

A simple two parameter distribution for modelling neuronal activity and capturing neuronal association

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Abstract

Recent developments in electrophysiological technology have lead to an increase in the size of electrophysiological datasets. Consequently, there is a requirement for new analysis techniques that can make use of these new datasets, while remaining easy to use in practice. In this work, we fit the Conway-Maxwell-binomial distribution to spiking data read from a mouse exposed to visual stimuli.

1 Introduction

Motivate by pointing out how much computational power it can require to calculate n th order correlations.

Point out that we don't necessarily need to measure correlations anyway.

2 Results

3 Discussion

4 Data

We used data collected by Nick Steinmetz and his lab ‘CortexLab at UCL’ [9]. The data can be found online ¹ and are free to use for research purposes.

Two ‘Phase3’ Neuropixels [4] electrode arrays were inserted into the brain of an awake, head-fixed mouse for about an hour and a half. These electrode arrays recorded 384 channels of neural data each at 30kHz and less than $7\mu\text{V}$ RMS noise levels. The sites are densely spaced in a ‘continuous tetrode’-like arrangement, and a whole array records from a 3.8mm span of the brain. One array recorded from visual cortex, hippocampus, and thalamus, the other array recorded from motor cortex and striatum. The data were spike-sorted automatically by Kilosort and manually by N. Steinmetz using Phy. In total 831 well-isolated individual neurons were identified.

4.1 Experimental protocol

The mouse was shown a visual stimulus on three monitors placed around the mouse at right angles to each other, covering about ± 135 degrees azimuth and ± 35 degrees elevation.

The stimulus consisted of sine-wave modulated full-field drifting gratings of 16 drift directions ($0^\circ, 22.5^\circ, \dots, 337.5^\circ$) with 2Hz temporal frequency and 0.08 cycles/degree spatial frequency displayed for 2 seconds plus a blank condition. Each of these 17 conditions were presented 10 times in a random order across 170 different trials. There were therefore 160 trials with a visual stimulus present, and 10 trials without a visual stimulus.

5 Methods

Details about all kinds of things here.

¹<http://data.cortexlab.net/dualPhase3/>

5.1 Binning data

We converted the spike times for each cell into spike counts by putting the spike times into time bins of a given ‘width’ (in seconds). We used time bins of 1ms, 5ms, and 10ms. We used different time bin widths to assess the impact of choosing a bin width.

5.2 Number of *active* neurons

To count the number of active neurons in each neuronal ensemble, we split the time interval for each trial into bins of a given width. We counted the number of spikes fired by each cell in each bin. If a cell fired *at least* one spike in a given bin, we regarded that cell as active in that bin. We recorded the number of active cells in every bin, and we recorded each cell’s individual spike counts.

It should be noted that when we used a bin width of 1ms, the maximum number of spikes in any bin was 1. For the wider time bins, some bins had spike counts greater than 1. Consequently when using a bin width of 1ms, the number of active neurons and the total spike count of a given bin were identical. But for wider bin widths, the total spike count was greater than the number of active neurons.

So for the 1ms bin width, the activity of a neuron and the number of spikes fired by that neuron in any bin can be modelled as a Bernoulli variable. But for wider time bins, only the activity can be modelled in this way.

5.3 Moving windows for measurements

When taking measurements (e.g. moving average over the number of active neurons) or fitting distributions (eg. the beta binomial distribution) we slid a window containing a certain number of bins across the data, and made our measurements at each window position. For example, when analysing 1ms bin data, we used a window containing 100 bins, and we slid the window across the time interval for each trial moving 10 bins at a time. So that for 2560ms of data, we made 246 measurements.

For the 5ms bin width data, we used windows containing 40 bins, and slid the window 2 bins at a time when taking measurements.

For the 10ms bin width data, we used windows containing 40 bins, and slid the window 1 bin at a time when taking measurements.

72 By continuing to use windows containing 40 bins, we retained statistical power
 73 but sacrificed the number of measurements taken.

74 **5.4 Fano factor**

The *Fano factor* of a random variable is defined as the ratio of the variable's variance to its mean.

$$F = \frac{\sigma^2}{\mu} \quad (1)$$

75 We measured the Fano factor of the spike count of a given cell by measuring the
 76 mean and variance of the spike count across trials, and taking the ratio of those two
 77 quantities. When calculated in this way the Fano factor can be used as a measure of
 78 neural variability. This is similar to the calculation used in [2].

79 **5.5 Probability Distributions suitable for modelling ensemble activity**

81 **5.5.1 Binomial distribution**

The binomial distribution is a discrete probability distribution that can be thought of as the sum of independent Bernoulli random variables, each with the same probability of success. The binomial distribution has two parameters, n , the number of Bernoulli variables, and p , the probability of success for each of these variables. A random variable with the binomial distribution can take values from $\{0, \dots, n\}$. The probability mass function is

$$P(k; n, p) = \binom{n}{k} p^k (1 - p)^{n-k} \quad (2)$$

If we have N samples from a binomial distribution and we know n but want to estimate p , we can maximise the log likelihood function

$$L(p) = \log P(\{k_1, \dots, k_N\}; n, p) \quad (3)$$

$$= \sum_{i=1}^N \log \binom{n}{k_i} + k_i \log p + (n - k_i) \log(1 - p) \quad (4)$$

82 If we do not know n there is no closed form way way of maximising this equa-
83 tion. Therefore the binomial distribution is generally only used in cases where we do
84 know n . Consequently, the distribution is practically a one parameter distribution.

85 **5.5.2 Beta-binomial distribution**

86 **5.5.3 Conway-Maxwell-binomial distribution**

87 **5.6 Goodness-of-fit**

88 **5.7 Spike count correlations**

89 **6 Discussion**

90 Point out that the Conway-Maxwell-binomial distribution could be used to measure
91 activity and association without having to sort the voltage traces into spikes. That
92 does defeat the purpose slightly, however.

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