

- **Exponential Smoothing** uses *all* the time series values to generate a forecast with lesser weights given to the observations further back in time.

Exponential Smoothing (1 of 2)

- A weighted moving average
 - Weights decline exponentially
 - Most recent observation weighted most
- Used for smoothing and short term forecasting (often one or two periods into the future)

- **Exponential smoothing is actually a way of “smoothing” out the data by eliminating much of the “noise” (random effects).**

Exponential Smoothing (2 of 2)

- The weight (smoothing coefficient) is α
 - Subjectively chosen
 - Range from 0 to 1
 - Smaller α gives more smoothing, larger α gives less smoothing
- The weight is:
 - Low value (closer to 0) for smoothing out unwanted cyclical and irregular components
 - Higher value (closer to 1) for forecasting, especially for smoother time series

Exponential Smoothing Model

1. Obtain the smoothed series:

$$\hat{x}_1 = x_1 \quad \hat{x}_t = (1-\alpha) \hat{x}_{t-1} + (\alpha)x_t \quad (0 < \alpha < 1; t = 2, 3, \dots, n)$$

2. From time n , the forecasts of future values are

$$\hat{x}_{n+h} = \hat{x}_n \quad (h = 1, 2, \dots)$$

where:

\hat{x}_t = exponentially smoothed value for period t

\hat{x}_{t-1} = exponentially smoothed value already
computed for period $t - 1$

x_t = observed value in period t

α = weight (smoothing coefficient), $0 < \alpha < 1$

Exponential Smoothing Example

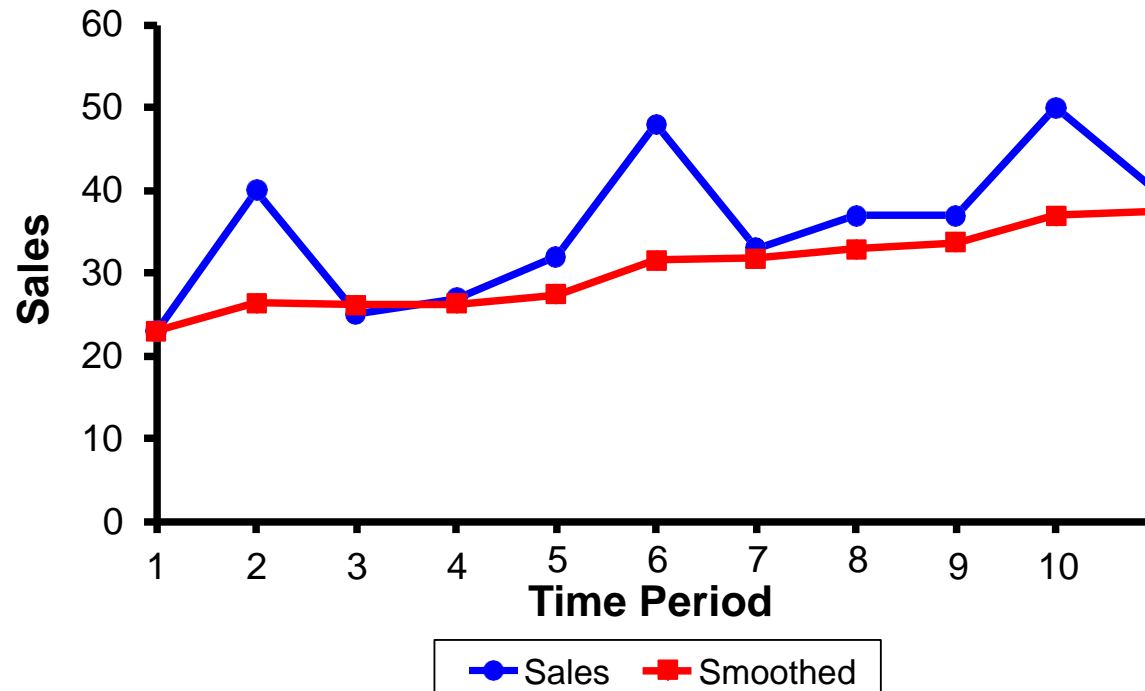
- Suppose we use weight $\alpha = 0.2$ $\hat{x}_t = (1 - 0.2)\hat{x}_{t-1} + 0.2x_t$

Time Period (t)	Sales (X_t)	Forecast from prior period (\hat{x}_{t-1})	Exponentially Smoothed Value for this period (\hat{x}_t)
1	23	--	23
2	40	23	$(.8)(23) + (.2)(40) = 26.4$
3	25	26.4	$(.8)(26.4) + (.2)(25) = 26.12$
4	27	26.12	$(.8)(26.12) + (.2)(27) = 26.296$
5	32	26.296	$(.8)(26.296) + (.2)(32) = 27.437$
6	48	27.437	$(.8)(27.437) + (.2)(48) = 31.549$
7	33	31.549	$(.8)(31.549) + (.2)(33) = 31.840$
8	37	31.840	$(.8)(31.840) + (.2)(37) = 32.872$
9	37	32.872	$(.8)(32.872) + (.2)(37) = 33.697$
10	50	33.697	$(.8)(33.697) + (.2)(50) = 36.958$
etc.	etc.	etc.	etc.

$\hat{x}_1 = x_1$
 since no
 prior
 information
 exists

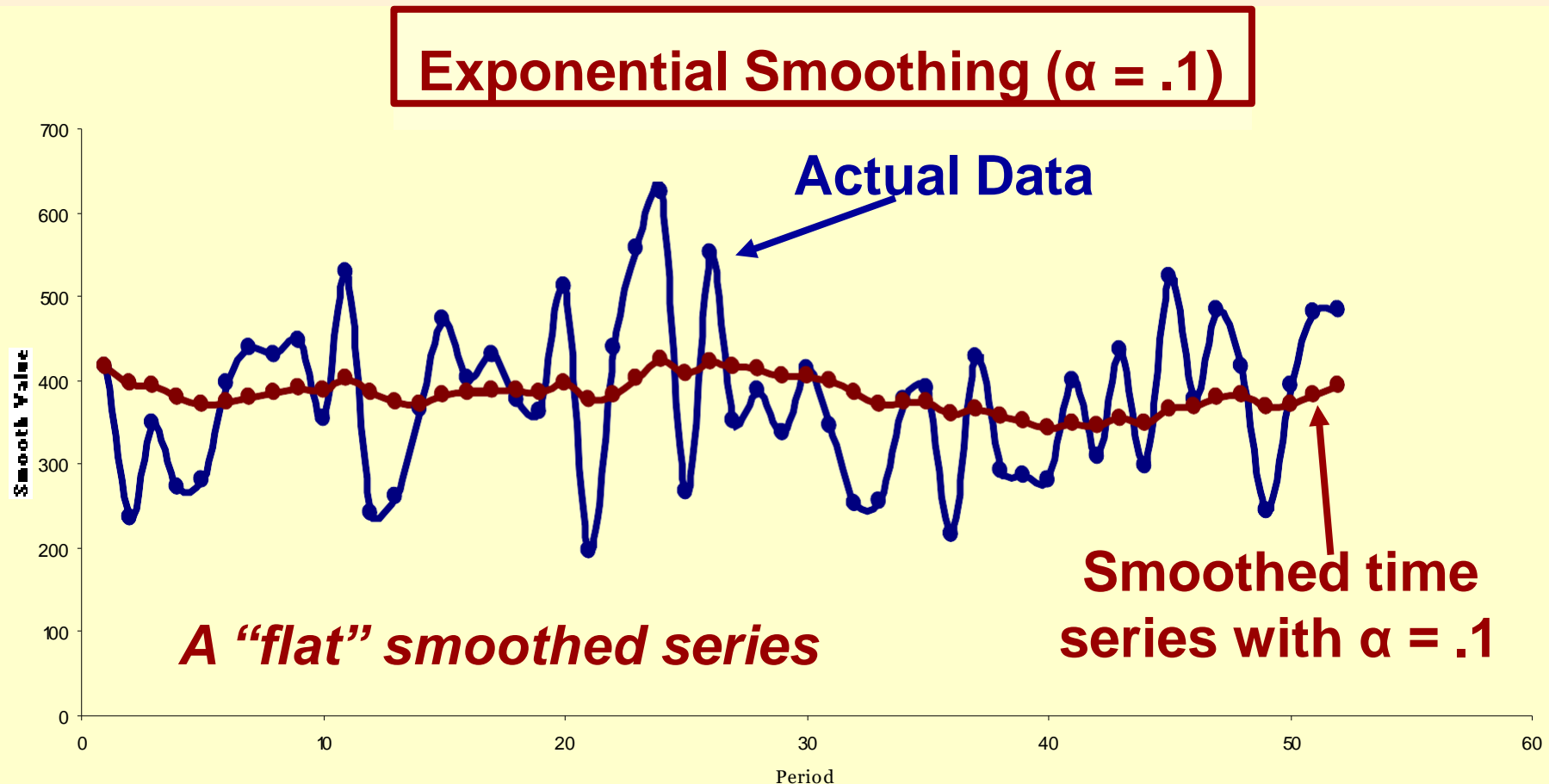
Sales vs. Smoothed Sales

- Fluctuations have been smoothed
- Note: the smoothed value in this case is generally a little low, since the trend is upward sloping and the weighting factor is only 0.2



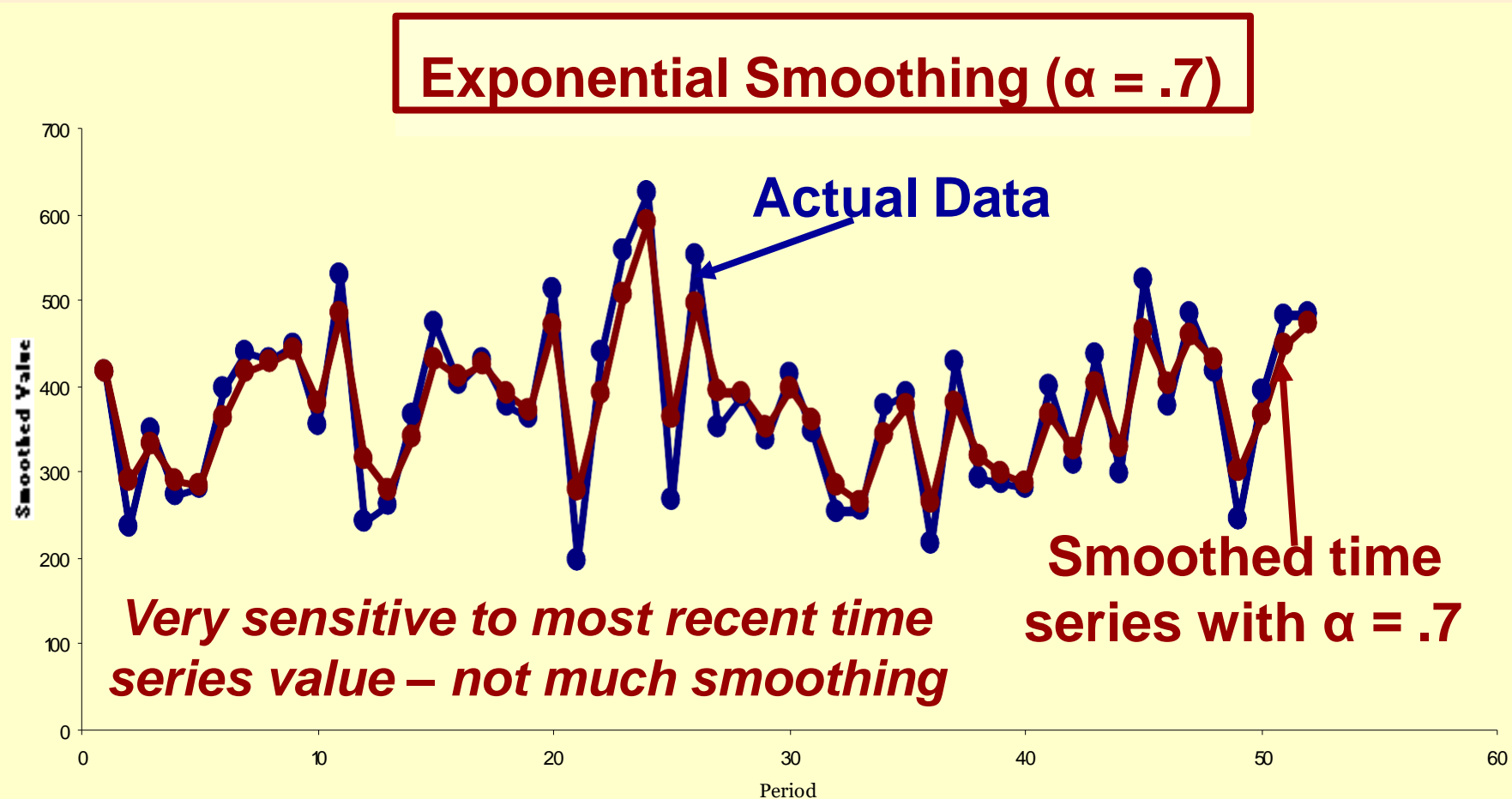
How Much Smoothing Is There?

- We said the lower the value of α , the more “smooth” the time series will become.



What About Larger Values of α ?

- Here is the “smoothed” series for $\alpha = .7$:



Forecasting Time Period ($t + 1$)

- The smoothed value in the current period (t) is used as the forecast value for next period ($t + 1$)
- Standing at time n , we obtain the forecasts of future values, X_{n+h} of the series

$$\hat{x}_{n+h} = \hat{x}_n \quad (h = 1, 2, 3 \dots)$$

Exponential Smoothing in Excel

- Use Data / Data Analysis / exponential smoothing
 - The “damping factor” is $(1-\alpha)$

