

1, Consider the time series model:

$$y_t = 4 + \epsilon_t - 0.5\epsilon_{t-1} + 0.8\epsilon_{t-2}$$

a) Calculate the mean for the model if it exists.

Mean = 4

b) Calculate the autocovariance functions for the model by hand if it exists . Show all your work.

The image shows handwritten work on lined paper for part b. It includes the model equation, the parameters θ_1 and θ_2 , and the calculations for $\gamma(0)$, $\gamma(1)$, and $\gamma(2)$.

$$y_t = 4 + \epsilon_t - 0.5\epsilon_{t-1} + 0.8\epsilon_{t-2}$$
$$\theta_1 = 0.5 \quad \theta_2 = -0.8$$
$$\gamma(0) = \sigma^2 [1 + (0.5)^2 + (-0.8)^2] = 1.89 \sigma^2$$
$$\gamma(1) = \sigma^2 [-0.5 + (0.5)(-0.8)] = -0.9 \sigma^2$$
$$\gamma(2) = \sigma^2 [-(-0.8)] = 0.8 \sigma^2$$

c) Calculate the autocorrelation functions for the model by hand if it exists . Show all your work.
Do NOT plot the ACF.

$$\rho(1) = \frac{-0.5 + (0.5)(-0.8)}{1 + (0.5)^2 + (-0.8)^2} = -0.476190$$

$$\rho(2) = \frac{0.8}{1 + (0.5)^2 + (-0.8)^2} = 0.423280$$

d) Is this a stationary time series process?

It is a weak stationary time series

2, Consider the time series model:

$$y_t = 200 + 0.7y_{t-1} + \epsilon_t$$

a) Is this a stationary time series process? Why or why not. Interpret your reason by the definition/theorem. (Do NOT use the time series plot or ACF to state your reason.)

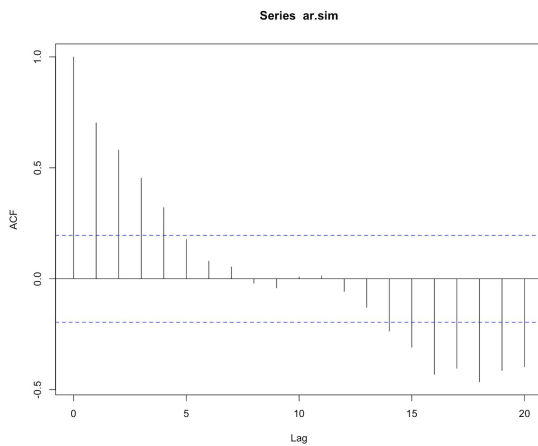
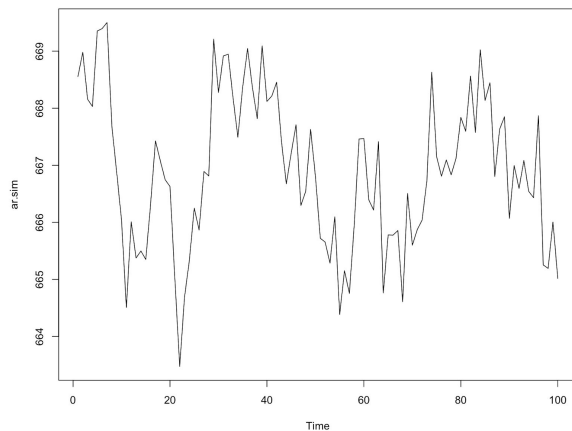
It is stationary time series process since $|\phi| = 0.7 < 1$.

b) What is the mean of the time series if it exists ?

$$\text{Mean} = \frac{\delta}{1-\phi} = \frac{200}{0.3} = 666.67$$

c) Give a realization of the model, and plot the ACF.

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ar.sim<-arima.sim(list(order=c(1,0,0),ar=0.7),n=100)+666.67
ts.plot(ar.sim)
ar.acf<-acf(ar.sim,type="correlation",plot=T)
ar.acf
```



d) Rewrite the expression of AR(1) model to an expression of MA model.

$$y_t = 666.67 + \varepsilon_t + 0.7 y_{t-1}$$

3, Consider the time series model:

$$y_t = 1.5y_{t-1} - 0.75y_{t-2} + \epsilon_t$$

a) Is this a stationary time series process? Why or why not. Interpret your reason by the theorem. (Do NOT use the time series plot or ACF to state your reason.)

This time series is stationary time series since it follows the 3 conditions for an AR(2) model to be stationary.

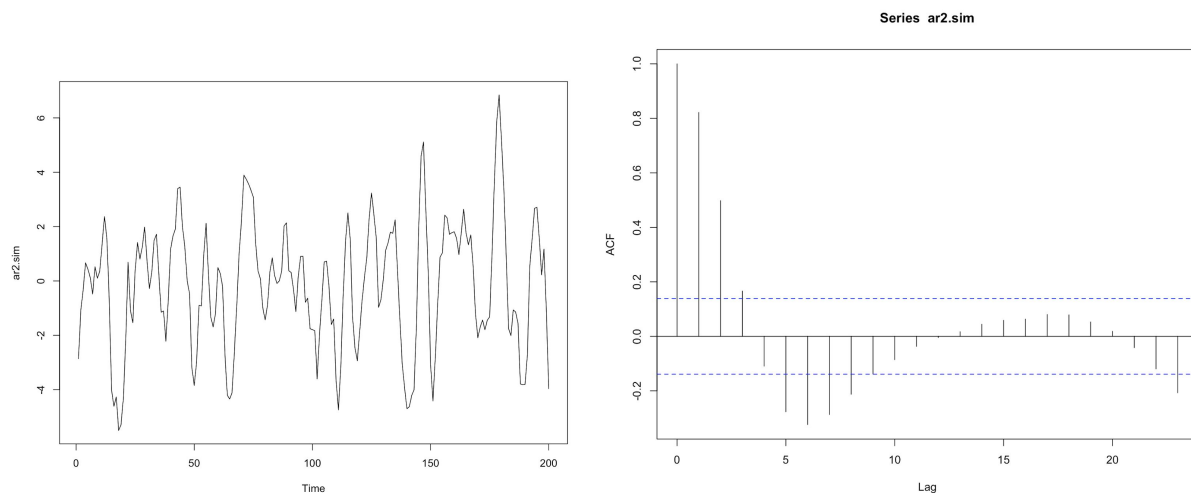
$$1. \quad \Phi_2 < 1 + \phi_1$$

2. $\Phi_2 < 1 - \phi_1$
3. $\Phi_2 > -1$

b) What is the mean of the time series if it exists?

The Mean = 0 since $\delta = 0$

c) Give a realization of the model, and plot the ACF.



4, Consider the time series model:

$$y_t = 2.5y_{t-1} - y_{t-2} + \epsilon_t$$

a) Is this a stationary time series process? Why or why not. Interpret your reason by the theorem. (Do NOT use the time series plot or ACF to state your reason.)

It is not a stationary time series since it does not satisfy the conditions for an AR(2) model to be stationary.

b) What is the mean of the time series if it exists ?

The Mean = 0 since $\delta = 0$