Problem Set 5

1 Huggett (1993)

Consider the Bellman equation corresponding to the Huggett model:

$$V(a,s) = \max_{-\phi \le a' \le (1+r)a+ws} U\left((1+r)a + ws - a'\right) + \beta \sum \pi(s,s')V(a',s'). \tag{1}$$

We choose the functional form $U(c) = \frac{c^{1-\gamma}-1}{1-\gamma}$.

$\pi(s,s')$ is given by the following transition matrix (in Matlab notation):

$$\begin{bmatrix} 0.7497 & 0.2161 & 0.0322 & 0.002 & 0 \\ 0.2161 & 0.4708 & 0.2569 & 0.0542 & 0.002 \\ 0.0322 & 0.2569 & 0.4218 & 0.2569 & 0.0322 \\ 0.002 & 0.0542 & 0.2569 & 0.4708 & 0.2161 \\ 0 & 0.002 & 0.0322 & 0.2161 & 0.7497 \end{bmatrix}$$

and $s \in S$, where S: [0.6177 0.8327 1.0000 1.2009 1.6188]

1. (For this subquestion only) Consider the following Markov transition matrix. Find its associated stationary distribution analytically.

$$\begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0.2 & 0.3 \\ 0 & 0.7 & 0.3 \end{bmatrix}$$

2. Simulate the Markov chain $\pi(s,s')$ and estimate its persistence and long-run variance. The goal of the simulation is to obtain a long history of realizations $s_N = (s_1, s_2, \ldots, s_N)$, for a large value of N. In order to do that, start from a random $s_1 \in S$. Then, generate a random number between 0 and 1 and compare it with the (cumulative) transition probabilities. For instance, if $s_1 = 0.6177$ and the random number you have generated is below 0.7497, $s_2 = 0.6177$, while if it is between 0.9658 and 0.9980, $s_2 = 1$. Repeat for a large number of times N and use the history you have computed to compute persistence and variance. You can generate random numbers from the uniform distribution on the $[0,1]$ interval with the Matlab function rand(.).

3. Numerically find the stationary distribution over S implied by the Markov chain: $\xi(s) = \pi(s, s')'\xi(s)$. You can use any of the methods described in Problem Set 3. Determine the total labor supply implied by the stationary distribution: $N = \sum \xi(s_i) * s_i$.

4. Let $\beta = 0.96$, $\gamma = 2$, $w = 1$, and $\phi = -2$. Solve for the stationary equilibrium of the Huggett economy. In particular:

   (a) First write down pseudo code that outlines the various steps of your algorithm.

   (b) Use the Value Function Iteration and the bisection method to find the equilibrium interest rate of this economy.

   - Given an interest rate, to find the ergodic distribution, simulate asset and income transitions for (100,1000,10000) households and (100,1000,10000) time periods.
   - You can plot the evolution of the total asset stock of the economy over these time periods to verify the convergence of the asset distribution.
   - To find the interest rate using the bisection method, you need to find a high enough interest rate r^H for which asset demand is positive, and a low enough interest rate r^L for which asset demand is negative. We then know that the interest rate that clears the asset market is between them. We then guess an interest rate $r^G = \frac{r^L+r^H}{2}$. If asset demand at r^G is positive (negative), we update $r^H = r^G$ ($r^L = r^G$) and then update the guess. We repeat until asset demand is close enough to zero.
   - Once you have found the equilibrium interest rate, plot a histogram of the asset distribution at the last time period of the simulated economy.