# ECON0057 Lecture 5

## Accounting for the U.S. Earnings and Wealth Inequality

Alan Olivi

UCL

2023

# Introduction

- Paper by Castaneda, Diaz-Gimenez and Rios-Rull
- Question: can standard HA models match the joint distribution of earnings and wealth (in the US)?
- Standard HA models would then provide a theory of earnings and wealth inequality

  $\rightarrow$ Based on optimal choices of ex ante identical households facing uninsured idiosyncratic productivity shocks
- Important to evaluate redistribution policies (here estate tax)

# Failure of Standard HA Models

TABLE 1
DISTRIBUTIONS OF EARNINGS AND OF WEALTH IN THE UNITED STATES AND IN SELECTED MODEL ECONOMIES

| | Gini | Bottom 40% | Top 5% | Top 1% |
|---|---|---|---|---|
| | | A. U.S. Economy | | |
| Earnings | .63 | 3.2 | 31.2 | 14.8 |
| Wealth | .78 | 1.7 | 54.0 | 29.6 |
| | | B. Aiyagari (1994) | | |
| Earnings | .10 | 32.5 | 7.5 | 6.8 |
| Wealth | .38 | 14.9 | 13.1 | 3.2 |
| | | C. Castañeda et al. (1998) | | |
| Earnings | .30 | 20.6 | 10.1 | 2.0 |
| Wealth | .13 | 32.0 | 7.9 | 1.7 |
| | | D. Quadrini (1998) | | |
| Earnings | ... | ... | ... | ... |
| Wealth | .74 | ... | 45.8 | 24.9 |
| | | E. Krusell and Smith (1998) | | |
| Earnings | ... | ... | ... | ... |
| Wealth | .82 | ... | 55.0 | 24.0 |
| | | F. Huggett (1996) | | |
| Earnings | .42 | 9.8 | 22.6 | 13.6 |
| Wealth | .74 | .0 | 33.8 | 11.1 |
| | | G. De Nardi (1999) | | |
| Earnings | ... | ... | ... | ... |
| Wealth | .61 | 1.0 | 38.0 | 15.0 |

# Failure of Standard HA Models

- *Dynastic Models* (eg. Aiyagari): infinitely lived household

  $\rightarrow$ Household care about their offspring's consumption as much as their own

  - Idiosyncratic Risk generates precautionary savings
  - No incentives to save once enough "buffer" is built

    $\rightarrow$ not much dispersion in wealth
  - Ratio of wealth to income too large at bottom, too small at top

- *Lifecycle models* (eg. Huggett 1996): working life + retirement

  - Large incentives to save during working life
  - No incentive to save at retirement (consume everything)

    $\rightarrow$ Models also fail to generate enough dispersion for high wealth households

# Key Features of Castaneda, Diaz-Gimenez and Rios-Rull

- CDR combine features of the *Dynastic* and *Lifecycle* models
  - Life Cycle: gives large incentive to save during working life
  - Dynastic: household care about their offspring so do not consume everything at retirement
    $\rightarrow$ might even want to save to insure against risk that "children" start life with low income
  - Combining the two potentially gives more incentives to accumulate wealth
- Issue: lots of hh with 0 wealth in the data despite those (hypothesized) incentives
  - CDR add Social Security
  - Government provides minimum level of income at retirement
  - This relaxes the need to save for poor households (Government "saves" for households)

# Skill/Life Cycle/Intergenerational Process

Mass 1 of households with identical preferences for consumption and labor

- State space: $s = (s_1, s_2)$ follows a Markov process
  - $s_1 = e$ during working life, $s_1 = r$ during retirement
  - $s_2 \in \mathcal{X} = \{x_1, x_2, \ldots, x_n\}$ skill of the agent
  - household's productivity $e(s) = x$ if $s_1 = e$, $e(s) = 0$ if $s_1 = r$
- Transition kernel described by $\Gamma(s, s')$
  - Retirement and death:

$$P(r \mid e,\ x_i) = P(r \mid e) = p_{e,\ell} \text{ probability of retiring, indep. of skills}$$
$$P(e \mid r,\ x_i) = P(e \mid r) = 1 - p_{\ell,\ell} \text{ probability of dying, indep. of skills}$$

  - Productivity during working life, $x$, follows a standard Markov process with kernel $Q(x, x')$:

$$\Gamma((e, x), (e', x')) = (1 - p_{e,\ell})Q(x, x') \text{ stops evolving at retirement}$$
$$\Gamma((e, x), (r, x')) = p_{e,\ell}\delta(x = x'), \quad \Gamma((r, x), (r, x')) = p_{\ell,\ell}\delta(x = x')$$

  - When hh with skill $x$ dies, replaced by its child, with skill $x'$ drawn from

$$(1 - \phi^1 - \phi^2)\gamma(x') + \phi^1\delta(x' = x) + \phi^2\delta(x' = x_1)$$

    where $\gamma$ is the stationary distribution of skills
    - $\phi^1$ helps control inter-generational mobility (child starts with same income than parent)
    - $\phi^2$ helps life-cycle (child starts with lowest skill $x_1$ and through $Q$ skill can increase during working life)

# Government

Government imposes income tax and estate tax when household dies and provides retirement insurance $\omega(s)$ to retired households:

- Income tax $\tau(y_t)$. Reason to include income tax:
  - Tax schedule progressive, disincentives to work especially at high incomes
  - Not including income tax would overstate model ability to generate dispersed earning distribution

  $\rightarrow$ Receipts of income tax are $T_t^y$

- Estate tax is $\tau_E(z_t)$ with $z_t$ savings at $t$ such that $s_{1,t} = r$, $s_{1,t+1} = e$
  $\rightarrow$

  $\rightarrow$ Receipts of estate tax $T_t^E$
- Retirement insurance, same for all households: $\omega(s) = \omega$ is $s_1 = r$

  $\rightarrow$ Cost of retirement insurance $T_t^R$
- Government budget constraint: $G_t + T_t^R = T_t^y + T_t^E$

  No debt, $G_t$ exogenous government expenses

# Household Problem

Bellman:

$$v(s,a) = \sup_{0\leq z, 0<c, 0\leq l\leq \ell} u(c, \ell - l) + \beta \sum_{s'} v(s', a(z))\Gamma(s, s')$$

$$\begin{aligned} s.t. \quad & c + z = y - \tau(y) + a \\ & y = ra + e(s)lw + \omega(s) \end{aligned}$$

$$a(z) = \begin{cases} z - \tau_E(z) & if \quad s_1 = r \text{ and } s_1' = e \\ z & otherwise \end{cases}$$

Household are altruistic: they care about the consumption of their children as much as their own

Retirement here can be interpreted as a large income shock

Note that labor supply is elastic

# Production

Competitive supply side as in Aiyagari. Representative firm production function:

$$Y_t = F(K_t, L_t)$$

Prices given by:

$$r_t = F_K(K_t, L_t) - \delta$$
$$w_t = F_L(K_t, L_t)$$

Note: elastic labor supply so $L_t$ is endogenous

# Equilibrium

A **Stationary Equilibrium** is defined by **1.** Policy functions $\{c(a,s), z(a,s), l(a,s)\}$; **2.** A distribution $\lambda(a,s)$; **3.** Government policy $\{G, \tau_E(z), \tau(y), \omega(s)\}$; **4.** Factor prices $\{r, w\}$, **5.** Aggregate quantities $\{K, L, T^R, T^y, T^E\}$ such that:

- $\{c(a,s), z(a,s), l(a,s)\}$ optimal given $r$, $w$, $\tau$, $\tau_E$, $\omega$
- tax receipts and gov spending are consistent with aggregation (e.g $T^y = \int \tau(y)\lambda(a,s)$)
- Factor prices given by Firm FOC given $K, L$
- Capital market and Labor market clear and Government budget satisfied
- $\lambda$ stationary given transition kernel induced by $\Gamma$ and hh choices

# Lorenz curves in the data

TABLE 2
DISTRIBUTIONS OF EARNINGS AND WEALTH IN THE U.S. ECONOMY (%)

| | Quintile | | | | | Top Groups (Percentile) | | |
|---|---|---|---|---|---|---|---|---|
| Gini | First | Second | Third | Fourth | Fifth | 90th–95th | 95th–99th | 99th–100th |
| | A. Distribution of Earnings | | | | | | | |
| .63 | −.40 | 3.19 | 12.49 | 23.33 | 61.39 | 12.38 | 16.37 | 14.76 |
| | B. Distribution of Wealth | | | | | | | |
| .78 | −.39 | 1.74 | 5.72 | 13.43 | 79.49 | 12.62 | 23.95 | 29.55 |

# Calibrated Skill Process

TABLE 4

TRANSITION PROBABILITIES OF THE PROCESS ON THE ENDOWMENT OF EFFICIENCY LABOR UNITS FOR WORKING-AGE HOUSEHOLDS THAT REMAIN AT WORKING AGE ONE PERIOD LATER, $\mathbf{\Gamma}_{\varepsilon\varepsilon}$ (%)

| | To $s'$ | | | |
|---|---|---|---|---|
| FROM $s$ | $s' = 1$ | $s' = 2$ | $s' = 3$ | $s' = 4$ |
| $s = 1$ | 96.24 | 1.14 | .39 | .006 |
| $s = 2$ | 3.07 | 94.33 | .37 | .000 |
| $s = 3$ | 1.50 | .43 | 95.82 | .020 |
| $s = 4$ | 10.66 | .49 | 6.11 | 80.51 |

TABLE 5

RELATIVE ENDOWMENTS OF EFFICIENCY LABOR UNITS, $e(s)$, AND THE STATIONARY DISTRIBUTION OF WORKING-AGE HOUSEHOLDS, $\boldsymbol{\gamma}^*_\varepsilon$

| | $s = 1$ | $s = 2$ | $s = 3$ | $s = 4$ |
|---|---|---|---|---|
| $e(s)$ | 1.00 | 3.15 | 9.78 | 1,061.00 |
| $\boldsymbol{\gamma}^*_\varepsilon$ (%) | 61.11 | 22.35 | 16.50 | .0389 |

# Matching Aggregate moments

TABLE 6

VALUES OF THE TARGETED RATIOS AND AGGREGATES IN THE UNITED STATES AND IN THE BENCHMARK MODEL ECONOMIES

| | $K/Y$ (1) | $I/Y$ (2) | $G/Y$ (3) | $Tr/Y$ (4) | $T_E/Y$ (5) | $h$ (6) | $CV_c/CV_l$ (7) | $e_{40/20}$ (8) | $\rho(f, s)$ (9) |
|---|---|---|---|---|---|---|---|---|---|
| Target (United States) | 3.13 | 18.6% | 20.2% | 4.9% | .20% | 30.0% | 3.00 | 1.30 | .40 |
| Benchmark | 3.06 | 18.1% | 20.8% | 4.4% | .20% | 31.2% | 3.25 | 1.09 | .25 |

NOTE.—Variable $h$ (col. 6) denotes the average share of disposable time allocated to the market. The statistic $CV_c/CV_l$ (col. 7) is the ratio of the coefficients of variation of consumption and of hours worked.

# Matching Lorenz Curve

TABLE 7
DISTRIBUTIONS OF EARNINGS AND OF WEALTH IN THE UNITED STATES AND IN THE BENCHMARK MODEL ECONOMIES (%)

| | | Quintile | | | | | Top Groups (Percentile) | | |
|---|---|---|---|---|---|---|---|---|---|
| Economy | Gini | First | Second | Third | Fourth | Fifth | 90th–95th | 95th–99th | 99th–100th |
| | | | | A. Distributions of Earnings | | | | | |
| United States | .63 | −.40 | 3.19 | 12.49 | 23.33 | 61.39 | 12.38 | 16.37 | 14.76 |
| Benchmark | .63 | .00 | 3.74 | 14.59 | 15.99 | 65.68 | 15.15 | 17.65 | 14.93 |
| | | | | B. Distributions of Wealth | | | | | |
| United States | .78 | −.39 | 1.74 | 5.72 | 13.43 | 79.49 | 12.62 | 23.95 | 29.55 |
| Benchmark | .79 | .21 | 1.21 | 1.93 | 14.68 | 81.97 | 16.97 | 18.21 | 29.85 |

TABLE 8
DISTRIBUTIONS OF CONSUMPTION IN THE UNITED STATES AND IN THE BENCHMARK MODEL ECONOMIES (%)

| | | Quintile | | | | | Top Groups (Percentile) | | |
|---|---|---|---|---|---|---|---|---|---|
| Economy | Gini | First | Second | Third | Fourth | Fifth | 90th–95th | 95th–99th | 99th–100th |
| United States: | | | | | | | | | |
| Nondurables | .32 | 6.87 | 12.27 | 17.27 | 23.33 | 40.27 | 9.71 | 10.30 | 4.83 |
| Nondurables+* | .30 | 7.19 | 12.96 | 17.80 | 23.77 | 38.28 | 9.43 | 9.69 | 3.77 |
| Benchmark: | | | | | | | | | |
| Wealthiest 1% excluded | .40 | 5.23 | 12.96 | 13.55 | 20.41 | 47.85 | 12.77 | 14.89 | 3.83 |
| Entire sample | .46 | 4.68 | 11.58 | 12.07 | 18.68 | 52.99 | 12.82 | 13.45 | 11.94 |

* Includes imputed services of consumer durables.

TABLE 9

EARNINGS AND WEALTH PERSISTENCE IN THE UNITED STATES AND IN THE BENCHMARK MODEL ECONOMIES: FRACTIONS OF HOUSEHOLDS THAT REMAIN IN THE SAME QUINTILE AFTER FIVE YEARS

| | Quintile | | | | |
|---|---|---|---|---|---|
| Economy | First | Second | Third | Fourth | Fifth |
| | A. Earnings Persistence | | | | |
| United States | .86 | .41 | .47 | .46 | .66 |
| Benchmark | .76 | .55 | .65 | .80 | .80 |
| | B. Wealth Persistence | | | | |
| United States | .67 | .47 | .45 | .50 | .71 |
| Benchmark | .81 | .80 | .80 | .75 | .89 |

TABLE 10
TARGETED MACROECONOMIC RATIOS AND AGGREGATES IN THE MODEL ECONOMIES

| | $K/Y$ (1) | $I/Y$ (2) | $G/Y$ (3) | $Tr/Y$ (4) | $T_E/Y$ (5) | $h$ (6) | $CV_c/CV_l$ (7) | $e_{40/20}$ (8) | $\rho(f, s)$ (9) |
|---|---|---|---|---|---|---|---|---|---|
| Benchmark | 3.06 | 18.1% | 20.8% | 4.4% | .20% | 31.2% | 3.25 | 1.09 | .25 |
| Match autocorrelation | 3.05 | 17.8% | 20.4% | 4.6% | .20% | 31.9% | 3.12 | 1.00 | .40 |
| Match life cycle | 3.07 | 18.1% | 20.5% | 4.6% | .20% | 31.8% | 3.15 | 1.30 | −.03 |

NOTE.—See note to table 6.

TABLE 11
DISTRIBUTIONS OF EARNINGS AND WEALTH IN THE MODEL ECONOMIES (%)

| | | Quintile | | | | | Top Groups (Percentile) | | |
|---|---|---|---|---|---|---|---|---|---|
| Economy | Gini | First | Second | Third | Fourth | Fifth | 90th–95th | 95th–99th | 99th–100th |
| | | | | A. Distributions of Earnings | | | | | |
| Benchmark | .63 | .00 | 3.74 | 14.59 | 15.99 | 65.68 | 15.15 | 17.65 | 14.93 |
| Match autocorrelation | .63 | .00 | 4.02 | 14.45 | 15.68 | 65.85 | 15.29 | 17.74 | 14.86 |
| Match life cycle | .62 | .00 | 3.71 | 14.65 | 16.66 | 64.98 | 13.79 | 18.21 | 14.45 |
| | | | | B. Distributions of Wealth | | | | | |
| Benchmark | .79 | .21 | 1.21 | 1.93 | 14.68 | 81.97 | 16.97 | 18.21 | 29.85 |
| Match autocorrelation | .80 | .18 | 1.12 | 1.64 | 14.25 | 82.80 | 17.38 | 18.63 | 30.00 |
| Match life cycle | .80 | .18 | .98 | 2.00 | 15.22 | 81.61 | 16.21 | 19.93 | 29.58 |

# Robustness

TABLE 12
EARNINGS AND WEALTH PERSISTENCE IN THE MODEL ECONOMIES: FRACTIONS OF HOUSEHOLDS THAT REMAIN IN THE SAME QUINTILE AFTER FIVE YEARS

| | Quintile | | | | |
|---|---|---|---|---|---|
| Economy | First | Second | Third | Fourth | Fifth |
| | A. Earnings Persistence | | | | |
| Benchmark | .76 | .55 | .65 | .80 | .80 |
| Match autocorrelation | .76 | .57 | .65 | .79 | .81 |
| Match life cycle | .76 | .57 | .67 | .82 | .78 |
| | B. Wealth Persistence | | | | |
| Benchmark | .81 | .80 | .80 | .75 | .89 |
| Match autocorrelation | .82 | .80 | .81 | .78 | .89 |
| Match life cycle | .80 | .79 | .78 | .73 | .89 |

# Counterfactual: Estate Tax

TABLE 13
TARGETED MACROECONOMIC RATIOS AND AGGREGATES IN THE MODEL ECONOMIES

| | $K/Y$ (1) | $I/Y$ (2) | $G/Y$ (3) | $Tr/Y$ (4) | $T_E/Y$ (5) | $h$ (6) | $CV_c/CV_l$ (7) | $e_{40/20}$ (8) | $\rho(f,s)$ (9) |
|---|---|---|---|---|---|---|---|---|---|
| Benchmark | 3.06 | 18.1% | 20.8% | 4.4% | .20% | 31.2% | 3.25 | 1.09 | .25 |
| No estate tax | 3.08 | 18.2% | 20.8% | 4.4% | .00% | 31.2% | 3.27 | 1.09 | .25 |

NOTE.—See note to table 6.

TABLE 14
DISTRIBUTIONS OF EARNINGS AND WEALTH IN THE MODEL ECONOMIES (%)

| | | Quintile | | | | | Top Groups (Percentile) | | |
|---|---|---|---|---|---|---|---|---|---|
| Economy | Gini | First | Second | Third | Fourth | Fifth | 90th–95th | 95th–99th | 99th–100th |
| | | | | A. Distributions of Earnings | | | | | |
| Benchmark | .63 | .00 | 3.74 | 14.59 | 15.99 | 65.68 | 15.15 | 17.65 | 14.93 |
| No estate tax | .60 | .00 | 3.75 | 14.59 | 15.98 | 65.68 | 15.14 | 17.68 | 14.89 |
| | | | | B. Distributions of Wealth | | | | | |
| Benchmark | .79 | .21 | 1.21 | 1.93 | 14.68 | 81.97 | 16.97 | 18.21 | 29.85 |
| No estate tax | .80 | .20 | 1.18 | 1.86 | 14.42 | 82.33 | 17.80 | 18.26 | 30.29 |

# Counterfactual: Estate Tax

TABLE 15
EARNINGS AND WEALTH PERSISTENCE IN THE MODEL ECONOMIES: FRACTIONS OF HOUSEHOLDS THAT REMAIN IN THE SAME QUINTILE AFTER FIVE YEARS

| | Quintile | | | | |
|---|---|---|---|---|---|
| Economy | First | Second | Third | Fourth | Fifth |
| | A. Earnings Persistence | | | | |
| Benchmark | .76 | .55 | .65 | .80 | .80 |
| No estate tax | .76 | .55 | .65 | .80 | .80 |
| | B. Wealth Persistence | | | | |
| Benchmark | .81 | .80 | .80 | .75 | .89 |
| No estate tax | .81 | .80 | .80 | .75 | .89 |