Numerical Methods (Math 385/685)

Homework 4

April 1, 2019

Due: Sunday, April 7th

Problems: Problem 1-4 are based on the linear system in Problem 5 of Homework 3.

1. For $n = 50$ states, use the power method with a tolerance of 10^{-5} and up to 100 iterations, to estimate the spectral radius of the iteration matrix $I - BA$. Let your initial eigen vector be $(1, 1, 1, \cdots, 1)$ and use difference between successive eigen values $|\lambda_k - \lambda_{k-1}|$ to check your tolerance.

   (a) Do this for $B = D^{-1}$ where D is the diagonal matrix formed from the diagonal entries of the coefficient matrix A.

   (b) Repeat part (a) for $B = (L + D)^{-1}$ where L is the strictly lower triangular entries of the coefficient matrix A.

   (c) Compare your calculated values with the true spectral radius computed using *Mathematica's Eigenvalues* function.

   (d) Comment on the convergence of the Jacobi and Gauss-Seidel methods for solving the linear system.

2. Implement the Jacobi method to solve the linear system with the stop tolerance of 10^{-2} .Use the normed error between successive estimates to check the tolerance. Display a line plot of the error against the iteration number.

3. Implement the Gauss-Seidel method to solve the linear system. Compare the number of iterations with the Jacobi method. Comment on

your results. Display a line plot of the error against the iteration number.

4. Now, set $n = 100, 1000, 5000, 10,000$. Apply the Gauss seidel method to solve the linear system in each case and calculate the cpu time using the Timing function. Plot the cpu time versus the size of your matrix and comment on your results. Compare your graph with your results in problem 5d in homework 3, using a direct solver.

5. Consider the function $f(x,y) = x^2 + xy + y^2$.

   - Perform one iteration of gradient decent by hand to estimate the minimum of $f(x,y)$ using the initial seed $(2,1)$.
   - Using the same seed, perform one iteration of the Hessian method by hand to estimate a minimum for $f(x,y)$
   - Comment on your results.

6. Consider the linear transformation

$$L(x,y,z,w) = \begin{pmatrix} 4 & -2 & 3 & -5 \\ 3 & 3 & 5 & -8 \\ -6 & -1 & 4 & 3 \\ -4 & 2 & -3 & 5 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 4x - 2y + 3z - 5w \\ 3x + 3y + 5z - 8w \\ -6x - y + 4z + 3w \\ -4x + 2y - 3z + 5w \end{pmatrix}$$

   (a) Use the maximal descent method to solve $L(x,y,z,w) = (1,1,1,-1)$. Take note of the following:
   - Use (5,5,5,5) for the initial estimate.
   - Use at least 35 iterations.
   - Use 10^{-5} as the tolerance.

   (b) Calculate a solution to the problem using Linear Solve. Compare this to your results in part (a). Comment on your results.