A Dynamic Programming Approximation for Energy Storage Arbitrage's Market Scalability with Price Impact and Renewables Growth

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Abstract: Integrating high levels of intermittent renewable energy may require large-scale deployment of energy storage, where energy arbitrage between time periods could form an increasingly important revenue stream. However, energy arbitrage at a massive scale would also compress the inter-period price spreads that enable such a strategy's profitability. Prior studies relying on purely historical prices ignore the price impact from high levels of energy arbitrage, while production cost based models ignore the potential for strategic bidding. For mathematical insight, analytic solutions are derived under a stylized framework to compare social planner optimization versus Cournot-Nash equilibrium. An efficient dynamic programming method is developed to quantify the scalability of energy arbitrage's profitability as a function of both storage and renewables deployment, incorporating realistic non-convex supply stacks. This method is used in a numerical case study of large-scale energy arbitrage in the CAISO wholesale market.

1. INTRODUCTION

Energy arbitrage as a revenue stream for energy storage projects represents substantial growth potential: while ancillary services such as frequency regulation have historically been the most lucrative, they have "a much smaller market potential due to the lower need for reserves capacity compared to energy capacity."[1] For example, CAISO's market rules define the frequency regulation requirement as a function of peak load, rather than scaling in response to higher solar growth.

Many energy storage studies examine energy arbitrage's marginal profitability for "price-taker" storage using purely historical electricity prices,[2,3,4,5,6,7,8,9,10] which fail to capture the diminishing returns effect due to price impact,[11] as well as future structural system changes including increased renewable supply.[12] Some theoretical studies impose a single linear or bilinear function for every pricing period to derive closed-form solutions with price impact,[13,14,15,16,17] but these ignore any inter-period changes of the generator supply stack. Production cost simulation models endogenously include price impact effects under a social welfare objective and have documented diminishing returns in energy storage's societal value,[1,18,19,20] but these cannot capture strategic bidding behavior by storage operators under existing market structures. Several studies utilize grid operator-published generator offer curve data as piecewise-linear price functions to account for price impact, within a mixed integer programming formulation,[21,22,23] but these only study single example days due to the integer variables' computational complexity. Similarly, bi-level optimization models account for strategic behavior along with detailed market clearing such as transmission constraints,[24,25] but their long computation times lead these papers to consider only a few days of analysis. c_t

Due to the decision problem involving multi-period linkages, energy arbitrage lends itself to a dynamic programming formulation, but this approach has previously been used only to study single example days using static historical prices.[26,27] Methodologically, this paper contributes an extension to the fast dynamic-programming approach to allow for generalized price functions without convexity restrictions, as well as generalized initial states of charge. Compared to more detailed approaches like bi-level programming, this paper's approximation method is very fast (~10 seconds to solve one year with hourly resolution), enabling flexible sensitivity analyses of storage and renewables deployment. For regulators and investors, this paper contributes to bridging the research gap between the social value of energy storage and the existing market remuneration mechanisms; it addresses the role of energy storage ownership models and the challenge of market power mitigation for energy storage.

2. PROBLEM FORMULATION

Energy storage offers a spectrum of services to the grid (Table 1). This paper focuses only on technology-neutral energy arbitrage, i.e. shifting energy between periods, so these other aspects are not within the paper's scope, including transmission constraints, ancillary services, and capital expenditures. Other unit commitment related costs and constraints not considered, a reasonable simplification for systems like California's where marginal thermal generators are predominantly natural gas units capable of flexible starts. To assess the maximum profitability achievable in theory, it is assumed that the load profile and generator supply curves are known ex-ante to storage operators deterministically. This assumption means that generators do not get to adjust their bids in response to storage, is reasonable given the maturity of market power mitigation rules such as default offer curves based on exogenous fuel costs.

Table 1. Grid services provided by energy storage.[28]

Bulk energy services	**- Electric energy time shift (arbitrage)** - Electric supply capacity
Ancillary services	- Regulation - Reserves - Voltage support - Black start
Transmission infrastructure services	- Transmission upgrade deferral - Transmission congestion relief
Distribution infrastructure services	- Distribution upgrade deferral - Voltage support
Customer energy management services	- Power quality - Power reliability - Retail electric energy time shift - Demand charge management - Increased solar PV self-consumption

For period t, let $c_t(d)$ denote the marginal cost of supplying a quantity d of net demand, i.e. $c_t(d)$ also represents the market-clearing energy price. In the absence of transmission and unit commitment constraints, the c_t supply curves (or price functions) simply consist of the generator marginal costs sorted in increasing order; this simple merit order logic embeds the minimum-production-cost economic dispatch.

Like generators, energy storage can bid into the energy market with price-based bids; this is equivalent to quantity decisions under the deterministic ex-ante generator curve assumption, because any bid curve translates to a cleared quantity amount. Let x_t denote the net injection from storage into the grid in hour t.

The energy arbitrage problem's objective depends on the ownership structure. For the social planner, each period's payoff function π^{social} consists of total production costs found by integrating the supply curve to cover net demand (load net of storage operations); here the social planner payoff is written as the negative of production costs, in terms of the value to be maximized. The storage decision quantity appears in the integral's upper limit.

$$\pi^{social}(t, x_t) = -\int_0^{D_t - x_t} c_t(u)du$$

For a monopoly storage owner, each period's payoff is the product of net injection with the resulting market-clearing electricity price, which is itself a function of the storage decision quantity.

$$\pi^{mono}(t, x_t) = x_t c_t(D_t - x_t)$$

Let f be a conversion efficiency function relating the grid net injection x_t to the corresponding change in storage state of charge ΔS_t, where a grid withdrawal results in an equal or lesser magnitude increase in the state of charge, and vice versa for grid injection. The roundtrip efficiency (where $0 < \eta \leq 1$) is assumed to be geometrically averaged between the charging and discharging steps.

$$f(x_t) = \Delta S_t = S_{t+1} - S_t$$
$$= \begin{cases} -\sqrt{\eta}x_t, & \text{if } x_t \leq 0 \text{ (withdraw, charge)} \\ \dfrac{-1}{\sqrt{\eta}}x_t, & \text{if } x_t > 0 \text{ (discharge, inject)} \end{cases}$$

For both social planner and monopolist, the overall objective function consists of summing all periods' payoff functions. Combining with storage-specific constraints results in a general formulation for the energy arbitrage problem. The first equality constraint represents conservation of energy across time. The next inequality constraint keeps the state of charge at all timesteps within the energy capacity. The initial S_0 state is allowed to vary, but must equal the final state due to the energy conservation constraint. The final inequality constraint limits the storage resource's available power capacity.

<u>*General energy arbitrage problem*</u>

$$\begin{aligned} &\max_{\{x_t\}, S_0} \sum_1^T \pi(t, x_t) \\ &\text{s.t.} \quad \sum_1^T f(x_t) = 0 \\ &\qquad 0 \leq S_0 + \sum_1^\tau f(x_\tau) \leq S_{max}, \quad \forall \tau = 1, \dots, T \\ &\qquad -X_{max} \leq x_t \leq X_{max}, \qquad \forall t = 1, \dots, T \end{aligned}$$

3. STYLIZED ANALYTIC FRAMEWORK

To gain mathematical insight into energy arbitrage's theoretical potential, consider a limiting case of relaxing the resource-specific capacity constraints (retaining only energy conservation), and assuming perfect efficiency $\eta = 1$. Further assume a linear functional form (only used in the analytic section) for each c_t with period-specific elasticity m_t. For notational simplicity, define the initial no-storage price p_t^0 as the market clearing price in the absence of any storage operations.

$$c_t(u) = c_t^0 + m_t u$$
$$p_t^0 \coloneqq c_t(D_t) = c_t^0 + m_t D_t$$

<u>*Social Planner*</u>

First, simplify the social planner payoff function, i.e. effectively the sum of areas of a rectangle and a triangle:

$$\int_0^{D_t - x_t} (c_t^0 + m_t u)du = c_t^0(D_t - x_t) + \tfrac{1}{2}m_t(D_t - x_t)^2$$

Linearity of the price function enables analytic derivation of the global optimum. The social planner Lagrangian (retaining only the energy conservation constraint) is:

$$\mathcal{L}^{social}(\{x_t\},\lambda) = -\sum_1^T \left[\int_0^{D_t - x_t} c_t(u)du\right] - \lambda \sum_1^T x_t$$

$$\frac{\partial \mathcal{L}^{social}}{\partial x_t} = c_t^0 + m_t(D_t - x_t) - \lambda$$
$$= p_t^0 - m_t x_t - \lambda = 0$$

The solution is maximal, since the slope is positive:

$$\frac{\partial^2 \mathcal{L}^{social}}{\partial x_t^2} = -m_t < 0$$

Rearranging the first order condition gives each period's storage decision, which depends on an "intertemporal lambda" representing the marginal value of energy across all time periods. In each period, if the initial market price is greater than the intertemporal lambda then x_t^* is positive, meaning energy storage should discharge and inject into the grid; and vice versa. Periods with higher slopes have their optimal storage decision attenuated by a larger denominator.

$$\boxed{x_t^{*social} = \frac{p_t^0 - \lambda^*}{m_t}}$$

Summing across periods, and substituting into the energy conservation constraint, reveals λ^* to be an elasticity-weighted average of the no-storage initial energy prices.

$$\sum x_t = \sum \frac{p_t^0 - \lambda}{m_t} = 0 \Rightarrow \sum \frac{p_t^0}{m_t} = \lambda \sum \frac{1}{m_t}$$

$$\boxed{\lambda^{*social} = \frac{\sum p_t^0/m_t}{\sum 1/m_t}}$$

Further assume that $k_t := 1/m_t$ inverse-slopes have negative covariance with the initial prices, i.e. higher initial prices correspond to higher elasticities, which is typically true for electricity markets. Using the definition of covariance then reveals that the simple arithmetic average of initial prices forms an upper bound on λ^*:

$$Cov(\boldsymbol{k},\boldsymbol{p^0}) = \tfrac{1}{T}\sum k_t p_t^0 - \left(\tfrac{1}{T}\sum k_t\right)\left(\tfrac{1}{T}\sum p_t^0\right) \le 0$$
$$\Longrightarrow \lambda^* = \frac{\sum k_t p_t^0}{\sum k_t} \le \tfrac{1}{T}\sum p_t^0$$

When elasticities are identical across periods, then λ^* equals the simple arithmetic average of initial prices (the upper bound). In such a case, the perfect competition or social planner's optimal storage operations will completely level out the prices across all periods, resulting in minimum social cost but also zero private storage profits. In more general cases the social planner solution can result in electricity prices that can differ depending on period-specific elasticities.

Cournot-Nash Equilibrium

The Cournot model represents imperfect competition where firms simultaneously choose output quantities. For simplicity assume there are N identical firms participating in energy arbitrage, where $x_t = \sum_{i=1}^N x_{ti}$. Rewrite the linear price function using the no-storage initial price as the intercept:

$$c_t(D_t - x_t) = p_t^0 - m_t x_t$$

In each period, each firm's payoff is the product of its individual net injection x_{ti} with the market price, which depends on the collective net injection x_t from energy storage overall. So the Lagrangian can be written:

$$\mathcal{L}_i^{olig}(\{x_{ti}\},\lambda_i) = \sum_{t=1}^T x_{ti}(p_t^0 - m_t x_t) - \lambda_i \sum_{t=1}^T x_{ti}$$
$$= \sum_{t=1}^T x_{ti}\left[p_t^0 - m_t\left(x_{ti} + \sum_{j\neq i} x_{tj}\right)\right] - \lambda_i \sum_{t=1}^T x_{ti}$$

Each firm only controls its own storage, so differentiate with respect to the firm-specific x_{ti}:

$$\frac{\partial \mathcal{L}_i^{olig}}{\partial x_{ti}} = p_t^0 - m_t x_t - m_t x_{ti} - \lambda_i = 0$$

By symmetry, the firms' inter-temporal $\lambda_i = \lambda$ must be identical, along with the optimal storage decisions, so:

$$x_{ti}^{olig} = \frac{1}{N} x_t^{olig}$$

Note that the above substitution cannot be made prior to the optimization differentiation step, since each firm only has individual control. A similar second derivative check as above verifies the solution is maximal. The first order condition becomes:

$$p_t^0 - \left(1 + \frac{1}{N}\right) m_t x_t - \lambda = 0$$

$$\boxed{x_t^{*olig} = \left(\frac{N}{N+1}\right)\frac{p_t^0 - \lambda^*}{m_t}}$$

Using the same energy conservation argument as above shows the Cournot λ is equal to the social planner's.

$$\boxed{\lambda^{*olig} = \lambda^{*social}}$$

Thus, with the stylized linear price function, the Cournot competition equilibrium results in the same decision structure as under the social planner except that the storage quantities are scaled by the factor $N/(N+1)$. In the monopoly case of $N = 1$ this scalar becomes 1/2, and in general strategic behavior results in lower magnitudes of energy arbitrage operation. In the perfect competition limit of $N \to \infty$ the outcome equals the social planner solution.

4. DYNAMIC PROGRAMMING METHOD

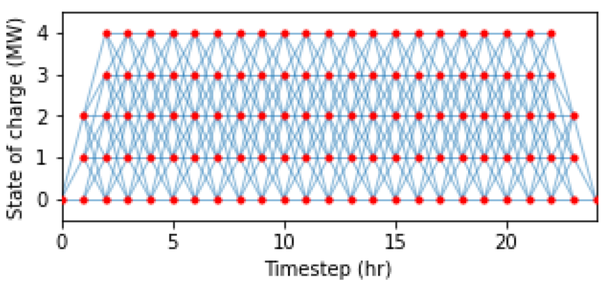
To set up the dynamic programming (DP) state space discretization, let R be the discretization resolution of the state of charge, and let $\sigma(t)$ be the set of feasible states of charge at each period consisting of positive integer multiples of R, constrained by "ramp up" and "ramp down" intervals to ensure starting and ending states at 0.

$$\sigma(t) = \left\{ \begin{matrix} s: \quad \exists k \in \mathbb{Z}^+, \\ s = kR \leq \min\{tX_{max}, S_{max}, (T-t)X_{max}\} \end{matrix} \right\}$$

Let $X(t,S)$ denote the set of feasible net injection values, conditional on the current timestep and state of charge. Note that since discretization is applied to the state of charge rather than net injection, in the case of imperfect efficiency the feasible net injections $X(t,S)$ can have different magnitude resolutions depending on whether it is withdrawal or injection.

$$X(t,S) = \{x: \quad S + f(x) \in \sigma(t)\}$$

The feasible states $\sigma(t)$ can be thought of as graph nodes and the feasible net injections $X(t,S)$ correspond to graph edges, as illustrated below in a toy example with $X_{max} = 2\text{MW}$, $S_{max} = 4\text{MWh}$, $\eta = 1$. Note that while there are on the order of $O((S_{max}/R)^2)$ edges between adjacent periods, there are only $O(2X_{max}/R)$ distinct payoff values requiring calculation.



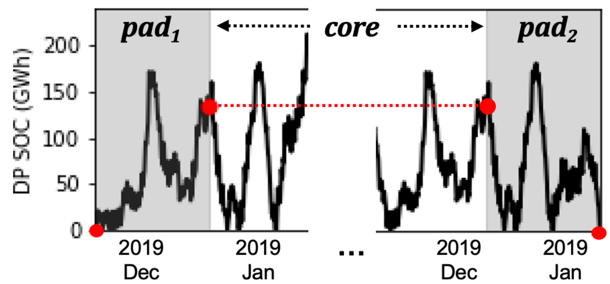
Let the value function $V(t,S)$ denote the total objective value achievable over the horizon from periods t to T, conditional on starting at state S. The objective can be based on either the social planner or monopoly payoffs as written earlier. The base case of the final period T has value 0 since no storage operations can be performed. Then the Bellman equation describes the inductive relationship between the optimal choices made during successive periods, where this period's decision should optimize the combination of the immediate payoff within this period and the resulting state's value for next period:

$$V(T,0) = 0$$
$$V(t,S_t) = \max_{x_t \in X(t,S_t)} \{\pi(t,x_t) + V(t+1, S_t + f(x_t))\}$$

It is evident that the energy arbitrage problem possesses an optimal substructure property. Therefore dynamic programming, which can be solved using standard backward induction, guarantees a globally optimal solution within the chosen discretized state space. DP places no restrictions on the payoffs' functional form, as they can be thought of as general weights of graph edges, i.e. the price functions c_t are now generalized. However, this method so far still restricts the starting (and ending) state S_0 to 0. The following section explains the required extension to endogenize SOC to any feasible value.

Existence of SOC Convergence

Augment the dynamic programming method by padding the optimization horizon with repeated starting and ending payoff periods to create a "wrapped" dataset; a sufficiently long padding length results in convergence, where the core horizon's starting SOC matches the ending SOC, as illustrated below (using a CAISO input dataset that will be explained in the next section):



Formal proof that convergence always exists is omitted; for this paper's purpose it suffices that all the numerical cases do reach convergence. Intuitively, payoffs that are too far in the past or future should not affect the optimal decision for a given period.

Correctness of DP with Endogenous Starting SOC

Once SOC convergence is reached, then the DP solution is feasible and optimal for the core horizon itself, as the following arguments prove. First, DP is trivially optimal over the entirety of the padded input, with the solution:

$$DP([\boldsymbol{D}_1, \boldsymbol{C}, \boldsymbol{D}_2]) = [\boldsymbol{x}_1^{DP}, \boldsymbol{x}_{core}^{DP}, \boldsymbol{x}_2^{DP}]$$

Then repeat the core horizon. The DP optimality property means the optimal solution for the padded double-core input must be the same as for the padded single-core input (otherwise one of the solutions could be improved):

$$DP([\boldsymbol{D}_1, \boldsymbol{C}, \boldsymbol{C}, \boldsymbol{D}_2]) = [\boldsymbol{x}_1^{DP}, \boldsymbol{x}_{core}^{DP}, \boldsymbol{x}_{core}^{DP}, \boldsymbol{x}_2^{DP}]$$

Repeat inductively:

$$DP([\boldsymbol{D}_1, (\boldsymbol{C})_{\times n}, \boldsymbol{D}_2]) = [\boldsymbol{x}_1^{DP}, (\boldsymbol{x}_{core}^{DP})_{\times n}, \boldsymbol{x}_2^{DP}]$$

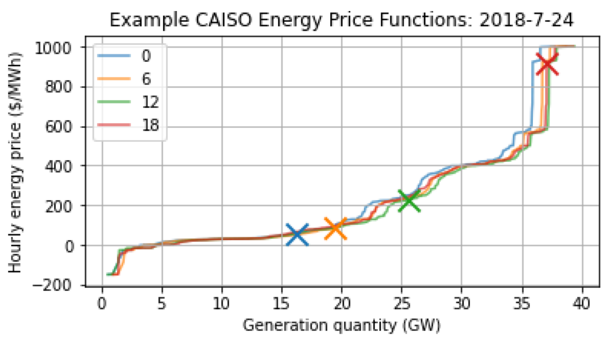
At the limit, the DP solution's core payoffs dominate the total sum of payoffs (pad plus core), meaning the padded infinitely-repeated-core DP solution also globally optimizes the core horizon payoffs by themselves:

$$\lim_{n\to\infty} \frac{1}{n} [\pi([\boldsymbol{x}_1^*, \boldsymbol{x}_2^*]) + n \cdot \pi(\boldsymbol{x}_{core}^*)] = \pi(\boldsymbol{x}_{core}^*)$$

Since the infinite-core DP's solution contains the same core-horizon path as in the single-core DP solution, then the single-core DP solution must be optimal for the core horizon. This padding method allows DP to find a global optimal energy arbitrage solution over the relevant horizon of interest, while endogenously optimizing S_0.

4. DATASET

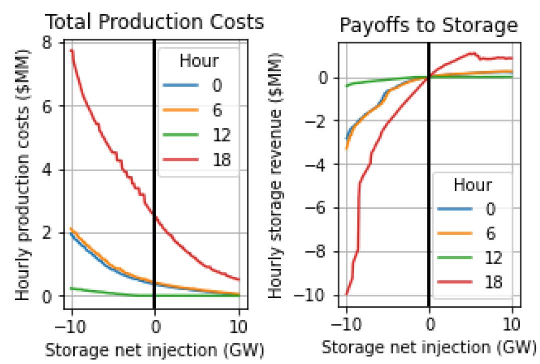
Construct hourly price functions $c_t(d)$ by sorting in merit order the day-ahead (DA) generator offer data published by CAISO, i.e. assuming economic dispatch in the absence of unit commitment and transmission constraints as described earlier. Use the intersection of each hourly supply curve with the realized DA energy prices' energy component (which is the same for all buses) to infer the no-storage implied demand level D_t. This ensures that a no-storage decision in one period would result in the correct historical DA price, ensuring model calibration.



The dataset from 2016-2019 is used. The supply curves embed historical profiles of imports from neighboring regions and weather-dependent generation including hydropower. The hourly supply curves can be revised to accommodate various levels of counterfactual renewables. Since most of California's renewables growth is expected to come from solar, for simplicity only scenarios of solar capacity growth are considered. The DA solar generation schedule published by CAISO (which only covers wholesale transmission-connected plants) is used as a proxy for combined utility-scale and behind-the-meter solar hourly generation (based on ratios of these two solar segments' monthly generation as published in EIA's Electric Power Monthly). Then the scaled solar profile is scaled up according to the specified scenario based on the peak GW of hourly generation reached within the year. The resulting hourly profiles of counterfactual increases in solar generation are incorporated into the supply curves as additional horizontal segments at $0/MWh price.

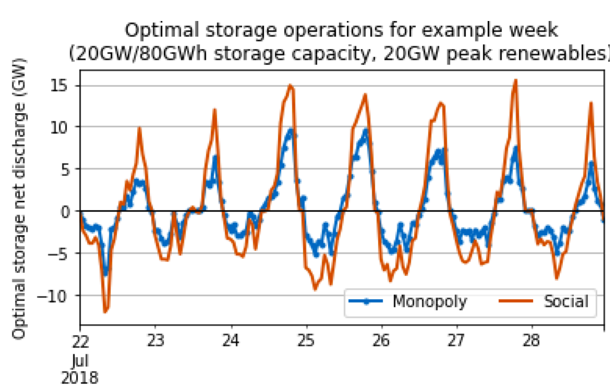
While the data procedure only considers future scenarios of solar buildout and otherwise retains the structure of historical generation and load profiles, the supply and demand inputs to the DP method in future studies can be adjusted to incorporate other changes as well (e.g. generation changes like retirements or expanded imports, electric vehicle charging, or carbon pricing evolution).

Given the price functions c_t, the above social planner (production cost) and storage monopoly payoff functions can be constructed as a matrix where prices are interpolated according to the chosen discretization scheme. The social planner production cost values can be calculated with discrete numerical integration of the generator supply curve segments. Payoff function examples (for four select hours on one day) are shown below, as constructed from the four price functions shown earlier. Note that with a monotonic price function, the storage profit payoff functions are not necessarily monotonic (due to the nonlinear product of price and quantity), highlighting an advantage of the DP approach.

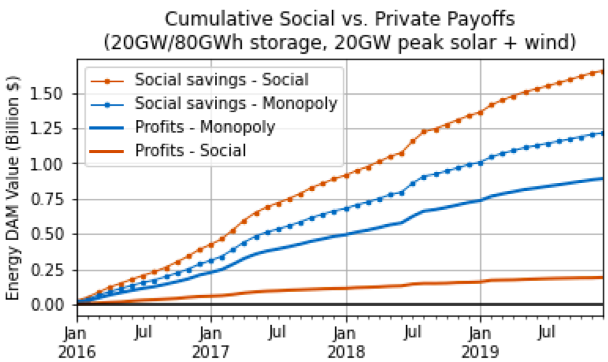


5. NUMERICAL RESULTS

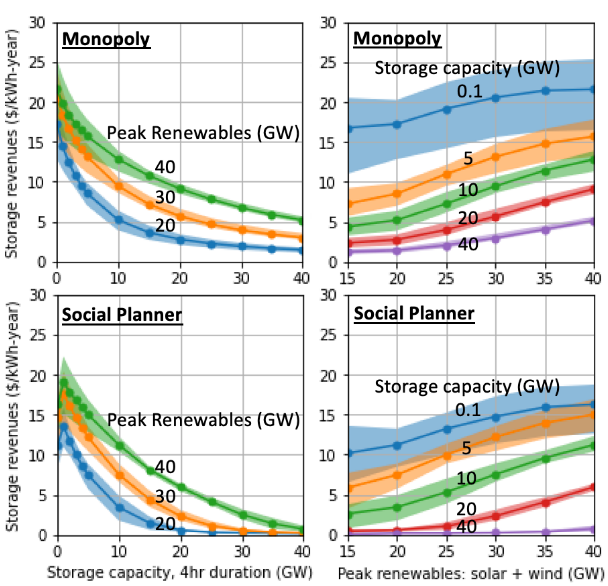
The DP approach efficiently quantifies the system-wide market scalability of energy arbitrage. This section assumes perfect conversion efficiency, focuses on 4-hour duration storage, and uses a discretization resolution of 100MW (these parameters can be generalized). The figure below illustrates an example week of optimal storage operations with a fixed storage capacity and renewables profile. As in the stylized results, the monopoly-optimized energy arbitrage operations exhibit smaller magnitudes than the socially-optimized profile.



Accumulated over time, the value of energy arbitrage can deviate significantly between social benefits (in terms of total production cost savings) versus private returns (via wholesale DA energy market revenues), and between the extremes of ownership structures. In this example covering 4 years, the theoretical limit of social savings accrue to $1.66 billion, while the corresponding market revenues under social planner operations are only $0.19 billion or 11%. Conversely, under a monopoly "merchant storage" model, social savings total $1.22 billion with $0.89 billion or 73% recoverable from the energy market.



The figure below traces the sensitivity of energy arbitrage revenues, showing the interactions between energy arbitrage and solar deployment. Solid lines indicate averages of annual values, and the shaded regions indicate ranges across the 2016-2019 dataset. In the left plots, moving rightwards long the horizontal axis demonstrates that market returns erode as more energy storage is deployed towards arbitrage; in the right plots, moving rightwards horizontally demonstrates the increasing energy arbitrage revenue opportunities created by additions of solar power.



For example, under social planner optimization, adding about 5GW of additional capacity in both storage and renewable capacity (from a starting point of 0.1GW and 15GW, respectively) diminishes the per-kWh energy arbitrage revenues by about 26% (from $10.17/kWh-year to $7.48/kWh-year). This net reduction in revenue, when both storage and solar power capacities increase at the same rate, suggests that the price impact effect outweighs the renewables-shaping effect.

6. DISCUSSION

Beyond indicating intuitions for directional effects, estimating energy arbitrage revenues in a forward timeframe with materially different storage and renewables deployment is highly relevant for energy industry practitioners and policymakers alike. The energy storage project investor (or ultimate offtaker of the wholesale market risk) needs to know what profits are recoverable from energy arbitrage, so that any remaining capital and financing costs can be covered from other revenue sources, thereby informing bidding prices during capacity market or other bilateral procurement constructs. Underestimating the future wholesale energy arbitrage value can lead to otherwise higher prices in the capacity pricing mechanism, leading consumers to ultimately pay more; conversely, overestimating the future wholesale revenue value potentially exposes the storage project (or ultimate risk owner) to undue financial risk. For the policymaker, estimating the amount of monetizable benefits can inform the level of potential extra-market incentives that might be deemed necessary to elicit the desired deployment level.

This paper demonstrates a fast dynamic programming method to quantify the system-level market depth of large-scale energy arbitrage, in terms of: 1) how quickly do marginal energy arbitrage profits under existing market rules erode with increasing energy storage deployment, and 2) how are these values impacted by renewables deployment? Energy arbitrage revenues are quantified to substantially diminish at higher deployment levels, even in the theoretically limiting case of monopoly ownership; this price impact or diminishing returns effect outweighs increased revenues from greater renewables deployment. Therefore, this paper contributes to the existing set of valuation tools and frameworks for energy storage, much of which still relies on purely historical electricity prices.

Methodologically, the improved DP method essentially accomplishes a similar goal to bilevel programming approaches, with the tradeoff being this paper attains very fast solution times (solving one year in about 10 seconds) by focusing on only the energy balance constraints instead of using fully detailed transmission system constraints for market clearing. Compared to MILP methods that implicitly iterate over the energy-power space to capture the generation supply stack, the DP method only iterates over time periods, resulting in solution times that scale deterministically and linearly with the time horizon considered. In addition, the functional form's flexibility further allows both using historical offer curves (with the advantage of inherent calibration to market realizations), as well as incorporating future generation stack changes (such as solar increases in this paper; or unit retirements, which are out of this paper's scope).

In addition, this paper contributes to the ongoing discussion about energy storage's market power and industrial organization, by analyzing the problem in both stylized closed-form solutions and using the flexibility of the DP method's objective function to study both monopoly and social planner extremes of ownership models. Energy arbitrage operations exhibit substantial potential for positive externalities, raising further policy questions about the role of capacity or other extra-market constructs to robustly and efficiently incentivize the investment and operation of energy storage resources.

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