Matrix Methods In Data Analysis, Signal Processing, And Machine Learning: 18.065
Massachusetts Institute of Technology
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Problem Set 4

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Problem 4-1.

Compute the eigenvalues and eigenvectors of A and A^{-1}. Check the trace!

$$A = \begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix} \quad \text{and} \quad A^{-1} = \begin{bmatrix} -1/2 & 1 \\ 1/2 & 0 \end{bmatrix}.$$

A^{-1} has the <u>same</u> eigenvectors as A. when A has eigenvalues λ_1 and λ_2, its inverse has eigenvalues $\underline{1/\lambda_1 \text{ and } 1/\lambda_2}$.

Problem 4-2.

The eigenvalues of A ***equal the eigenvalues of*** A^T. This is because det$(A - \lambda I)$ equals det$(A^T - \lambda I$. That is true because **you can pick a row in** $A - \lambda I$ **to compute the determinant and a column in** $A^T - \lambda I$**, which is the same line**. Show by an example that the eigenvectors of A and A^T are *not* the same.

Solution:

A simple 2-by-2 matrix is able to show that, we can use the A in the first problem, the eigenvalues and eigenvectors are shown by coding below:

```
Eigenvalues for B: [-1.  2.]
Eigenvectors for B
 [[-0.89442719 -0.70710678]
 [ 0.4472136  -0.70710678]]
Eigenvalues for B.T: [-1.  2.]
Eigenvectors for B.T
 [[-0.70710678 -0.4472136 ]
 [ 0.70710678 -0.89442719]]
```

Problem 4-3.

(a)Factor these two matrices into $A = X \Lambda X^{-1}$:

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} \quad \text{and} A = \begin{bmatrix} 1 & 1 \\ 3 & 3 \end{bmatrix}$$

(b)If $A = X\Lambda X^{-1}$ then $A^3 = \underline{(X)(\Lambda^3)(X^{-1})}$ and $A^{-1} = \underline{(X)(\Lambda^{-1})(X^{-1})}$

Solution:

(a)The matrix $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ has eigenvalues 1 and 3, and eigenvectors $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$, which gives the factorized matrices :

$$X = \begin{bmatrix} 1 & 1/\sqrt{2} \\ 0 & 1/\sqrt{2} \end{bmatrix}, \quad \Lambda = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}.$$

The matrix $A = \begin{bmatrix} 1 & 1 \\ 3 & 3 \end{bmatrix}$ has eigenvalues 0 and 4, and eigenvectors $\begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$ and $\begin{bmatrix} -1/\sqrt{10} \\ -3/\sqrt{10} \end{bmatrix}$, which gives the factorized matrices :

$$X = \begin{bmatrix} -1/\sqrt{2} & -1/\sqrt{10} \\ 1/\sqrt{2} & -3/\sqrt{10} \end{bmatrix}, \quad \Lambda = \begin{bmatrix} 0 & 0 \\ 0 & 4 \end{bmatrix}.$$