Model-free control of underactuated robots with vision-based monitoring

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Abstract

1 Dynamic model

The double pendulum is modeled with the parameters described figure 1 and table 1. The kinetic energy

$$T = \frac{1}{2}m_1 v_G^2 + \frac{1}{2}I_G\dot{\alpha}^2 + \frac{1}{2}m_1 v_G^2 + \frac{1}{2}I_G\dot{\beta}^2$$

$$T = \frac{1}{2}K\dot{\alpha}^2 + \frac{1}{2}L\dot{\beta}^2 + M\dot{\alpha}\dot{\beta}\cos(\alpha-\beta)$$

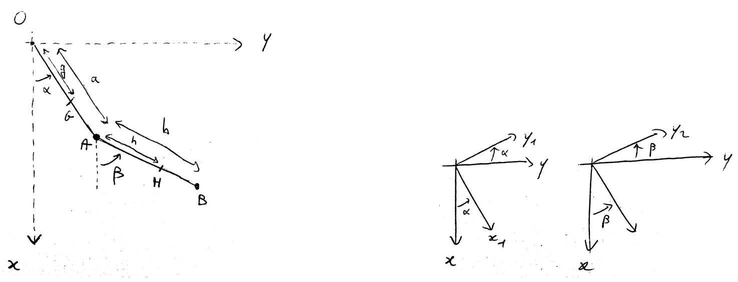
where $K = m_1c^2 + m_2a^2 + I_G$, $L = m_2b^2 + I_H$ and $M = m_2ab$. The potential energy

$$V = -m_1gx_G - m_2gx_H = P\cos\alpha + Q\cos\beta$$

where $P = -g(m_1c + m_2a)$ and $Q = -m_2g$. The lagrangian $\mathscr{L}(\alpha,\dot{\alpha},\beta,\dot{\beta},t) = T - V$ leads to the equations of motion $\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial\mathscr{L}}{\partial\dot{\alpha}} = \frac{\partial\mathscr{L}}{\partial\alpha}$ (idem for β) which gives

$$K\ddot{\alpha} + M\dot{\beta}^2\sin(\alpha-\beta) + M\ddot{\beta}\cos(\alpha-\beta) - P\sin(\alpha) = 0$$

$$L\ddot{\beta} - M\dot{\alpha}^2\sin(\alpha-\beta) + M\ddot{\alpha}\cos(\alpha-\beta) - Q\sin(\beta) = 0$$



(a) Parameterization (b) Local basis

Figure 1: Double pendulum

parameter	description
O	first hinge
A	second hinge
G	center of mass of the first mobile
H	center of mass of the second mobile
α	angle of the first mobile with respect to the reference frame
β	angle of the first mobile with respect to the reference frame
m_1	mass of the first mobile
m_2	mass of the second mobile
I_G	inertia of the first mobile about its center of mass
I_H	inertia of the second mobile about its center of mass
a	distance OA
b	distance AB
c	distance OG
d	distance AH

Table 1: Geometric and inertial parameters

and can be rewritten

$$\ddot{\alpha} = \frac{PL\sin\alpha - MQ\sin\beta\cos(\alpha-\beta) - ML\dot{\beta}^2\sin(\alpha-\beta) - M^2\dot{\alpha}^2\sin(\alpha-\beta)\cos(\alpha-\beta)}{LK - M^2\cos^2(\alpha-\beta)}$$

$$\ddot{\beta} = \frac{KQ\sin\beta - MP\sin\alpha\cos(\alpha-\beta) + MK\dot{\alpha}^2\sin(\alpha-\beta) + M^2\dot{\beta}^2\sin(\alpha-\beta)\cos(\alpha-\beta)}{LK - M^2\cos(\alpha-\beta)}$$