Problem Set 2

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Problem 2-1.

Suppose a and b are column vectors with components $a_1, \cdots, a_m$ and $b_1, \cdots, b_p$. Can you multiply a times b^T (yes or no)? What is the shape of the answer ab^T? What number is in row i, column j of ab^T? What can you say about aa^T?

Solution:

Yes, an m by 1 matrix multiplied with a 1 by p matrix leads to a m by p rank-1 matrix.

The entry in row i, column j of ab^T is $a_i \times b_j$.

aa^T is not only a rank-1 matrix, but also symmetric.

Problem 2-2.

If A has columns $\mathbf{a_1}, \mathbf{a_2}, \mathbf{a_3}$ and $B = I$ is the identity matrix, what are the rank one matrices $\mathbf{a_1 b_1^T}, \mathbf{a_2 b_2^T}, \mathbf{a_3 b_3^T}$? They should add to $AI = A$.

Solution:

$$\mathbf{a_1 b_1^T} = \begin{bmatrix} \vdots & \vdots & \vdots \\ \mathbf{a_1} & 0 & 0 \\ \vdots & \vdots & \vdots \end{bmatrix}, \mathbf{a_2 b_2^T} = \begin{bmatrix} \vdots & \vdots & \vdots \\ 0 & \mathbf{a_2} & 0 \\ \vdots & \vdots & \vdots \end{bmatrix}, \mathbf{a_3 b_3^T} = \begin{bmatrix} \vdots & \vdots & \vdots \\ 0 & 0 & \mathbf{a_3} \\ \vdots & \vdots & \vdots \end{bmatrix},$$

all three matrices adds up to A.