ECON0057 Lecture 9

BHA meets DMP

Alan Olivi

UCL

2022

Standard Incomplete Markets Model vs DMP

- Bewley-Huggett-Aiyagari (BHA) model: consumers face idiosyncratic earnings risks, are risk-averse and can only insure partially-through saving-against these risks
- Risk plays a central role, as does the idea that poorer consumers are less well insured and thus more vulnerable to adverse outcomes
- In a typical BHA model, heterogeneity is driven by exogenous employment shocks
- DMP focuses on the determinants of aggregate and individual unemployment
- Challenge: DMP models generally rely on risk-neutral consumers
- Households face significant risks, but they do not suffer from these risks

  $\rightarrow$ asset positions do not influence how they are affected by/deal with these risks.

Why does it matter?

- Optimal level of government-provided unemployment insurance (UI)
- Trade-off between insurance and job creation

  $\rightarrow$ high unemployment benefits provide better insurance for the unemployed

  $\rightarrow$ hurts chances for reemployment since there is less job creation.
- BHA risk is exogenous optimal level of insurance is full insurance.
- In DMP, a higher UI raises wages lowers incentives for firms to open new vacancies

  $\rightarrow$ rise in unemployment

  $\rightarrow$ Steady-state output in the DMP model is maximized when there is no UI at all.

Model Population, preferences and technology

- Time is discrete and infinite, mass 1 of households either employed or unemployed
- Households are risk averse (concave utility u), discount factor β and do not value leisure
- Each worker produces $zF(k)$ with $F()$ increasing, concave production function

  $\rightarrow$ z aggregate productivity, k capital stock used by the worker
- One firm for each "job", firm are symmetric and act competitively.

  $\rightarrow$ Because of symmetry, in equilibrium the same amount of k is used at each filled job.
- Vacant jobs and unemployed randomly matched each period according to matching function:

$$M(u, v)$$

  - Prob to fill vacancy: $\lambda_f = M(u, v)/v = M(1/\theta, 1) = q(\theta)$
  - Prob to find job: $\lambda_w = M(u, v)/u = (v/u)M(u/v, 1) = \theta q(\theta)$
  - U-rate transition: $u' = (1 - \lambda_w)u + s(1 - u)$

Asset structure

- No insurance markets for unemployment risk
- HHs can hold only two kinds of asset

  $\rightarrow$ capital k, used as an input for production

  $\rightarrow$ equity x, claim to the firm's profit, total amount of equity normalized to one
- No arbitrage between capital/equity gives:

$$p = \frac{d + p}{1 + r_k - \delta}$$

  d dividends, p price of equity, r_k return to capital, δ depreciation
- Capital and Equity are equivalent for households

  $\rightarrow$ we do not have to keep track of the portfolio composition
- As if households save in a single asset a with price $q = 1/(1 + r_k - \delta)$

Households

Value function of employed:

$$\tilde{W}(w, a) = max_{a'} u(c) + \beta[\ sU(a') + (1-s)W(a')]$$

$$\text{s.t.: } c + qa' = a + w, \text{ with } a' \geq \underline{a}$$

with $W(a) = \tilde{W}(\omega(a), a)$ and $w = \omega(a)$ resulting from Nash bargaining

Policy function:

$$a' = \tilde{\Psi}_e(w, a) = \tilde{\Psi}_e(\omega(a), a) = \Psi_e(a)$$

Value function of unemployed:

$$U(a) = max_{a'} u(c) + \beta[\ (1-\lambda_w)U(a') + \lambda_w W(a')]$$

$$\text{s.t.: } c + qa' = a + h, \text{ with } a' \geq \underline{a}$$

Policy function:

$$a' = \Psi_u(a)$$

- Firms create jobs, rent capital from consumers, and produce
- To create a job, a firm first posts a vacancy. Cost of posting a vacancy denoted by ξ

Value of posting a vacancy, V:

$$V = -\xi + q\left[(1-\lambda_f)V + \lambda_f \int J(\Psi_u(a))\frac{f_u(a)}{u}da\right]$$

In equilibrium, firms post new vacancies until $V = 0$

Value of a filled job, given wage w

$$\tilde{J}(w,a) = max_k zF(k) - r_k k - w + q\left[sV + (1-s)\tilde{J}(\tilde{\Psi}_e(w,a))\right]$$

- With prob. $1-s$, firm still matched with worker, whose next period asset level depends on a $\rightarrow$ J depends on a, $J(a) := \tilde{J}(\omega(a), a)$
- First-order condition implies that $r_k = zF'(k)$

Wage determination

- Wages are set period by period
- (Generalized) Nash bargaining (with $\gamma \in (0,1)$ the bargaining power of the worker):

$$max_w(\tilde{W}(w,a) - U(a))^{\gamma}(\tilde{J}(w,a) - V)^{1-\gamma}$$

- Denote the solution $w = \omega(a)$
- Future wages are at their equilibrium level $\omega(a)$ during bargaining for the current wage
- Dependence of w on a stems from $\tilde{W}(w,a) - U(a)$ depending on a

Computation

- Functional fixed-point problem

  instead of one-dimensional fixed-point problem in the interest rate in Aiyagari

- HHs need to know the entire wage function $\omega(a)$

- Fixed point problem in $\omega(a), r_k, \theta$

1. Guess on $\omega(a)$, along with r and θ
2. Use $M(u, v)$ and θ to compute λ_w
3. Solve HH problem given prices and transition prob.
4. Monte-carlo simulation of HHs to obtain distribution
5. Do VFI on firm value functions
6. Given all value functions, perform Nash bargaining

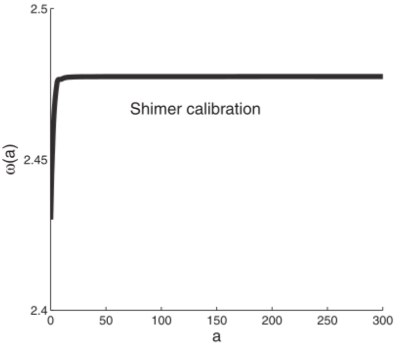
- This gives you update of $\omega(a)$ (from 6), capital stock (from 4), and θ (from 5)
- Iterate on 1-6 until convergence in $\omega(a), r, \theta$

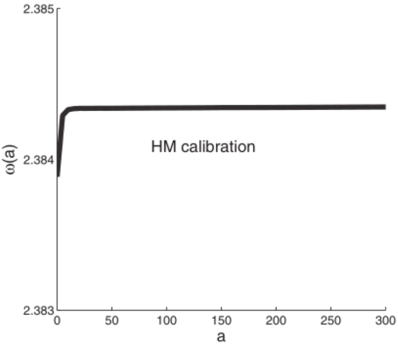
Calibration

- Period: 6 weeks
- $\alpha = 0.3$, $\delta = 0.01$ and $\beta = 0.995$ to match:

  capital share of 0.3, investment-output ratio of 0.2, annual return on capital of 0.04
- $\underline{a} = 0$
- $u(c) = log(c)$
- DMP: Either Shimer (2005) or Hagedorn & Manovskii (2008) calibration
  - Shimer (2005): household production parameter h is 40% of the wage, $\gamma = 0.72$
  - Hagedorn& Manovskii (2008) household production parameter h is 96% of the wage, $\gamma = 0.052$

Equilibrium Wage Function





Welfare effects of UI

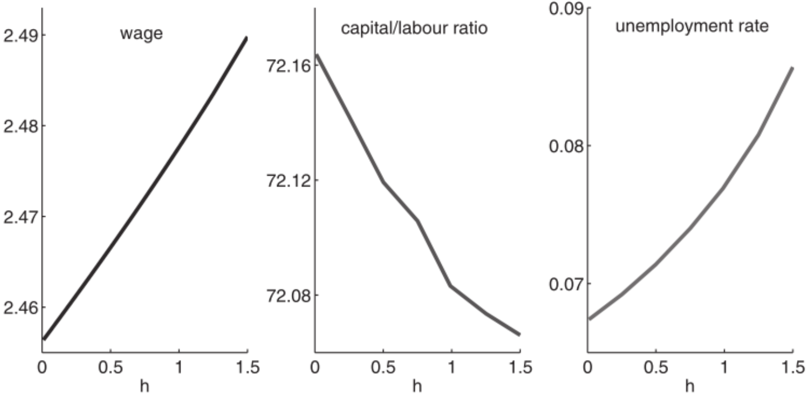
- Assume that UI is financed using taxes that are proportional to wages
- No home production here so all the income for the unemployed comes from the UI
- Budget constraints become:

$$c + qa' = a + (1-\tau)w \quad \textit{and} \quad c + qa' = a + (1-\tau)h$$

- For a given h, the government sets τ to balance its period-by-period budget constraint:

$$\tau \int \omega(a) f_e(a) da = (1-\tau)uh$$

Wages, capital/labour ratio, the unemployment rate as a function of h



Effects on quantities

- A higher h increases relative value of the outside option workers and increases the wage
- This reduces the firm's incentive to post vacancies, and hence unemployment rises
- Improvement in insurance also reduces the capital-labour ratio
- Steady-state output is the highest with zero (or even negative) UI

Welfare

- Compare steady states with different levels of UI
- high UI levels are associated with low long-run levels of capital

  $\rightarrow$ average utility across steady states will not do justice to UI
- Imagine moving each of the HHs in the benchmark economy, along with her employment status and asset holdings, into each of the other SS and then comparing utilities
- Consider an unemployed consumer moves to a high UI economy

  $\rightarrow$ she will benefit in the short run from higher income

  $\rightarrow$ but she will also suffer in the future from having a lower wage when employed
- Welfare measure (consumption equivalents):

$$E_0[\sum_{t=0}^{\infty} \beta^t log((1+\lambda(a,w))c_t)] = E_0[\sum_{t=0}^{\infty} \beta^t log(\tilde{c}_t)]$$

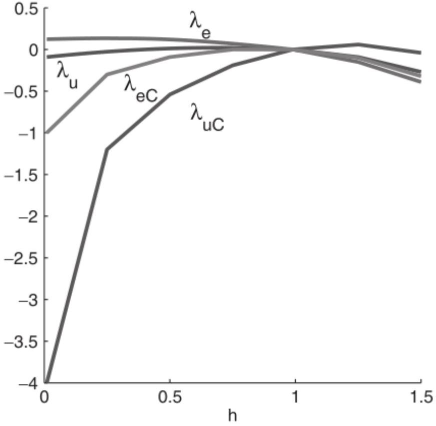
c_t is consumption under the benchmark case and $\tilde{c}_t$ is consumption under a different h

Average values of $\lambda(a, w)$

Average values of λ, compared with the benchmark of $h = 0.99$

h from 0.99 to	Total (%)	Unemployed (%)	Employed (%)	% gaining	Poorest unemployed (%)	Poorest employed (%)
0.01	0.11	−0.09	0.13	92.10	−4.0	−1.0
0.25	0.12	−0.02	0.14	92.29	−1.2	−0.3
0.50	0.11	0.01	0.12	99.22	−0.54	−0.09
0.75	0.07	0.02	0.07	99.96	−0.2	0.00
1	0	0	0	0	0	0
1.25	−0.14	−0.09	−0.15	0.00	0.06	−0.1
1.50	−0.38	−0.27	−0.39	0.00	−0.04	−0.32

$\lambda(a, w)$ for $\{e, ue\} \times \{a = 0, a > 0\}$



Welfare

- Almost everyone gains from lowering h to 0.75

  except the poorest unemployed workers who are constrained

- h that maximizes the average gain is about 0.30

- Lower h has three effects:

  1. profitability of a match increases, induces more vacancy posting and reduces unemployment
  2. (before-tax) wage rate declines because of Nash bargaining
  3. less insurance for unemployed workers

Welfare counterfactuals - BHA economy

- Assume that the employment process is exogenously given

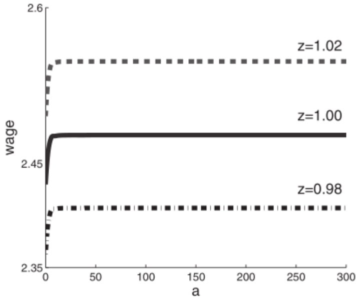
  job-finding rate at fixed 0.6, separation rate at 0.05

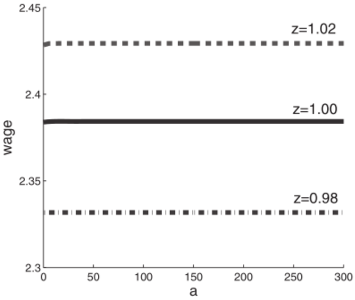
- Unemployment rate is fixed at 7.69% regardless of the value of h
- Welfare is monotonically increasing in h for the unemployed in that economy
- The employed experience very small welfare changes
- Perfect insurance ($w = h$) achieves a Pareto efficient outcome
- Because the employment process is given exogenously, UI and taxes are non- distortionary

- How large are fluctuations in θ in response to z shocks?
- Does adding precautionary motivs change the results of Shimer (2005)?
- Interaction of curvature in the wage function/distribution of HHs might affect vacancy creation
- Consider a 2% deviation in aggregate productivity (permanent)

Productivity with Shimer calibration

	u (%)	v	θ	$\bar{k}$	p	d	w
$z = 0.98$	7.79	−3.6	−4.8	−3.0	−3.9	−3.9	−2.8
$z = 1.02$	7.60	+3.6	+4.9	+3.0	+3.4	+3.4	+2.9





Productivity with HM calibration

	u (%)	v	θ	$\overline{k}$	p	d	w
$z = 0.98$	16.09	−21.3	−61.8	−11.6	−21.4	−21.4	−2.2
$z = 1.02$	6.23	+21.6	+52.4	+4.6	+21.5	+21.5	+1.9

- Firm profits change more with z, leading to a larger change in θ and v, resulting in a large response in u
- When u changes substantially, the marginal product of capital changes and, as a result, k also changes

Productivity with Shimer vs HM vs Linear Model

		y	u (%)	v	θ	$\bar{k}$
Shimer calibration	z = 0.98-linear	−2.94	7.79	−3.6	−4.8	−3.0
	z = 0.98-incomplete	−2.94	7.79	−3.6	−4.8	−3.0
	z = 1.02-linear	+2.97	7.60	+3.6	+4.9	+3.0
	z = 1.02-incomplete	+2.97	7.60	+3.6	+4.9	+3.0
HM calibration	z = 0.98-linear	−11.2	16.10	−21.4	−62.0	−11.6
	z = 0.98-incomplete	−11.2	16.10	−21.3	−61.8	−11.6
	z = 1.02-linear	+4.0	6.23	+21.5	+52.4	+4.6
	z = 1.02-incomplete	+4.0	6.23	+21.6	+52.4	+4.6

Productivity and labour-market outcomes

- Linear model is very similar to the incomplete-markets model with log utility
- Robust to more curvature in utility and higher wealth inequality
- Intuition:
  - With Nash bargaining, the wage function moves with z in a parallel manner
  - Thus the impact of z on a firm's profit is uniform across different matches