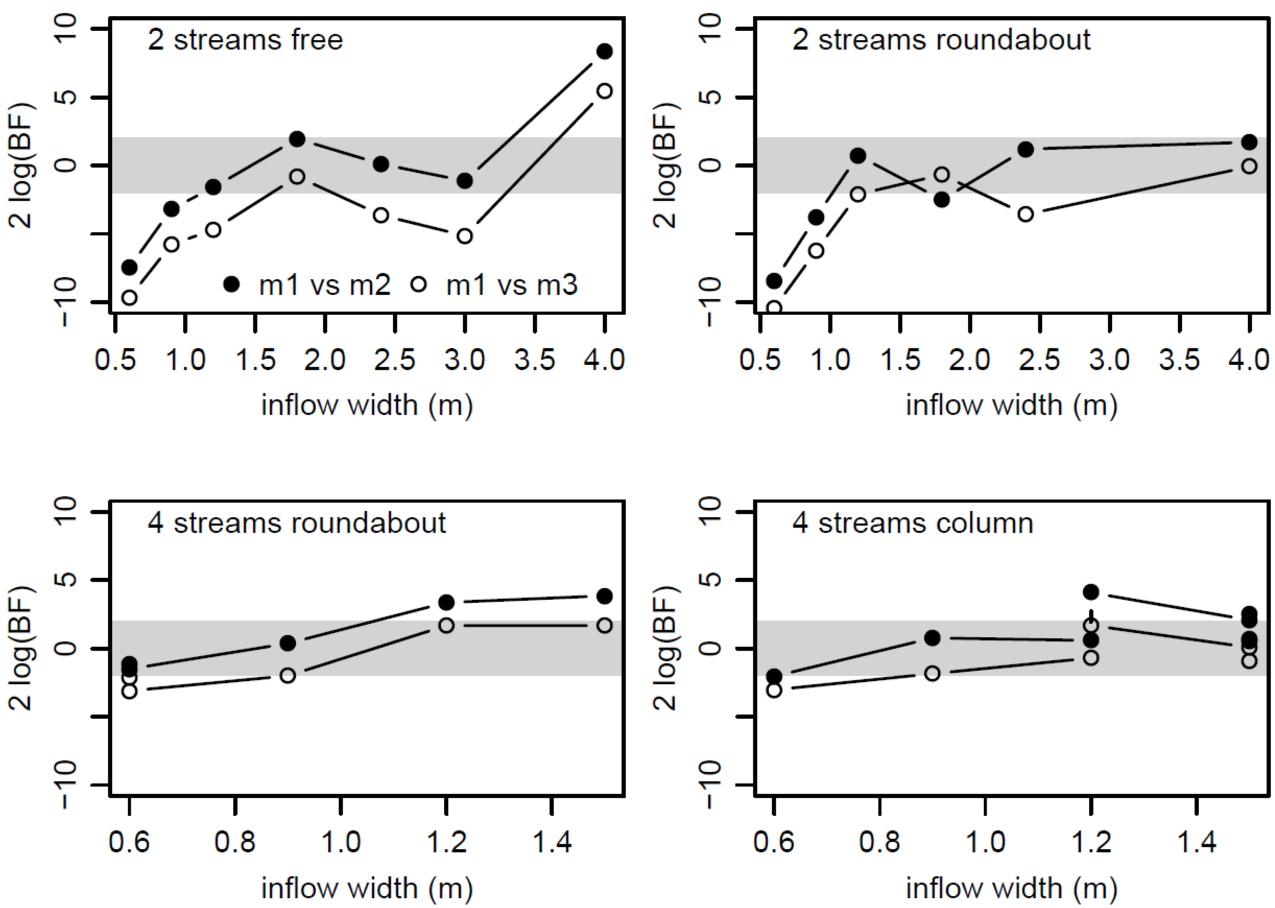
Model Selection using ABC



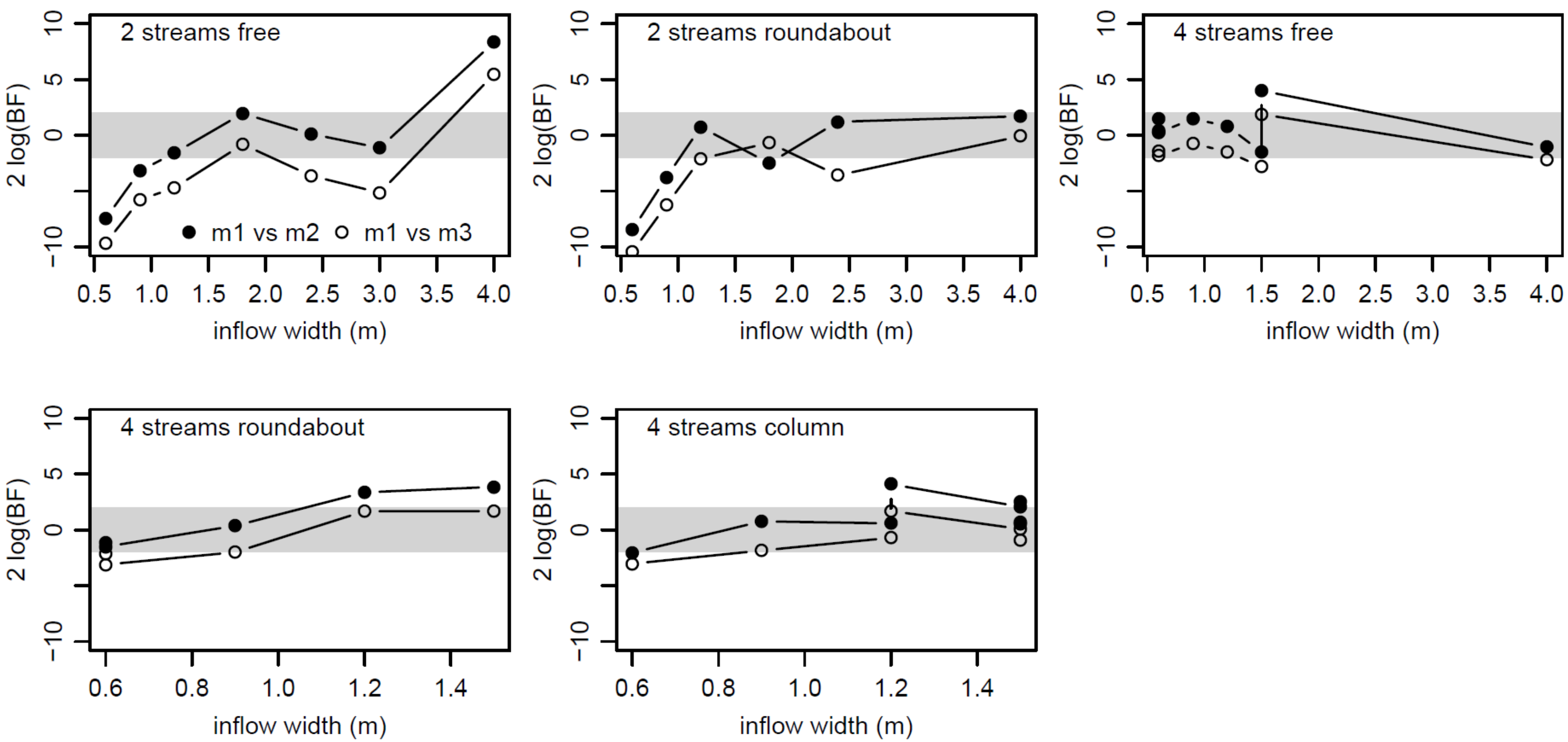
Using acceptance rate of priors for different models in ABC, we can approximate the Bayes factor,

$$BF = \frac{P(data|M_a)}{P(data|M_b)}$$

Model Selection using ABC



Model Selection using ABC



Pedestrian discussion

Population dynamics models for intersecting pedestrian streams

Macroscopic analysis of dynamics

Models pre-dict different dynamics

ABC allows model fitting and selection

Starting point for general insights?

Acknowledgements

Munich

Michael Seitz

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Jülich

Stefan Holl

Mohcine Chraibi

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Through Research, Protection

Thank you!