Stability of Modified Max Pressure Controller with Application to Signalized Traffic Networks

Thomas Pumir, Leah Anderson, Dimitrios Triantafyllos, and Alexandre M. Bayen

Abstract—This work describes a type of distributed feedback control algorithm that acts on a vertical queueing network where flow dynamics may greatly outpace the rate of feedback and actuation. The modeled network has a known, finite set of feasible actuations for the binary controllers located at each network node. It also has known expected demands, split ratios, and maximum service rates. Previous work proposed the application of a max pressure controller to maximize throughput on such a network without the need for centralized computation of a control policy. Here, we extend the max pressure controller to a practical scenario . Specifically, we extend the controller so as to satisfy some practical requirements. The set of allowable controllers in this setting is extended to any convex combination of available signal phases to account for signal changes within a single signal "cycle". Moreover the controller cannot be updated every time step. We show that this proposed extended max pressure controllers stabilize the network (queue lengths remain bounded in expectation) given slight restrictions on admissible network demand flows. This work is motivated by the application of controlling traffic signals on arterial road networks. Max pressure provides an intriguing alternative to existing feedback control systems due to its theoretical guarantees, but cannot be directly applied as originally formulated due to hardware and safety constraints. We ultimately apply our extension of max pressure to a simulation of an existing arterial roadway and provide comparison to the control policy that is currently deployed on this site.

Keywords—*max pressure, vertical queueing network, network stability, adaptive signal control, arterial modeling*

I. INTRODUCTION

This article investigates the design and stability of decentralized controller for vertical queueing networks. In a *vertical queueing network*, agents traveling across the network are stored in "point queues" which do not inhabit a "horizontal" position along the length of a network edge, but instead are considered to be "vertical" stacks at the front end of each edge. In the present work, we examine a vertical queueing network that can be controlled at junctions. Such networks are good models for urban traffic, but are inherited from other fields such as supply chain management or internet routing. The flows passing through each node of this network are constrained by a controller, similar to how a traffic light dictates allowable flows across a traffic intersection: only a finite set of non-conflicting turning movements or *phases* can be permitted to flow simultaneously across each node. At some regular interval, each of the the node controllers selects one element from its set of feasible phases to actuate. Hence flows through the network are dictated largely by the control policy applied at the nodes. A signal control policy is said to be *stabilizing* if it ensures that the mean length of the queues waiting at each of the nodes remains bounded for all time. One such stabilizing control policy that is known to maximize network throughput is the *max pressure* controller. Here, we show that such network stabilization is still achieved given various extensions of the original max pressure controller. These extensions are motivated by the practical physical constraints imposed on realistic traffic signal controllers, and they present new mathematical problems in the analysis of queue dynamics which are investigated in this article.

The general problem of a stabilizing controller in this context was first considered from the point of view of multihop radio networks in Tassiulas et Ephremides [1]. The control of a network is a broad subject with many applications such as wireless networks [2][3], oscilatory networks [4], or queuing networks [5][6]. In the particular context of arterial traffic control, little work has been made on the approximation equilibrium of the network, at the exception of Osorio and Bierlaire [7][8]. Most of the literature focuses on the design of feedback responsive control. Comprehensive on this topic are provided in Baras and Levine [9][10], Mirchandani and Head [11], Papageorgiou et al. [12], Osorio and Bierlaire [7], and Xie et al. [13].

The max pressure traffic controller [14] is a specific distributed network signal control policy derived from the concept of a "back pressure controller", which was first studied in the context of routing packets through a communications network (see for example the pioneering work of Tassiulas and Ephremides [1]). The ideas has been applied to road traffic management more recently by Varaiya [14], as well as in the work of Wongpiromsarn et al. [15].

The concept of max pressure control is intuitive: at each intersection, give priority to the phase which will be able to service the most traffic while taking into account the subsequent feasibility of downstream demand queues. Variaya's original formulation of this controller in [14], however, does not fully consider the practical limitations on the rate of queue measurement and signal actuation in practical traffic networks. Using Variaya's stability proof as a starting point, we prove that two extended max pressure controllers that account for these constraints will still provide queue stability. Specifically, we address the fact that a practical control policy cannot be updated at the same time scale as that at which queues form: due to hardware limitations and safety constraints, most

T. Pumir and A. Bayen are with the Department of Electrical Engineering and Computer Science, University of California, Berkeley, CA; L. Anderson is with the Department of Civil and Environmental Engineering, University of California, Berkeley. Corresponding e-mail: thomas.pumir@berkeley.edu

existing signal controllers must cycle through all allowable sets of phases while adhering to a fixed cycle length and strict limitations on the minimum and maximum allowable actuation time for each phase within the cycle. During the course of one signal cycle, however, vehicle queues may aggregate (or dissipate) significantly. Therefore, we first show that a "non-updated" max pressure controller, or one which only received occasional feedback upon which to update its actuation policy, will still stabilize network queues. We then show that under slightly stronger conditions on admissible input flows, a "relaxed" max pressure controller that allocates an entire actuation cycle between each signal phase given occasional feedback will also stabilize the network. We also examine the effect that each of these limitations on control will have on the ultimate bound on queue length relative to the chosen cycle time and network parameters.

The remainder of this article is organized as follows: first, Section II describes our modeling framework and Section III gives a mathematical definition of controllers and the concept of network stability. Section IV describes the problem the max pressure encounters in practice. Sections V describe the new controller we propose to solve those problems and expose the proof of stability for this new controller. Finally, Section VI outlines numerical results provided by this controller using a micro-simulation running in the Aimsum platform of a real arterial traffic network within the I-15 corridor which was part of a SANDAG pilot program ran in San Diego, CA.

II. MODEL FRAMEWORK

Definitions

We consider a network of arterial roads with infinite storage capacity, modeled topologically as a graph with road links being edges and intersections being vertices. An individual link $l \in \mathcal{L}$ can be either at the entry of the network ($l \in \mathcal{L}_{\text{ent}}$) or in the interior of the network ($l \in \mathcal{L} \backslash \mathcal{L}_{\text{ent}}$). The inflow on entry links is a defined entirely by a random demand d_l, while the input flows of all other links depend on queues on upstream links and the relevant set of physical flow constraints are defined within the network. We require that each link has an exit path, that is, a continuous set of subsequent links on which vehicles can travel from the link to eventually exit the network. Each link in the network model can have multiple *queues* corresponding to individual *movements*: all vehicles in a given queue on any link are intending to advance onto the same subsequent link (though not necessarily the same subsequent queue).

We describe the dynamics of these queues as a discrete time dynamical model using the following notation:

- A *movement* (l, m) distinguishes an intention to travel from link l to link m, (in that case, say that $m \in Out(l)$ where $Out(l)$ is the set of links immediately downstream l);
- A *queue* $x(l, m)(t)$ is the number of vehicles on link l waiting to enter link m at timestep t, and $X(t)$ is the set (vector or matrix) of all the queue lengths on the network at timestep t;

- A *saturation flow* $c(l, m)$ is the expected number of vehicles that can travel from link l to link m per time step given maximum demand for the queue $x(l, m)$, and $C(l, m)(t)$ is the *realized saturation flow* at time t;
- The *turn ratio* $r(l, m)$ is the expected proportion of vehicles that are leaving l which are intending to enter m, and $R(l, m)(t)$ is the *realized turn ratio* at time t;
- The *demand vector* d of dimension $|\mathcal{L}_{\text{ent}}|$ specifies demands at network entry links;
- The *flow vector* f of dimension $|\mathcal{L}|$ denotes flows on all links of the network such that f_l is the flow within link l.

Note that there is necessarily a linear relationship between the flow vector f and the demand vector d: $f = dP$ where the matrix P depends only on routing proportions.

Controller

A road intersection is modeled as a node in our framework. Controllers (traffic signals) are placed at every node to limit the set of queues permitted to discharge at any given time. A set of movements that can be simultaneously actuated without flow conflicts is called a *phase*. Each permissible phase for a given intersection can be represented as a binary control matrix S defined as follows:

$$S(l, m) = \begin{cases} 1 & \text{if movement } (l, m) \text{ is activated} \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

We denote U_n the known finite set of permissible control matrices for node n. Note that in this article, we often drop the subscript n for ease of notation.

While more than one phase cannot realistically be simultaneously actuated, a "relaxed" controller can also be defined as a matrix representing the fraction of each time step allocated to each phase:

$$S^r(l, m) = \lambda_{l,m} \in [0, 1] \quad (2)$$

Such a relaxed controller can be seen as a convex combination of all possible control matrices,

$$S^r = \sum_{S \in U} \lambda_S S \quad (3)$$

We suppose that at each model time step t, a controller selects a single control matrix $S(t)$ that encodes which set of queues approaching the intersection are permitted to discharge during that time step. In general, the determination of such a controller is based on the state of the network at a previous time step.

Queue Dynamics

The evolution of queue state $X(t)$ can be seen as a Markov chain: the state of the network at time $t + 1$ is a function of only the network state at time t and external demand vector d,

$$X(t + 1) = F(X(t), d) \quad (4)$$

Define $[a \wedge b] := \min\{a, b\}$. To describe queue dynamics explicitly, we must make a distinction between entry links and internal links: if $l \in \mathcal{L}_{\text{ent}}$,

$$x(l,m)(t+1) = x(l,m)(t) + d_l(t+1) \tag{5}$$
$$- [C(l,m)(t+1)S(l,m)(t+1) \wedge x(l,m)(t)]$$

and if $l \in \mathcal{L} \backslash \mathcal{L}_{\text{ent}}$,

$$x(l,m)(t+1) = x(l,m)(t)+ \tag{6}$$
$$\sum_k [C(k,l)(t+1)S(k,l)(t+1) \wedge x(k,l)(t)]R(l,m)(t+1)$$
$$- [C(l,m)(t+1)S(l,m)(t+1) \wedge x(l,m)(t)]$$

Stability conditions

We focus on networks for which the boundary inflow demands $d = (d_l)_{(l \in \mathcal{L}_e)}$ are *feasible*—that is, the network is servicing a distribution of inflows for which it is possible to find a controller that allows *in average* more departures than arrivals at each link. For a specific sequence of control matrices $\overline{S} = \{S(1), S(2), ..., S(t), ...\}$, we define the *long-term control proportion* matrix $M_{\overline{S}}$ as follows:

$$M_{\overline{S}}(l,m) = \liminf_T \frac{1}{T} \sum_{t=1}^{T} S(l,m)(t) \tag{7}$$

We define $co(U)$ as the convex hull of the set of permissible control matrices U. The following property can be shown:

Property 1: As shown in [14], $M \in co(U)$ if and only if $\exists$ a sequence of control matrices $\overline{S} = \{S(1), S(2), ..., S(t), ...|S(\cdot) \in U\}$ such that $\forall(l,m)$

$$M(l,m) = \liminf_T \frac{1}{T} \sum_{t=1}^{T} S(l,m)(t) \tag{8}$$

Property 2: As defined in [14], a demand is feasible if and only if $\exists M_{\overline{S}} \in co(U)$ and $\varepsilon > 0$ such that

$$c(l,m)M_{\overline{S}}(l,m) > f_l r(l,m) + \varepsilon. \tag{9}$$

Furthermore, we say that a network is *stable* if the following quantity is bounded:

$$\frac{1}{T} \sum_{t=1}^{T} \mathbb{E}\Big\{|X(t)|_1\Big\} \tag{10}$$

where $|X|_1 = \sum_{l,m} |x(l,m)|$. If the demand on a network is feasible, then it is possible to find a controller that stabilizes this network as shown by Varaiya [14].

III. STANDARD MAX PRESSURE CONTROLLER: IMMEDIATE FEEDBACK

This section summarizes results from [14] and [1] used later in this article.

Principle

We consider a weight assigned to each queue (l,m) as a function of all network queue lengths X:

$$w(l,m)(X(t)) = x(l,m)(t) - \sum_{p \in Out(m)} r(m,p)x(m,p)(t) \tag{11}$$

where $Out(m)$ is the set of all links receiving flow from link m. The original *immediate feedback* max pressure controller actuates the phase $S^* \in U$ which alleviates the most *pressure* at the intersection. The pressure $\gamma(S)$ that is potentially alleviated by a control action S is defined as follows:

$$\gamma(S)(X(t)) = \sum_{l,m} c(l,m)w(l,m)(X(t))S(l,m)(t) \tag{12}$$

$$= \sum_{l,m:S(l,m)(t)=1} c(l,m)w(l,m)(X(t)) \tag{13}$$

Explicitly, $u^*(X)$ therefore simply choses the phase $S^* \in U$ that maximizes $\gamma(S)(X(t))$ at each time step:

$$S^*(t) = u^*(X(t)) = \arg\max\{\gamma(S)(X(t))|S \in U\} \tag{14}$$

First Stability Result

Varaiya [14] shows that the following stability result holds for the immediate feedback max pressure controller:

Theorem 3: The max pressure control u^* is stabilizing whenever the average demand vector $d = \{d_l\}$ is within the set of feasible demands, called D^0. There is no stabilizing control when d is not within the set of feasible demands D^0.

IV. CYCLE MAX PRESSURE

Cycle max pressure enables the control to act at a slower time scale than the queue dynamics, as would be the case in a practical traffic application. Suppose that we are given the model dynamics $X(t)$ as in (5)-(6), but the controller $S^*(t)$ can only be updated every τ model time steps (or once per *cycle*). The "τ-non updated" control sequence is therefore composed of control matrices repeated for at least τ model time steps: for a fixed cycle size τ and integer n,

$$S(n\tau+1) = S(n\tau+2) = \ldots = S((n+1)\tau) = S^*(n\tau+1)$$
$$= \arg\max\{\gamma(S)(X(n\tau+1))|S \in U\} \tag{15}$$

Physically, the controller that maximizes the pressure at time step $n\tau+1$ is continuously applied until time step $(n+1)\tau$.

τ-admissible flows

We first prove that this set of τ-admissible flows (demands that can be accommodated using τ-non updated sequences) is in fact the same set of flows that is admissible under typical updated control sequences, defined in equation (9). Define the following sets:

- U is the set of admissible control matrices as in (9),
- $U_{\mathbb{N}}$ is the set of control sequences $\{S(1), S(2) \ldots S(t) \ldots |S(\cdot) \in U\}$ where elements are applied at consecutive time steps,

- $U_{\tau\mathbb{N}}$ is the set of control sequences $\{S(1), S(1), \ldots, S(\tau + 1), S(\tau + 1), \ldots, S(n\tau + 1), S(n\tau + 1), \ldots | S(\cdot) \in U\}$ where controls are updated only once every τ steps,
- $conv(U) = \Big\{ \liminf_T \frac{1}{T} \sum_{t=1}^T S(t) | \{S(1), S(2), \ldots, S(t), \ldots\} \in U_{\mathbb{N}} \Big\}$
- $conv(U_\tau) = \Big\{ \liminf_T \frac{1}{T} \sum_{t=1}^T S(t) | \{S(1), S(1), \ldots, S(\tau + 1), S(\tau + 1), \ldots\} \in U_{\tau\mathbb{N}} \Big\}$

Obviously, $conv(U_\tau) \subset conv(U)$. But we can also show that $conv(U) \subset conv(U_\tau)$:
Suppose $M \in conv(U)$, so $\exists \{S(1) \ldots S(t) \ldots\}$ such that $M = \liminf_T \frac{1}{T} \sum_{t=1}^T S(t)$.

$$M = \liminf_T \frac{1}{T} \sum_{t=1}^T S(t)$$
$$= \liminf_T \frac{1}{\tau T} \sum_{t=1}^{\tau T} \tilde{S}(t)$$
$$\text{with } \tilde{S} = \{S(1), \ldots, S(1), \ldots, S(t), \ldots, S(t), \ldots\}$$
$$= \liminf_T \frac{1}{T} \sum_{t=1}^T \tilde{S}(t)$$

Trivially, $\tilde{S} \in conv(U_\tau)$. Because $conv(U_\tau) \subset conv(U)$ and $cont(U) \subset conv(U_\tau)$, it must hold that $conv(U) = conv(U_\tau)$. This establishes Property 4:

Property 4:

$$conv(U) = conv(U_\tau)$$

This property implies that a τ-non updated control sequence can accommodate the same set of flows as a control sequence updated at every time step. The equivalence becomes intuitive when one considers that our definition of feasible flows considers only the long-term average of demand and service rates: note that a τ control matrix in $\tilde{S}$ is simply the average of the corresponding τ matrices in $\overline{S}$, such that $M_{\tilde{S}} = M_{\overline{S}}$. Hence,

$$f_l r(l, m) < c(l, m) M_{\overline{S}}(l, m) \implies \tag{16}$$
$$f_l r(l, m) < c(l, m) M_{\tilde{S}}(l, m).$$

However, as we will show in the following sections, the bound on queue lengths when the cycle controller is applied will be larger than in the immediate feedback setting.

Stability of cycle max pressure

Here we extend the previous proof of stability of an immediate feedback max pressure controller to the case of a cycle max pressure controller:

Theorem 5: A cycle max pressure controller updated every τ iterations stabilizes the network whenever the demand is within the set of feasible demands D^0.

In Appendix **??**, we prove Theorem 5 by showing that

$$\frac{1}{\tau T} \sum_{t=1}^{\tau T} \mathbb{E}\Big\{|X(t)|\Big\} < \frac{1}{2\varepsilon\eta\tau T} \mathbb{E}\Big\{|X(1)|^2\Big\} \tag{17}$$
$$+ \frac{1}{2\varepsilon\eta\tau} \sum_{p=0}^{\tau-1} (B(p) + \tau K)$$

where ε, η and K are constants which depend on network topology and demand.

Increased bound on queue length

Max pressure controllers that are only updated every τ model time steps will stabilize a network; however the resulting bound on the queues will be higher than in the immediate feedback setting. Comparing (**??**) and (17), note that the constants are increased by a factor of

$$\frac{1}{\varepsilon\tau} \sum_{p=1}^{\tau-1} B(p) = \frac{1}{\varepsilon\tau} \frac{\tau(\tau-1)}{2} \cdot (\text{constant}) = \frac{\tau-1}{2\varepsilon} \cdot (\text{constant}) \tag{18}$$

The relative cost to queue bounds is therefore linear in τ.

V. ALLOCATED MAX PRESSURE

Next, we want to reformulate the controller so that instead of a single phase being chosen for actuation at every model step, all feasible phases are actuated in parallel during the time step at a proportionally reduced rate. The control output will no longer be a binary matrix, but rather will be a *relaxed* control matrix $S^r(t)$ which is a convex combination of all elements in the set of feasible control phases U. Each element $S^r(t)(i, j) \in [0, 1]$ will represent fraction of the flow capacity at time step t that is allocated to movement (i, j). Physically, one could consider one model time step to represent an entire light cycle, during which each signal phase will be sequentially actuated for some proportional amount of the period. (Although this would suggest a serial division of the time period between independent phases rather than the parallel implementation that is represented here mathematically.)

Minimum cycle time

Consider that the length T of the controller update time period (signal cycle) is not pre-defined. However, there is a known amount of *lost time* L per cycle during which no phases can be actuated to account for physical clearance of the intersection between actuated phases. Hence it must be the case that $T > L$. We furthermore impose that each feasible phase has an associated minimum proportion constraint: phase S must be actuated for at least $\lambda_S T$ seconds during any one cycle. We then want to determine the minimum control period or *cycle length* for which a fixed demand flow can be served while the stated phase proportion constraints are satisfied.

Unlike the previous case where we constrained our analysis to flows which could be served in average over an arbitrary long-term time horizon, here our proof of stability depends on the assumption that the average demand is served *within a*

single cycle. As suggested in [14], we then pose the selection of a cycle length as a convex optimization problem with constraints applied to enforce the desired minimum phase proportions κ:

$$\underset{\lambda=(\lambda_S)_S}{\text{minimize}} \quad \sum_{S \in U} \lambda_S$$
$$\text{subject to} \quad \lambda_S \geq \kappa_S \tag{19}$$
$$f_l r(l,m) < \sum_S \lambda_S c(l,m) S(l,m)$$

where $\kappa_S \in [0,1] \; \forall S \in U$, and $\sum_S \kappa_S < 1$.

Let us denote Λ^* to be the optimum of (19). If $\Lambda^* > 1$, the demand is not feasible under the set of control constraints $\{\kappa_S\}$ for any length time step. If $\Lambda^* < 1$, then the flow is admissible for a time step of length

$$T > \frac{L}{1 - \Lambda^*} \tag{20}$$

We can now use this lower bound to select an appropriate cycle length.

Phase distribution within a cycle

We next investigate how the max pressure controller must be altered to allow the minimum proportion constraints on each phase in the set of allowable phases $\mathcal{U}$.

Let T be the cycle length satisfying (20) and κ_S be the minimum proportion of the cycle which must be allocated to each $S \in U$. The allocated max pressure controller selects a set of λ_S that maximizes alleviated pressure under the following constraints:

- $\lambda_S \geq \kappa_S$
- $f_l r_{l,m} < \sum_S \lambda_S c(l,m) S(l,m)$
- $\sum_S \lambda_S = 1 - \frac{L}{T}$

The desired controller is therefore determined by the solution to the following linear program:

$$\{\lambda_S^*\} = \underset{\lambda_1, \ldots, \lambda_{|U|}}{\arg\max} \quad \sum_{S \in \mathcal{U}} \lambda_S \left(\sum_{l,m} w(l,m)(t) c(l,m) S(l,m) \right)$$
$$\text{subject to} \quad \lambda_S \geq \kappa_s$$
$$\sum_S \lambda_S \leq 1 - \frac{L}{T} \tag{21}$$

The solution to (21) selects coefficients $\{\lambda_S^*\}$ which form a corresponding relaxed control matrix

$$S^{r*} = \sum_{S \in \mathcal{U}} \lambda_S^* S \tag{22}$$

Stability of allocated max pressure

We now show that the allocated max pressure controller S^{r*} is stable in the sense of (**??**) under conditions slightly modified from those assumed in the stability proofs of immediate feedback and cycle max pressure.

Define $conv_\kappa$ as the set of convex combinations of control matrices with coefficients larger than κ:

$$conv_\kappa = \left\{ \sum_S \lambda_S S | \; \lambda_S > \kappa_S \; \forall S \in U \right\} \tag{23}$$

Also define a set of reduced admissible demands D_κ which can in average be served in a single cycle with a relaxed control matrix that maintains the minimum time allocation for a given cycle time (as in (19)):

$$d \in D_\kappa \text{ iff } \exists \, S^r \in conv_\kappa$$
$$\text{such that } f_l r(l,m) < c(l,m) S^r(l,m) \tag{24}$$

The following Theorem is proven in Appendix **??**:

Theorem 6: The sequence of allocated max pressure controller stabilizes the network whenever the average demand vector $d = \{d_l\}$ is within the set of feasible demands D_κ.

VI.   CYCLE-ALLOCATED MAX PRESSURE

Definition

We now combine the cycle max pressure and allocated max pressure: once every τ model time steps, a relaxed controller S^r is chosen and then applied at each of the subsequent τ time steps. In other words, a single relaxed controller $S^r(t)$ is applied for the set of time steps $\{t+1, t+2, \ldots, t+\tau-1\}$. Then, at time $t+\tau$, a new relaxed controller $S^r(\tau)$ is calculated to be applied at $\{t+\tau+1, \ldots, t+2\tau-1\}$.

Stability of cycle-allocated max pressure

Theorem 7: The sequence of cycle-allocated max pressure controller stabilizes the network whenever the average demand vector $d = \{d_l\}$ is within the set of feasible demands D_κ defined above.

The stability of the cycle-allocated max pressure control follows from a logical combination of the stability proofs of cycle and allocated variations. Explicitly, the result for cycle max pressure in Appendix **??** still holds in the case of a relaxed controller when one assumes that demand flow satisfies the modified cycle-by-cycle admissibility condition introduced for allocated max pressure in Section V.

VII.   NUMERICAL IMPLEMENTATION

A cycle-allocated max pressure controller was implemented on a network of 11 signalized intersections modeled in AIM-SUN, a mico-simulation platform commonly used by practitioners. The present simulation was generated as part of the I-15 Integrated Corridor Management project, led by SANDAG in San Diego, CA. The controller was set to run with a cycle length of 90 seconds and minimum time constraints of 10 seconds for each of the 3-4 available standard signal control stages.

The rest of this section provides a comparison of the performance of the max pressure algorithm with two other algorithms described below. While it makes sense to compare the numerical performance of these algorithms, it should be
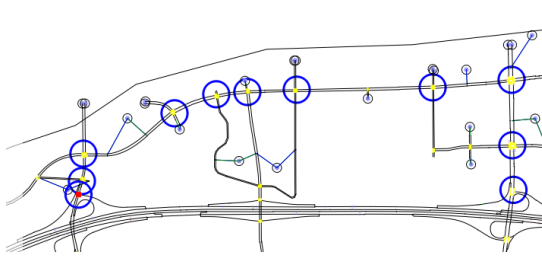
Fig. 1. The chosen network was calibrated to represent realistic demands and physical parameters observed on a stretch of Black Mountain Road near the I-15 freeway in San Diego, California.

noted that the max pressure algorithm is the only algorithm, to the best of our knowledge, which provides theoretical guarantees: while the other algorithms seem to eprform well in simulation, they come with no proof or guarantees, as they are heuristics commonly used in the practioners field. Various performance metrics were compared between model runs using the max pressure controller and two alternative controllers: a fixed-time control plan that divides each signal cycle equally between all available phases, and a "fully-actuated" control system such as that which is currently operational on the real road network represented by the model. The fully actuated-controller is essentially a flexible fixed-time plan in which green times can be shortened or extended in real time to promote continuity of flows in response to instantaneous link demand measurements. The comparison of network vehicle counts in Figure 2 suggests that the uniformly-allocated fixed time controller caused significantly fewer vehicles to be served than the other two controllers, which were comparable to each other in vehicle service rates. This difference made it difficult to fairly compare other performance metrics between the uniform fixed-time controller and the other two options.
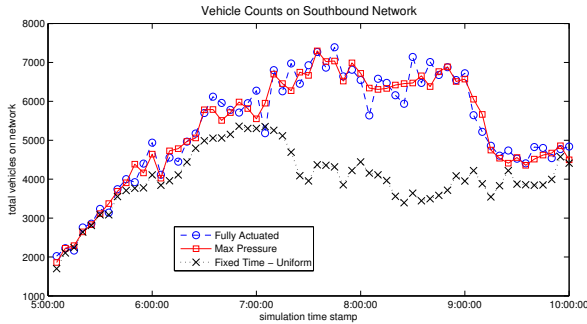

Fig. 2. During congestion, cycle-allocated max pressure demonstrated service rates that are higher than the uniformly allocated fixed-time controller and consistent with a fully-actuated control system.

Differences between the fully-actuated and max pressure controllers were observed in measurements of delays and queue lengths: while the fully-actuated system appeared to produce less delay and shorter queues than max pressure during periods of relatively low network demand, max pressure was equally as effective or even more effective at reducing delays and queue lengths given larger demands. Figure 3 shows how delays were reduced and had less variance over time when
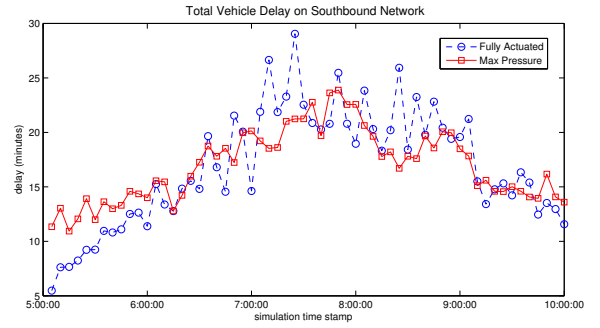

Fig. 3. Max pressure outperforms the fully actuated system during periods of high demand.

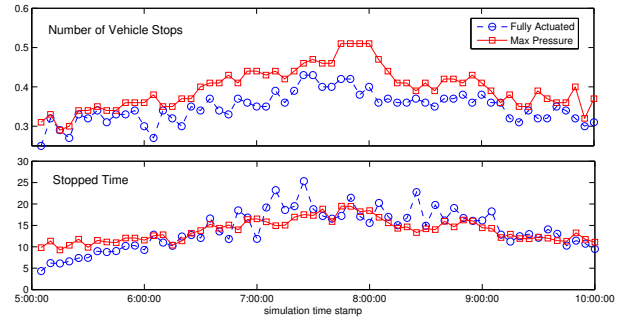the max pressure controller was applied than with the fully-actuated controller.

Fig. 4. While max pressure caused more vehicle stop events, stoppage times were similar to those observed using the standard fully-actuated controller.

Max pressure consistently induced more stops during a vehicle's journey across the network, which is expected given the design objectives of the fully-actuated system. As illustrated in Figure 4, total stoppage times were higher with max pressure given low demand, but improved over the existing controller during peak demand.

VIII. CONCLUSION

In this work we have defined three extensions of the max pressure controller required in order to be applied to a real network of signalized traffic intersections. Given only the constraint that the network demands are (in average) serviceable, we have proven that updating a max pressure controller at a slower rate than that which governs traffic flow will not destroy the stabilizing properties of the controller. We have also formulated a max pressure controller which allocates a fixed minimum proportion of service to each permissible phase, and shown that such a controller will also stabilize a network.

While there are additional modifications to the control model that must be addressed before max pressure can be considered a feasible algorithm for real-time operation of traffic signal controllers, our implementation in the rfeerence micro-simulation platform demonstrates how it could comply with the hardware and communications constraints commonly encountered in existing roadway infrastructure. These numerical simulations furthermore reveal that an implementation

of cycle-allocated max pressure competes with the existing standard feedback controller in many performance metrics, especially during periods of high congestion. This comes on top of theoretical proofs it provides, unlike algoriithms used in practice. It also appears to provide less variance in delays than the existing alternative.

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