EDHEC BUSINESS SCHOOL

FEN_842 Risk Measurement

Lecture 2
Market Risk Measures

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Make an impact

Outline of lecture 2

Market risk measures

- **the importance of risk measurement**
- typology of portfolios
- typology of risk measures
- Value-at-Risk
- Expected Shortfall
- decomposing risk : volatility, loss, VaR, ES

Benjamin Graham

"The essence of

investment management is the

management of **risks**,

not the management of returns"

Importance of risk measures

1. Regulation

satisfying external risk limits :

banking : BIS : Expected Shortfall (replaced Value-at-Risk as per 2023)

asset managers : ESMA : Value-at-Risk

2. Formal risk monitoring of client portfolios → “second line of defense”

satisfying internal risk limits

client risk appetite

3. Risk analysis & monitoring of client portfolios → “first line of defense”

client risk appetite

Global Assets under Management

Some figures

- the world's 500 largest asset managers manage almost **USD 128 trillion** (as per end 2023)
- of which about **34%** is managed **passively**
- compared to 10 years ago, **nearly half** of the names on this list is **new**
- as per Mar-2024 :

	Company	Country	Total AuM, USD bln
1.	BlackRock	U.S.	10,473
2.	Vanguard Group	U.S.	9,300
3.	Fidelity Investments	U.S.	5,303
4.	State Street Global	U.S.	4,340
5.	Morgan Stanley	U.S.	3,629
6.	J.P. Morgan Chase	U.S.	3,564
7.	Credit Agricole	France	2,858
8.	Goldman Sachs	U.S.	2,848
9.	UBS Group	Switzerland	2,620
10.	Capital Group	U.S.	2,600

USD 47.6 trn
= 37% of total

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Measuring market risk

It depends .....

types of investment portfolios :

1. **index**, ETFs, “passive”, “beta”

2. indexed, benchmarked, **relative return** → outperform index, *Information Ratio*
   benchmark index + deviation (active share, tracking error) : “tilted passive”, “ beta + alpha”
   - enhanced indexing, benchmark hugging <1% deviation (equities)
   - up to 5% deviation (equities)

3. **total return** → capital appreciation + income, *Sharpe Ratio*
   reap (multi-) asset risk + factor premia (“smart beta”), net long positions
   benchmark agnostic, “anything goes”
   there may be a reference index (passive alternative) for performance benchmarking

4. **absolute return** → deliver positive return regardless of market conditions
   net long positions, net short positions, or “zero beta” (long/short)
   performance benchmark is cash
   includes hedge funds

Measuring market risk

It depends .....

types of **investment portfolios :**

1. **index**
2. **relative return**
3. **total return**
4. **absolute return**

increasing complexity

trading portfolios :

- held by financial institutions
- because of transactions for clients, and hedging
- short term horizon
- concentrated positions
- high leverage

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Market risk measures

- the importance of risk measurement
- typology of portfolios
- **typology of risk measures**
    1. absolute versus relative risk measures
    2. full domain versus partial domain risk measures
    3. market risk vs exposure measures
- Value-at-Risk
- Expected Shortfall
- decomposing risk : volatility, loss, VaR, ES

Typology of market risk measures

Link risk measure to investment strategy

1. absolute versus relative risk measures :

p = portfolio
b = benchmark, index

absolute $\leftrightarrow$ **relative**
$risk(r_p) \leftrightarrow risk(r_p - r_b)$

for example : $\sigma(r_p) \leftrightarrow \sigma(r_p - r_b)$ = Tracking Error Volatility TEV

given investment weights {*w*} :

$$
\begin{aligned}
TE_p &= r_p - r_b \\
&= \sum_{i \in p} w_{pi} \cdot r_i - \sum_{i \in b} w_{bi} \cdot r_i \\
&= \sum_{i \in p,b} \underbrace{\left(w_{pi} - w_{bi} \right)}_{\text{active weights}} \cdot r_i
\end{aligned}
$$

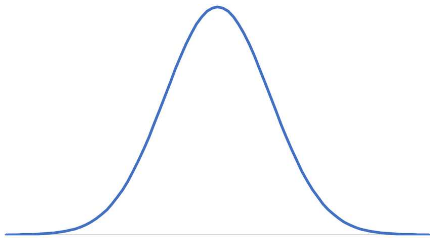
Typology of market risk measures

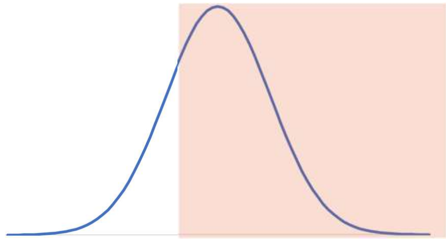
Downside risk, upside potential

1. absolute versus relative risk measures

2. full domain versus partial domain risk measures :

central moments $\leftrightarrow$ Lower Partial Moments (LPMs)





for example : variance $\leftrightarrow$ semi-variance

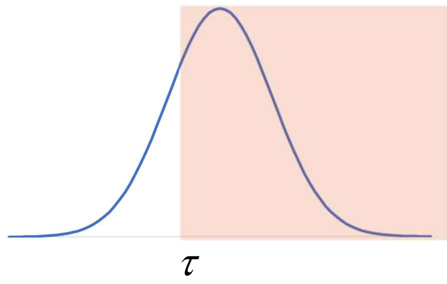
$$\sigma^2(\tilde{r}) = E\left[(\tilde{r} - \overline{r})^2\right] \quad \leftrightarrow \quad SV = E\left[\min(\tilde{r} - \overline{r}, 0)^2\right]$$

$$\sigma^2(r) = \frac{1}{T}\sum_{t=1\ldots T}(r_t - \overline{r})^2 \quad \leftrightarrow \quad SV = \frac{1}{T}\sum_{t=1\ldots T}\min(r_t - \overline{r}, 0)^2$$

Deeper dive into LPMs

General downside risk measures

Lower Partial Moments :



two parameters :
1. "minimal acceptable return" τ
2. order n of statistical moment of returns below τ

$$LPM_n(\tau) = \int_{-\infty}^{\tau} (\tau - r)^n \, dF(r)$$

F(r) : cumulative distribution function
dF(r) = f(r)dr
f(r) : density function

discrete, non-parametric : $LPM_n(\tau) = \dfrac{1}{T}\sum_{t=1}^{T} \max(\tau - r, 0)^n$

note 1 : set all returns above τ equal to 0, so averaging is over **all** T periods with weights $1/T$

note 2 : $\max(\tau - r, 0) = -\min(r - \tau, 0)$

note 3 : semi-variance is $LPM_2(\tau = \overline{r})$

Typology of market risk measures

Directional & conditional risk measures

1. absolute versus relative risk measures

2. full domain versus partial domain risk measures

3. market risk vs exposure measures : sensitivity to underlying market factor

- equities : $\text{return}_{\text{stock}} = \alpha + \beta \cdot \text{return}_{\text{index}} + \text{error}$

- govt bonds : $\text{return}_{\text{bond}} \approx -\text{modified\_duration} \cdot \Delta \text{interest\_rate}$
- credits : $\text{return}_{\text{credit}} \approx -\text{modified\_duration} \cdot \Delta \text{interest\_rate} - \text{spread\_duration} \cdot \Delta \text{credit\_spread}$

- derivatives : $\Delta \text{option\_value} \approx \text{option\_delta} \cdot \Delta \text{stock\_price}$

Q : *what is fundamental difference between risk measures and exposure measures ?*

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- **Value-at-Risk**
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Value-at-Risk

History

late 1980s, early 1990s : turbulent financial environment

1987 stock market crash, US Savings & Loan crisis (home mortgages), US recession

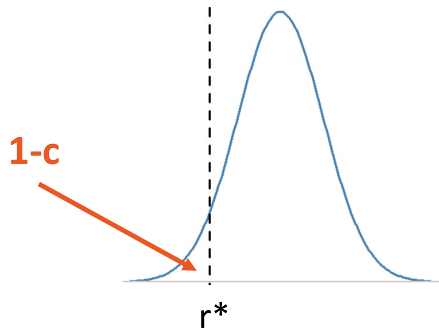
1990 Iraq invades Kuwait, oil prices surge

JPMorgan CEO Dennis Weatherstone requested a 1-pager "4:15 report",

combining all firm risk in one number :

"given 95% confidence, what is the maximum amount we can loose over one day"

= 95% 1-day Value-at-Risk (VaR)



- consider returns r over some holding period T
- define confidence level c at which losses are not to be exceeded
- denote this quantile of the return distribution as r^* : $\Pr\{r \le r^*\} = 1 - c \quad \Rightarrow \quad VaR = -r^*$

VaR in returns or money

We will work in terms of returns

- Consider a portfolio *p* with current value V_p
- the change in portfolio value over some holding period equals : $\Delta V_p \equiv r_p V_p$
- The quantile return *r** induces a change in portfolio value of : $\Delta V_p^* \equiv r_p^* V_p$
- The portfolio VaR is defined in terms of losses and hence given by : $VaR_p \equiv -\Delta V_p^*$

Example : The current 95% 1-day portfolio VaR has been estimated as 0.65%.
If the current portfolio value is € 100 mln,
then the VaR is equivalently 0.65%*€ 100 mln = € 650,000

VaR parameters

Given the portfolio composition

- **confidence interval :** typically 95%, 99% ***Q :*** *why not 99.9% ?*

- **holding period :** typically 1, 10, 20 days ***Q :*** *why not 1 year ?*

  portfolio is constant over holding period ***Q :*** *why necessary ?*

VaR under normality

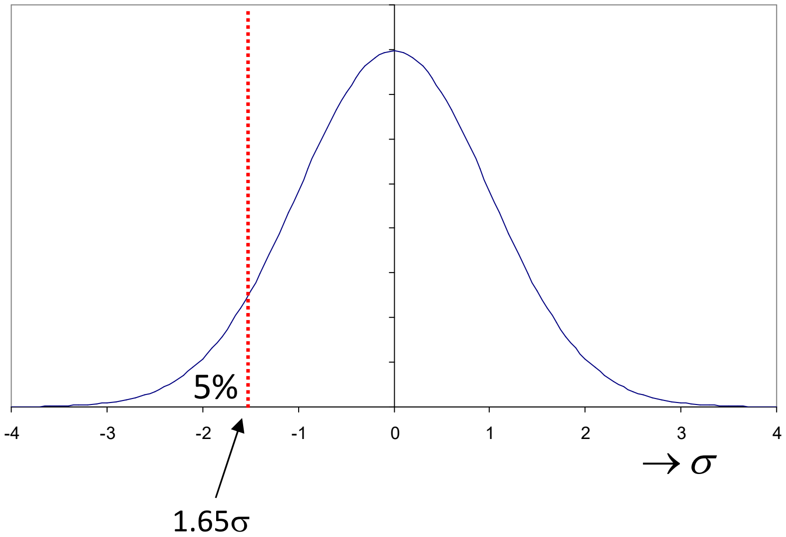
Parametric VaR : "normal VaR"

in terms of losses :

- VaR is defined by the quantile : $\Pr\left\{r \le r^*\right\} = 1 - c \quad \Rightarrow \quad VaR = -r^*$

- under normality : $-r^* = -\bar{r} + z_c \cdot \sigma$ with $z_c = N^{-1}(c)$

  where $N^{-1}(\cdot)$ = cumulative normal distribution



c	z_c
95 %	1.65
97.5%	1.96
99 %	2.33

example :
σ = 20% p.a.

1-day 95% VaR is :

$$20\% \cdot \sqrt{1/260} \cdot 1.65 = 2.05\%$$

Normal VaR over different horizons

Assuming constant portfolio composition …

time-scaling VaR : adjust normal VaR estimate to apply to a longer horizon

- start with 1-day VaR → use daily returns to estimate (cf. lecture 3)

- **iff** returns on successive days
  - have **identical normal** distributions
  - are **independent**
  - have **mean of zero**,

  "i.i.d." normal

then :

$$T\text{-}day\ \ VaR \ = \ 1\text{-}day\ \ VaR \cdot \sqrt{T}$$

"square root of T-rule"

***Q :** under these conditions, why does it work ?*

Evaluation of VaR as risk measure (1)

Coherence

Artzner ea [1999] present & justify 4 desirable properties of a risk measure :

1. monotonicity :

if for all states of the world, portfolio A has systematically lower returns than portfolio B, then the risk of A is larger than the risk of B

2. translation invariance :

adding a cash amount of C to a portfolio should reduce it risk [in €] by C

3. homogeneity :

scaling a portfolio by a factor k should scale its risk by the same factor

4. sub-additivity

Evaluation of VaR as risk measure (2)

"VaR is not sub-additive"

4. sub-additivity :

merging portfolios cannot increase risk (because of **diversification**),
for a conservative risk estimate : simply add risks of individual trading desks, businesses etc.

- **VaR violates sub-additivity** : VaR(A+B) > VaR(A) + VaR(B) is possible ....
  hence, VaR is not a coherent risk measure …

- "normal VaR" is sub-additive : because underlying volatility is sub-additive (and coherent)

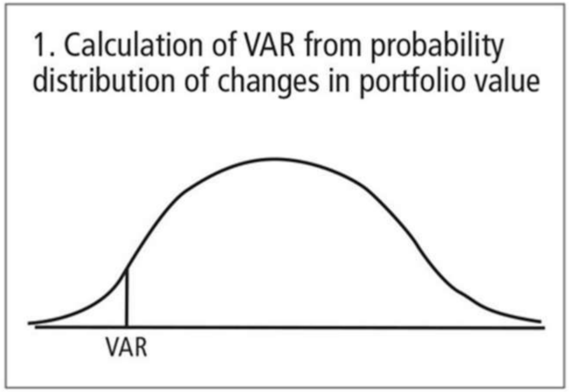
- … and in general, **non sub-additivity is a mathematical problem** that only arises in pathological examples (some discrete distributions)

- for "regular" distributions in our investment context, there is sub-additivity in the tails
- hence, non sub-additivity of VaR is **NO problem !**

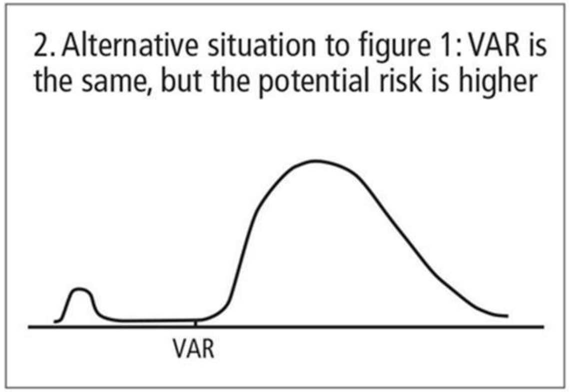
Evaluation of VaR as risk measure (3)

VaR is an incomplete risk measure

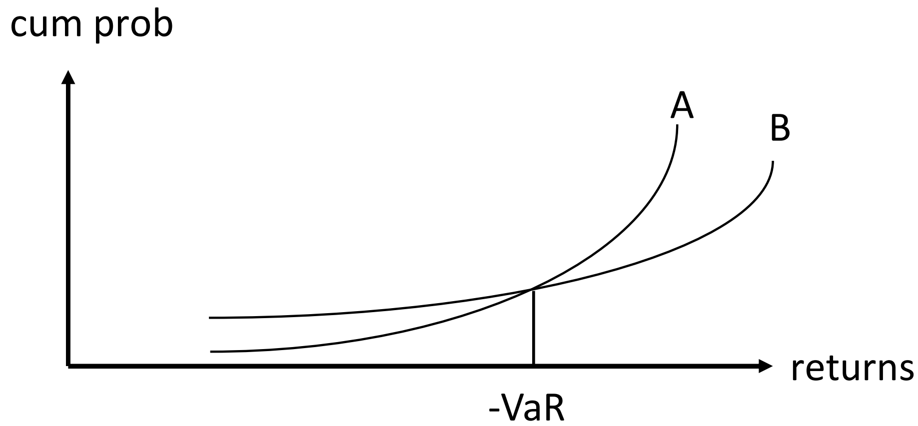
Consider two cumulative pdfs : A & B have same VaR ... but B has thicker tail beyond VaR



1. Calculation of VAR from probability distribution of changes in portfolio value



2. Alternative situation to figure 1: VAR is the same, but the potential risk is higher



what happens in the left tail if VaR is exceeded ?

quantile value → it's the best of worse case scenarios, hence under-estimating potential losses

"an airbag that works all the time, except when you have a car accident"

Q : *how could we mitigate this problem ?*

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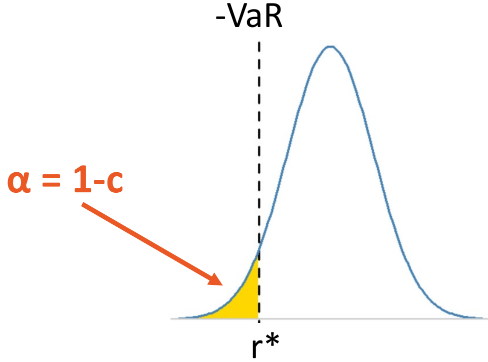
Expected Shortfall (ES)

Looking beyond VaR

"what is the average loss if the VaR is exceeded ?"

same parameters as VaR, a "95% 1-day ES", e.g.

just as VaR, defined in terms of loss :



$$ES = -E\left(r \middle| r \le -VaR\right) = E\left(-r \middle| -r \ge VaR\right)$$

ES is a conditional expectation, adjust for the total probability over which the averaging takes place :

$$ES = \frac{1}{\alpha} E\left[\max(-VaR - r, 0)\right]$$

where α = 1 – confidence level c
= exceedance probability

$$ES = -\frac{1}{\alpha} \int_{-\infty}^{VaR_\alpha} r\, dF(r) = \frac{1}{\alpha} \int_0^{\alpha} VaR_q \, dq$$

→ for this reason, ES is also called "Average VaR"

All these definitions are **equivalent** !

ES under normality

Parametric ES

If returns are distributed normally, then it can be derived that :

$$ES_c = -\bar{r} + \sigma \frac{n\left(N^{-1}(c)\right)}{1-c}$$

if volatility scales with sqrt(T), then so does ES

where c = confidence level = 1 – exceedance probability α

$n(.)$ = normal density function

$N^{-1}(\cdot)$ = inverse of the normal distribution function

you don't have to know this formula for the exam

example : σ = 20% p.a.

for 95% ES we have : $N^{-1}(95\%) = 1.65$ and $n(1.65)$ = 0.103, so 0.103/(1-95%) = 2.06

1-day 95% ES is : $20\% \cdot \sqrt{1/260} \cdot 2.06 = 2.56\%$ > 95% VaR = 2.05%

Comparing ES to VaR

- **ES complements VaR** : "VaR gives boundary between normal markets & extreme events,
  ES tells how bad it can be if we reach that boundary"

- under same parameters : ES ≥ VaR

  assume normality : $VaR_c = \boxed{N^{-1}(c)} \cdot \sigma$ and : $ES_c = \boxed{\frac{n\left(N^{-1}(c)\right)}{1-c}} \cdot \sigma$

  volatility multipliers :

c	ES	VaR	ratio ES/VaR
0.9	1.755	1.282	1.369
0.95	2.063	1.645	1.254
0.99	2.665	2.326	1.146
0.995	2.892	2.576	1.123
0.999	3.367	3.090	1.090

ES : A rose by any other name ...

Beware :

- ES is also called : Average VaR
  Conditional VaR
  Tail VaR, Expected Tail Loss
  Tail Conditional Expectation
  Conditional Tail Expectation ....

- confusing notation regarding :

  confidence levels : c
  exceedance probabilities : $\alpha = 1 - c$

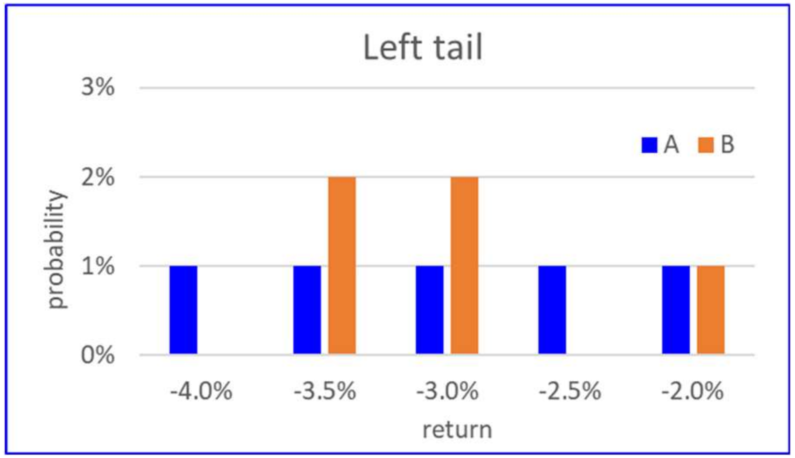
Evaluation of ES as risk measure

- coherent risk measure (always sub-additive)
- provides more information than VaR,
  as per 1-Jan-2023, 97.5%-ES replaces 99%-VaR for regulatory capital requirements in Basle IV
- although losses in the tail are taken into account, they are averaged;
  so the distribution (concentration or spread) of losses is irrelevant as long as their average remains the same.
  This implies assuming **risk neutrality** to losses beyond the VaR level

example :

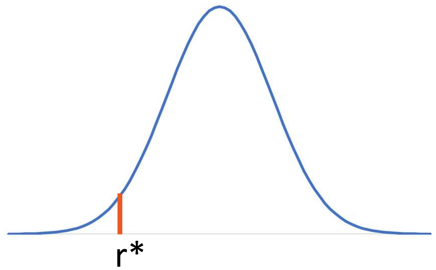
- first 5 percentiles of return distribution, these are the lowest returns
- 2 assets, A & B
- 95% VaR = 2% for both
- 95% ES = 3% for both

Q : *are A & B really equivalent ?*



LPMs, some special cases : VaR

$n = 0$: Value-at-Risk



- consider returns r over some holding period T
- define confidence level at which losses are not to be exceeded
- denote this quantile of the return distribution as r^* : $\Pr\{r \le r^*\} = 1 - c \quad \Rightarrow \quad VaR = -r^*$

so **VaR is defined by the zero-order LPM :**

$$LPM_0(r^*) = \int_{-\infty}^{r^*} (r^* - r)^0 \, dF(r) \equiv \Pr\{r \le r^*\} = 1 - c \quad \Rightarrow \quad -r^* = -LPM_0^{-1}(1-c)$$

LPMs, some special cases : ES

n = 1 : Expected Shortfall

First-order LPM with target rate r^* is :

$$LPM_1(r^*) = \int_{-\infty}^{r^*} (r^* - r)^1 \, dF(r) = E\left[\max(r^* - r, 0)^1\right]$$

ES with confidence level c and $VaR_c = -r^*$ is defined by :

$$ES = \frac{1}{1-c} \cdot E\left[\max(\, r^* - r, 0\,)\right]$$

note that c% of the observations are put to zero, hence the adjustment term 1/(1-*c*)

Hence, **ES is defined by the first-order LPM :**

$$ES = \frac{1}{1-c} \cdot LPM_1(-VaR)$$

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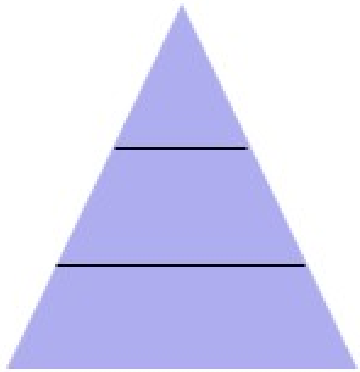
Importance of risk decomposition

Active investment management :



- **allocation** over asset classes, sectors, industries

- **selection** of individual securities = “micro-allocation”

Important difference between :

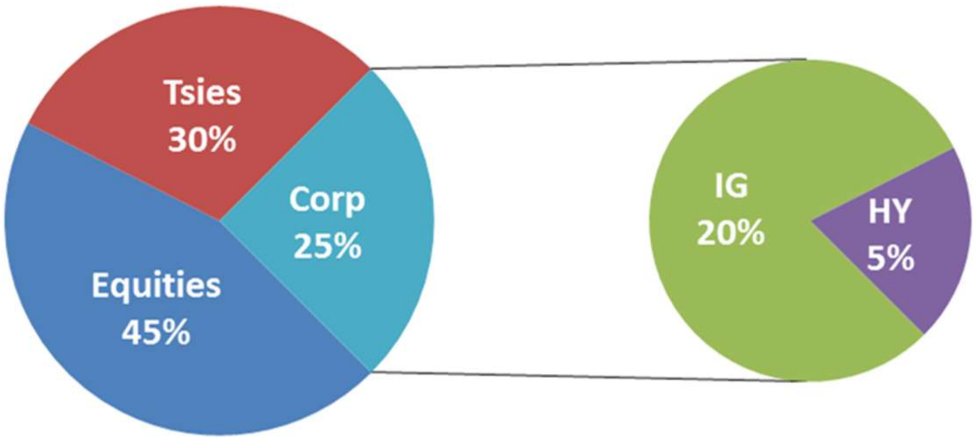
%-money allocation → how you construct the portfolio

%-risk allocation → how you evaluate the portfolio

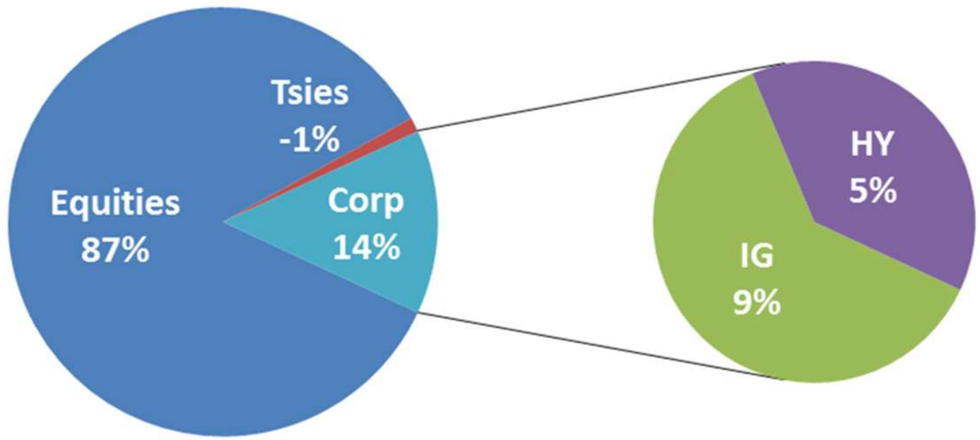
Example : US market

risk is proxied by monthly volatility

US market cap allocation :



risk allocation :



Qs : **> how do portfolio constituents contribute to diversified portfolio risk ?**

> are there pockets of risk concentrations ? → **hot spots :** impair portfolio diversification

Risk decomposition …

... or risk attribution … or risk drill-down

1. **measure total portfolio risk**

2. **slice & dice** total portfolio risk along components :

    what is contribution of **component** i to diversified portfolio risk ?

    what is the impact of **marginal** portfolio change on portfolio risk ?

we require that : $$\sum_{i \in p} component\ contribution_i = total\ portfolio\ risk$$

we will see that : $$component\ contribution_i = marginal\ contribution_i \times weight_i$$

Slicing and dicing returns

Returns and expected returns are linear in weights

opportunity set of N securities with stochastic (~) returns : $\tilde{r}_{it}$

portfolio p with weights : $\{w_i\}_{i \in p}$

budget restriction : $\sum_{i=1}^{N} w_i = 1$

no short sales : $w_i \geq 0,\ \forall i \in p$

portfolio return : $\tilde{r}_p = \sum_{i \in p} w_i \tilde{r}_i$

Excel :

SUMPRODUCT([w] , [r$_t$])

marginal : $\tilde{r}_i$

component : $w_i\,\tilde{r}_i$

contribution of security i to portfolio return

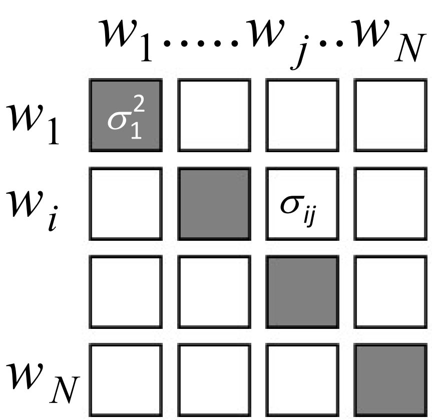
A. Vola decomp via variance (1)

Volatility is non-linear : diversification effects (1)

portfolio variance : $\sigma_p^2 = \sum_i \sum_j w_i w_j \sigma_{ij}$

covariance : $\sigma_{ij} = \rho_{ij}\sigma_i\sigma_j$

correlation : ρ_{ij}



$$\sigma_p^2 = \mathbf{w}'\Sigma\mathbf{w}$$

→ (1) multiply all cells with weights, (2) sum

Vola decomp via variance (2)

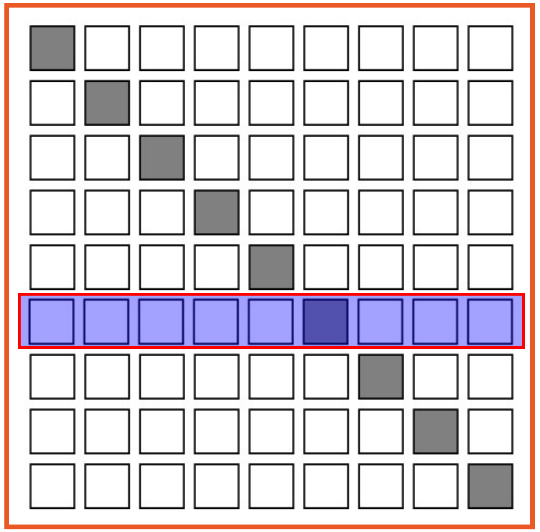
Volatility is non-linear : diversification effects (2)

contribution of security *i* to portfolio **variance** :

$$\operatorname{var}\left(\tilde{r}_p\right) \equiv \operatorname{cov}\left(\tilde{r}_p, \tilde{r}_p\right) = \operatorname{cov}\left(\sum_i w_i \tilde{r}_i, \tilde{r}_p\right)$$
$$= \sum_i w_i \operatorname{cov}\left(\tilde{r}_i, \tilde{r}_p\right) = \sum_i w_i \sigma_{ip}$$

$$\sigma_p^2 = \sum_i w_i \sigma_{ip}$$

σ_p^2



contribution to portfolio **volatility** :

$$\sigma_p = \sum_i w_i \frac{\sigma_{ip}}{\sigma_p}$$

marginal effect for changes in weight w_i

component effect, given weight w_i

Vola decomp via variance (3)

Be careful in switching between variance & volatility

- decompose variance :

$$\sigma_p^2 = \sum_i w_i \, \sigma_{ip}$$

$$\sigma_p^2 = \mathbf{w}' \left[\Sigma \mathbf{w} \right]$$

- divide by volatility to obtain volatility decomposition :

$$\sigma_p = \sum_i w_i \frac{\sigma_{ip}}{\sigma_p}$$

$$\sigma_p = \mathbf{w}' \frac{\Sigma \mathbf{w}}{\sqrt{\mathbf{w}' \Sigma \mathbf{w}}}$$

- divide again by volatility to obtain percentage decomposition :

$$100\% = \sum_i w_i \frac{\sigma_{ip}}{\sigma_p^2}$$

$$100\% = \mathbf{w}' \frac{\Sigma \mathbf{w}}{\mathbf{w}' \Sigma \mathbf{w}}$$

Vola decomp via Euler (1)

A formal perspective via linear homogeneity

Definition :

If a risk measure $f(\cdot)$, as a function of portfolio weights ***w***, is linearly (or 1st-order) homogeneous,

then it exhibits multiplicative scaling behavior : $f(k \cdot \boldsymbol{w}) = k^1 \cdot f(\boldsymbol{w})$ for constant *k*

Observation :

Proper risk measures satisfy **linear homogeneity** :

- volatility
- loss
- VaR
- ES

Q : *Why is it important that portfolio risk measures should be linearly homogeneous in the weights ?*

Vola decomp via Euler (2)

Euler's Theorem

Definition :

If a risk measure $f(\cdot)$, as a function of portfolio weights ***w***, is linearly (or 1st-order) homogeneous,

then it exhibits multiplicative scaling behavior : $f(k \cdot \boldsymbol{w}) = k^1 \cdot f(\boldsymbol{w})$ for constant $k > 0$

Implication : **Euler's Theorem :**

$$1 \cdot f(\mathbf{w}) = \sum_i \frac{\partial f(\mathbf{w})}{\partial w_i} w_i$$

marginal contribution

component contribution

1. the risk function can be written as the sum of the component risk contributions
2. component risk contribution = partial derivative * weight

Vola decomp via Euler (3)

Applying Euler’s Theorem to volatility

$$\sigma_p^2 = \mathbf{w}'\Sigma\mathbf{w}$$

Invoking Euler’s Theorem : $\sigma_p = \sum_i w_i \frac{\partial \sigma_p}{\partial w_i}$

$$\frac{\partial \sigma_p}{\partial \mathbf{w}} = \frac{\Sigma\mathbf{w}}{\sqrt{\mathbf{w}'\Sigma\mathbf{w}}}$$

We estimate volatility by variance, hence : $\frac{\partial \sigma_p}{\partial w_i} = \frac{\partial \left(\sigma_p^2\right)^{½}}{\partial w_i} = ½\left(\sigma_p^2\right)^{-½} \cdot \frac{\partial \sigma_p^2}{\partial w_i}$

Since : $\sigma_p^2 = \sum_i \sum_j w_i w_j \sigma_{ij}$ we find that : $\frac{\partial \sigma_p^2}{\partial w_i} = 2 \cdot \sum_j w_j \sigma_{ij} = 2 \cdot \sigma_{ip}$

Hence : $\frac{\partial \sigma_p}{\partial w_i} = ½\left(\sigma_p^2\right)^{-½} \cdot 2 \cdot \sigma_{ip} = \frac{\sigma_{ip}}{\sigma_p}$ and : $\sigma_p = \sum_i w_i \frac{\sigma_{ip}}{\sigma_p}$

$$\sigma_p = \mathbf{w}' \frac{\Sigma\mathbf{w}}{\sqrt{\mathbf{w}'\Sigma\mathbf{w}}}$$

Vola decomp : interim recap

Decomposing volatility : via Euler's Theorem or via variance

- conform Euler's Theorem :

$$\sigma_p = \sum_i w_i \frac{\partial \sigma_p}{\partial w_i}$$

calculate derivative

$$\sigma_p = \sum_i w_i \frac{\sigma_{ip}}{\sigma_p}$$

- decompose variance :

$$\sigma_p^2 = \sum_i w_i \sigma_{ip}$$

divide by portfolio vola

$$\sigma_p = \sum_i w_i \sigma_i \rho_{ip}$$

$$\Updownarrow$$

$$\sigma_p = \sum_i w_i \sigma_i$$

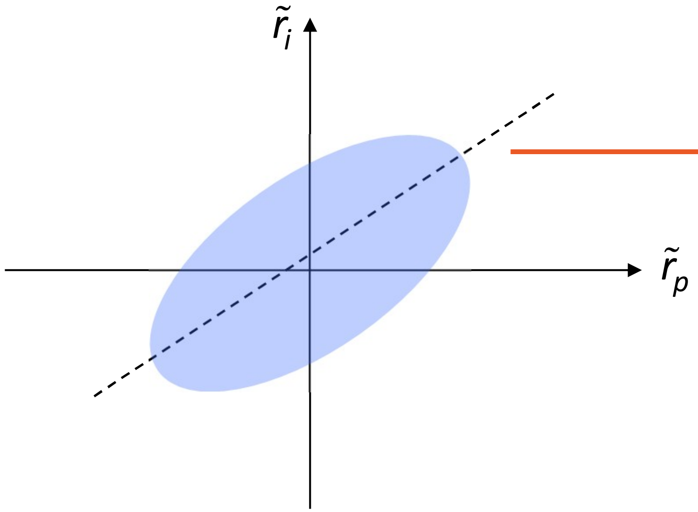
Simplifying %-vola contributions

It's about conditional expectations

relative (or %) contribution of security *i* to portfolio risk :

$$1 = \sum_i w_i \boxed{\frac{\sigma_{ip}}{\sigma_p^2}} = \sum_i w_i \beta_{ip} \qquad \beta_{ip} = \frac{\sigma_{ip}}{\sigma_p^2} = \rho_{ip}\frac{\sigma_i}{\sigma_p} \quad \text{: OLS slope}$$

regression of security return on portfolio return :



$$\tilde{r}_i = \alpha_i + \beta_{ip}\tilde{r}_p + \tilde{\varepsilon}_i \qquad E\left(\tilde{\varepsilon}_i\right) = E\left(\tilde{r}_{pf}\tilde{\varepsilon}_i\right) = 0$$

"*the* regression function" : $E\left(\tilde{r}_i \middle| \tilde{r}_p\right) = \alpha_i + \beta_{ip}\tilde{r}_p$

= conditional expectation

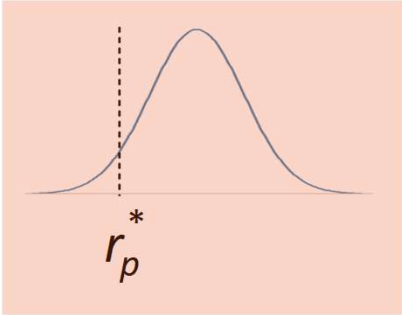
Excel :

Tools / Data Analysis / Regression ;

SLOPE(y,x) ; INTERCEPT(y,x)

B. Loss decomposition (1)

Parametric : using linear conditional expectations



choose some negative portfolio return level $r_p^* < 0$

so $-r_p^*$ is chosen **loss** level (or $-r_p^* \cdot V_p$ in monetary units : see slide 16)

what is contribution of portfolio components to this portfolio loss ?

1. "trick" : realize that : $r_p^* = E\left(\tilde{r}_p \middle| r_p^*\right)$ and $\tilde{r}_p = \sum_{i \in p} w_i \, \tilde{r}_i$

so : $$r_p^* = E\left(\tilde{r}_p \middle| r_p^*\right) = E\left(\sum_i w_i \, \tilde{r}_i \middle| r_p^*\right) = \sum_i w_i \underbrace{E\left(\tilde{r}_i \middle| r_p^*\right)}_{\text{conditional expectation}}$$ **completely general !**

Loss decomposition (2)

Estimating cond'l expectations via linear regression

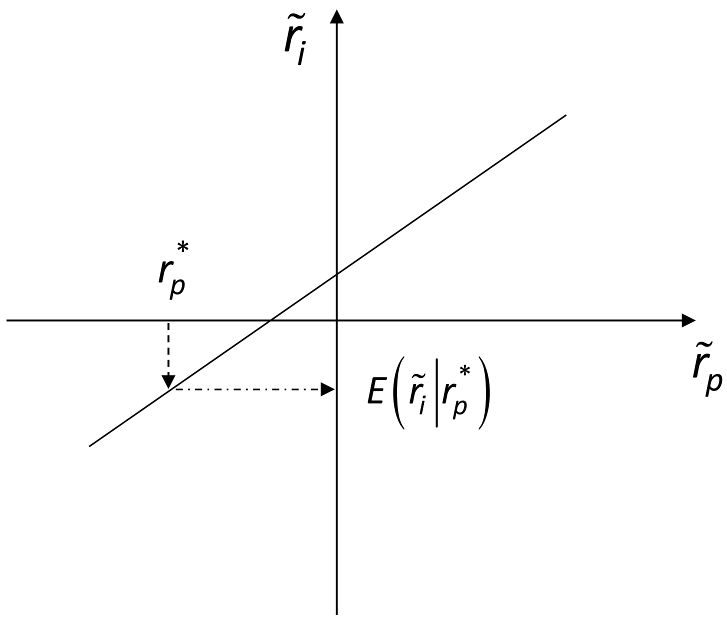
1. "trick" : $r_p^* = \sum_i w_i E\left(\tilde{r}_i \middle| r_p^*\right)$

2. conditional security return :

from regression function : $\tilde{r}_i = \alpha_i + \beta_{ip}\tilde{r}_p + \tilde{\varepsilon}_i$

conditional security return :

$$E\left(\tilde{r}_i \middle| r_p^*\right) = \alpha_i + \beta_{ip} r_p^*$$



Loss decomposition (3)

Equivalence of loss decomp and vola decomp

combine 1 & 2 : $$r_p^* = \sum_i w_i \, E\left(\tilde{r}_i \middle| r_p^*\right) = \sum_i w_i \left(\alpha_i + \beta_{ip} r_p^*\right)$$

ignore conditional average return : negligible at short horizons

define portfolio loss : $$L_p \equiv -r_p^*$$

hence : $$L_p = \sum_i w_i \, \beta_{ip} \, L_p$$

Component Loss : $$CL_{ip} = w_i \, \beta_{ip} \, L_p$$ with $$L_p = \sum_i CL_{ip}$$

%-**loss** contribution of security *i* : $$\frac{CL_{ip}}{L_p} = w_i \, \beta_{ip}$$ = %-**volatility** contribution !

Loss decomposition (4) – alternative view

Measure size of loss in terms of standard deviations

return generating process :

$$r_p = \bar{r}_p + \sigma_p \cdot \varepsilon$$

with ε zero-mean unit-variance

ignore average return : negligible at short horizons

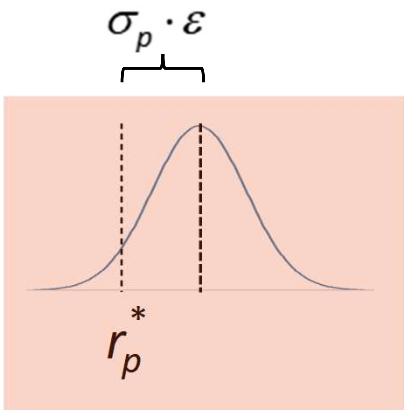
remember volatility decomposition :

$\sigma_p \cdot \varepsilon$

$$\sigma_p = \sum_i w_i \cdot \beta_{ip} \cdot \sigma_p$$

hence, perturbing the volatility with ε yields :

$$\sigma_p \cdot \varepsilon = \sum_i w_i \cdot \beta_{ip} \cdot \sigma_p \cdot \varepsilon \qquad \Leftrightarrow \quad 1 = \sum_i w_i \cdot \beta_{ip}$$



so %-**realized return** decomposition = %-**loss** decomposition
= %-**volatility** contribution

C. Decomposing normal VaR & ES

Conform decomposing volatility

1. consider some risk measure that is a multiple of volatility : $Q = a \cdot \sigma_p$ where a = a constant

2. we know how to decompose vola : $\sigma_p = \sum_i w_i \beta_{ip} \sigma_p$

marginal
component

3. hence : $Q = a \cdot \sum_i w_i \beta_{ip} \sigma_p$

$= \sum_i w_i \beta_{ip} Q \qquad \Leftrightarrow \; 100\% = \sum_i w_i \beta_{ip}$

4. **normal VaR** : $VaR_c = z_c \cdot \sigma_p \qquad \Rightarrow \quad VaR_p = \sum_i w_i \beta_{ip} VaR_p = \sum_i CVaR_i$

5. **normal ES** : $ES_c = \frac{n(N^{-1}(c))}{1-c} \cdot \sigma_p \qquad \Rightarrow \quad ES_p = \sum_i w_i \beta_{ip} ES_p = \sum_i CES_i$

Putting it all together

Summary overview

risk (volatility) :

$$\sigma_{pf} = \sum_i \left(w_i \frac{\sigma_{ipf}}{\sigma_{pf}} \right)$$

loss :

$$L_{pf} = \sum_i w_i \, \beta_{ip} \, L_{pf}$$

VaR :

$$VaR_p = \sum_i w_i \, \beta_{ip} VaR_p$$

ES :

$$ES_p = \sum_i w_i \, \beta_{ip} ES_p$$

money allocation

$$100\% = \sum_i w_i$$

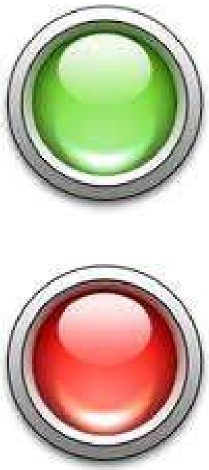
$$100\% = \sum_i w_i \, \beta_{ip}$$

risk allocation

What risk measure to choose ?

As simple as possible - but not too simple ....

dashboard 1 :



dashboard 2 :

What risk measure to choose ?

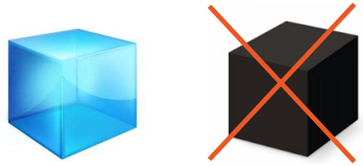
Transparency is key

- risk is multifarious, so use multiple risk measures

- risk measures should be able to catch the risks of the portfolio

- prefer robustness over complexity (remember Occam)

- prefer transparency over hidden complexity



- do you fully understand the risk measures ?
- do you fully understand the deficiencies of the risk measures ?

- do the risk measures allow for easy communication ? (remember Keppler)