Homework 2 (with answers)

1. Evaluate and simplify the following expressions:
   - $10^5 10^{-3}$
   - $\log_{10} 1000$
   - $\log_2 32$
   - $25^{3/2}$

Answers:

- $10^5 10^{-3} = 100$
- $\log_{10} 1000 = 3$
- $\log_2 32 = 4$
- $25^{3/2} = 125$

In [16]:
```
from sympy import log
```

Out[16]: 100.0

In [22]:
```
10**5 * 10**(-3)
```

Out[22]: 100.0

In [17]:
```
log(1000, 10)
```

Out[17]: 3

In [18]:
```
log(32,2)
```

Out[18]: 5

In [21]:
```
25**(3/2)
```

Out[21]: 125.0

2. Label each of these as either rational or irrational:
   - 67.2715882509
   - e
   - $sqrt2$
   - π
   - 25/2
   - 2/3
   - 1577

Answers:

- 67.2715882509 **(RATIONAL)**
- $sqrt2$ **(IRRATIONAL)**
- π **(IRRATIONAL)**
- 25/2 **(RATIONAL)**
- 2/3 **(RATIONAL)**
- 1577 **(RATIONAL)**

3. Evaluate this expression:

$$\lim_{x \to 15} \left(\frac{3}{(x-15)^2} + 5 \right)$$

In [7]:
```
from sympy import *

x = symbols('x')
f = 3 / (x-15)**2 + 5

limit(f,x,15)
```

Out[7]: ∞

4. Evaluate this expression:

$$\lim_{x \to \infty} \left(\frac{3}{(x-15)^2} + 5 \right)$$

In [8]:
```
from sympy import *

x = symbols('x')
f = 3 / (x-15)**2 + 5

limit(f,x,oo)
```

Out[8]: 5

5. Calculate the slope of this function at $x = 3$

$$f(x) = 2x^3 + 1$$

In [10]:
```
from sympy import *

x = symbols('x')
f = 2*x**3 + 1

diff(f,x).subs(x,3)
```

Out[10]: 54

6. Perform gradient descent on this function to find its minimum.

$$f(x) = 5x^2 + 1$$

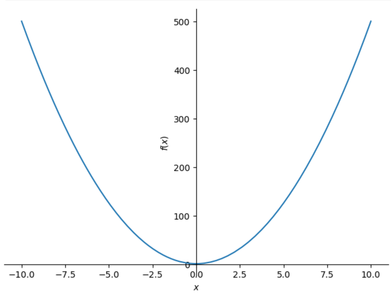
Calculate the derivative.

In [26]:
```
from sympy import *

x = symbols('x')
f = 5*x**2 + 1

dx_f = diff(f,x)
dx_f
```

Out[26]: $10x$

Plot the function for verification

In [27]:
```
plot(f)
```



Out[27]: <sympy.plotting.plot.Plot at 0x118de8bd0>

Perform gradient descent.

In [24]:
```
def f(x): return 5*x**2 + 1
def dx_f(x): return 10*x

L = .05
x = 0
for i in range(1000):
    x -= L*dx_f(x)

print(x, f(x))
```

```
0.0 1.0
```

7. Calculate the under this function from $x = -3$ through $x = 5$.

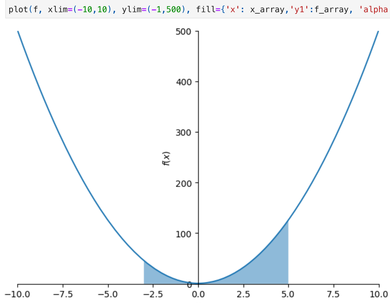
$$f(x) = 5x^2 + 1$$

In [28]:
```
from sympy import *

x = symbols('x')
f = 5*x**2 + 1

integrate(f,(x,-3, 5))
```

Out[28]: $\frac{784}{3}$

Plot area just for good measure.

In [29]:
```
from sympy import *
import numpy as np


x_array = np.linspace(-3, 5, 1000)
f_array = lambdify(x, f)(x_array)

plot(f, xlim=(-10,10), ylim=(-1,500), fill={'x': x_array,'y1':f_array, 'alpha': .5})
```



Out[29]: <sympy.plotting.plot.Plot at 0x11ebeb550>