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Volatility Managed Portfolios

Report

Group 3

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Introduction

In the volatile world of finance, where political tensions and economic fluctuations are always present, volatility management is a promising strategy that can be useful. Echoing John Maynard Keynes' wisdom, "Day-to-day fluctuations in the profits of existing investments, which are obviously of an ephemeral and non-significant character, tend to have an altogether excessive, and even an absurd, influence on the market," we find a lesson applicable to finance: volatility acceptance is the main ingredient of it. Strategy of volatility management serves a pragmatic solution for the management of market uncertainties by providing investors with an opportunity to reduce risks even if their overall expectation remains unchanged. Through accepting volatility, investors can create a stable and adaptable investment practice.

It has been often repeated in the recent academic studies that strategies, which are focused on the volatility, significantly outperformed the traditional techniques like the mean-variance optimization. Volatility-targeted strategies, affirming the belief that volatilities are predictable just because common returns are not, have gained a lot of the market's attention, and players like hedge funds and CTAs dominate the industry.

The fundamentals of volatility targeting are simple yet powerful: leverage positions during times of low predictability of volatility and scale down the exposure when volatility is expected to be high. With these strategies, the risk management techniques attempt to increase the portfolio return whilst at the same time, decreasing the effect of tail risk and negative turnaround.

The main goal of the study is to assess the performance and practical implementation of volatility-timing strategies. To do this, we based our analysis on the FAMA-FRENCH factors as they are widely used by practitioners. Formed by Eugene Fama and Kenneth French, these factors - market risk (the excess returns of the market over the risk-free rate), size (small vs. extended market capitalization (small vs. large companies), and book-to-market ratios (high vs. low book-to-market) - all these factors have now become critical elements in building pricing models for assets and in making portfolio decisions.

The goal of introducing the Fama-French factors into our research is to widen the knowledge base of the efficiency of volatility management strategies by using a data set that is

acknowledged and used by investors within the capital industry. The inclusion of the above factors gives a solid foundation that matches the industry standards, thus making the analysis based on the methodologies that are recognized by both the professionals and academics.

We faced troubles in the application of some forecasting methods because of the complicated nature of the volatility prediction. The challenge of developing forecasting models, which had high complexity associated with them, demanded one to be very careful with the details and to have a great understanding of the mathematical aspect. Despite difficulties we were able to advance thanks to the thorough research which allowed us to step over the obstacles like the complexities of coding and extract invaluable understanding of the management of volatility.

In this research project, we will be examining in detail the ways in which volatility management strategies are implemented. Next, we will lay out the plan, drawing on Moreira and Muir's theories. Next, we will explain what we have observed about the concept of volatility, the role it plays in returns and risk, and how these changes during good and bad times. Finally, we will give a detailed look at how the transaction costs and leverage effect should be handled when developing this strategy before giving our global recommendation.

1. Theory behind volatility-targeting strategies

Among stock risk metrics, volatility is the most used. The primary goal of the volatility targeting technique is to control the exposure of the portfolio so that the volatility of the portfolio approaches the target value as much as feasible. A. Moreira and T. Muir's research reveals interesting findings: the correlation between lagged volatility and average returns is minimal, while a significant correlation exists between lagged volatility and current volatility. Consequently, the trade-off between risk and return diminishes during periods of elevated volatility. In terms of portfolio decisions, this suggests that a conventional mean-variance investor might consider adjusting their exposure to risk based on the attractiveness of the risk-return trade-off; for instance, increasing risk during periods of low volatility and decreasing it during periods of high volatility.

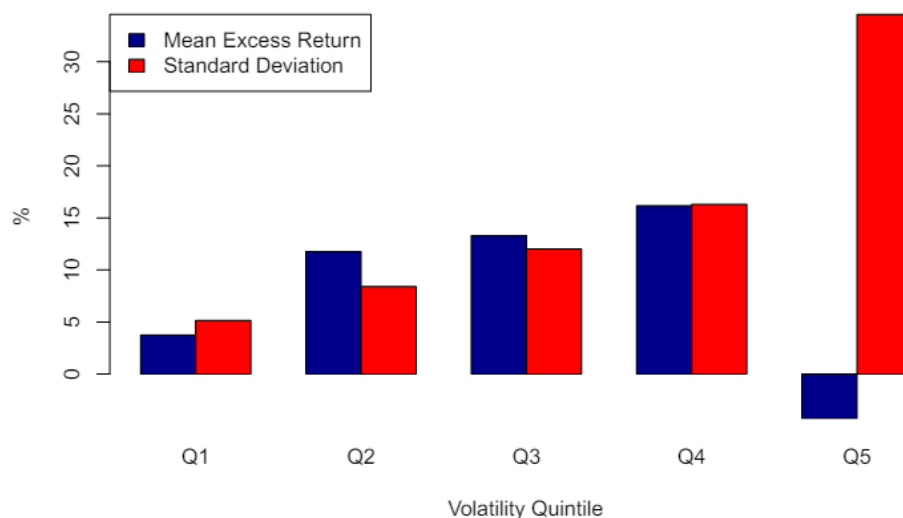


Chart 1: Volatility Quintile (The Performance of the Volatility Targeting Strategy, Magdalena Anna Godek 2021)

The analysis of annualized volatility and mean excess return, based on data from U.S. equities spanning 1926 to 2020, categorizes volatility into five levels: very low (5%), low (8%), medium (12%), high (16%), and very high (35%). The returns across these quintiles do not exhibit a consistent trend; they range from 4% in the first quintile to -4% in the highest volatility quintile, indicating that higher volatility does not necessarily correlate with higher returns. In fact, the

highest volatility is associated with negative returns, suggesting a lack of a direct relationship between equity market volatility and returns.

We differentiate between two types of volatility. Based on historical data, the first one is the historical volatility. The implied volatility is the second. Historical volatility is predicated on the idea that we can forecast the future by understanding the past. Conversely, implied volatility adjusts for the volatility that current market prices imply.

1. Data

1.1. Data Overview (single factor)

For this research project, we opted to utilize European Fama-French data and the Momentum Factor Mom.

This includes:

- **Market factor (R_m):** the difference between the expected return on the market and the risk-free rate.
- **Small minus Big (SMB):** explains the difference in returns between small and large businesses, which is determined by the market capitalization of the company.
- **High minus low (HML):** commonly referred to as the value factor, it explains the difference in returns between value and growth stocks and suggests that growth stocks - those with lower book-to-market ratios - perform worse than value stocks, which have higher figures.
- **Robust minus Weak (RMW):** explains the variation in returns between profitable and unprofitable companies.
- **Conservative minus aggressive (CMA):** represents the variation in profits between companies with high and low investment policies. Aggressive firms exhibit a higher degree of investment, while conservative firms have lower investment policies.
- **Momentum Factor Mom:** the average return on the two high prior return portfolios minus the average return on the two low prior return portfolios. As shown on the above graph, Mom has been constantly outperforming other factors since the 90s.

Using European Fama-French factors instead of their US counterparts offers several advantages. Firstly, as European students, we found the use of European Fama-French factors especially compelling because of their connection to the dynamics of the regional markets and our need for a more sophisticated understanding of European investment landscapes. Secondly, most of the papers that have been done regarding this subject mostly use US data and we therefore thought it was more interesting to test this kind of analysis on European stocks.

While monthly data are frequently available for a longer period, all the data used in this study are daily. Research supports our choice to use daily data; for example, Fleming et al. (2003) used intraday returns, which improved the strategy's performance. Dividends are included in the total returns on stocks. Kenneth French's online data library are the sources of the equity data.

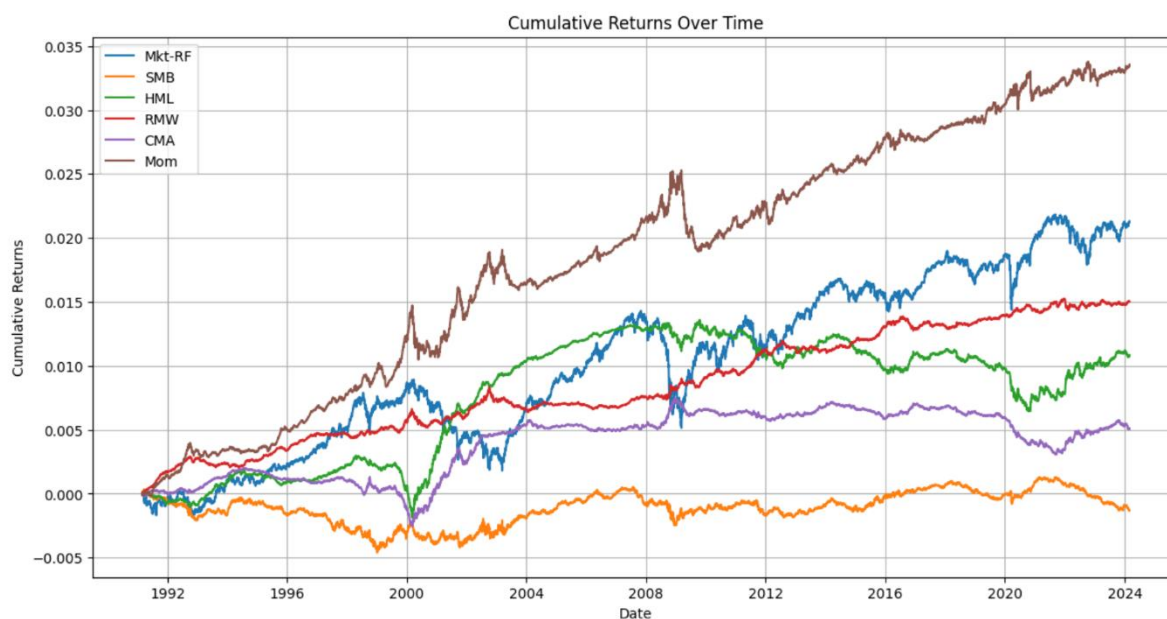


Figure 1: Cumulative Returns Over Time, including Momentum Factor

1.2. Data Overview (portfolio with multiple factors)

To go further into our analysis, we opted to evaluate the model using portfolios we constructed based on the Fama-French factors.

Here's a breakdown of our portfolios:

- **FF3 (Fama-French 3-Factor Model):** $\text{Mkt-RF} + \text{SMB} + \text{HML}$

FF3 provides a basic framework for assessing portfolio performance beyond just market returns. By incorporating SMB and HML, it accounts for size and value factors, which are crucial in understanding the sources of stock returns.

- **FF5 (Fama-French 5-Factor Model):** $\text{Mkt-RF} + \text{SMB} + \text{HML} + \text{RMW} + \text{CMA}$

FF5 further enhances the explanatory power of the model by incorporating two additional factors (RMW and CMA) that capture different dimensions of stock returns beyond size and value. This helps in better understanding the drivers of portfolio performance.

- **FF3 + Mom (Fama-French 3-Factor Model with Momentum):** $\text{Mkt-RF} + \text{SMB} + \text{HML} + \text{MOM}$

Adding momentum to the FF3 model allows for the examination of whether recent trends in stock performance persist or reverse. Momentum is often considered a behavioural anomaly in financial markets and including it provides insights into short-term trends in stock returns.

- **FF5 + Mom (Fama-French 5-Factor Model with Momentum):** $\text{Mkt-RF} + \text{SMB} + \text{HML} + \text{RMW} + \text{CMA} + \text{MOM}$

Like FF3 Mom, FF5 Mom extends the FF5 model by including momentum. This allows for a more comprehensive analysis of stock returns by incorporating both fundamental and behavioural factors.

Once we determined the constituents for each portfolio, the next step was to determine weight allocation. To achieve a sharper analysis, we opted for Mean Variance Optimization instead of equally weighting the assets.

Overall, these portfolios based on the Fama-French factors offer a structured approach to dissecting stock returns, helping us to better understand the underlying drivers of portfolio performance beyond just market movements.

2. Description of the Strategy

2.1. Global description

In a similar approach to that adopted by Moreira and Muir in their seminal paper "Volatility Managed Portfolios," we constructed our volatility-managed factors and portfolios of factors. This was achieved by scaling the excess returns by the inverse of their conditional volatilities. By implementing this method, we ensure that each factor's exposure is adjusted dynamically based on the prevailing market conditions, as reflected by their respective volatilities. This scaling not only helps in managing the inherent risk associated with each factor but also aligns the portfolio's performance more closely with its risk-adjusted return objectives. Through this technique, our investment strategy seeks to optimize the balance between risk and return by modulating exposure according to fluctuations in underlying volatilities, thereby enhancing the potential for improved, stable returns over time:

$$f_{t+1}^{\sigma} = \frac{c}{\sigma_t^2} \times f_{t+1}$$

f_{t+1} is the factor or the portfolio excess return, σ_t^2 is a proxy for the factor or the portfolio's conditional variance and we choose c so f^{σ} has the same unconditional volatility as f .

Then we use **mean-variance optimization** to combine:

- The original (without timing) factor or portfolio excess return
- The volatility-managed version of this factor or portfolio excess return

By doing this strategy in two steps we construct combined portfolios of unconditional and conditional factors that should perform better in and out-of-sample than the unconditional original factors.

2.2. In-sample Implementation (Moreira, A., Muir, T., 2016) and Conditional Volatility Estimates

In this section, as we initiated our analysis, we began by determining the parameter c using the full dataset, which inadvertently introduced a look-ahead bias. This potential flaw in our initial setup is addressed and rectified in the subsequent section of our analysis. Concerning the estimation of conditional variance, we employed three distinct methodologies. The first method calculates the variance based on the realized variance from the previous month, treating daily returns as equally weighted. The second method extends this approach by considering the variance of daily returns over the last six months. Lastly, the third method involves a more dynamic variance forecasting model, specifically the Exponentially Weighted Moving Average (EWMA), which adapts more responsively to changes in market volatility.

Using an equal-weighted (EW) variance forecast in financial markets often encounters challenges, particularly due to the dynamics of financial data and the importance of capturing the most relevant information. The first major shortcoming of the EW estimator arises from its non-adaptive nature at the front end. In this model, each observation within the estimation window is assigned a constant weight. Specifically, each day's observation within a 21-trading day window receives a fixed weight of approximately $1/21$, or about 4.76%, without any consideration for the recency of the data. This approach fails to reflect the more immediate reality of financial markets, where more recent observations are generally more indicative of current and future market conditions. This static weighting scheme does not accommodate the often-rapid changes in market volatility, potentially leading to delayed responses to fresh market data, thereby limiting the effectiveness of timely risk assessments.

Moreover, the EW estimator also suffers from the "ghost effect" at the rear end. This phenomenon is particularly noticeable during periods of significant market upheavals, such as the Global Financial Crisis (GFC) or the COVID-19 pandemic. As these events age out of the 21-day window, there can be sudden and misleading reductions in the estimated volatility, which appear as if volatility is "miraculously" decreasing. This effect can lead to underestimations of risk precisely as older, more volatile periods drop out of the calculation window. This dropout effect fails to capture the persistent nature of high volatility and its potential impacts over a longer term, thereby misleading investors about the actual risk levels.

Given these weaknesses, an EW variance forecast lacks several desirable properties for effective financial risk management. A more sophisticated model would ideally incorporate features such as short-term adaptiveness, to quickly adjust to changes in market conditions; volatility clustering, to account for the persistence and autocorrelation in volatility data; and long-run mean reversion, to acknowledge that while markets may experience periods of high or low volatility, they typically revert to a long-term average level over time. These aspects are critical for capturing the true dynamics of market volatility and providing more accurate and responsive risk assessments.

In contrast, models like the **Exponentially Weighted Moving Average (EWMA)** offer a more refined approach by assigning exponentially decreasing weights to older observations. This method enhances responsiveness to recent market changes, mitigating some of the inherent limitations of the equal-weighted model, especially in terms of adaptiveness and handling the ghost effect. The flexibility to adjust the decay factor in EWMA allows for better customization to specific market conditions, making it a valuable alternative for volatility forecasting and risk management. We have shown in *Figure 2* that EWMA volatility forecast can be used to construct 95% confidence intervals on daily returns on the Mkt-RF factor. The graph highlights the **short-term adaptiveness** and the **stickiness** of this model.

In Table 3, we present a detailed comparison of the three volatility forecasting models: EWMA, RV 6M, and RV 1M. The results indicate that the EWMA model outperforms the others, exhibiting the lowest Mean Squared Error and Mean Absolute Error. This superior performance underscores the effectiveness of the EWMA model in capturing and predicting market volatility more accurately than the alternative methods assessed.

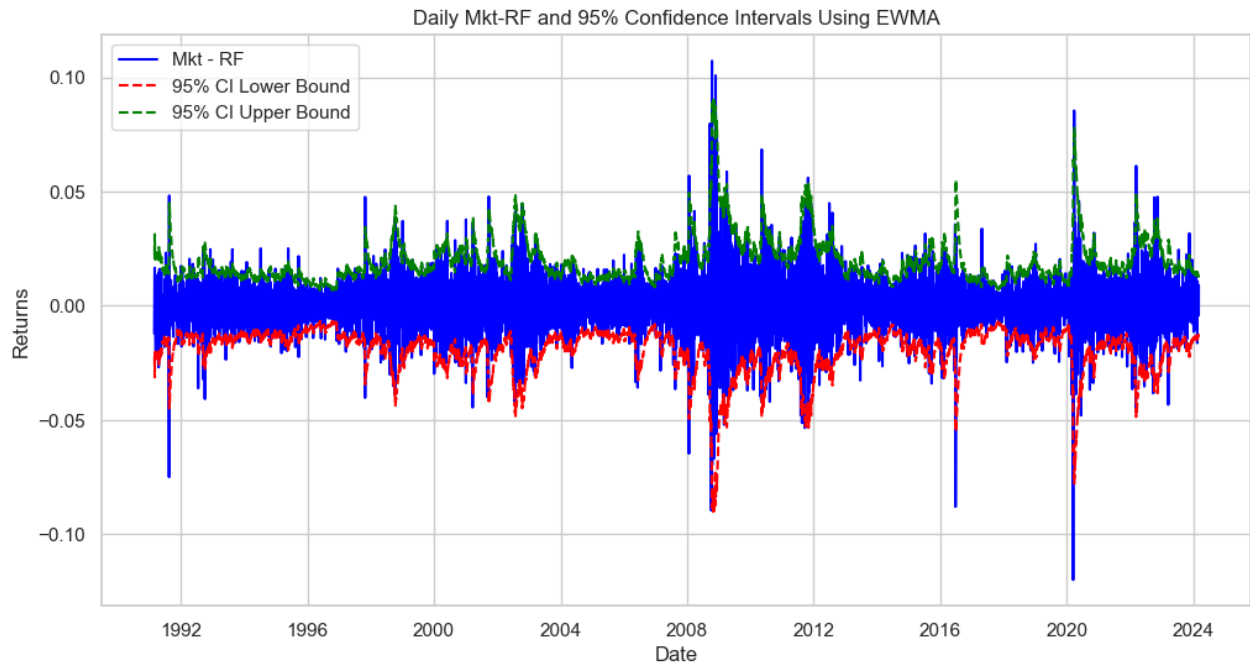


Figure 2: Market Returns and 95% Confidence Intervals Using EWMA

	Mean Squared Error	Mean Absolute Error
Model		
EWMA	0.2169	0.4211
RV 6M	0.2178	0.4214
RV 1M	0.2170	0.4214

Table 1: Comparison of Volatility Forecast Models

We study and compare the results on our in-sample strategy using the 3-volatility forecast above in **section 3**.

2.3. Out-of-Sample Implementation (Dion Bongaerts, Xiaowei Kang & Mathijs van Dijk, 2020)

To effectively implement our strategy in an out-of-sample context, which is crucial since in-sample strategies are not practicable for real-world applications, we employed a rolling window approach. This method was utilized to calculate our target volatility and to estimate our

forecasts accurately. For the analysis of both equity markets and equity factors, we set the volatility target as of a specific date t , using all available return data up to that point. The long-term volatility of each asset was then defined based on this historical data, thereby helping to avoid any look-ahead bias or arbitrary determinations regarding volatility targets. A requisite dataset, spanning at least 10 years of returns, was necessary to compute this long-term volatility. Therefore, our analysis for volatility-targeting began 10 years after the start dates of the two datasets involved in our study. This approach ensures that our volatility assessments are solidly anchored in a substantial historical context and are devoid of any forward-looking information **(Dion Bongaerts, Xiaowei Kang & Mathijs van Dijk, 2020)**.

Furthermore, look-ahead bias is a potential concern when determining the weights in the "combined" strategy, as moments of asset returns are calculated using the entire dataset, not just historical data. By adopting the rolling window technique, we were able to create an out-of-sample combined portfolio. The performance metrics of this portfolio are thoroughly examined in Section 3 of our report.

Regarding in-sample implementation, we evaluated the effectiveness of our approach by comparing performance metrics across three models of volatility forecasting. This comparison helped to illuminate the strengths and weaknesses of each model under controlled, historical conditions, providing deeper insights into the robustness of our forecasting techniques.

3. Main Results

3.1. In-sample Vs Out-of-Sample Results

We initially set out to compute Sharpe ratios for both the in-sample and out-of-sample analysis and compared these to the Sharpe ratios of the original factors and portfolios. This preliminary assessment aimed to determine whether our strategy could enhance the Sharpe ratios, and importantly, if its effectiveness persisted when applied to out-of-sample data. By doing so, we sought to establish not only the potential for improved risk-adjusted returns within the confines of our historical data but also the robustness and applicability of our strategy in real-world scenarios.

	Mkt-RF	SMB	HML	RMW	CMA	Mom	MVE - FF3	MVE - FF5	MVE - FF3 Mom	MVE - FF5 Mom
SR unconditional	0.342874	-0.041360	0.416253	0.923817	0.277231	0.814582	0.549479	1.530076	1.194123	1.713912
SR conditional in sample (last month estimation)	0.393282	0.566082	0.634638	1.190817	0.298232	2.048802	0.986164	1.865588	1.916962	2.310730
SR conditional in sample (last 6 month estimation)	0.490630	0.026119	0.622203	1.166694	0.446138	1.647672	0.952413	1.766952	1.751685	2.156195
SR conditional in sample (EWMA estimation)	0.775590	0.497062	0.736363	1.511411	0.429930	2.548013	1.443450	2.169525	2.474025	2.788315
SR conditional out-of-sample (last month estimation)	0.093811	0.711506	1.045883	0.698375	0.158942	1.589811	1.361844	1.473612	1.680152	1.952855
SR conditional out-of-sample (last 6 months estimation)	0.225827	0.031778	0.923067	0.783422	0.266341	1.220975	1.133601	1.492394	1.403447	1.827746
SR conditional out-of-sample (EWMA estimation)	0.567767	0.777424	1.380243	1.056841	0.165773	2.213169	1.963325	1.769684	2.340275	2.443116

Table 2: Conditional and Unconditional Sharpe Ratios

The analysis highlights the efficacy of the Exponentially Weighted Moving Average (EWMA) model, particularly evident in its ability to adapt to recent market changes and enhance portfolio performance. For instance, the EWMA estimation improves the Sharpe ratio for the 'MVE - FF5 Mom' portfolio to 2.788 in-sample, up from an unconditional Sharpe ratio of 1.713. Similarly, the 'MVE - FF3 Mom' sees an increase to 2.474 in-sample from its unconditional counterpart of 1.194. These improvements underscore EWMA's capacity to effectively adjust to volatility, providing more accurate risk assessments.

Comparatively, while the performance is notably better in the in-sample analysis, which is expected due to the model being calibrated on this data, it is imperative to note that in-sample results, though ideal, cannot be practically implemented. The out-of-sample results, crucial for real-world application, also show promising improvements, albeit slightly lower than the in-sample. For example, the 'MVE - FF5 Mom' portfolio still achieves a Sharpe ratio of 2.443 out-of-sample under EWMA estimation, and the 'MVE - FF3 Mom' achieves 2.340, demonstrating robustness and the practical utility of the EWMA model. This consistent performance in the out-of-sample setting is particularly encouraging, affirming the model's potential for real-time financial decision-making, despite the inherent challenges of transitioning from theoretical to actual market conditions.

As we continue with our study, the analysis will exclusively use **out-of-sample testing with Exponentially Weighted Moving Average (EWMA) volatility forecasts**. This approach ensures that our findings are both theoretically sound and practically relevant. Out-of-sample testing is essential as it assesses the model's effectiveness on data not used during its development, providing a true measure of its predictive ability and reliability.

3.2. Performance Analysis of the Out-of-Sample EWMA Portfolio

In our analysis, we conducted time-series regressions for each volatility-managed factor and portfolio against its corresponding unmanaged factor, using the regression model $f\sigma_t = \alpha + \beta f_t + \epsilon_t$, where f_t represents the return of the benchmark portfolio. This regression approach allowed us to isolate the alpha, which measures the excess return of the volatility-managed factors and portfolios over the benchmark, independent of market movements. Additionally, we calculated the appraisal ratio, which assesses the performance of the managed factors and portfolios relative to the variability of the alpha, providing a measure of risk-adjusted excess performance. These statistical measures are crucial for evaluating the effectiveness of volatility management in enhancing the performance of the factors and portfolios beyond their unmanaged counterparts.

	Mkt-RF	SMB	HML	RMW	CMA	Mom	MVE - FF3	MVE - FF5	MVE - FF3 Mom	MVE - FF5 Mom
Alpha	0.108937	0.492295	1.229118	0.271280	0.002082	3.673187	4.010552	0.685967	5.400242	2.949642
Std Error	0.291378	0.412019	0.614591	0.293317	0.214523	0.783220	0.822225	0.472809	0.876789	0.714784
Appraisal Ratio	0.059355	0.189690	0.317499	0.146831	0.001541	0.744552	0.774372	0.230332	0.977809	0.655134
R-squared	0.105285	0.100715	0.042489	0.438109	0.283149	0.142678	0.102699	0.475895	0.145747	0.345956
RMSE	29.135371	41.198548	61.454098	29.329252	21.450555	78.315571	82.215714	47.276972	87.671744	71.472563

Table 3: Regression Results

The regression analysis centred on alpha and appraisal ratio metrics for volatility-managed factors and portfolios provides key insights into their performance relative to original portfolios. Notably, the Momentum (Mom) portfolio and MVE - FF3 Mom stand out with high alphas of 3.673 and 5.400, respectively, each more than twice their respective tracking errors. This marked outperformance indicates highly effective volatility management, capturing significantly higher returns compared to their original portfolio counterparts.

In terms of risk-adjusted performance, the appraisal ratios highlight this effectiveness. The Mom portfolio achieves an appraisal ratio of 0.744 and MVE - FF3 Mom records a ratio of 0.978, both indicating robust returns that adequately compensate for the risks undertaken. In contrast, other factors like SMB and HML, though showing significant alphas of 0.492 and 1.229, do not achieve twice their tracking errors, reflecting more moderate enhancements relative to their original portfolios.

Overall, these findings underline the success of the volatility management strategies implemented, especially in portfolios like Mom and MVE - FF3 Mom where high alpha is achieved far beyond the associated risk levels. This suggests that these strategies are not only effective in enhancing returns but also in managing the underlying investment risks efficiently.

3.3. Tail risks and Global Performance

The analysis of the managed portfolios reveals a significant enhancement in both annualized mean returns and Sharpe ratios, indicating that the risk-adjusted returns are notably improved. For example, the Sharpe Ratio in 'Mom' portfolio increased from 0.716 to 2.213, underscoring the effectiveness of the volatility management strategies in optimizing performance.

	Mkt-RF	SMB	HML	RMW	CMA	Mom	MVE - FF3	MVE - FF5	MVE - FF3 Mom	MVE - FF5 Mom
Original Annualized Mean	0.060098	0.009805	0.037359	0.038381	0.022962	0.091408	0.121595	0.775448	0.476303	0.959904
Managed Annualized Mean	0.174898	0.337774	0.866902	0.413542	0.042002	1.872089	1.704173	1.155772	2.220082	2.159313
Original Sharpe Ratio	0.308625	0.107036	0.462278	0.777049	0.426324	0.715863	0.601909	1.423782	1.115409	1.555230
Managed Sharpe Ratio	0.567814	0.777488	1.380356	1.056927	0.165787	2.213351	1.963486	1.769830	2.340467	2.443317
Original Max Drawdown	-0.630847	-0.270590	-0.523519	-0.177498	-0.361414	-0.474258	-0.718970	-0.828298	-0.859857	-0.843181
Managed Max Drawdown	-0.653291	-0.714756	-0.959377	-0.731268	-0.768758	-0.771071	-0.983647	-0.812657	-0.900409	-0.845176
Original Expected Shortfall	-0.029782	-0.014367	-0.011613	-0.007070	-0.007558	-0.020805	-0.030583	-0.073851	-0.066854	-0.087759
Managed Expected Shortfall	-0.043474	-0.063012	-0.091137	-0.054174	-0.037594	-0.115518	-0.109987	-0.086050	-0.118353	-0.111791
Original Skew	-0.237402	-0.650726	0.146634	0.176173	0.140722	-1.083777	-0.476591	1.141087	-0.524925	0.745396
Managed Skew	-0.106971	0.443895	0.751536	0.140335	1.066769	0.492143	1.520594	0.187618	1.263602	0.385169
Original Kurt	8.072162	7.931063	5.549565	9.014857	3.507507	11.501668	7.580184	34.882473	8.656979	38.116479
Managed Kurt	17.664460	5.862706	11.702443	2.083333	28.905972	4.721818	16.090641	2.056039	9.557036	2.138513

Table 4: Global Performance Metrics and Tail Risk

However, a critical examination of the tail risk metrics-Max Drawdown and Expected Shortfall-raises concerns. Ideally, volatility management techniques should not only enhance returns but also mitigate tail risks. Contrary to expectations, the data shows that managed portfolios exhibit increased Max Drawdowns and Expected Shortfalls. For instance, the Max Drawdown in the 'Mom' portfolio deepened from -0.474 to -0.771, and the Expected Shortfall worsened from -0.021 to -0.116, indicating larger potential losses in adverse market conditions.

The increase in tail risks is problematic as it suggests that the managed strategies, while effective in pursuing higher returns, may be exposing portfolios to greater financial risks during market downturns. This is contrary to one of the fundamental goals of risk management which is to control and limit losses in extreme scenarios.

Additionally, the changes in skewness and kurtosis toward more positive values and higher peaks respectively might indicate that while the frequency of negative returns is reduced, the potential for experiencing severe negative outcomes when they do occur might have increased.

3.4. Business cycle risks

To analyse the performance of the volatility-managed strategy during the expansion and recession phases of the USA business cycle, we'll consider the periods outlined in the question along with the NBER (National Bureau of Economic Research) business cycle indicators.

We will focus on the EWMA managed portfolios.

a) Expansion Phases:

- June 2009 to February 2020:

During this prolonged expansion phase following the recession that commenced in December 2007, various macroeconomic indicators demonstrate a robust recovery. Key indicators such as GDP growth, employment rates, consumer spending, and industrial production typically exhibit positive trajectories, reflecting a strengthening economy. Additionally, inflation remains moderate, signalling stable price levels conducive to economic growth.

Against this backdrop, financial markets tend to thrive, characterized by buoyant investor sentiment and robust asset performance. Equities often experience steady gains, buoyed by optimistic corporate earnings outlooks and increased consumer confidence. Moreover, credit conditions may loosen, facilitating capital investments and business expansions.

In such an environment, a volatility-managed strategy becomes imperative to navigate the evolving market dynamics. While aiming to capitalize on the overall upward trend, prudent risk management strategies are essential to mitigate the impact of increased market volatility. This strategy may involve dynamic asset allocation, wherein portfolio weights are adjusted based on prevailing market conditions and risk profiles.

	Mkt-RF	SMB	HML	RMW	CMA	Mom	MVE - FF3	MVE - FF5	MVE - FF3 Mom	MVE - FF5 Mom
Original Sharpe Ratio	0.418062	0.193961	-0.347410	0.981892	-0.133067	0.980177	0.162201	1.306024	0.877749	1.492602
Managed Sharpe Ratio	0.356973	0.784450	-0.485385	1.074194	-0.129143	2.259419	0.156150	1.427206	1.394285	1.975309
Original Max Drawdown	-0.303919	-0.138708	-0.356830	-0.104701	-0.149749	-0.176432	-0.543592	-0.492523	-0.533440	-0.519944
Managed Max Drawdown	-0.845587	-0.695102	-0.925445	-0.567836	-0.406218	-0.650683	-0.995684	-0.892171	-0.946314	-0.913087
Original Expected Shortfall	-0.026954	-0.010297	-0.010181	-0.006651	-0.005238	-0.014717	-0.029245	-0.053356	-0.052966	-0.063129
Managed Expected Shortfall	-0.050711	-0.060634	-0.045093	-0.048033	-0.016193	-0.094284	-0.083453	-0.091814	-0.128289	-0.123196

Table 5: Portfolio metrics results (June 2009 to February 2020)

	Mkt-RF	SMB	HML	RMW	CMA	Mom	MVE - FF3	MVE - FF5	MVE - FF3 Mom	MVE - FF5 Mom
Alpha	-0.027749	0.362848	-0.128731	0.088445	-0.007625	2.377144	0.031644	0.251349	1.123164	1.121827
Tracking Error	0.283058	0.330249	0.232193	0.179944	0.059058	0.575183	0.498944	0.375477	0.724230	0.603416
Appraisal Ratio	-0.015565	0.174446	-0.088026	0.078039	-0.020498	0.656187	0.010070	0.106285	0.246232	0.295180
R-squared	0.258334	0.430306	0.393937	0.739900	0.671193	0.409493	0.169610	0.739013	0.383300	0.628236
RMSE	28.300739	33.018970	23.215193	17.991235	5.904771	57.508004	49.885491	37.540980	72.410134	60.330888

Table 6: Portfolio performance results (June 2009 to February 2020)

- April 2020 to March 2024:

Spanning from April 2020 to March 2024, this period encapsulates the aftermath of the COVID-19 pandemic-induced recession, characterized by a rollercoaster ride of economic and market dynamics. In its initial stages, macroeconomic indicators painted a bleak picture, with sharp contractions in GDP, historic spikes in unemployment rates, disrupted supply chains, and plummeting consumer spending. Financial markets mirrored this uncertainty, experiencing extreme volatility and significant selloffs as investors grappled with the unprecedented shock to global economies.

During these tumultuous times, a volatility-targeted strategy would have prioritized nimble risk management techniques to navigate the heightened market turbulence effectively. Moreover, dynamic asset allocation strategies would have been crucial to capitalize on market dislocations and identify pockets of relative strength amidst the turmoil.

As the recovery efforts gained momentum and fiscal stimulus measures took effect, macroeconomic indicators gradually started to show signs of improvement. GDP growth rebounded, driven by robust fiscal policies and accommodative monetary measures. Unemployment rates began to recede as businesses reopened and hiring resumed, while consumer confidence saw a gradual uptick, leading to a resurgence in spending.

Amidst this evolving economic landscape, a volatility-targeted strategy would have adapted its focus to capture the emerging growth opportunities while remaining vigilant against residual volatility risks. This could involve reallocating capital towards sectors poised to benefit from the recovery, while maintaining a diversified portfolio to mitigate sector-specific risks.

Furthermore, as central banks maintained their commitment to accommodative monetary policies, including low interest rates and asset purchase programs, investors would have adjusted their portfolio allocations accordingly.

	Mkt-RF	SMB	HML	RMW	CMA	Mom	MVE - FF3	MVE - FF5	MVE - FF3 Mom	MVE - FF5 Mom
Original Sharpe Ratio	0.699414	-0.288380	0.592048	0.423057	-0.067238	0.327006	0.867168	1.331341	1.041186	1.615964
Managed Sharpe Ratio	-0.080377	-0.913219	-0.506073	0.851597	-0.054326	1.348871	0.035361	1.851154	1.970604	2.315491
Original Max Drawdown	-0.337177	-0.233835	-0.165699	-0.086505	-0.204064	-0.207464	-0.276058	-0.416570	-0.477037	-0.437517
Managed Max Drawdown	-0.525082	-0.860778	-0.474637	-0.337985	-0.388699	-0.270331	-0.643301	-0.326258	-0.237652	-0.405460
Original Expected Shortfall	-0.026488	-0.007935	-0.015106	-0.007655	-0.008429	-0.025084	-0.032706	-0.071271	-0.080639	-0.080755
Managed Expected Shortfall	-0.030498	-0.041075	-0.024166	-0.034427	-0.023482	-0.066044	-0.051643	-0.054834	-0.060201	-0.077206

Table 7: Portfolio metrics results (April 2020 to March 2024)

	Mkt-RF	SMB	HML	RMW	CMA	Mom	MVE - FF3	MVE - FF5	MVE - FF3 Mom	MVE - FF5 Mom
Alpha	-0.119445	-0.181707	-0.035715	0.136584	-0.006220	0.473525	-0.018157	0.318891	1.059645	0.816496
Tracking Error	0.150401	0.180542	0.126893	0.121694	0.061054	0.428145	0.293181	0.178226	0.411726	0.350740
Appraisal Ratio	-0.126134	-0.159847	-0.044702	0.178256	-0.016179	0.175657	-0.009836	0.284174	0.408757	0.369727
R-squared	0.416801	0.631580	0.304573	0.796934	0.859363	0.116312	0.171548	0.844392	0.137971	0.639643
RMSE	15.032730	18.045360	12.683048	12.163420	6.102413	42.793569	29.303739	17.813866	41.152462	35.056819

Table 8: Portfolio performance results (April 2020 to March 2024)

- Expansion periods results:

From June 2009 to February 2020, the original portfolio exhibited relatively positive Sharpe ratios, indicating favourable risk-adjusted returns, but was susceptible to significant drawdowns during market downturns. This vulnerability underscored the need for proactive management strategies. Interventions aimed at enhancing risk-adjusted returns yielded improvements, particularly in factors like "SMB" (Small Minus Big) and "Mom" (Momentum). However, challenges persisted in effectively managing downside risks, highlighting the importance of further refinement in risk mitigation techniques.

Transitioning to the period from April 2020 to March 2024, the original portfolio continued to deliver decent risk-adjusted returns but faced notable drawdowns and expected shortfall, signalling its susceptibility to market volatility. Despite these challenges, the managed portfolio demonstrated mixed results. While improvements were observed in factors such as "Mom" and "MVE - FF5 Mom" (Market Value of Equity minus Book Value, Fama-French 5-factor Momentum), it grappled with negative Sharpe ratios and deeper drawdowns compared to the original portfolio.

The observed struggles suggest that while volatility-managed portfolios hold potential for enhancing risk-adjusted returns and resilience, they require ongoing adjustments to address evolving market dynamics and uncertainties. This entails refining risk management techniques, optimizing asset allocation strategies, and incorporating new insights into portfolio construction.

b) Recession Phases:

- December 2007 to June 2009:

From December 2007 to June 2009, the global economy plunged into the depths of the Great Recession, marked by severe declines in economic activity and unprecedented market volatility. Key macroeconomic indicators painted a grim picture, with GDP contracting sharply, unemployment rates soaring, consumer spending plummeting, and financial markets experiencing tumultuous declines.

In response to this challenging environment, a volatility-targeted strategy during this recessionary phase would have prioritized capital preservation and risk mitigation while adhering to predefined volatility thresholds. This necessitated reducing exposure to risk assets, such as equities, which were particularly vulnerable to the economic downturn.

The primary focus of a volatility-targeted strategy during the Great Recession would have been on preserving wealth and minimizing losses during the downturn, while actively managing portfolio volatility to stay within predefined bounds. This involved a cautious approach to portfolio management, emphasizing liquidity, diversification, and risk management, while continuously monitoring and adjusting portfolio allocations to maintain the desired volatility target.

Looking ahead, the experience of the Great Recession underscores the importance of incorporating a volatility target strategy into investment decision-making during periods of heightened market uncertainty. By setting and adhering to volatility targets, investors can proactively manage risk while seeking to achieve their long-term financial objectives in a disciplined and systematic manner.

	Mkt-RF	SMB	HML	RMW	CMA	Mom	MVE - FF3	MVE - FF5	MVE - FF3 Mom	MVE - FF5 Mom
Original Sharpe Ratio	-0.811262	-0.230208	-0.351590	0.931973	0.755638	-0.176119	-1.123919	0.381272	-1.101690	0.091935
Managed Sharpe Ratio	-0.214474	0.644006	-0.091780	0.494661	0.474890	2.253234	0.488005	0.115364	0.156312	-0.022021
Original Max Drawdown	-0.620459	-0.198059	-0.200228	-0.045700	-0.129744	-0.414361	-0.670532	-0.674162	-0.821940	-0.819833
Managed Max Drawdown	-0.481576	-0.141315	-0.613210	-0.228174	-0.425802	-0.237497	-0.823622	-0.628600	-0.863752	-0.746636
Original Expected Shortfall	-0.054560	-0.026740	-0.015555	-0.009325	-0.010520	-0.039645	-0.045347	-0.115740	-0.077516	-0.119274
Managed Expected Shortfall	-0.090022	-0.035833	-0.063430	-0.034896	-0.044553	-0.037969	-0.158248	-0.097945	-0.100866	-0.103344

Table 9: Portfolio metrics results (December 2007 to June 2009)

	Mkt-RF	SMB	HML	RMW	CMA	Mom	MVE - FF3	MVE - FF5	MVE - FF3 Mom	MVE - FF5 Mom
Alpha	-0.380736	0.206731	0.077695	-0.058680	-0.051296	1.054131	-0.211519	-0.004359	1.811690	0.201487
Tracking Error	0.275922	0.194084	0.315877	0.145293	0.086056	0.276288	0.614284	0.415313	0.574361	0.560739
Appraisal Ratio	-0.219314	0.169295	0.039093	-0.064191	-0.094739	0.606401	-0.054728	-0.001668	0.501334	0.057110
R-squared	0.756985	0.458059	0.524723	0.625585	0.924091	0.024823	0.681136	0.626145	0.407406	0.463099
RMSE	27.558659	19.384865	31.549323	14.511631	8.595186	27.595272	61.353758	41.480889	57.366349	56.005781

Table 10: Portfolio performance results (December 2007 to June 2009)

- March 2020 to April 2020:

March 2020 to April 2020 stands out as a brief yet pivotal period marking the onset of the recession triggered by the COVID-19 pandemic. During this time, market volatility surged to unprecedented levels, driven by widespread uncertainty and panic selling. Asset prices experienced rapid declines, reflecting investor fears and economic upheaval.

In response to the heightened market volatility, a volatility-targeted strategy would have swiftly implemented aggressive risk management measures to protect capital and mitigate losses. The primary objective would have been to manage portfolio volatility within predefined thresholds while navigating the turbulent market conditions.

To achieve this, the strategy would likely have reduced exposure to high-risk assets, such as equities, which tend to be particularly sensitive to market downturns. Allocating capital towards

safe-haven assets, such as cash, high-quality government bonds, or gold, would have provided stability and acted as a hedge against further market declines.

Given the rapid and unpredictable nature of the market movements during this period, maintaining a disciplined approach to risk management was paramount. Constant monitoring of portfolio volatility and adjusting allocations accordingly would have been essential to ensure that the portfolio remained aligned with the predefined volatility target.

Despite the challenges posed by the extreme market conditions, a volatility-targeted strategy aimed to provide investors with a structured and disciplined approach to navigating turbulent times. By prioritizing risk management and adhering to volatility targets, investors could seek to preserve capital and position themselves for potential opportunities that may arise amidst the market turmoil.

	Mkt - RF	SMB	HML	RMW	CMA	Mom	MVE - FF3	MVE - FF5	MVE - FF3 Mom	MVE - FF5 Mom
Original Sharpe Ratio	-0.859001	-0.736065	-5.073756	3.690576	-5.271324	0.541022	-3.127816	-2.687335	-3.281929	-3.073070
Managed Sharpe Ratio	0.098840	0.662221	-1.054305	3.647978	-3.946257	2.770874	1.932215	-4.553149	1.660867	-4.707163
Original Max Drawdown	-0.299617	-0.098457	-0.165481	-0.012262	-0.073562	-0.079279	-0.422973	-0.687481	-0.551492	-0.714317
Managed Max Drawdown	-0.140286	-0.082602	-0.083844	-0.063667	-0.155428	-0.158192	-0.376870	-0.455289	-0.089195	-0.390317
Original Expected Shortfall	-0.092000	-0.027850	-0.029350	-0.007100	-0.010300	-0.031850	-0.121678	-0.214384	-0.188007	-0.242516
Managed Expected Shortfall	-0.043761	-0.029723	-0.048862	-0.031337	-0.026100	-0.032138	-0.139500	-0.097046	-0.044261	-0.082820

Table 11: Portfolio metrics results (March 2020 to April 2020)

	Mkt - RF	SMB	HML	RMW	CMA	Mom	MVE - FF3	MVE - FF5	MVE - FF3 Mom	MVE - FF5 Mom
Alpha	-0.263638	-0.006104	-0.559918	0.001762	0.273068	0.465128	-0.743768	-0.671403	-0.325884	-0.678250
Tracking Error	0.093436	0.179592	0.213461	0.075909	0.050794	0.449689	0.272006	0.163487	0.246835	0.241935
Appraisal Ratio	-0.453089	-0.005458	-0.421210	0.003728	0.863275	0.166094	-0.439089	-0.659468	-0.212007	-0.450177
R-squared	0.905052	0.413601	0.313272	0.933309	0.942913	0.173190	0.951949	0.918453	0.391877	0.703369
RMSE	9.236853	17.753918	21.102103	7.504119	5.021362	44.454922	26.889683	16.161807	24.401375	23.916982

Table 12: Portfolio performance results (March 2020 to April 2020)

- **Recession results:**

Comparing the performance of the original and managed portfolios during the December 2007 to June 2009 period and the March 2020 to April 2020 period reveals significant disparities in

outcomes, underscoring the effectiveness of the volatility target strategy in navigating volatile market conditions.

In the first period, spanning the Great Recession, the original portfolio struggled with negative Sharpe ratios, substantial drawdowns, and elevated expected shortfall, indicative of poor risk-adjusted returns and vulnerability to market downturns. The heightened market volatility exposed the original portfolio to significant downside risks, resulting in pronounced losses during the economic turmoil. However, the implementation of our managed volatility target strategy led to notable improvements in risk-adjusted returns, downside protection, and expected shortfall mitigation. By dynamically adjusting portfolio allocations, hedging against downside risks, and reallocating capital to defensive assets, the managed strategy successfully preserved capital and mitigated losses amidst the turbulent market conditions. These enhancements underscored the importance of proactive risk management and strategic portfolio optimization in navigating challenging market environments.

Conversely, during the second period, marked by the onset of the COVID-19 pandemic, both the original and managed portfolios faced challenges associated with heightened market volatility and uncertainty. However, the managed portfolio demonstrated superior performance compared to the original portfolio. Despite the pervasive market turmoil, the managed portfolio exhibited positive Sharpe ratios, reduced drawdowns, and lower expected shortfall, highlighting the efficacy of the volatility target strategy in mitigating downside risks and preserving capital during periods of extreme market stress.

Overall, the performance disparity between the original and managed portfolios underscores the critical role of dynamic risk management and proactive portfolio optimization in achieving superior investment outcomes. By implementing a volatility target strategy, investors can effectively navigate volatile market conditions, preserve capital, and position themselves for long-term success amidst evolving economic landscapes.

4. Practical implementation of the strategy

4.1. Turnover and Transaction Costs

To determine the impact of the transaction costs, we focus on the portfolio with EWMA volatility. We calculated the turnover of each strategy and multiplied them by 10, 30 and 50 bps, then deduced that cost from each return.

$$R_t - \text{Turnover} \times \text{TC}$$

We then calculated the results of the portfolios while considering the impacts. The turnover is a great estimate of each step's weight change as we calculated using the mean of the daily weight changes.

The selection of 10, 30, and 50 basis points (bps) as the varying levels of transaction costs is grounded in both historical precedent and contemporary market realities. The choice of 50 bps stems from seminal studies such as those conducted by **Stoll and Whaley (1983)**, **Bhardwaj and Brooks (1992)**, and **Lesmond, Ogden, and Trzcinka (1999)**, which have established this figure as a reasonable approximation of transaction costs for individual stocks on the NYSE. This benchmark reflects the substantial expenses associated with trading in traditional equity markets, as evidenced by empirical research spanning several decades. Conversely, the consideration of 10 to 20 bps aligns with more recent observations, particularly in the context of trading large-cap stocks. This narrower range reflects advancements in market infrastructure, technological innovations, and increased competition among market participants, resulting in lower transaction costs for certain asset classes. By encompassing both historical benchmarks and contemporary trends, the chosen range of transaction costs provides a comprehensive framework for assessing the sensitivity of your volatility target strategy to varying levels of trading frictions, thereby enhancing the robustness and relevance of the analysis.

	Mkt-RF	SMB	HML	RMW	CMA	Mom	MVE - FF3	MVE - FF5	MVE - FF3 Mom	MVE - FF5 Mom
no TC	0.108937	0.492295	1.229118	0.271280	0.002082	3.673187	4.010552	0.685967	5.400242	2.949642
with 10 bps TC	-0.301181	0.094385	0.767387	-0.130564	-0.416493	3.215240	3.595125	0.292029	4.983011	2.545536
with 30 bps TC	-0.541417	-0.121436	0.423925	-0.354251	-0.673642	2.879345	3.344271	0.084155	4.728548	2.317325
with 50 bps TC	-0.781653	-0.337256	0.080463	-0.577937	-0.930792	2.543450	3.093417	-0.123720	4.474084	2.089113

Table 13: Impact of transaction costs on alpha

Analysing the nuanced effects of varying transaction costs on your volatility target strategy illuminates the intricate relationship between cost dynamics and portfolio performance across different risk factors. Initially, in the absence of transaction costs, the strategy showcases robust performance metrics, with positive returns observed across key factors such as HML, Mom, MVE - FF3, and MVE - FF5 Mom. These favourable outcomes underscore the strategy's efficacy in generating risk-adjusted returns in a cost-free environment. However, as transaction costs are introduced at 10 bps, a subtle yet discernible impact on performance emerges. While some factors experience marginal declines, notably in Mkt-RF, RMW, and CMA, the overall effect remains relatively contained, suggesting that the strategy can absorb modest transaction costs without significant impairment.

However, as transaction costs escalate to 30 bps and 50 bps, the deleterious effects become more pronounced and widespread. Across all factors, a notable deterioration in performance is evident, signalling heightened sensitivity to transaction frictions. Notably, CMA emerges as the most vulnerable factor, experiencing the most substantial decline in returns across all levels of transaction costs. This suggests that the strategy's performance is disproportionately affected by trading costs, particularly in factors with higher turnover or trading activity.

Furthermore, the escalation of transaction costs underscores the importance of cost management strategies in preserving the strategy's efficacy. As costs increase, the need for more efficient execution techniques, such as minimizing turnover or leveraging low-cost trading venues, becomes imperative to mitigate the adverse impact on performance.

Moreover, the analysis highlights the dynamic trade-off between transaction costs and portfolio optimization. While transaction costs are an inevitable aspect of portfolio management, their effective management is essential to maintaining the strategy's competitiveness in generating risk-adjusted returns. Therefore, adopting a holistic approach that integrates cost considerations into the portfolio construction process is crucial for navigating the complexities of modern financial markets and sustaining long-term performance objectives. Ultimately, by understanding and managing transaction costs effectively, your volatility target strategy can adapt to evolving market conditions and maintain its competitive edge in pursuit of superior risk-adjusted returns.

Possible improvement methods of our implementation:

- Method 1: Longer rebalancing periods / threshold fixed

There are two possible improvements using this approach to mitigate the impact of transaction costs on portfolio performance: longer rebalancing periods and fixed threshold rebalancing. Longer rebalancing periods involve adjusting the portfolio less frequently, such as on a monthly or quarterly basis, to reduce the number of transactions and, consequently, transaction costs. On the other hand, fixed threshold rebalancing triggers portfolio adjustments only when asset allocations deviate significantly from predetermined thresholds, minimizing unnecessary transactions. Both approaches aim to strike a balance between managing transaction costs and maintaining portfolio alignment with investment objectives.

- Method 2: including the turnover at each step in the optimization function

In this method instead of using the mean-variance optimization classical approach to determine at each step the weight we can use this optimization function instead to incorporate the potential transactions costs impacts on the portfolio weights.

The objective optimization function:

$$\text{Max}(w) : \left[\frac{1}{\gamma} \times w^T \times \mu - w^T \times V \times w - TC \times ||w - w_o|| \right], \text{ with } \sum_{n=1}^N w_n = 1$$

Applying this method is quite demanding in terms of computing power, which is why we restrained from implementing it, but it is far more accurate than the first approach since it does not interfere with the volatility target strategy rebalance frequency or put constraints on the weights.

4.2. Leverage Constraints

To assess the impact of leverage constraints, two distinct weighting schemes are employed: one based on the inverse of variance (the original without constraint) and the other on the inverse of standard deviation (with constraint) (**Dion Bongaerts, Xiaowei Kang & Mathijs van Dijk, 2020**). Following portfolio construction, a comprehensive performance analysis is conducted to assess the effectiveness of each strategy. Performance metrics such as Sharpe ratio, max drawdown, expected shortfall, alpha, appraisal ratio are computed for each portfolio. By comparing the performance metrics between the two portfolios constructed using different weighting schemes, we gain insights into the impact of leverage constraint on various aspects

of portfolio performance. Changes in metrics such as Sharpe ratio indicate the risk-adjusted returns achieved by each strategy, while metrics like max drawdown and expected shortfall provide insights into capital preservation and risk management. Additionally, metrics like alpha and tracking error offer valuable information regarding portfolio outperformance and alignment with the benchmark. Overall, this structured methodology enables a thorough examination of how leveraging influences risk-adjusted returns, capital preservation, and overall portfolio performance in our volatility target strategy.

Note that we performed the detailed analysis on the EWMA Volatility portfolio.

Metric	Mkt-RF	SMB	HML	RMW	CMA	Mom	MVE - FF3	MVE - FF5	MVE - FF3 Mom	MVE - FF5 Mom
Original Sharpe Ratio	0.342894	-0.041363	0.416277	0.923870	0.277247	0.814629	0.549511	1.530165	1.194192	1.714012
Managed Sharpe Ratio	0.287305	0.634745	1.014296	1.066344	0.100609	2.184186	1.692368	1.756299	2.165291	2.326682
Original Max Drawdown	-0.630847	-0.406010	-0.523519	-0.177498	-0.367941	-0.474258	-0.718970	-0.828298	-0.859857	-0.843181
Managed Max Drawdown	-0.710025	-0.712944	-0.986680	-0.762760	-0.779375	-0.813774	-0.995613	-0.856398	-0.981538	-0.916590
Original Expected Shortfall	-0.027405	-0.013973	-0.011054	-0.006738	-0.007650	-0.019406	-0.027559	-0.068520	-0.061821	-0.080344
Managed Expected Shortfall	-0.038976	-0.051662	-0.082383	-0.054529	-0.037394	-0.113887	-0.115650	-0.086629	-0.120725	-0.110490
Alpha	0.019910	0.348162	0.665706	0.300439	-0.015888	3.313762	3.113437	0.735332	4.380153	2.694255
Tracking Error	0.269160	0.339694	0.521938	0.308231	0.210442	0.768852	0.821070	0.496814	0.837582	0.720463
Appraisal Ratio	0.011744	0.162716	0.202488	0.154745	-0.011986	0.684250	0.601999	0.234977	0.830229	0.593694
R-squared	0.100297	0.111180	0.036309	0.425701	0.305858	0.153938	0.119215	0.435638	0.147133	0.307723
RMSE	26.913810	33.966643	52.189521	30.820574	21.042452	76.878869	82.100285	49.677332	83.751294	72.040427

Table 14: Performance metrics for weights as 1/std

Across most metrics, leverage constraints, as indicated by the transition from 1/Variance to 1/Std, tends to result in mixed effects. In terms of risk-adjusted returns, leverage constraints generally lead to declines in the managed Sharpe ratios compared to the original ratios for both portfolio constructions. For instance, the Sharpe ratio for the Mkt-RF factor decreases by approximately 16.2% when transitioning from the 1/Variance portfolio to the 1/Std portfolio, indicating a reduction in risk-adjusted returns. However, there are exceptions where certain factors exhibit improvements in the managed Sharpe ratios with leverage. Notably, the effect on maximum drawdowns varies across factors, with some experiencing increases and others decreases, suggesting a nuanced impact of leverage on downside risk. For instance, the maximum drawdown for the SMB factor increases when transitioning from the 1/Variance portfolio to the 1/Std portfolio, indicating higher downside risk with leverage. Expected shortfall, which provides a measure of extreme downside risk, indicates higher potential losses. Alpha, representing excess returns, shows declines in the managed portfolios compared

to the original portfolios for both constructions, reflecting a reduction in the ability to generate excess returns with leverage. For example, the alpha for the HML factor decreases by approximately 45.8% when transitioning from the 1/Variance portfolio to the 1/Std portfolio, indicating a reduction in the excess returns generated by this factor with leverage. The appraisal ratio, indicating risk-adjusted returns relative to tracking error, exhibits mixed changes with leverage. Some factors show improvements, while others show declines, suggesting varying efficiency in generating excess returns relative to risk taken. Overall, leverage constraint impacts performance and risk metrics differently across factors.

In the assessment of leverage constraints within the construction of a volatility-managed portfolio, we have observed that reducing leverage exposure can significantly influence the relative performance of the portfolio. Specifically, while leveraging can enhance potential returns by amplifying exposure to favourable market movements, it also increases the portfolio's risk. Therefore, introducing or tightening leverage constraints tends to reduce this risk but at the cost of diminishing the portfolio's potential performance. In practical scenarios, where regulatory and risk management considerations often necessitate lower leverage levels, this could lead to further subdued performance. Consequently, it is crucial to carefully balance the benefits of leverage against its risks, ensuring that leverage strategies align with the overall risk tolerance and investment objectives of the portfolio.

Conclusion and Recommendations

After conducting a comprehensive quantitative assessment of the performance of a volatility-managed strategy on European factor portfolios, our findings necessitate a cautious approach. While our analysis shows that volatility timing can enhance performance compared to a Minimum Variance Portfolio (MVP), the effectiveness of this strategy significantly hinges on the composition of the portfolio. Notably, portfolios incorporating the momentum factor consistently produced the highest alpha, indicating a strong dependency on this particular factor for achieving superior results.

However, practical challenges associated with trading the momentum factor - such as high transaction costs and operational complexities - may diminish the benefits observed in our theoretical models. Additionally, our study reveals that volatility management strategies do not uniformly reduce risk; in some cases, transaction costs and leverage constraints can negate the gains in alpha and improvements in the Sharpe ratio.

An interesting aspect of our study was the resilience of the volatility-managed strategy during different phases of the business cycle. This resilience suggests potential for further investigation, particularly in understanding how different economic conditions affect the performance of such strategies.

Given these insights, we recommend cautious implementation of the volatility-managed strategy. It appears prudent to limit the application of this strategy to portfolios that include the momentum factor, provided that the trading of this factor is viable in practice. Before proceeding with broader implementation, a thorough evaluation of transaction costs and potential operational hurdles is essential to ensure that the strategy delivers tangible benefits without undue risk. Further research into its cyclic resilience could also provide deeper insights into optimizing strategies across various economic environments.

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Appendix

