Problem Set 1

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Problem 1-1.

Give an example where a combination of three nonzero vectors in $\mathbf{R}^4$ is the zero vector. Then write your example in the form $Ax = 0$. What are the shapes of A and x and 0?

Solution:

$$A = \begin{bmatrix} 2 & 1 & 3 & 6 \\ 4 & 3 & 1 & 8 \\ 3 & 4 & 4 & 11 \\ 7 & 5 & 2 & 14 \end{bmatrix}, x = \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{bmatrix}$$

The above example satisfies $Ax = 0$, where A has shape 4×4 and x has shape 4×1 and 0 has shape 4×1.

Problem 1-2.

Suppose A is the 3 by 3 matrix **ones**(3,3) of all ones. Find two independent vectors x and y that solve $Ax = 0$ and $Ay = 0$. Write that first equation $Ax = 0$ (with numbers)as a combination of the columns of A. Why don't I ask for a third independent vector with $Az = 0$?

Solution:

Let's say $x = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, y = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$, then $Ax = 1 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + 0 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - 1 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

Because the solution space is $\mathbf{R}^2$, so z is in the plane spanned by x and y, which makes it a **dependent** vector.

Problem 1-3.

Suppose the column space of an m by n matrix is all of $\mathbf{R}^3$. What can you say about m? What can you say about n? what can you say about the rank r?

Solution:

If the column space is the whole $\mathbf{R}^3$, then the dimension of the column space or the rank r is 3, i.e., $r = 3$. To span a $\mathbf{R^3}$ space, $n \geq 3$, which means there are at least three vectors left for selection for three independent vectors, and $m = 3$ so that the spanned space is the whole $\mathbf{R}^3$.

Problem 1-4.

If $A = CR$, what are the CR factors of the matrix $\begin{bmatrix} 0 & A \\ 0 & A \end{bmatrix}$?

Solution: The idea is to think of matrix multiplication as the combination of columns!

$$\begin{bmatrix} 0 & A \\ 0 & A \end{bmatrix} = \begin{bmatrix} C \\ C \end{bmatrix} \begin{bmatrix} 0 & R \end{bmatrix}$$