ECON0057 Lecture 7

Diamond-Mortensen-Pissarides

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- Last three lectures: equilibrium theory of unemployment
- In the previous lectures, income shocks were *exogenous*

  $\rightarrow$ households become less productive, they earn less
- In reality, unemployment spells account for an important part of "negative income shocks"
- What determines the level of unemployment? The length of unemployment spells?

Matching Function

- Mass 1 of workers, fraction u_t unemployed, firms open v_t vacancies
- Number of jobs created captured by reduced form matching function:

$$M(u_t, v_t)$$

- Main friction in the model: it takes time to match unemployed to job openings.
  $\rightarrow$ Matching function summarizes how much input (u_t, v_t) is needed to create M jobs
- M satisfies same assumption as prod. function: concave, increasing in both arguments

CRS Matching Function

- Matching Function often assumed to be constant return to scale

  $\rightarrow$ reasonable given aggregate evidence

- In that case the probability of a vacancy being filled (match per vacancy) is:

$$\frac{M(u_t, v_t)}{v_t} = M\left(\frac{u_t}{v_t}, 1\right) \equiv q\left(\frac{v_t}{u_t}\right) = q(\theta_t)$$

  where $\theta_t = v_t / u_t$ is called **labor market tightness**

- $q(\theta)$ is decreasing (the more vacancy per unemployed, the longer to fill a given vacancy)

- The probability of finding a job is given by:

$$\frac{M(u_t, v_t)}{u_t} = \frac{v_t}{u_t} M\left(\frac{u_t}{v_t}, 1\right) = \theta_t q(\theta_t)$$

- $\theta q(\theta)$ increasing in θ (the more vacancy per unemployed, the faster it is to find a job)

- Market tightness determines the evolution of unemployment
- Noting s_t the separation rate (fraction of workers who lose their jobs):

$$u_{t+1} = u_t + s_t(1 - u_t) - \theta_t q(\theta_t) u_t$$

- So in steady state we have:

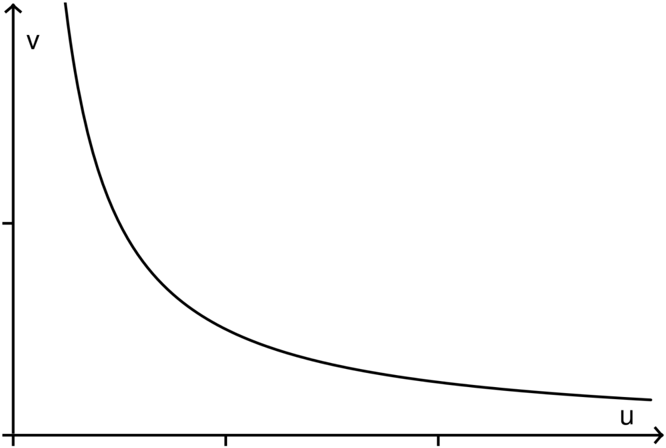
$$u = \frac{s}{s + \theta q(\theta)}$$

- Equation defines the **Beveridge curve**: number of vacancies $v(u)$ needed to reach u:

$$su + M(u, v(u)) = s$$

- $v(u)$ is decreasing and convex (note $M_{uu} < 0$, $M_{vv} < 0$, $M_{vu} > 0$):

$$\frac{\partial v(u)}{\partial u} = -\frac{s + M_u}{M_v} < 0 \quad \frac{\partial^2 v(u)}{\partial u^2} = -\frac{(M_{uu} + M_{vu} v_u) M_v - (M_{vu} + M_{vv} v_u)(M_u + s)}{M_v^2} > 0$$


v
u

Labor Demand

- Simple accounting: given a separation rate and job finding rate we have equilibrium unemployment
- Matching function (and Beveridge Curve) relates ss unemployment to vacancy creations
- Now we model firm labor demand to understand how vacancy are created
- Firms have CRS prod. function each worker produces y output (labor only input)

  $\rightarrow$ no meaningful distinction between "job" and "firm"
- Firm pay c to keep a vacancy open and can stop searching ("exit") at 0 cost
- When they find a worker they get $y - w$ profit until the job is terminated (at rate s)
- Discount future profit/cost at rate $1/R$

Firm Problem

- Call J the NPV of a firm profits/cost when the job is filled, V when the firm is searching:
- Value of a filled job:

$$J = y - w + \frac{1}{R}((1-s)J + sV)$$

- Value of a vacancy (assuming $V \geq 0$):

$$V = -c + \frac{1}{R}(q(\theta)J + (1-q(\theta))V)$$

- Job can freely enter the labor market

  $\rightarrow$ firms enter until the value of opening an additional vacancy is 0:

$$\frac{J}{R} = \frac{y-w}{s+r} \quad \textit{NPV of profits}$$

$$\frac{J}{R} = \frac{c}{q(\theta)} \quad \textit{Expected costs}$$

Job Creation

- Using the two equations, we obtain the labor demand curve (or Job Creation curve) :

$$w = y - \frac{(s+r)c}{q(\theta)}$$

$\rightarrow$ For a given wage w the equation determines the equilibrium θ

$\rightarrow$ θ pins down v (through beveridge curve), the number of vacancies opened

$\rightarrow$ Labor Demand

- Note that the expected time it takes to fill a vacancy is

$$\sum tq(\theta)(1-q(\theta))^{t-1} = 1/q(\theta)$$

- The equation simply equalizes expected profits and expected costs

- θ increases with per period profits $y - w$, decreases with vacancy maintenance costs c, separation rate s and the interest rate r

Wage Determination

- To close the model, we need to describe how wages are determined
- The relationship between matched firm and workers is purely bilateral
  - There's not a single way to determine wages
  - Any wage can be part of an equilibrium as long as:
    1. Hiring the worker is weakly better than creating a new vacancy for the firm
    2. Taking the job is weakly better than going back to unemployment for the worker
- To determined the set of equilibrium wages, we need:
  - The **joint surplus** (sum of the gains for worker and firm) of creating a job
  - The **outside option** of the worker (going back to unemployment)

Worker's Problem

- Workers are risk neutral and discount the future at rate $\beta = 1/R$
- They earn w when hired, z when unemployed

  $\rightarrow$ value of leisure, unemployment benefits, home production etc.
- Calling E and U the NPV of utility of employed and unemployed workers we have:

$$E = w + \frac{1}{R}((1-s)E + sU)$$
$$U = z + \frac{1}{R}(\theta q(\theta)E + (1-\theta q(\theta))U)$$

- The worker surplus is then $E - U$ (the 2 equations allow to express it in terms of w and θ)

Total Surplus

- A match provides surplus to the worker of $E - U$ and surplus to the firm of $J - V = J$
- The total match surplus is $S = (E - U) + J$
- The wage determines what fraction of the surplus S goes to the worker (and to the firm)
- A match creates a job if the wage is to be such that both the firm and the worker gain
  - $\rightarrow$ $E - U \geq 0$ and $J \geq 0$
  - $\rightarrow$ The lowest possible wage gives all the surplus to the firm ($E = U$, $J = S$)
  - $\rightarrow$ The highest possible wage gives all the surplus to the worker ($E - U = S$, $J = 0$)
  - $\rightarrow$ Any wage between these bounds could be an equilibrium wage
- To close the model we now analyze a commonly used mechanism to determine wages: the generalized Nash bargaining solution

Nash Bargaining

- The idea of Nash Bargaining is that the firm and worker sequentially make wage offers until a wage can be agreed upon

- The solution of the bargaining game is summarized by a bargaining parameter ϕ and is:

$$\sup_{E-U,J} (E-U)^{\phi} J^{1-\phi} \quad s.t. \quad S = E - U + J$$

- This simplifies to

$$\sup_{E-U} \phi log(E-U) + (1-\phi) log(S-(E-U))$$

- Taking FOCs gives

$$E - U = \phi S, \quad J = (1-\phi)S$$

- The larger ϕ the larger the share of the total surplus commanded by the worker

- ϕ is called the **bargaining power** of workers

Determining w

From the worker's problem, we have:

$$E - U = w - z + \frac{1 - s - \theta q(\theta)}{R}(E - U)$$
$$\frac{E - U}{R} = \frac{w - z}{s + r + \theta q(\theta)}$$

Recall that we have

$$\frac{J}{R} = \frac{y - w}{s + r} = \frac{c}{q(\theta)}$$

So using $(1-\phi)\phi S = (1-\phi)(E-U) = \phi J$:

$$(1-\phi)\frac{w - z}{s + r + \theta q(\theta)} = \phi\frac{y - w}{s + r}$$
$$(1-\phi)\frac{w - z}{s + r + \theta\frac{c(s+r)}{y-w}} = \phi\frac{y - w}{s + r}$$
$$(1-\phi)(w - z) = \phi(y - w) + c\theta\phi$$
$$w = (1-\phi)z + \phi(y + c\theta)$$

Wage equation

Rewriting the equation we have:

$$w = z + \phi(y - z + c\theta)$$

Workers are compensated for the loss of leisure proportionally to their bargaining power:

- workers capture a fraction ϕ of the production net of leisure $y - z$ and of the cost of vacancy per unemployed worker (cv/u)
- the value of unemployment increases with θ (reducing the length of unemployment spell) which leads to higher wages

The Key Equations

We have three variables to determine $\{w, \theta, u\}$

θ and w are determined by the job creation curve and the wage equation:

$$w = y - \frac{(s+r)c}{q(\theta)}$$
$$w = (1-\phi)z + \phi(y + c\theta)$$

Finally the Beveridge curve gives the ss level of unemployment:

$$u = \frac{s}{s + \theta q(\theta)}$$

- The Job creation curve is strictly decreasing in θ (it takes more time to fill a vacancy)
- The wage equation is strictly increasing in θ (quicker to find a job)
- There's a unique w^*, θ^* as long as

$$w_{JC}(0) = y > w_{WE}(0) = (1-\phi)z + \phi y \quad \Rightarrow z < y$$

(otherwise working is inefficient!)

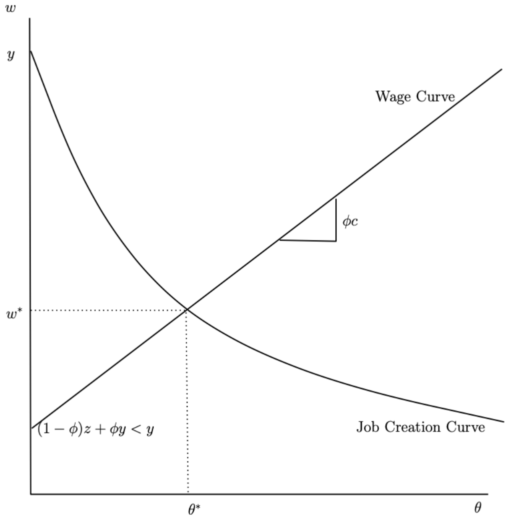


Figure: Job Creation and Wage Curve

Graphical Representation

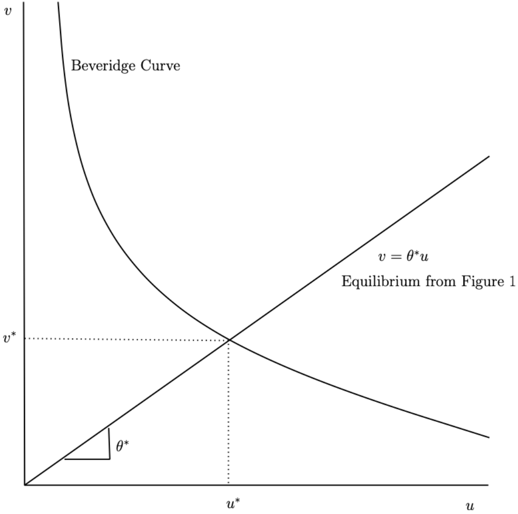


Figure: Beveridge Curve and Equilibrium Job Creation

Comparative Statics

- Higher labor productivity shifts the JC curve in Figure 1 up by y
- Wage equation shifts by $\phi y < y$ so the result is an increase in θ (more job creation)
- In figure 2 we see this causes JC to rotate counter clockwise leading to higher v and lower u
- More job creation and less unemployment
- From a modelling point of view this is undesirable, since it implies unemployment will disappear with economic growth.
- Solution: make z proportional to y
- Example: unemployment insurance, or some outside option that also increase with productivity (value of time)
- Exercise: think of the comparative statics with respect to ϕ, c, z, s and r