

EMAT30007 Applied Statistics

Lab 9: Experimental Design and ANOVA

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Parts (1) and (2) of this lab cover the two types of ANOVA we looked at in lectures. In addition, we will make an explicit comparison between ANOVA and Linear models. Part (3) is a group exercise where you will have to address a statistical experimental design challenge.

(1) One-way ANOVA

We will analyse the data we have already seen in lecture 9. The data are from a study of the strength of structural beams in Hogg (1987). Reference:

[Hogg, R. V., and J. Ledolter. *Engineering Statistics*. New York: MacMillan, 1987]

You can copy the data out of the document below, or find it online under the link shown below.

```
% one-way ANOVA example

% https://uk.mathworks.com/help/stats/anova1.html

strength = [82 86 79 83 84 85 86 87 74 82 ...
            78 75 76 77 79 79 77 78 82 79];
alloy = {'steel','steel','steel','steel','steel',...
         'steel','steel','steel','alloy1','alloy1',...
         'alloy1','alloy1','alloy1','alloy1',...
         'alloy2','alloy2','alloy2','alloy2','alloy2','alloy2'};
```

The vector `strength` measures deflections of beams in thousandths of an inch under 3000 pounds of force. The vector `alloy` identifies each beam as steel ('steel'), alloy 1 ('alloy1'), or alloy 2 ('alloy2'). Although `alloy` is sorted in this example, grouping variables do not need to be sorted.

Plot the mean strength for the three different alloys tested using the Matlab command 'boxplot'.

This plot should show that the deflection of alloys 1 and 2 seems lower than the deflection of steel beams.

1.1. `anova1()`

In this experiment, we only consider one factor ('alloy'). This means the experiment is equivalent to a completely randomised design and we can use a one-way ANOVA to analyse the data. Use the command `anova1` in Matlab to perform a one-way ANOVA on this data, making sure that the command shows an ANOVA table. Use the Matlab help page or the link above if you get stuck.

You should get the same ANOVA table as the one shown in lecture 9. Remember, this only shows that the mean strength of beams for at least two alloys differs.

1.2. Analysis using a Linear Model

In lecture 9 we saw that performing ANOVA is equivalent to analysing data using Linear Models. Fit the linear model corresponding to the analysis in 1.1. to the data. Remember that you will have to tell Matlab that the variable 'alloy' is qualitative. You may find it easier to create a vector with numbers indicating the alloy type.

Compare the F-test of this model to the test result shown in the ANOVA table above. They should be the same. Why?

The parameter-specific t-tests in the Linear Model summary table also give an indication of how alloy1 and alloy2 differ in the strength measure compared to the baseline (which is steel here).

(2) Two-way ANOVA

In this section we will use the popcorn data that is included in Matlab to perform a two-way ANOVA.

In Matlab, read in the data using the code below:

```
load popcorn
popcorn
```

The Matlab help pages tell us that the data is from a study of popcorn brands and popper types (Hogg 1987). The three columns of the matrix `popcorn` are the brands, Gourmet, National, and Generic, respectively. The rows are popper types: oil and air. To get the data researchers popped a batch of each brand three times with each popper, that is, the number of replicates is 3. The first three rows correspond to the oil popper, and the last three rows correspond to the air popper. The response values are the yield in cups of popped popcorn.

2.1. `anova2()`

Use the Matlab command `anova2` to analyse the data. Have a look at the input arguments of this function - you need to let it know how many replicates are included in the rows of the data matrix `popcorn`.

You should obtain the same ANOVA table as shown in the lecture notes. Notice how `anova2` tests for interactions between factors by default.

2.2. Analysis using a Linear Model

The lecture notes show what the linear model equivalent to a two-way ANOVA is. Fit this model to the popcorn data. You will have to re-format the data into separate predictor and response vectors.

Compare the F-test provided in this model fit to the tests in the ANOVA table produced by `anova2`. None of the tests are the same. Why?

(3) Group exercise

Ideally, statisticians are already involved when experiments are designed, but often they only get approached by other scientists to help with analysing experimental data that has already been collected. Sometimes it is possible to develop an elegant statistical analysis *post hoc*, but sometimes the data does not lend itself to statistical analysis and sometimes a statistician does not have the experience or expertise to deal with an already existing data set. It is an important skill to identify and admit when this is the case.

In groups, have a look at one of the following examples of proposed experimental designs. Come up with answers to the following questions, if possible:

- a) Can you use a statistical model we have covered already to assess the statistical design described? (write down a formula for the structure of the model)
- b) If you could change the experimental design, what would you do and why? (assume you cannot change the number of data points)
- c) What statistical model would you use for your new experimental design? (write down a formula for the structure of the model)

Example 1 - computer chips

A computer chip manufacturer wants to assess the efficacy of a new photolithographic printing process to form transistors and circuits on silicon wafers. The company owns 10 printing machines and each machine can be programmed to use the old or new process. Past experience shows that not all machines perform identically when new software is introduced. The company decides to test two machines. For each machine, the silicon wafer printing is first performed twice using the old process and then four times using the new process. The measure the company assesses is the number of viable chips a printed wafer produces (in hundreds of chips).

Answer:

Example 2 - indoor drones

A magazine dedicated to electronic toys wants to investigate the responsiveness of small indoor drones of a particular make. The team have a total of 10 drones each from two different manufacturers available for testing - the products by both manufacturers are known to vary a lot in quality, even across items. In the responsiveness experiment, a test pilot has to wait for a remotely activated light signal and then steer the drone as quickly as possible from a starting hovering position across a finishing line. The experiment is filmed at a high frame rate and the time it takes for the drone to cross the finishing line after the light signal is activated is measured (in milliseconds). As all 20 staff of the magazine are keen to act as test pilots, they are each randomly assigned a drone to test.

Answer:

Example 3 - health apps

A health agency wants to assess the accuracy of two apps that use image analysis to determine people's blood pressure. One hundred patients are included into the experiment. Half of them have high blood pressure and the other half low blood pressure. In each of these patient groups, half of the patients are randomly assigned to either app. The health agency workers measure the blood pressure of all patients using established and reliable approaches and subsequently record a binary measure indicating whether high/low blood pressure was identified correctly.

Answer:

Example 4

A gas company wants to reduce the peak in demand on winter mornings (most people's boilers are programmed to start heating at 6am). The company has obtained permission to perform a trial with twelve comparable sets of ten households each. In the trial, the following factors are considered in a complete factorial design: price inducement (3 levels: low, medium, high), educational campaign (2 levels: present, not present). Households are randomly allocated to the treatments, ensuring that the same number of households per treatment. The company measures peak demand in cubic feet for each set of households.

Answer: