

Seasonal variation

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Seasonal variation

To find seasonal variations in a time series, time intervals should be measured in weeks, months or quarters because if data are expressed annually there is no seasonal variation.

According to season, the seasonal variation may be negative or positive and the total seasonal variation is zero for the year because total negative seasonal fluctuation is equal to total positive seasonal fluctuation.

On the basis of past and current seasonal variations, one can easily forecast short-term seasonal variations as they generally follow similar pattern.

They are calculated in the form of seasonal indices after eliminating the effect of trend, cyclical and irregular variations.

Seasonal variation

Seasonal variation are measured in relative terms – mostly in percentage called seasonal index.

A separate seasonal index is calculated for each time period (say for each month or quarter).

Multiplicative model is applied in calculation of seasonal indices.

Removal of seasonal effect is known as deseasonalization of time series.

Formula for deseasonalization is

Seasonally adjusted value or deseasonalized value $= \frac{Y}{S} = \frac{T * S * C * I}{S} = T * C * I$

In order to obtain annual seasonal data, seasonal index numbers are multiplied by the number of time period in a year.

Quarterly data multiplied by 4, monthly data multiplied by 12.

✓ $Y = T + S + C + I$
✓ $Y - S = T + C + I$

Measurement of Seasonal Variations

- Seasonal variations are those rhythmic changes in the time series data that occur regularly each year
- Origin in climatic or institutional factors that affect either supply or demand or both
- Variations be measured accurately for three reasons
 - Eliminate seasonal variations from the data
 - Seasonal pattern aid in planning future operations
 - Remove the cause of seasonal or are attempting to mitigate the problem by diversification

Methods of Measuring Seasonal Variations

1. Method of Simple Averages (Weekly, Monthly or Quarterly).
2. Ratio-to-Trend Method.
3. Ratio-to-Moving Average Method.

Methods of Simple Average

	Q ₁	Q ₂	Q ₃	Q ₄
2001				
2002				
1				
2005				
Total	20			
	5			

= \bar{x}

This is the simplest method of obtaining a seasonal index. The following steps are necessary for calculating the index :

- I. Average the unadjusted data by years and months or quarters if quarterly data are given.
- II. Find totals of January, February etc. or Q₁, Q₂ etc
- III. Divide each total by the number of years for which data are given.
- IV. Obtain an average of monthly or quarterly averages by dividing the total of monthly averages by 12 or 4
- V. Taking the average of monthly average, compute the percentage of various monthly averages

Problem 1

Calculate the seasonal index for the following data:

Year	Q1	Q2	Q3	Q4
2005	75	83	94	82
2006	71	88	83	72
2007	72	85	84	90
2008	84	72	86	79
2009	90	79	81	90

Problem 1

Calculate the seasonal index for the following data:

$$SI = \frac{Avg Q_i}{Avg \& Avg} \times 100$$

Year	Q1	Q2	Q3	Q4
2005 ✓	75	83	94	82
2006 ✓	71	88	83	72
2007 ✓	72	85	84	90
2008 ✓	84	72	86	79
2009 ✓	90	79	81	90
Totals ✓	392 ✓	407 ✓	428 ✓	413 ✓
Quarterly Avg ✓	78.4 ✓	81.4 ✓	85.6 ✓	82.6 ✓
Seasonal Index	95.61%	99.27%	104.39%	100.73%

Average of Quarterly Average:

$$= (78.4 + 81.4 + 85.6 + 82.6) / 4 = 82$$

$$\text{Seasonal Index of Q1} = (78.4 / 82) * 100 = 95.61\%$$

$$\text{Seasonal Index of Q2} = (81.4 / 82) * 100 = 99.27\%$$

$$\text{Seasonal Index of Q3} = (85.6 / 82) * 100 = 104.39\%$$

$$\text{Seasonal Index of Q4} = (82.6 / 82) * 100 = 100.73\%$$

1200

400

Ratio-to-moving average method

- The method of monthly totals or monthly averages does not give any consideration to the trend which may be present in the data.
- The ratio-to-moving-average method is one of the simplest of the commonly used devices for measuring seasonal variation

Consideration: The steps to compute seasonal variation are as follows

- ✓ 1. Arrange the unadjusted data by years and months
2. Compute the trend values by the method of moving averages ✓
- ✓ 3. Express the data for each month as a percentage ratio of the corresponding moving-average trend value
4. Arrange these ratios by months and years
5. Aggregate the ratios for January, February, etc.,
6. Find the average ratio for each month 100
7. Adjust the average monthly ratios found in step (4) so that they will themselves average 100 percent.

Y	M/Q	4-Y	MA ₄	CMA
2001	Q ₁	Q ₁		Q ₁
	Q ₂	Q ₂		Q ₂
	Q ₃	Q ₃		Q ₃
	Q ₄	Q ₄		Q ₄
2002				

Ratio to Moving Average (or percentage of Moving Average) method Steps involved:

(i) Take centered 12 monthly (or 4 quarterly) moving average values.

(ii) Express the original data as a percentage of the centered moving average values.

✓ (iii) Arrange these percentage season wise for all the years. Average these percentages. These values are the preliminary seasonal indices.

✓ (iv) Add these seasonal indices. If the sum is not 1200 or 400 for monthly or quarterly figures respectively. Then, multiply each value by the adjustment factor as explained below.

✓ Adjustment factor = $1200 / \text{sum of monthly indices}$.

or

✓ Adjustment factor = $400 / \text{sum of quarterly indices}$.

This gives adjusted seasonal indices. ✓

Problem 1

Calculate the seasonal index in the Ratio to Moving Average Method

Year	Quarter	Data
2005	Q1	68
	Q2	62
	Q3	61
	Q4	63
2006	Q1	65
	Q2	58
	Q3	66
	Q4	61
2007	Q1	68
	Q2	63
	Q3	63
	Q4	67

Problem 1

Calculate the seasonal index in the Ratio to Moving Average Method

Step 1:

Year	Quarter	Data	Quarterly Total	Quarterly Average	Average Centered	Ratio
2005	Q1	68				
	Q2	62	254	63.5		
	Q3	61	251	62.75	63.125	96.63%
	Q4	63	247	61.75	62.25	101.20%
2006	Q1	65	252	63	62.375	104.21%
	Q2	58	250	62.5	62.75	92.43%
	Q3	66	253	63.25	62.875	104.97%
	Q4	61	258	64.5	63.875	95.50%
2007	Q1	68	255	63.75	64.125	106.04%
	Q2	63	261	65.25	64.5	97.67%
	Q3	63				
	Q4	67				

$$= \frac{61}{63.125} \times 100$$

Problem 1

Calculate the seasonal index in the Ratio to Moving Average Method

Step 2:

Year	Quarter	Data	Ratio
2005	Q1	68	
	Q2	62	
	Q3	61	96.63%
	Q4	63	101.20%
2006	Q1	65	104.21%
	Q2	58	92.43%
	Q3	66	104.97%
	Q4	61	95.50%
2007	Q1	68	106.04%
	Q2	63	97.67%
	Q3	63	
	Q4	67	

Year	Q1	Q2	Q3	Q4	
2005	-	-	96.63%	101.20%	
2006	104.21%	92.43%	104.97%	95.50%	
2007	106.04%	97.67%	-	-	
Total	210.25%	190.10%	201.60%	196.70%	Total:
Average	105.13%	95.05%	100.80%	98.35%	399.33%
Adjusted Seasonal Index					

$$\text{Correction Factor} = 400 / 399.33 = \cancel{100.17\%} = 1.0017 \checkmark$$

Ratio-to-trend Method

- The ratio-to-trend method is similar to ratio-to-moving-average method
- The only difference is the way of obtaining the trend values
- Whereas in the ratio-to-moving-average method, the trend values are obtained by the method of moving averages, in the ratio-to-trend method
- The steps in the calculation of seasonal variation are as follows :
 1. Arrange the unadjusted data by years and months.
 2. Compute the trend values for each month with the help of least squares equation.
 3. Express the data for each month as a percentage ratio of the corresponding trend value.
 4. Aggregate the January's ratios, February's ratios, etc., computed previously
 5. Find the average ratio for each month.
 6. Adjust the average ratios found in step (v) so that they will themselves average 100 per cent.

Problem 1

Calculate the Seasonal Index of the mentioned data with Ratio to Trend Method

Year	Q1	Q2	Q3	Q4
1991	36	34	38	32
1992	38	48	58	42
1993	42	56	50	52
1994	56	74	68	62
1995	82	90	88	80
Total				

Problem 1

$$Y = a + bx$$

$$Y = 56.3 + 11.85x$$

$$a = \frac{\sum Y}{n} = \frac{281.5}{5} = 56.3$$

Calculate the Seasonal Index of the mentioned data with Ratio to Trend Method

Step 1:

$$X = t - 1993$$

$$b = \frac{\sum XY}{\sum X^2} = \frac{118.5}{10} = 11.85$$

Year	Q1	Q2	Q3	Q4	Total	Average	TimeCode		
t						$\checkmark Y \checkmark$	$X \checkmark$	XY	X^2
1991	36	34	38	32	$= 140$	$= 35$	-2	-70	4
1992	38	48	58	42	$= 186$	$= 46.5$	-1	-46.5	1
1993	42	56	50	52	$= 200$	$= 50$	0	0	0
1994	56	74	68	62	$= 260$	$= 65$	1	65	1
1995	82	90	88	80	$= 340$	$= 85$	2	170	4
Total						56.3	0	118.5	10

$$\sum Y = 281.5$$

$$\sum XY =$$

+ trend

32.6

44.45

56.3

68.15

80

Problem 1

Calculate the Seasonal Index of the mentioned data with Ratio to Trend Method

Step 1:

Year	Q1	Q2	Q3	Q4	Total	Average	TimeCode		
						Y	X	XY	X^2
1991	36	34	38	32	140	35	-2	-70	4
1992	38	48	58	42	186	46.5	-1	-46.5	1
1993	42	56	50	52	200	50	0	0	0
1994	56	74	68	62	260	65	1	65	1
1995	82	90	88	80	340	85	2	170	4
Total						56.3		118.5	10

Slope:	=Sum(XY)/Sum(X^2)	=118.5/10	11.85
Y-Intercept	=Mean(Y)	56.3	

$y = 11.85x + 56.3$ is the trend line

Problem 1

$$\phi I = \frac{11.85}{4} = 2.96 = 3M$$

Calculate the Seasonal Index of the mentioned data with Ratio to Trend Method

Step 2: $y = 11.85x + 56.3$ is the trend line

trend value

Year	TimeCode	Trend Value
	X	Y = 11.85*X + 56.3
1991	-2	=11.85*-2 + 56.3 32.6
1992	-1	=11.85*-1 + 56.3 44.45
1993	0	=11.85*0 + 56.3 56.3
1994	1	=11.85*1 + 56.3 68.15
1995	2	=11.85*2 + 56.3 80

Year	Q1	Q2	Q3	Q4
1991	=32.6-1.5-3 ✓ 28.1	=32.6-1.5 ✓ 31.1	=32.6+1.5 ✓ 34.1	=32.6+1.5+3 ✓ 37.1
✓ 1992	✓ 39.95	✓ 42.95	✓ 45.95	✓ 48.95
1993	51.8	54.8	57.8	60.8
1994	63.65	66.65	69.65	72.65
1995	75.5	78.5	81.5	84.5

$$44.45 - 1.5 + 3 = 45.95$$

Q1 Q2 Q3 Q4

$$3M, 3M, 3M, 3M$$

Q1 Q2 Q3 Q4

Problem 1

$$\frac{36}{28.1} \times 100 = 128.11$$

Calculate the Seasonal Index of the mentioned data with Ratio to Trend Method

Step 3:

$$AF = \frac{400}{403.44} = 0.9914$$

given ✓

	Actual				Trend				Trend			
Year	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4
1991	36	34	38	32	28.1	31.1	34.1	37.1	128.11%	109.32%	111.44%	86.25%
1992	38	48	58	42	40	43	46	48.95	95.12%	111.76%	126.22%	85.80%
1993	42	56	50	52	51.8	54.8	57.8	60.8	81.08%	102.19%	86.51%	85.53%
1994	56	74	68	62	63.7	66.7	69.7	72.65	87.98%	111.03%	97.63%	85.34%
1995	82	90	88	80	75.5	78.5	81.5	84.5	108.61%	114.65%	107.98%	94.67%
Total									500.90%	548.95%	529.77%	437.60%
Average									100.18%	109.79%	105.95%	87.52%

$$\frac{500.90}{5}$$

SI

ASI

$$100.18 \times 0.9914 = 99.32, 108.85$$

$$= 403.44$$