Numerical Methods (Math 385/685)

Homework 1

January 28, 2019

Due: Sunday, February 10th

Problems:

1. Write an algorithm and implement a Mathematica code that generates all odd integers from 1 to n. Your program should work for any choice of integer n. Make sure that if n is an even number, the largest generated odd number is n-1.

2. Write an algorithm and implement a Mathematica code that computes the sum of the integers from 1 up to and including n. **Do not use the Sum function in Mathematica**. Compare the result with the famous formula $\frac{n(n+1)}{2}$

3. Consider the function
$$y(t) = v_0 t - \frac{1}{2} g t^2$$
with $v_0 = 10$ and $g = 9.8$. Implement a Mathematica code that performs the following operations:

   (a) For any interger n, generate n+1 uniformly spaced t values throughout the interval $[0, 2v_0/g]$.

   (b) Create a function y(t).

   (c) Evaluate your function at each t value from part (a) and use a For loop to produce a table of t and $y(t)$. Show a heading and grid lines in your table.

(d) Use a *While* loop to produce the same table. **Hint** Because of potential round-off errors, you may need to adjust the upper limit of the *while* loop to ensure that the last point ($t = 2v_0/g, y = 0$) is included. Your program should work for any positive integer n.

4. Consider the smoothed Heaviside function

$$H_\epsilon(x) = \begin{cases} 0 & \text{if } x < -\epsilon \\ 1/2 + \frac{x}{2\epsilon} + \frac{1}{2\pi}\sin(\frac{\pi x}{\epsilon}) & \text{if } -\epsilon \le x \le \epsilon \\ 1 & \text{if } x > \epsilon \end{cases}$$

Implement a Mathematica code to perform the following operations:

(a) Define the function $H_\epsilon(x)$ with ϵ and x as input variables. **Do not use the Piecewise function in Mathematica**.

(b) Set $\epsilon = 3$ and use it to evaluate

$$H_\epsilon(-5), H_\epsilon(-2), H_\epsilon(0), H_\epsilon(2), H_\epsilon(5)$$

(c) Plot your function for $x \in [-5, 5]$. Make sure you label your plot appropriately.

(d) [**Graduate Students**] Discuss the differentiability of the smoothed Heaviside function over the open interval $(-5, 5)$.

5. Consider the function $f(x, y) = \frac{(x+y)^2 - 2xy - y^2}{x^2}$. We expect that if $x \neq 0$, then $f(x, y) = 1$.

(a) Set $y = 10^3$ and evaluate $f(x, y)$ for

$$x = 10.0^{-1}, 10.0^{-2}, 10.0^{-3}, 10.0^{-4}, 10.0^{-5}, 10.0^{-6}, 10.0^{-7}, 10.0^{-8}.$$

For each value of x compute the absolute error. Present your results in a table.

(b) Repeat part (a) with $g(x, y) = \frac{(x+y)^2}{x^2} - \frac{2xy}{x^2} - \frac{y^2}{x^2}$.

(c) Are the two functions identical? Which one are you most likely to implement to perform calculations in Mathematica involving small values of x? Give reasons for your answer.

(d) What role does loss of significance error play in explaining your results obtained in part (a) and (b)

(e) Repeat part (a) using $x = 10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}, 10^{-6}, 10^{-7}, 10^{-8}$. Why are the absolute errors smaller?