The First-Order Moving Average Process, [MA(1)]

we know that moving average process of order q [MA(q)] is given as

$$\boxed{y_t = \mu + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \dots - \theta_q \varepsilon_{t-q}} \quad -(1)$$

The simplest first order MA model is obtained when $q=1$ in (1)

$$\boxed{y_t = \mu + \varepsilon_t - \theta_1 \varepsilon_{t-1}} \quad -(2)$$

For the first order moving average or MA(1) model,

Variance of MA(1) process is

$$\boxed{\gamma_y(0) = \sigma^2 (1 + \theta_1^2)} \quad -(3)$$

[∵ See MA(q) model Variance]

Autocovariance function of MA(1) process is

$$\boxed{\gamma_y(1) = -\theta_1 \sigma^2} \quad -(4)$$

$\gamma_y(k) = 0$, $k > 1$ [because process is MA(1)]

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Similarly, $\boxed{\text{ACF}}$ {autocorrelation function of MA(1) process is}

$$\rho_y(1) = \frac{\gamma_y(1)}{\gamma_y(0)}$$

$$\rho_y(1) = \frac{-\theta_1 \sigma^2}{\sigma^2(1+\theta_1^2)}$$

$$\boxed{\rho_y(1) = \frac{-\theta_1}{1+\theta_1^2}} \quad -(5)$$

$$\rho_y(k) = 0 \quad , \quad k > 1.$$

from eqn (5), we can see that the first lag autocorrelation in MA(1) is bounded as

$$|\rho_y(1)| = \frac{|\theta_1|}{1+\theta_1^2} \leq \frac{1}{2} \qquad \frac{2}{1+c}$$

and autocorrelation functions cuts off after lag 1.

The Second-order Moving Average Process, MA(2)

we know that moving average process of order q [MA(q)] is given as

$$y_t = \mu + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \cdots - \theta_q \varepsilon_{t-q} \quad -(1)$$

~~The~~ Second-order Moving Average Process model [MA(2)] is given as

$$\boxed{y_t = \mu + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2}} \quad -(2)$$

or In terms of backshift operator

$$y_t = \mu + \varepsilon_t - \theta_1 B \varepsilon_t - \theta_2 B^2 \varepsilon_t$$

$$y_t = \mu + (1 - \theta_1 B - \theta_2 B^2)\, \varepsilon_t$$

For the second order moving average MA[2] process,

Variance of MA[2] process is

$$\boxed{\gamma_y(0) = \sigma^2(1 + \theta_1^2 + \theta_2^2)} \quad -(3)$$

Auto covariance function of MA[2] process is

$$\left.\begin{aligned} \gamma_y(1) &= \sigma^2(-\theta_1 + \theta_1\theta_2) \\ \gamma_y(2) &= \sigma^2(-\theta_2) \\ \gamma_y(k) &= 0 \quad , \quad k > 2 \end{aligned}\right\} \quad -(4)$$

Similarly, $\boxed{\text{ACF}}$, Auto correlation function of MA[2] process is

$$\rho_k(1) = \frac{\gamma_y(1)}{\gamma_y(0)}$$

$$\rho_k(1) = \frac{-\theta_1 + \theta_1\theta_2}{1 + \theta_1^2 + \theta_2^2}$$

$$\rho_{y}(2) = \frac{\gamma_y(2)}{\gamma_y(0)}$$

$$\rho_{y}(2) = \frac{\sigma^2(-\theta_2)}{\sigma^2(1+\theta_1^2+\theta_2^2)}$$

$$\rho_{y}(2) = \frac{-\theta_2}{1+\theta_1^2+\theta_2^2}$$

$$\rho_y(k) = 0 \quad , \quad k > 2$$