5.3.1 Rules

A rule has the form

$$B_1, \ldots, B_n \rightarrow A$$

where $A, B_1, \ldots, B_n$ are atomic formulas. A is the *head* of the rule, and $B_1, \ldots, B_n$ are the *premises* of the rule. The set $\{B_1, \ldots, B_n\}$ is referred to as the *body* of the rule.

The commas in the rule body are read conjunctively: if B_1 and B_2 and ... and B_n are true, then A is also true (or equivalently, to prove A it is sufficient to prove all of $B_1, \ldots, B_n$).

Note that variables may occur in $A, B_1, \ldots, B_n$. For example,

$$loyalCustomer(X), age(X) > 60 \rightarrow discount(X)$$

This rule is applied for *any* customer: if a customer happens to be loyal and over 60, then she gets the discount. In other words, the variable X is implicitly universally quantified (using $\forall X$). In general, all variables occurring in a rule are implicitly universally quantified.

In summary, a rule r

$$B_1, \ldots, B_n \rightarrow A$$

is interpreted as the following formula, denoted by $pl(r)$:

$$\forall X_1 \ldots \forall X_k((B_1 \wedge \ldots \wedge B_n) \rightarrow A)$$

or equivalently,

$$\forall X_1 \ldots \forall X_k(A \vee \neg B_1 \vee \ldots \vee \neg B_n)$$

where $X_1, \ldots, X_k$ are all variables occurring in $A, B_1, \ldots, B_n$.