Protocol for Canteen Dilemma

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1 Background

The Canteen Dilemma experiment is meant to investigate what happens when people have to coordinate their actions without common knowledge. In the research literature, group G have *common knowledge* that A when everyone in group G knows A, and everyone in group G knows that everyone in group G knows A, and everyone in group G knows that everyone in group G knows that everyone in group G knows A etc. etc. Common knowledge is an essential aspect of social cognition, i.e. reasoning about the reasoning of others. In everyday use of the concept, common knowledge often refers to either something that everyone knows (called *mutual knowledge* in the research literature) or even just something that most people believe. The importance of common knowledge is not always intuitive. The Canteen Dilemma is intended to investigate the extend to which people recursively model the mental states of others and how important they think this is in coordination games.

The Canteen Dilemma is structurally similar to the Consecutive Numbers example in (Ditmarsch et al., 2008). In this example, two agents, a (Anne) and b (Bill), face each other and can each see a number on the other's head. The numbers are consecutive numbers n and *n+1* for a certain $n \in \mathbb{N}$. For our example, assume the possible numbers are 0 to 10. These facts are common knowledge for Anne and Bill. As an example, assume that Anne has the number 3 and Bill has 2. Now imagine that we ask them if it is common knowledge that they both have numbers higher than 0. If one of them had a 0, the other would see this and conclude that their own number must be a 1. So our question is equivalent to asking if it's common knowledge that none of them know their number.

Let us check. Anne and Bill both know their numbers must be higher than 0, Since Anne sees a 2 and Bill sees a 3, so $E_{\{a,b\}}(\neg a_0 \wedge \neg b_0)$, where E denotes some proposition is mutual knowledge for some agents. Now notice that since Anne can see a 2, she thinks it's possible she has 1 and Bill has 2: $\hat{K}_a(a_1 \wedge b_2)$ (where $\hat{K}_a A$ means that a considers A possible, defined as $\hat{K}_a \varphi := \neg K_a \neg \varphi$). In that situation, Bill might not know his own number, since he sees a 1, but since he might have a 0 in that situation, he considers it possible that Anne might know her number. So we have: $E_{\{a,b\}}(\neg a_0 \wedge \neg b_0) \wedge \neg E_{\{a,b\}} E_{\{a,b\}}(\neg a_0 \wedge \neg b_0)$. So while both Anne and Bill know that their numbers are higher than 0, not everyone knows this epistemic fact, so it is not common knowledge.

This is important in some cases, for example if we told Anne and Bill that they have to cooperate their actions in the following way. Assume that they get a large reward if both choose B while none of them have a 0, a smaller reward for both choosing A regardless of their number, and a large penalty for either making different choices or one choosing B when one of them has a 0. In our case $(a_3 \wedge b_2)$, both know they would get the largest reward if they both chose B. But since Anne sees a 2 on Bill, she thinks she might have a 1. If she has a 1, Bill would think he might have a 0, and he knows that Anne has to choose A if he has a 0, making him choose A as well. So since they don't have common knowledge about their numbers, the only safe strategy is choosing A. This holds no matter what number they have. This is highly unintuitive and it is this aspect of theoretical social cognition that the Canteen Dilemma is meant to capture.

Respondents in the Canteen Dilemma are given a similar story to the Consecutive Numbers game. Players are told that they and their colleague arrive at work every morning between 8:00 am and 9:10 am and that they always arrive 10 minutes apart. If they both arrive before 9:00 am, they have time to go to the canteen for a coffee. If either arrive at 9:00 am or after, they have to go to their offices. Players are rewarded if they both go to the canteen before 9:00 am and get a smaller reward if they both go to their offices at any time. They get penalized if they either go to different places or if one goes to the canteen at 9:00 am or later. Much like the consecutive numbers game, players only have common knowledge that they arrive 10 minutes apart, but no common knowledge about their arrival time.

The structure of the Canteen Dilemma can be found in coordination problems in everyday life when people communicate through text. In such cases, people might have mutual knowledge but no proper common knowledge. This experiment is intended to investigate the way people act and reason in such cases.

2 Research Questions

Common knowledge is an important theoretical concept in epistemic logic. It can be difficult for people to imagine cases where the difference between mutual knowledge and common knowledge has practical significance however. Our experiment exhibits a structure where the lack of common knowledge makes a tangible difference. Our research questions and hypotheses relies on this fact and can be expressed like this:

1. The theoretical concept of common knowledge does not match our everyday notions of common knowledge. The infinite recursive modeling of other's beliefs does not occur in everyday situations and at most 2-3 iterations happen.

2. Most respondent will reason to approximately the same depth, which is 2-3 iterations of social reasoning. In other words, players will at most reason about others, reasoning about them.

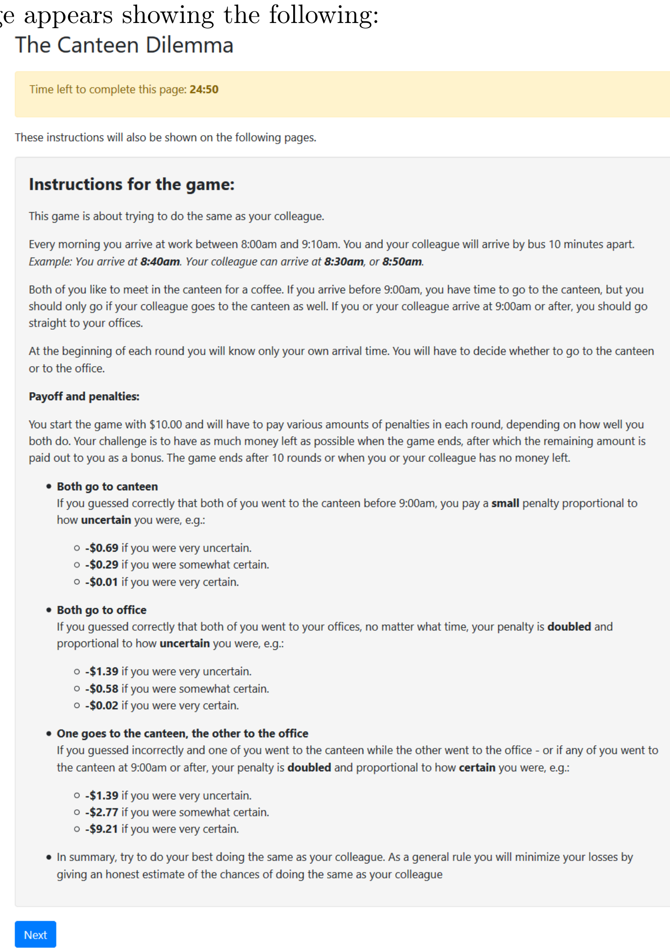
3. Respondents will intentionally limit their reasoning depth because they find this depth sufficient for the task at hand. In other words, a limited reasoning depth is not necessarily due to lack of capacity in respondents but also due to the belief that a more iterations are not necessary to succeed in the game. I.e. respondents have the false belief that arriving early enough warrants that it is safe to go the canteen.

Our general research questions are: What are the respondents intuition about common knowledge? Do they really reason about each other's knowledge as modeled in epistemic logic? How well do the respondents do in terms of payoffs, i.e. how optimised are their strategies?

3 Experimental Design and Setup

3.1 Instructions to player (requires formatting layout of screencaps)

The experimental setup is facilitated through Amazon's Mechanical Turk. After accepting the Amazon Mechanical Turk 'hit' all participants are asked to provide informed consent during the registration process. Upon being provided with a link to a UCPH-server at https://cibs.mef.sc.ku.dk and entering the experimental platform, participants wait until the choice room is available. A web page appears showing the following:


The Canteen Dilemma

Time left to complete this page: **24:50**

These instructions will also be shown on the following pages.

Instructions for the game:

This game is about trying to do the same as your colleague.

Every morning you arrive at work between 8:00am and 9:10am. You and your colleague will arrive by bus 10 minutes apart.
*Example: You arrive at **8:40am**. Your colleague can arrive at **8:30am**, or **8:50am**.*

Both of you like to meet in the canteen for a coffee. If you arrive before 9:00am, you have time to go to the canteen, but you should only go if your colleague goes to the canteen as well. If you or your colleague arrive at 9:00am or after, you should go straight to your offices.

At the beginning of each round you will know only your own arrival time. You will have to decide whether to go to the canteen or to the office.

Payoff and penalties:

You start the game with $10.00 and will have to pay various amounts of penalties in each round, depending on how well you both do. Your challenge is to have as much money left as possible when the game ends, after which the remaining amount is paid out to you as a bonus. The game ends after 10 rounds or when you or your colleague has no money left.

- **Both go to canteen**
  If you guessed correctly that both of you went to the canteen before 9:00am, you pay a **small** penalty proportional to how **uncertain** you were, e.g.:
  - **-$0.69** if you were very uncertain.
  - **-$0.29** if you were somewhat certain.
  - **-$0.01** if you were very certain.
- **Both go to office**
  If you guessed correctly that both of you went to your offices, no matter what time, your penalty is **doubled** and proportional to how **uncertain** you were, e.g.:
  - **-$1.39** if you were very uncertain.
  - **-$0.58** if you were somewhat certain.
  - **-$0.02** if you were very certain.
- **One goes to the canteen, the other to the office**
  If you guessed incorrectly and one of you went to the canteen while the other went to the office - or if any of you went to the canteen at 9:00am or after, your penalty is **doubled** and proportional to how **certain** you were, e.g.:
  - **-$1.39** if you were very uncertain.
  - **-$2.77** if you were somewhat certain.
  - **-$9.21** if you were very certain.
- In summary, try to do your best doing the same as your colleague. As a general rule you will minimize your losses by giving an honest estimate of the chances of doing the same as your colleague

Next

After seeing the instructions, the following web page appears.

Round 1

It is Monday morning and you arrive at your workplace at 8.10 am.

Where will you go?

Canteen Office

After making their choice, they have to give an estimate of their certainty that the other player made the same choice.

How certain are you that your colleague has made the same choice as you?

- Very uncertain
- Slightly certain
- Somewhat certain
- Quite certain
- Very certain

Next

Each round, after having made their choice and estimated their certainty, respondents see the following result page:

Results (after round 4)

Round	You went to the	at	Your colleague went to the	at	How certain you were that colleague did the same as you	**Penalty**
1	office	8.10	office	8.20	very certain	**-$0.02**
2	office	9.00	office	8.50	somewhat certain	**-$0.58**
3	canteen	8.20	canteen	8.10	quite certain	**-$0.13**
4	office	8.50	canteen	8.40	somewhat certain	**-$2.77**

You went to the office while your colleague did not.

You lost **-$2.77** this round. Your total losses are **-$3.50**. You still have **$6.50**.

Next

The game stops if either player loses their initial $10 bonus

3.2 Questions

The experiment involves two key questions and five supplementary ones. Each round involves these two questions:

- It is Monday morning, and you have arrived at your workplace at 8:10 am. Where will you go? (Either the canteen or the office).
- How certain are you that your colleague has made the same choice as you? (Very uncertain, slightly certain, somewhat certain, quite certain, very certain).

If the game is lost because either respondent has lost their initial bonus, they are given the question:

- The game is over. Do you think it was your fault it is over, your colleagues fault, or do you think it was because of some other reason? (My fault, my colleagues fault, other reason)

When the game is either over after 10 rounds or is lost, the respondents receive the following questions:

- What strategy did you use while playing the game? (Free text)
- Assume that you prior to the game could have decided a cut-off point with your colleage which you would both stick to while giving a *very certain* estimation of making the same choice. This means .meaning that you will only go to the office if you arrive at this time or later. At what time would this cut-off point be? (Do not know, none of these times, 8:00, 8:10, 8:20, 8:30, 8:40, 8:50, 9:00, 9:10).
- Question about everyday notion of common knowledge
- Question about theoretical notion of common knowledge

3.3 Study Type

The experiment will be done online on the Amazon Mechanical Turk Platform. Participants will be from all over the world. The software platform is otree 2.1.7

3.4 Inclusion and exclusion rules on Mturk

No player is allowed to play the game more than once which we will secure by giving each respondent a permanent qualification which excludes them from further participation.

4 Treatments / Conditions

We only employ one treatment so far, the Canteen Dilemma with a logarithmic scoring rule.

5 Payoff Structure

5.1 General structure:

Respondents receive a participation fee of \$2 and an initial \$10 bonus. Every round they will pay a penalty which is taken out of their bonus depending on the choices they make and how certain they are that their colleague has made the same choice. Payoffs range from \$0.01 to \$9.21. In order to get respondents to answer honestly, logarithmic scoring is used as a proper scoring rule.

If both players go to the office, their penalty is $\ln(player\_certainty) \cdot 2$. If they both go to the canteen it is $\ln(player\_certainty)$. If they go to different places it is $\ln(1 - player\_certainty) \cdot 2$. We expect that many will lose their entire bonus, while other's might keep some of their bonus if their co-player loses all their bonuses early and ends the game.

(Probably skip next section of examples of scores)

5.2 Payoff for both going to the office:

Calculated by $\ln(player\_certainty) \cdot 2$

- Very uncertain (50%) = - \$1.39
- Slightly uncertain (62.5%) = -\$0.94
- Somewhat certain (75%) = -\$0.57
- Quite certain (87.5%) = -\$0.27
- Very certain (99%) = -\$0.02

5.3 Payoff for both going to the canteen

Calculated by $\ln(player\_certainty)$

- Very uncertain (50%) = - \$0.69
- Slightly uncertain (62.5%) = -\$0.47
- Somewhat certain (75%) = -\$0.29
- Quite certain (87.5%) = -\$0.13
- Very certain (99%) = -\$0.01

5.4 Payoff for both going to the canteen

Calculated by $\ln(1 - player\_certainty) \cdot 2$

- Very uncertain (50%) = - \$1.39
- Slightly uncertain (62.5%) = -\$1.96
- Somewhat certain (75%) = -\$2.77
- Quite certain (87.5%) = -\$4.16
- Very certain (99%) = -\$9.21

6 Theoretical Approach and Methods

We have a a few assumptions in our experiment, some stronger than others. We strongly expect that respondents always choose to go to the office when they arrive at 9:00 am or 9:10 am. Our theoretical considerations are modeled in Epistemic Logic. Through mostly self-explanatory notation, we can say that for any player a, if a arrives at 9:00 am or 9:10 am, it holds that $K_a(late)$, where $(late)$ is understood atomically as denoting a situation, consisting of a pair arrival times, where both players lose if either goes to the canteen. In other words, we assume that $K_a(late)$ entails that a will go to the office. It's more complicated what $K_a\neg(late)$ entails for a's decision.

Imagine a arrives at 8:50 am. She then knows b arrived at 8:40 am or 9:00 am, i.e. a doesn't know if it's too late to go to the canteen or not: $\neg K_a(\neg late) \wedge \neg K_a(late)$. We can read $\neg K_a\neg(late)$ as expressing that a considers it possible that it is too late to go to the canteen, written as $\hat{K}_a(late)$, given this definition $\hat{K}_a\varphi := \neg K_a\neg\varphi$. Continuing this line of reasoning, assume a arrives at 8:40 am. In this case, a knows b arrived at 8:40 or 8:50, which means he knows it is not too late to go to the canteen: $K_a(\neg late)$. But a doesn't know if b knows this, since she considers it possible that b arrived at 8:50 am, in which case b would not know if it is too late to go to the canteen, so we have $\hat{K}_a\hat{K}_b(late)$ or otherwise expressed as $K_a\neg E_{\{a,b\}}(\neg late)$. In this case, a knows it's not too late to go to the canteen, but he does not know if b knows this.

Similar reasoning applies for a arriving at 8:30 am. In this case we have $K_a(\neg late)$ and given that b can arrive at 8:40 am at the latest, we have $K_b(\neg late)$ and a knowing this epistemic fact: $K_aK_b(\neg late)$. This implies $E_{\{a,b\}}(\neg late)$. Since a knows that both a and b knows it's not too late to go to the canteen, we have $K_aE_{\{a,b\}}(\neg late)$. But if b arrives at 8:40, it holds that everyone knows it's not too late to go to the canteen, but that not everyone knows this very fact! I.e. $E_{\{a,b\}}(\neg late) \wedge \neg E_{\{a,b\}}E_{\{a,b\}}(\neg late)$. In fact, while there are many situations where $K_a\neg(late)$, there is no situation where this is common knowledge, i.e. $C_{\{a,b\}}\neg(late)$ is false for all possible situations in the Canteen Dilemma. However, we do not hypothesize that $C_{\{a,b\}}\neg(late)$ is necessary for a person to choose to go the canteen.

The interesting aspect is what knowledge will be sufficient for a player to go to the canteen. If player a arrives at 8:50, she will consider it possible that it is too late to go to the canteen, i.e. $\hat{K}_a(late)$. We might think that$\hat{K}_a(late)$ entails that a will go to the office. But if a arrives at 8:40 am, she will consider it possible that b considers it possible that it's too late: $\hat{K}_a\hat{K}_b(late)$. If $\hat{K}_a(late)$ if entails a going to the office, and if a assumes b reasons like herself, then $\hat{K}_a\hat{K}_b(late)$ would also entail a going to the office. While $K_a(late)$ should entails a going to the office, it's not so certain if $\hat{K}_a(late)$ entails it as well, or if $\hat{K}_a\hat{K}_b(late)$ does or any other iteration of social reasoning entails it.

Iterated social reasoning has been shown to be quite intuitive. We expect respondents to be limited to around 2 levels of social reasoning. And even if a respondent is a logician who is aware that there is no common knowledge about arriving before 9:00 am, they do not know if others are aware of this and so they might divert from a safe office-only strategy as well. This means that it might be problematic to elicit belief from respondents choices. We counteract this problem in part by asking respondents how certain they are that their colleague makes the same choice as them. This question can also help us in in avoiding conclusions about fixed bounds on social cognition, i.e. respondents can reason up to order n but not $n+$. In other words, we allow respondents to show semi-continuous bounds on their social cognition by asking for their estimates of the probability of success. This might also support modeling of different types of reasoners with different resource bounds on social cognition, since it allows differentiation between respondents not just by a reasoning depth of n but also by a probability estimate.

We can state a few assertions about the Canteen Dilemma. We can assert that any strategy in the game which includes going to the canteen also includes the risk for players to go to different places. We can also assert that if two players were to choose beforehand to go the canteen before some specific time, the optimal cut-off point would be as close to 9:00 am as possible, i.e. 8:50 am, in order to maximise the amount of possible canteen-canteen pairs of arrival times.

We can also say that a symmetrical office-only strategy has the highest expected payoff. If both players go to the office with a 'very certain' estimate every time, they will be certain to get a bonus of \$9.80. If players arrive before 9:00 am at all times and go to the canteen with a 'very certainty' estimate, they get a bonus of \$9.90, only \$0.10 more than the safe office-only strategy. But any such strategy is likely to be given arrival times at 8:50 am and 9:00 am, where the strategy results in large penalties. There is also an interesting case for an asymmetrical office-only strategy. If player a arrives at 8:00 am, she knows player b will arrive at 8:10 am. If player a believes both that player b will go to the canteen with a 'very high' certainty estimate and that b has a bonus left of $<$\$9.21, player a might choose office to intentionally end the game. In that case, player a would get her remaining bonus minus the \$1.39 penalty for that round, which might be high compared to expected payoff for attempting at making it through 10 rounds. It is also highly unlikely that any respondent employs this reasoning, but it remains as a possible rational strategy.

To re-iterate, the only strong assumption we have is that for any agent m, $K_m(late)$ entails that m will go to the office. Besides that, we assume that players will have a reasoning depth of around 2 levels of theory of mind, i.e. going to the office at around 8:40 am the earliest. We also assume that certainty estimates are distributed around this expected reasoning depth.

7 Existing knowledge

Most of current research literature on social reasoning suggest the cognitive human capacity is limited at 2 iterations, i.e. I reason about you, reasoning about me, while few reports find higher levels of reasoning [26] [5]

Research literature on eliciting belief also show that forecasts elicited from observers through proper scoring rules are significantly more accurate and calibrated then those elicited from observers using an improper scoring rule. Calibrated is defined as such: "a set of probabilistic predictions are *calibrated* if p percent of all predictions reported at probability p are true."[23, p. 275].

Forecasts elicited by the logarithmic scoring rule also seem to have significantly less dispersion than quadratic scoring rules even though both are proper scoring rules[20, **?**].

8 AMT Type

9 Ethical Considerations

9.1 Approval by the Institutional Review Board

We provide participants with a consent page before the experiment starts and participants need to check a box in order to proceed. If not, they are not allowed to participate in the experiment.

9.2 Anonymity and data management

We will follow the faculty's guidelines in accordance with the Danish Data Protection Act, cf. requirements from the Danish Data Protection Agency and the faculty's Institutional Review Board.

The only personal information that will be available to the researchers is what is publicly available on respondents MTurk profile. This information will not be shared with any individuals who are not part of the research team. The experiment only records answers to a series of questions.

Data will be stored in a separate database at https://cibs.mef.sc.ku.dk, where the experiment will take place. No data will be stored in the cloud.

10 Additional Considerations

10.1 Time Plan

10.2 Total Cost and Financing

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