Thomas Pattara
Stat 560 Time Series

1. *Analyze the data ex1. See attached.*
a.) *Plot the production data in ex1 and its ACF and PACF.*

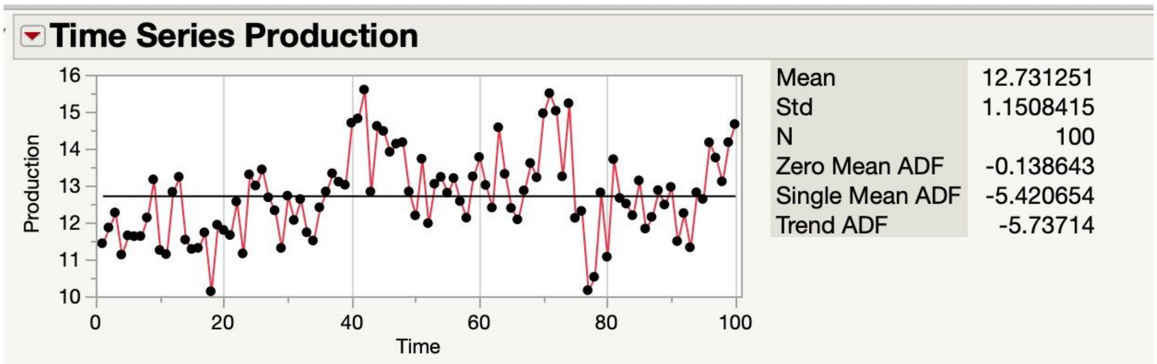


Figure 1

Lag	AutoCorr	Ljung-Box Q	p-Value	Lag	Partial
0	1.0000	.	.	0	1.0000
1	0.5119	26.9988	<.0001*	1	0.5119
2	0.4562	48.6571	<.0001*	2	0.2631
3	0.3402	60.8306	<.0001*	3	0.0488
4	0.2581	67.9083	<.0001*	4	-0.0036
5	0.1220	69.5056	<.0001*	5	-0.1119
6	0.0373	69.6569	<.0001*	6	-0.0857
7	0.0701	70.1956	<.0001*	7	0.0964
8	0.0097	70.2062	<.0001*	8	-0.0105
9	0.0905	71.1235	<.0001*	9	0.1292
10	0.0583	71.5090	<.0001*	10	-0.0034
11	0.1579	74.3647	<.0001*	11	0.1110
12	0.0636	74.8333	<.0001*	12	-0.1052
13	0.0797	75.5789	<.0001*	13	-0.0202
14	0.0838	76.4120	<.0001*	14	0.0318
15	0.1208	78.1619	<.0001*	15	0.1083
16	0.0947	79.2503	<.0001*	16	0.0135
17	0.0083	79.2588	<.0001*	17	-0.1027
18	0.0476	79.5409	<.0001*	18	-0.0123
19	-0.0538	79.9058	<.0001*	19	-0.1015
20	-0.0895	80.9260	<.0001*	20	-0.0919
21	-0.1033	82.3045	<.0001*	21	0.0348
22	-0.1051	83.7480	<.0001*	22	-0.0132
23	-0.0290	83.8591	<.0001*	23	0.1538
24	-0.0494	84.1869	<.0001*	24	-0.0284
25	0.0568	84.6262	<.0001*	25	0.0654

Figure 2

b.) *Fit an ARIMA(p,d,q) model to the production by ML estimate methods. (Hint: choose the model which has the minimum aic.) Specify the model*

Table 1

Model Comparison

Report	Graph	Model	DF	Variance	AIC	SBC	RSquare	-2LogLH	Weights	MAPE	MAE
		AR(2)	97	0.914854	278.34377	286.15929	0.325	272.34377	0.278507	6.234054	0.788601
		ARMA(1, 1)	97	0.9234249	279.25024	287.06575	0.319	273.25024	0.177011	6.188474	0.782409
		AR(3)	96	0.9223851	280.13498	290.55566	0.327	272.13498	0.113732	6.212497	0.785602
		ARMA(2, 1)	96	0.9225321	280.15040	290.57108	0.327	272.1504	0.112858	6.216114	0.786062
		ARIMA(2, 1, 2)	94	0.9166869	280.17607	293.15167	0.325	270.17607	0.111419	6.236817	0.789325
		ARMA(2, 2)	95	0.921483	281.05349	294.07934	0.334	271.05349	0.071850	6.260308	0.791883
		IMA(1, 1)	97	0.9790051	281.17802	286.36826	0.273	277.17802	0.067513	6.274145	0.790651
		ARIMA(1, 1, 1)	96	0.9890568	283.14727	290.93263	0.273	277.14727	0.025221	6.293290	0.793089
		AR(1)	98	0.9822435	284.30294	289.51328	0.270	280.30294	0.014152	6.303655	0.797538
		ARI(2, 1)	96	1.0016498	284.34006	292.12542	0.264	278.34006	0.013892	6.353501	0.800693
		ARI(1, 1)	97	1.0155273	284.67872	289.86896	0.246	280.67872	0.011728	6.539766	0.826014
		MA(2)	97	1.0111499	288.12373	295.93924	0.257	282.12373	0.002095	6.361182	0.805958
		MA(1)	98	1.1210418	297.32076	302.53110	0.169	293.32076	0.000021	6.533314	0.827013
		I(1)	98	1.262934	305.05506	307.65018	0.053	303.05506	0.000000	6.972124	0.881097
		ARIMA(0, 0, 0)	99	1.3378144	313.88639	316.49156	-0.00	311.88639	0.000000	7.143400	0.905056

Table 2

Model: AR(2)

Model Summary

DF	97	Stable	Yes
Sum of Squared Errors	88.7408339	Invertible	Yes
Variance Estimate	0.91485396		
Standard Deviation	0.95647998		
Akaike's 'A' Information Criterion	278.343775		
Schwarz's Bayesian Criterion	286.159285		
RSquare	0.32548413		
RSquare Adj	0.31157659		
MAPE	6.23405442		
MAE	0.78860093		
-2LogLikelihood	272.343775		

Parameter Estimates

Term	Lag	Estimate	Std Error	t Ratio	Prob>|t|	Constant Estimate
AR1	1	0.385786	0.0961087	4.01	0.0001*	4.26387237
AR2	2	0.279732	0.0969573	2.89	0.0048*	
Intercept	0	12.747696	0.2740860	46.51	<.0001*	

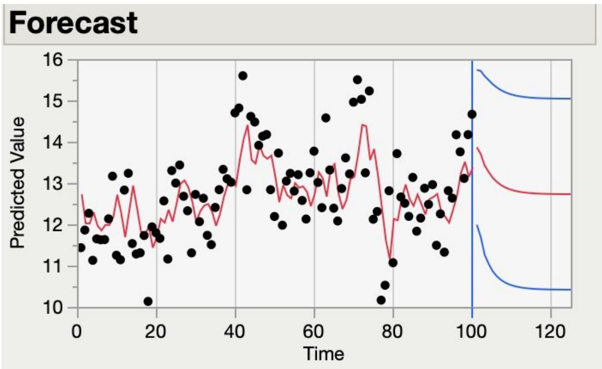


Figure 3

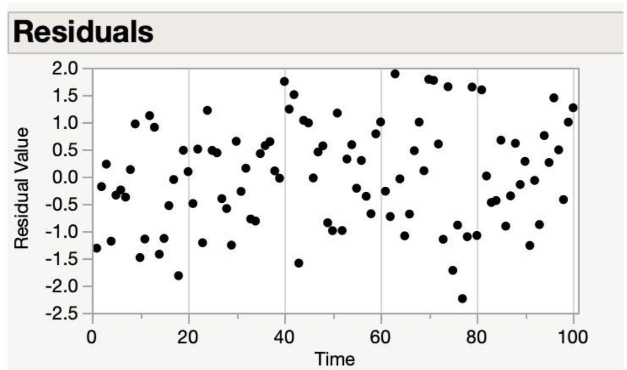


Figure 4

Table 3

Iteration History

Iter	Iteration History	Step	Delta-Criterion	Obj-Criterion
0	244.34692124	Initial	.	1.35205381
0	244.33597112	BFGS	.	1.27483219
1	244.32607771	BFGS	.	1.20249666
2	244.24604115	BFGS	.	0.11164698
3	244.245808	BFGS	.	0.09361527
4	244.24554343	BFGS	.	0.07560436
5	244.24525394	BFGS	.	0.05162916
6	244.2451319	BFGS	.	0.03042582
7	244.24511067	BFGS	.	0.0222385
8	244.24510738	BFGS	.	0.02065212
9	244.24510302	BFGS	.	0.01821158
10	244.24509605	BFGS	.	0.01319606
11	244.24509006	BFGS	.	0.00674992
12	244.24508786	BFGS	.	0.00214985
13	244.2450876	BFGS	.	0.00041853
14	244.24508759	BFGS	.	0.00004214
15	244.24508759	BFGS	.	2.39696e-6
16	244.24508759	BFGS	.	6.44554e-7
17	244.24508759	BFGS	.	3.80882e-7
18	244.24508759	BFGS	.	5.77425e-7

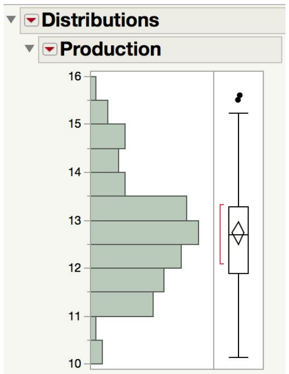
Lag	AutoCorr	-.8-.6-.4-.2 0 .2 .4 .6 .8	Ljung-Box Q	p-Value	Lag	Partial	-.8-.6-.4-.2 0 .2 .4 .6 .8
0	1.0000		.	.	0	1.0000	
1	-0.0153		0.0241	0.8765	1	-0.0153	
2	-0.0251		0.0898	0.9561	2	-0.0254	
3	0.0617		0.4907	0.9209	3	0.0610	
4	0.0774		1.1276	0.8899	4	0.0790	
5	-0.0493		1.3886	0.9256	5	-0.0442	
6	-0.1100		2.7007	0.8454	6	-0.1131	
7	0.0333		2.8226	0.9009	7	0.0182	
8	-0.0984		3.8951	0.8665	8	-0.1037	
9	0.0335		4.0211	0.9100	9	0.0543	
10	-0.0413		4.2143	0.9372	10	-0.0333	
11	0.1634		7.2747	0.7764	11	0.1700	
12	-0.0450		7.5098	0.8222	12	-0.0480	
13	-0.0267		7.5934	0.8690	13	-0.0238	
14	0.0018		7.5938	0.9094	14	-0.0429	
15	0.0948		8.6714	0.8941	15	0.0941	
16	0.0628		9.1498	0.9071	16	0.0685	
17	-0.0727		9.7996	0.9118	17	-0.0165	
18	0.0930		10.8762	0.8995	18	0.0592	
19	-0.0466		11.1495	0.9187	19	-0.0446	
20	-0.0536		11.5160	0.9317	20	-0.0810	
21	-0.0819		12.3821	0.9287	21	-0.0614	
22	-0.0627		12.8961	0.9360	22	-0.1007	
23	0.0254		12.9815	0.9524	23	0.0751	
24	-0.1276		15.1651	0.9158	24	-0.0979	
25	0.0397		15.3798	0.9321	25	0.0515	

Figure 5

We used the AR(2) model because it had the lowest minimum AIC. The model is:

$$y_t = 4.263872 + 0.38578\ y_{t-1} + 0.27973\ y_{t-2}$$

***c.)** Use MOM (Method of Moments) to estimate the coefficients of the model that you selected in part b). (Hint: Calculate the coefficients by hand. Type your results. You can use the acf and pacf function in R to obtain the corresponding estimators.)*



Quantiles

100.0%	maximum	15.59632
99.5%		15.59632
97.5%		15.35711725
90.0%		14.565353
75.0%	quartile	13.28644
50.0%	median	12.70057
25.0%	quartile	11.8793125
10.0%		11.307731
2.5%		10.35256075
0.5%		10.13265
0.0%	minimum	10.13265

Summary Statistics

Mean	12.731251
Std Dev	1.1566393
Std Err Mean	0.1156639
Upper 95% Mean	12.960753
Lower 95% Mean	12.501748
N	100

Figure 6

Table 4

```
> var(ex1$production)
[1] 1.337814
> mean(ex1$production)
[1] 12.73125
>
```

$\gamma_{(0)} = 1.337$
$\hat{p}_{(1)} = 0.5119$
$\hat{p}_{(2)} = 0.4562$

$0.5119 = \Phi_{(1)}+\Phi_{(2)}(0.44562)$
$0.4562 = \Phi_{(2)}(0.5119) +\Phi_{(2)}$

$\Phi_{(1)} = 0.3772$
$\Phi_{(2)} = 0.2631$

$\bar{y} = 12.7313$

$\delta = 12.7313(1-0.3772 - 0.2631)$
$\delta = 4.579$

$$\mathbf{y_{(t)} = 4.579+0.377\, y_{(t-1)} + 0.263\, y_{(t-2)} + \varepsilon_t}$$

d.) *Use the model in part b) to obtain the fitted values.*

Table 5

Predicted Production
12.747695854
12.043793423
12.037704854
12.31363359
11.987934602
11.870118103
12.006873385
12.002374849
12.197257926
12.736224438

e.*)* *Calculate the residuals. But don't print all values, list only the first 10 residuals.*

Table 6

Residual Production
-1.314195854
-0.183553423
0.2283451458
-1.18608359
-0.340254602
-0.245098103
-0.377083385
0.1291151512
0.9675120738
-1.486014438

f.) *Analyze the residuals. Your analysis should include: Box-Ljung test and interpretation (5 points), ACF, PACF, and their interpretation (5 points) and 4-in-1 plot of residuals and interpretation.*

Lag	AutoCorr	-.8-.6-.4-.2 0 .2 .4 .6 .8	Ljung-Box Q	p-Value	Lag	Partial	-.8-.6-.4-.2 0 .2 .4 .6 .8
0	1.0000		.	.	0	1.0000	
1	-0.0153		0.0241	0.8765	1	-0.0153	
2	-0.0251		0.0898	0.9561	2	-0.0254	
3	0.0617		0.4907	0.9209	3	0.0610	
4	0.0774		1.1276	0.8899	4	0.0790	
5	-0.0493		1.3886	0.9256	5	-0.0442	
6	-0.1100		2.7007	0.8454	6	-0.1131	
7	0.0333		2.8226	0.9009	7	0.0182	
8	-0.0984		3.8951	0.8665	8	-0.1037	
9	0.0335		4.0211	0.9100	9	0.0543	
10	-0.0413		4.2143	0.9372	10	-0.0333	
11	0.1634		7.2747	0.7764	11	0.1700	
12	-0.0450		7.5098	0.8222	12	-0.0480	
13	-0.0267		7.5934	0.8690	13	-0.0238	
14	0.0018		7.5938	0.9094	14	-0.0429	
15	0.0948		8.6714	0.8941	15	0.0941	
16	0.0628		9.1498	0.9071	16	0.0685	
17	-0.0727		9.7996	0.9118	17	-0.0165	
18	0.0930		10.8762	0.8995	18	0.0592	
19	-0.0466		11.1495	0.9187	19	-0.0446	
20	-0.0536		11.5160	0.9317	20	-0.0810	
21	-0.0819		12.3821	0.9287	21	-0.0614	
22	-0.0627		12.8961	0.9360	22	-0.1007	
23	0.0254		12.9815	0.9524	23	0.0751	
24	-0.1276		15.1651	0.9158	24	-0.0979	
25	0.0397		15.3798	0.9321	25	0.0515	

Figure 7

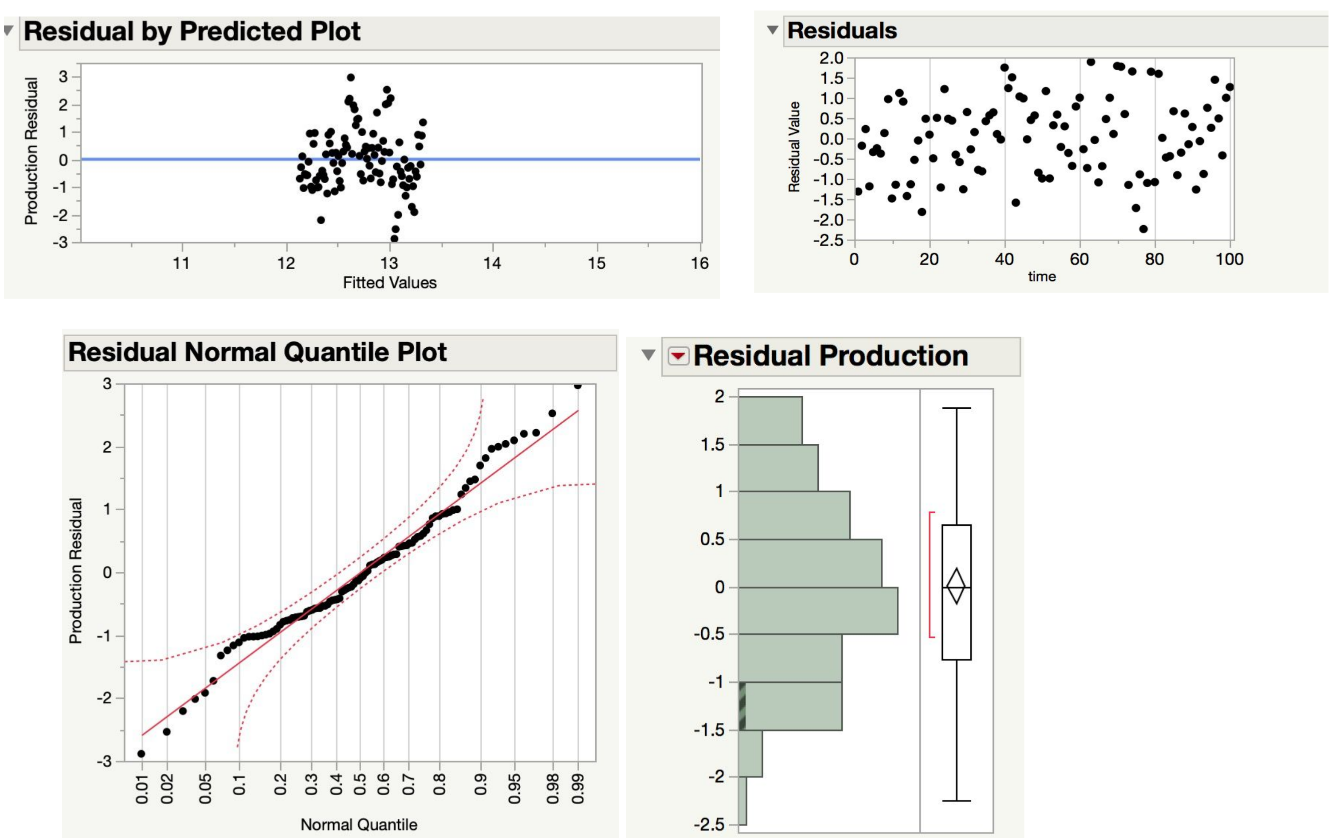


Figure 8

In Figure 8, we can see a normal distribution of residuals in both the histogram and the normal quantile plot. The ACF and PACF for each lag are within the confidence interval and fluctuates around 0, between -0.2 and 0.2 . Also, with the Residual by row plot we can finally conclude the time series is stationary.

The Ljung–Box test is uncommonly used in (ARIMA) modeling. Note that it is applied to the residuals of a fitted ARIMA model, not the original series, and in such applications the hypothesis actually being tested is that the residuals from the ARIMA model have no autocorrelation. Hence from Figure 7, none of the Ljung–Box Q values have significant p-values. Therefore we accept the hypothesis that residuals from the ARIMA model have no autocorrelation.

g.) *Plot the observations and fitted values in the same graph. Are you satisfactory with your model?*

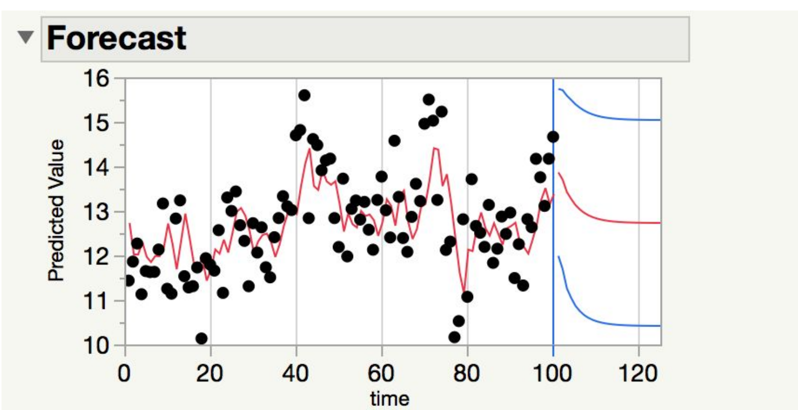


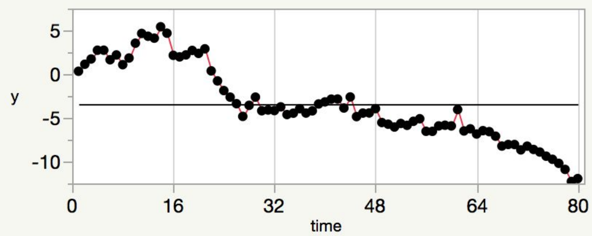
Figure 9

Yes, we are satisfied with our model. The observations seem to follow the fitted values.

2. *Analyze the data ex3. See attached.*

a.) *Plot the data y in ex3, and its ACF and PACF.*

Time Series y



Mean	-3.428952
Std	4.3234587
N	80
Zero Mean ADF	0.8450434
Single Mean ADF	-0.093817
Trend ADF	-2.910101

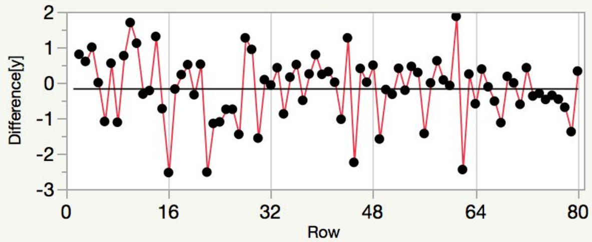
Time Series Basic Diagnostics

Lag	AutoCorr	Ljung-Box Q	p-Value	Lag	Partial
0	1.0000	.	.	0	1.0000
1	0.9484	74.6901	<.0001*	1	0.9484
2	0.8962	142.242	<.0001*	2	-0.0323
3	0.8491	203.658	<.0001*	3	0.0225
4	0.8029	259.295	<.0001*	4	-0.0166
5	0.7547	309.107	<.0001*	5	-0.0434
6	0.7157	354.512	<.0001*	6	0.0658
7	0.6764	395.625	<.0001*	7	-0.0295
8	0.6359	432.468	<.0001*	8	-0.0275
9	0.5890	464.527	<.0001*	9	-0.0870
10	0.5364	491.488	<.0001*	10	-0.0913
11	0.4830	513.671	<.0001*	11	-0.0376
12	0.4230	530.933	<.0001*	12	-0.1118
13	0.3707	544.387	<.0001*	13	0.0371
14	0.3273	555.036	<.0001*	14	0.0369
15	0.2909	563.578	<.0001*	15	0.0342
16	0.2571	570.352	<.0001*	16	0.0101
17	0.2297	575.848	<.0001*	17	0.0349
18	0.2021	580.169	<.0001*	18	-0.0059
19	0.1737	583.415	<.0001*	19	-0.0111
20	0.1517	585.931	<.0001*	20	0.0582
21	0.1255	587.682	<.0001*	21	-0.0668
22	0.1062	588.959	<.0001*	22	0.0457
23	0.0869	589.828	<.0001*	23	-0.0420
24	0.0665	590.347	<.0001*	24	-0.0567
25	0.0520	590.669	<.0001*	25	0.0338

Figure 10

b.) *Plot the first difference of y and its ACF and PACF. Interpret your outputs.*

Time Series Difference[y]



Mean	-0.155411
Std	0.9120215
N	79
Zero Mean ADF	-9.479364
Single Mean ADF	-9.764913
Trend ADF	-9.80365

Time Series Basic Diagnostics

Lag	AutoCorr	Ljung-Box Q	p-Value	Lag	Partial
0	1.0000	.	.	0	1.0000
1	-0.1072	0.9422	0.3317	1	-0.1072
2	-0.0003	0.9422	0.6243	2	-0.0119
3	-0.0211	0.9798	0.8061	3	-0.0227
4	0.0745	1.4532	0.8349	4	0.0706
5	-0.1292	2.8977	0.7158	5	-0.1160
6	0.0085	2.9040	0.8208	6	-0.0164
7	0.1041	3.8679	0.7949	7	0.1071
8	0.1232	5.2364	0.7320	8	0.1410
9	-0.0518	5.4817	0.7905	9	-0.0080
10	-0.0036	5.4829	0.8567	10	-0.0230
11	0.1381	7.2782	0.7761	11	0.1332
12	-0.2197	11.8864	0.4548	12	-0.1948
13	-0.1662	14.5636	0.3354	13	-0.1957
14	-0.0815	15.2181	0.3634	14	-0.1510
15	0.0761	15.7972	0.3957	15	-0.0014
16	-0.2145	20.4720	0.1997	16	-0.2019
17	0.0452	20.6826	0.2408	17	-0.0383
18	-0.0742	21.2603	0.2665	18	-0.1392
19	0.1710	24.3771	0.1821	19	0.1590
20	-0.2051	28.9415	0.0889	20	-0.0521
21	-0.0265	29.0189	0.1135	21	-0.0359
22	0.0362	29.1658	0.1402	22	0.0638
23	-0.0112	29.1801	0.1744	23	0.0691
24	-0.1091	30.5653	0.1667	24	-0.0193
25	0.0665	31.0898	0.1861	25	-0.0251

Figure 11

The first difference transforms the initial non stationary times series into a stationary time series, all the ACF and PACF values lie between the upper and lower limits.

c.) *Fit an ARIMA(p,d,q) model to the y by ML estimate methods. Specify the model and its coefficients.*

Model Comparison

Report	Graph	Model	DF	Variance	AIC	SBC	RSquare	-2LogLH	Weights	.2 .4 .6 .8	MAPE	MAE
☑	☐	I(1)	78	0.8424471	211.64180	214.01125	0.956	209.6418	0.321567		28.859894	0.708050
☑	☐	IMA(1, 1)	77	0.843235	212.70841	217.44731	0.956	208.70841	0.188651		29.078591	0.703738
☑	☐	ARI(1, 1)	77	0.84341	212.72434	217.46324	0.956	208.72434	0.187154		29.067227	0.703497
☑	☐	ARIMA(2, 1, 2)	74	0.7820896	213.04459	224.89183	0.959	203.04459	0.159463		27.487774	0.693176
☑	☐	ARIMA(1, 1, 1)	76	0.8543134	214.70693	221.81528	0.956	208.70693	0.069452		29.082631	0.704060
☑	☐	ARI(2, 1)	76	0.8543702	214.71198	221.82033	0.956	208.71198	0.069277		29.082800	0.703833
☑	☐	AR(1)	78	0.8732183	221.68141	226.44546	0.937	217.68141	0.002124		48.107917	0.761075
☑	☐	AR(2)	77	0.8806664	223.37076	230.51684	0.936	217.37076	0.000913		48.607339	0.759368
☑	☐	ARMA(1, 1)	77	0.8809226	223.39233	230.53841	0.936	217.39233	0.000903		48.590004	0.759742
☑	☐	ARMA(2, 2)	75	0.8552723	224.58811	236.49824	0.937	214.58811	0.000497		49.130474	0.772073
☑	☐	I(2)	77	1.874702	271.36702	273.72373	0.901	269.36702	0.000000		38.500909	1.029355
☑	☐	MA(2)	77	3.3125841	328.02072	335.16680	0.820	322.02072	0.000000		54.869477	1.529641
☑	☐	MA(1)	78	6.654127	381.97421	386.73826	0.644	377.97421	0.000000		72.169920	2.127034

Figure 12

Model: ARIMA(2, 1, 2)

Model Summary

DF	74	Stable	Yes
Sum of Squared Errors	57.874629	Invertible	Yes
Variance Estimate	0.78208958		
Standard Deviation	0.88435829		
Akaike's 'A' Information Criterion	213.044586		
Schwarz's Bayesian Criterion	224.891826		
RSquare	0.95896877		
RSquare Adj	0.95675087		
MAPE	27.4877742		
MAE	0.69317615		
-2LogLikelihood	203.044586		

Parameter Estimates

Term	Lag	Estimate	Std Error	t Ratio	Prob>|t|	Constant Estimate
AR1	1	1.536987	0.0712308	21.58	<.0001*	-0.0452739
AR2	2	-0.820221	0.0853553	-9.61	<.0001*	
MA1	1	1.739716	0.0566906	30.69	<.0001*	
MA2	2	-0.999960	0.0551029	-18.15	<.0001*	
Intercept	0	-0.159846	0.0879117	-1.82	0.0731	

Model: IMA(1, 1)

Model Summary

DF	77	Stable	Yes
Sum of Squared Errors	64.9290947	Invertible	Yes
Variance Estimate	0.843235		
Standard Deviation	0.91827828		
Akaike's 'A' Information Criterion	212.708415		
Schwarz's Bayesian Criterion	217.44731		
RSquare	0.95615336		
RSquare Adj	0.95558392		
MAPE	29.0785906		
MAE	0.70373751		
-2LogLikelihood	208.708415		

Parameter Estimates

Term	Lag	Estimate	Std Error	t Ratio	Prob>|t|	Constant Estimate
MA1	1	0.1098417	0.1128434	0.97	0.3334	-0.1573595
Intercept	0	-0.1573595	0.0909428	-1.73	0.0876	

Model: ARI(1, 1)

Model Summary

DF	77	Stable	Yes
Sum of Squared Errors	64.9425688	Invertible	Yes
Variance Estimate	0.84340998		
Standard Deviation	0.91837355		
Akaike's 'A' Information Criterion	212.724345		
Schwarz's Bayesian Criterion	217.46324		
RSquare	0.95614453		
RSquare Adj	0.95557498		
MAPE	29.0672265		
MAE	0.70349659		
-2LogLikelihood	208.724345		

Parameter Estimates

Term	Lag	Estimate	Std Error	t Ratio	Prob>|t|	Constant Estimate
AR1	1	-0.1077329	0.1121228	-0.96	0.3396	-0.1741245
Intercept	0	-0.1571899	0.0921966	-1.70	0.0922	

Model: I(1)

Model Summary

DF	78	Stable	Yes
Sum of Squared Errors	65.7108765	Invertible	Yes
Variance Estimate	0.84244713		
Standard Deviation	0.91784919		
Akaike's 'A' Information Criterion	211.6418		
Schwarz's Bayesian Criterion	214.011248		
RSquare	0.95563299		
RSquare Adj	0.95563299		
MAPE	28.8598943		
MAE	0.70805012		
-2LogLikelihood	209.6418		

Parameter Estimates

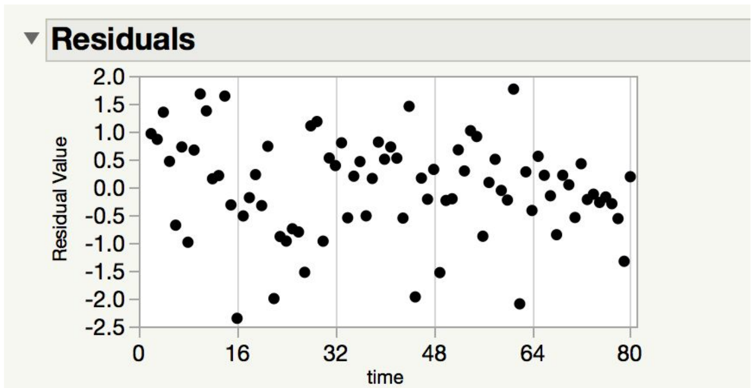
Term	Lag	Estimate	Std Error	t Ratio	Prob>|t|	Constant Estimate
Intercept	0	-0.1554110	0.1025851	-1.51	0.1338	-0.155411

Figure 13

The model chosen is ARIMA(2,1,2) since this model has the best fit under the maximum likelihood function (minimum -*2LogLikelihood*). All the parameter estimates can be seen significant for ARIMA(2,1,2), whereas in Figure 12, there are three models which have lower AIC than ARIMA(2,1,2), but from Figure 13 we see that these models do not have significant parameters. The ARIMA(2,1,2) model equation can be written as

$$y_t = -0.045273 + 1.53698\, y_{t-1} - 0.8202\, y_{t-2} + 1.73971\, \varepsilon_{t-1} - 0.9999\, \varepsilon_{t-2}$$

***d.)** Plot a realization of your fitted model, also show its ACF and PACF. Compare them with the results in part a).*



Lag	AutoCorr	Ljung-Box Q	p-Value	Lag	Partial
0	1.0000	.	.	0	1.0000
1	-0.0038	0.0012	0.9725	1	-0.0038
2	0.0788	0.5176	0.7720	2	0.0788
3	0.0371	0.6334	0.8887	3	0.0379
4	0.0942	1.3903	0.8459	4	0.0890
5	-0.1158	2.5490	0.7691	5	-0.1221
6	-0.0146	2.5677	0.8608	6	-0.0318
7	0.0685	2.9845	0.8864	7	0.0817
8	0.0881	3.6834	0.8845	8	0.0973
9	-0.0599	4.0112	0.9107	9	-0.0501
10	-0.0063	4.0149	0.9467	10	-0.0420
11	0.1232	5.4436	0.9078	11	0.1109
12	-0.2014	9.3166	0.6757	12	-0.2007
13	-0.1715	12.1694	0.5138	13	-0.1704
14	-0.1070	13.2956	0.5034	14	-0.1012
15	0.0250	13.3581	0.5747	15	0.0379
16	-0.2372	19.0739	0.2648	16	-0.1693
17	-0.0026	19.0746	0.3243	17	-0.0124
18	-0.0982	20.0858	0.3280	18	-0.1261
19	0.1320	21.9445	0.2870	19	0.1368
20	-0.1901	25.8640	0.1704	20	-0.0954
21	-0.0205	25.9105	0.2099	21	-0.0599
22	0.0373	26.0670	0.2488	22	0.0407
23	-0.0067	26.0721	0.2975	23	0.0407
24	-0.0971	27.1696	0.2966	24	-0.0370
25	0.0557	27.5372	0.3296	25	-0.0248

Figure 14

The non stationary time series in part a is transformed to a stationary time series. The model is stable with all ACF and PACF values in between the upper and lower limits.