# Fully connected networks

In the previous notebook, you implemented a simple two-layer neural network class. However, this class is not modular. If you wanted to change the number of layers, you would need to write a new loss and gradient function. If you wanted to optimize the network with different optimizers, you'd need to write new training functions. If you wanted to incorporate regularizations, you'd have to modify the loss and gradient function.

Instead of having to modify functions each time, for the rest of the class, we'll work in a more modular framework where we define forward and backward layers that calculate losses and gradients respectively. Since the forward and backward layers share intermediate values that are useful for calculating both the loss and the gradient, we'll also have these function return "caches" which store useful intermediate values.

The goal is that through this modular design, we can build different sized neural networks for various applications.

In this HW #3, we'll define the basic architecture, and in HW #4, we'll build on this framework to implement different optimizers and regularizations (like BatchNorm and Dropout).

### Modular layers

This notebook will build modular layers in the following manner. First, there will be a forward pass for a given layer with inputs (x) and return the output of that layer (out) as well as cached variables (cache) that will be used to calculate the gradient in the backward pass.

```
def layer_forward(x, w):
    """ Receive inputs x and weights w """
    # Do some computations ...
    z = # ... some intermediate value
    # Do some more computations ...
    out = # the output

cache = (x, w, z, out) # Values we need to compute gradients
    return out, cache
```

The backward pass will receive upstream derivatives and the cache object, and will return gradients with respect to the inputs and weights, like this:

```
def layer_backward(dout, cache):
    """
    Receive derivative of loss with respect to outputs and cache,
    and compute derivative with respect to inputs.
    """
    # Unpack cache values
    x, w, z, out = cache
```

```
# Use values in cache to compute derivatives
          dx = \# Derivative of loss with respect to x
          dw = # Derivative of loss with respect to w
          return dx, dw
In [ ]:
         ## Import and setups
         import time
         import numpy as np
         import matplotlib.pyplot as plt
         from nndl.fc net import *
         from utils.data_utils import get_CIFAR10_data
         from utils.gradient_check import eval_numerical_gradient, eval_numerical_gradient_arm
         from utils.solver import Solver
         %matplotlib inline
         plt.rcParams['figure.figsize'] = (10.0, 8.0) # set default size of plots
         plt.rcParams['image.interpolation'] = 'nearest'
         plt.rcParams['image.cmap'] = 'gray'
         # for auto-reloading external modules
         # see http://stackoverflow.com/questions/1907993/autoreload-of-modules-in-ipython
         %load ext autoreload
         %autoreload 2
         def rel error(x, y):
           """ returns relative error """
           return np.max(np.abs(x - y) / (np.maximum(1e-8, np.abs(x) + np.abs(y))))
        The autoreload extension is already loaded. To reload it, use:
          %reload ext autoreload
In [ ]:
         # Load the (preprocessed) CIFAR10 data.
         data = get CIFAR10 data()
         for k in data.keys():
           print('{}: {} '.format(k, data[k].shape))
        X_train: (49000, 3, 32, 32)
        y_train: (49000,)
        X_val: (1000, 3, 32, 32)
        y val: (1000,)
        X_test: (1000, 3, 32, 32)
```

## **Linear layers**

y\_test: (1000,)

In this section, we'll implement the forward and backward pass for the linear layers.

The linear layer forward pass is the function affine\_forward in nndl/layers.py and the backward pass is affine\_backward.

After you have implemented these, test your implementation by running the cell below.

#### Affine layer forward pass

Implement affine forward and then test your code by running the following cell.

```
In [ ]:
         # Test the affine_forward function
         num inputs = 2
         input\_shape = (4, 5, 6)
         output dim = 3
         input size = num inputs * np.prod(input shape)
         weight size = output dim * np.prod(input shape)
         x = np.linspace(-0.1, 0.5, num=input size).reshape(num inputs, *input shape)
         w = np.linspace(-0.2, 0.3, num=weight_size).reshape(np.prod(input_shape), output_dim)
         b = np.linspace(-0.3, 0.1, num=output dim)
         out, _ = affine_forward(x, w, b)
         correct_out = np.array([[ 1.49834967, 1.70660132, 1.91485297],
                                 [ 3.25553199, 3.5141327,
                                                             3.77273342]])
         # Compare your output with ours. The error should be around 1e-9.
         print('Testing affine_forward function:')
         print('difference: {}'.format(rel_error(out, correct_out)))
```

Testing affine\_forward function: difference: 9.769849468192957e-10

#### Affine layer backward pass

Implement affine\_backward and then test your code by running the following cell.

```
In [ ]:
         # Test the affine_backward function
         x = np.random.randn(10, 2, 3)
         w = np.random.randn(6, 5)
         b = np.random.randn(5)
         dout = np.random.randn(10, 5)
         dx num = eval numerical gradient array(lambda x: affine forward(x, w, b)[0], x, dout)
         dw_num = eval_numerical_gradient_array(lambda w: affine_forward(x, w, b)[0], w, dout)
         db num = eval numerical gradient array(lambda b: affine forward(x, w, b)[0], b, dout)
         _, cache = affine_forward(x, w, b)
         dx, dw, db = affine_backward(dout, cache)
         # The error should be around 1e-10
         print('Testing affine backward function:')
         print('dx error: {}'.format(rel error(dx num, dx)))
         print('dw error: {}'.format(rel_error(dw_num, dw)))
         print('db error: {}'.format(rel_error(db_num, db)))
```

Testing affine\_backward function: dx error: 1.4468800761634958e-10 dw error: 2.217717455215615e-09 db error: 1.5708548011392253e-11

### **Activation layers**

In this section you'll implement the ReLU activation.

#### **ReLU forward pass**

Implement the relu\_forward function in nndl/layers.py and then test your code by running the following cell.

Testing relu\_forward function: difference: 4.999999798022158e-08

#### **ReLU backward pass**

Implement the relu\_backward function in nndl/layers.py and then test your code by running the following cell.

Testing relu\_backward function: dx error: 3.2756090330175045e-12

### Combining the affine and ReLU layers

Often times, an affine layer will be followed by a ReLU layer. So let's make one that puts them together. Layers that are combined are stored in nndl/layer\_utils.py .

#### Affine-ReLU layers

We've implemented affine\_relu\_forward() and affine\_relu\_backward in nndl/layer\_utils.py . Take a look at them to make sure you understand what's going on. Then run the following cell to ensure its implemented correctly.

```
In [ ]:
         from nndl.layer_utils import affine_relu_forward, affine_relu_backward
         x = np.random.randn(2, 3, 4)
         w = np.random.randn(12, 10)
         b = np.random.randn(10)
         dout = np.random.randn(2, 10)
         out, cache = affine relu forward(x, w, b)
         dx, dw, db = affine_relu_backward(dout, cache)
         dx_num = eval_numerical_gradient_array(lambda x: affine relu forward(x, w, b)[0], x,
         dw num = eval numerical gradient array(lambda w: affine relu forward(x, w, b)[0], w,
         db num = eval numerical gradient array(lambda b: affine relu forward(x, w, b)[0], b,
         print('Testing affine_relu_forward and affine_relu_backward:')
         print('dx error: {}'.format(rel error(dx num, dx)))
         print('dw error: {}'.format(rel_error(dw_num, dw)))
         print('db error: {}'.format(rel_error(db_num, db)))
        Testing affine relu forward and affine relu backward:
```

dx error: 1.268297939177043e-10

dw error: 9.469295601105647e-10 db error: 3.275562045851031e-12

#### Softmax losses

You've already implemented it, so we have written it in layers.py . The following code will ensure its working correctly.

```
In []:
    num_classes, num_inputs = 10, 50
    x = 0.001 * np.random.randn(num_inputs, num_classes)
    y = np.random.randint(num_classes, size=num_inputs)

dx_num = eval_numerical_gradient(lambda x: softmax_loss(x, y)[0], x, verbose=False)
    loss, dx = softmax_loss(x, y)

# Test softmax_loss function. Loss should be 2.3 and dx error should be 1e-8
    print('\nTesting softmax_loss:')
    print('loss: {}'.format(loss))
    print('dx error: {}'.format(rel_error(dx_num, dx)))
Testing softmax loss:
```

loss: 2.30251094386135 dx error: 7.1057208864523925e-09

### Implementation of a two-layer NN

In nndl/fc\_net.py , implement the class TwoLayerNet which uses the layers you made here. When you have finished, the following cell will test your implementation.

```
In [ ]:
         N, D, H, C = 3, 5, 50, 7
         X = np.random.randn(N, D)
         y = np.random.randint(C, size=N)
         std = 1e-2
         model = TwoLayerNet(input_dim=D, hidden_dims=H, num_classes=C, weight_scale=std)
         print('Testing initialization ... ')
         W1_std = abs(model.params['W1'].std() - std)
         b1 = model.params['b1']
         W2_std = abs(model.params['W2'].std() - std)
         b2 = model.params['b2']
         assert W1 std < std / 10, 'First layer weights do not seem right'
         assert np.all(b1 == 0), 'First layer biases do not seem right'
         assert W2_std < std / 10, 'Second layer weights do not seem right'</pre>
         assert np.all(b2 == 0), 'Second layer biases do not seem right'
         print('Testing test-time forward pass ... ')
         model.params['W1'] = np.linspace(-0.7, 0.3, num=D*H).reshape(D, H)
         model.params['b1'] = np.linspace(-0.1, 0.9, num=H)
         model.params['W2'] = np.linspace(-0.3, 0.4, num=H*C).reshape(H, C)
         model.params['b2'] = np.linspace(-0.9, 0.1, num=C)
         X = np.linspace(-5.5, 4.5, num=N*D).reshape(D, N).T
         scores = model.loss(X)
         correct_scores = np.asarray(
           [[11.53165108, 12.2917344,
                                        13.05181771, 13.81190102, 14.57198434, 15.33206765,
            [12.05769098, 12.74614105, 13.43459113, 14.1230412, 14.81149128, 15.49994135,
            [12.58373087, 13.20054771, 13.81736455, 14.43418138, 15.05099822, 15.66781506,
         scores_diff = np.abs(scores - correct_scores).sum()
         assert scores diff < 1e-6, 'Problem with test-time forward pass'
         print('Testing training loss (no regularization)')
         y = np.asarray([0, 5, 1])
         loss, grads = model.loss(X, y)
         correct loss = 3.4702243556
         assert abs(loss - correct loss) < 1e-10, 'Problem with training-time loss'</pre>
         model.reg = 1.0
         loss, grads = model.loss(X, y)
         correct loss = 26.5948426952
         assert abs(loss - correct loss) < 1e-10, 'Problem with regularization loss'</pre>
         for reg in [0.0, 0.7]:
           print('Running numeric gradient check with reg = {}'.format(reg))
           model.reg = reg
           loss, grads = model.loss(X, y)
           for name in sorted(grads):
             f = lambda : model.loss(X, y)[0]
             grad num = eval numerical gradient(f, model.params[name], verbose=False)
             print('{} relative error: {}'.format(name, rel_error(grad_num, grads[name])))
```

```
Testing initialization ...

Testing test-time forward pass ...

Testing training loss (no regularization)

Running numeric gradient check with reg = 0.0

W1 relative error: 1.8336562786695002e-08

W2 relative error: 3.201560569143183e-10

b1 relative error: 9.828315204644842e-09

b2 relative error: 4.329134954569865e-10

Running numeric gradient check with reg = 0.7

W1 relative error: 3.11964217216572e-07

W2 relative error: 7.976652806155026e-08

b1 relative error: 1.3467619389117099e-08

b2 relative error: 9.089617896905665e-10
```

#### Solver

We will now use the utils Solver class to train these networks. Familiarize yourself with the API in utils/solver.py . After you have done so, declare an instance of a TwoLayerNet with 200 units and then train it with the Solver. Choose parameters so that your validation accuracy is at least 50%.

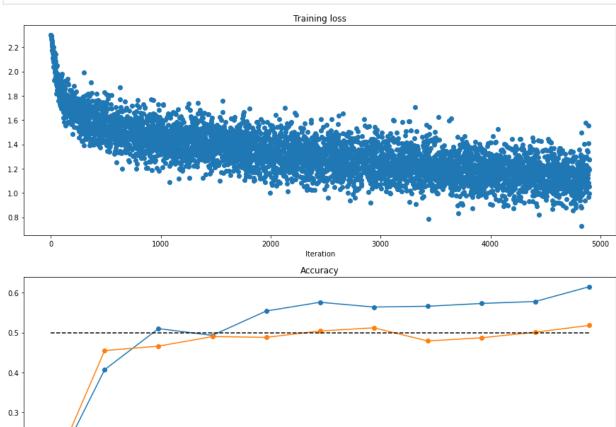
```
In [ ]:
      model = TwoLayerNet()
      solver = None
      # YOUR CODE HERE:
        Declare an instance of a TwoLayerNet and then train
        it with the Solver. Choose hyperparameters so that your validation
        accuracy is at least 50%. We won't have you optimize this further
        since you did it in the previous notebook.
      solver = Solver(model, data,
                   update rule='sgd',
                   optim config={
                    'learning rate': 1e-3,
                   1r decay=0.95,
                   num epochs=10, batch size=100,
                   print every=100)
      solver.train()
      # END YOUR CODE HERE
      # ------ #
```

```
(Iteration 1 / 4900) loss: 2.299471
(Epoch 0 / 10) train acc: 0.137000; val acc: 0.142000
(Iteration 101 / 4900) loss: 1.708394
(Iteration 201 / 4900) loss: 1.601478
(Iteration 301 / 4900) loss: 1.993499
(Iteration 401 / 4900) loss: 1.607356
(Epoch 1 / 10) train acc: 0.407000; val acc: 0.455000
(Iteration 501 / 4900) loss: 1.746897
(Iteration 601 / 4900) loss: 1.579854
(Iteration 701 / 4900) loss: 1.467449
(Iteration 801 / 4900) loss: 1.523966
(Iteration 901 / 4900) loss: 1.307409
(Epoch 2 / 10) train acc: 0.510000; val acc: 0.466000
(Iteration 1001 / 4900) loss: 1.495136
(Iteration 1101 / 4900) loss: 1.484715
(Iteration 1201 / 4900) loss: 1.527117
(Iteration 1301 / 4900) loss: 1.408847
(Iteration 1401 / 4900) loss: 1.455489
(Epoch 3 / 10) train acc: 0.493000; val acc: 0.490000
(Iteration 1501 / 4900) loss: 1.439727
(Iteration 1601 / 4900) loss: 1.363553
(Iteration 1701 / 4900) loss: 1.382937
(Iteration 1801 / 4900) loss: 1.261643
(Iteration 1901 / 4900) loss: 1.377745
(Epoch 4 / 10) train acc: 0.554000; val acc: 0.488000
(Iteration 2001 / 4900) loss: 1.389566
(Iteration 2101 / 4900) loss: 1.534006
(Iteration 2201 / 4900) loss: 1.423061
(Iteration 2301 / 4900) loss: 1.391580
(Iteration 2401 / 4900) loss: 1.024386
(Epoch 5 / 10) train acc: 0.576000; val acc: 0.504000
(Iteration 2501 / 4900) loss: 1.272888
(Iteration 2601 / 4900) loss: 1.267780
(Iteration 2701 / 4900) loss: 1.470104
(Iteration 2801 / 4900) loss: 1.286965
(Iteration 2901 / 4900) loss: 1.163836
(Epoch 6 / 10) train acc: 0.564000; val_acc: 0.512000
(Iteration 3001 / 4900) loss: 1.070258
(Iteration 3101 / 4900) loss: 1.343257
(Iteration 3201 / 4900) loss: 1.279531
(Iteration 3301 / 4900) loss: 1.343413
(Iteration 3401 / 4900) loss: 1.198661
(Epoch 7 / 10) train acc: 0.566000; val acc: 0.479000
(Iteration 3501 / 4900) loss: 1.268137
(Iteration 3601 / 4900) loss: 1.343852
(Iteration 3701 / 4900) loss: 1.233011
(Iteration 3801 / 4900) loss: 1.159400
(Iteration 3901 / 4900) loss: 1.040336
(Epoch 8 / 10) train acc: 0.573000; val acc: 0.487000
(Iteration 4001 / 4900) loss: 1.150482
(Iteration 4101 / 4900) loss: 1.219391
(Iteration 4201 / 4900) loss: 1.241482
(Iteration 4301 / 4900) loss: 1.169890
(Iteration 4401 / 4900) loss: 1.290256
(Epoch 9 / 10) train acc: 0.578000; val acc: 0.501000
(Iteration 4501 / 4900) loss: 1.102325
(Iteration 4601 / 4900) loss: 1.272343
(Iteration 4701 / 4900) loss: 1.118803
(Iteration 4801 / 4900) loss: 1.098855
(Epoch 10 / 10) train acc: 0.615000; val acc: 0.518000
```

```
In []: # Run this cell to visualize training loss and train / val accuracy

plt.subplot(2, 1, 1)
plt.title('Training loss')
plt.plot(solver.loss_history, 'o')
plt.xlabel('Iteration')

plt.subplot(2, 1, 2)
plt.title('Accuracy')
plt.plot(solver.train_acc_history, '-o', label='train')
plt.plot(solver.val_acc_history, '-o', label='val')
plt.plot([0.5] * len(solver.val_acc_history), 'k--')
plt.xlabel('Epoch')
plt.legend(loc='lower right')
plt.gcf().set_size_inches(15, 12)
plt.show()
```



Epoch

# Multilayer Neural Network

Now, we implement a multi-layer neural network.

Read through the FullyConnectedNet class in the file nndl/fc\_net.py .

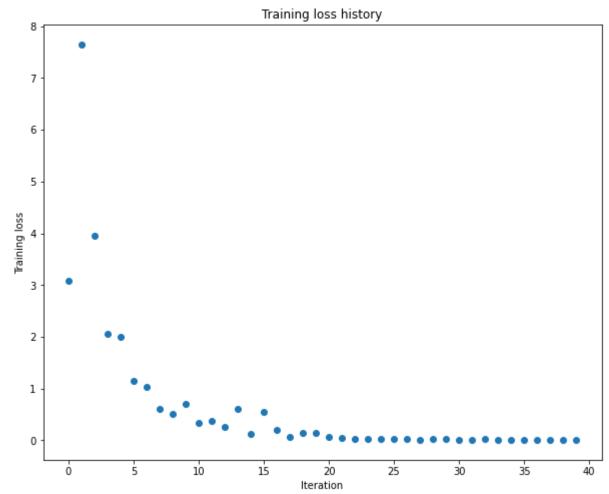
0.2

10

Implement the initialization, the forward pass, and the backward pass. There will be lines for batchnorm and dropout layers and caches; ignore these all for now. That'll be in HW #4.

```
In [ ]:
         N, D, H1, H2, C = 2, 15, 20, 30, 10
         X = np.random.randn(N, D)
         y = np.random.randint(C, size=(N,))
         for reg in [0, 3.14]:
           print('Running check with reg = {}'.format(reg))
           model = FullyConnectedNet([H1, H2], input dim=D, num classes=C,
                                      reg=reg, weight scale=5e-2, dtype=np.float64)
           loss, grads = model.loss(X, y)
           print('Initial loss: {}'.format(loss))
           for name in sorted(grads):
             f = lambda _: model.loss(X, y)[0]
             grad_num = eval_numerical_gradient(f, model.params[name], verbose=False, h=1e-5)
             print('{{}} relative error: {{}}'.format(name, rel error(grad num, grads[name])))
        Running check with reg = 0
        Initial loss: 95.85407094785221
        W1 relative error: 2.406647686142172e-06
        W2 relative error: 8.40967738222611e-09
        W3 relative error: 0.07345422900104145
        b1 relative error: 1.1891545513306533e-07
        b2 relative error: 7.018782682414375e-09
        b3 relative error: 0.013252669686837895
        Running check with reg = 3.14
        Initial loss: 2003.408003999365
        W1 relative error: 1.9874040374137043e-07
        W2 relative error: 9.7946185246163e-07
        W3 relative error: 9.023619271452891e-07
        b1 relative error: 2.366295008918749e-08
        b2 relative error: 4.395167944654979e-07
        b3 relative error: 2.668324250522108e-05
In [ ]:
         # Use the three layer neural network to overfit a small dataset.
         num train = 50
         small data = {
           'X_train': data['X_train'][:num_train],
           'y_train': data['y_train'][:num_train],
           'X_val': data['X_val'],
            'y_val': data['y_val'],
         #### !!!!!!
         # Play around with the weight scale and learning rate so that you can overfit a small
         # Your training accuracy should be 1.0 to receive full credit on this part.
         weight scale = 2e-2
         learning_rate = 1e-2
         model = FullyConnectedNet([100, 100],
                       weight_scale=weight_scale, dtype=np.float64)
         solver = Solver(model, small_data,
                         print every=10, num epochs=20, batch size=25,
```

```
(Iteration 1 / 40) loss: 3.073480
(Epoch 0 / 20) train acc: 0.160000; val acc: 0.080000
(Epoch 1 / 20) train acc: 0.320000; val acc: 0.151000
(Epoch 2 / 20) train acc: 0.540000; val acc: 0.174000
(Epoch 3 / 20) train acc: 0.720000; val acc: 0.160000
(Epoch 4 / 20) train acc: 0.800000; val acc: 0.163000
(Epoch 5 / 20) train acc: 0.880000; val acc: 0.189000
(Iteration 11 / 40) loss: 0.336315
(Epoch 6 / 20) train acc: 0.960000; val acc: 0.196000
(Epoch 7 / 20) train acc: 0.960000; val acc: 0.177000
(Epoch 8 / 20) train acc: 0.980000; val acc: 0.165000
(Epoch 9 / 20) train acc: 1.000000; val acc: 0.181000
(Epoch 10 / 20) train acc: 1.000000; val_acc: 0.187000
(Iteration 21 / 40) loss: 0.064427
(Epoch 11 / 20) train acc: 1.000000; val acc: 0.196000
(Epoch 12 / 20) train acc: 1.000000; val acc: 0.191000
(Epoch 13 / 20) train acc: 1.000000; val acc: 0.192000
(Epoch 14 / 20) train acc: 1.000000; val acc: 0.194000
(Epoch 15 / 20) train acc: 1.000000; val acc: 0.192000
(Iteration 31 / 40) loss: 0.014815
(Epoch 16 / 20) train acc: 1.000000; val_acc: 0.189000
(Epoch 17 / 20) train acc: 1.000000; val acc: 0.190000
(Epoch 18 / 20) train acc: 1.000000; val acc: 0.185000
(Epoch 19 / 20) train acc: 1.000000; val_acc: 0.190000
(Epoch 20 / 20) train acc: 1.000000; val acc: 0.190000
```



In [ ]:

In [ ]: