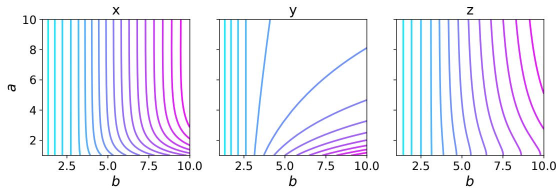
Unsupervised learning of parametric optimization problems

$$
\begin{aligned}
\min_{x,z,y} \quad & \frac{1}{xyz} & & \\
\text{s.t.} \quad & (xy + xz + yz) - a & \leq 0, & \\
& y^b - x & \leq 0, & \qquad (1)\\
& x & > 0, & \\
& y & > 0, & \\
& z & > 0, &
\end{aligned}
$$

for $a \in \mathbb{R}_{++}$ and $b \in \mathbb{R}$.

Parametric optimization problems and their solutions play an increasingly important role in our daily lives as well as in industrial applications. A popular example is model predictive control, where the optimal inputs to apply to a system are obtained by solving an optimization problem that has the current state of the system as parameters. However, even modern computing capacities are not always sufficient to solve these optimization problems fast enough for real-time applications. The optimal solution to a parametric optimization problem can be expressed as a function of the parameters that define the problem, e.g., $\boldsymbol{x}^*(\boldsymbol{\theta})$ with $\boldsymbol{x}^* = [x^*, y^*, z^*]^\top$ and $\boldsymbol{\theta} = [a, b]^\top$ for $a > 0$ in the case of problem 1. To circumvent the computational capacity bottleneck, artificial neural networks (ANNs) can be used to learn this mapping between parameters and optimal solutions. However, supervised training of ANNs requires labeled training data and thus the solutions to these parametric optimization problems. Obtaining this data can be difficult and computationally expensive. In this project, students are therefore investigating an unsupervised training algorithm to learn the mapping between the parameter defining the non-linear convex optimization problem 1 and the corresponding optimal solutions [1].



Solution map for problem 1. The contour plots show how the optimal solution changes as a function of the parameters a and b.

Task description

In this project, students will investigate and compare an unsupervised and a supervised approach to train ANNs to solve problem 1. For the supervised approach, the first major task is to generate training and validation data by solving problem 1 for different parameter combinations (a, b). It is recommended to use the Python library cvxpy for this purpose. Then, students have to use the obtained data to train an ANN that predicts the solution of problem 1 by classical supervised training.

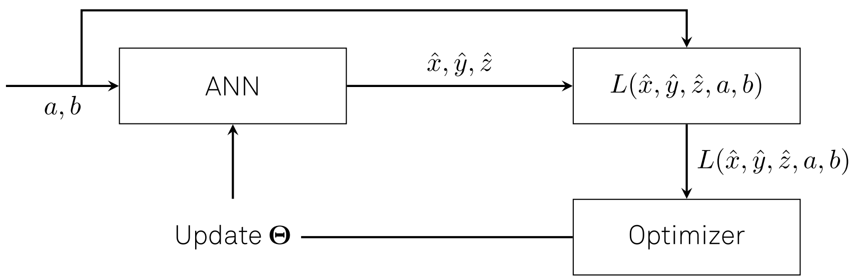


Figure 1: Illustration of the unsupervised training algorithm. The ANN computes a prediction of the optimal solution $(\hat{x}, \hat{y}, \hat{z})$ based on the parameter input (a, b). The predicted solution is then used along with the parameter input to evaluate the custom loss function L, which represents the parametric optimization problem [1].

The second approach investigated for learning the mapping between parameters and optimal solutions is unsupervised, i.e., no labeled data is used for training. Figure 1 illustrates the unsupervised

training algorithm. For the unsupervised approach, a penalty formulation of the problem is used as a loss function instead of the mean square error to train the ANN.
Students have the task of formulating their own custom loss function representing problem 1. To accomplish this, the constrained optimization problem must be reformulated into an unconstrained form. This can be achieved by adding the constraints as penalty terms to the cost function. The custom loss function should then be used to train a model using the algorithm shown in Figure 1. Once the unsupervised and supervised approaches are implemented, the models trained with these approaches should be compared in terms of their performance on a validation dataset. Experiments on the training efficiency of the two approaches should also be conducted and the results visualized.

Mandatory tasks

The following tasks **have to be completed** in order to pass the project.

- **Supervised Approach** - Generate data by solving the optimization problem
- **Supervised Approach** - Train a model with supervised training and evaluate its performance.
- **Unsupervised Approach** - Reformulate the optimization problem into an unconstrained form.
- **Unsupervised Approach** - Train a model with the unsupervised approach and evaluate its performance.
- Compare both approaches in terms of model performance and training efficiency and visualize your results.

Additional tasks

Below are **suggested** additional tasks to obtain good or excellent grades for the project. We want to emphasize that students are encouraged to come up with their own ideas for additional investigations and not all of the suggestions below must be included for an excellent grade.

- Tune and investigate hyper-parameters of the proposed methods.
- Investigate the influence of weighting factors in your loss function
- Formulate alternative loss functions, e.g., by reformulating inequality constraints into equality constraints using slack variables.

Deliverables

The following materials have to be submitted **before the deadline** communicated via Moodle:

- **Recorded final presentation** (video screencast). The presentation must be **5-7 minutes** (for the entire group) and the file should not exceed **200 mb**. Highlight on the slides which group member(s) are responsible.
- **Written report** to present and discuss the obtained results. You must use the supplied template on Moodle and write no more than **3-4 pages** (for the entire group). Highlight which group member worked on which section.
- **Source code** of your project. Please ensure that the code is executable and optionally add a short explanation of the structure (readme).

Please ensure that all formal conditions (e.g. page limits, highlight responsible author) are satisfied, as we will deduct points for significant violations. Please submit all deliverables via Moodle.

Responsible tutor

Please address questions to:

Name	Contact
Felix Brabender	felix.brabender@tu-dortmund.de

References

[1] Rasoul Nikbakht, Anders Jonsson, and Angel Lozano. “Unsupervised Learning for Parametric Optimization”. In: ***IEEE Communications Letters*** 25.3 (Mar. 2021), pp. 678–681. ISSN: 1089-7798, 1558-2558, 2373-7891. DOI: 10.1109/LCOMM.2020.3027981. URL: https://ieeexplore.ieee.org/document/9210010/.