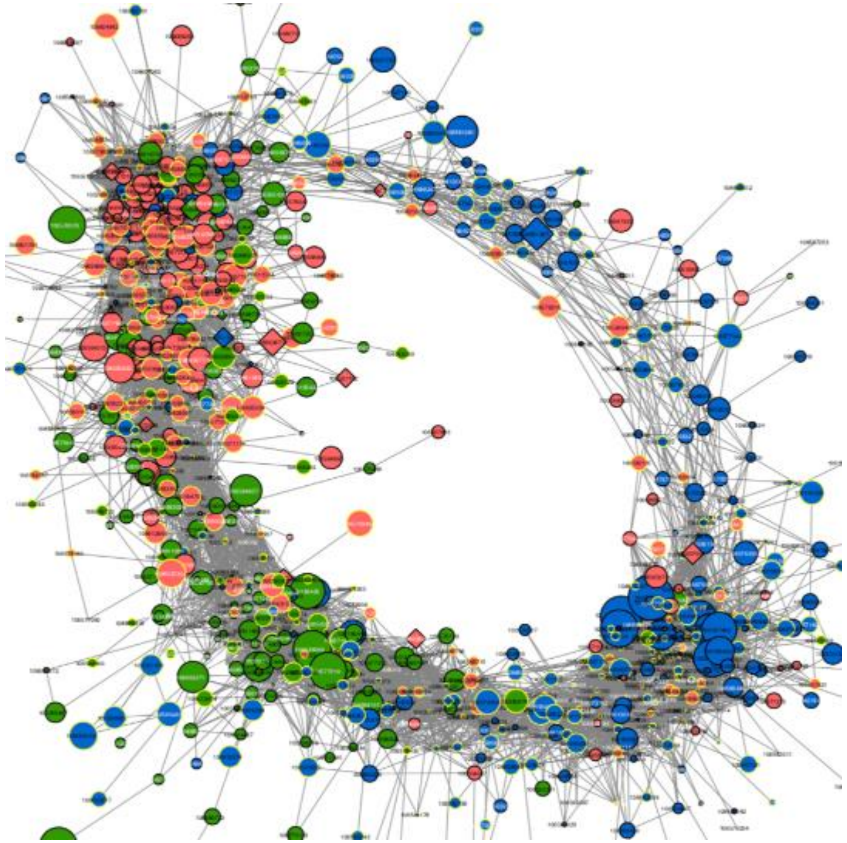
Path analysis



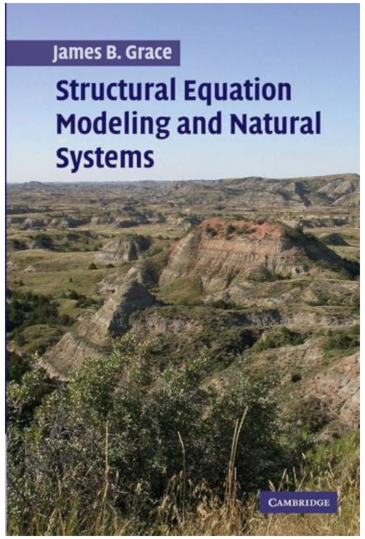
Some fun reading!

Sarah Cubaynes

Kate Layton-Matthews

Jon Lefcheck

`https://jslefche.github.io/sem_book/`



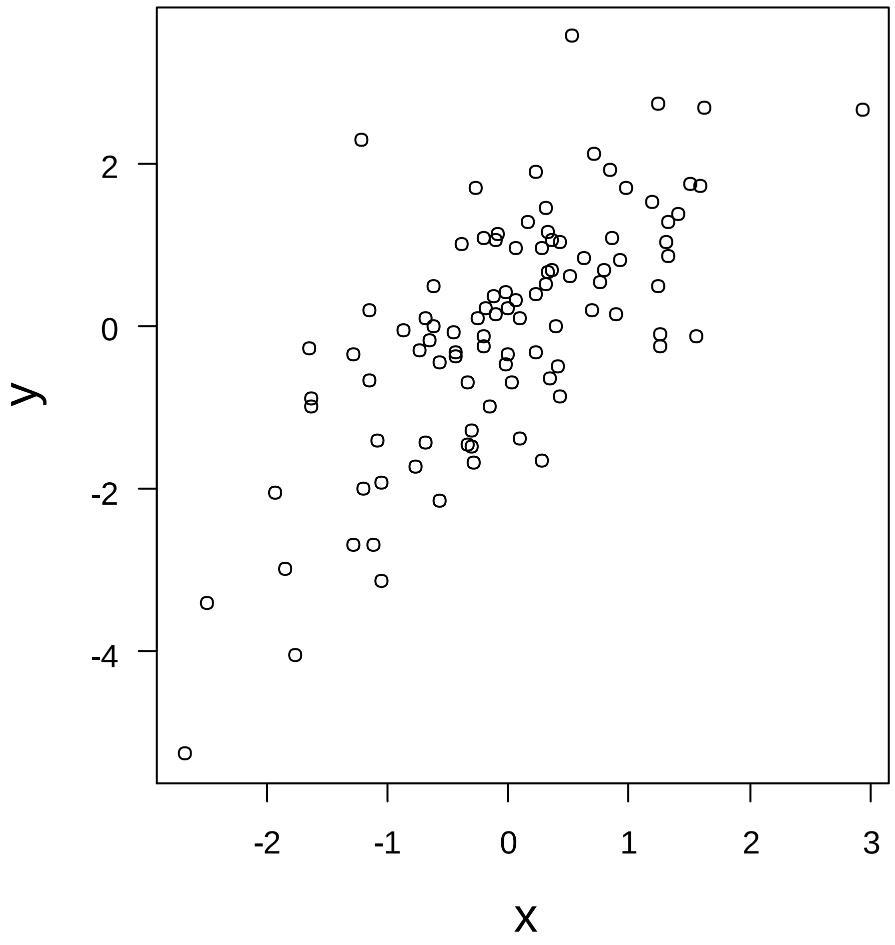
James Grace

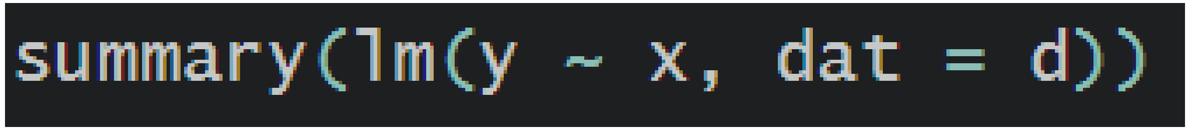
Grace **(2006)** *Cambridge University Press*

Cubaynes et al. **(2014)** *Journal of Animal Ecology*

Layton-Matthews et al. (**2019**) *Journal of Animal Ecology*

Simulating data (n = 100)



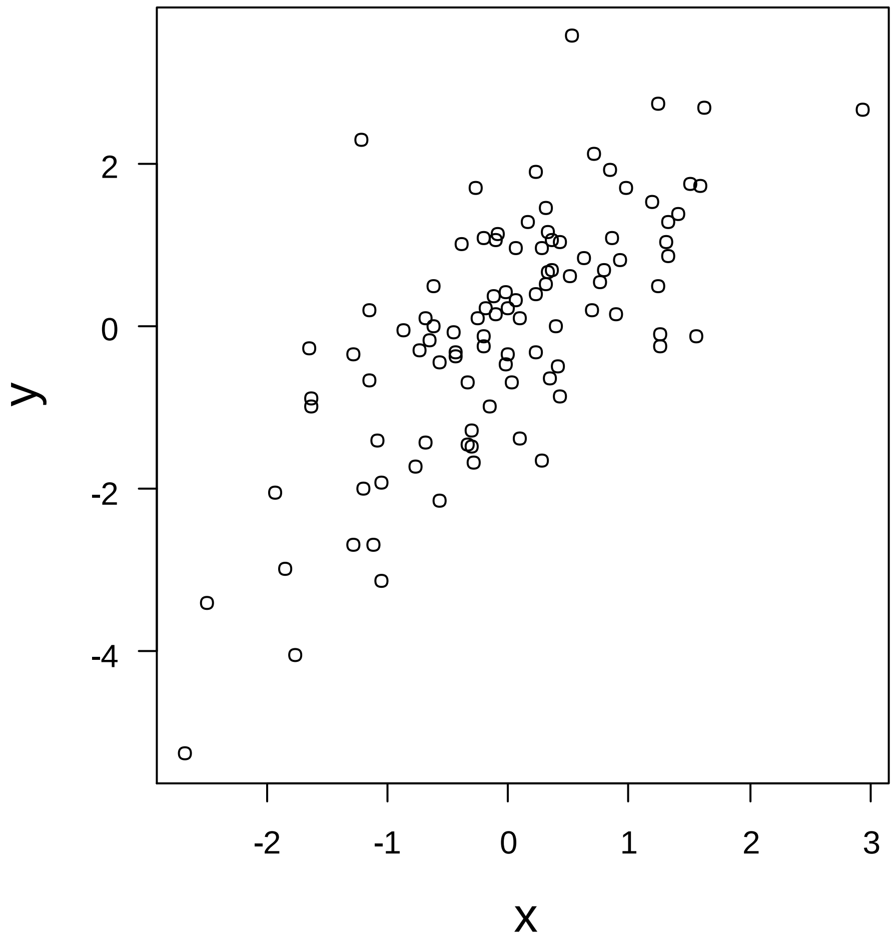


```
summary(lm(y ~ x, dat = d))
```

```
x ~ normal(0,1)
y ~ normal(x, 1)
```

Grace **(2006)**

Variance-covariance matrix



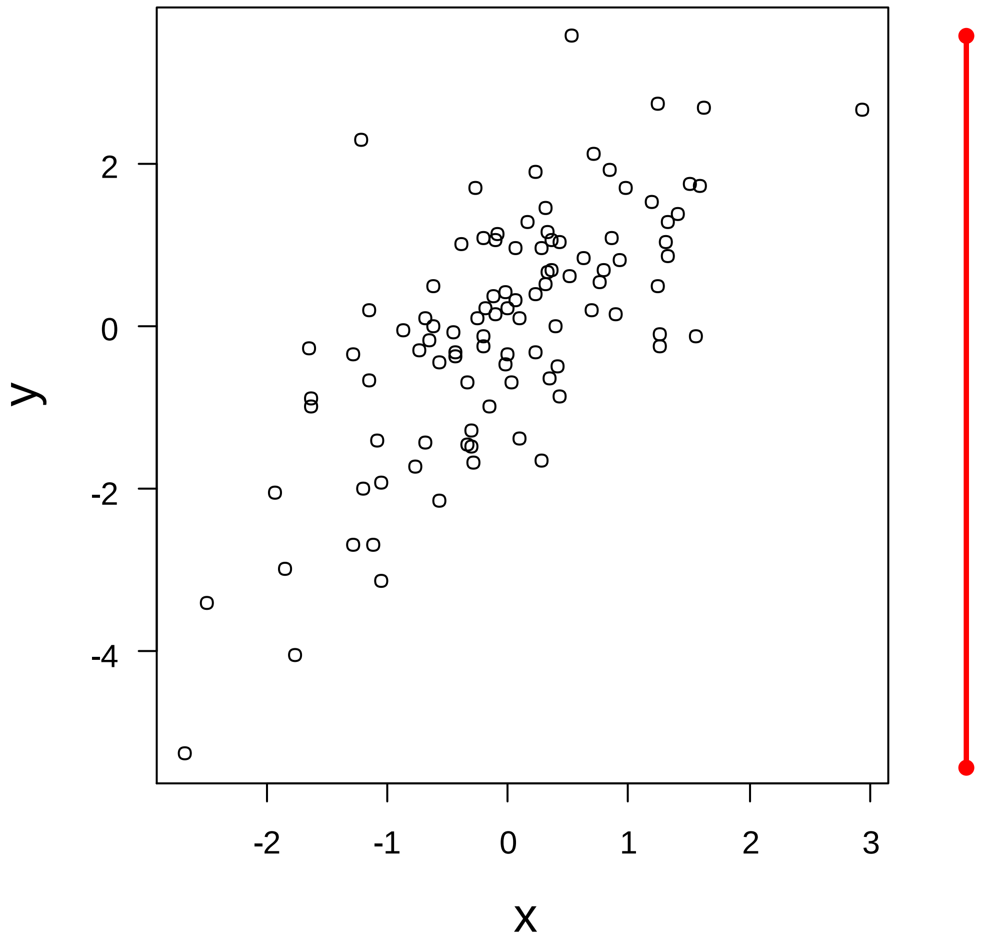
$$\Sigma_{yx} = \begin{bmatrix} \sigma_y^2 & \sigma_{yx} \\ \sigma_{yx} & \sigma_x^2 \end{bmatrix}$$

$$\Sigma_{yx} = \begin{bmatrix} 2.247 & 1.035 \\ 1.035 & 0.946 \end{bmatrix}$$

Grace **(2006)**

Variance of y

`var(x)`



$$\Sigma_{yx} = \begin{bmatrix} \boldsymbol{\sigma_y^2} & \sigma_{yx} \\ \sigma_{yx} & \sigma_x^2 \end{bmatrix}$$

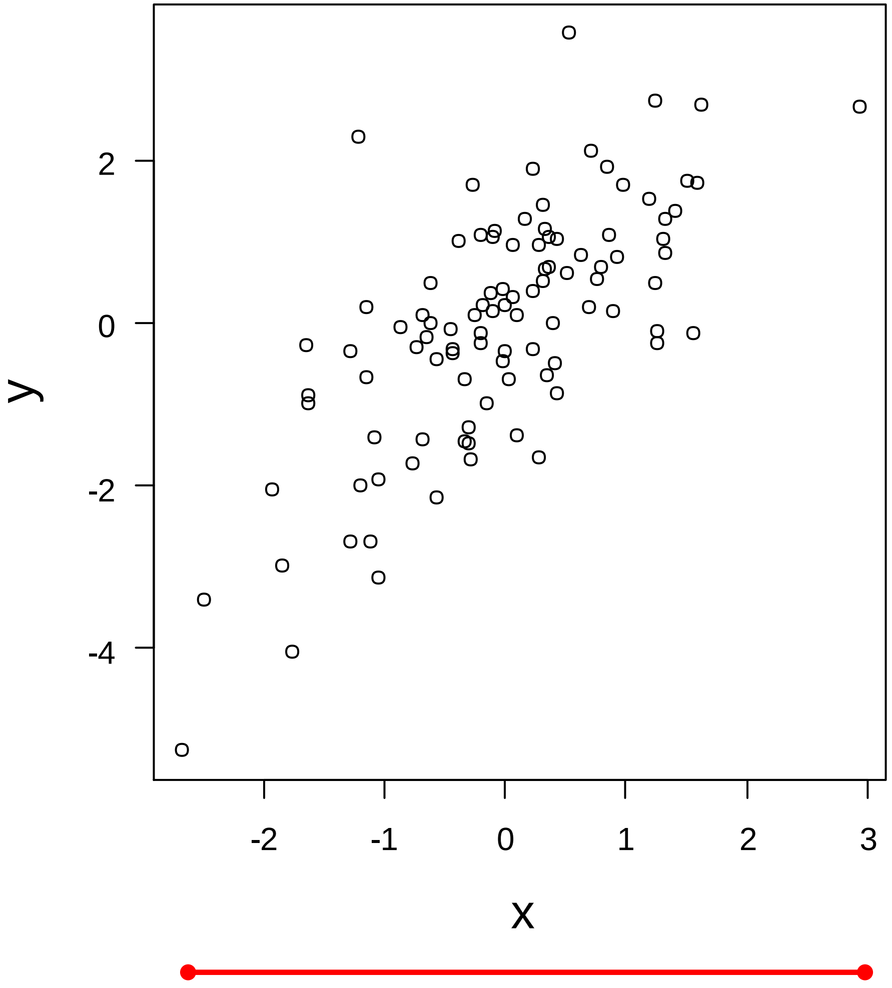
Variance of y

$$\sigma_y^2 = \frac{\sum (y_i - \bar{y})^2}{n-1}$$

Grace **(2006)**, eqs. 3.1 & 3.2

Variance of x

`var(x)`



$$\Sigma_{yx} = \begin{bmatrix} \sigma_y^2 & \sigma_{yx} \\ \sigma_{yx} & \boldsymbol{\sigma_x^2} \end{bmatrix}$$

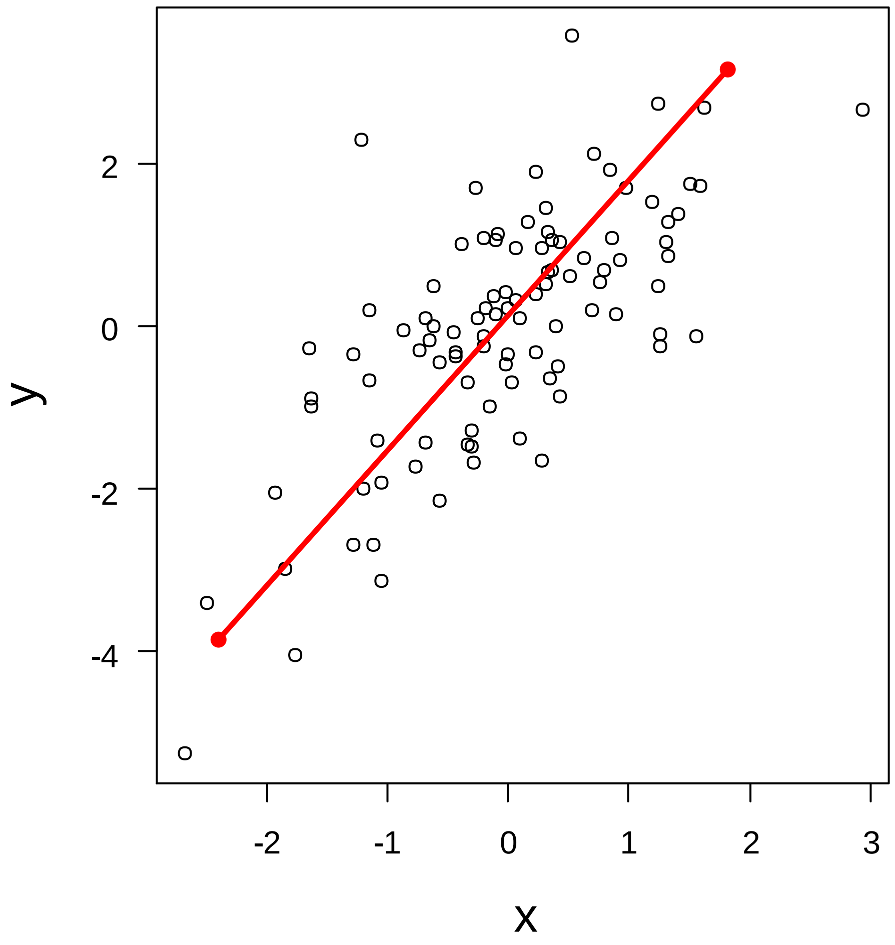
Variance of x

$$\sigma_x^2 = \frac{\sum(x_i - \bar{x})^2}{n-1}$$

Grace **(2006)**, eqs. 3.1 & 3.2

Covariance

`cov(x,y)`



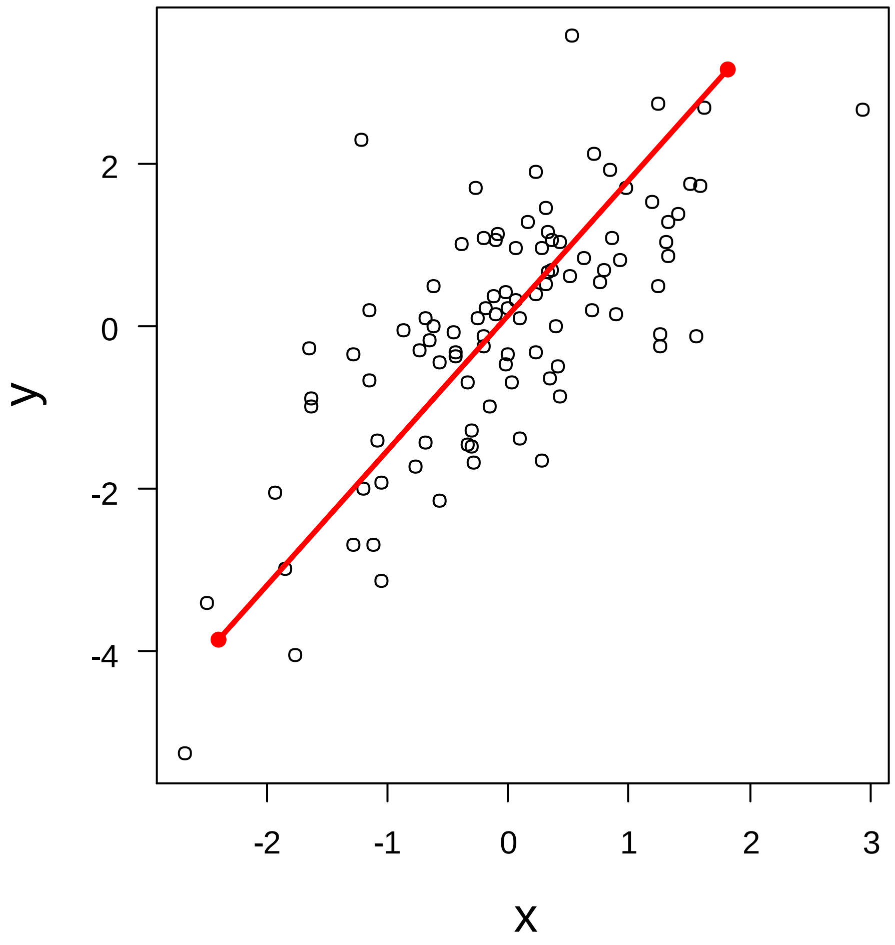
$$\Sigma_{yx} = \begin{bmatrix} \sigma_y^2 & \boldsymbol{\sigma_{yx}} \\ \boldsymbol{\sigma_{yx}} & \sigma_x^2 \end{bmatrix}$$

Covariance

$$\sigma_{xy} = \frac{\sum (y_i - \bar{y})(x_i - \bar{x})}{n-1}$$

Correlation

`cor(x,y)`



Covariance

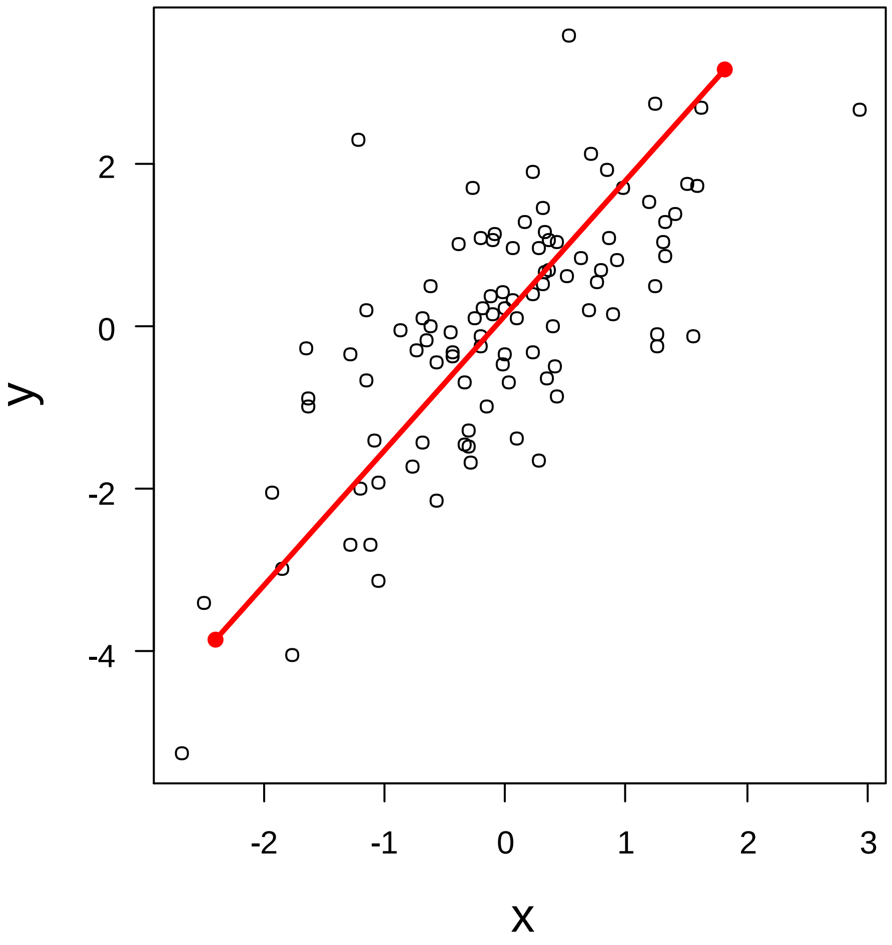
$$\sigma_x \sigma_y \rho = \sigma_{xy} = \frac{\sum (y_i - \bar{y})(x_i - \bar{x})}{n-1}$$

Correlation

$$r = \rho = \frac{\sigma_x \sigma_y \rho}{\sigma_x \sigma_y}$$

Slope

```
lm(y ~ x)
```



Covariance

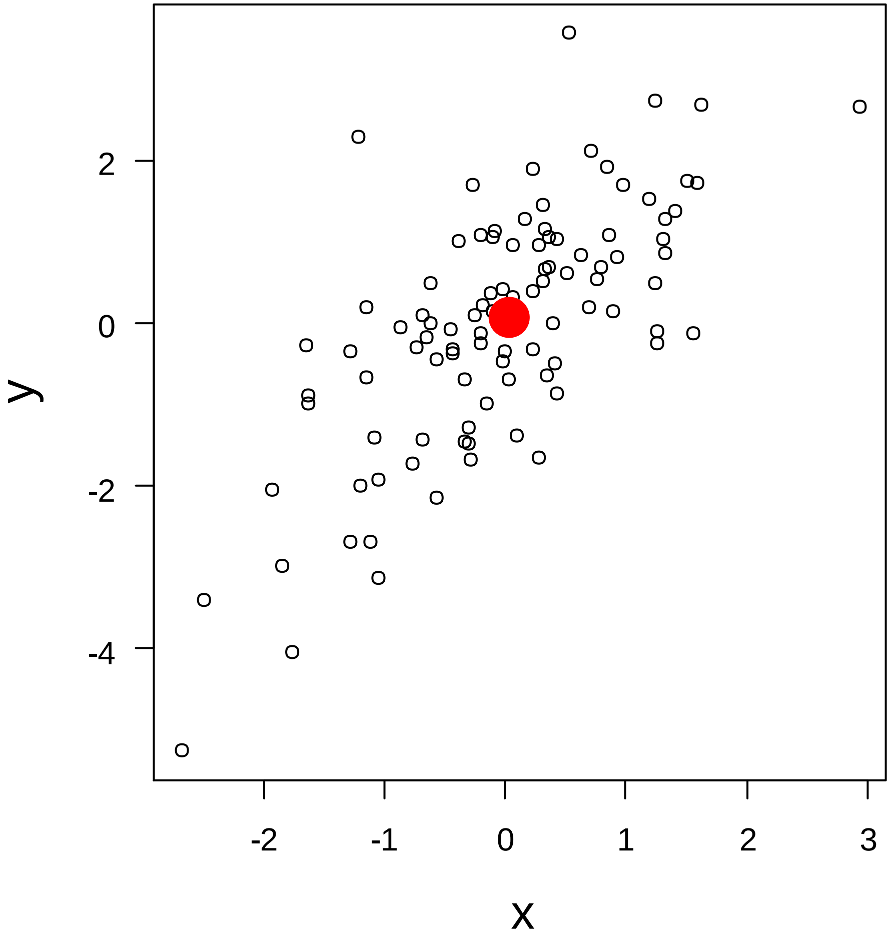
$$\sigma_x \sigma_y \rho = \sigma_{xy} = \frac{\sum (y_i - \bar{y})(x_i - \bar{x})}{n-1}$$

Slope

$$\beta_1 = \frac{\sigma_x \sigma_y \rho}{\sigma_x^2}$$

Intercept

`lm(y ~ x)`



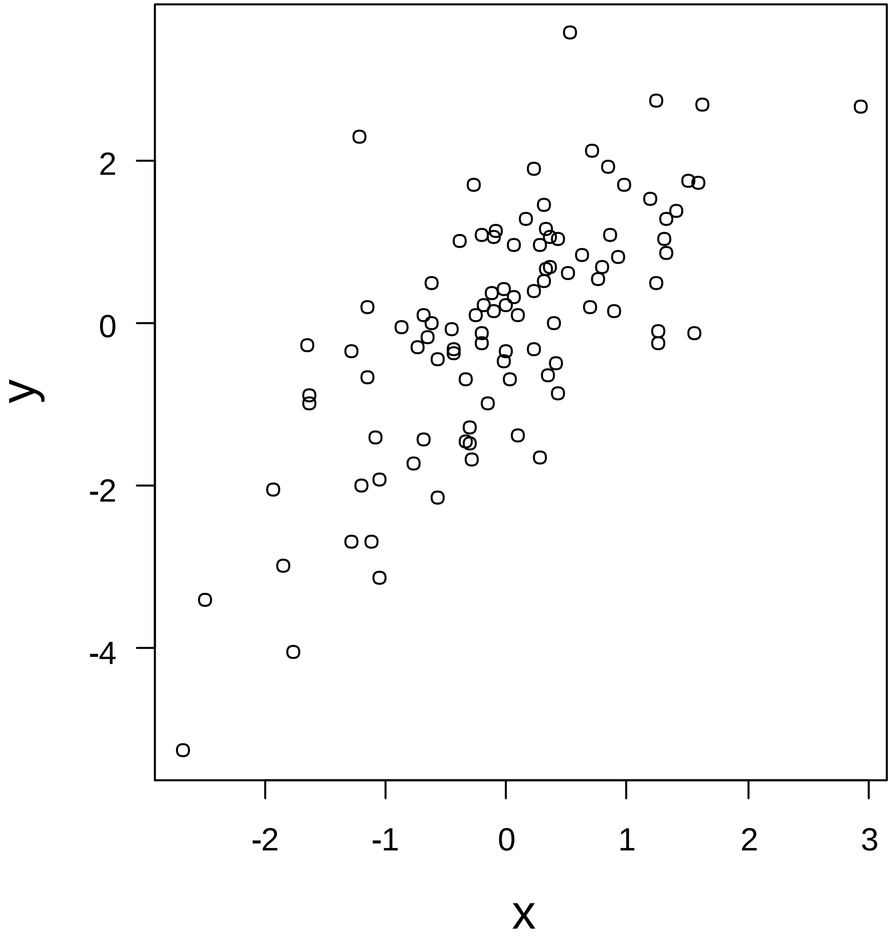
Slope

$$\beta_1 = \frac{\sigma_x \sigma_y \rho}{\sigma_x^2}$$

Intercept

$$\beta_0 = \bar{y} - \beta_1 \times \bar{x}$$

An easy-peasy linear model



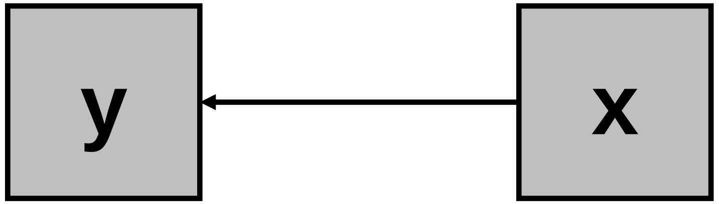
R code

```
lm(y ~ x)
```

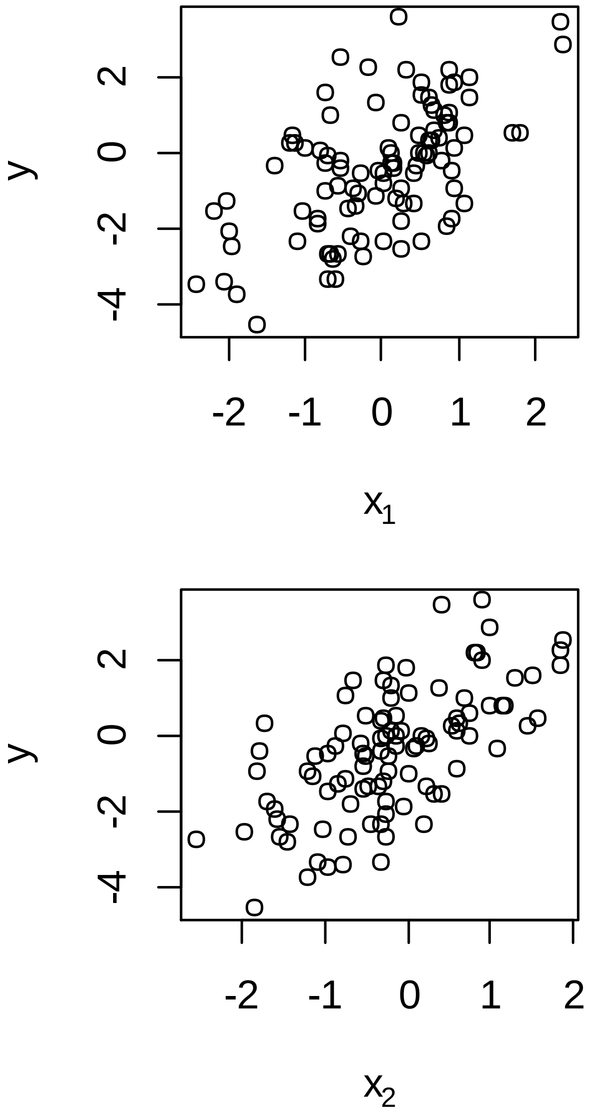
Model

$$y_i \sim \text{normal}(\beta_0 + \beta_1 x_i, \sigma_y^2)$$

Causal diagram



Expanding to two explanatory variables



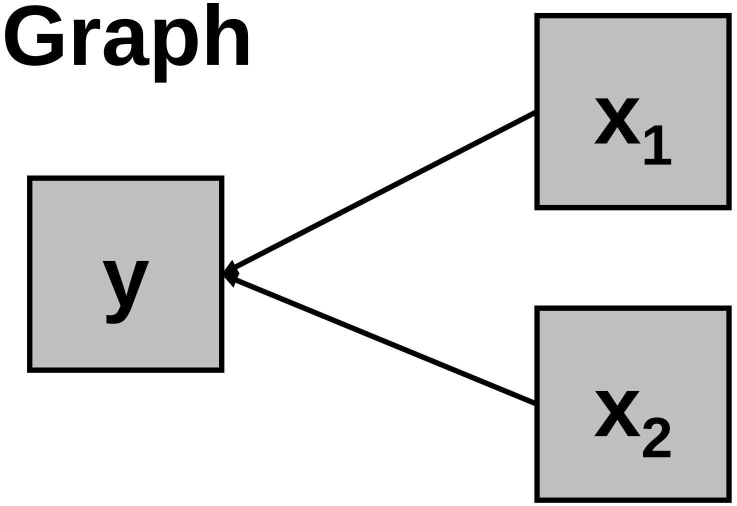
R code

```
lm(y ~ x1 + x2)
```

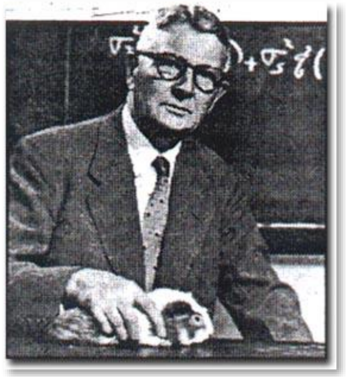
Model

$$y_i \sim \text{normal}(\beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i}, \sigma_y^2)$$

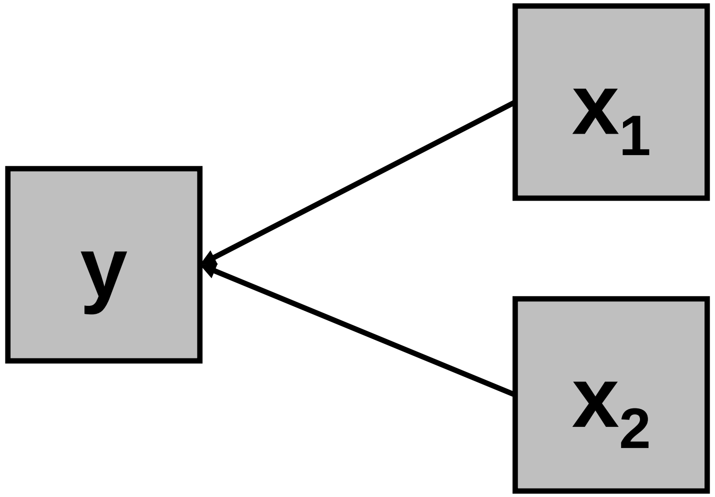
Graph



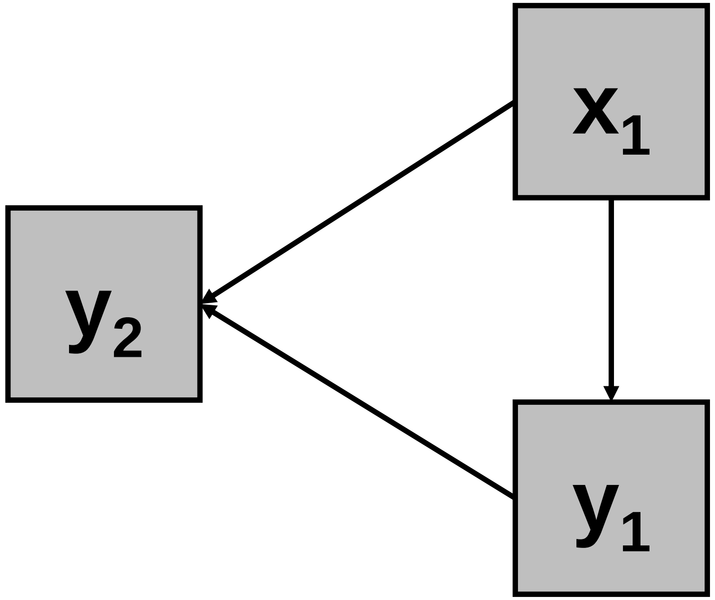
Sewall Wright’s breakthrough



Multiple regression

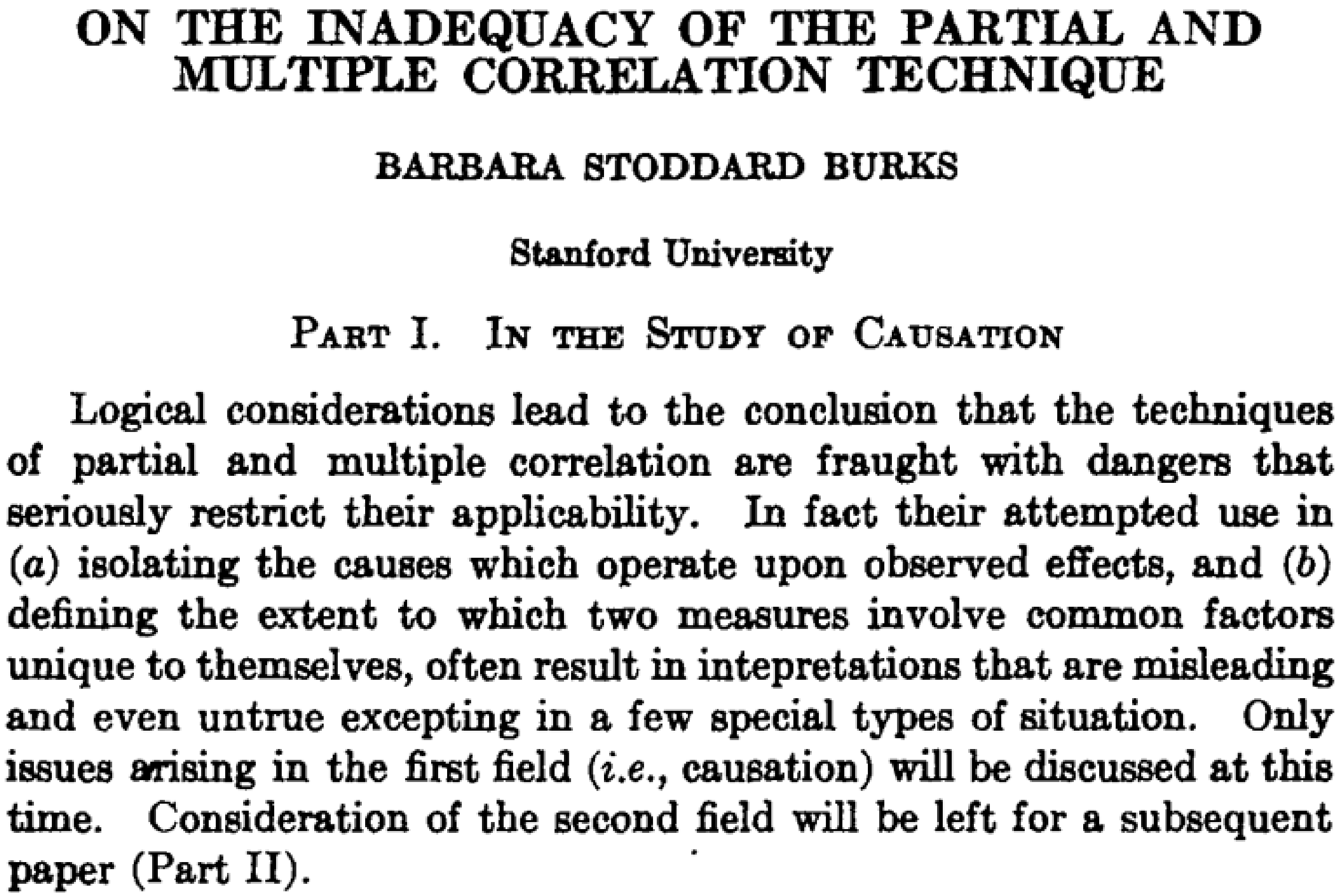


Path analysis



Wright **(1921; 1934)**

Barbara Burks: nature vs. nurture



ON THE INADEQUACY OF THE PARTIAL AND MULTIPLE CORRELATION TECHNIQUE

BARBARA STODDARD BURKS

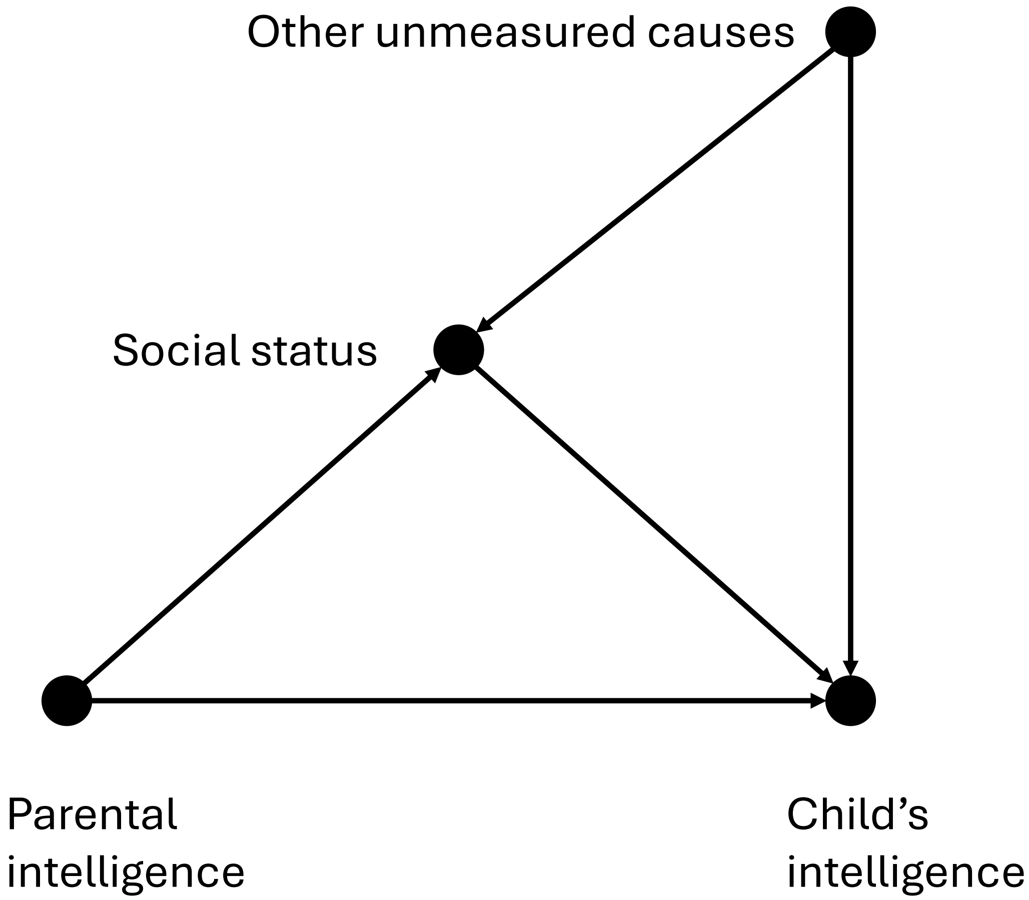
Stanford University

Part I. In the Study of Causation

Logical considerations lead to the conclusion that the techniques of partial and multiple correlation are fraught with dangers that seriously restrict their applicability. In fact their attempted use in (*a*) isolating the causes which operate upon observed effects, and (*b*) defining the extent to which two measures involve common factors unique to themselves, often result in intepretations that are misleading and even untrue excepting in a few special types of situation. Only issues arising in the first field (*i.e.*, causation) will be discussed at this time. Consideration of the second field will be left for a subsequent paper (Part II).

Burks **(1926)**

Barbara Burks: collider bias

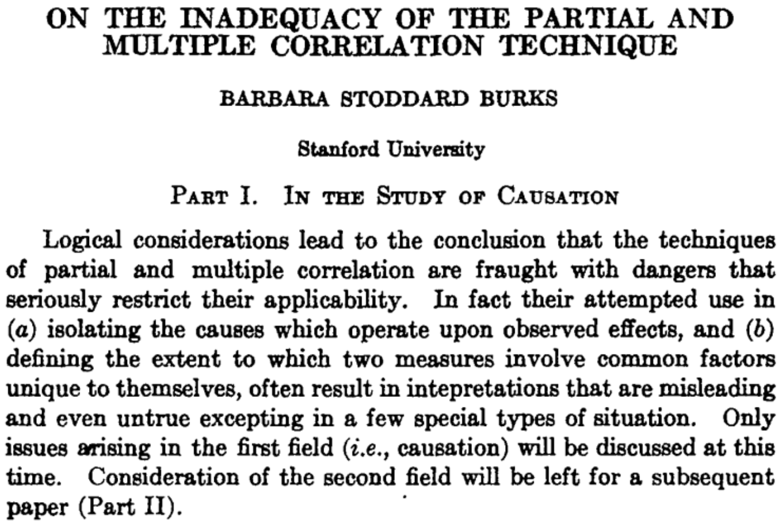


Other unmeasured causes

Social status

Parental intelligence

Child's intelligence



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PART I. IN THE STUDY OF CAUSATION

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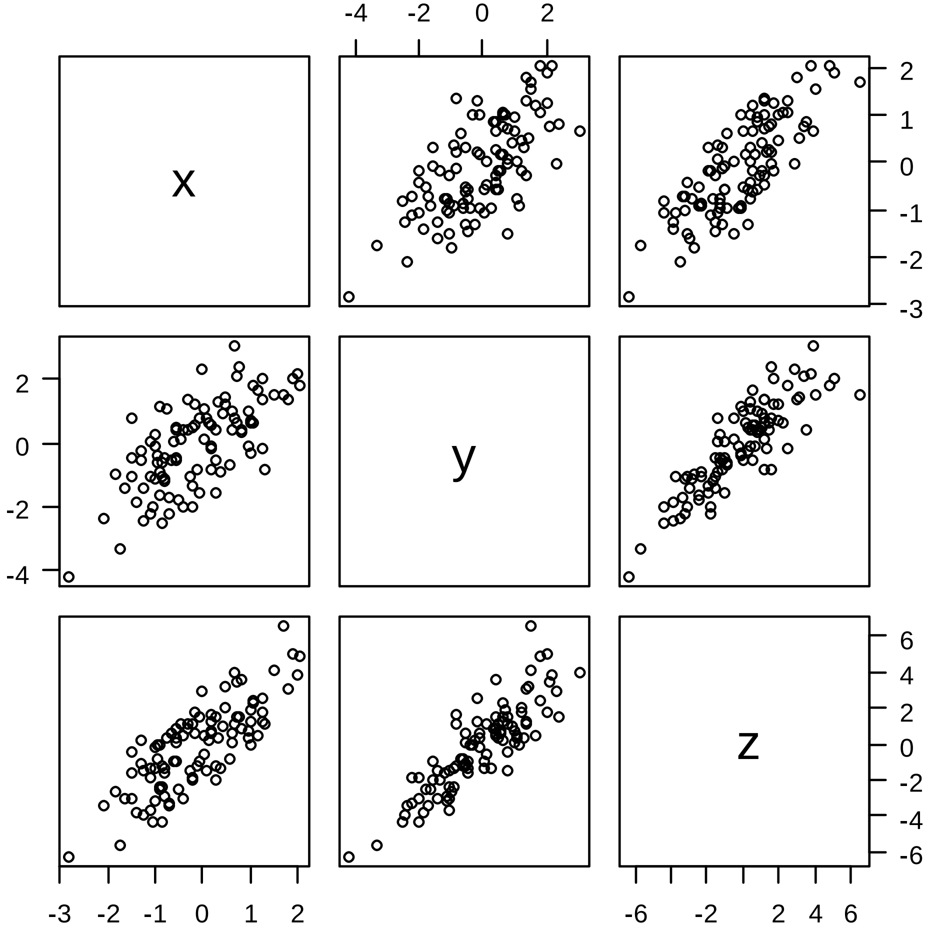
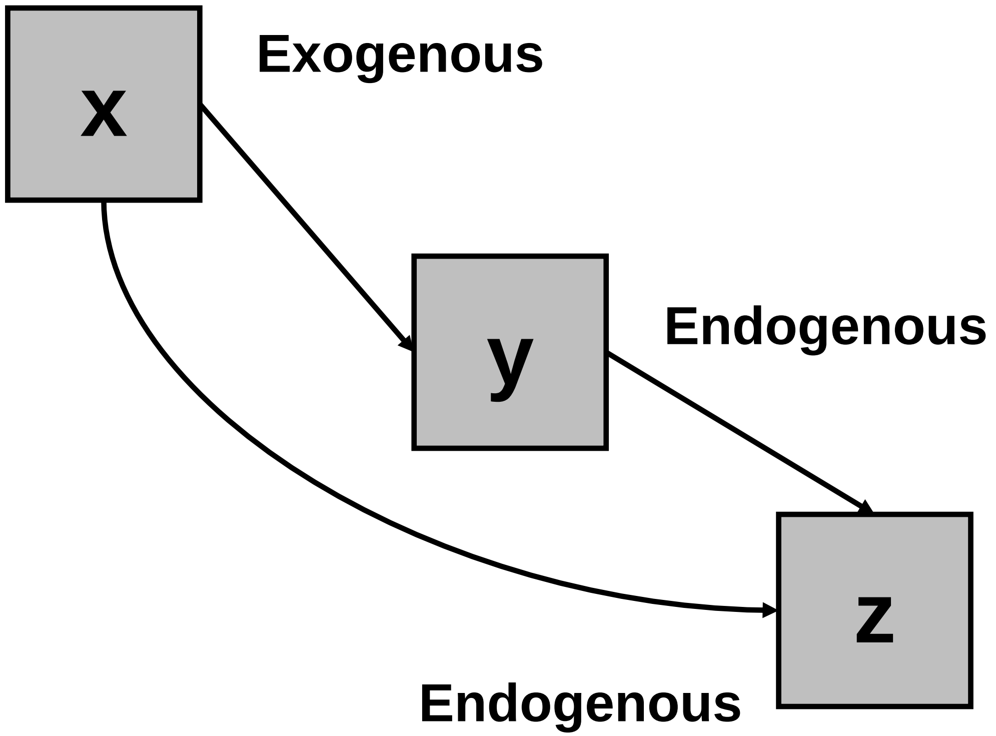
Burks **(1926);** Pearl **(2001)** Figure 9.2

Simulating data (n = 100)

```
x ~ normal(0,1)
y ~ normal(x, 1)
z ~ normal(x + z, 1)
```



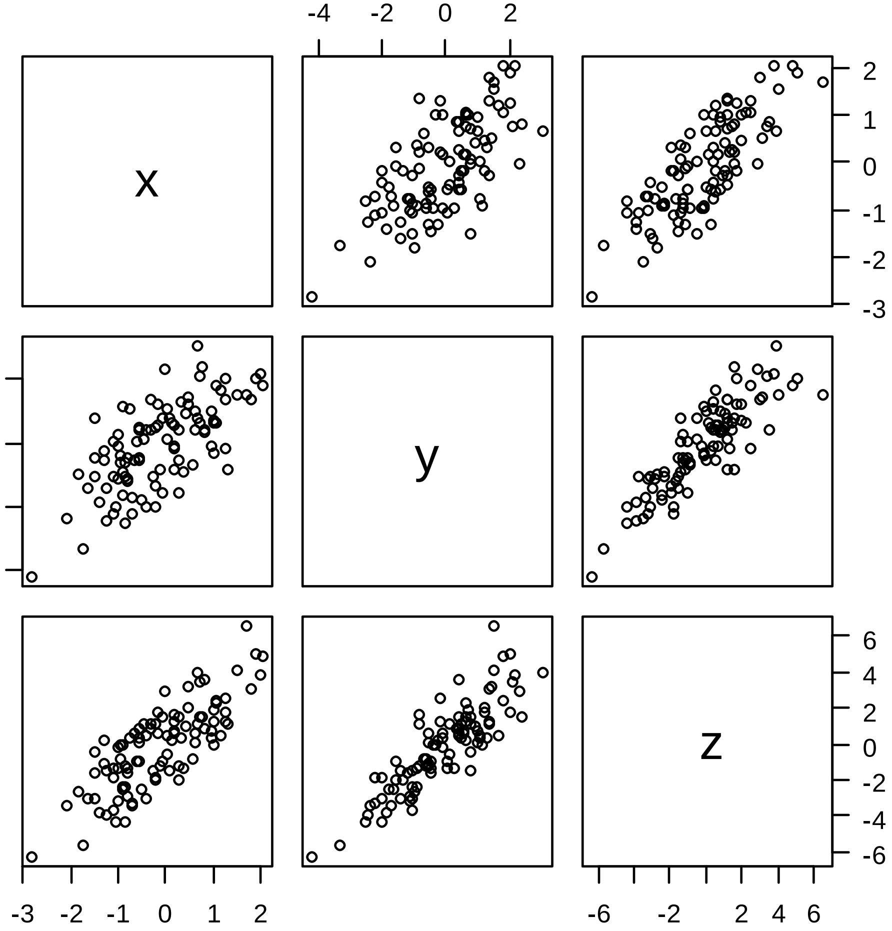


Exogenous

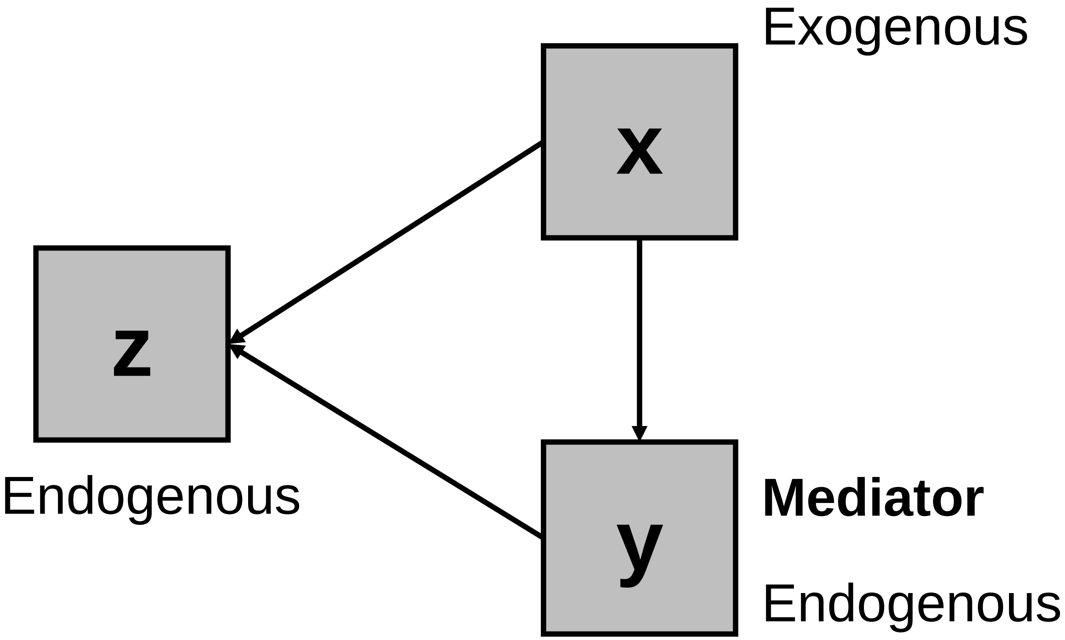
Endogenous

Endogenous

Simulating data (n = 100)

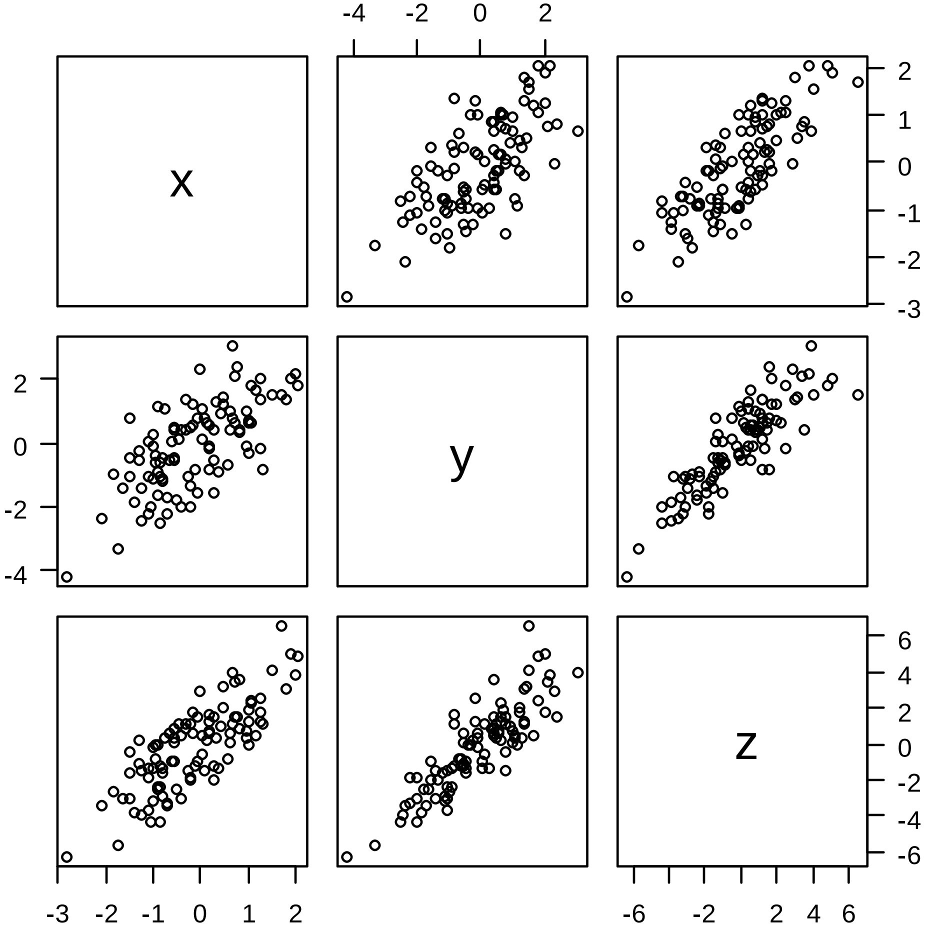


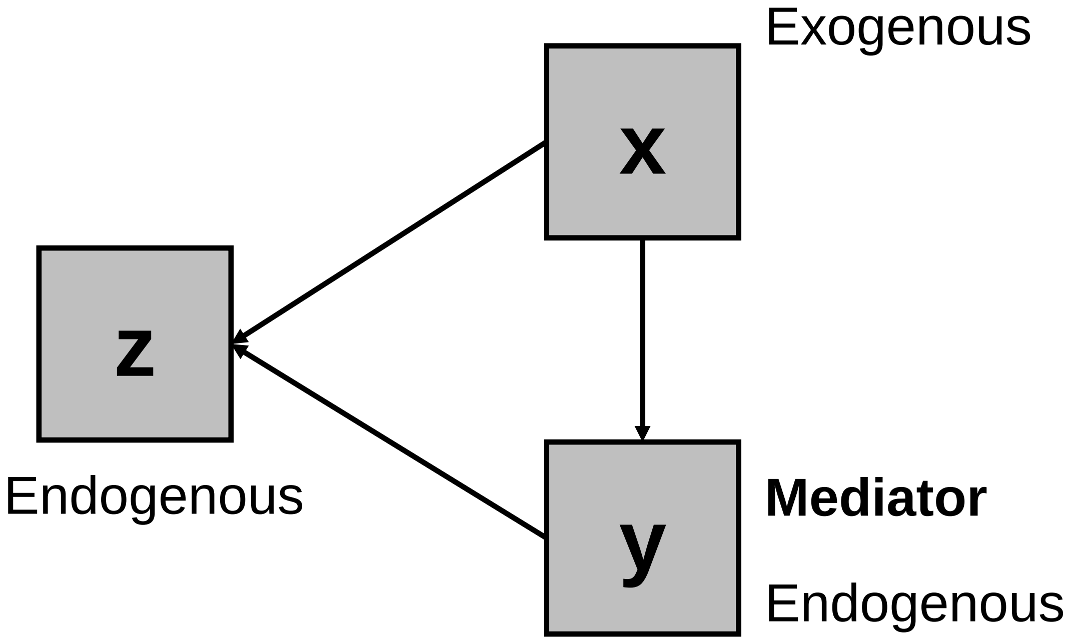
```
x ~ normal(0,1)
y ~ normal(x, 1)
z ~ normal(x + z, 1)
```



Simulating data (n = 100)

```
x ~ normal(0,1)
y ~ normal(x, 1)
z ~ normal(x + z, 1)
```





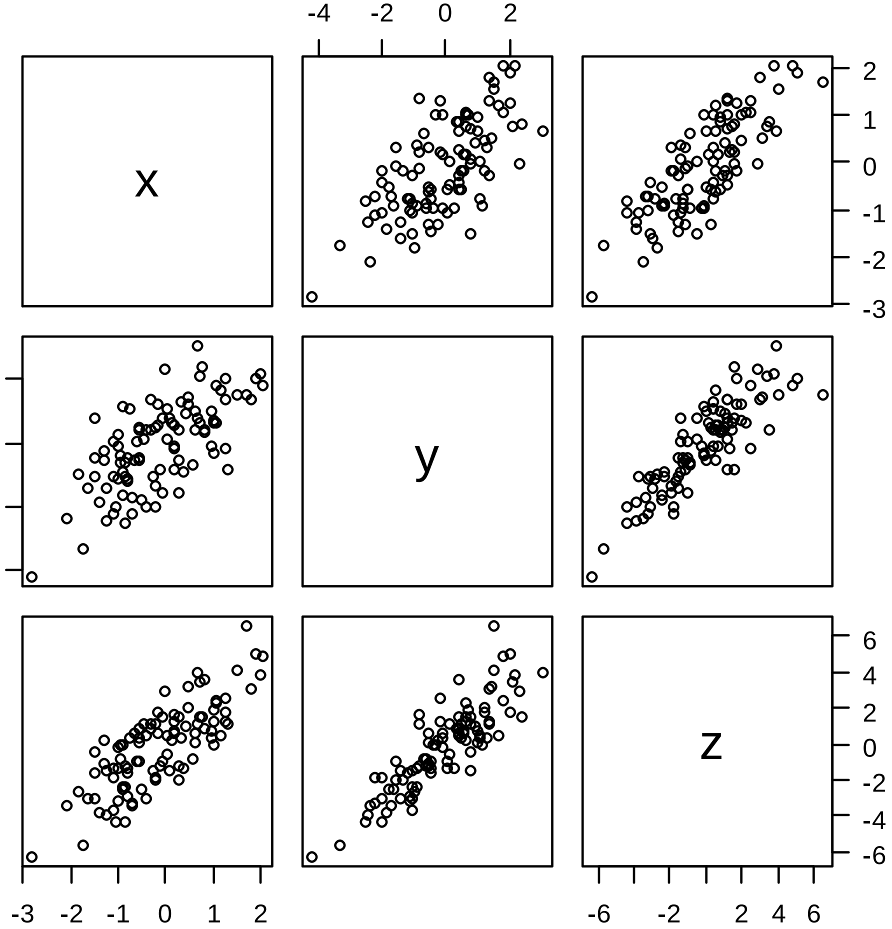
Exogenous

Endogenous

Mediator

Endogenous

Variance-covariance and correlation matrices



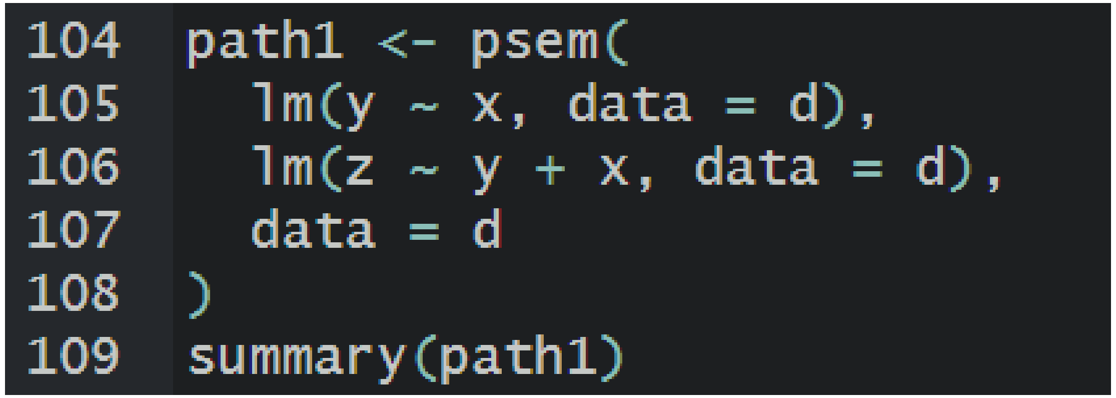
Variance-covariance matrix

$$\Sigma_{xyz} = \begin{bmatrix} 0.992 & 0.925 & 1.884 \\ 0.925 & 1.869 & 2.798 \\ 1.884 & 2.798 & 5.625 \end{bmatrix}$$

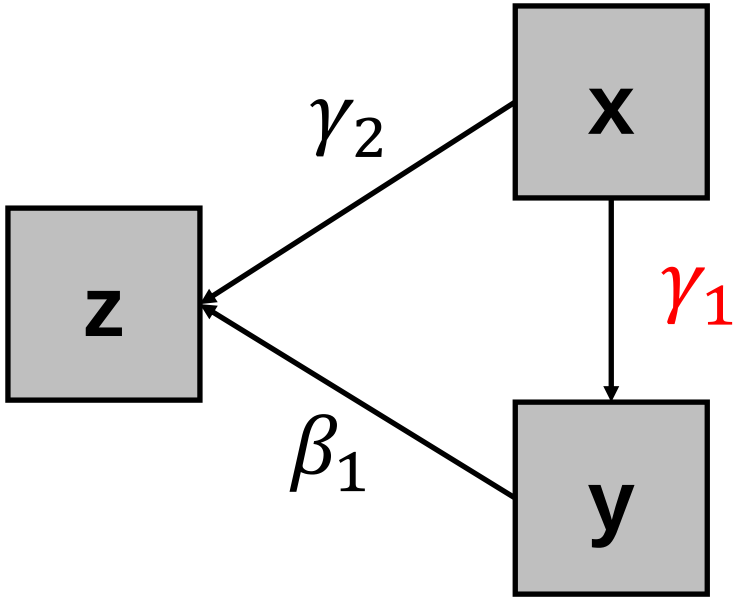
Correlation matrix

$$\mathrm{R}_{xyz} = \begin{bmatrix} 1 & 0.679 & 0.798 \\ 0.679 & 1 & 0.863 \\ 0.798 & 0.863 & 1 \end{bmatrix}$$

Let’s fit a model



```r
path1 <- psem(
  lm(y ~ x, data = d),
  lm(z ~ y + x, data = d),
  data = d
)
summary(path1)
```



Variance-covariance matrix

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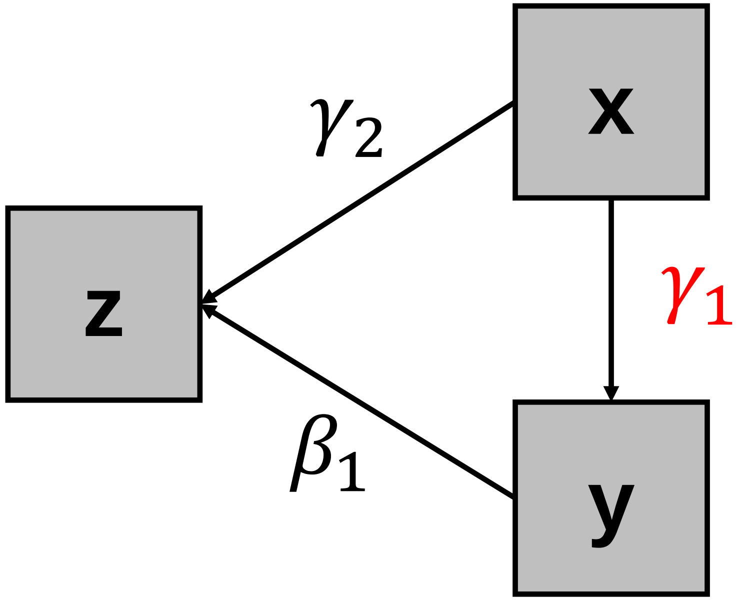
Correlation matrix

$$\mathrm{R}_{xyz} = \begin{bmatrix} 1 & 0.679 & 0.798 \\ 0.679 & 1 & 0.863 \\ 0.798 & 0.863 & 1 \end{bmatrix}$$

The relationship between y ~ x

$$\gamma_1 = \frac{\Sigma_{xy}}{\sigma_x^2} = (R_{12})\frac{\sigma_y}{\sigma_x}$$



Variance-covariance matrix

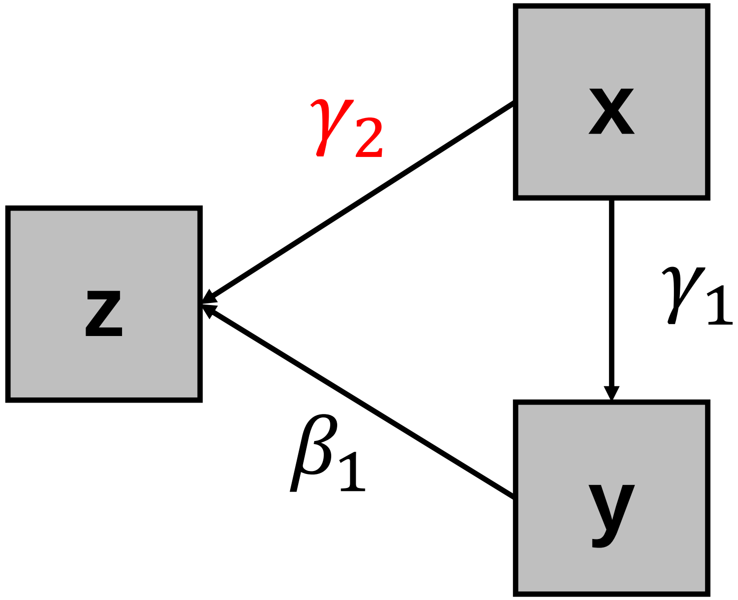
$$\Sigma_{xyz} = \begin{bmatrix} 0.992 & 0.925 & 1.884 \\ 0.925 & 1.869 & 2.798 \\ 1.884 & 2.798 & 5.625 \end{bmatrix}$$

Correlation matrix

$$\mathrm{R}_{xyz} = \begin{bmatrix} 1 & 0.679 & 0.798 \\ 0.679 & 1 & 0.863 \\ 0.798 & 0.863 & 1 \end{bmatrix}$$

The relationship between z ~ x|y

$$\gamma_2 = \frac{R_{xz} - (R_{xy} \times R_{yz})}{1 - R_{xy}^2} \times \frac{\sigma_z}{\sigma_x}$$



Variance-covariance matrix

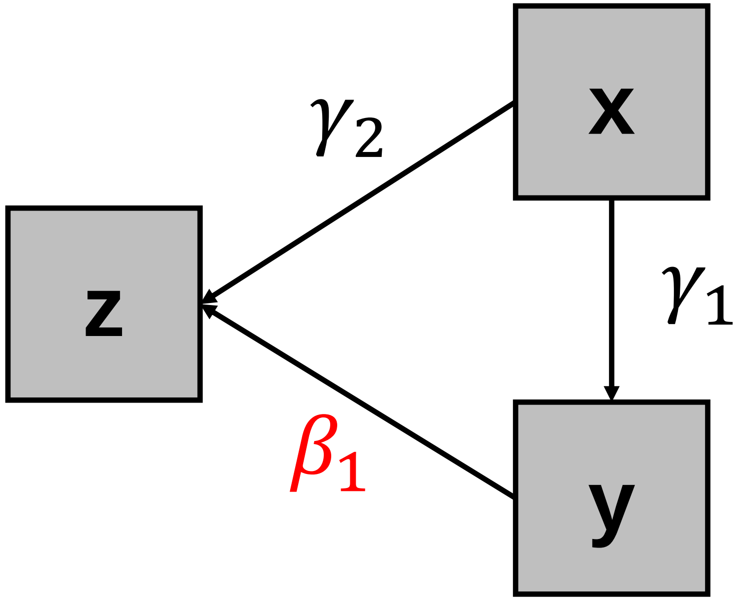
$$\Sigma_{xyz} = \begin{bmatrix} 0.992 & 0.925 & 1.884 \\ 0.925 & 1.869 & 2.798 \\ 1.884 & 2.798 & 5.625 \end{bmatrix}$$

Correlation matrix

$$\mathrm{R}_{xyz} = \begin{bmatrix} 1 & 0.679 & 0.798 \\ 0.679 & 1 & 0.863 \\ 0.798 & 0.863 & 1 \end{bmatrix}$$

The relationship between z ~ y|x

$$\beta_1 = \frac{R_{yz} - (R_{xy} \times R_{xz})}{1 - R_{xy}^2} \times \frac{\sigma_z}{\sigma_x}$$



Variance-covariance matrix

$$\Sigma_{xyz} = \begin{bmatrix} 0.992 & 0.925 & 1.884 \\ 0.925 & 1.869 & 2.798 \\ 1.884 & 2.798 & 5.625 \end{bmatrix}$$

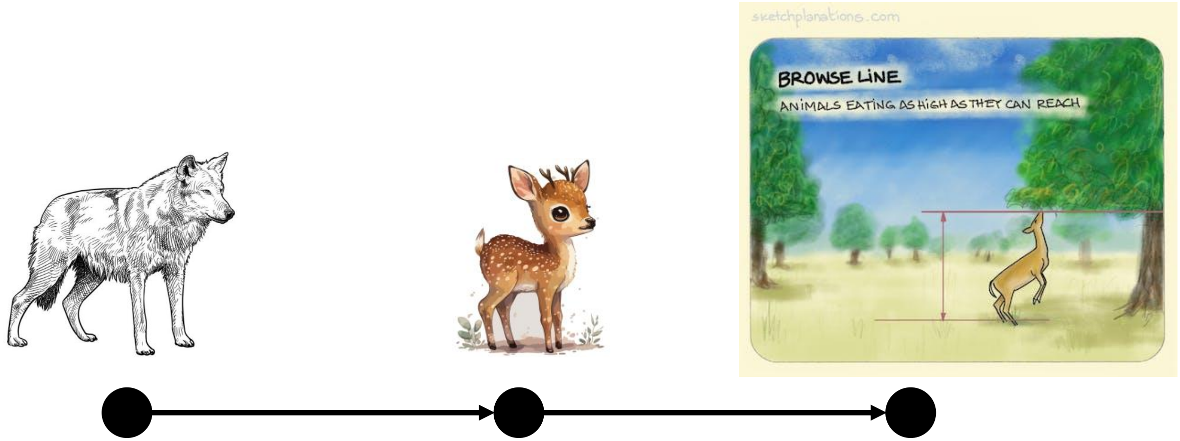
Correlation matrix

$$\mathrm{R}_{xyz} = \begin{bmatrix} 1 & 0.679 & 0.798 \\ 0.679 & 1 & 0.863 \\ 0.798 & 0.863 & 1 \end{bmatrix}$$

This was 100 years ago…

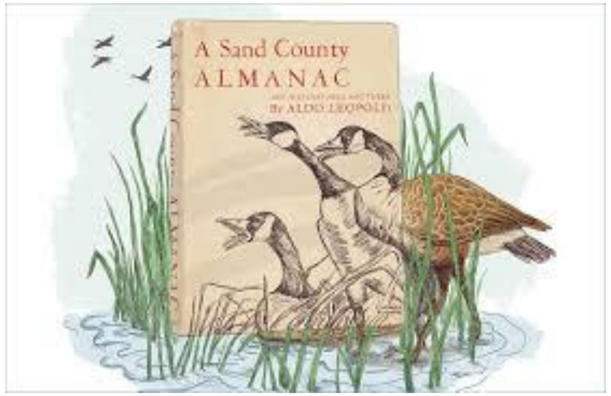
They just figured it out… by hand.

One variable can influence another through a third!

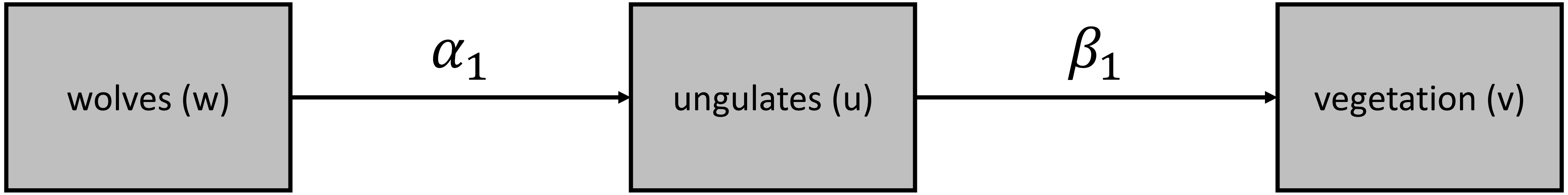


We have always ‘**known**’ that when we discuss and teach principles of wildlife management and ecology, **we can also formally model it.**

Our first example



Available browse (***v***; vegetation) as a function of ungulate (***u***) and predator (***w***; wolf) abundance.

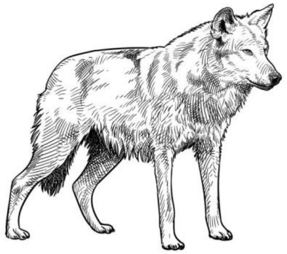

wolves (w)

α_1

ungulates (u)

β_1

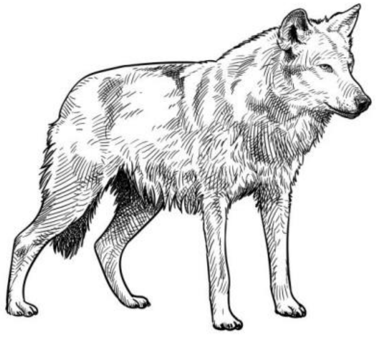
vegetation (v)

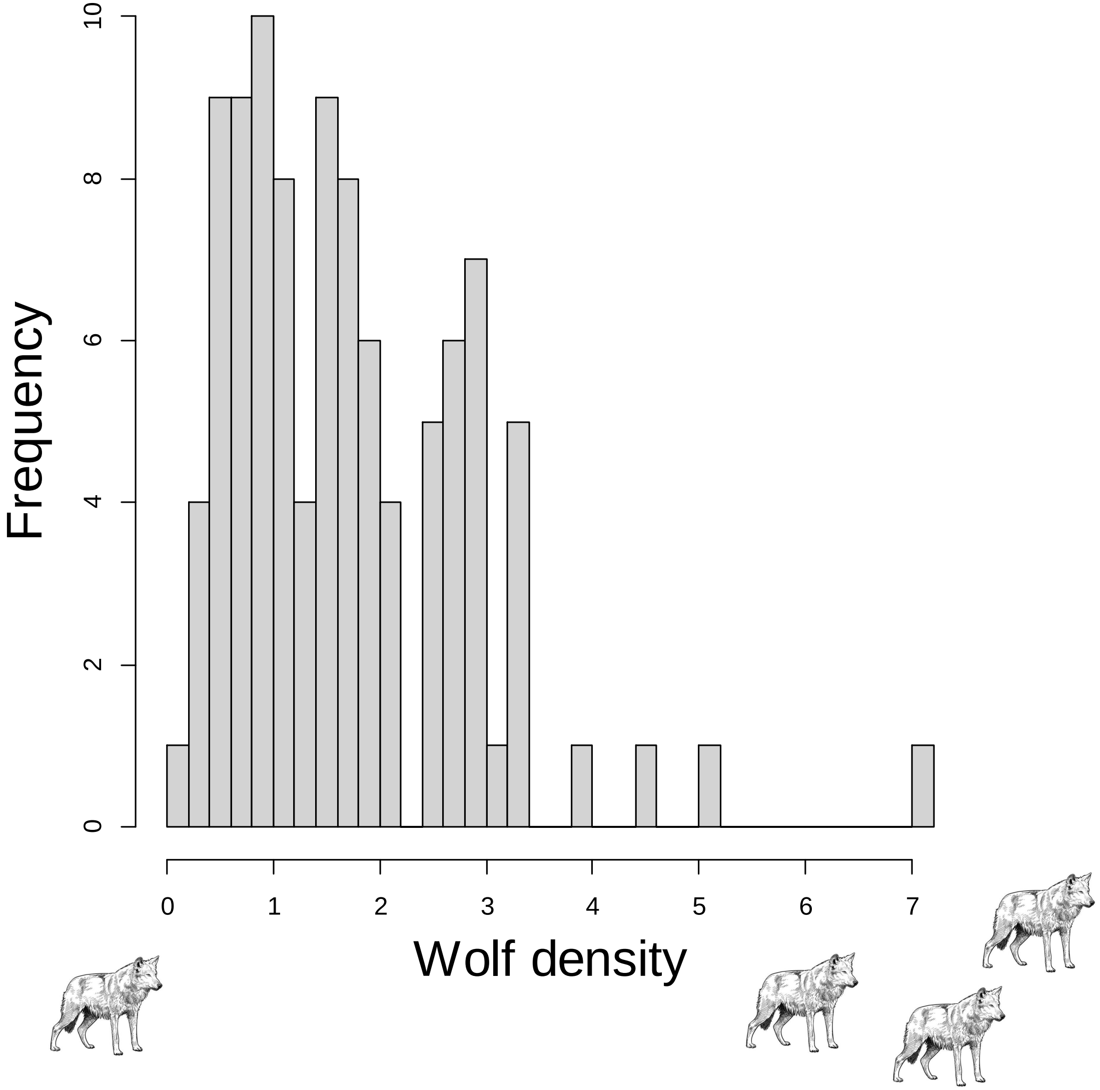


Lecture2b_wolf_rsf.R

First, we'll simulate random variation in wolf density (*w*)

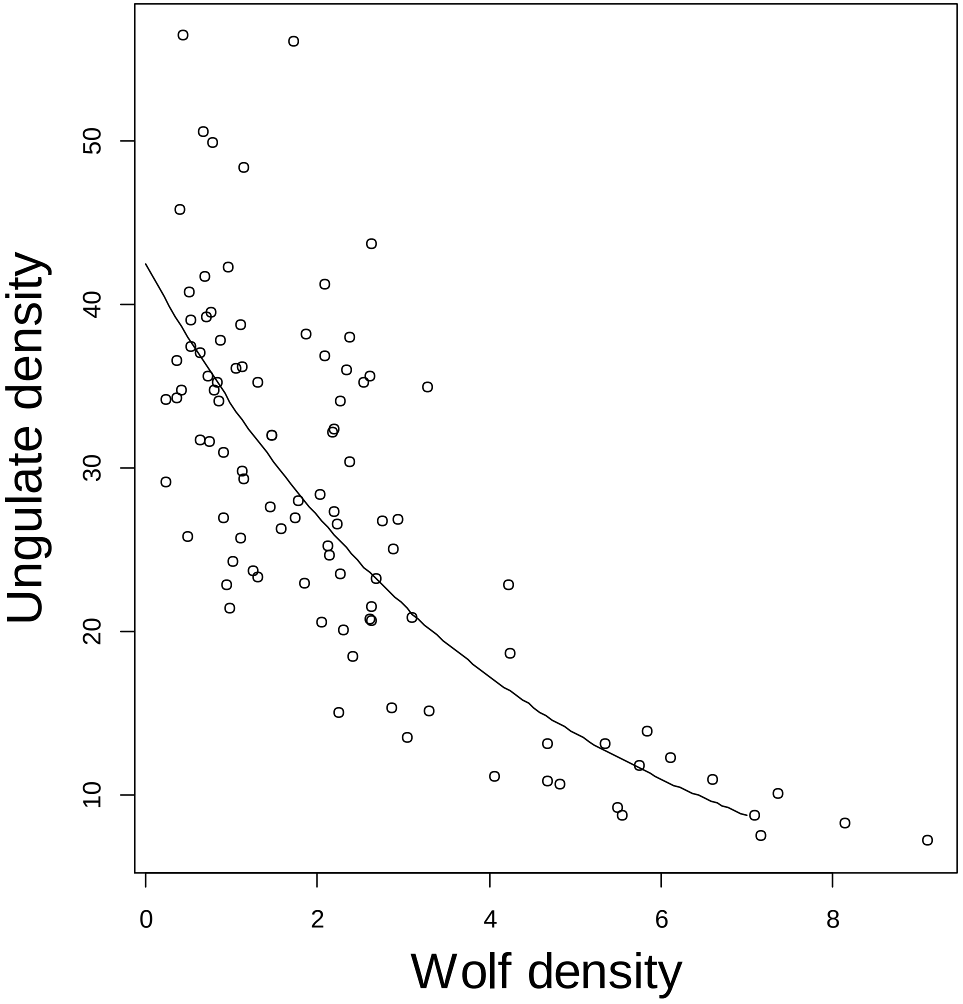
$$w_i \sim \text{lognormal}(0.35, 0.75)$$





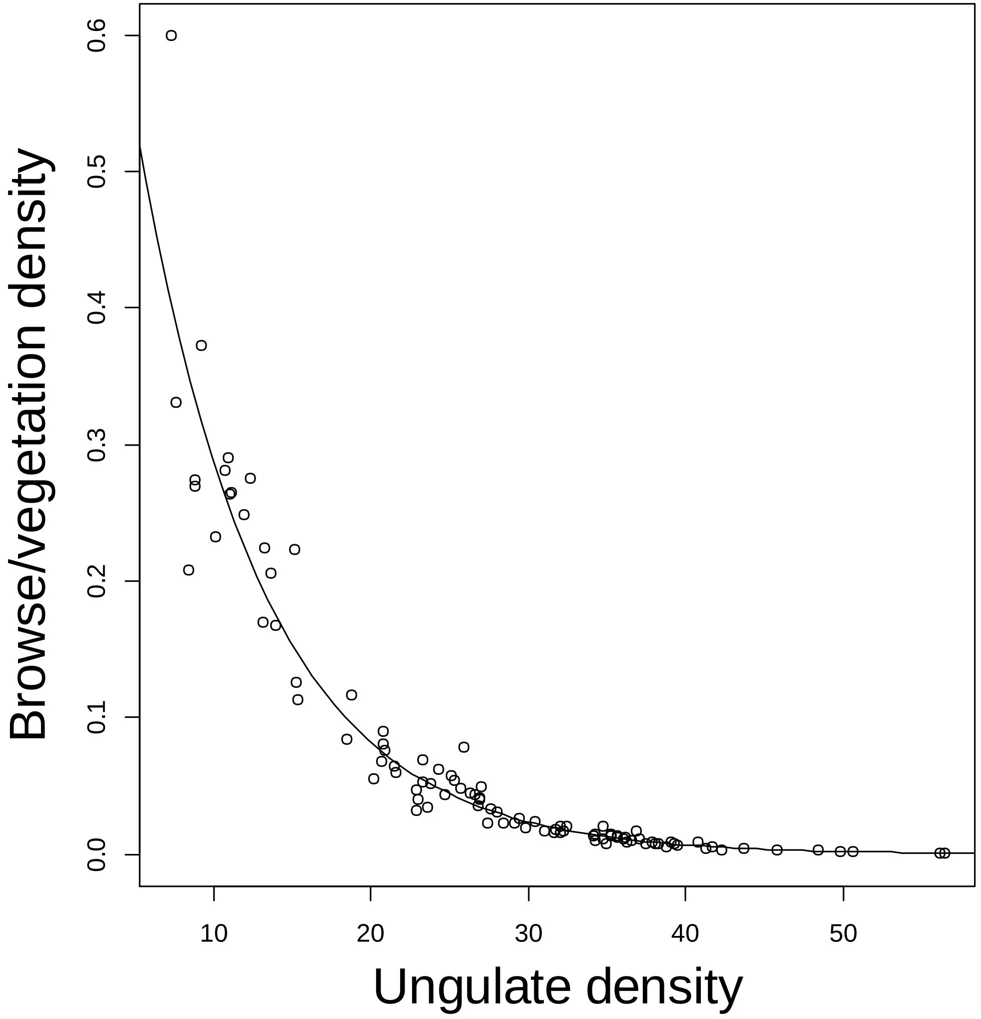
Second, we’ll simulate ungulate density (*u*) as a function of *w*

$$u_i \sim \text{lognormal}(\alpha_0 + \alpha_1 w_i, \sigma_v^2)$$



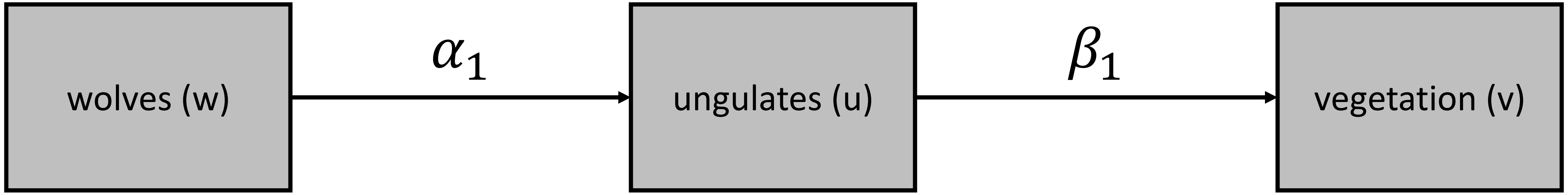
Third, we'll simulate browse density (*v*) as a function of *u*

$$v_i \sim \mathrm{lognormal}(\beta_0 + \beta_1 u_i, \sigma_v^2)$$

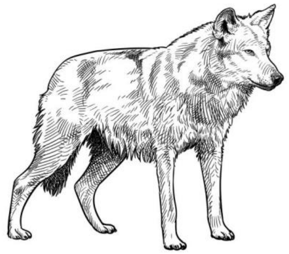


We’ll use simple linear models to analyze the data

1. Identity link functions (i.e., normal distributions) in R



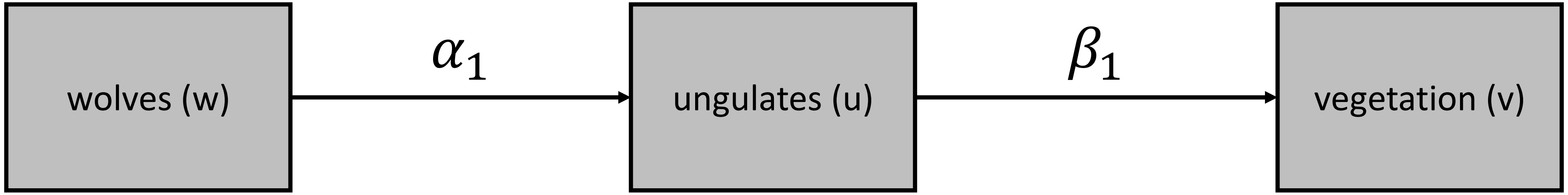
wolves (w) $\xrightarrow{\alpha_1}$ ungulates (u) $\xrightarrow{\beta_1}$ vegetation (v)

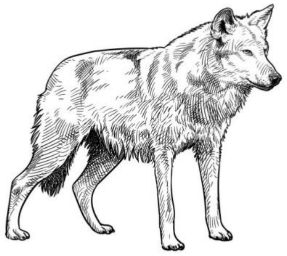


First, we’ll use ‘identity’ link functions in R [lm()]

$$u_i \sim \text{normal}(\alpha_0 + \alpha_1 w_i, \sigma_u^2)$$

$$v_i \sim \text{normal}(\beta_0 + \beta_1 u_i, \sigma_v^2)$$



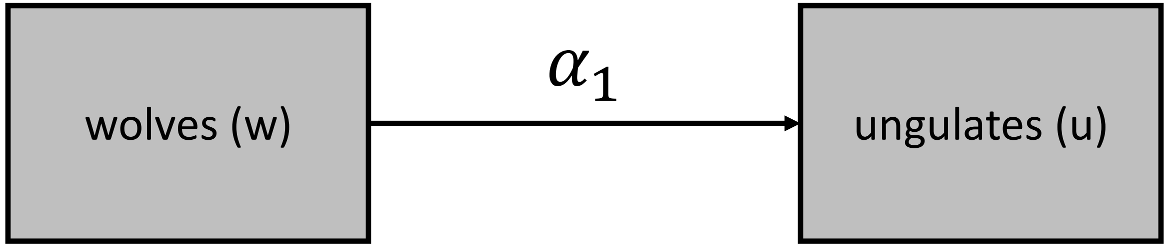


It really is this simple

Something to think about

$$u_i \sim \text{normal}(\alpha_0 + \alpha_1 w_i, \sigma_u^2)$$

$$v_i \sim \text{normal}(\beta_0 + \beta_1 u_i, \sigma_v^2)$$

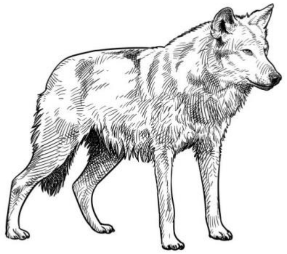


wolves (w)

α_1

ungulates (u)

```
lm(u ~ w)
```

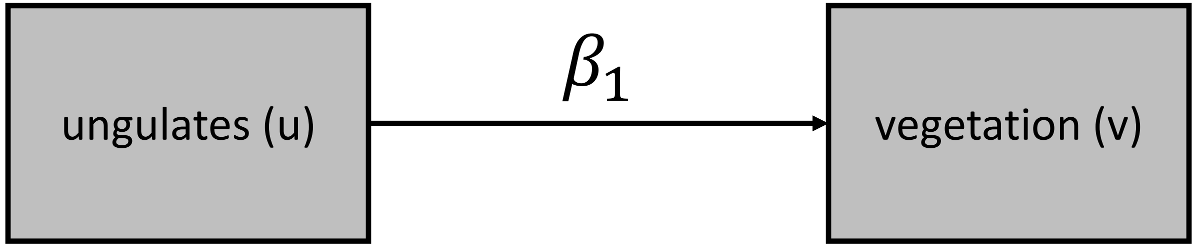


It’s easy to calculate the effect of wolves on ungulate (α_1)!

Something to think about

$$u_i \sim \text{normal}(\alpha_0 + \alpha_1 w_i, \sigma_u^2)$$

$$v_i \sim \text{normal}(\beta_0 + \beta_1 u_i, \sigma_v^2)$$



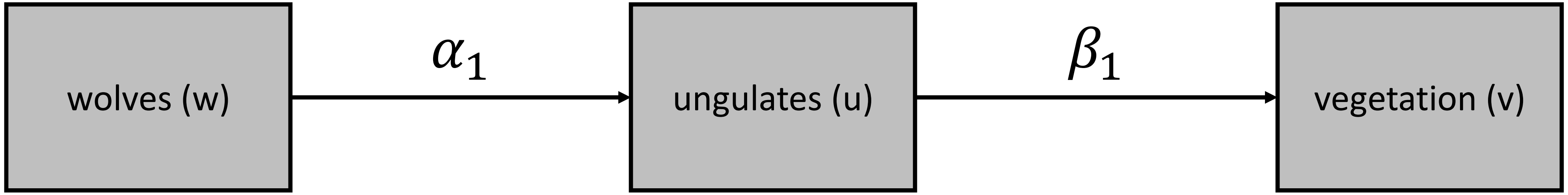
```
lm(v ~ u)
```

It’s also easy to calculate the effect of ungulates on vegetation (β_1)

Something to think about (we'll discuss this shortly!)

$$u_i \sim \text{normal}(\alpha_0 + \alpha_1 w_i, \sigma_u^2)$$

$$v_i \sim \text{normal}(\beta_0 + \beta_1 u_i, \sigma_v^2)$$



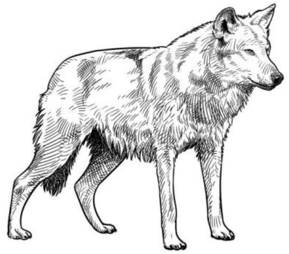
wolves (w)

α_1

ungulates (u)

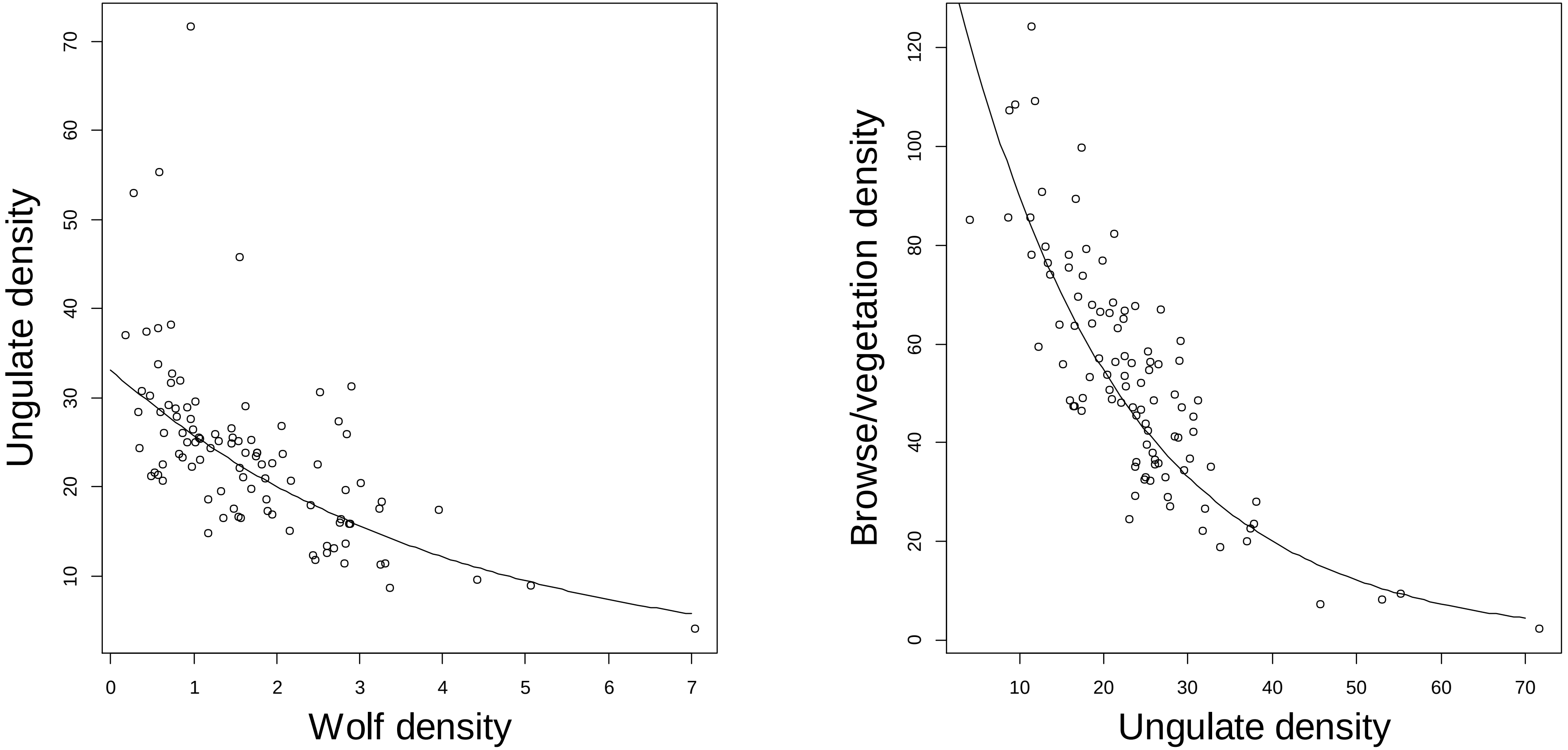
β_1

vegetation (v)



How would we calculate the effect of wolves on browse?!

How would we calculate the effect of wolves on browse?!



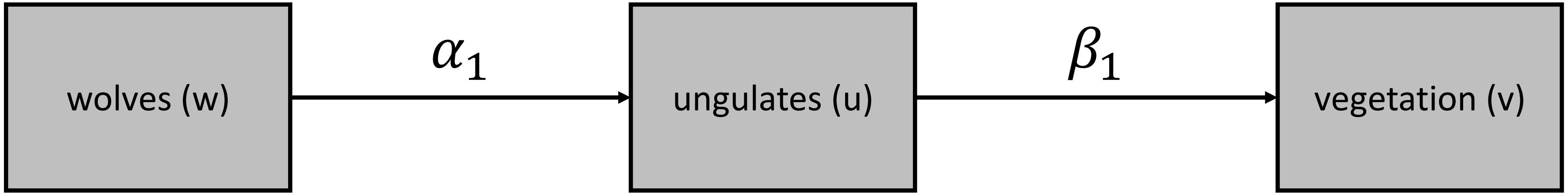
Talk to your neighbor

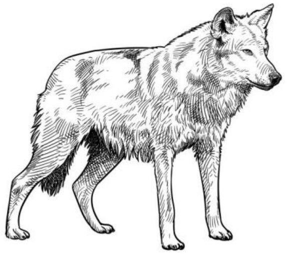
Let’s imagine there is a hypothetical 101st study site with a current density of 2 wolves per ‘unit’

1. What is the current ML estimate of ungulates and browse?

2. What would happen to ungulate density if wolves were extirpated?

3. What would happen to browse if wolves were extirpated?

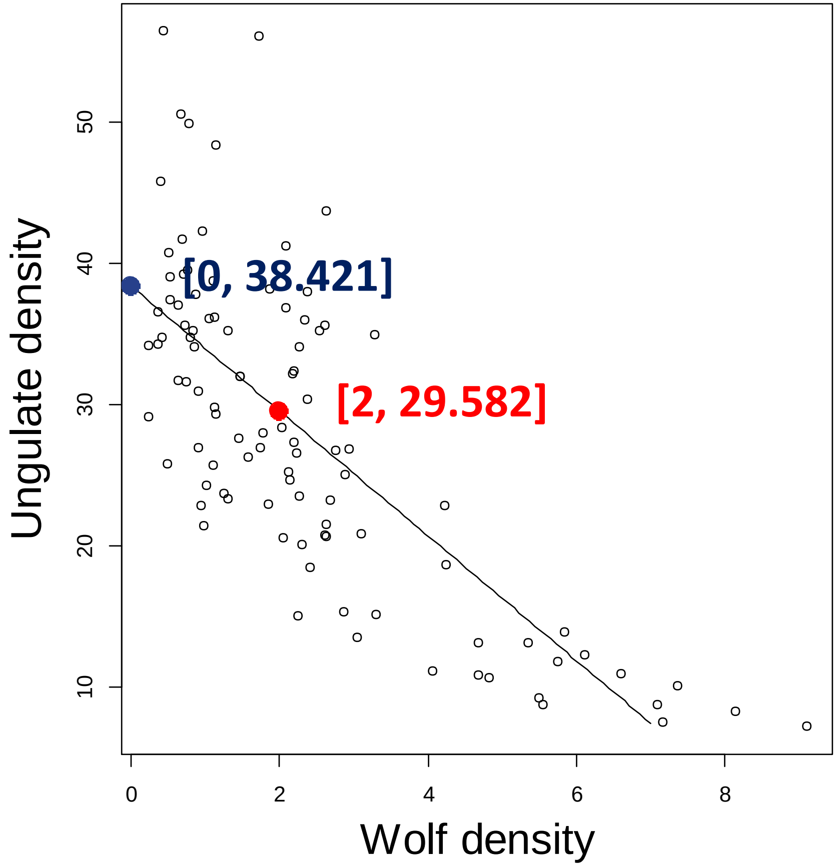




Let’s visualize an indirect effect!

$$u_i \sim \text{normal}(\alpha_0 + \alpha_1 w_i, \sigma_u^2)$$

$$v_i \sim \text{normal}(\beta_0 + \beta_1 u_i, \sigma_v^2)$$



We would gain about 8.83 ungulates, going from 29.582 to 38.421.

$$\alpha_0 = 38.4211$$

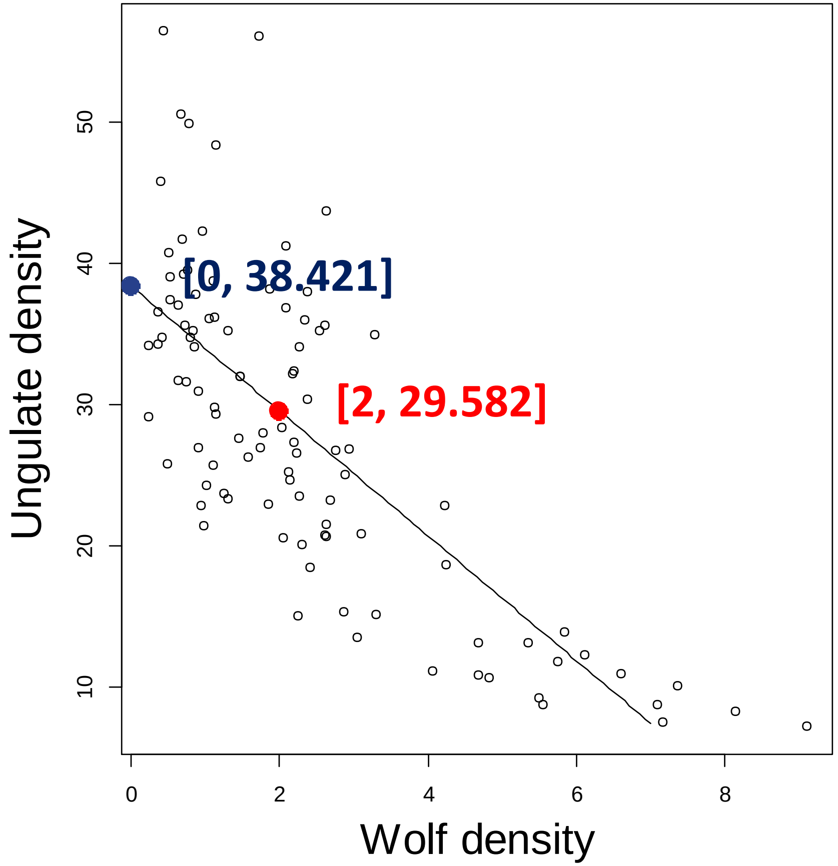
$$\alpha_1 = -4.4194$$

What would happen if we went from 2 wolves to 0?

Let’s visualize an indirect effect!

$$u_i \sim \text{normal}(\alpha_0 + \alpha_1 w_i, \sigma_u^2)$$

$$v_i \sim \text{normal}(\beta_0 + \beta_1 u_i, \sigma_v^2)$$



We would gain about 8.83 ungulates, going from 29.582 to 38.421.

$$\alpha_0 = 38.4211$$

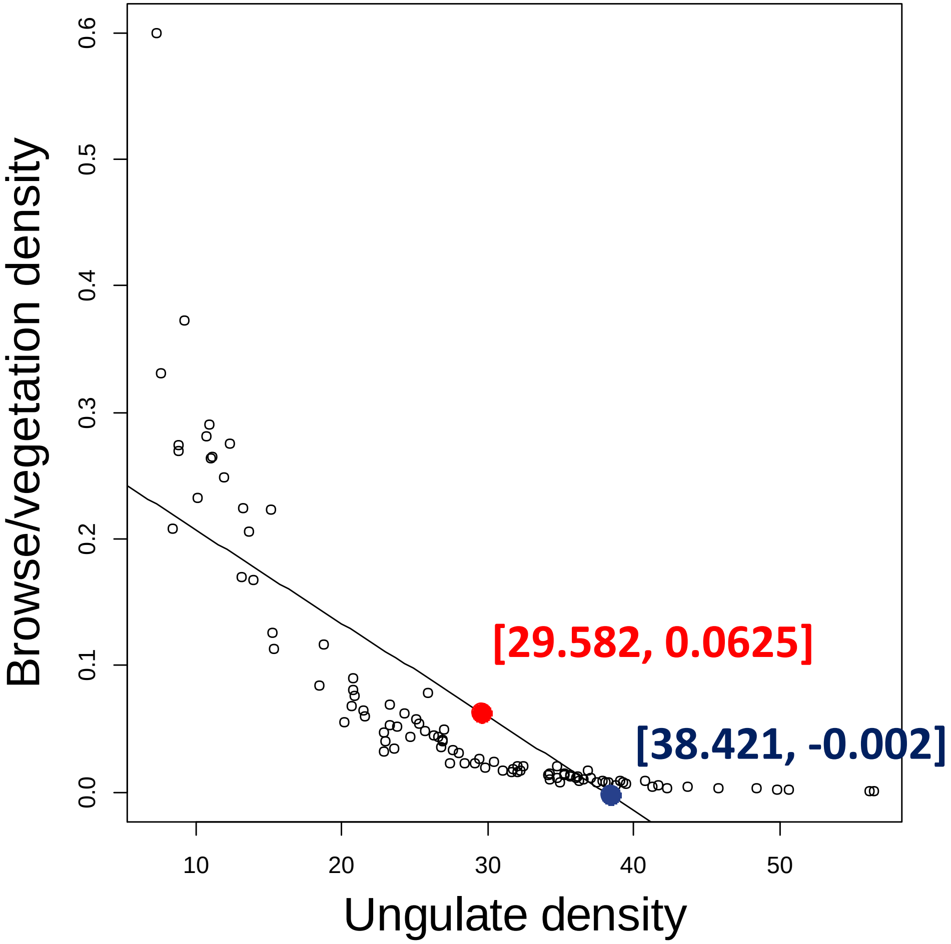
$$\alpha_1 = -4.4194$$

How would that 8.83 gain in ungulates affect browse?

Let’s visualize an indirect effect!

$u_i \sim \text{normal}(\alpha_0 + \alpha_1 w_i, \sigma_u^2)$

$v_i \sim \text{normal}(\beta_0 + \beta_1 u_i, \sigma_v^2)$



If we gain about 8.83 ungulates, we lose ‘browse’.

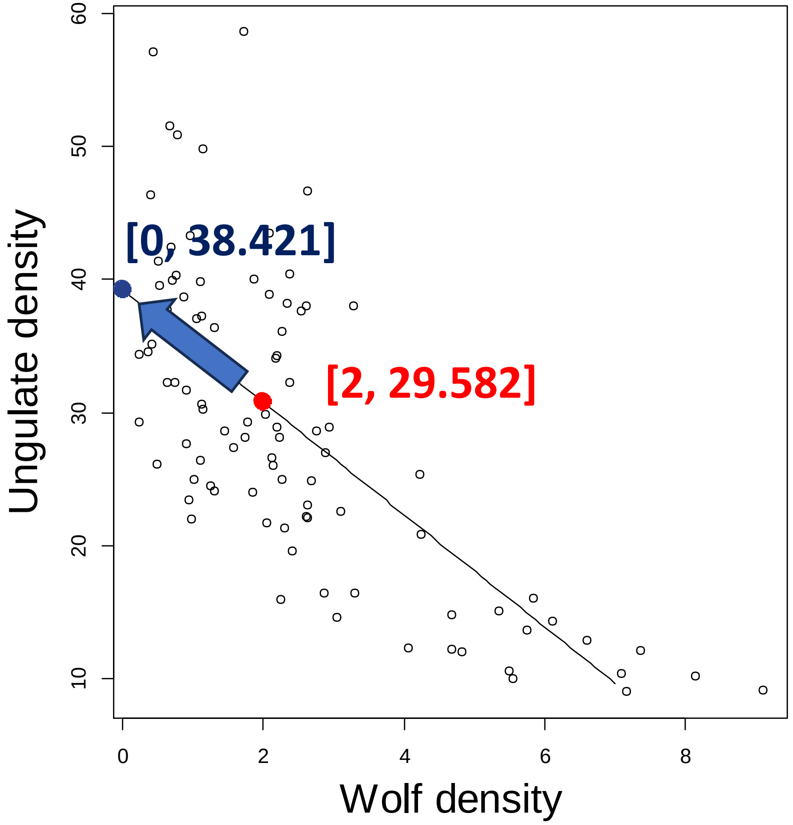
$$\beta_0 = 0.2809$$

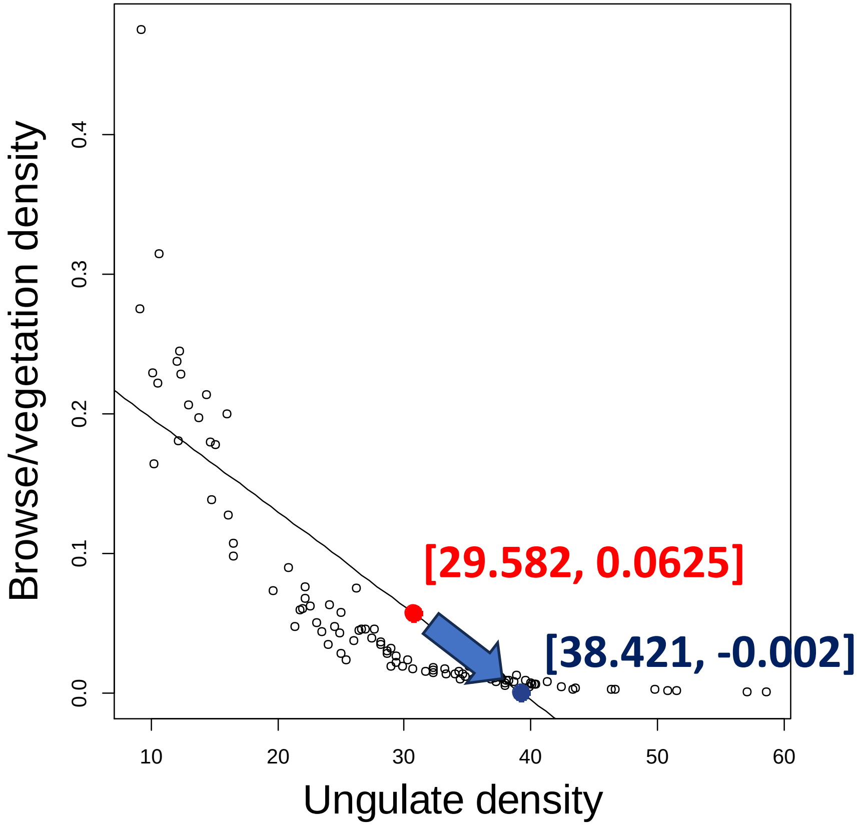
$$\beta_1 = -0.00738$$

Let’s visualize an indirect effect!

$$u_i \sim \text{normal}(\alpha_0 + \alpha_1 w_i, \sigma_u^2)$$

$$v_i \sim \text{normal}(\beta_0 + \beta_1 u_i, \sigma_v^2)$$



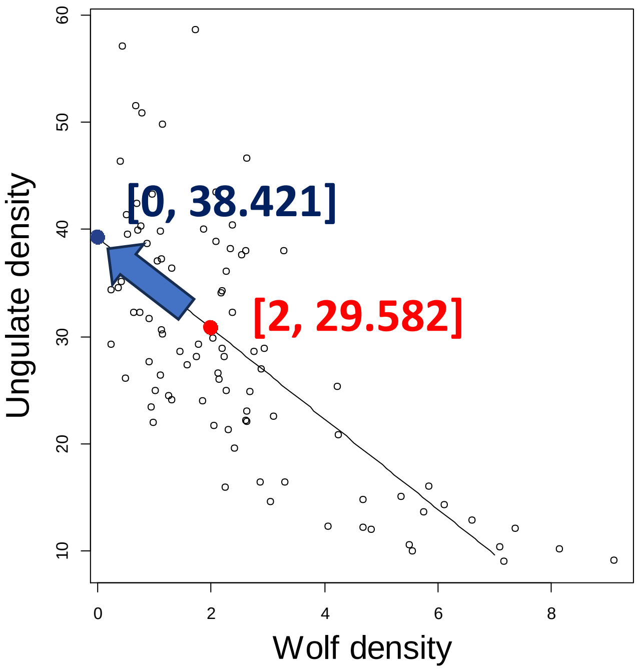


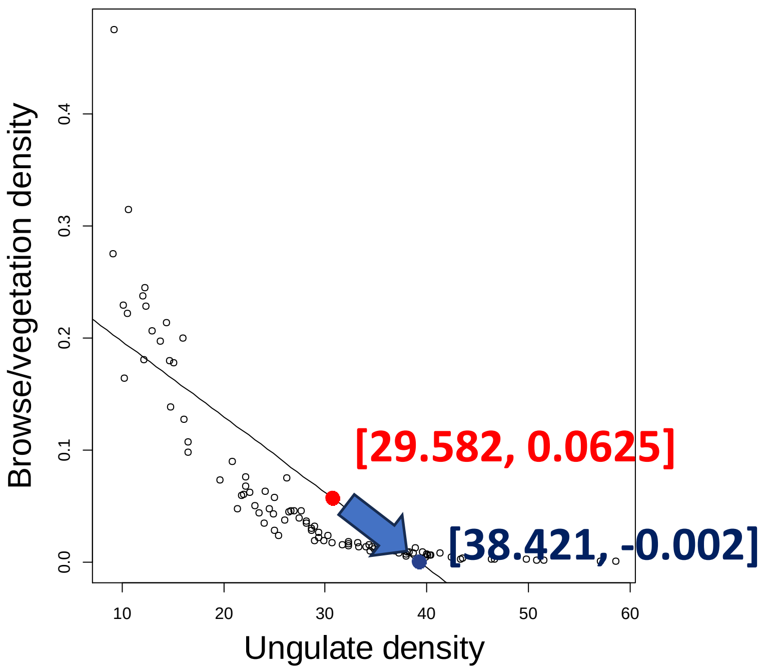
Losing wolves -> more deer; more deer -> less browse

Let’s visualize an indirect effect!

$$u_i \sim \text{normal}(\alpha_0 + \alpha_1 w_i, \sigma_u^2)$$

$$v_i \sim \text{normal}(\beta_0 + \beta_1 u_i, \sigma_v^2)$$





$$\beta_1 = -0.00738$$

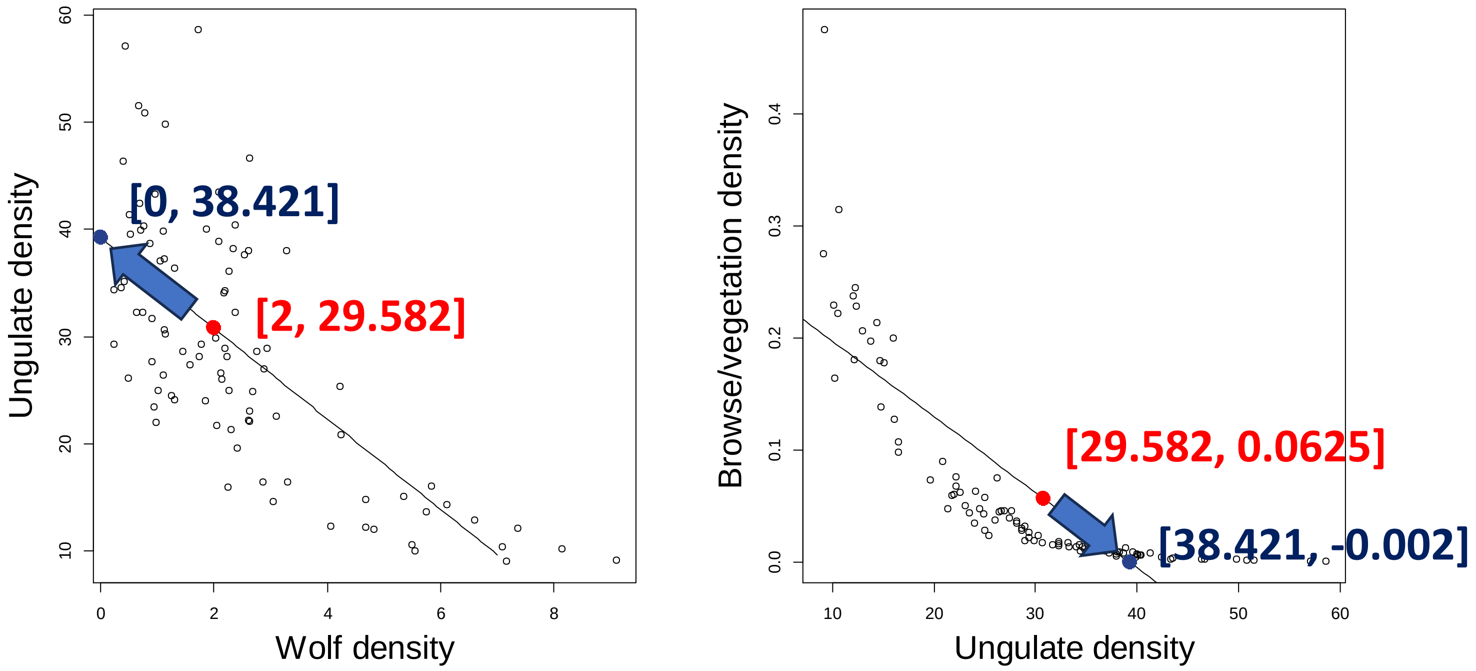
$$\alpha_1 = -4.4194$$

$$\mathbf{-0.06524} = \boxed{\mathbf{-0.002\ -\ 0.0625}}$$

Change in browse

Let's visualize an indirect effect!

$u_i \sim \text{normal}(\alpha_0 + \alpha_1 w_i, \sigma_u^2)$ $v_i \sim \text{normal}(\beta_0 + \beta_1 u_i, \sigma_v^2)$



$$\beta_1 = -0.00738$$

$$\alpha_1 = -4.4194$$

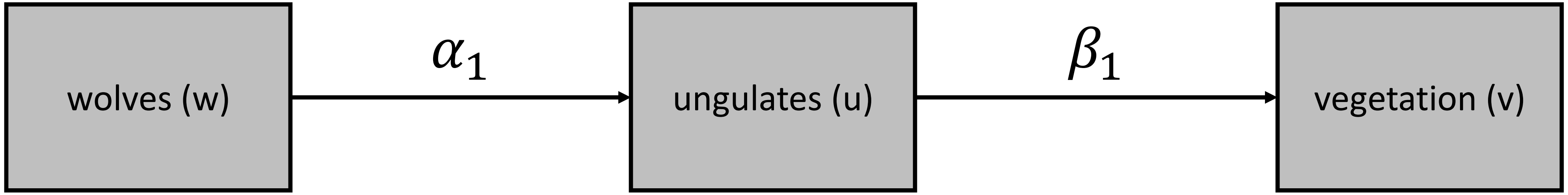
$$\mathbf{-0.06524 = \boxed{-0.002 \ - \ 0.0625} = \alpha_1(\beta_1)(-2)}$$

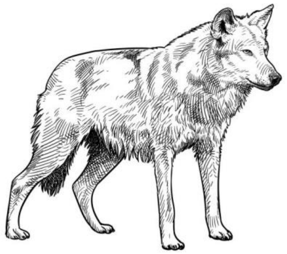
Change in browse

Take-home 1: indirect effects aren’t as ‘horrifying’ as they seem

$$u_i \sim \text{normal}(\alpha_0 + \alpha_1 w_i, \sigma_u^2)$$

$$v_i \sim \text{normal}(\beta_0 + \beta_1 u_i, \sigma_v^2)$$



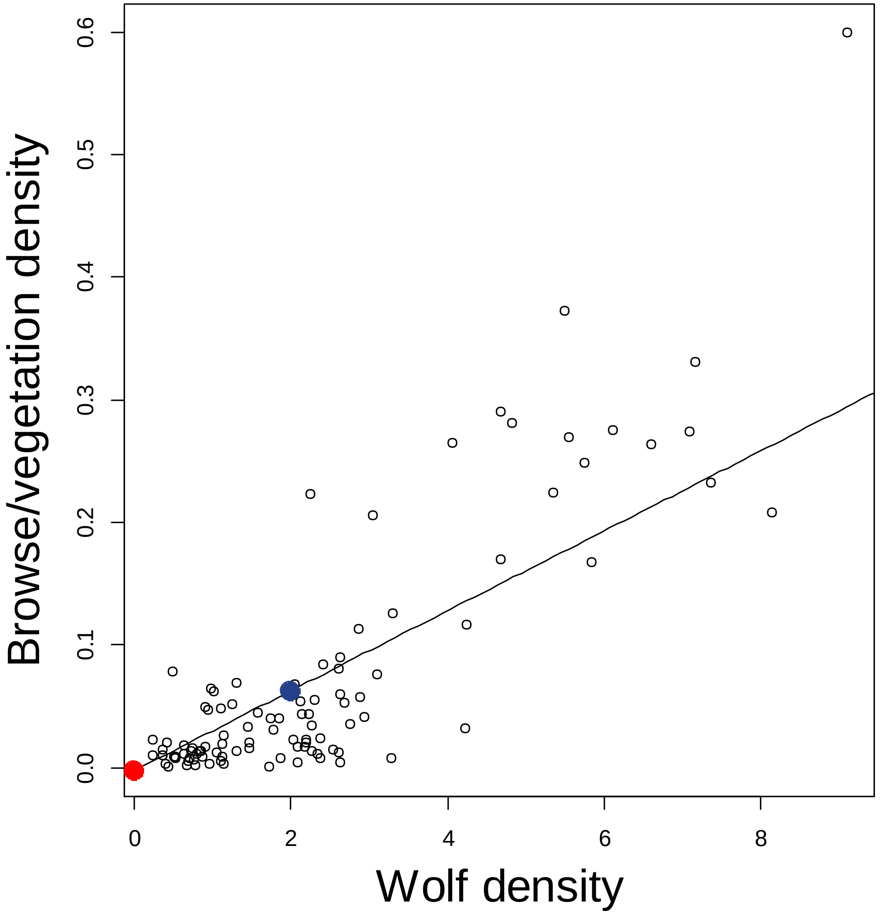


i.e., if identity link, then effect of wolves on veg = $\alpha_1\beta_1$

$$\mathrm{E}(v_i|w_i) = [\boldsymbol{\beta_0} + \boldsymbol{\beta_1}(\boldsymbol{\alpha_0})] + \alpha_1\beta_1 w_i$$

slope $= \alpha_1\beta_1$

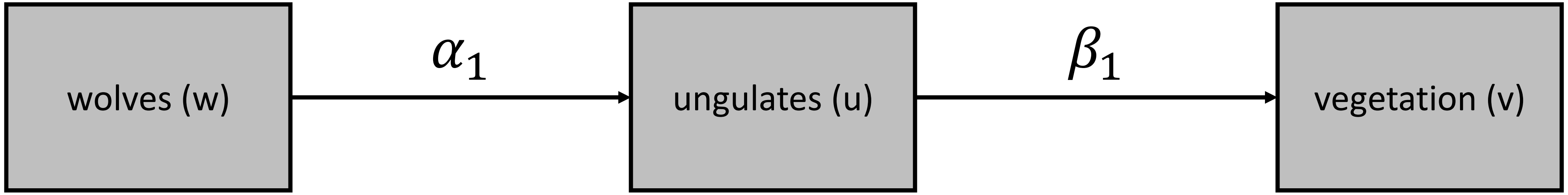
intercept =
what value of veg would be given number of deer if there were no wolves



Take-home 1: this is easy?!

$$u_i \sim \mathrm{normal}(\alpha_0 + \alpha_1 w_i, \sigma_u^2)$$

$$v_i \sim \mathrm{normal}(\beta_0 + \beta_1 u_i, \sigma_v^2)$$



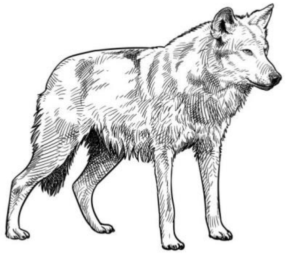
wolves (w)

α_1

ungulates (u)

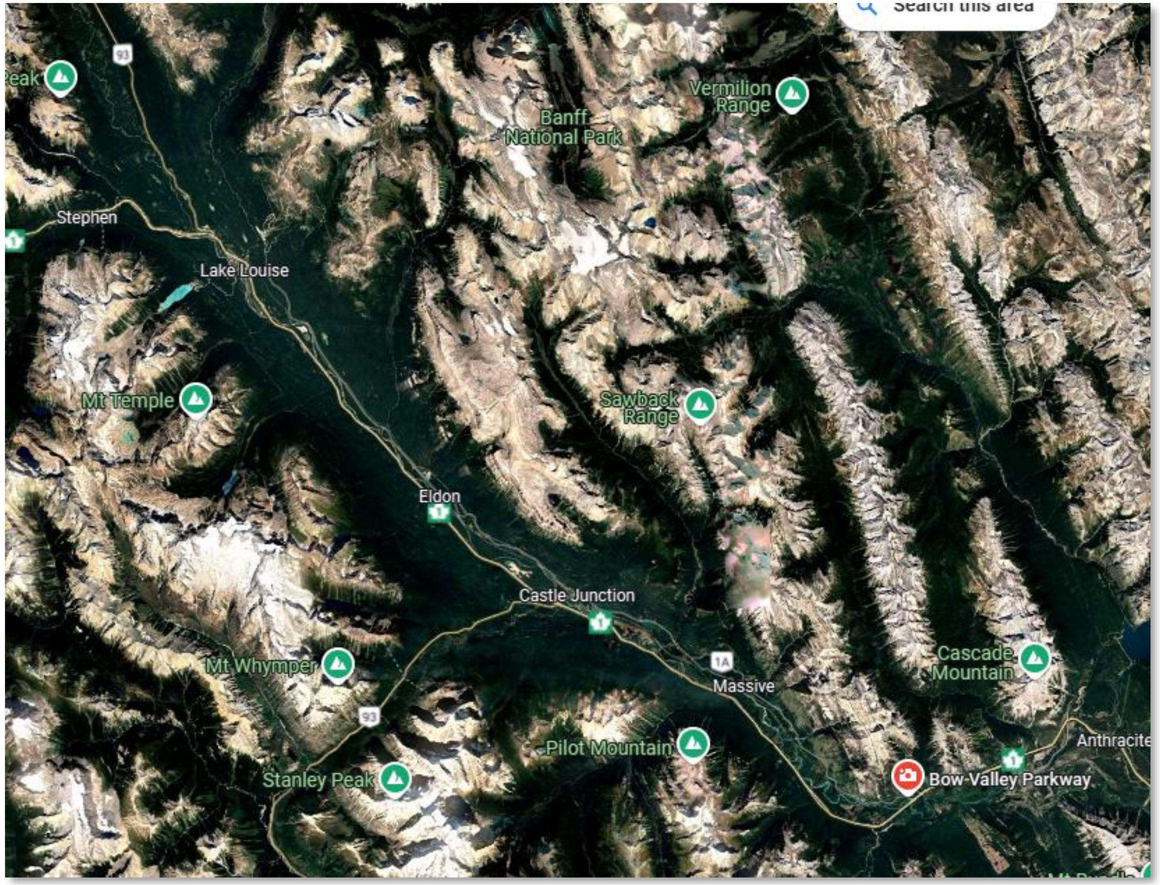
β_1

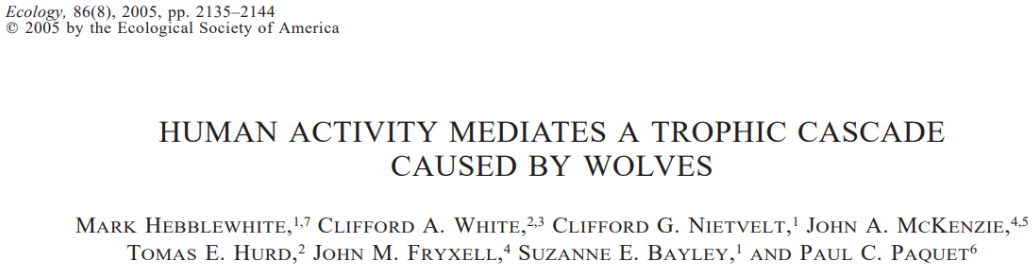
vegetation (v)



i.e., we’re more or less ‘stapling’ two linear models together

Our first analysis: wolf resource selection in Bow Valley





Ecology, 86(8), 2005, pp. 2135–2144
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HUMAN ACTIVITY MEDIATES A TROPHIC CASCADE
CAUSED BY WOLVES

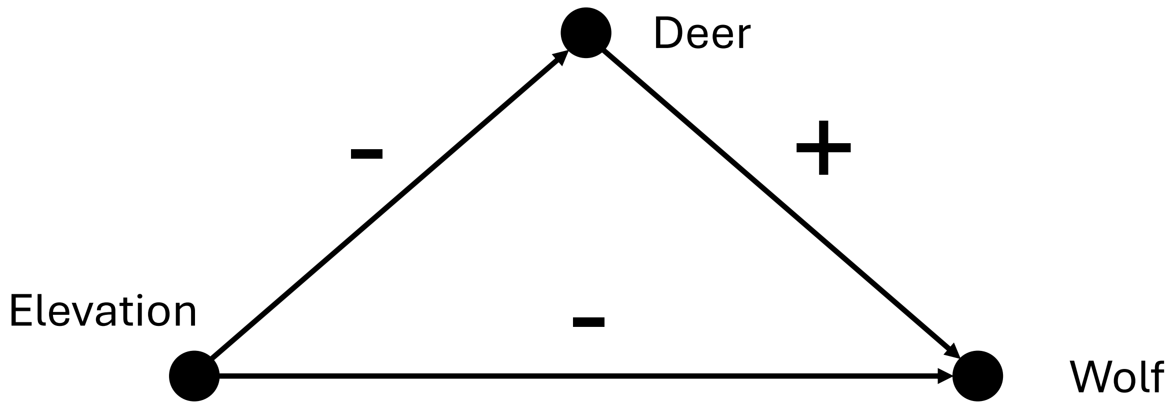
Mark Hebblewhite,[1,7] Clifford A. White,[2,3] Clifford G. Nietvelt,[1] John A. McKenzie,[4,5]
Tomas E. Hurd,[2] John M. Fryxell,[4] Suzanne E. Bayley,[1] and Paul C. Paquet[6]

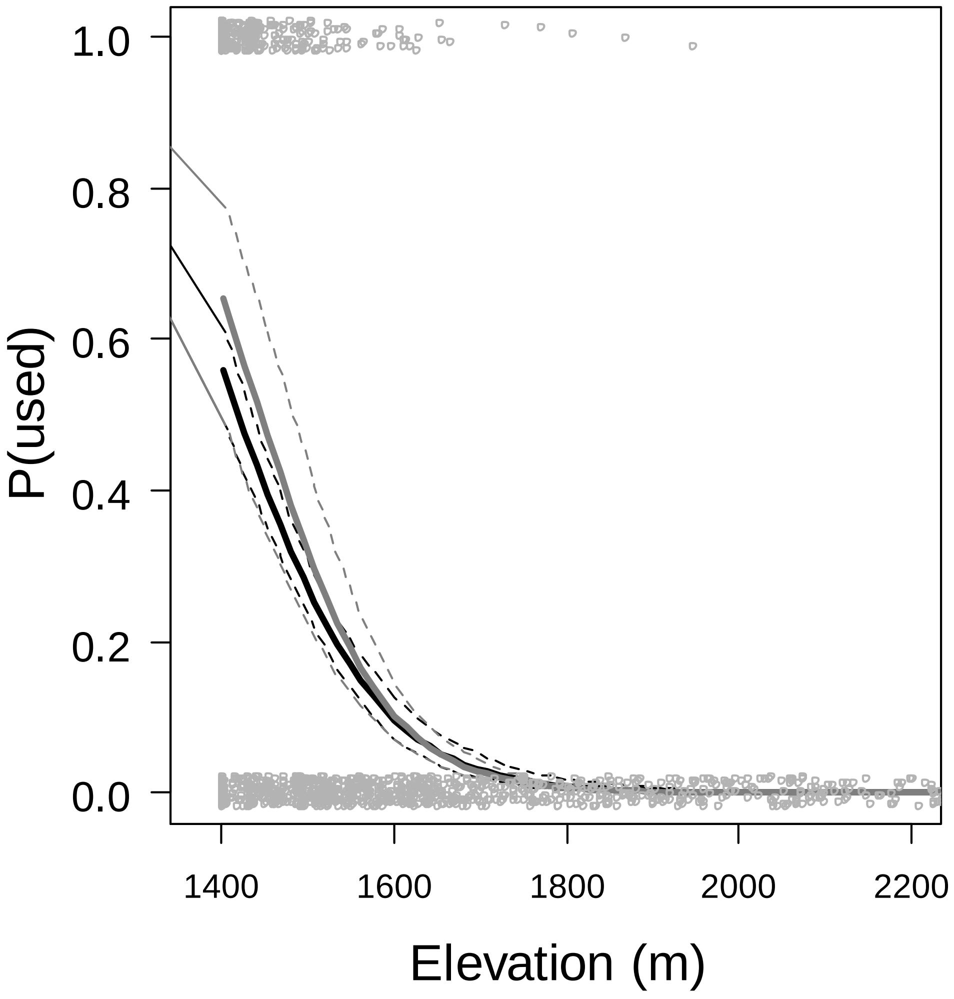
Lecture2c_wolf_rsf.R

Hebblewhite **(2000)**

Collider bias: wolf resource selection in Bow Valley

- *The black line is the direct effect of elevation holding deer abundance constant at the mean.

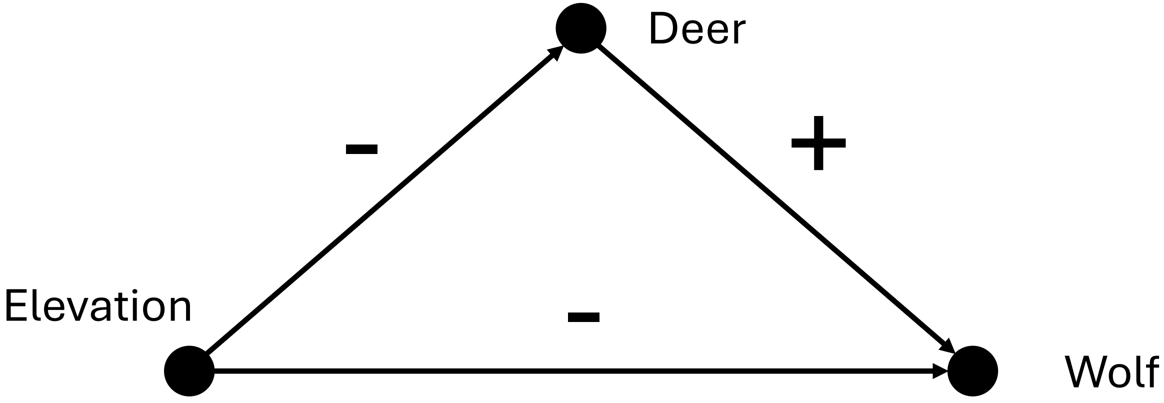


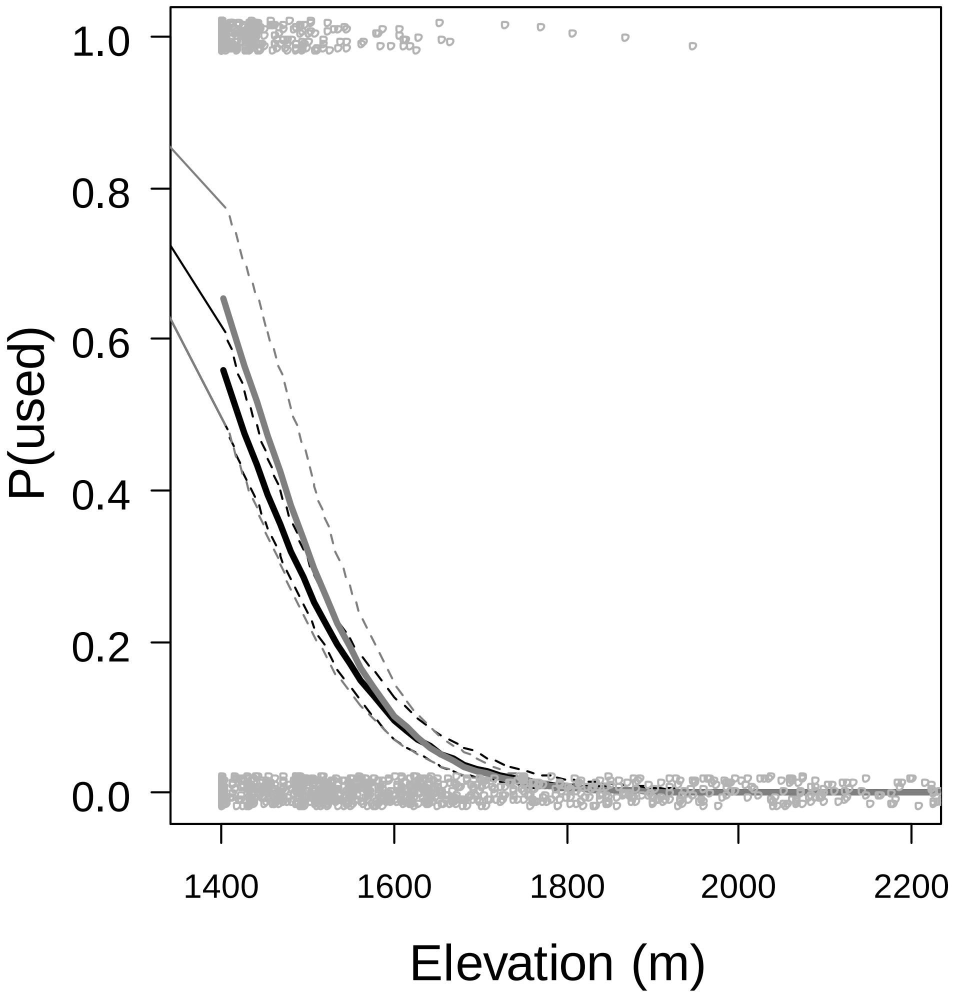


*Deer abundance will not remain constant as elevation changes

- The black line is the direct effect of elevation holding deer abundance constant at the mean.

- The grey line is the total effect of elevation via formally acknowledging that increases in elevation will reduce deer abundance.

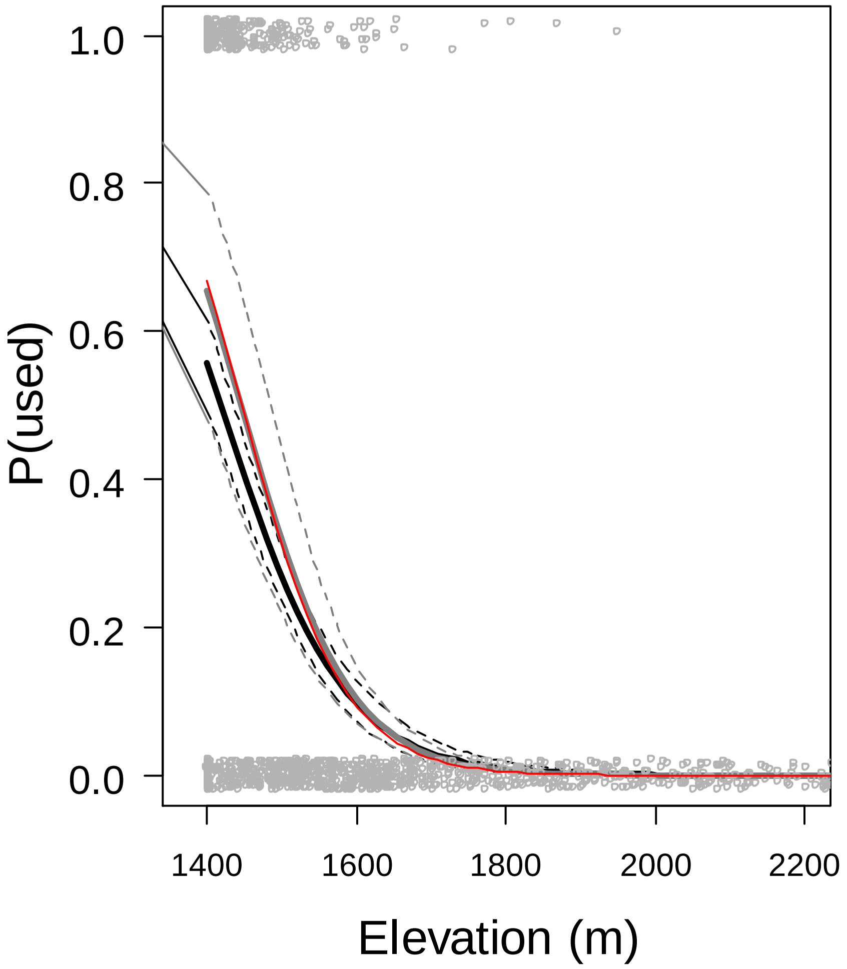


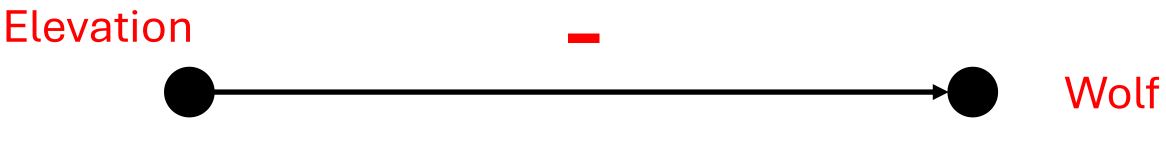


Note that a path analysis and simple glm(used ~ elevation) are similar.

- The grey line is the total effect of elevation via formally acknowledging that increases in elevation will reduce deer abundance.

- The red line is the stand-alone effect of elevation from a model w/o deer





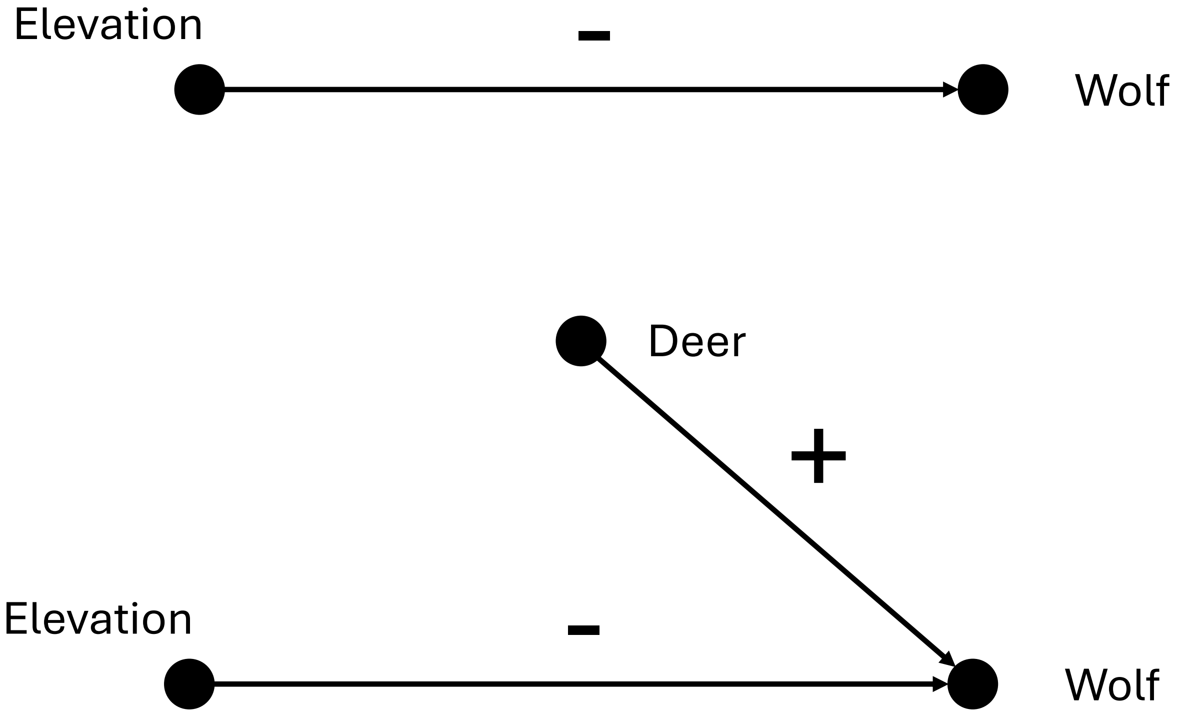
Elevation

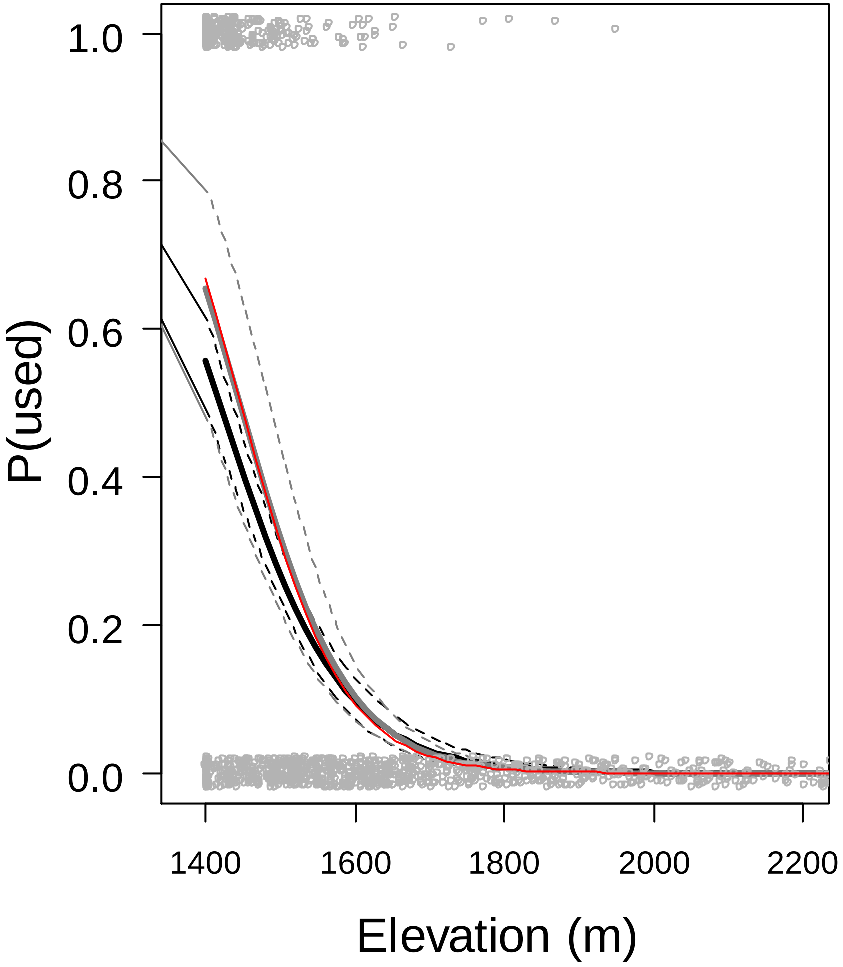
–

Wolf

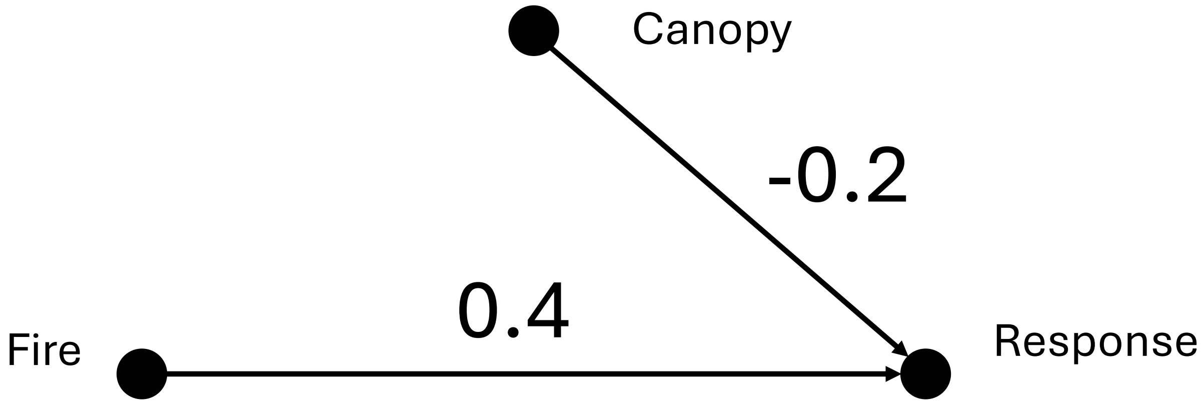
Why is this important?

- AIC (deer + elevation) = 892.93
- AIC (elevation) = 937.86





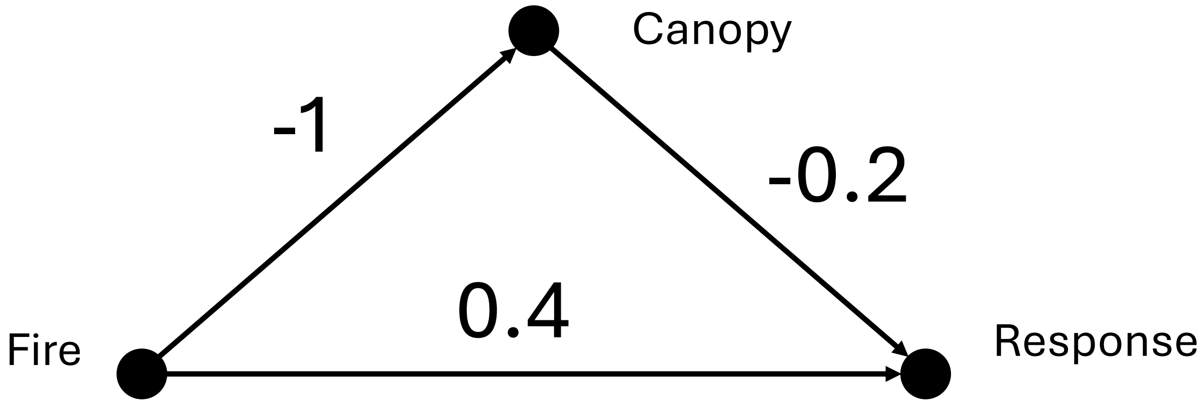
Why is this important?



Prescribed burns were associated with an increase in our ecological response variable of interest (**β = 0.4**; p < 0.01). Canopy cover negatively effected our response variable of interest (-0.2, p < 0.01).

Imagine you’re interested in the impact of a management action

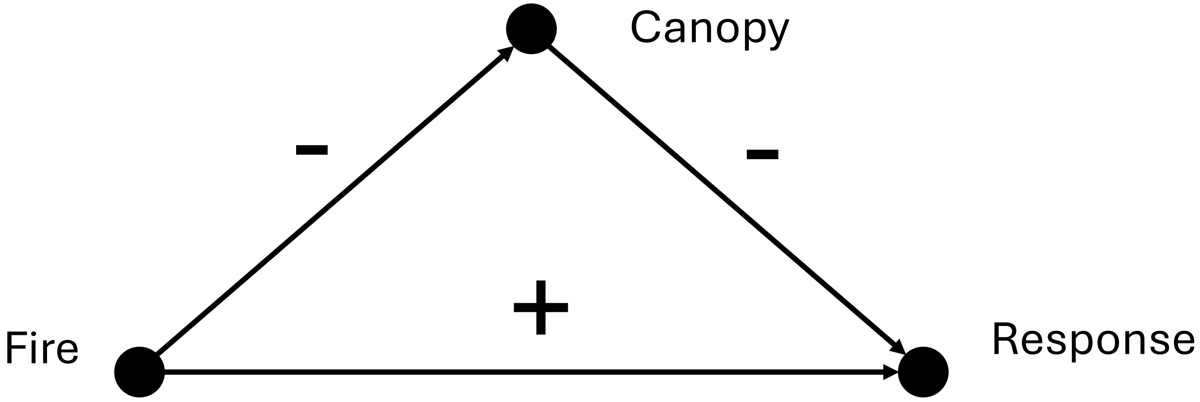
Why is this important?



Prescribed burns had a positive direct effect on our ecological response variable of interest (β = 0.4; $p < 0.01$). Burns also reduced canopy cover (β = -1; $p < 0.01$), which negatively effected our response variable of interest (-0.2, $p < 0.01$) **leading to a total increase of 0.6.**

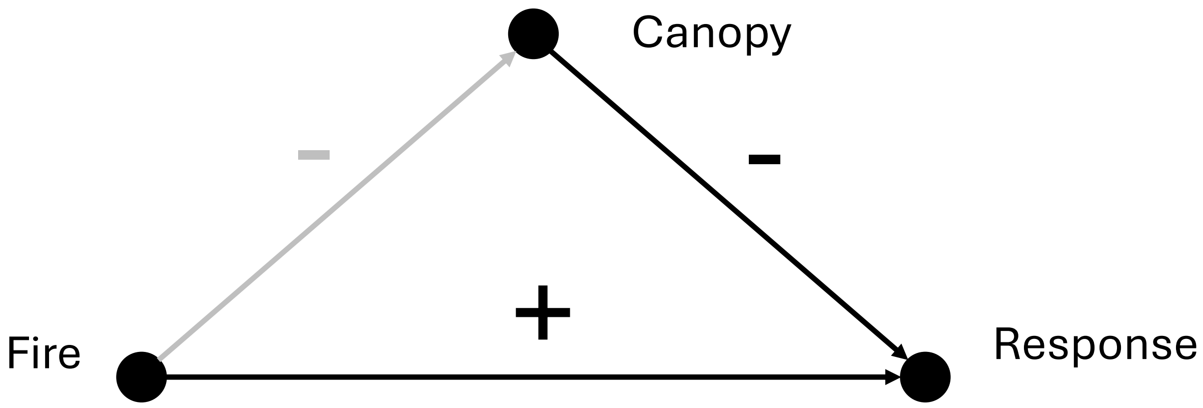
The total effect is 50% greater than the direct effect!

Why is this important?



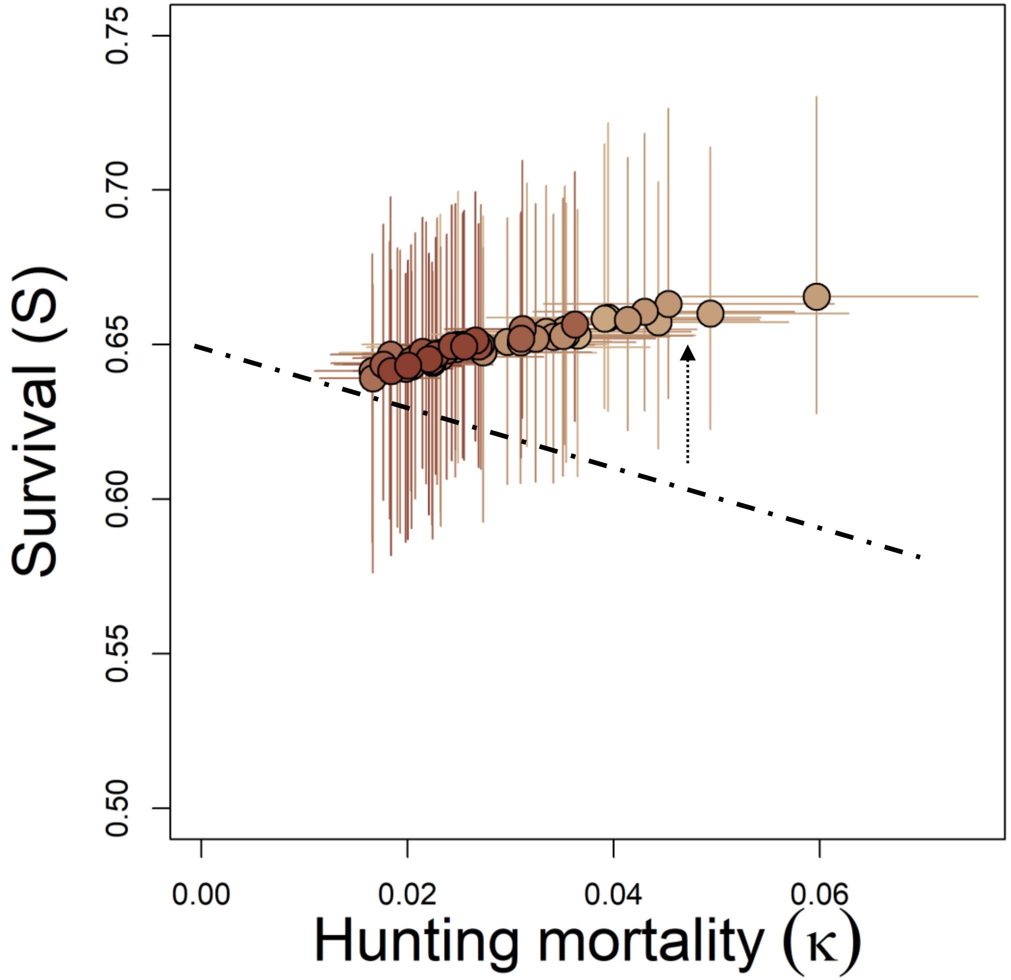
Inference can change substantially via causal hypotheses

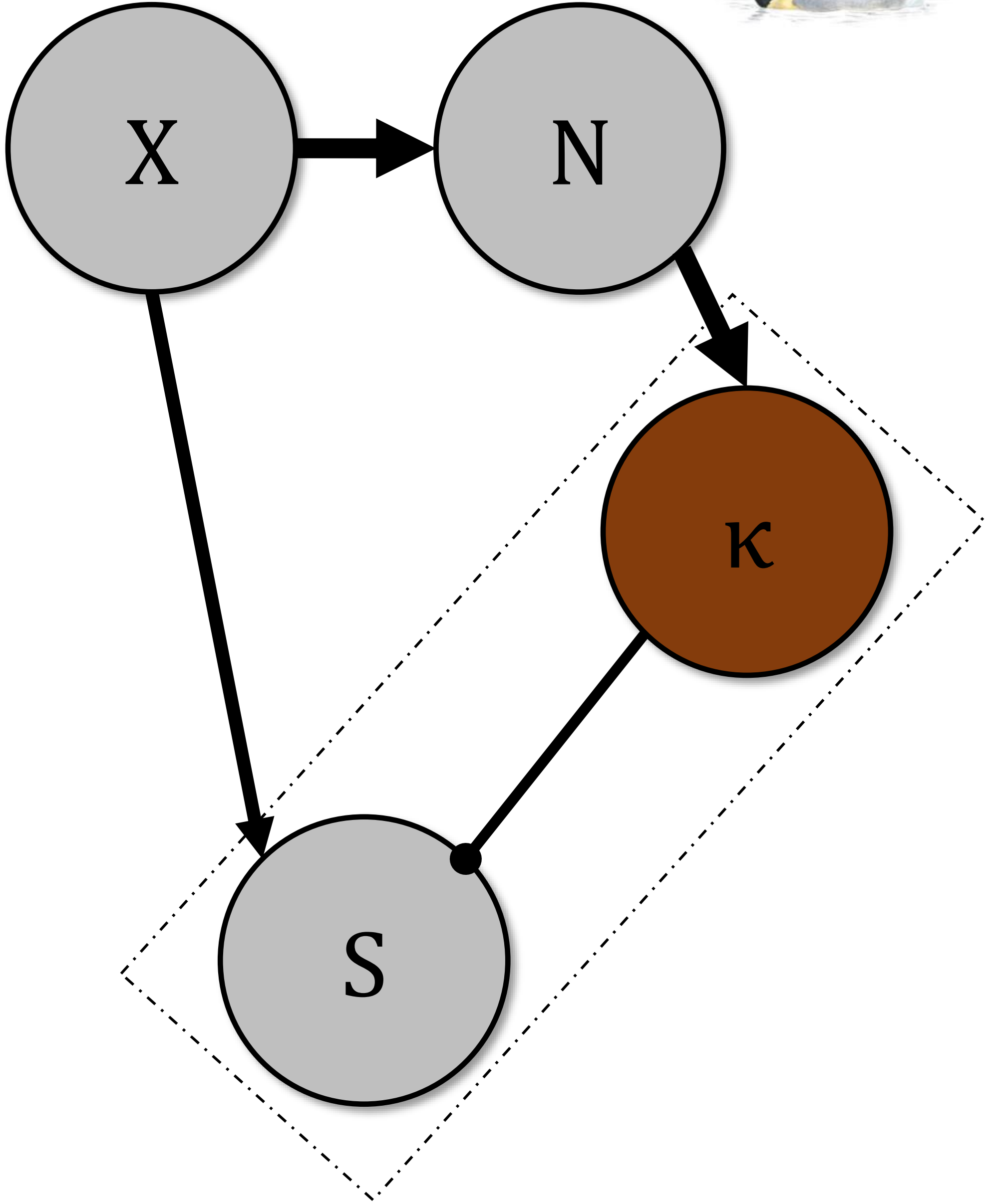
When we ‘fix’ independent variables to estimate effects



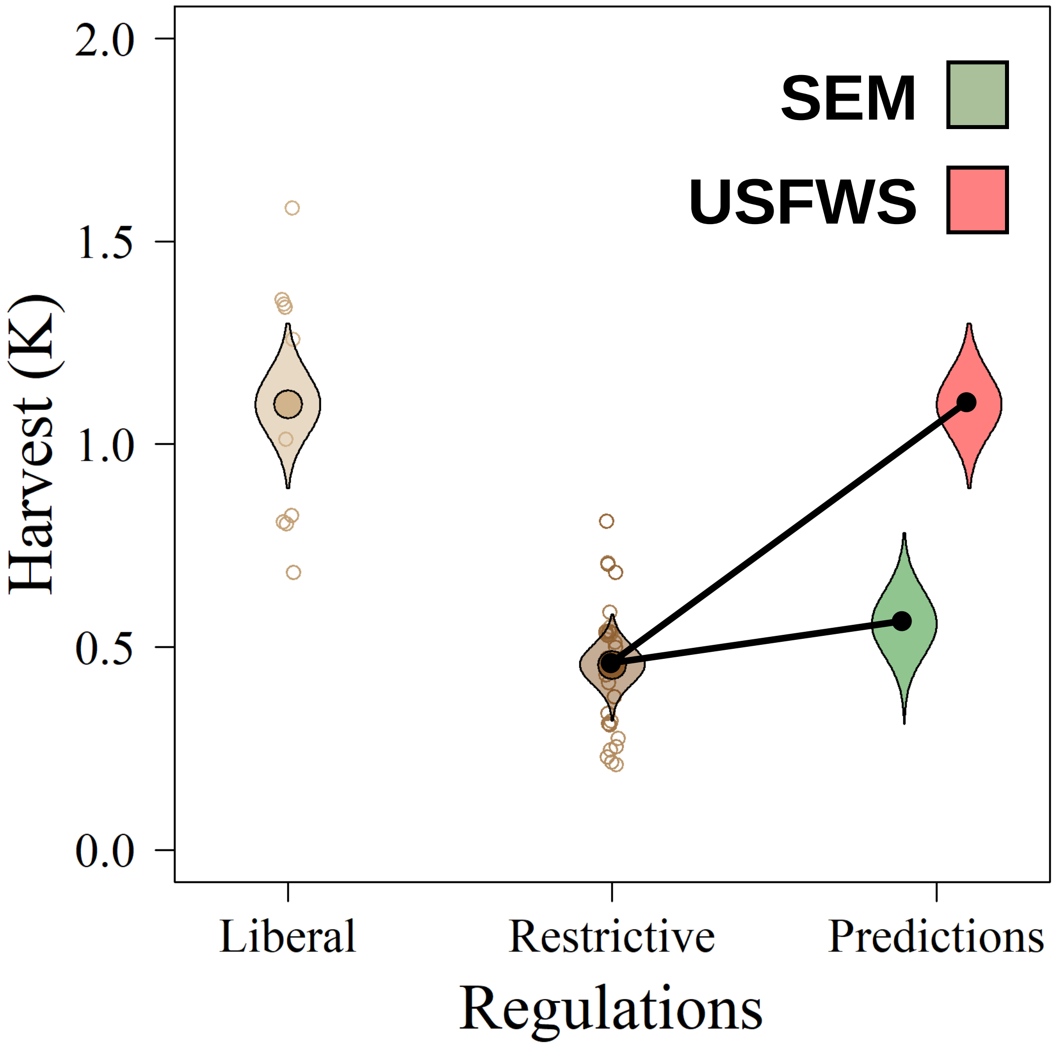
We ignore the fact that changes in one covariate can affect another

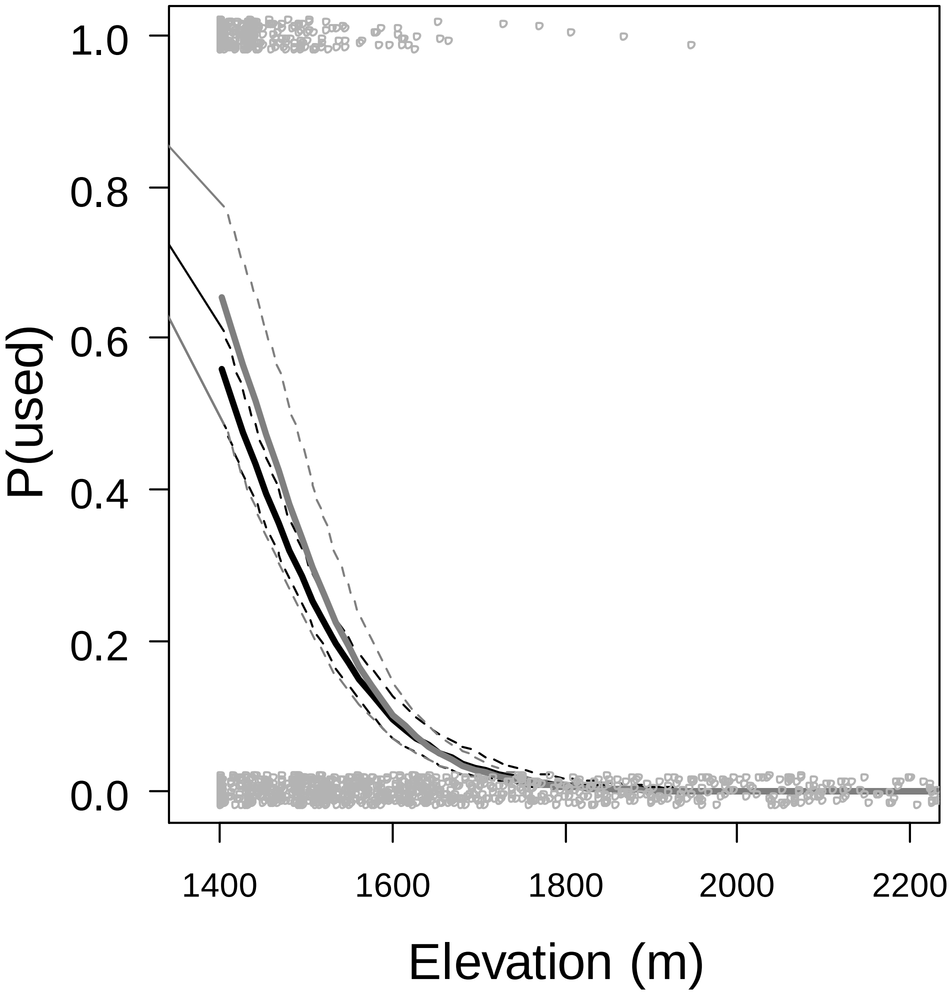
This can lead to absurdity!



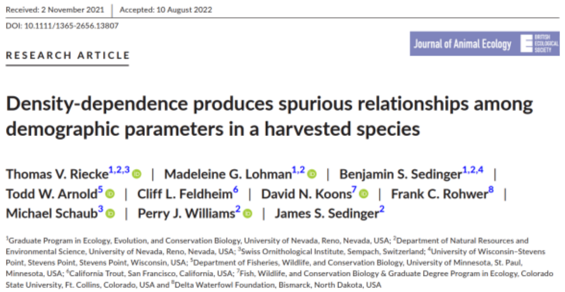


It’s more likely that it will lead to smaller yet important changes





We can build substantially more complex models.



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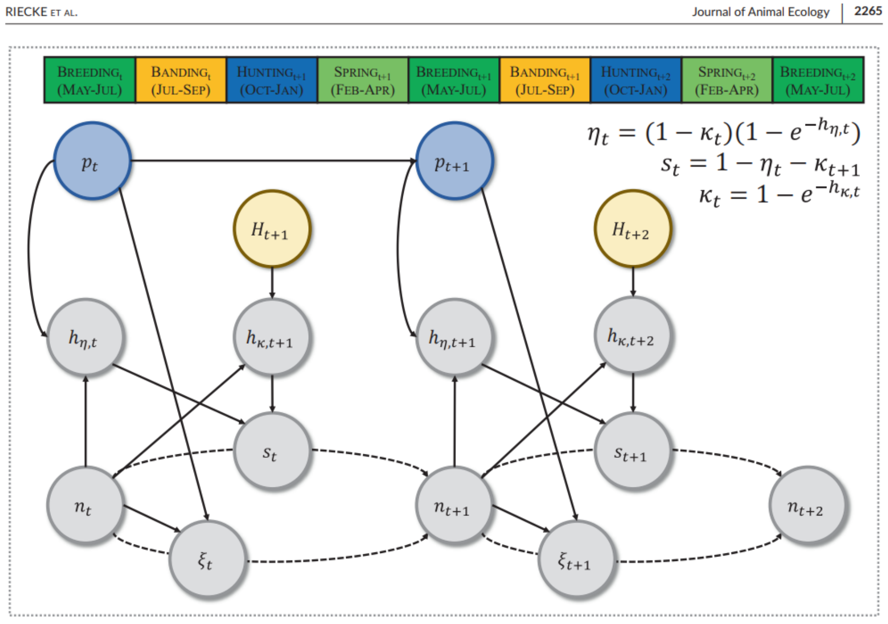
Journal of Animal Ecology

RESEARCH ARTICLE

Density-dependence produces spurious relationships among demographic parameters in a harvested species

Thomas V. Riecke[1,2,3] | Madeleine G. Lohman[1,2] | Benjamin S. Sedinger[1,2,4] | Todd W. Arnold[5] | Cliff L. Feldheim[6] | David N. Koons[7] | Frank C. Rohwer[8] | Michael Schaub[3] | Perry J. Williams[2] | James S. Sedinger[2]

[1]Graduate Program in Ecology, Evolution, and Conservation Biology, University of Nevada, Reno, Nevada, USA; [2]Department of Natural Resources and Environmental Science, University of Nevada, Reno, Nevada, USA; [3]Swiss Ornithological Institute, Sempach, Switzerland; [4]University of Wisconsin–Stevens Point, Stevens Point, Stevens Point, Wisconsin, USA; [5]Department of Fisheries, Wildlife, and Conservation Biology, University of Minnesota, St. Paul, Minnesota, USA; [6]California Trout, San Francisco, California, USA; [7]Fish, Wildlife, and Conservation Biology & Graduate Degree Program in Ecology, Colorado State University, Ft. Collins, Colorado, USA and [8]Delta Waterfowl Foundation, Bismarck, North Dakota, USA



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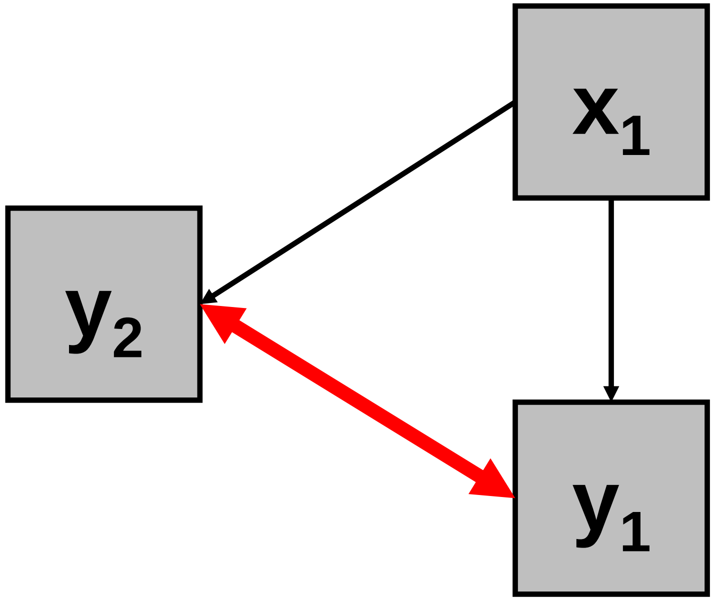
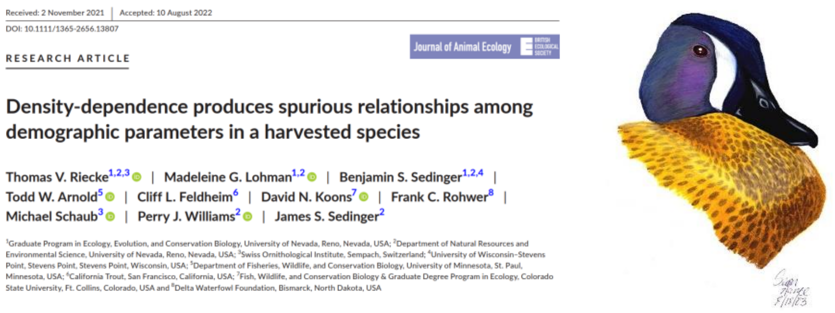
BREEDING$_t$ (MAY-JUL)
BANDING$_t$ (JUL-SEP)
HUNTING$_{t+1}$ (OCT-JAN)
SPRING$_{t+1}$ (FEB-APR)
BREEDING$_{t+1}$ (MAY-JUL)
BANDING$_{t+1}$ (JUL-SEP)
HUNTING$_{t+2}$ (OCT-JAN)
SPRING$_{t+2}$ (FEB-APR)
BREEDING$_{t+2}$ (MAY-JUL)
$\eta_t = (1 - \kappa_t)(1 - e^{-h_{\eta,t}})$
$s_t = 1 - \eta_t - \kappa_{t+1}$
$\kappa_t = 1 - e^{-h_{\kappa,t}}$
p_t
p_{t+1}
H_{t+1}
H_{t+2}
$h_{\eta,t}$
$h_{\kappa,t+1}$
$h_{\eta,t+1}$
$h_{\kappa,t+2}$
s_t
s_{t+1}
n_t
n_{t+1}
n_{t+2}
ξ_t
ξ_{t+1}

FIGURE 2 A directed acyclic graph demonstrating the relationships among abundance (**n**), ponds (**p**; blue), fecundity (ξ), hunting mortality hazard rate (**h**$_\kappa$), natural mortality hazard rate (**h**$_\eta$), survival (**s**) and the number of duck hunters (**H**; brown) for blue-winged teal breeding in the North American Prairie Pothole Region across the annual cycle (1973–3016). Solid arrows represent estimated directional relationships, and dashed arrows represent processes leading to changes in population abundance.

The same assumptions inherent to linear models apply, plus…



The importance of causal diagrams



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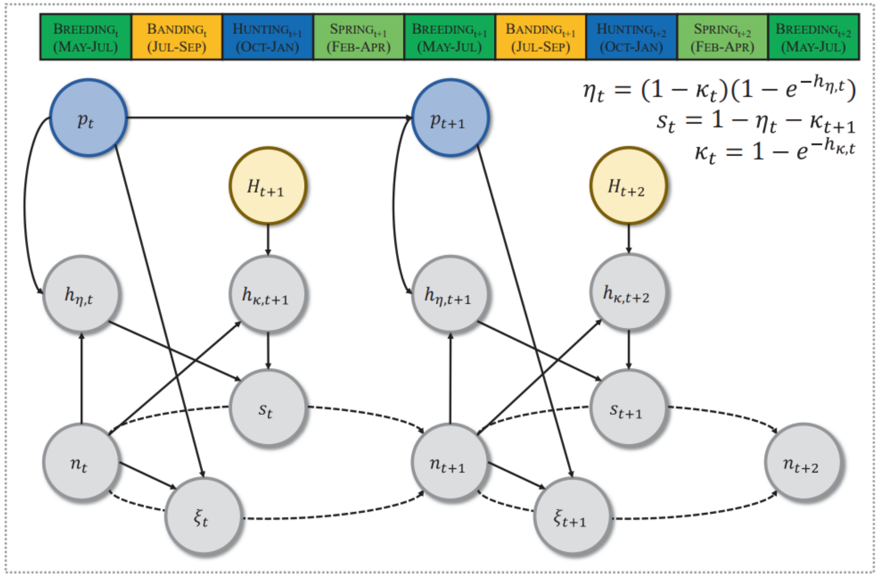
BREEDING$_t$ (MAY-JUL)
BANDING$_t$ (JUL-SEP)
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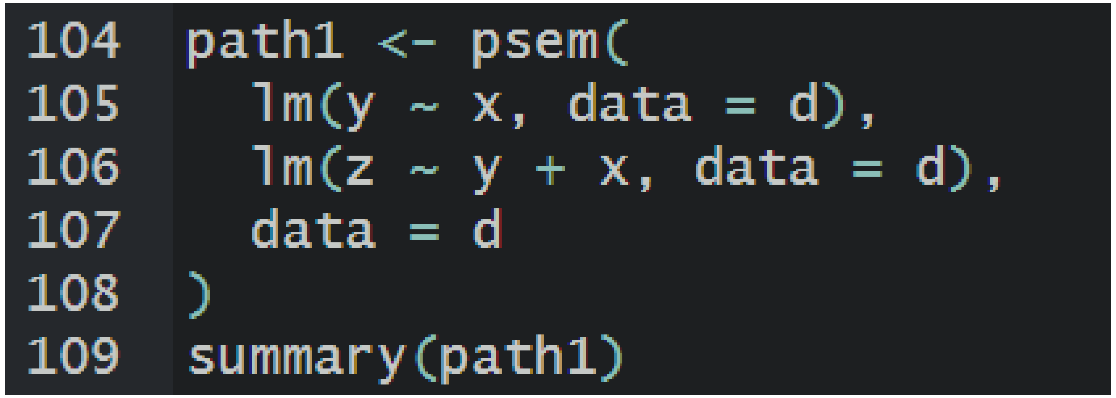
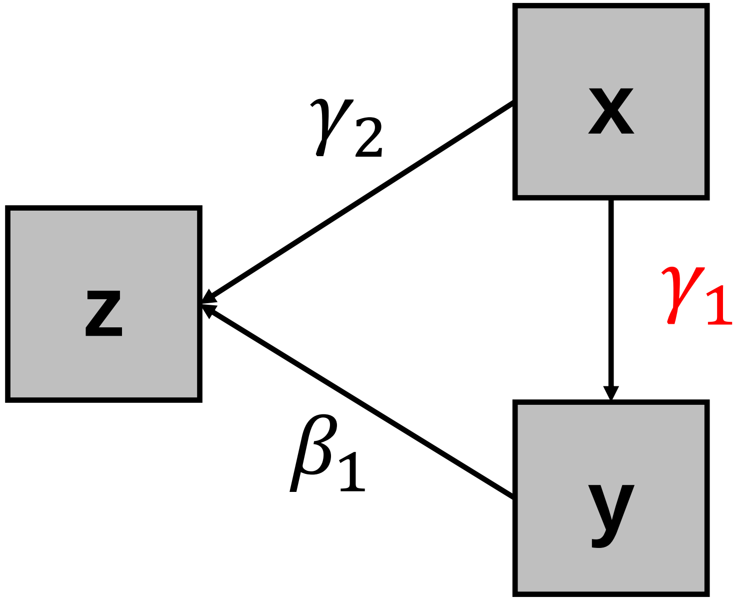
BREEDING$_{t+2}$ (MAY-JUL)
$\eta_t = (1-\kappa_t)(1-e^{-h_{\eta,t}})$
$s_t = 1 - \eta_t - \kappa_{t+1}$
$\kappa_t = 1 - e^{-h_{\kappa,t}}$
p_t
p_{t+1}
H_{t+1}
H_{t+2}
$h_{\eta,t}$
$h_{\kappa,t+1}$
$h_{\eta,t+1}$
$h_{\kappa,t+2}$
s_t
s_{t+1}
n_t
n_{t+1}
n_{t+2}
ξ_t
ξ_{t+1}

FIGURE 2 A directed acyclic graph demonstrating the relationships among abundance (***n***), ponds (***p***; blue), fecundity (*ξ*), hunting mortality hazard rate (***h***$_\kappa$), natural mortality hazard rate (***h***$_\eta$), survival (**s**) and the number of duck hunters (***H***; brown) for blue-winged teal breeding in the North American Prairie Pothole Region across the annual cycle (1973–3016). Solid arrows represent estimated directional relationships, and dashed arrows represent processes leading to changes in population abundance.



```
path1 <- psem(
  lm(y ~ x, data = d),
  lm(z ~ y + x, data = d),
  data = d
)
summary(path1)
```



Next up, latent variables…