The Monty Hall Problem

The Monty Hall Problem

DOOR 1

DOOR 2

DOOR 3

Choose a door, one has a prize.
Two others have goats.

The Monty Hall Problem

DOOR 1

DOOR 2

DOOR 3

You choose **Door 1**.
What is the probability it has the prize?

The Monty Hall Problem

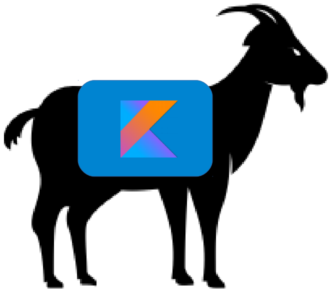
DOOR 1	DOOR 2	DOOR 3
33.33%	**33.33%**	**33.33%**

You choose **Door 1**.
What is the probability it has the prize? **33%**

The Monty Hall Problem

DOOR 1

DOOR 2

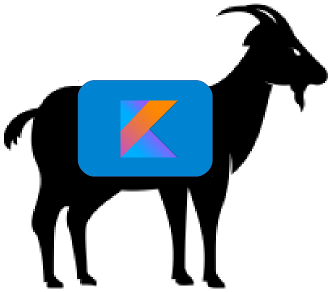


Twist! **Door 3** was just opened. It’s a goat.
Did you want to switch from **Door 1** to **Door 2**?

The Monty Hall Problem

DOOR 1
?%

DOOR 2
?%

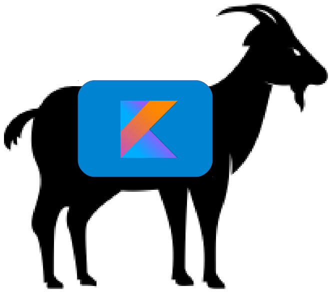


What is the prize probability of each door now?

The Monty Hall Problem

DOOR 1
?%

DOOR 2
?%



HINT: The prize probability of either door is not 50%

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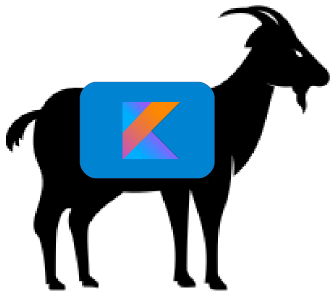
DOOR 1
33.33%

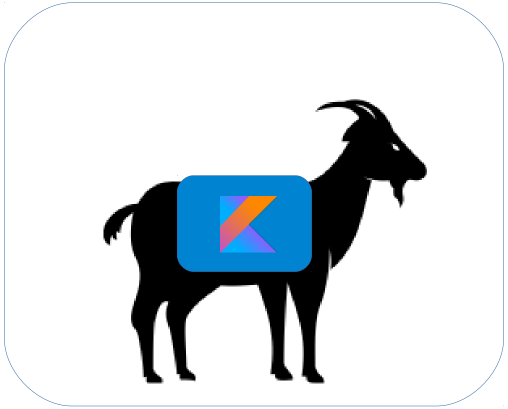
DOOR 2
66.66%



The probability of **Door 1** is **33.33%** while **Door 2** is now **66.66%**.
You should switch! But why?

The Monty Hall Problem

33.33%



66.66%

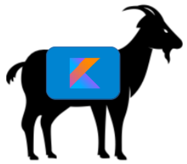
33.33%

The Monty Hall Problem

According to Bayes Theorem…

DOOR 1
33.33%

DOOR 2
66.66%



$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

$$P(P_1|D_2) = \frac{P(D_2|P_1)P(P_1)}{P(D_2)} = \frac{(.5)(.33)}{(.5)} = .33$$

$$P(P_2|D_2) = \frac{P(D_2|P_2)P(P_2)}{P(D_2)} = \frac{(1)(.33)}{(.5)} = \boxed{.66}$$

The Monty Hall Problem

$$D_2 = \text{Probability of door 2 being left} = .5$$

$$P_1 = \text{Probability door 1 contains prize} = .33$$

$$P_2 = \text{Probability door 2 contains prize} = .5$$

$$P(P_1|D_2) = \text{Probability door 1 contains prize given door 2 is left} = .33$$

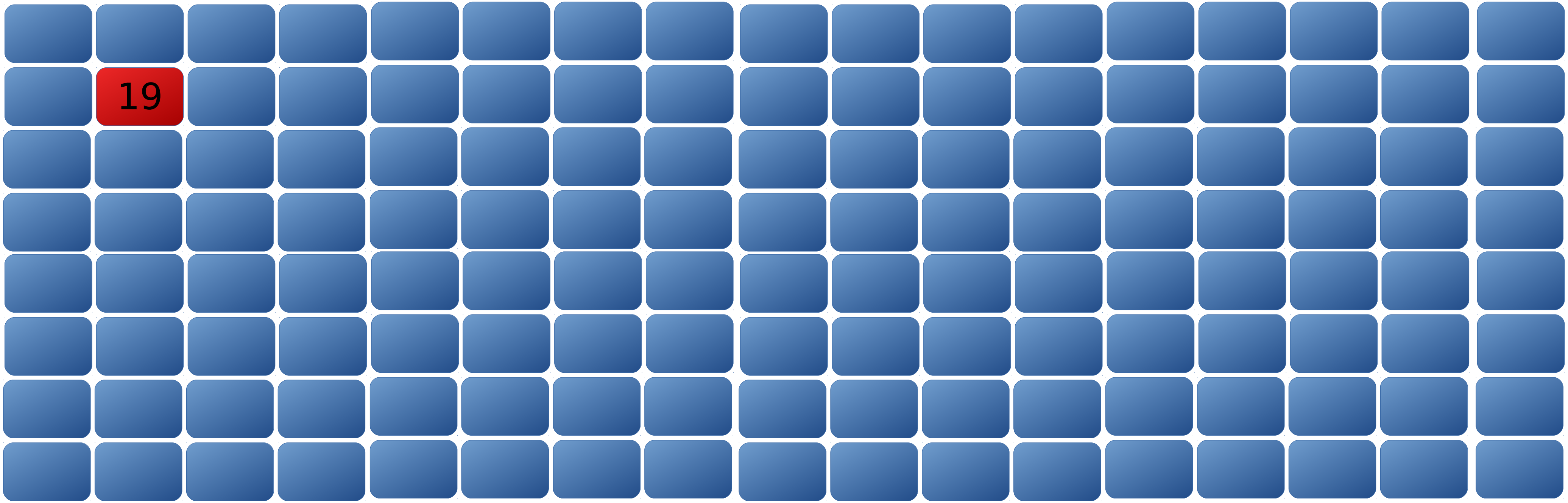
$$P(P_2|D_2) = \text{Probability door 2 contains prize given door 2 is left} = .66$$

$$P(D_2|P_1) = \text{Probability door 2 is left given door 1 has prize} = .5$$

$$P(D_2|P_2) = \text{Probability door 2 is left given door 2 has prize} = 1.0$$

The Monty Hall Problem

Still confused? Hyperbolize! Imagine you had 1000 doors, and you chose **Door #19**.



The Monty Hall Problem

All other doors are opened but yours and **Door #77**. Inclined to switch now?

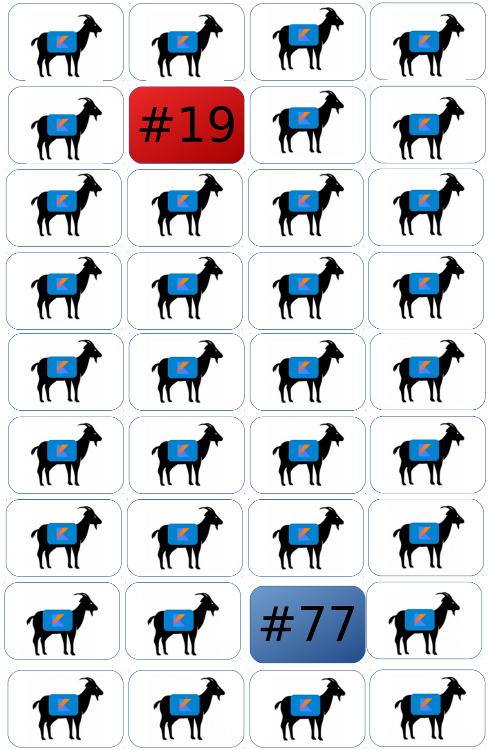
The Monty Hall Problem

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

$$P(P_{19}|D_{77}) = \frac{P(D_{77}|P_{19})P(P_{19})}{P(D_{77})} = \frac{\frac{1}{999} * \frac{1}{1000}}{\frac{1}{1000}} = \frac{1}{999}$$

$$P(P_{77}|D_{77}) = \frac{P(D_{77}|P_{77})P(P_{77})}{P(D_{77})} = \frac{\frac{999}{1000} * \frac{1}{1000}}{\frac{1}{1000}} = \boxed{\frac{999}{1000}}$$

Yes, you should switch!



Monty Hall Simulation in Kotlin

Monte Carlo simulation of the Monty Hall Problem
https://gist.github.com/thomasnield/7fe76d27a57afbea49939dc1879c9883

Modeling Uncertainty and Approximation

The Monty Hall problem encapsulates the critical (and misunderstood) nuances of.

As programmers, we thrive in certainty and exactness.

But the valuable, high-profile problems today often tackle uncertainty and approximation.

- Users and businesses expect “smarter” applications that will predict what they want.
- Machine learning and optimization is nondeterministic and unavoidably has error.
- Many search spaces are too large to exhaustively explore.