Numerical Methods (Math 385/685)

Homework 2

February 13, 2019

Due: Sunday, February 24th

Problems:

1. Plot the function $f(x) = xe^{-x} - 0.16064$.

   (a) Use *FindRoot* to locate a root near $x = 3$

   (b) Write a program in *Mathematica* implementing Newton's method.Use your program to approximate the root near $x = 3$. Set the maximal number of iterations to 100 and the *kick out threshold* to 10^{-5}. Display in a table the iteration number, the approximate root, the residual and $|x_n - x_{n-1}|$ at each iteration. How many iterations does your program actually execute before it stops?

   (c) Redo (b) with a kick out threshold set to 10^{-2}. Using the result of (a) as the actual and this result as the computed, calculate the relative absolute error. Display your results in a table.

2. Figure 1.3.5 (from text) shows the graph of $f(x) = \frac{x}{x^2+1}$ together with the point $(1.5, f(1.5))$.

   (a) Use FindRoot to solve $f(x) = 0$ starting at 1.5. What happens? Why?

   (b) Write your own program to execute Newton's method staring at $x = 1.5$. What is the output for the first 10 iterations?

(c) Plot f along with the tangent at the 4th iteration. Put both plots on the same axis. How does the tangent line help explain your results from part (b)?

3. Consider $f(x) = (x-1)^2 - 2$ which has a root at $1+\sqrt{2}$.

   (a) Use *FindRoot* to estimate the root of $f(x)$ with seeds at 2 and 3. Note this will default to the Secant method.

   (b) Repeat the exercise with Newton's method with initial seed 3.

   (c) Estimate the absolute error using the roots from Newton's method and Secant method.

   (d) Use the true root to calculate the actual absolute error and verify that actual absolute error is less than the estimated absolute error. This demonstrates the result from Theorem 1.4.2 in the text.

4. Consider the function $f(x) = xe^{-x} - 0.16064$

   (a) use *FindRoot* to locate a root between $x = 2$ and $x = 3$.

   (b) Write a program in *Mathematica* implementing the secant method (with bracketing). Use your program to locate a root near $x = 3$. Set the maximal number of iterations to 100 and the kick out threshold to 10^{-5}. Display the approximate root, residual and $|x_n - x_{n-1}|$ at each iteration in a table.

   (c) How many iterations does your program actually execute before it stops? Comment on the speed of convergence of the Newton's method in problem 1 with the Secant method?

   (d) Use *ListPlot* to display 5 Newton's method estimates along with 5 secant method estimates. Use different symbols for the two plots. Comment on your results

Graduate

5. (a) Complete the proof of Theorem 1.4.1 in the text.

   (b) Use your results to prove that if f has a root in $[a, b]$ and no critical or inflection points in the interval, then the sequence of Newton method approximate roots is a convergent sequence.

(c) Complete the proof of Theorem 2 (class notes) by showing that if $x_n \to \alpha$ then

$$\lim_{n\to\infty} \frac{\alpha - x_{n+1}}{(\alpha - x_n)^2} = -\frac{f''(\alpha)}{2 \cdot f'(\alpha)}$$