

TRAFFIC SYSTEM

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Abstract

How much time do we waste stuck in traffic? This question inspired our team to investigate possible ways to analyze traffic and optimize the roads for our society to be more efficient. Our research project focuses on the study of transportation systems, specifically the simulation of traffic flow on a highway. We use queuing models to simulate an increasing traffic flow with different capacity constraints determined by the level of highway demand. We aim to help civil engineers model toll booths, potential construction costs, and other road elements for both existing and new projects to maximize efficiency through determining the equilibrium between the cost of each element and its expected benefit.

1 Value Opportunity

Our target audiences are primarily the government and construction companies working on the road development although we feel that the topic is important to the society as a whole. Each minute an individual spends stuck in a traffic jam is time which they can use to do something productive. This project might be especially appealing to political economists and other academics because it allows people to save time and in turn be more productive, which increases a regions GDP output.

Highways, and roads in general, are one of the fundamental parts of America's infrastructure as over time, a lot more people became car owners due to government policies that aimed to connect the country through a national highway system. Old infrastructure can still manage this flow, but at what cost? Would it be better to upgrade existing systems and implement new technologies in the building of new roads? Our model works for many kinds of roads and allows maximum flexibility in design and planning. Running our algorithm on all existing roads is possible with some future development. In the long run, a more thorough work, based on our work in this project would be capable of automating the highway design and planning process to eliminate human bias and allow for an equilibrium point between saving peoples' time and saving government budget money.

2 Model Proposals

We started with a one lane highway model to simulate the traffic in one single direction highway (by symmetry of the

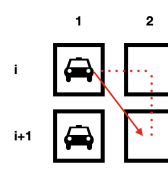


Figure 1. Case 1

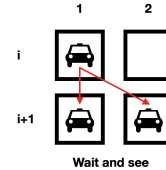


Figure 2. Case 2

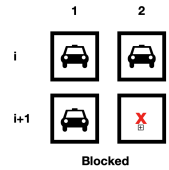


Figure 3. Case 3

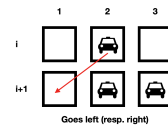


Figure 4. Case 4

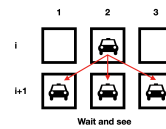


Figure 5. Case 5

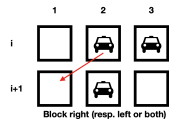


Figure 6. Case 6

problem we always focused on one direction only), then we launch a two lane highway model which allows the car on the highway to switch roads between these two.

This upgrade aims for a more realistic and more efficient traffic flow. Indeed, if a car is stuck behind another car without any other car on its direct sides (namely at the same level of the road, same row of server) the car might want to switch lane. To do so, if the server on the row $i+1$, on the other lane is free with no cars on its direct sides, the car will naturally decide to switch. If the server on row $i+1$ is taken, with still no cars on the direct sides, the car will wait to see which car goes faster on row $i+1$: the car on the same lane or on the next lane. The same logic is applied to N lanes (with some special cases extremities of the road).

Finally, we tried to add priority request to the highway transportation system. When a car wants to change lane, its priority is inferior to the cars that are already in the requested lane. This was in order to have a system closer to reality, respecting a universal driving rule.

2.1 Modeling Methods

We simulate a highway road with length 1 kilometers for one single-direction lane, and equally divide this road into 100 segments (the toll booth location is at $\text{floor}(n/2) - 1$, $n=100$ here) with 10 meters each where each 10 meter segment is regarded as a server. In real cases, this 10 meters segment is composed of a length of a car and some safe driving distance (at low speed, but that we will keep constant) between the car in front of it and the car behind it,

so for one single-direction lane, we modeled it as multiple identical servers in sequence. Then for the intensity of transportation system simulation, we inherit the arrival process with the Poisson process. For the car in the transportation system, their service time for each segment depends on the availability of the next server in front of them and the servers parallel to them. To simplify and differentiate the real driving speed case, we leverage different uniform distributions to simulate their driving time in each segment (aka. service time).

2.2 Transportation System Simulation Cases

2.2.1 One Unidirectional Lane

In this situation, when a car arrives at service and requests a service, there are 2 situations: if there is no car in front of it (next segment service is idle), then it drives through in **faster speed** and moves into the next server; if there is a car in front of it, it **drives slower** and enters the next server until it is available. To change the speed we change the service time and therefore the uniform distributions.

In the case of a single direction lane, we found that a car could pass through the highway in about a minute and six seconds, which meant that there was an average speed of 15 meters per second or 54 km/h (or 33 mph). This became our baseline expectations for speed.

2.2.2 Two Unidirectional Lanes

Under the two unidirectional lane model, if a car arrives, it randomly selects which lane to enter, so we model that for a 50% chance it enters the first lane, and otherwise second lane. Then, when the car drives through the segment services, there are 3 cases (cf. Figure 1,2,3) that could happen: 1) if there is no car in front of it, it stays in the current lane and move forward; 2) when there is a car in front of it, then if there is no car on the parallel server in the other lane (no car alongside) and meanwhile there is no traffic in the next server for the other lane (no car in front for the neighboring lane), the car switches to the other lane and move forward; 3) if there are all busy servers for the server in front of us in the same lane, the server parallel us in the other lane, and the server in the next position in the other lane, then the car does not change lanes. In this case, cars went a bit slower going through the highway in a minute and 14 seconds, or approximately 48km/h (30 mph).

2.2.3 Multi Unidirectional Lane

The multi unidirectional case generalizes the models created previously to the case with, n , a chosen amount of lanes. Generalizing from before, we give the car a uniform chance to join any lane and keep the 3 cases for travel within the highway. In this case, when $n = 3$ cars passed through the highway faster than all previous cases as they made it through in 54 seconds with an average speed of 65km/h (41mph).

3 Varying Arrival Rates

To simulate the traffic flows during a day, we have realized that the number of vehicles using a highway is significantly different in different times of a day. For example, the majority of people would use highway during the commute hours, and not a lot of them would drive on the highway around midnight. Therefore, we want to simulate the system with varying arrival rate, that addresses both the rush hours, as well as times that the highway is not much busy. In general, the traffic in a city would peak at around 7 in the morning, where people usually drive from home to office. At night, as the time to drive home is more flexible for many commuters, some of them would leave earlier or later to avoid congestion on the road. Therefore, the level of traffic is not as high as the one in the morning rush hours. Still, the peak of evening rush hour would take place at around 7pm. In our simulation, we simplify the case by setting the rush hour traffic pattern to be similar for both morning and evening. We use a sinusoidal function to address for the arrival rate that peaks at 7am and 7pm, and stays at a relatively low level in the middle of a day and during night.

As time in our system is measured in the unit of seconds, running a system of 24 hours would take an extremely long amount of time and disk space. Therefore, we have decided to simulate for 240 seconds, where each hour in a day is represented by 10 seconds. Although this approximation is not an accurate representation of the traffic flow in a day, we can still observe the change of time and speed across busy and non-busy periods of time. We believe the number of arrivals in the 240-second simulation can represent the average arrivals during a day. Thus, we track the number of cars that arrive in our simulation, and calculate the daily arrivals by multiplying by 360 (as there are $360 * 240$ seconds in a day). The arrival rate function we apply follows the shape as the following graph indicates, where region marked in yellow represents the rush hours in the morning and evening.

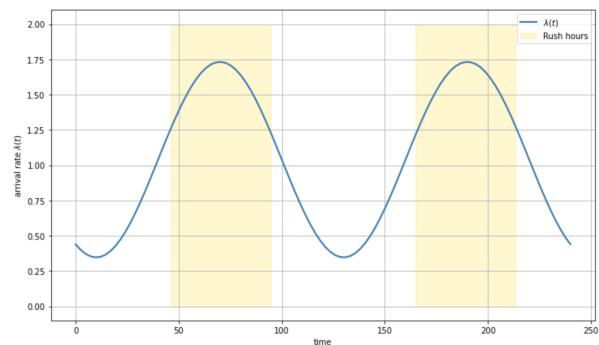


Figure 7. Arrival rate of the Non Homogeneous Poisson Process

We also want to model the effect of varying arrival rate based on different number of lanes. In reality, when the government builds a wider highway, more residents along the highway would be willing to use the highway, resulting in higher arrival rates. However, we also notice that this relationship is not linear. As the number of lanes gets

large, the number of vehicles using the highway will not increase much any more, as the all traveling needs have been satisfied. Therefore, we approximate the relationship between arrival rates and number of lanes as the arrival rates is proportional to the square root of number of lanes.

As a result, we set the arrival rate function as

$$\lambda(t) = \frac{\sin(\frac{t/10-4}{6}\pi) + 3}{5} * \sqrt{\text{lanes}} \quad (1)$$

, to address all the assumptions above, and the integral of this arrival function is

$$\Lambda(t) = \left(\frac{3t}{5} - \frac{12\cos(\frac{t/10-4}{6}\pi)}{\pi} - \frac{6}{\pi} \right) * \sqrt{\text{lanes}} \quad (2)$$

As a result of modeling such traffic flow, we can see that, by plotting the average time spent on the highway by each car, people spend more time on the road during rush hours (Figure 2). This aligns with our expectation as cars spend more time when there is congestion (especially because of the toll booth process time that slows the car down). We did not consider any "annoyment cost" (people not taking the highway anymore because of low traveling speed) in our financial optimization.

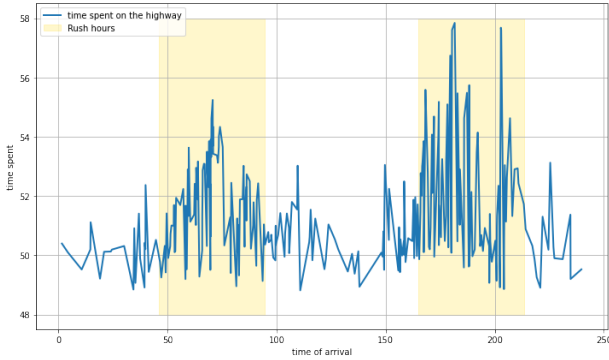


Figure 8. Average time spent on the road by cars during a day

4 Model Setup

4.1 Passing Time

As the previous section indicates, our model serves as a good approximation to a traffic system in the real world. To add more realism to our simulation, we have set up the parameters in our model to fit the cases in the actual world.

If a vehicle enters a server (road segment of 10 meters long), and the server in front of it is empty, then we allow the vehicles to speed up to move through the server faster. We set the time taken to pass through the 10m segment to follow a uniform distribution between 0.4 and 0.6 seconds, representing a speed between 60-90km/h. When there is congestion ahead of the vehicles, they either slow down if they were previously sped up or remain their previous velocity. In this case, the time needed to cross this 10m segment will follow a uniform distribution between

0.8 and 1.2 seconds, which is equivalent to 30-45km/h. When the vehicle drives through a segment, and there is still a car ahead, it will look to switch lanes if it thinks the neighboring lane will allow them to go faster. Otherwise, it will come to a complete stop and wait for the cars in front of it to move.

4.2 Toll Booth

We have also added a toll booth in the "middle" of our highway to simulate the effect of toll booth on traffic flows. The toll booth is on the 49th server for all lanes. Vehicles need to slow down in the toll booth to complete their payment before they can move on to the next server, thus takes longer time than a normal segment. We set the time to pass a toll booth to a uniform random variable between 3 to 5 seconds. It is expected that the setup of a toll booth will slow the traffic, and causing a congestion for the road segments ahead of the booth.

4.3 Construction

Moving on from the previous cases, we tried to add more realism to the model by creating a case where a server needs to be repaired, perhaps due to a pothole, and construction needs to be done to fix it. In this case, we say that when construction starts, that part of the highway will be blocked, and all the cars have to get around the construction site. We assume that construction will only takes place at one server (10 meters for one lane). When construction ends, the server will be immediately available to all the vehicles again. To make sure the entire traffic system works properly during construction, we assume there can be at most one incident of construction on the highway at any time, and the gap between two construction projects follows an exponential distribution with a rate of 120. Since our system only simulates the arrivals within 240 seconds, we assume each construction takes 60 seconds to be completed.

5 Visualization

As our project is based on the study of a phenomenon where visualization can be a key part to understanding its principles and behaviors, we tried to print every element implemented above during the period (240 seconds).

To do so, we built a "road", represented by a $n \times (n_{\text{lanes}} + 1)$ matrix. We also had to generate construction blockages in the "simulate" function in order to have the time they would happen in the "service" function and therefore be able to print those when needed.

For simplification, the length of the road printed is 100m (10 servers) with only 4 lanes. Cars are represented by a "o" symbol, constructions with "X" and on the very left column, server numbers are shown as well as the location of toll booths "TB" (the "-" is the barrier of the toll booth).

Looking at this 5 min long video (Click Here for the video) we can see some important things :

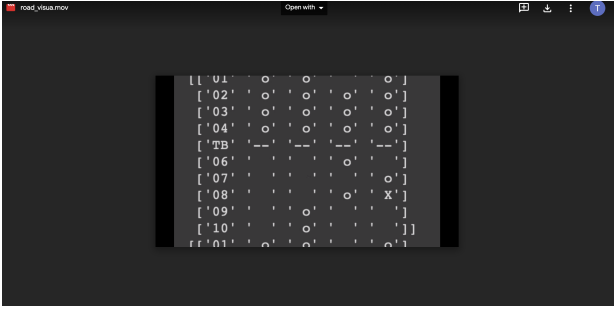


Figure 9. Screenshot from the linked video of the traffic modeling

- The non-homogeneous Poisson process causes traffic congestion. For instance, during the first rush hour (around 1min) we can see that way more cars arrive at the toll booth (that cannot keep up with the arrivals). Thus, the cars are getting blocked in traffic. Subsequently, at empty hours (beginning and end of the video), traffic flow is lighter and the toll booth serves all cars with ease.
- When there is congestion in front of cars, they try to pass the congestion by switching lanes to the left or right.
- Construction blockages happen for a certain amount of time (the first 60 seconds) then are fixed and disappear. Cars avoid construction sites.

N.B:

Looking closely, it could seem that some cars disappear. They don't. Prints happen when they are in between servers or requests. They always show up again in the following prints.

6 Conclusions

6.1 Financial Optimization

We begin under the assumption that the first lane costs \$1.75 million per kilometer to build [1] as there will likely be upfront costs to beginning construction and assume that further lanes will cost an additional million per kilometer. Which makes our initial cost $(1.75 + (1 * (n_{lanes} - 1))) * 1,000,000$ per kilometer where n_{lanes} is the amount of lanes. Next, we assume that the amount will be paid by a loan that will be taken out with a compounding interest rate of 4% [2]. These total costs are broken down into daily and yearly costs in our analysis. Additional costs include: a yearly maintenance fee of \$40,000 [3] per lane per kilometer which is the cost to maintain the infrastructure already built, and the salaries of toll booth employees, which amounts to \$20 [4] per hour per lane built. Expanding on this, the toll booth workers can also be taken as revenue sources as they are collecting money from cars that are passing through the highways. According to [4],

the average toll is around \$0.08 a kilometer which allows us to add this to the revenue of the highway for each car that passes. Therefore, we will calculate the revenue of the highway, which is equal to the tolls collected per day, subtracted by the average loan payment and maintenance fee for each day.

For most highways in the US, the number of lanes in one direction ranges from 2 to 6. Therefore, we run the simulation of a highway with 2 lanes to 6 lanes, and compare the outputs of each system. Financially, the system with 4 lanes produces the most daily profit, which amounts to about \$4,196. per day. This result makes sense as building more lanes would increase construction costs, maintenance costs, as well as salaries for toll booth employees, while the number of vehicles using the highway will not experience a huge increase.

N.B: The numbers taken in those assumptions have been round up to have the most extreme case and to make sure our model would resist to easier conditions.

6.2 Time Spent in System

Besides the financial aspect of our system, we also want to generate some other features to compare among different configurations. Therefore, we track the passing time of each vehicles in the system by calculating how much time they stay in our system (from entering to leaving). We then calculate the average speed of a vehicle in this highway through out the whole period, and compare the speeds generated from different models. A comparison of profits and average speed is shown below.

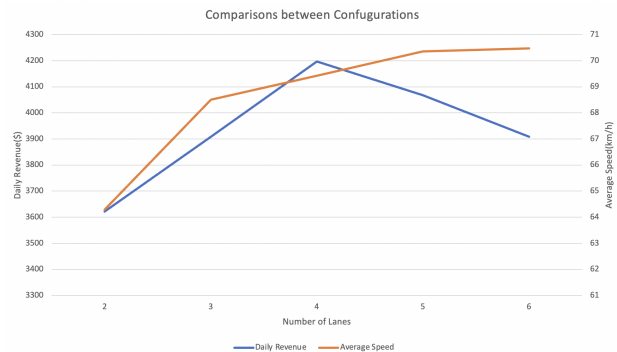


Figure 10. Comparison between Different Configurations

As the graph indicates, the average speed of vehicles experience a huge increase as the highway expands from 2 lanes to 3 lanes. In general, as the number of lanes increases, the average speed of vehicles always increases, although the effect will diminish as we further expand the width of highway.

6.3 Choosing Number of Lanes

Deciding the size of a highway is always a difficult project for civil engineers. As the analysis above suggests, a potential decision making framework to use to solve this is to find the amount of lanes that strikes a balance between

profitability and efficiency. In reality, there are more factors, which are harder to model, that policy makers and engineers need to consider before making a final decision. For example, the traveling needs of a highway is dependent on the number of residents along the highway, the cities that the highway connects, as well as the layout of other highways in the region, alternative transportation methods, etc. These factors will affect the arrival rate function, which may be more complicated than a sinusoidal function. The engineers will also need to consider how many trucks or buses will use the highway, as they normally drive in a slower speed, thus might require the highway to add more lanes to avoid congestion. Before the government decides to build a new highway, they should conduct research into these areas, so that they can have a better estimate of the arrival rate function, passing speed, and other variables that we use in our model.

However, one thing that our model cannot reflect, is that expanding a highway also means occupying land that could be used for other purposes. Therefore, simply comparing the data output from different configurations does not necessarily generate the best solution in and of itself. Decision makers must have a broad picture of city development and planning before the construction of a highway, as the fundamental purpose of an infrastructure project is to better serve the development of a city.

6.4 Model Limitations

Our initial models randomize the arrival times of drivers while our more robust ones model arrivals in a sinusoidal fashion. While we feel that it is a good start, this approach does not include potential changes in behavioral patterns based on reactions to time spent in traffic. Given lots of traffic, many individuals may decide to switch from cars to alternative forms of transportation, consequently decreasing the traffic flow, which would then decrease the profitability of the highway. Furthermore, the arrival rate and traffic speed will also have a seasonal component, depending on the location of the highway, as less people are inclined to drive when ice is on roads.

Additionally, we do not account for the increased wear on roads and higher expected maintenance in cases of increased number of arrivals and a higher average speed as high-speed roads may need more maintenance. In addition, the expected number of accidents can potentially increase at higher speeds. If this happens, we will observe a decrease in average speed of the road as certain servers will be congested and will need to be cleared as well as high priority vehicles needing to go through the highway to clear the vehicles.

Despite attempted randomization in our model, we do not represent some types of vehicles. There are motorbike, heavyweight trucks, such as those carrying cargo for companies, that are slower than average cars and they require more road maintenance. Without an additional fee, they might impact our forecast negatively. There also are sport cars and dangerous drivers who tend to go over the speed limit and make multiple lane changes. Their presence in-

creases the chance of accident, while simultaneously increasing the average speed of traffic flow.

Also, on a real-world highway, there will be entrances and exits on the side of highway (usually on the right for countries that drive on the right). The existence of an exit usually means that there will be an additional lane before the exit to lead the vehicles away from the highway, and the opposite case in the event of an entrance to the highway. In addition, if a vehicle intends to leave a highway in the next exit, it will start switching lanes to the right (or left for left-driving countries) beforehand to avoid accidents if it attempts to turn right at the last second. In this case, the drivers will decide the best timing to switch lanes, and lane switching will happen even if there is no traffic. Our model only simulates a piece of highway that has no entrance or exit, so we do not address the cases where vehicles join or leave the traffic system in the middle. However, we believe this model is still a close simulation of a real-world highway, because it depicts the actions that vehicles would take in different level of traffic intensities, which can be applied to simulate different types of highways.

Finally, in a way, modeling potential benefits and detriments from a highway in a way that hopes to be accurate to the real world can potentially be much more difficult than we've laid out. For instance, there could be incidental benefits to businesses that advertise along the highway which could lead to an increase in revenue for the city in the sense that the business would get more revenue which would lead to the government getting more money when the business is taxed. This tax revenue increase can thus partially be attributed to the highway. Furthermore, at least in America, when major road projects have been built, they've mostly been built through and around areas populated by those with low incomes [5] which disrupts communities and in the long term causes medical issues such as impaired lung function, heart disease, and cognitive decline due to a concentration of pollutants being emitted from the vehicles that drive the highway [6]. While we were able to capture things that are directly calculable from the installation of a highway, it's much harder to generally model externalities that may arise from the highway. Therefore, we feel that it's important for policy makers to at least pause and consider where they build the highway and the various trade offs associated with the construction, outside of the calculations presented.

Contributions

Andrew Johnson: Transportation System Simulation Cases, Model Limitations, Construction, Financial Optimization

Leonid Kozlov: Abstract, Value Opportunity, Model Limitations,

Thomas Michel: Implementation of the Traffic System , Model Proposals, Varying Arrival Rates, Visualization, Financial Optimization

Adrien Simonet: Implementation of the Traffic System and Analysis of the results

Endong Teng: Model Proposals

Chang Zhou: Implementation of the Traffic System, Design and Implementation of Lane-switching Rules, Model Setup, Varying Arrival Rates, Analysis

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[road-highway-costs/](#)

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