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Analog IC Design - Homework #4

TP24222

"pledged"

Problem 1:

a) For proper current mirror operation, the transistors must be in saturation.

So $V_{DS} \geq \overbrace{V_{GS} - V_T}^{V_{OV}}$.

$$V_{in_1} = V_{GS6} + V_{GS5} = V_{OV6} + V_T + V_{OV5}$$

$$\boxed{V_{in_1} = 2V_{OV} + V_T}$$

$$\boxed{V_{in_2} = V_{OV} + V_T}$$

b) $R_{out} = r_{o4}\left(1 + (g_{m4} + g_{mb4})R_{d3}\right) + R_{d3}$ $\qquad R_{d3} = r_{o3}\left(1 + \cancel{(g_{m3} + g_{mb3})\,0}\right) \to 0$

$$R_{d3} = r_{o3}$$

$$\boxed{\begin{aligned} R_{out} &= r_{o4}\left(1 + (g_{m4} + g_{mb4})r_{o3}\right) + r_{o3} \\ R_{out} &\approx (g_{m4} + g_{mb4})r_o^2 \end{aligned}}$$

$$\boxed{V_{OV,MIN} = 2V_{OV}}$$

c) $R_{out} = \dfrac{1}{\lambda I_{out}} \Rightarrow \lambda = \dfrac{1}{R_{out} I_{out}}$

$$V_{DS3} = V_{OV} \qquad V_{DS1} = V_T$$

$$\epsilon = (V_{DS3} - V_{DS1})\lambda = \frac{V_{OV} - V_T}{R_{out} I_{out}} = 0$$

$$\boxed{V_{OV} = V_T} \text{ or } \boxed{V_{DS3} = V_{DS1}}$$

d) $$\lambda_{WSCCM} = \frac{\frac{g_{ds4}}{I_D}}{\frac{g_{m4}}{I_D} + \frac{g_{mb4}}{I_D} + \frac{g_{ds4}}{I_D} + \lambda}\,\lambda \quad , \quad \lambda = \frac{1}{I_D r_{o3}}$$

$$\lambda_{WSCCM} = \frac{1}{R_o I_D} = \frac{1}{\left(r_{o4}\left(1 + (g_{m4} + g_{mb4}) r_{o3}\right) + r_{o3}\right) I_D}$$

$$= \frac{1}{\left(r_{o4}\left(1 + g_{m4} r_{o3} + g_{mb4} r_{o3}\right) + r_{o3}\right) I_D}$$

$$= \frac{1}{I_D r_{o4} + I_D r_{o4} g_{m4} r_{o3} + g_{mb4} r_{o3} I_D r_{o4} + r_{o3} I_D}$$

$$= \frac{\frac{1}{I_D}}{r_{o4} + r_{o4} g_{m4} r_{o3} + g_{mb4} r_{o3} r_{o4} + r_{o3}}$$

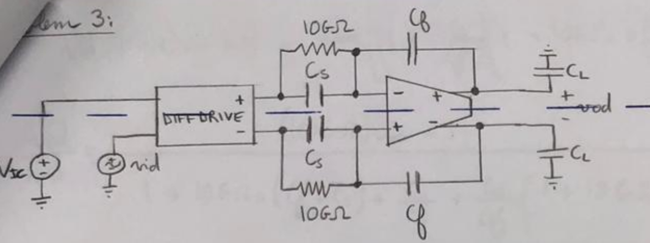
$$= \frac{\frac{1}{I_D r_{o4}}}{1 + g_{m4} r_{o3} + g_{mb4} r_{o3} + \frac{r_{o3}}{r_{o4}}}$$

$$= \frac{\frac{g_{ds4}}{I_D}}{\frac{1}{I_D r_{o3}}\left(1 + g_{m4} r_{o3} + g_{mb4} r_{o3} + \frac{r_{o3}}{r_{o4}}\right)} \quad \frac{1}{I_D r_{o3}}$$

$$= \frac{g_{ds4}/I_D}{\frac{1}{I_D r_{o3}} + \frac{g_{m4}}{I_D} + \frac{g_{mb4}}{I_D} + \frac{1}{I_D r_{o4}}} \quad \frac{1}{I_D r_{o3}}$$

$$\boxed{\lambda_{WSCCM} = \frac{\frac{g_{ds4}}{I_D}}{\frac{g_{m4}}{I_D} + \frac{g_{mb4}}{I_D} + \frac{g_{ds4}}{I_D} + \lambda}\,\lambda}$$

lem 3:

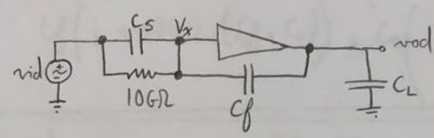


$C_s = 10pF$

$C_f = 10pF$

$C_L = 5pF$

$V_{ic} = 1.1V$



$$(v_{id} - v_x)sC_s + \frac{(v_{id} - v_x)}{10G\Omega} = (v_x - v_{od})sC_f$$

$$v_{id}sC_s - v_x sC_s + \frac{v_{id}}{10G\Omega} - \frac{v_x}{10G\Omega} = v_x sC_f - v_{od}sC_f$$

$$v_x\left(sC_s + \frac{1}{10G\Omega} + sC_f\right) = v_{id}\left(sC_s + \frac{1}{10G\Omega}\right) + v_{od}sC_f$$

$$v_x = \frac{v_{id}\left(sC_s + \frac{1}{10G\Omega}\right) + v_{od}sC_f}{sC_s + \frac{1}{10G\Omega} + sC_f} = \frac{v_{id}\left((10G\Omega)sC_s + 1\right) + v_{od}sC_f(10G\Omega)}{(10G\Omega)(sC_s + sC_f) + 1}$$

$$(v_x - v_{od})sC_f = v_{od}sC_s$$

$$v_x = v_{od}\left(1 + \frac{sC_s}{sC_f}\right)$$

$$\frac{v_{id}\left((10G\Omega)(sC_s) + 1\right) + v_{od}sC_f(10G\Omega)}{(10G\Omega)(sC_s + sC_f) + 1} = v_{od}\left(1 + \frac{\cancel{s}C_s}{\cancel{s}C_f}\right)$$

$$v_{id}\left((10G\Omega)sC_s+1\right) = v_{od}\left[\left(1+\frac{C_s}{C_f}\right)\left(1+10G\Omega\, s\left(C_s+C_f\right)\right) - sC_f(10G\Omega)\right]$$

$$\frac{v_{od}}{v_{id}} = \frac{(10G\Omega)sC_s+1}{1+10G\Omega s\left(C_f+C_s\right)+\frac{C_s}{C_f}+\frac{C_s}{C_f}\left(1+10G\Omega s\left(C_s+C_f\right)\right)-sC_f(10G\Omega)}$$

$$\boxed{\frac{v_{od}}{v_{id}} = \frac{C_f\left((10G\Omega)sC_s+1\right)}{C_f\left(1+10G\Omega s\left(C_f+C_s\right)\right)+C_s\left(1+\left(1+10G\Omega s\left(C_s+C_f\right)\right)\right)-sC_f^2\,10G\Omega}}$$

$$\boxed{W_{-3dB} = \frac{2C_s+C_f}{C_s^2(10G\Omega)+20G\Omega\left(C_sC_f\right)}}$$

$$W_u = \frac{2}{10G\Omega\left(C_s+C_f\right)} = \boxed{\frac{1}{5G\Omega\left(C_s+C_f\right)}}$$

$$v_{id}\left((10G\Omega)sC_s+1\right) = v_{od}\left[\left(1+\frac{C_s}{C_f}\right)\left(1+10G\Omega\, s\left(C_s+C_f\right)\right) - sC_f(10G\Omega)\right]$$

$$\frac{v_{od}}{v_{id}} = \frac{(10G\Omega)sC_s+1}{1+10G\Omega s\left(C_f+C_s\right)+\frac{C_s}{C_f}+\frac{C_s}{C_f}\left(1+10G\Omega s\left(C_s+C_f\right)\right)-sC_f(10G\Omega)}$$

$$\boxed{\frac{v_{od}}{v_{id}} = \frac{C_f\left((10G\Omega)sC_s+1\right)}{C_f\left(1+10G\Omega s(C_f+C_s)\right)+C_s\left(1+\left(1+10G\Omega s\left(C_s+C_f\right)\right)\right)-sC_f^2 10G\Omega}}$$

$$\boxed{W_{-3dB} = \frac{2C_s+C_f}{C_s^2(10G\Omega)+20G\Omega\left(C_sC_f\right)}}$$

$$W_u = \frac{2}{10G\Omega\left(C_s+C_f\right)} = \boxed{\frac{1}{5G\Omega\left(C_s+C_f\right)}}$$

...em 4: Textbook Problem 8.28b) (Local shunt-shunt feedback amplifier).

$R_f = 100\,k\Omega$

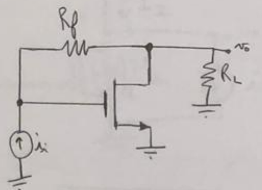
$R_L = 15\,k\Omega$

$I_D = 0.5\,mA$

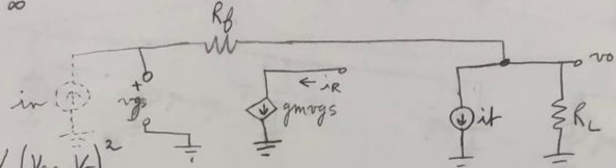
$\frac{W}{L} = 100$

$k' = 180\,\mu A/V^2$

$r_o = \infty$



Calculate: - loop gain.
- closed-loop gain.



$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_T)^2$$

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \mu_n C_{ox}\frac{W}{L}(V_{GS}-V_T) = k'\frac{W}{L}V_{ov} = \frac{2I_D}{V_{ov}}$$

$$V_{ov}^2 = \frac{2I_D}{k'\frac{W}{L}} \Longrightarrow V_{ov} = \sqrt{\frac{2I_D}{k'\frac{W}{L}}} = 0.2357\,V$$

$$g_m = \frac{2I_D}{V_{ov}} = 4.243\,mS$$

$$i_t = \frac{0 - v_o}{R_L} = -\frac{v_o}{R_L} \qquad i_R = g_m v_{gs} = g_m v_o$$

$$T = -\frac{i_R}{i_t} = \frac{\cancel{-}\,g_m \cancel{v_o}}{\cancel{-}\,\frac{\cancel{v_o}}{R_L}} \Longrightarrow \boxed{\text{Loop gain} = T = g_m R_L = 63.645}$$

$$A = A_\infty \frac{T}{T+1} + \frac{d}{T+1}$$

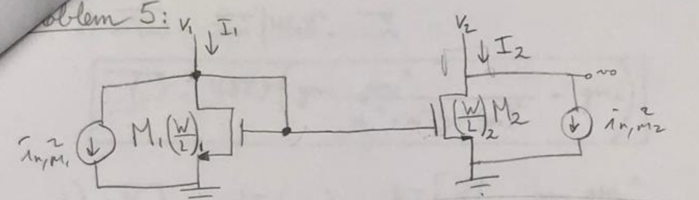
$$\boxed{\text{Closed loop gain} = A_\infty = -R_f = -100\,k\Omega}$$

$$A_\infty = \left.\frac{v_o}{i_n}\right|_{g_m=\infty} = -R_f$$

when $g_m = \infty \Rightarrow v_{gs} = 0$

$$i_n = \frac{0 - v_o}{R_f} \Rightarrow \frac{v_o}{i_n} = -R_f$$

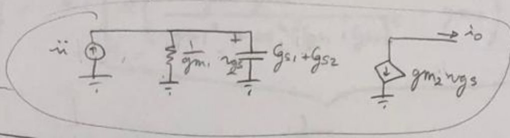
Problem 5:

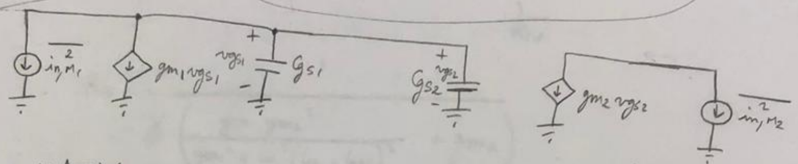


$L_1 = L_2$

$W_2 = M * W_1$

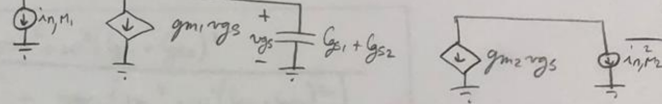
$\gamma_1 = \gamma_2 = \gamma$





$$\overline{i_{n,M_1}^2} = 4kT\gamma g_{m_1}$$

$$\overline{i_{n,M_2}^2} = 4kT\gamma g_{m2}$$



$$i_o = -g_{m2} v_{gs} = -g_{m2}\, i_i \left(\frac{1}{g_{m_1}} \,//\, \frac{1}{s(C_{gs1}+C_{gs2})}\right)$$

$$H(s) = \frac{i_o(s)}{i_i(s)} = -g_{m2}\left(\frac{1}{g_{m_1}} \,//\, \frac{1}{s(C_{gs1}+C_{gs2})}\right)$$

$$H(s) = -g_{m2} \frac{\frac{1}{g_{m_1} s(C_{gs1}+C_{gs2})}}{\frac{1}{g_{m_1}} + \frac{1}{s(C_{gs1}+C_{gs2})}} = \frac{-g_{m2}}{g_{m1} + s(C_{gs1}+C_{gs2})}$$

$$|H(s)| = \frac{g_{m2}}{\sqrt{g_{m_1}^2 + \omega^2 (C_{gs1}+C_{gs2})^2}}$$

$$|H(s)|^2 = \frac{g_{m2}^2}{g_{m_1}^2 + \omega^2 (C_{gs1}+C_{gs2})^2}$$

a) $\overline{i_o^2(\omega)} = \overline{i_{n,M_1}^2}\, |H(s)|^2 + \overline{i_{n,M_2}^2}$

a) $\overline{i_o^2(\omega)} = \overline{i_{n,M_1}^2}\,|H(s)|^2 + \overline{i_{n,M_2}^2}$

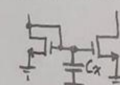
$$\boxed{\overline{i_o^2(\omega)} = 4kT\gamma\left(gm_1 \frac{gm_2^2}{gm_1^2+\omega^2(C_{gs1}+C_{gs2})^2} + gm_2\right)}$$

b) $\overline{i_o^2(\omega)} = 4kT\left(\frac{1}{R_N}\right) = 4kT\underbrace{\left[\gamma\left(\frac{gm_1\, gm_2^2}{gm_1^2+\omega^2(C_{gs1}+C_{gs2})^2} + gm_2\right)\right]}_{\frac{1}{R_N}}$

$$R_N = \frac{1}{\gamma\left(\frac{gm_1\, gm_2^2}{gm_1^2+\omega^2(C_{gs1}+C_{gs2})^2} + gm_2\right)}$$

$$\boxed{R_N = \frac{1}{\gamma}\,\frac{gm_1^2+\omega^2(C_{gs1}+C_{gs2})^2}{gm_1\, gm_2^2 + gm_2\left(gm_1^2+\omega^2(C_{gs1}+C_{gs2})^2\right)}}$$

- To minimize the noise, you want to maximize R_N.
- Maximizing R_N entails maximizing gm_1 and minimizing gm_2.
- So you want to minimize M, since $M \propto \frac{gm_2}{gm_1}$

c) To minimize the overall output noise contribution of the current mirror, attach a large capacitor (C_x) between the gate voltage and ground $\Rightarrow$ 

$$R_{N\,resulting} = \frac{1}{\gamma}\,\frac{gm_1^2+\omega^2(C_{gs1}+C_{gs2}+C_x)^2}{gm_1\, gm_2^2 + gm_2\left(gm_1^2+\omega^2(C_{gs1}+C_{gs2}+C_x)^2\right)}$$

Assume $C_x \gg C_{gs1}+C_{gs2} \Longrightarrow R_{N\,resulting} \approx \frac{1}{\gamma}\,\frac{\cancel{\omega^2 C_x^2}}{gm_2\,\cancel{\omega^2 C_x^2}} \approx \frac{1}{\gamma\, gm_2}$

d) The noise of this mirror would be much greater than that of a resistor that provides the same output resistance as the mirror.

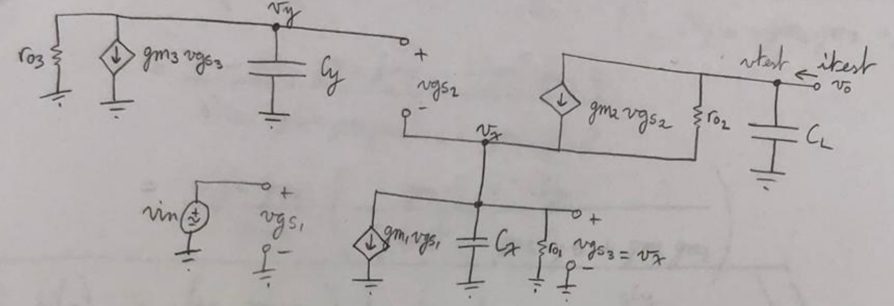
$$\Rightarrow \overline{i_{n,mirror}^2} = \frac{4kT}{R_N}$$

$$\overline{i_{n,R_o}^2} = \frac{4kT}{r_o}$$

$\underline{\underline{R_N \ll r_o, \text{ so } \overline{i_{n,R_o}^2} \ll \overline{i_{n,mirror}^2}}}$

However, replacing the mirror with such a resistor would completely annihilate the headroom at the output of the circuit, since the value of r_o is usually on the order of hundreds of $k\Omega$. So the voltage drop accross r_o would probably be the same as the supplied voltage.

lem 6:



a) $v_x = \left(-g_{m1} v_{in} + g_{m2}(v_y - v_x)\right)\frac{1}{sC_x}$ $\quad v_o = -g_{m2}(v_y - v_x)\frac{1}{sC_L}$

$v_x = \left(-g_{m1} v_{in} + g_{m2}\left(-\frac{g_{m3} v_x}{sC_y} - v_x\right)\right)\frac{1}{sC_x}$

$\frac{v_x}{v_{in}} = \frac{-g_{m1}\, sC_y}{sC_y sC_x + g_{m2}(g_{m3} + sC_y)}$ $\quad v_o = -g_{m2}\left(-\frac{g_{m3} v_x}{sC_y} - v_x\right)\frac{1}{sC_L}$

$v_y = -g_{m3} v_x \frac{1}{sC_y}$

$v_o = \frac{-g_{m2}}{sC_L} v_x \left(-\frac{g_{m3}}{sC_y} - 1\right)$

$\frac{v_y}{v_x} = \frac{-g_{m3}}{sC_y}$

$$H(s) = \frac{v_o}{v_{in}} = \frac{v_o}{v_x}\frac{v_x}{v_{in}} = \frac{g_{m2}}{sC_L}\left(\frac{g_{m3}}{sC_y} + 1\right)\frac{-g_{m1}\, sC_y}{s^2 C_y C_x + g_{m2}(g_{m3} + sC_y)}$$

$$= \left(\frac{g_{m2} g_{m3}}{C_L C_y s^2} + \frac{g_{m2}}{sC_L}\right)\frac{-g_{m1}\, sC_y}{s^2 C_y C_x + g_{m2} g_{m3} + g_{m2} sC_y}$$

$$= \frac{g_{m2}}{sC_L}\left(\frac{g_{m3}}{sC_y} + 1\right)\frac{-g_{m1}\, sC_y}{s^2 C_y C_x + g_{m2} g_{m3} + g_{m2} sC_y}$$

$$= -\frac{g_{m1} g_{m2} C_y}{C_L}\left(\frac{g_{m3}}{sC_y} + 1\right)\frac{1}{s^2 C_y C_x + g_{m2} g_{m3} + g_{m2} sC_y}$$

$$H(s) = \left(-\frac{g_{m1} g_{m2} g_{m3}}{sC_L} - \frac{s g_{m1} g_{m2} C_y}{sC_L}\right)\left(\frac{1}{s^2 C_y C_x + g_{m2} g_{m3} + g_{m2} s C_y}\right)$$

$$= \frac{-g_{m1} g_{m2} g_{m3} - s g_{m1} g_{m2} C_y}{s^3 C_L C_y C_x + g_{m2} g_{m3} C_L s + g_{m2} C_y C_L s^2}$$

$$= \frac{-g_{m1} g_{m2}}{sC_L}\left(\frac{g_{m3} + sC_y}{C_y C_x s^2 + g_{m2} C_y s + g_{m2} g_{m3}}\right)$$

$$\boxed{H(s) = \underbrace{-\left(g_{m1} g_{m2} g_{m3}\right)}_{\text{DC gain}} \underbrace{\frac{1}{sC_L}\left(\frac{1 + \frac{sC_y}{g_{m3}}}{C_y C_x s^2 + g_{m2} C_y s + g_{m2} g_{m3}}\right)}_{\text{Freq. Dependence}}}$$

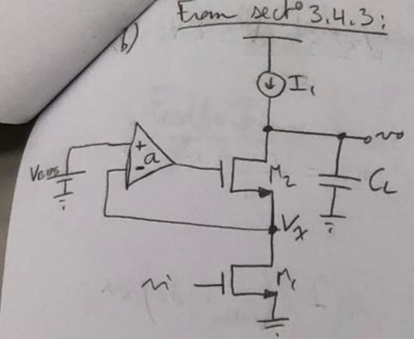
$$H(s) = \underbrace{-g_{m1}\frac{1}{sC_L}}_{\text{Dominant pole}}\left(\frac{1 + \frac{sC_y}{g_{m3}}}{\frac{C_y C_x}{g_{m2} g_{m3}} s^2 + \frac{C_y}{g_{m3}} s + 1}\right)$$

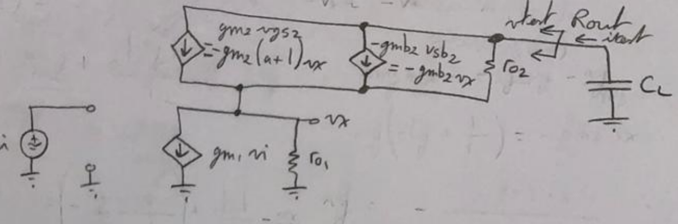
Dominant pole $\frac{-g_{m1}}{sC_L} = 1$

Assume the 2 other poles and the zero appear far after the dominant pole.

$$\boxed{\omega_u = \frac{g_{m1}}{C_L}}$$

b) From sect° 3.4.3:





$$V_{gs2} = V_{g2} - V_{s2} = -a v_x - v_x = -v_x(1+a)$$

$$i_{test} = \frac{v_{test} - v_x}{r_{o2}} - g_{m2}(a+1)v_x - g_{mb2} v_x$$

$$i_{test} = \frac{v_x}{r_{o1}}$$

$$\frac{v_x}{r_{o1}} = \frac{v_{test}}{r_{o2}} - \frac{v_x}{r_{o2}} - g_{m2}(a+1)v_x - g_{mb2} v_x$$

$$v_x\left(\frac{1}{r_{o1}} + \frac{1}{r_{o2}} + g_{mb2} + g_{m2}(a+1)\right) = \frac{v_{test}}{r_{o2}}$$

$$v_x = \frac{1}{r_{o2}\left(\frac{1}{r_{o1}} + \frac{1}{r_{o2}} + g_{mb2} + g_{m2}(a+1)\right)} v_{test}$$

$$i_{test} = \frac{v_x}{r_{o1}} = \frac{1}{r_{o1} r_{o2}\left(\frac{1}{r_{o1}} + \frac{1}{r_{o2}} + g_{mb2} + g_{m2}(a+1)\right)} v_{test}$$

$$\frac{v_{test}}{i_{test}} = R_{out} = r_{o2} + r_{o1} + r_{o1} r_{o2}\left(g_{mb2} + g_{m2}(a+1)\right)$$

$$\boxed{\begin{gathered} R_{out} \approx r_{o1} r_{o2}\left(g_{mb2} + g_{m2}(a+1)\right) \\ Z_{out} = R_{out} \,//\, \frac{1}{sC_L} \end{gathered}}$$

c) Unity gain frequency of the feedback loop is $\omega_u = \frac{g_{m3}}{C_y}$

<u>Feedback circuit</u> $\Rightarrow$ $\frac{v_y}{v_x} = -\frac{g_{m3}}{1+g_{m3}(0)} \frac{1}{sC_y}$

$$\frac{v_y}{v_x} = -\frac{g_{m3}}{sC_y}$$

$$\frac{v_x}{v_y} = \frac{g_{m2}\frac{1}{sC_x}}{1+g_{m2}\frac{1}{sC_x}} = \frac{1}{1+\frac{sC_x}{g_{m2}}}$$

Unity gain frequency $= \frac{g_{m3}}{C_y}$

From $\frac{v_x}{v_y}$ expression, second pole activates at $\frac{g_{m2}}{C_x}$.

So to have 45° phase margin, 2^{nd} pole must kick in after ω_u.

$$\boxed{\frac{g_{m2}}{C_x} \geqslant \frac{g_{m3}}{C_y}}$$