ECON0057 Lecture 2

Income Fluctuation Problem I

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Introduction

- Two lectures on the Income Fluctuation Problem
- Goal:
  - Start from an income process (preferably estimated)
  - impose a consumption/savings decision problem
  - See if we can replicate the joint dynamics of income, wealth and consumption
- Important for policy
  - How income inequality impacts consumption inequality
  - How government can redistribute (if needed)
- This lecture: "old" view of consumption/savings, certainty equivalence (see [LS] chapter 17)

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Deterministic Case

Consumption-savings problem under certainty:

$$\sup_{\{c_t\}_{t\geq 0}} \sum_{t\geq 0} \beta^t u(c_t)$$

$$s.t. \quad a_{t+1} = R(a_t + y_t - c_t)$$

$$a_0 \geq -\sum_{t\geq 0} R^{-t} y_t \quad and \quad \lim_{t\to\infty} R^{-t} a_t = 0$$

- Condition on a_0 ensures consumption is positive
- Condition on limit ensures debt is sustainable (No Ponzi condition)
- Often $a_{t+1} = Ra_t + y_t - c_t$ (no real difference)

Expanding the sequential budget constraint:

$$R^{-T}a_{t+T} - a_t = \sum_{s=0}^{T-1} R^{-s}\left(y_{t+s} - c_{t+s}\right)$$

Using No Ponzi we get the NPV (or lifetime) budget constraint at t:

$$\sum_{s\geq 0} R^{-s}c_{t+s} = a_t + \sum_{s\geq 0} R^{-s}y_{t+s}$$

Euler equation:

$$u'(c_t) = \beta R u'(c_{t+1})$$

Special case $\beta R = 1$:

$$u'(c_t) = u'(c_{t+1})$$

c_t is constant! Using the NPV at t:

$$c_t = \frac{R-1}{R}\left(a_t + \sum_{s\geq 0} R^{-s}y_{t+s}\right) = \frac{R-1}{R}\left(a_0 + \sum_{t\geq 0} R^{-t}y_t\right)$$

Version of Permanent Income Hypothesis (PIH): consumption determined by lifetime wealth

Stochastic Case

Now uncertain income: $y_t \geq 0$ is a general Markov process:

$$\sup_{\{c_t\}_{t\geq 0}} \mathbb{E}_0 \left(\sum_{t\geq 0} \beta^t u(c_t) \right)$$

$$s.t. \quad a_{t+1} = R(a_t + y_t - c_t)$$

$$a_0 \geq 0 \quad and \quad \lim_{t\to\infty} \mathbb{E}_0(R^{-t} a_t^2) = 0$$

Tempting: just replace y_{t+s} by $\mathbb{E}_t(y_{t+s})$:

$$c_t = \frac{R-1}{R} \left(a_t + \mathbb{E}_t \left(\sum_{s\geq 0} R^{-s} y_{t+s} \right) \right)$$

Not true except in very special case!

Certainty Equivalence with Quadratic Preferences

Assume $u(c) = b_1 c - \frac{b_2}{2} c^2$ with $b_1 > 0,\ b_2 > 0$ and keep $\beta R = 1$. Euler equation:

$$c_t = \mathbb{E}_t(c_{t+1})$$

c_t is a martingale: constant on average $c_{t+1} = c_t + \epsilon_{t+1}$ with $\mathbb{E}_t(\epsilon_{t+1}) = 0$

Expanding the sequential budget constraint:

$$\mathbb{E}_t(R^{-s} a_{t+s}) = a_t + \mathbb{E}_t\left(\sum_{0 \leq k \leq s-1} R^{-k} y_{t+k}\right) - \sum_{0 \leq k \leq s-1} R^{-k} \mathbb{E}_t(c_{t+k})$$

Using No Ponzi and Euler:

$$c_t = \frac{R-1}{R}\left(a_t + \mathbb{E}_t\left(\sum_{s \geq 0} R^{-s} y_{t+s}\right)\right)$$

Very strong predictions:

- c_t is a linear function of a_t and the "MPC" is small

$$\frac{\partial c_t}{\partial a_t} = \frac{R-1}{R} \approx 0.04\,! \quad \textit{if} \quad R = 1.04$$

- Consumption growth $\Delta c_{t+1} = c_{t+1} - c_t$ only depends on "income innovation":

$$\Delta c_{t+1} = (R-1)\left(\sum_{s\geq 1} R^{-s}\mathbb{E}_{t+1}\left(y_{t+s}\right) - \sum_{s\geq 1} R^{-s}\mathbb{E}_{t}\left(y_{t+s}\right)\right)$$

  Consumption growth does not react to predicted change in income ($\mathbb{E}_{t+1}(y_{t+s}) = \mathbb{E}_t(y_{t+s})$)

- Consider $y_t = y_t^p + u_t$ with $y_t^p = y_{t-1}^p + v_t$:
  u_t iid transitory shocks, v_t iid permanent shocks with $\mathbb{E}(v_t) = \mathbb{E}(u_t) = 0$

$$\Delta c_{t+1} = v_{t+1} + \frac{R-1}{R}u_{t+1}, \quad c_t = y_t^p + \frac{R-1}{R}(a_t + u_t)$$

  - Consumption growth reacts 1 to 1 to innovations in permanent income (v_t)
  - Pass through of transitory shocks small ($\partial \Delta c_{t+1}/\partial u_{t+1} = (R-1)/R$)

Older (70s and 80s) literature tests either this model or slightly more general versions (log linearized Euler with CRRA utility) on aggregate data:

- Hall (1978) regress c_t on c_{t-1} and lags of income $y_{t-1}, y_{t-2}\dots$
  - Consistent with the theory, the coefficients on lag of income are not significant
  - Indeed $c_t = \beta R c_{t+1} + \epsilon_{t+1}$ with $\mathbb{E}_t(\epsilon_{t+1}) = 0)$
  - Reject old Keynesian theory $c_t = b_0 y_t + b_1 y_{t-1} + \dots + b_T y_{t-T}$
- Campbell and Deaton (1989) estimate an AR(1) process (in diff):

$$\Delta y_t = 8.2 + 0.442 \Delta y_{t-1} + \epsilon_t \quad with\ \sigma_\epsilon = 25.2$$

  We can rewrite our PIH consumption growth:

$$\Delta c_{t+1} = R \left(\sum_{s \geq 1} R^{-s} \mathbb{E}_{t+1} \left(\Delta y_{t+s} \right) - \sum_{s \geq 1} R^{-s} \mathbb{E}_t \left(\Delta y_{t+s} \right) \right)$$

Empirical Tests

- Plugging Δy_t, we get:

$$\Delta c_{t+1} = \frac{R}{R - 0.442}\epsilon_{t+1} = 1.78\epsilon_{t+1} \quad \textit{for } R = 1.01$$

  - Standard deviation of Δc_{t+1} should be 1.78 times σ_ϵ
  - Data: 27.3 for total, 12.4 for non durables: less than $\sigma_\epsilon = 25.2 \rightarrow$ Excess Smoothness
- Campbell and Mankiw (1990) estimate a model where:
  - Fraction λ of the population just consumes y_t (hand-to-mouth)
  - $1 - \lambda$ follows PIH
  - To explain the data: $\lambda \approx 0.5$, Excess Sensitivity to predictable current income
- Flavin (1981) find that consumption overreact to transitory shocks
- We will see next time how to address these issues

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Blundell & Preston

- Excellent illustration of how to use theory and data to learn about he world
- BP want to understand what part of the increases in inequality are due to:
  - Increases in permanent shocks to income
  - Transitory shocks to income
- Important for policy:
  - Transitory shocks mostly smoothed (maybe no need to raise taxes to redistribute)
  - Permanent shocks generate consumption inequality (and welfare inequality)
    $\rightarrow$ warrants more redistributive policies
- Use joint dynamics of consumption/income within birth cohorts (people born the same year)
  - Assumes that income shocks are independent across individuals born in the same cohorts
  - In a cohort: hh have the same planning horizon, face the same prices (wages, interest rates etc.)
- Cannot distinguish permanent/transitory shocks with cross sections of only income distribution

Income Process

Income of cohort k at age t is given by:

$$y_{kt} = y^p_{kt} + u_{kt}$$

$$y^p_{kt} = y^p_{k,t-1} + v_{kt}$$

v_t and u_t are independently distributed overtime, satisfy $cov(u_{kt}, v_{ks}) = 0$ and $\mathbb{E}(v_t) = \mathbb{E}(u_t) = 0$

Permanent shocks v_t accumulate over time, transitory u_t shocks appear only for one period:

$$y_{kt} = v_{kt} + v_{k,t-1} + \dots + v_{k0} + u_{kt}$$

So income growth given by:

$$y_{kt} = y_{k,t-1} + v_{kt} + u_{kt} - u_{k,t-1}$$

Income Variance

- Variance:

$$var_{kt}(y) = var_{k,t-1}(y) + var_{kt}(v) + var_{kt}(u) - var_{k,t-1}(u)$$

  - Variance of sum:
  $$var(\sum_i X_i) = \sum_i var(X_i) + 2\sum_{i<j} cov(X_i, X_j)$$
  - From our assumptions:
  $$\begin{aligned} 2cov(y_{k,t-1}, u_{k,t-1}) &= -2cov(y_{k,t-2} + v_{k,t-1} - u_{k,t-2}, u_{k,t-1}) - 2var(u_{k,t-1}) \\ &= -2var(u_{k,t-1}) \end{aligned}$$

- Gives the following expression for the growth in income variance for a cohort

$$\Delta var_{kt}(y) = var_{kt}(v) + \Delta var_{kt}(u)$$

(We can't decompose this from cross-sectional income data alone)

- Growth in income variance results from:
  - permanent inequality
  - growth in transitory inequality

Joint Dynamics of Consumption and Income

Households live for T periods, have quadratic utility with $\beta R = 1$

We can adapt our previous formulas (check it!) to show that:

$$\Delta c_{k,t} = v_{kt} + \frac{R-1}{R}\frac{1}{\rho_t}u_{kt}$$

where $\rho_t = 1 - R^{-(T-t+1)}$ corrects for the finite lifetime

From this we get:

$$\Delta var_{kt}(c) = var_{kt}(v) + \left(\frac{R-1}{R}\frac{1}{\rho_t}\right)^2 var_{kt}(u) \tag{1}$$

$$\Delta var_{kt}(y) = var_{kt}(v) + \Delta var_{kt}(u) \tag{2}$$

which gives:

$$\Delta var_{kt}(y) - \Delta var_{kt}(c) = \left(1 - \left(\frac{R-1}{R}\frac{1}{\rho_t}\right)^2\right) var_{kt}(u) - var_{k,t-1}(u) \tag{3}$$

with slightly more algebra:

$$\Delta cov_{kt}(c, y) = \Delta \left\{ \frac{R-1}{R} \frac{1}{\rho_t} var_{kt}(u) \right\} + var_{kt}(v) \tag{4}$$

To see why, verify that we have

$$cov_{kt}(c, y) = cov_{k,t-1}(c, y) + var_{kt}(v) + \frac{R-1}{R} \frac{1}{\rho_t} var_{kt}(u) - cov_{k,t-1}(c, u)$$

$$cov_{k,t-1}(c, u) = \frac{R-1}{R} \frac{1}{\rho_t} var_{k,t-1}(u)$$

If we measure the variance of consumption and income by cohort:

→ (1) (2) and (4) allow us to solve for $var_{kt}(v)$, $var_{kt}(v)$ and $var_{k,t-1}(u)$

Discussion

Note that for young cohorts ($T - t$ large, if $R - 1$ small) we have:

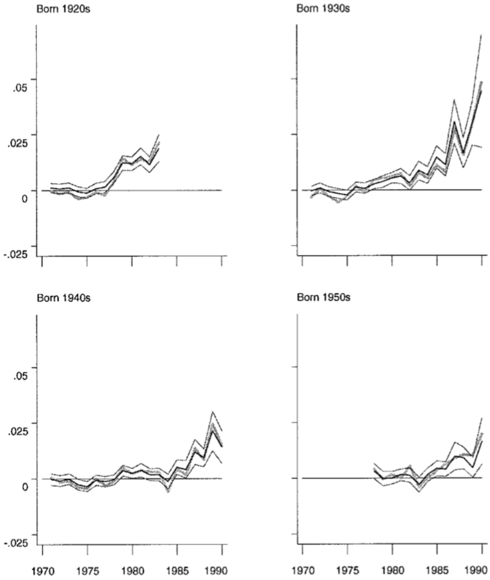
$$\Delta var_{k,t}(c) = var_{kt}(v)$$
$$\Delta covar_{k,t}(c, y) = var_{kt}(v)$$
$$\Delta var_{k,t}(y) - \Delta var_{k,t}(c) = \Delta var_{kt}(u)$$

- If growth in income variance is higher than growth in consumption variance

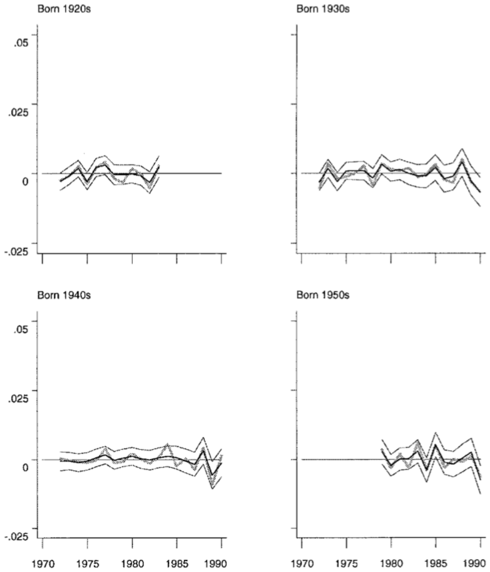
  $\Rightarrow$ there has been growth in transitory uncertainty (can also be seen from (3))

- Larger growth rate of consumption variance $\rightarrow$ larger variance of permanent uncertainty

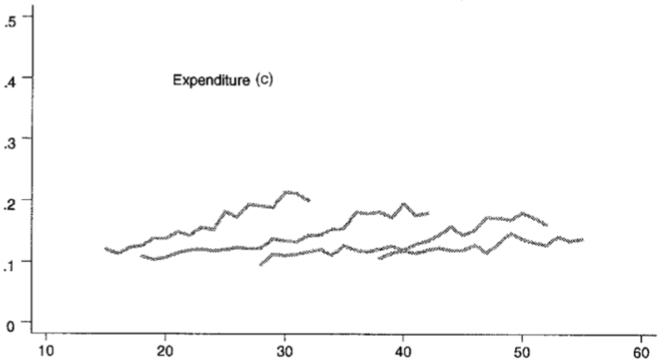
$\Delta var_{k,t}(y) - \Delta var_{k,t}(c)$ (grey) vs $\Delta var_{k,t}(u)$ (black)



$\Delta^2 var_{k,t}(c)$ (grey) vs $\Delta var_{k,t}(v)$ (black)



$var_{k,t}(c)$ across cohorts



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Baker (2018)

- In our model no interaction between assets and income
- A wealthy agent and a poor agent reacts the same way to a bad income shocks!
- Baker tests whether assets are used to smooth consumption

  (also tests whether debt matters by itself but find little evidence)
- Intro for next time: models with "liquidity constraints"
- Baker uses "financial account data" (apps that help you manage your finance)
- Econometric model:

$$\Delta log(Spending_{it}) = \beta_0 + \beta_1 \Delta log(Income_{it}) + \beta_2 \Delta log(Income_{it}) \times \frac{Debt}{Income}_{it} + \beta_3 \Delta log(Income_{it}) \times \frac{Asset\ category}{Income}_{it} + h_i + t_t + \epsilon_{it}$$

- Instrument *income* with firm shocks (layoffs, sales and acquisition etc.)

Results

Effects of Balance Sheet Holdings on ΔLog(Spending) Following Income Shocks

	Sample				
	All			Nondurables	Durables
	IV (1)	IV (2)	IV (3)	IV (4)	IV (5)
ΔLog(Income)	.315***	.343***	.346***	.319***	.414***
	(.031)	(.026)	(.023)	(.022)	(.021)
ΔLog(Income) × (Debt/Income)	.076***	.071***	.051***	.049***	.063***
	(.024)	(.023)	(.016)	(.015)	(.021)
ΔLog(Income) × (Total Assets/Income)		−.049***			
		(.014)			
ΔLog(Income) × (Liquid Assets/Income)			−.074***	−.069***	−.101***
			(.014)	(.016)	(.018)
ΔLog(Income) × (Illiquid Assets/Income)			−.028***	−.024**	−.037***
			(.010)	(.011)	(.015)
Observations	3,014,721	3,014,721	3,014,721	3,014,721	3,014,721
Period fixed effects	Yes	Yes	Yes	Yes	Yes
Household fixed effects	Yes	Yes	Yes	Yes	Yes
Instrumented variables	Income	Income	Income	Income	Income

Note.—All columns instrument for ΔLog(Income) (and interactions with ΔLog(Income)) with positive and negative shocks to a household's employer in the prior quarter as well as interactions of firm shocks and leverage and credit measures. The dependent variable in cols. 1–3 is ΔLog(Spending) by household-quarter. Columns 4 and 5 use a dependent variable of ΔLog(Nondurables Spending) and ΔLog(Durables Spending), respectively. Liquid assets measure savings accounts, checking accounts, cash holdings, and nonretirement equity accounts. Illiquid assets include pensions and retirement accounts, housing wealth, and other property holdings. All regressions are weighted by CPS-derived household frequency weights. Regressions span January 2008–December 2013. All standard errors are clustered at an employer level.

* $p < .1$.
** $p < .05$.
*** $p < .01$.