Engineering Maths EMAT 30007, Hypothesis Testing (1)

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1 Testing a population mean:

MATLAB function [h, p, ci, stats] = ttest(x, m, alpha, tail) performs a one sample t-test of the null hypothesis $H_0 : \mu = m$. It returns value $h = 0$ if the null hypothesis is accepted and $h = 1$ if it is rejected. The argument `tail` can be either 'both', 'right' or 'left' indicating that the alternate hypothesis is either two-tailed $\mu \neq m$, right-tailed $\mu > m$ or left-tailed $\mu < m$.
If the population variance is known, MATLAB function [h, p, ci, zval] = ztest(x, m, sigma, alpha, tail, dim) can be used.

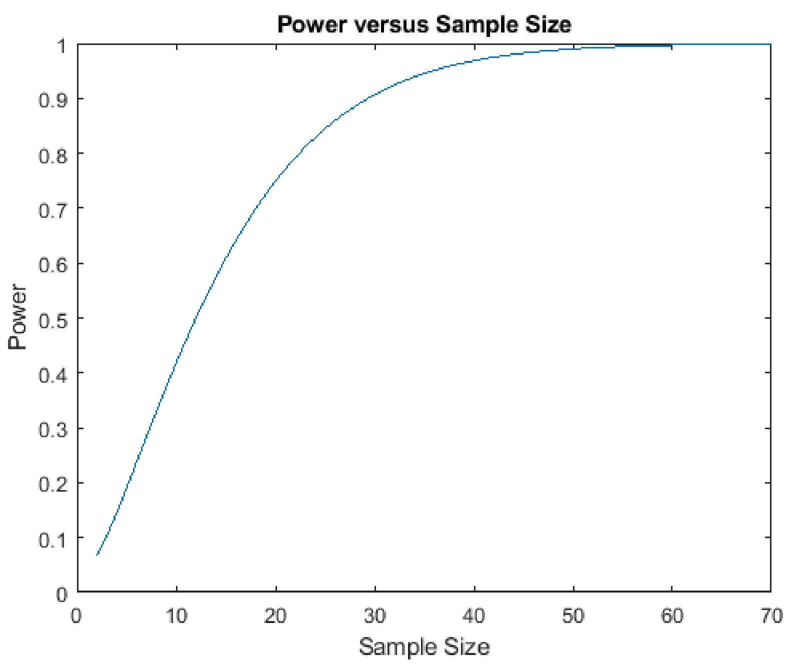
(a) The building specifications in a certain city require that the sewer pipe have a mean breaking strength of more that 2500 pounds per lineal foot. A manufacturer who would like to supply the city with the pipes has submitted a bid and hired an independent contractor. The contractor randomly selected seven sections of pipes and tested each for breaking strength with the following results: 2610, 2750, 2420, 2510, 2540, 2490, 2680.
Work out:

- Is there sufficient evidence to conclude that the manufacturer's sewer pipe meets the required specifications? Use a significance level (probability of Type 1 error) of $\alpha = 0.1$.
  What is H_0 and H_A?

  ```
  >> x=[2610, 2750, 2420, 2510, 2540, 2490, 2680]); % sample
  >> [h,p,ci,stats] = ttest(x, 2500, 0.1, 'right'); % one-tailed hypothesis test
  >> tv=tinv(0.1, 6) % rejection region of t-distribution with 6 degrees of freedom
  ```

  Use two methods for testing hypothesis: (1) analyse p-value and (2) perform significance test by comparing test statistic (`tstat` output in MATLAB) with the rejection region.

- Compute the `power` of the test (1 - probability Type 2 error) and interpret it.
  ```
  >> power = sampsizepwr('t',[2500 115.13],2571.4,[],7, 'Tail','right','Alpha',0.1);
  ```
- Determine the sample size required for the `power` = 0.9.
  ```
  >> sample_size = sampsizepwr('t',[2500 115.13],2571.4,0.9,[],'Tail','right','Alpha',0.1);
  ```
- Generate a power curve to visualize how the sample size affects the power of the test.
  ```
  >> nn = 1:70;
  >> plot_power = sampsizepwr('t',[2500 115.13],2571.4,[],nn)
  >> plot(plot_power)
  >> title('Power versus Sample Size')
  >> xlabel('Sample Size')
  >> ylabel('Power')
  ```



- Change significance level α to 0.05 and visualize a power curve. Will you need a larger sample size for the `power` $= 0.9$? How does decreasing the significance level affect Type 2 error?

(b) When asked "How much time will you require to complete the task?", people normally underestimate the time required. Would the opposite theory hold if the question was phrased "How much work could be completed in a given period of time?". For one study conducted by the psychologists students were asked how many minutes it would take to read a 32-page technical report. In a second study, 42 students were asked how many pages of a report they could read in 48 minutes. The data from the experiment are presented in the table below.

	Estimated time (in minutes)	Estimated number of pages
Sample size, n	40	42
Sample mean	60	28
Sample standard deviation	41	14

- The psychologists determined that the actual mean time it takes to read the report is $\mu = 48$ minutes. Is there evidence to support the theory that the students on average will overestimate the time it takes to read the report? Use $\alpha = 0.1$.
- The researchers determined that the acual mean number of pages of the report that are read within the allocated time is $\mu = 32$ pages. Is there evidence to support the claim that students on average will underestimate the number of report pages that can be read? Use $\alpha = 0.1$.
- The psychologists noted that the distributions for both estimated time and estimated number of pages is highly skewed (not normally distributed). Does it impact the conclusions in two previous items?

(c) Download file "CHEEKTEETH.xls" from Blackboard. Researchers recorded the dentary depth of molars for a sample of 18 cheek teeth extracted from primate skulls. Anthropologists know that the mean dentary depth of molars in species called "A" is 15 millimeters. Is there any evidence that the sample of 18 cheek teeth come from other primate species (other than species "A")?

2 Testing a population proportion:

(a) The Computer Security Institute (CSI) conducts an annual survey of computer crime in USA. CSI sends survey questionnaires to computer security personnel at USA corporations. Out of 351 organizations, 144 admitted unauthorized use of computer systems.

- Work out a point estimate $\hat{p}$
- Does the proportion of organizations with unauthorized use of computer systems differs from 0.35? Write down H_0 and H_A.
- Calculate `test statistic` using $Z = \frac{\hat{p}-p_0}{\sqrt{p_0 q_0/n}}$ Answer: Z=2.36
- Find the rejection region if $\alpha = 0.05$ (use `zinv` function)
- Using significance test (not p-value), decide weather H_0 hypothesis should be rejected or not
- Work out the p-value. Do you get the same conclusion? Answer: p-value is 0.0182
- Was the sample size large enough ($n\hat{p} \geq 4$ and $n\hat{q} \geq 4$) in order to use z-scores from normal distribution?

3 Kolmogorov-Smirnov test: `kstest`:

Kolmogorov-Smirnov test answers the following questions:
Do my data come from a normal distribution?
Do my data come from exponential distribution?
Do my data come from Weibull distribution?

*E*xample 1 The code below does not reject H_0 that the simulated distribution is a normal distribution.

```
> x = normrnd(1,2,35,1);
>> X=-6:0.5:8;
>> Y=normcdf(X,1,2);
>> cdf=[X' Y'];
>> H = kstest(x,cdf)
H =
     0
```

Example 2 The code below does rejects H_0 that the simulated distribution is a Weibull distribution.

```
>> x = unifrnd(0,4,40,1);
>> X=0:0.5:4;
>> Y=wblcdf(X,1,2);
>> cdf=[X' Y'];
>> H = kstest(x,cdf,0.01)
H =
     1
```

Download file from Blackboard "ANIMONY.xls". Use Kolmogorov-Smirnov test to verify if the values in the column "Strength" normally distributed? What are H_0 and H_A? Do not forget to visually inspect the distribution as well (e.g., use `normplot` function).