

$$C(X) \rightarrow D(X)$$

$$D(X) \rightarrow C(X)$$

and similarly for *equivalentProperty*. Transitivity of a property  $P$  is easily expressed as

$$P(X, Y), P(Y, Z) \rightarrow P(X, Z)$$

Now we turn to Boolean operators. We can state that the intersection of classes  $C_1$  and  $C_2$  is a subclass of  $D$  as follows:

$$C_1(X), C_2(X) \rightarrow D(X)$$

In the other direction, we can state that  $C$  is a subclass of the intersection of  $D_1$  and  $D_2$  as follows:

$$C(X) \rightarrow D_1(X)$$

$$C(X) \rightarrow D_2(X)$$

For union, we can express that the union of  $C_1$  and  $C_2$  is a subclass of  $D$  using the following pair of rules:

$$C_1(X) \rightarrow D(X)$$

$$C_2(X) \rightarrow D(X)$$

However, the opposite direction is outside the expressive power of Horn logic. To express the fact that  $C$  is a subclass of the union of  $D_1$  and  $D_2$  would require a disjunction in the head of the corresponding rule, which is not available in Horn logic. Note that there are cases where the translation is possible. For example, when  $D_1$  is a subclass of  $D_2$ , then the rule  $C(X) \rightarrow D_2(X)$  is sufficient to express that  $C$  is a subclass of the union of  $D_1$  and  $D_2$ . The point is that there is not a translation that works in all cases.

Finally, we briefly discuss some forms of restriction in OWL. The OWL statement