Axioms of Probablity

Axiom 1: Non-negativity$P(A) \geqslant 0$

Axiom 2: Normalization $P(\Omega)=1$

Axiom 3: **P(A U B)** = P(A) + P(B)

if A and B are disjoint, or mutualy exclusive
then P(A n B) = 0

Probablity Rules:

1. **Addition rule**: P(A or B) = P(A u B) = P(A) + P(B) + P(A n B)

if A and B are mutually execlusive(if they dont occure at the same time) then P(A n B) = 0

example: Probablity of geting Head or Tail on a single coin toss

Example: getting Head or tail on first trial

P(H or T) = P(H u T) = p(H) + P(T) + P(H n T)   but p(H n T) = 0

1/2 + 1/2 = 1

2. **multiplication rule** (product rule) or Joint probablity

P(A and B) = P(A n B) = P(A/B) P(B)

if A is independent of B (if the outcome of A does not depend on B) then

P(A|B) = P(A)

3. **Bays theorm P(A|B)** = P(B|A) P(A) / P(B)

Question:
Probablity of geting 2 heads in 3 consecative tosses

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<tr><td rowspan="4">H</td><td rowspan="2">H</td><td>H</td></tr>
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HHH, HHT, HTH, **HTT**, THH, **THT**, TTH, **TTT** **3/8**

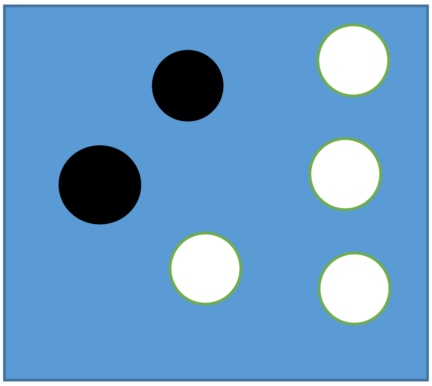
Probablity of hetting HT

P(HH) = 1/2 x 1/2

this is like joint porbablity(multiplication rule)

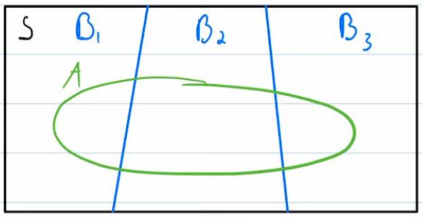
P(A n B) = P(A/B) P(B) But since the outcome of the first doesnot depend on the second one
we can say P(A n B) = P(A) P(B)

1. If I have a box that contains 6 balls where 2 are black and 4 are white, what is the probability of getting a white ball on the second draw without replacement?



Black and White	2/6 x 4/5 = 8/30	20/30 = 2/3
or	+	
white and black	4/6 x 3/5 = 12/30	

Total Probablity



$A = (A \cap B_1) \cup (A \cap B_2) \cup (A \cap B_3)$

$P(A \cap B) = P(A|B)\, P(B)$

$$P(A) = P(A \cap B_1) + P(A \cap B_2) + P(A \cap B_3)$$

suppose B1= Brazil, B2 = Bolivia, and B3 = Bangladesh are countries..
Fb (the green one) = who can play footbal

supose

10% of Brazil, = p(fb | brazil)

5% percent of bolivia, p(fb | bolivia)

7% Bangladesh. p(fb | bangladesh)

what is the probablity that a randomly selected person from those 3 countries can play football. (Assuming a Country can be selected equally likely)

p(fb) = p(fb n Brazil) + p(fb n bolivia) + p(fb n bangladesh)

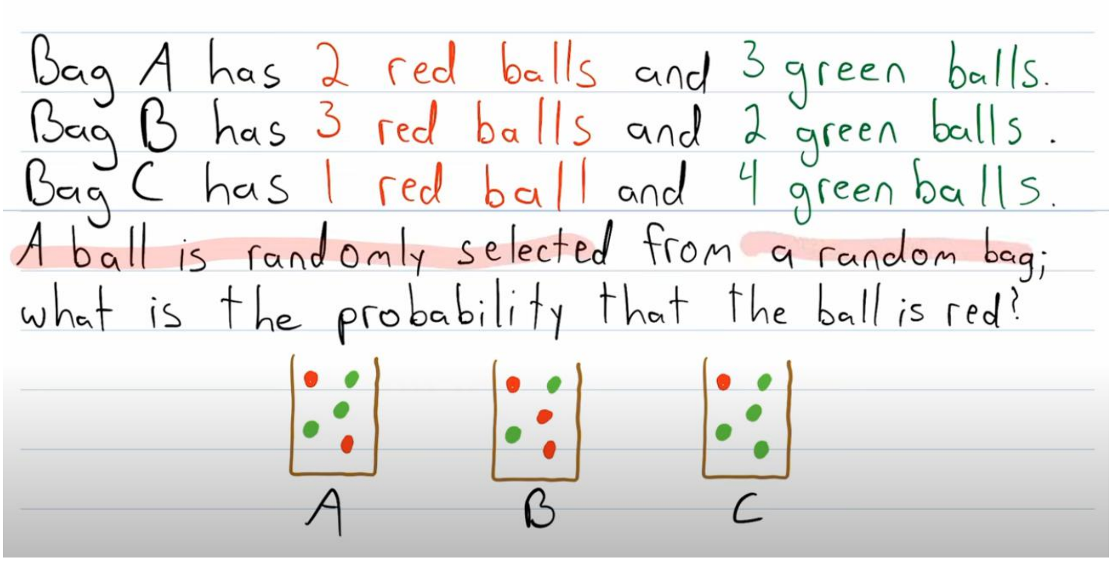
but according to multiplication rule

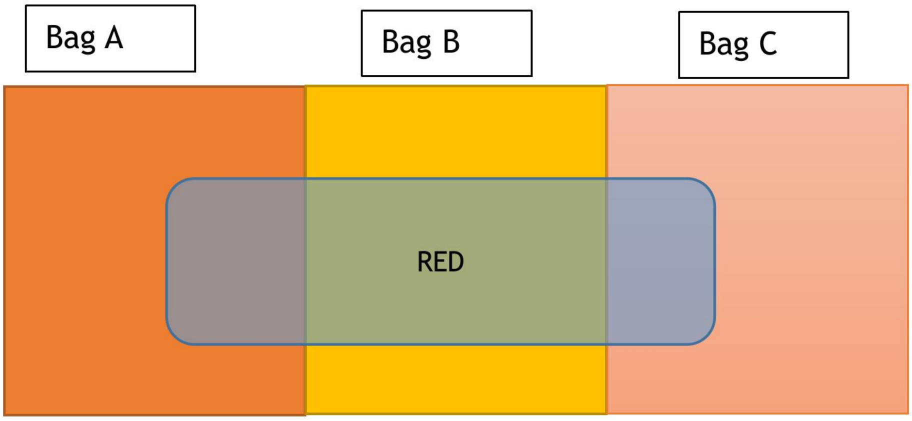
p(fb n Brazil) = p(fb | Brazil) p(Brazil)

but p(Brazil) = p(bangladesh) = p(bangladesh) = 1/ 3

:-

1.the Law of total probablity





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P(red) = P(Bag A n Red) + P(Bag B n Red) + P(Bag C n Red)

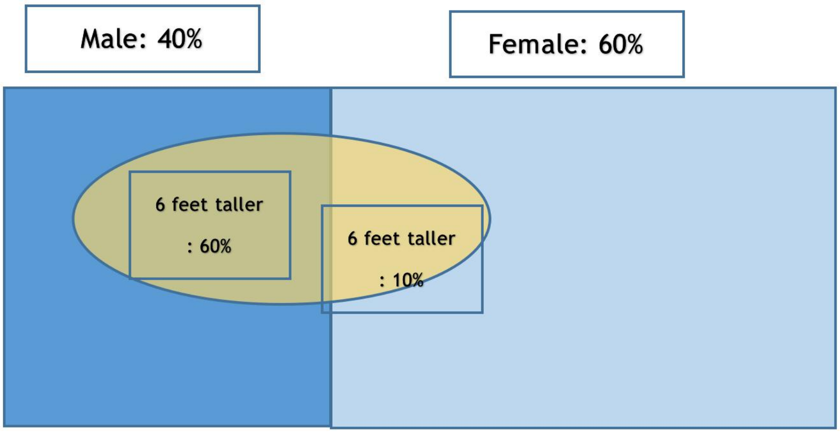
= P(BagA).P(RED| Bag A) + P(Bag B).P(RED| Bag B) + P(BagA).P(RED| Bag B)

= (1/3 X 2/5) + (1/3 X 3/5) + (1/3 X 1/5)

=

2. The law of total probality

In a class, 40% of students are male and 60% are female. Sixty percent of the males are taller than 6 feet, and 10% of the females are taller than 6 feet. What percent of the class is shorter than 6 feet?



P(taller than 6ft) = P(6ft_taller ∩ male) + P(6ft_taller ∩ female)

= 0.4 x 0.6 + 0.6 x 0.1

= 0.7

p(shorter than 6 feet) = 1 - p (taller than 6ft)

= 1 - 0.7 = 0.3 = 30%

3. Bayes thorem

$P(D) = 1\% = 0.01$

$P(+/Dc) = 10\% = 0.1$

$P(-/D) = 5\% = 0.05$

$P(D/+) = ?$

$P(D/+) = P(+|D)P(D) / P(+)$

BUT $P(+|D) = 100\% - 5\% = 95\%$

$P(+) = P(+ \text{ n } D) + P(+ \text{ n } Dc)$

$= P(+|D)P(D) + P(+|Dc)P(Dc)$

$= 0.95 \times 0.01 + 0.1 \times (1 - 0.01)$

$= 0.0095 + 0.099$

$= 0.1085$

$P(D/+) = 0.0095/0.1085$

=

4. Bays theorem Question:

A screening test has been developed for a very rare form of cancer.

Given the following information:

1. The prevalence of the cancer in the general population is **0.1%**.
2. The test is **99% accurate** at detecting the cancer when it is present. (I.e., the probability of a positive test result given a person has the disease is 0.99).
3. The probability of a false positive is **1%**. (I.e., the probability of a positive test result given a person does *not* have the disease is 0.01).

Question:

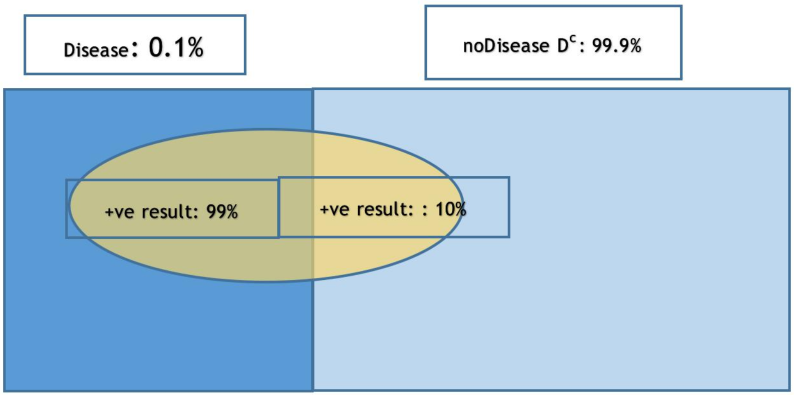
A person is randomly selected from the population and receives a positive test result. What is the probability that this person **actually has the cancer**?

Event / Definition	**Probability (Prior / Likelihood)**	**Value**
D - Has the Disease (Cancer)	P(D)	0.001 (0.1%)
D^c - Does Not Have the Disease	P(Dc)	0.999 (99.9%)
+ - Positive Test Result (given disease)	P(+ | D)	0.99 (99%)
+ - Positive Test Result (given no disease)	P(+ |Dc)	0.01 (1%)
Goal: Probability of Disease given Positive Test	P(D | +)	?

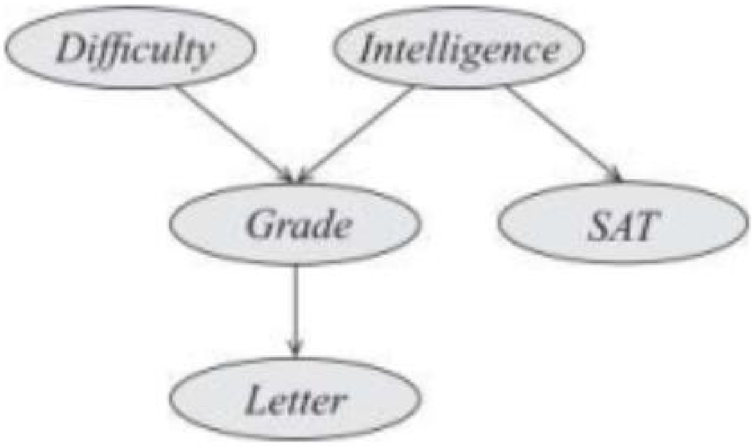
$$P(D \mid +) = \frac{P(+ \mid D)\ P(D)}{P(+)}$$

but P(+) is not given so we have to compute P(+) using total probablity

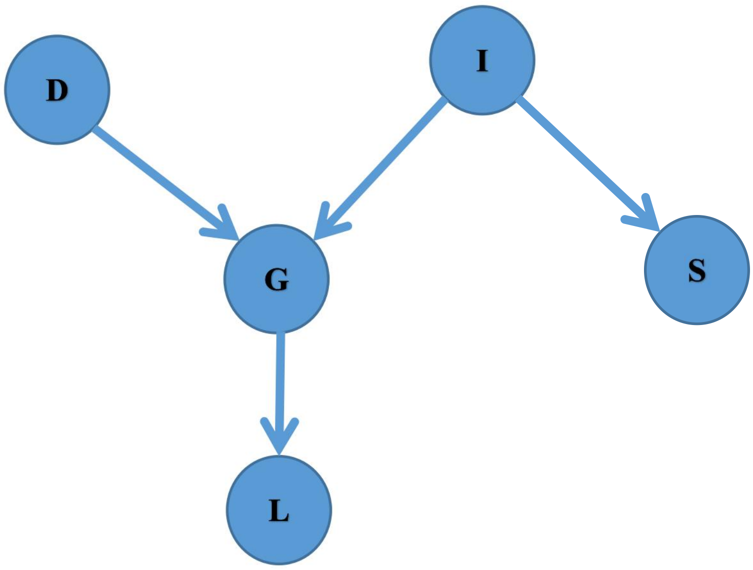
$P(+) = P(+) = P(+ \mid D)P(D) + P(+ \mid D^c)P(D^c)$



Factorize this Baysian Network



$$P(D,I,G,S,L) = P(D)P(I)P(G \mid D,I)P(S \mid I)P(L \mid G)$$



Structure	When BLOCKED?	Example
CHAIN	Condition on middle	$X \rightarrow A \rightarrow Y$ + A = BLOCKED
FORK	Condition on middle	$X \leftarrow A \rightarrow Y$ + A = BLOCKED
COLLIDER	DON'T condition on middle	$X \rightarrow A \leftarrow Y$ + no A = BLOCKED

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$P(D \perp I, S) =$ True

becasue D->G->I is a collider

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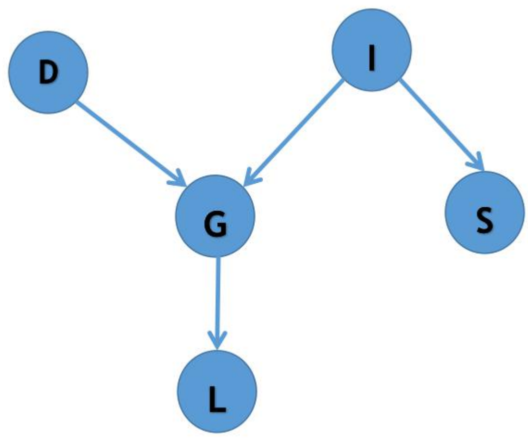

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$P(D \perp I, S \mid G) =$ False

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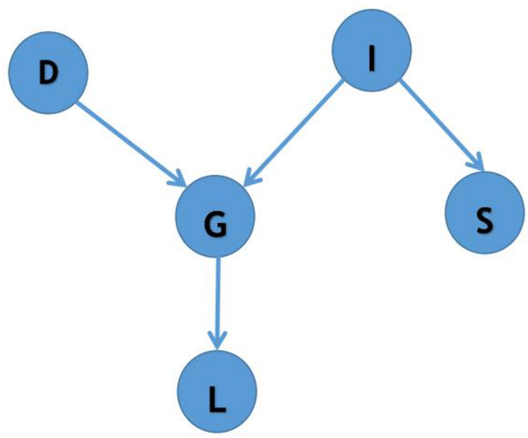

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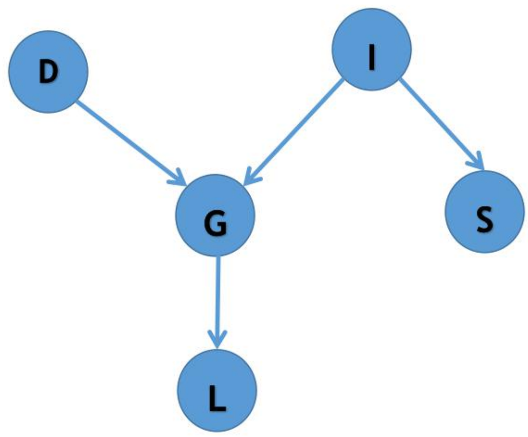
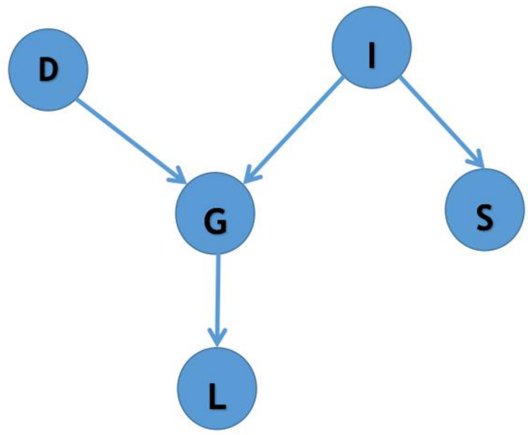
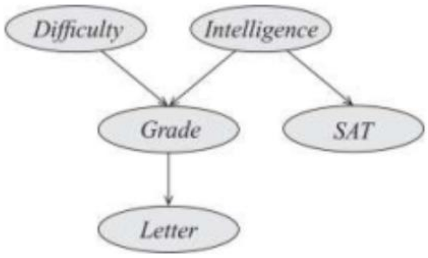
special case **P(D ⊥ I | L)** **False (Special case)** **collider G will be opend up because of L**	
P(D ⊥ I) = True **P(D ⊥ I | G) = False**	

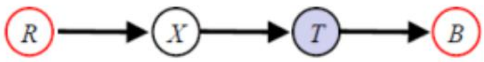
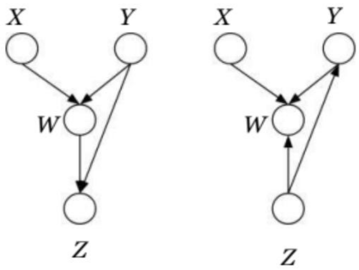
$G \perp S \mid \emptyset?$

$D \perp L \mid G?$

$D \perp I, S \mid \emptyset?$

$D, L \perp S \mid I, G?$



▪ Query: $R \perp\!\!\!\perp B \mid T$  	True
do these DAGs have the same set of CIs? no!  $X \perp Z \mid W$	