Problem Set 7

1 Krueger and Perri (2006)

Consider a simple exchange economy with endogenous constraints due to limited enforceability of inter-temporal contracts.

Time is discrete, $t = 1, ..., \infty$. There are two *ex ante* identical households $i = 1, 2$, and a single nonstorable consumption good per period. Households' endowments come from two sources. The first is stochastic labour income. The stochastic process is such that if one household receives $1 + \varepsilon$ then the other household receives $1 - \varepsilon$. Denote by $s_t \in S = \{1, 2\}$ the household who has the high endowment (the currently rich household) in period t. Let s_t be i.i.d. with $\pi(s_t = 1) = \pi(s_t = 2) = \frac{1}{2}$. The parameter $\varepsilon \in [0, 1)$ measures the variability of labour income. In addition to labour income, each household has capital income r per period.

In this economy households are free to engage in risk pooling. In addition, households are free to break any risk sharing contracts they have made, subject to the penalty of being excluding from all future risk sharing *plus* loss of access to the capital market (in this example, loss of capital income r).

Let $s^t = (s_0, s_1, ..., s_t)$ be a history of labour income shocks and $\pi(s^t)$ be the time zero probability of the history s^t. An allocation across households $c = (c^1, c^2)$ maps the histories s^t into individual consumption histories which the households value according to

$$U(c^i) = (1 - \beta)E_0 \sum_{\tau=t}^{\infty} \beta^{\tau-t} log(c_\tau),$$

where $0 < \beta < 1$. Note that the per period utility flow is given by

$$(1 - \beta)log(c_t).$$

1. What is the feasibility constraint in any period t in which both agents have kept their risk sharing agreements up to, and including, this date?

2. What is the first best (perfect insurance) allocation, $c^{FB}(s_t)$? In other words, if an unconstrained social planner could allocate consumption to the two households each period as a function of their current endowment, what would this allocation look like?

3. Derive the first best lifetime utility $U^{FB}(r)$ associated with this allocation.

4. Now, define the autarkic allocation as the allocation in which each household simply consumes their labour endowment (but no capital income) each period. What is the autarkic allocation, $c^A(s_t)$?

5. Derive the lifetime utility associated with autarky for the currently high endowment, $U^A(1+\varepsilon)$, and the currently low endowment, $U^A(1-\varepsilon)$, households.

6. Verify that $U^A(1+\varepsilon)$ is strictly increasing in ε at $\varepsilon = 0$; $U^A(1+\varepsilon)$ is strictly decreasing in ε as $\varepsilon \to 1$; and $U^A(1+\varepsilon)$ is strictly concave in ε. Verify that $U^A(1-\varepsilon)$ is strictly decreasing in ε.

7. What is a constrained efficient allocation? What constraint(s) must be satisfied for this allocation to be sustainable?

8. Discuss, possibly using a diagram, how and why the ability to share risk (either fully or partially) in this economy depends on r, β, and ε. In other words, how does the relationship between income inequality and consumption inequality depend on r, β, and ε in an economy with limited commitment?