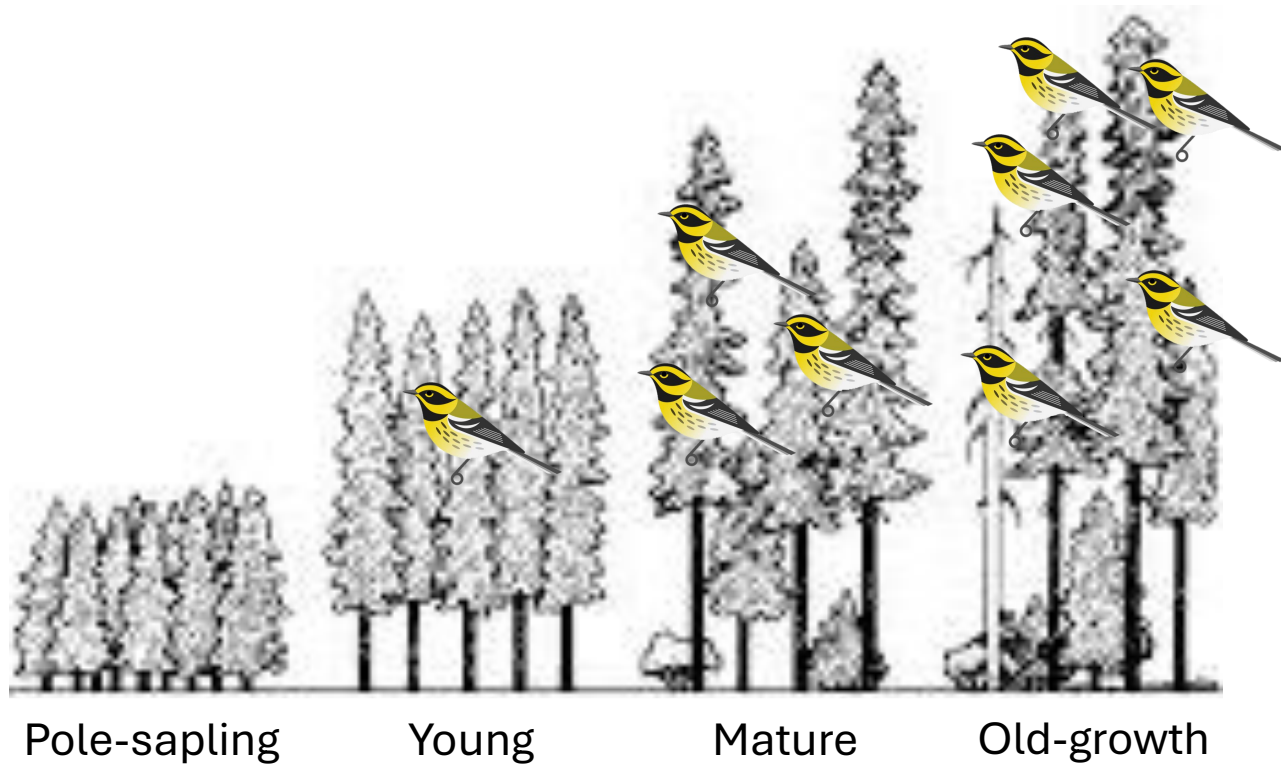
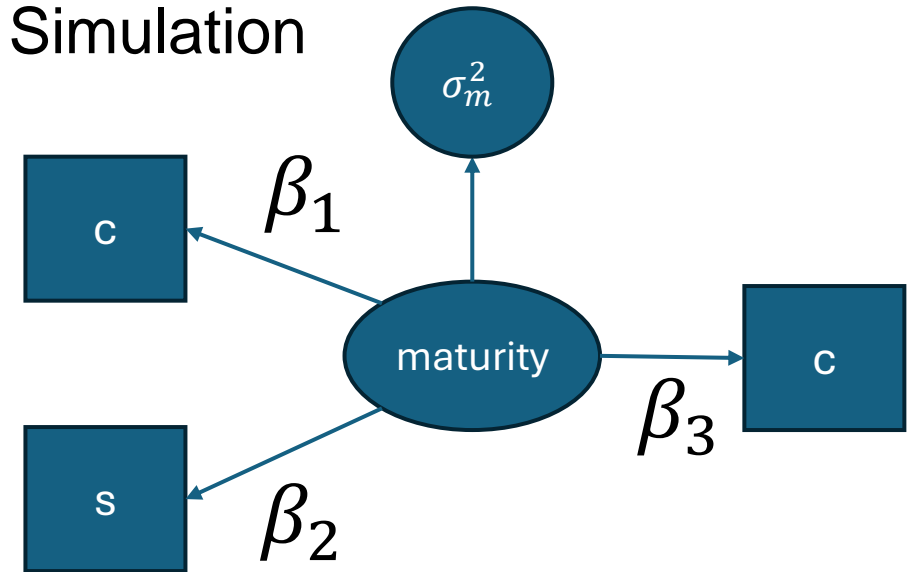


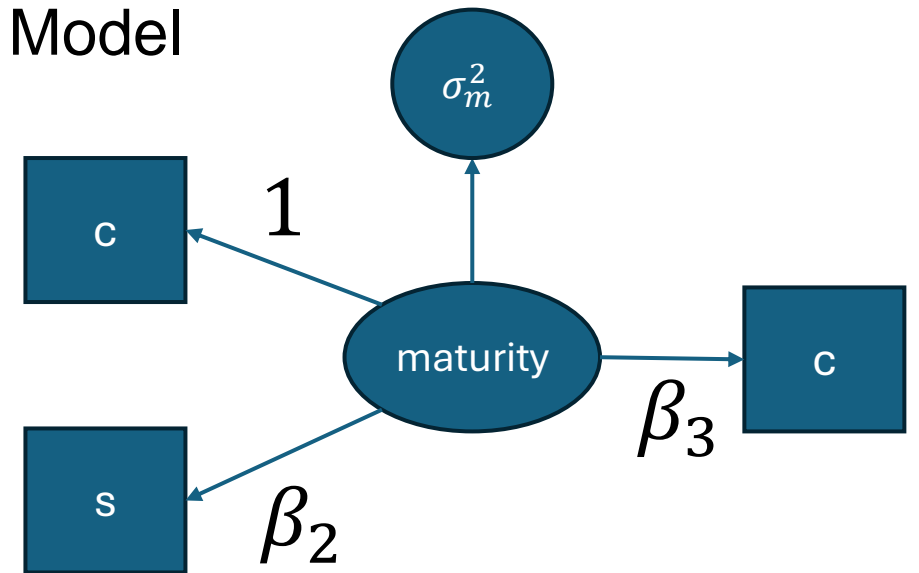
Latent variables: part II



Simulation



Model



What is a latent (or hidden) variable?



A random variable that is unmeasured but not necessarily unmeasurable.

-P Spirtes (2001)

A variable that is not directly observable but is inferred from other variables that can be measured

-Generative AI (yesterday)

Variables that can only be inferred indirectly through a mathematical model from other observable variables

-Wikipedia (also yesterday)

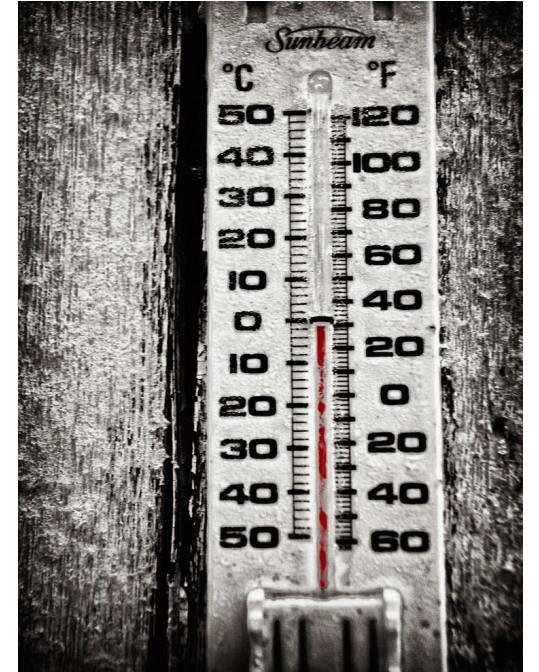
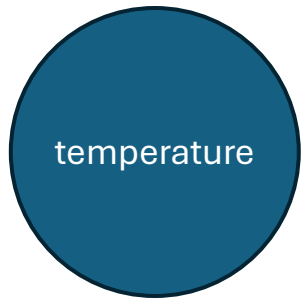
What is a latent (or hidden) variable?



Everything is a latent variable – **LA Dyer**



Is temperature a latent variable?

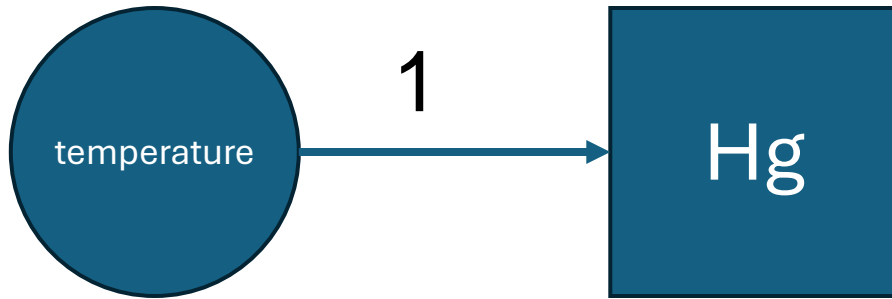


A random variable that is unmeasured but not necessarily unmeasurable.

-P Spirtes (2001)

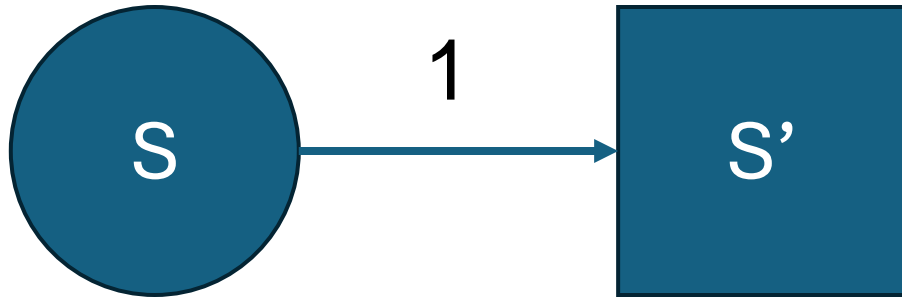
Temperature is the average kinetic energy of particles

Temperature is a latent variable



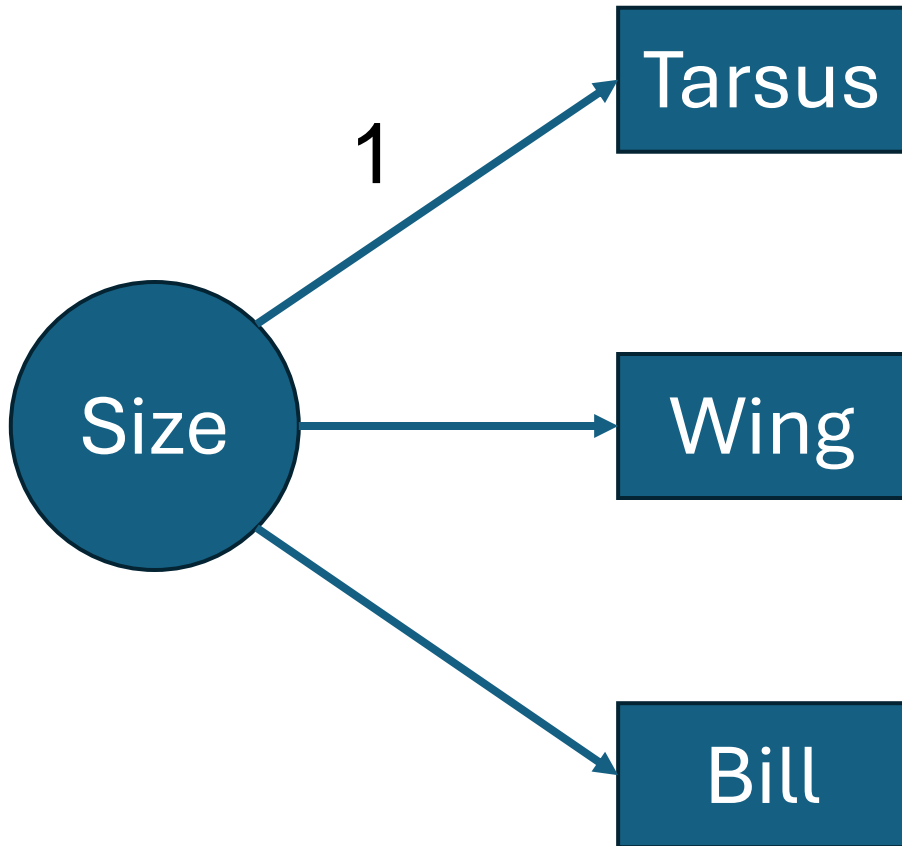
Temperature is the average kinetic energy of particles

Survival



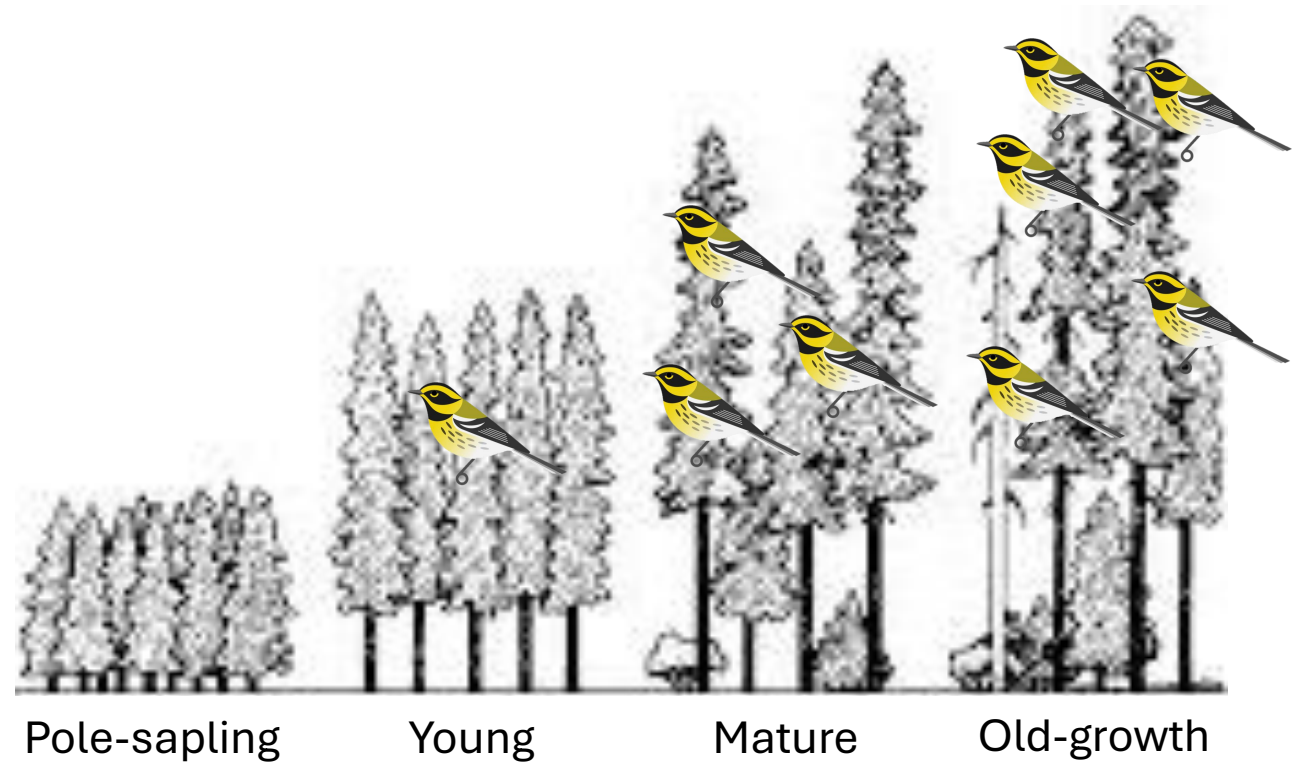
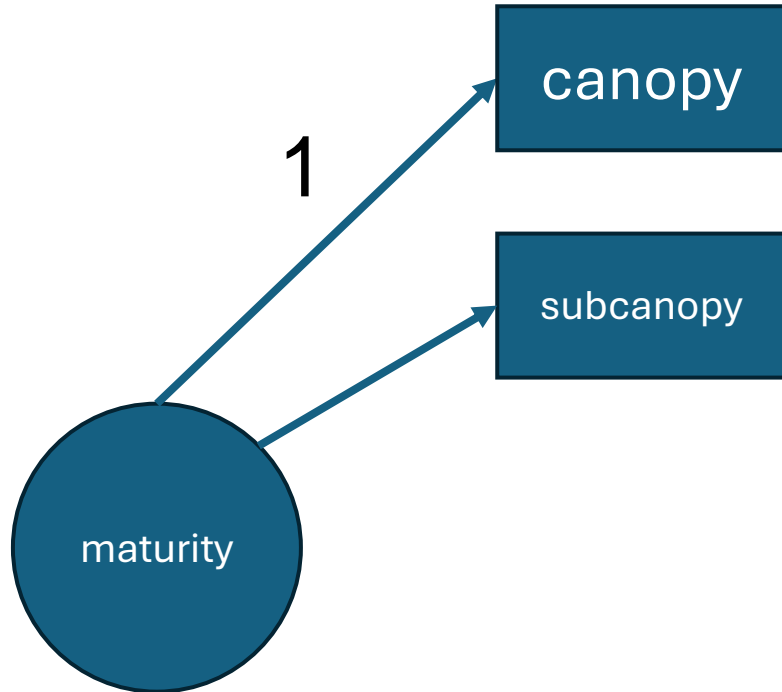
S: survival of a population, S': survival of a marked sample

Size



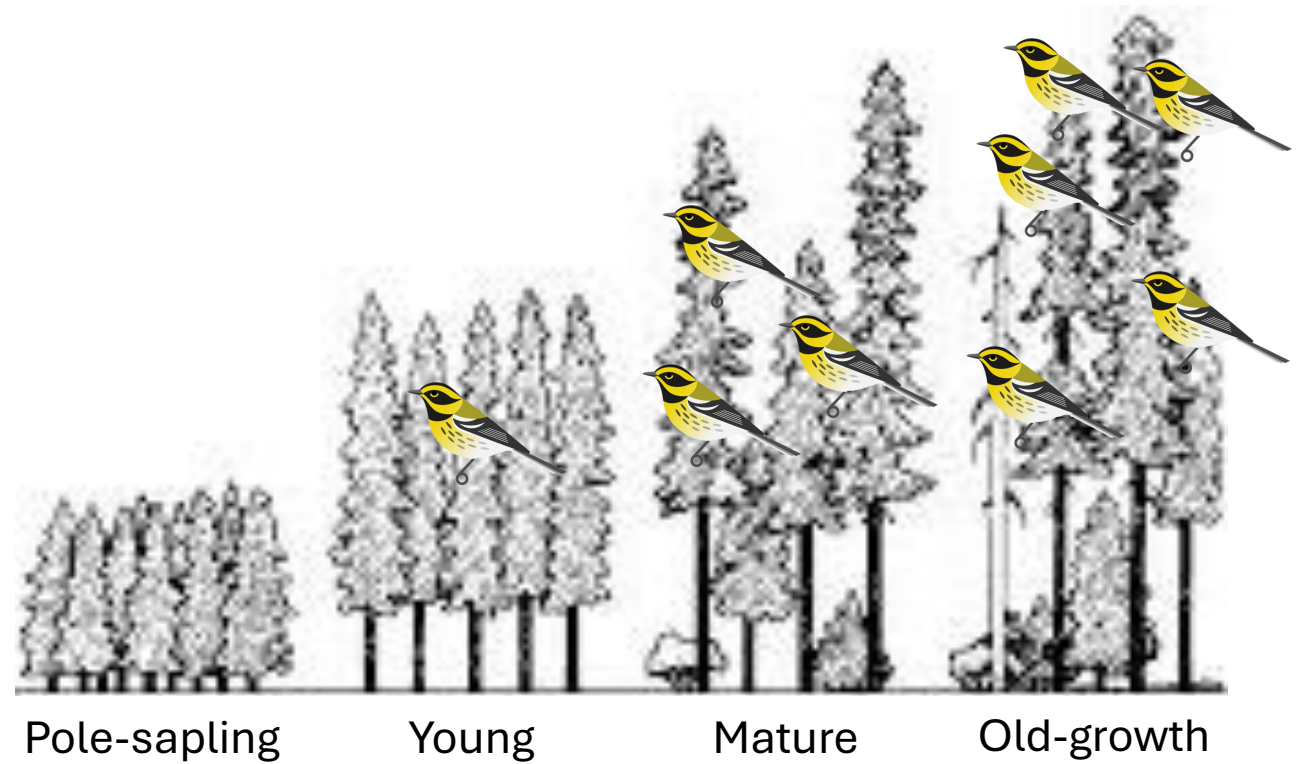
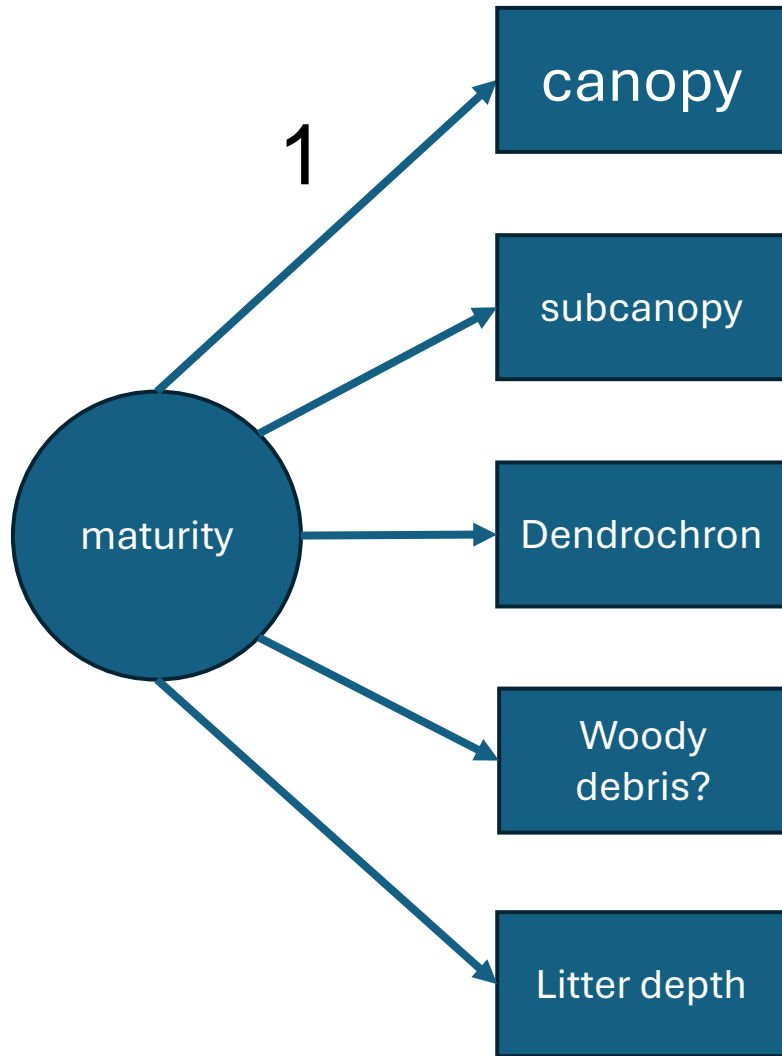
'Size' is a human construct

Forest maturity

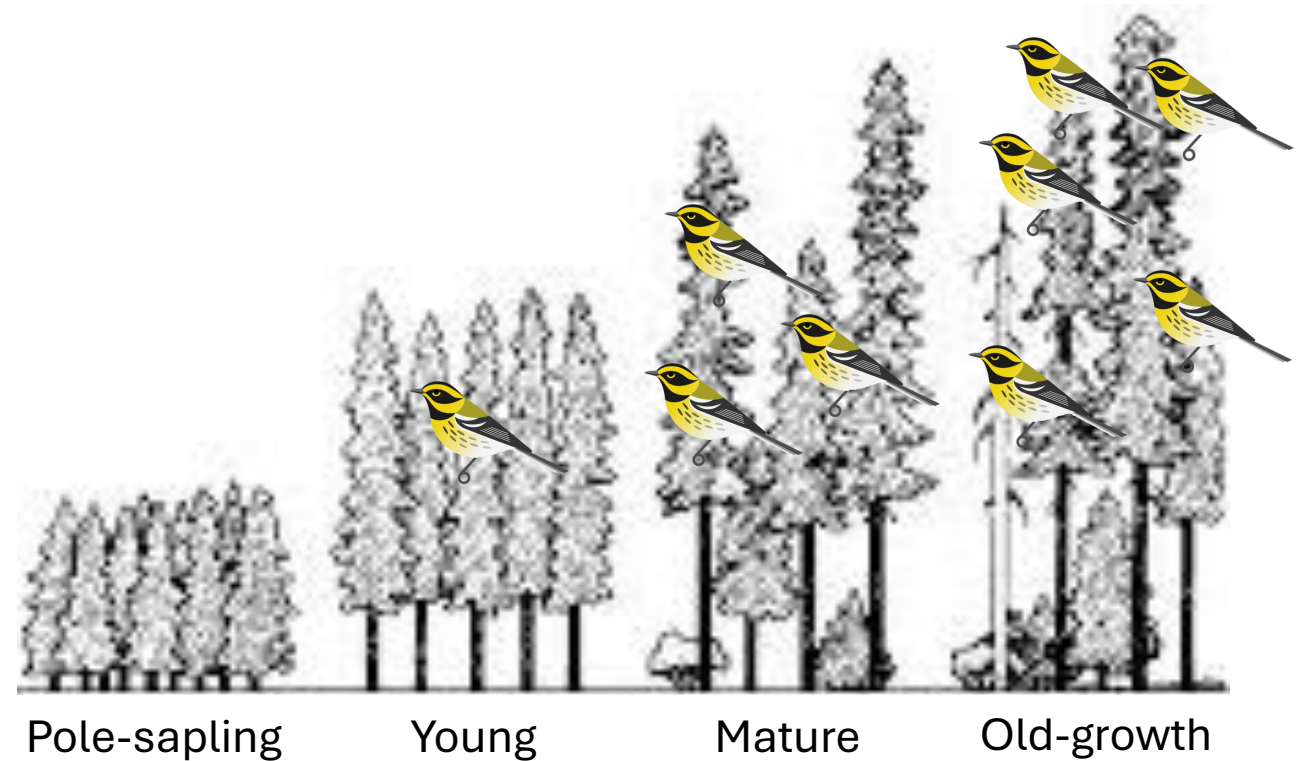
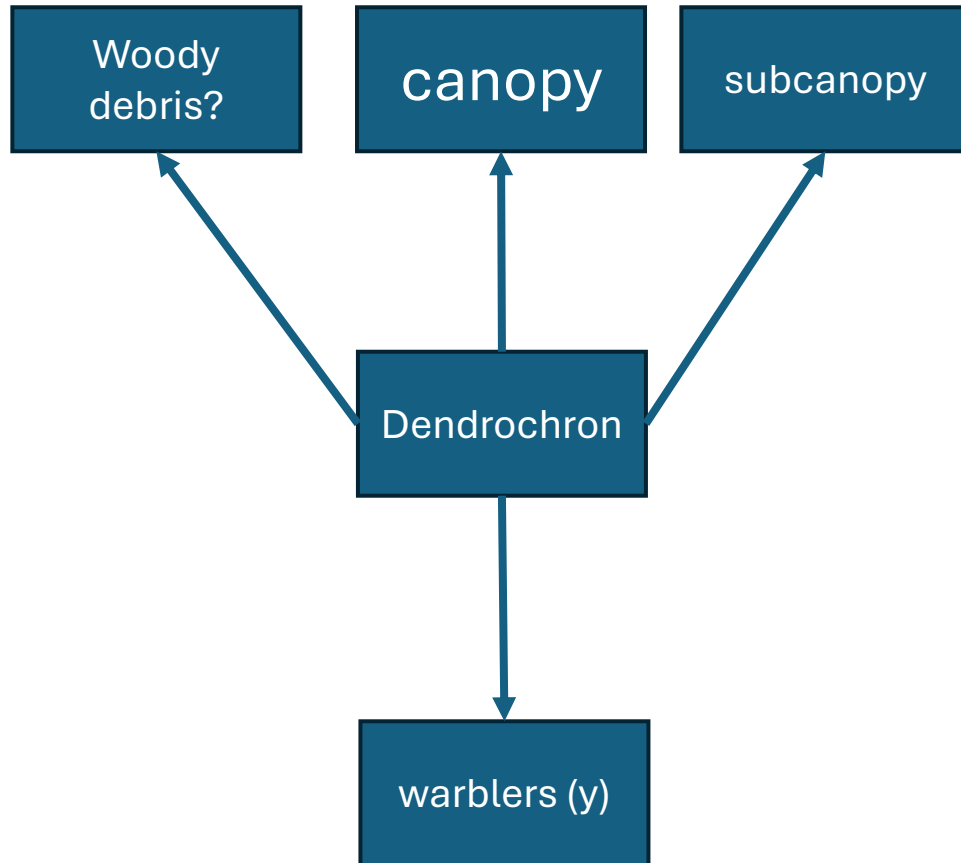


These 'seral stages' are human constructs

Forest maturity [expanded]

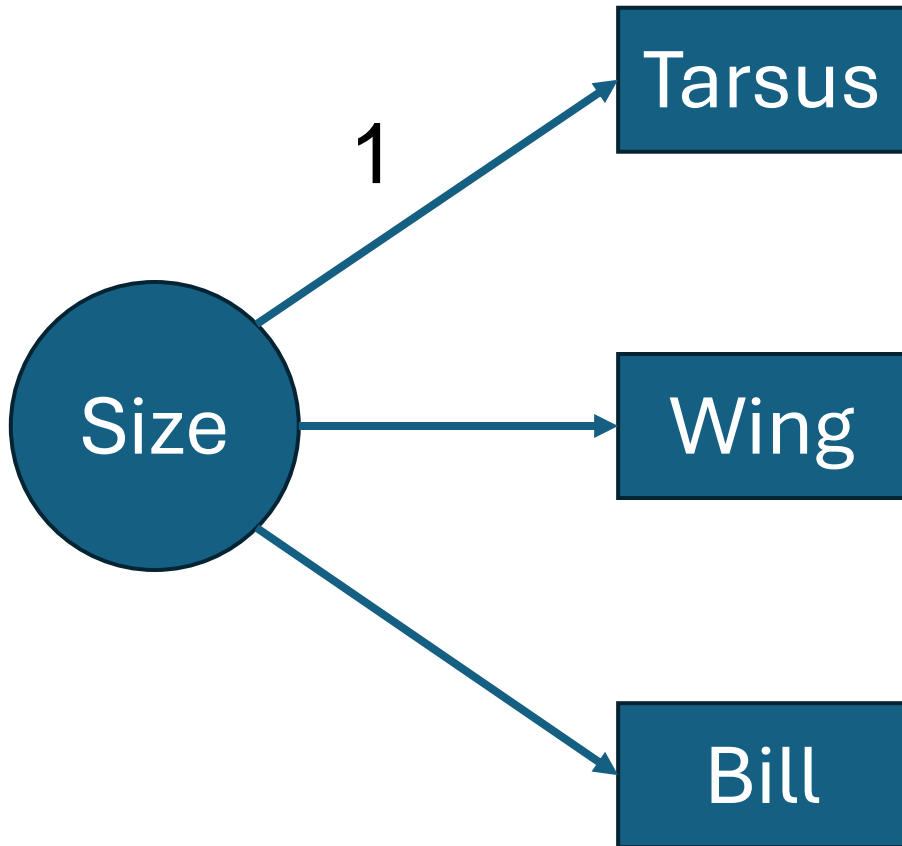


We could structure this differently



We don't have to use latent variables!

They're just very useful...



We can make some assumptions about our latent variable(s)

We generally assume they're normally distributed

$$\mathbf{m} \sim \text{normal}(\mu, \sigma_m^2)$$

We assume that they are zero-centered b/c they're human constructs

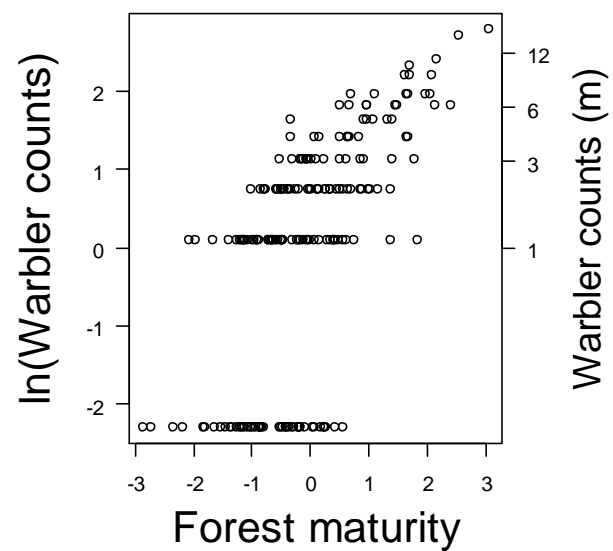
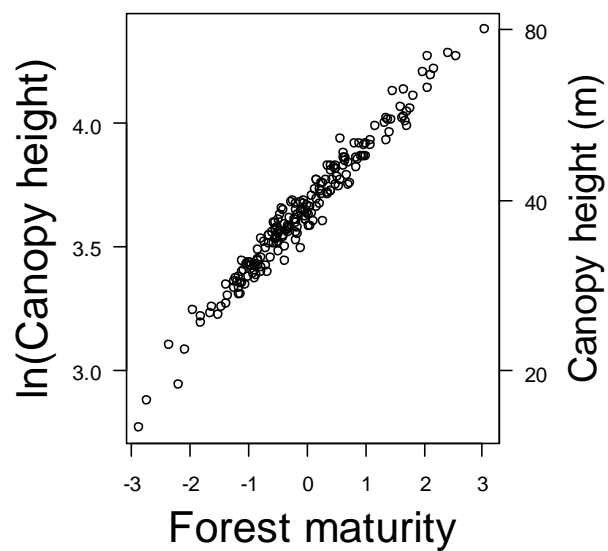
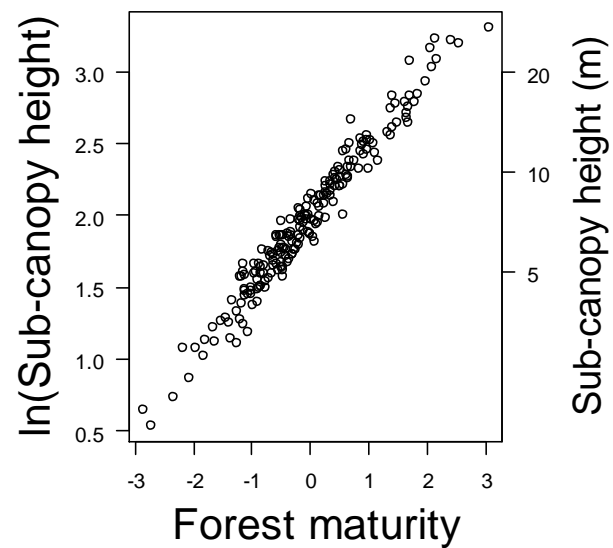
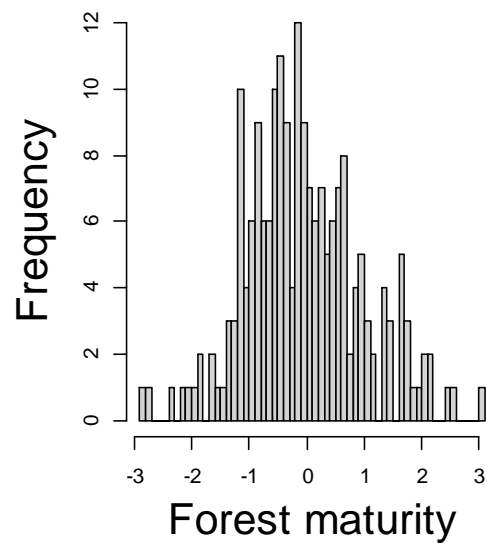
i.e., what should the scale of forest maturity be?

$$\mathbf{m} \sim \text{normal}(0, \sigma_m^2)$$

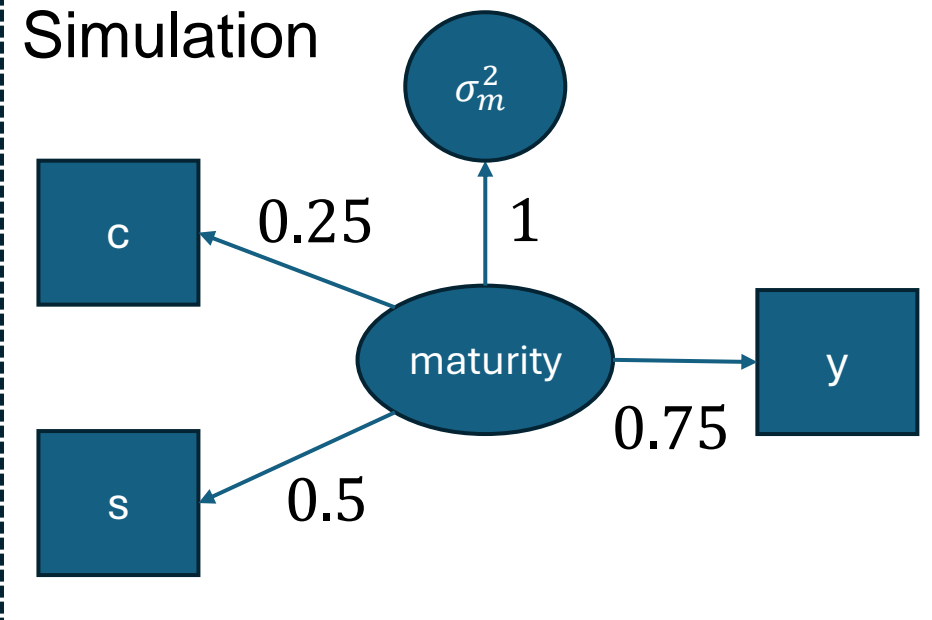
Assigning an intercept would be entirely subjective, plus the math is easier if $\mu = 0$

So let's talk about this 'fixing a loading to 1' thing

Let's simulate some data

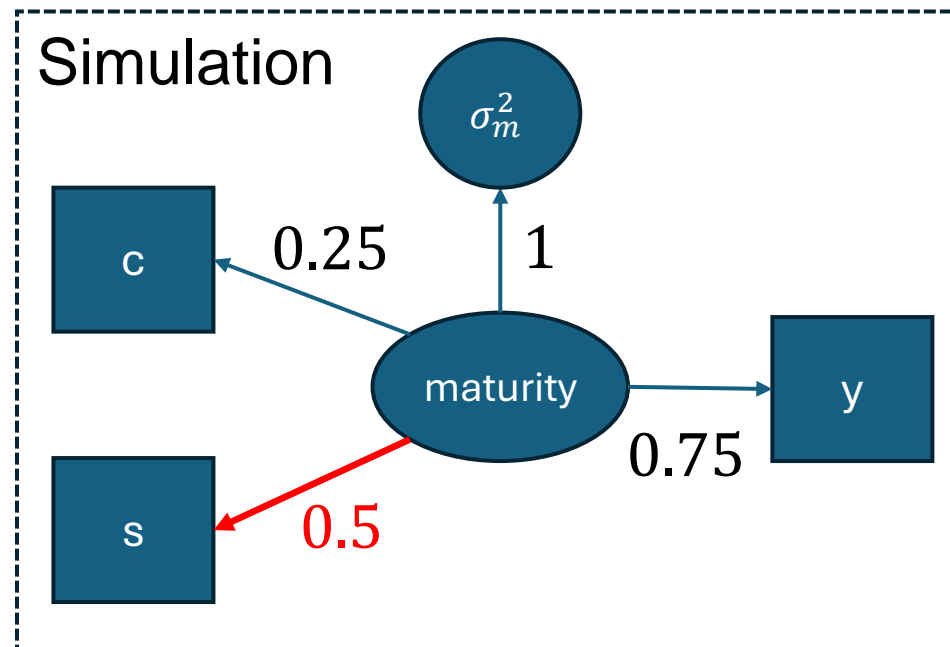
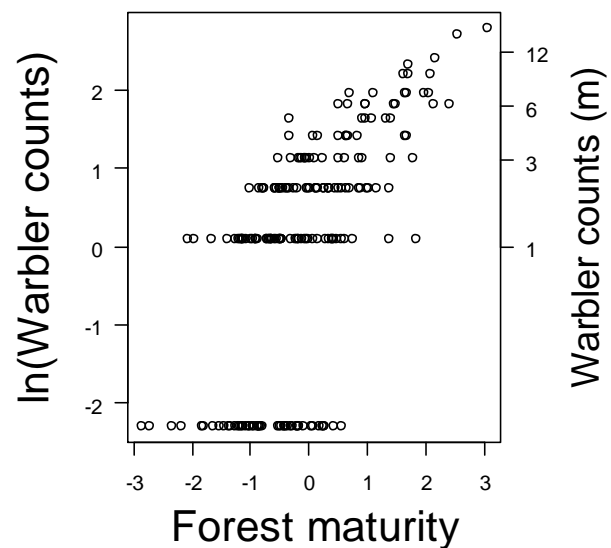
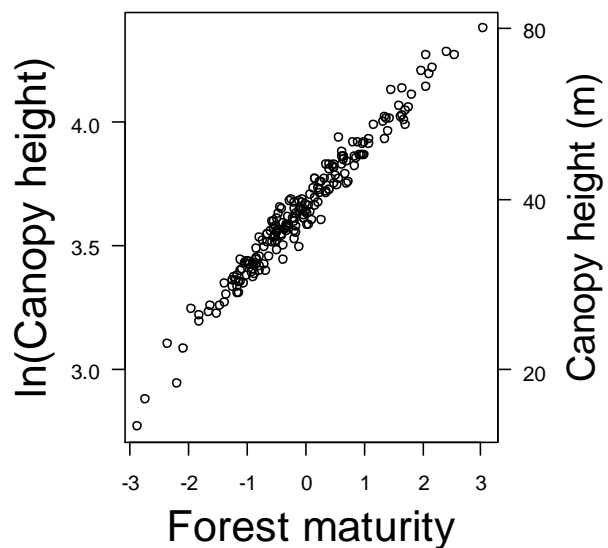
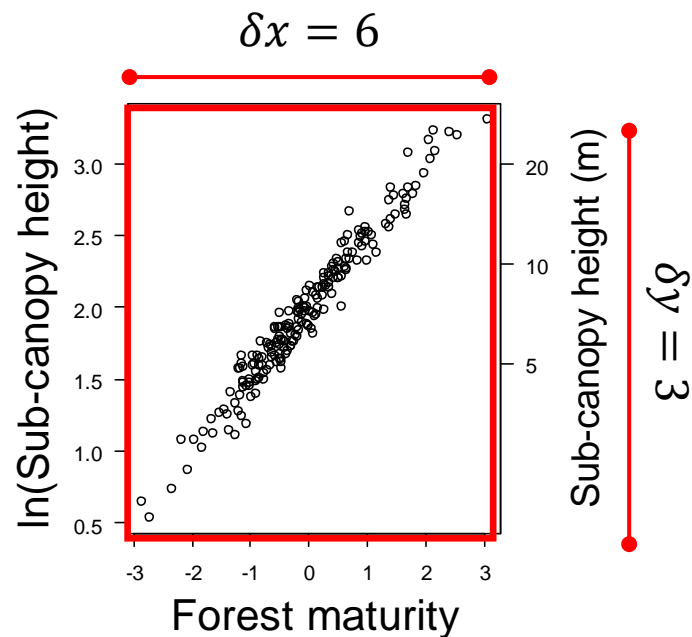
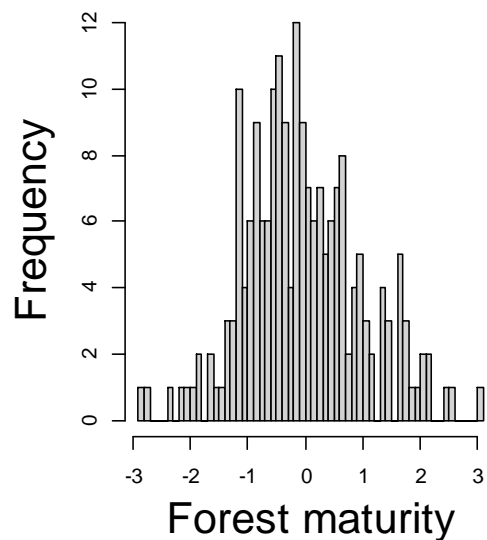


Simulation



$$\delta y = \delta x \beta$$

Clarifying a loading...

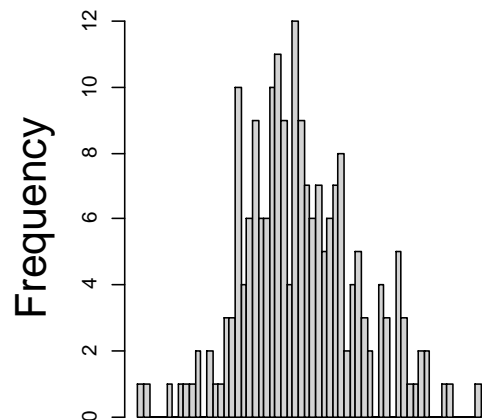


$$\delta y = \delta x \beta$$

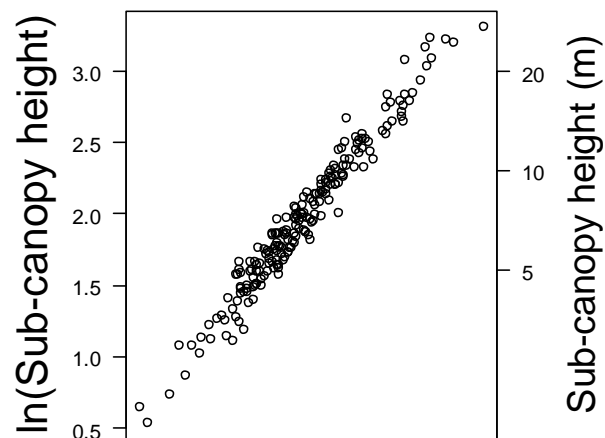
$$3 = 6\beta$$

Now, let's build a model

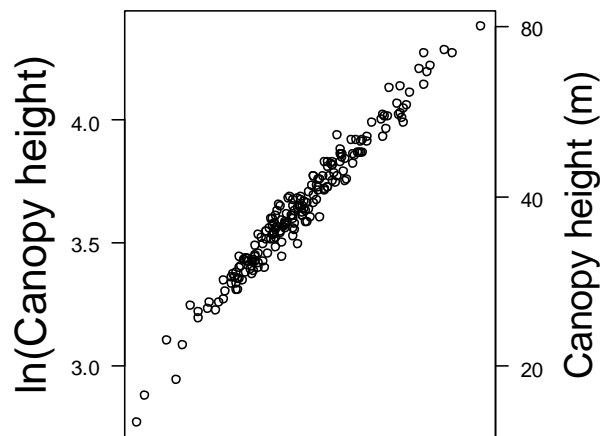
Shoot...



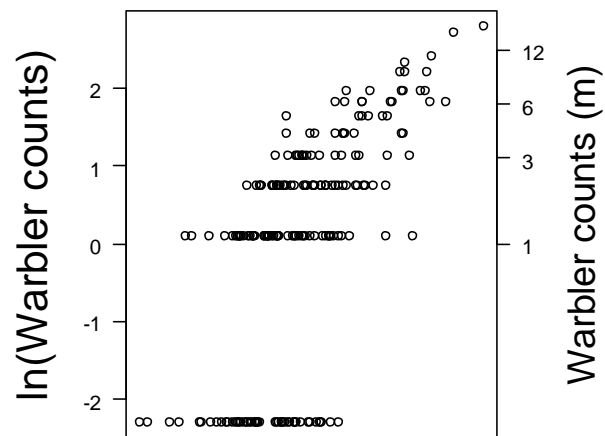
Forest maturity



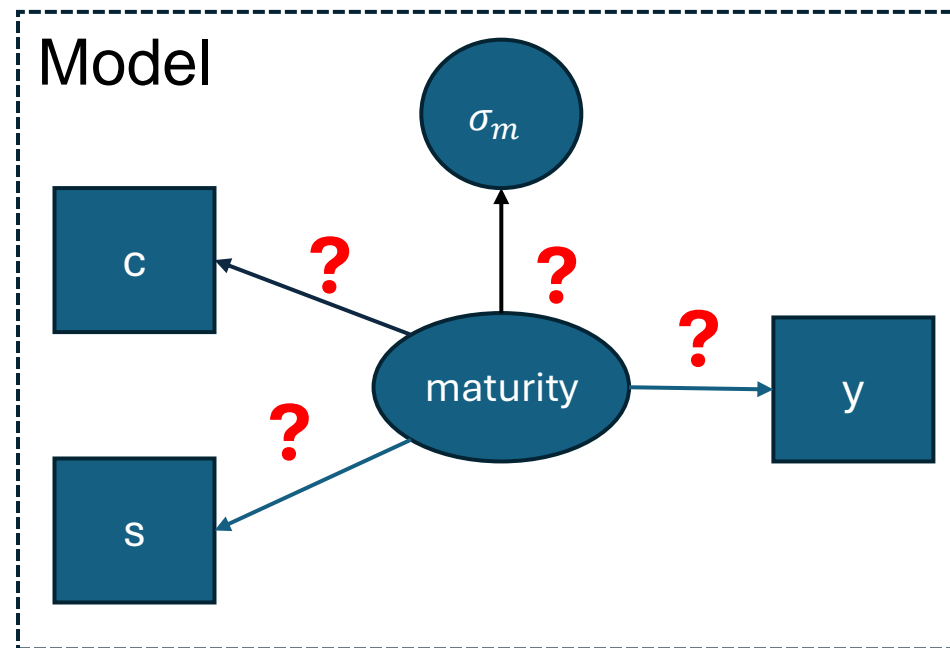
Forest maturity



Forest maturity



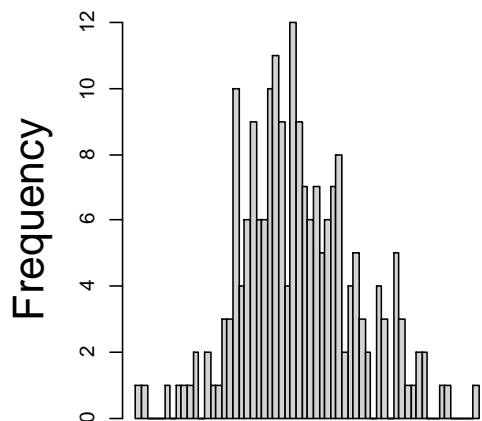
Forest maturity



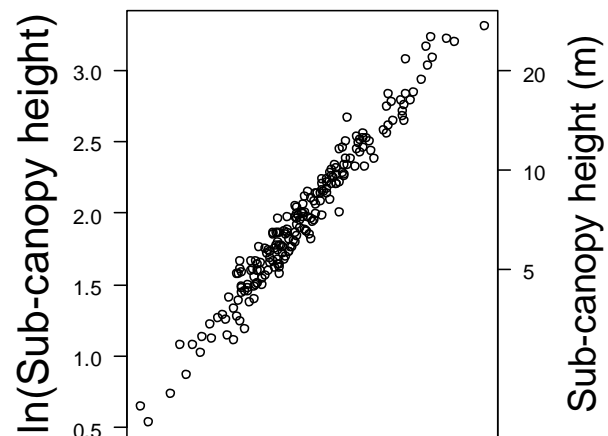
There's a big problem:
We don't know the range of maturity

$$m \sim \text{normal}(0, \sigma_m^2)$$

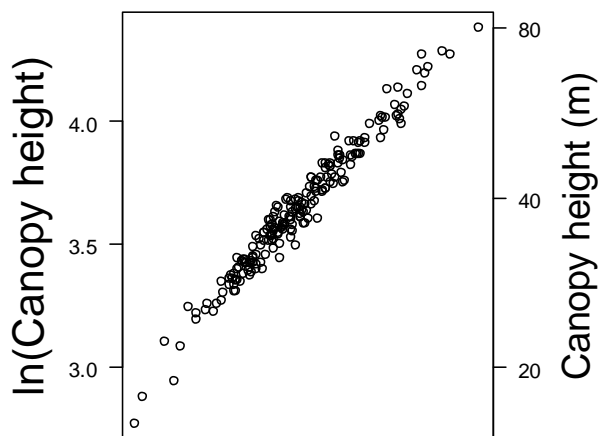
Small groups!



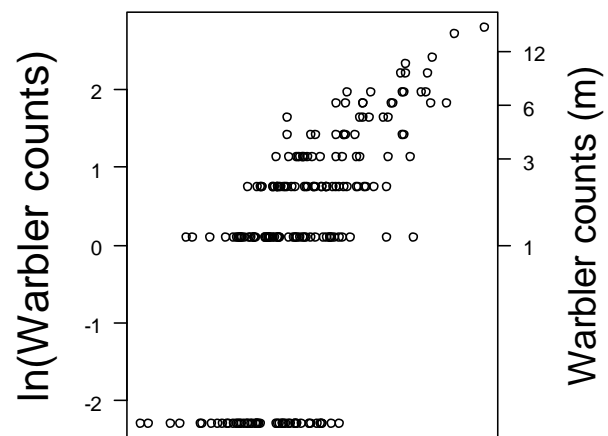
Forest maturity



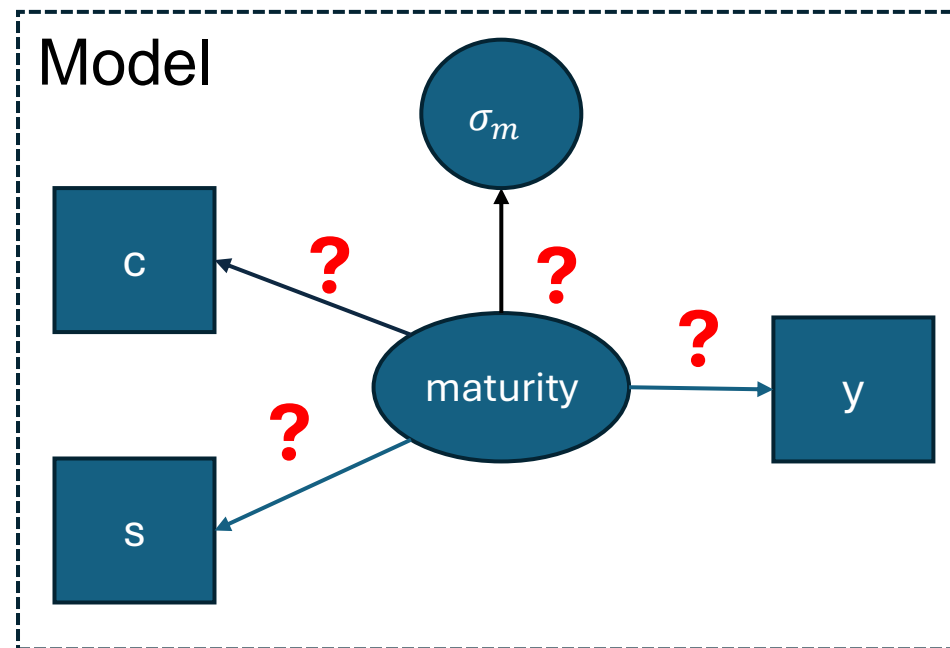
Forest maturity



Forest maturity



Forest maturity

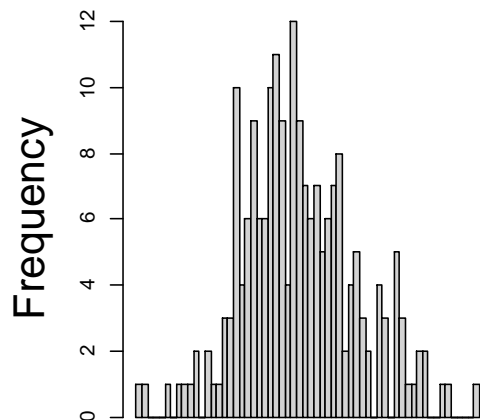


How could we scale maturity, i.e., what should the minimum and maximum values of maturity be?

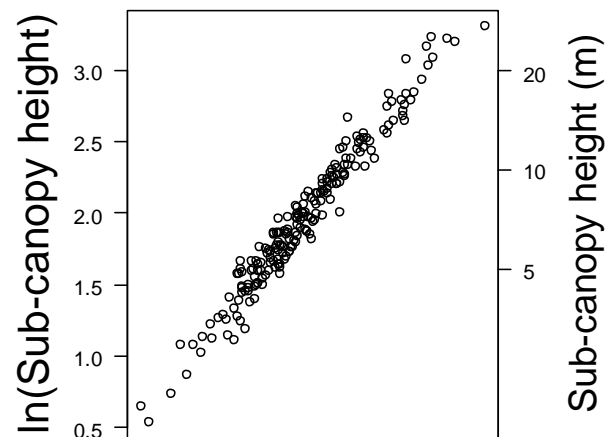
$$\delta y = \delta x \beta$$

$$m \sim \text{normal}(0, \sigma_m^2)$$

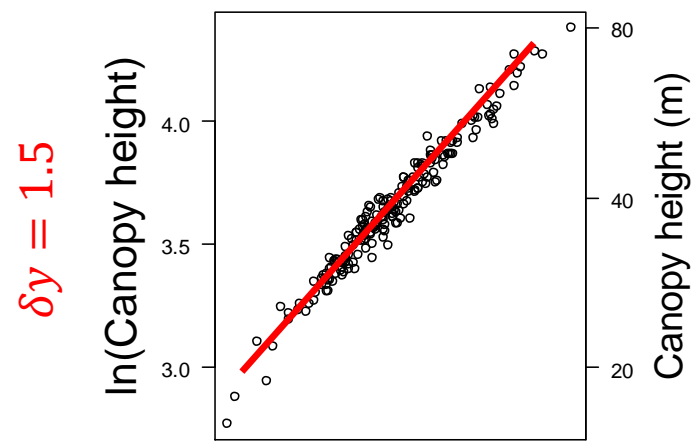
What if we fix a beta?



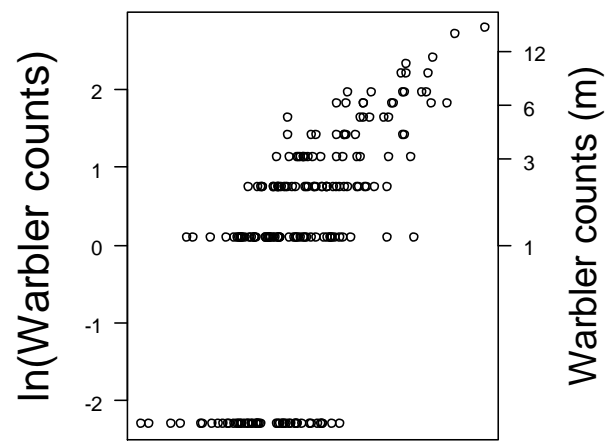
Forest maturity



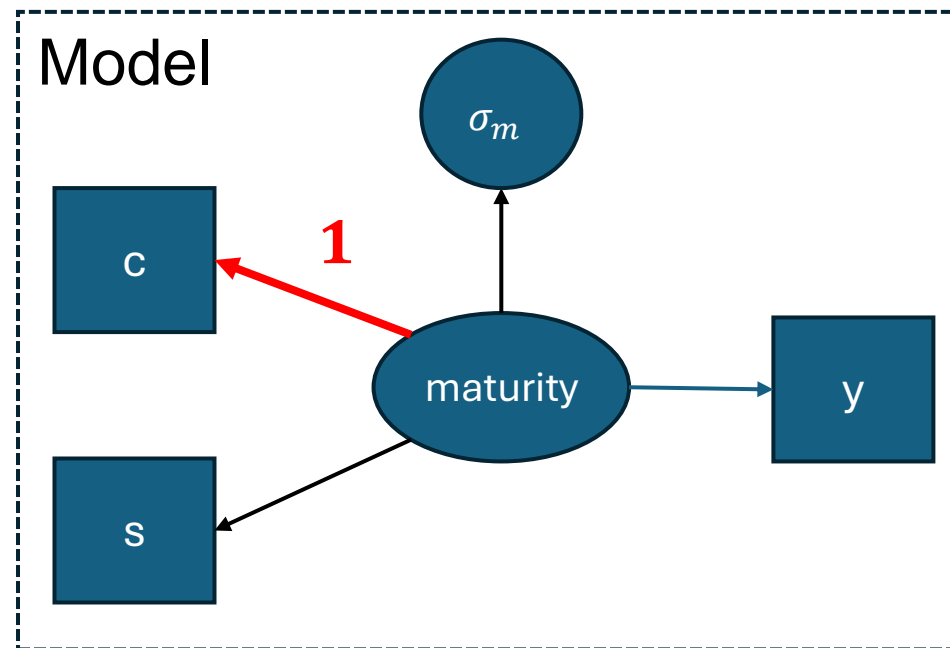
Forest maturity



Forest maturity



Forest maturity

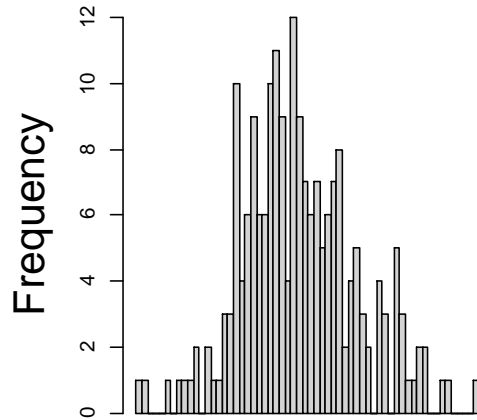


$$\delta y = \delta x \beta$$

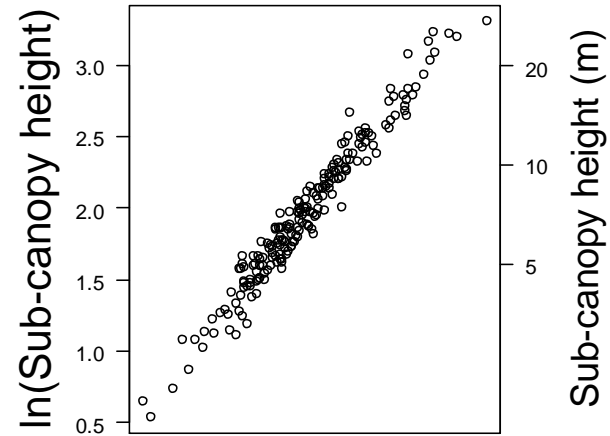
$$1.5 = ? \times 1$$

$$\mathbf{m} \sim \text{normal}(0, \sigma_m^2)$$

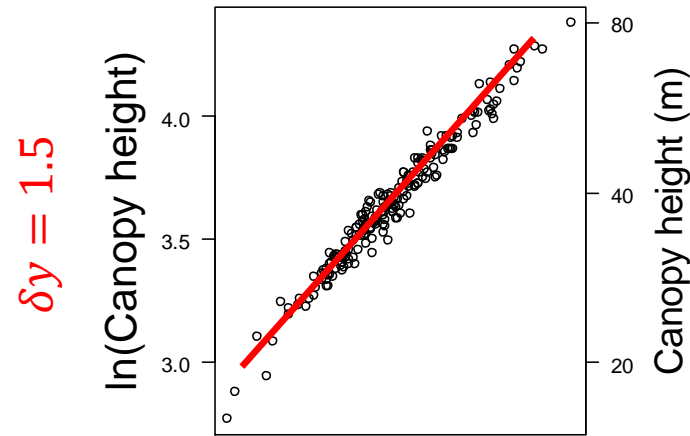
Now we can calculate σ_m



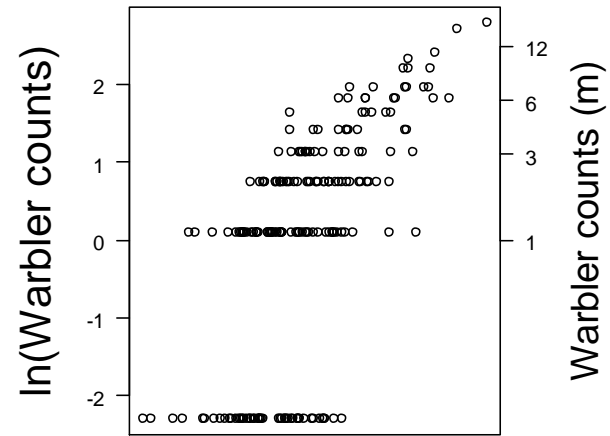
Forest maturity



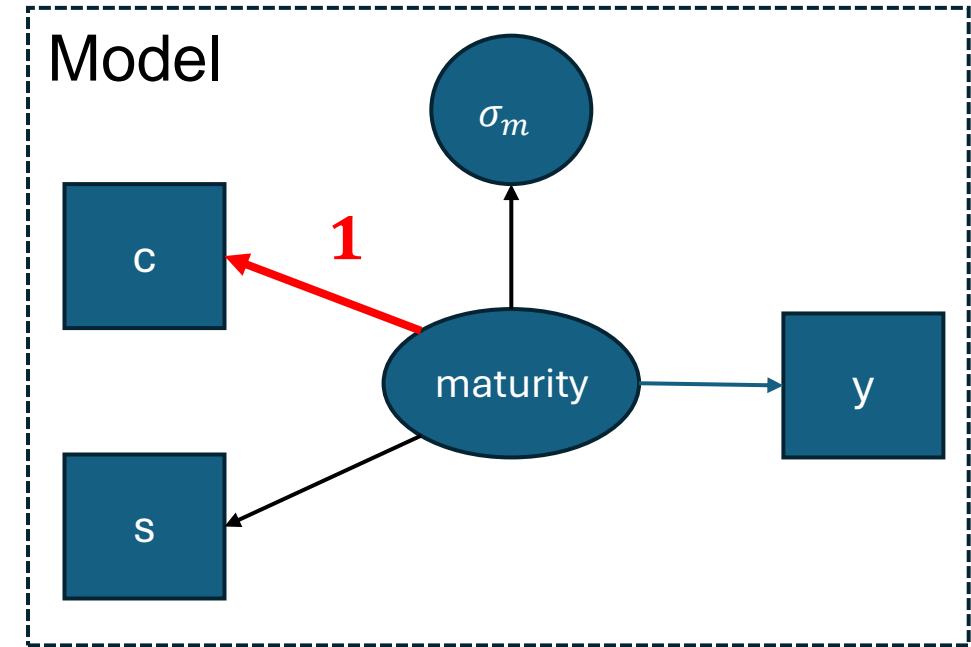
Forest maturity



Forest maturity



Forest maturity

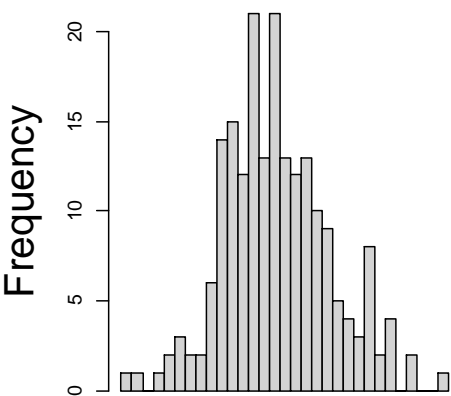


$$\delta y = \delta x \beta$$

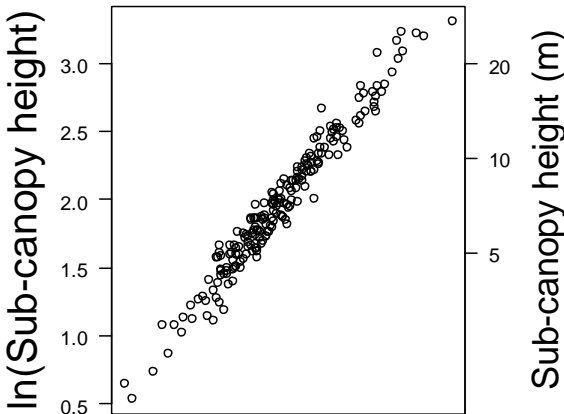
$$1.5 = ? \times 1$$

Our x-axis must have a range of 1.5!

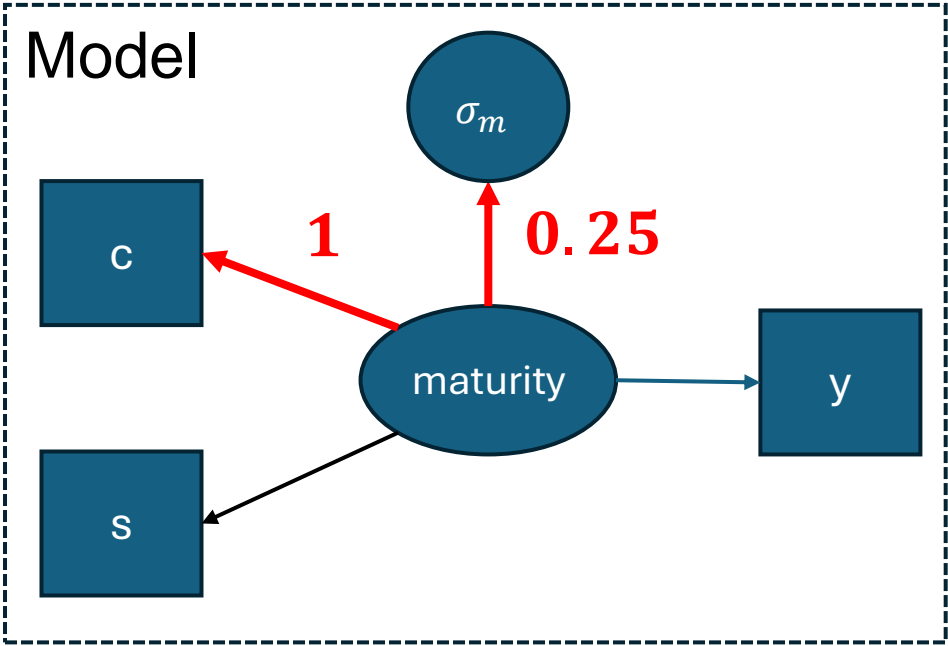
Now we have a scale!!



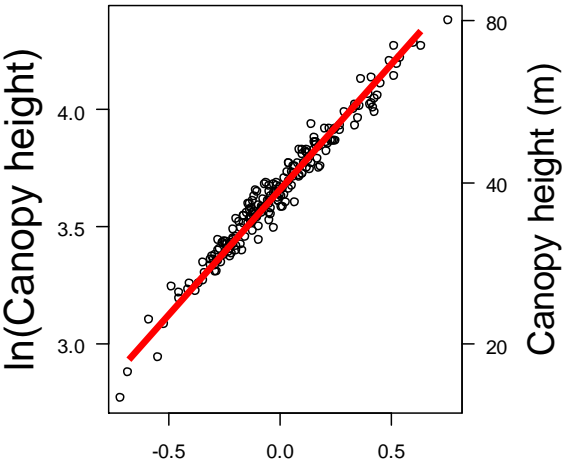
Forest maturity



Forest maturity

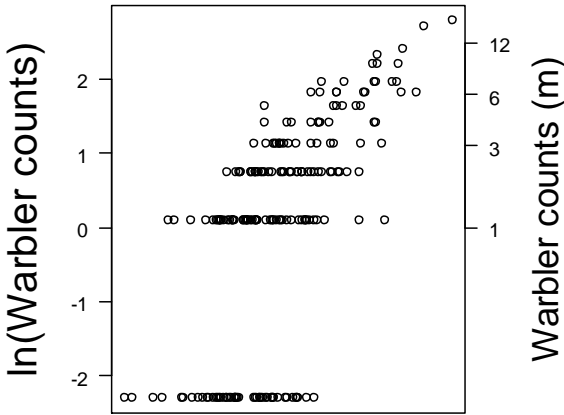


$\delta y = 1.5$

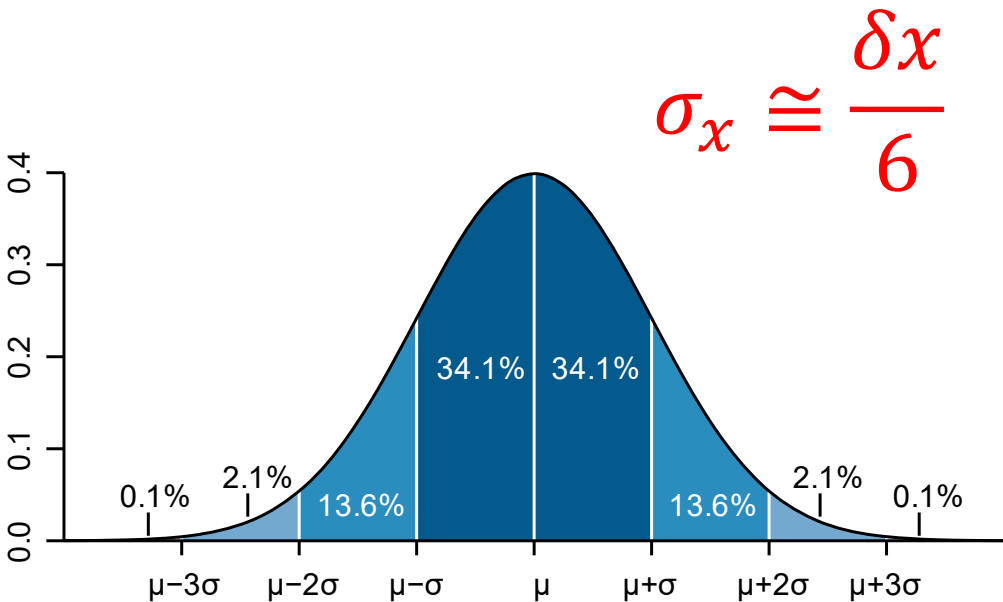


Forest maturity

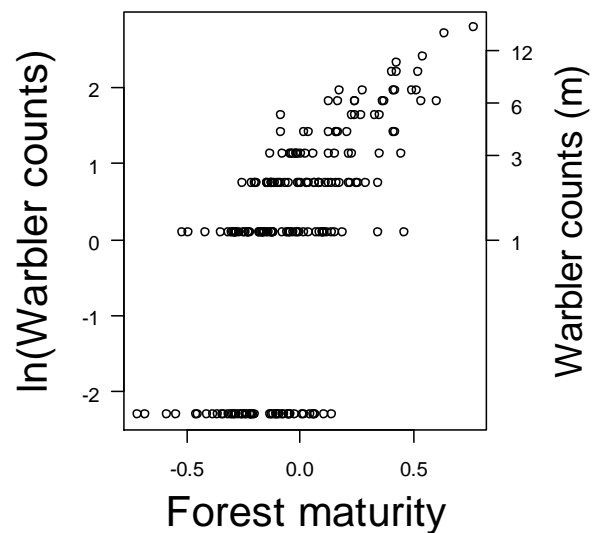
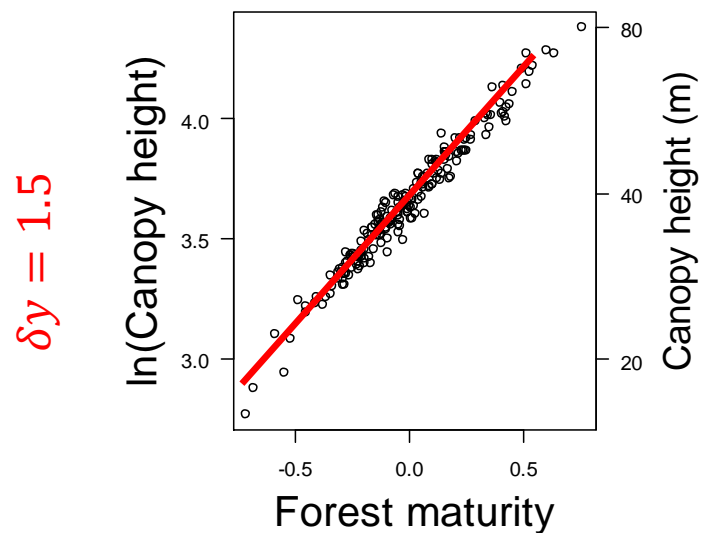
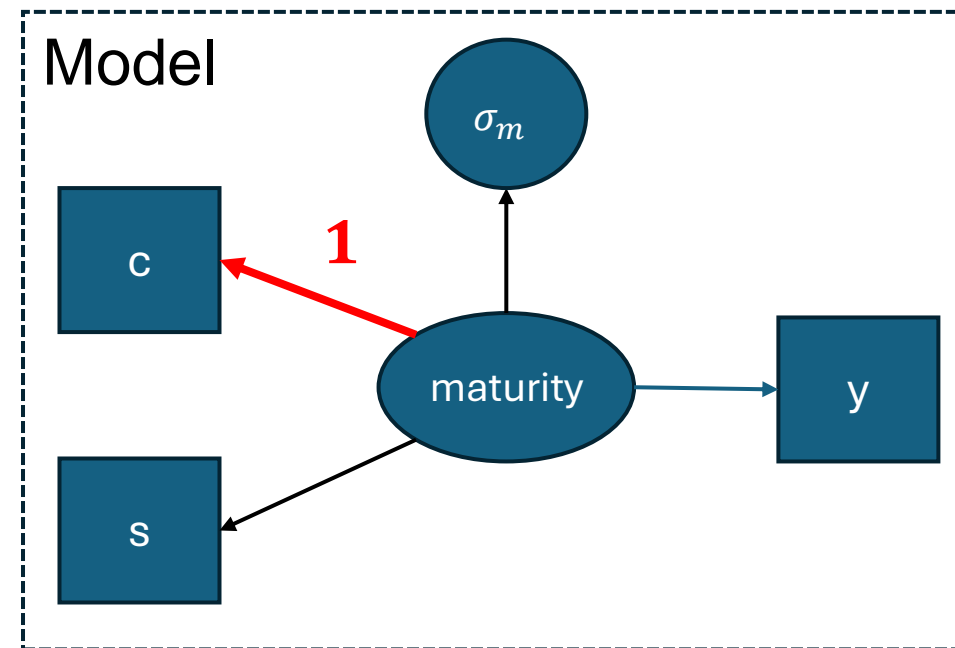
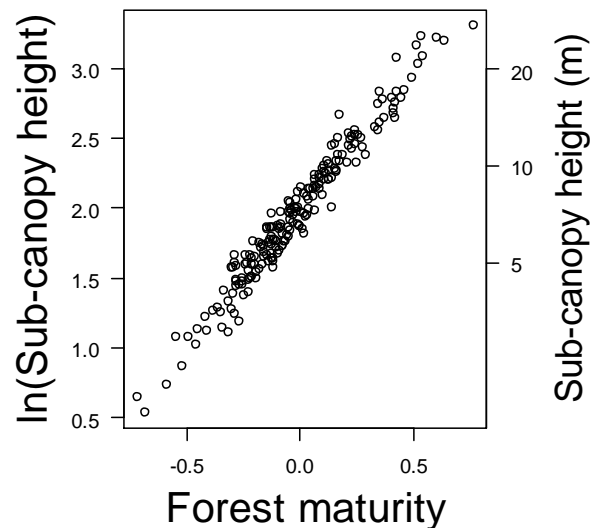
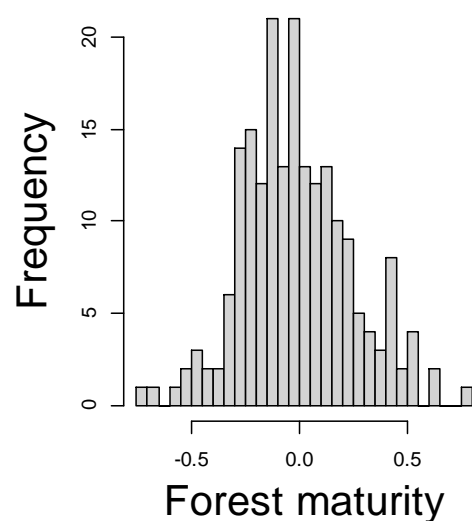
$\delta x = 1.5$



Forest maturity



Now we know the x-axis for everything!

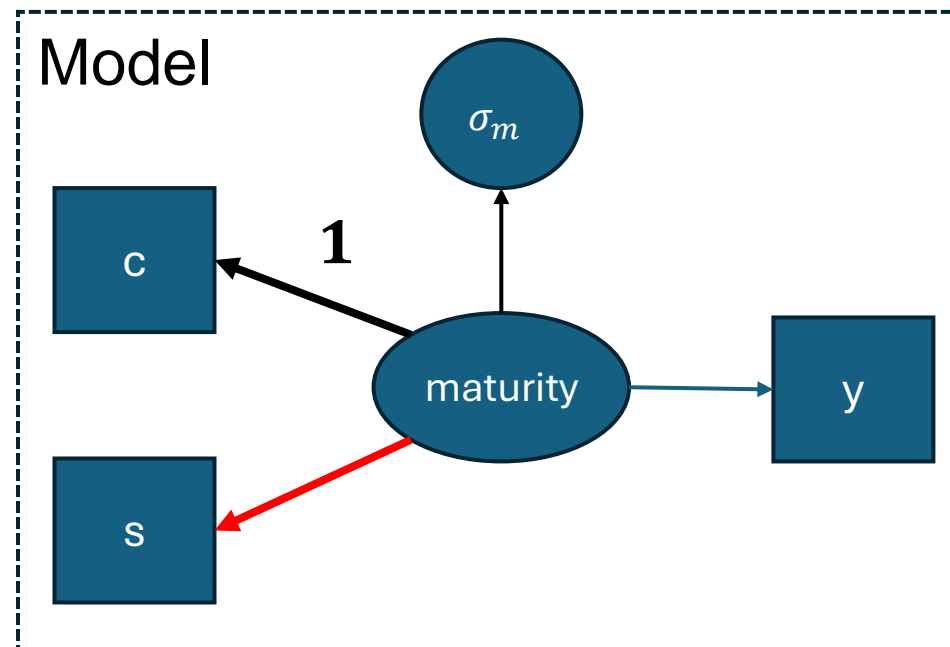
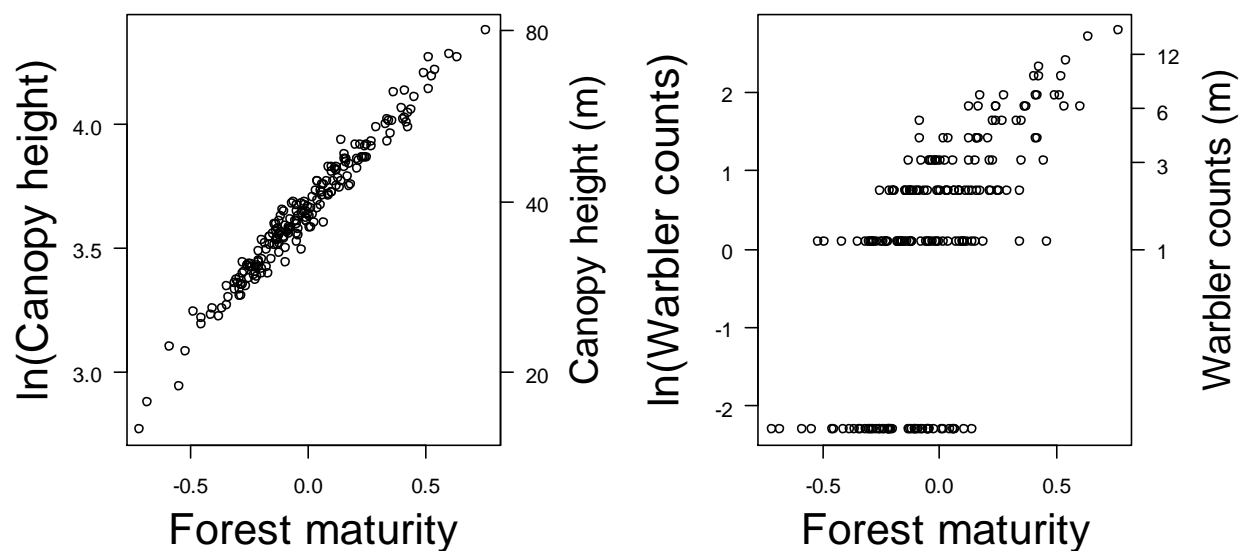
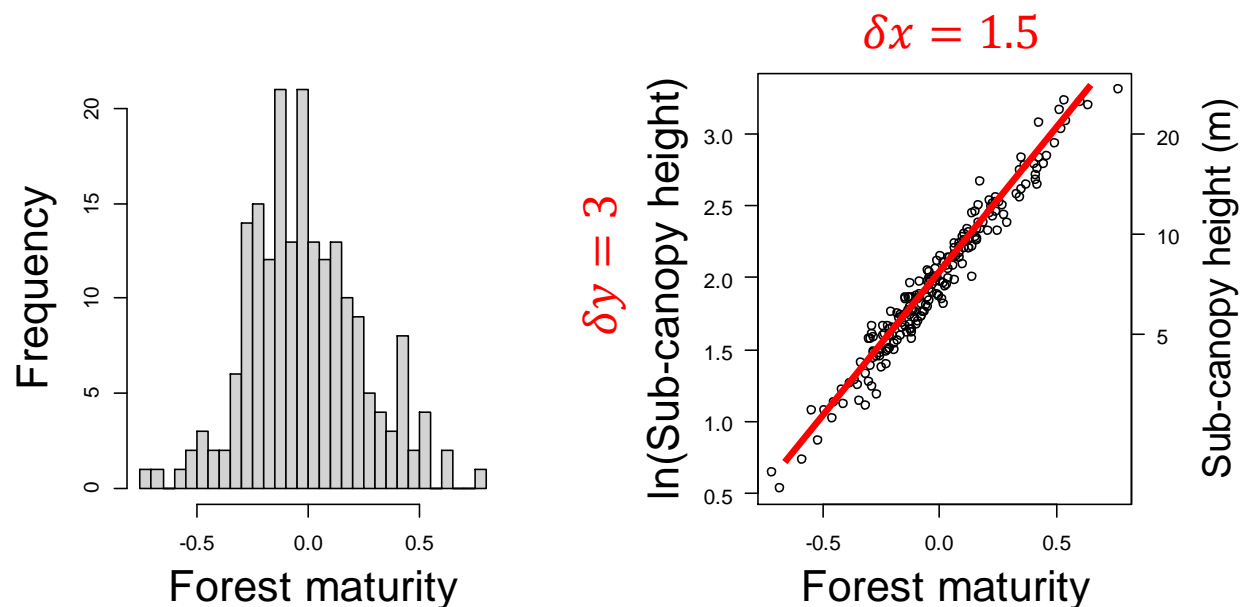


$$\delta y = \delta x \beta$$

$$1.5 = 1.5 \times 1$$

$$\delta x = 1.5$$

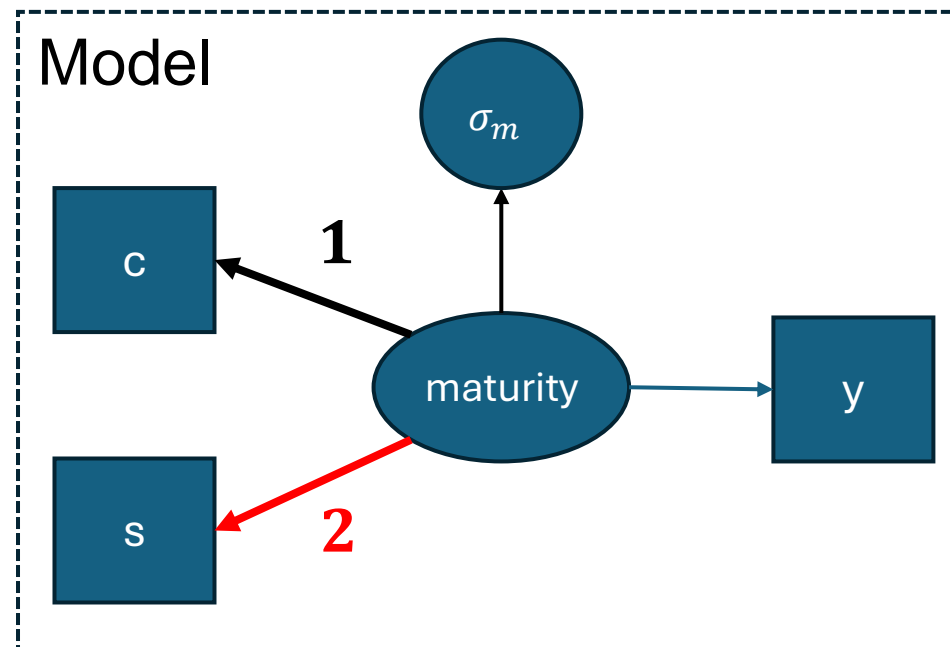
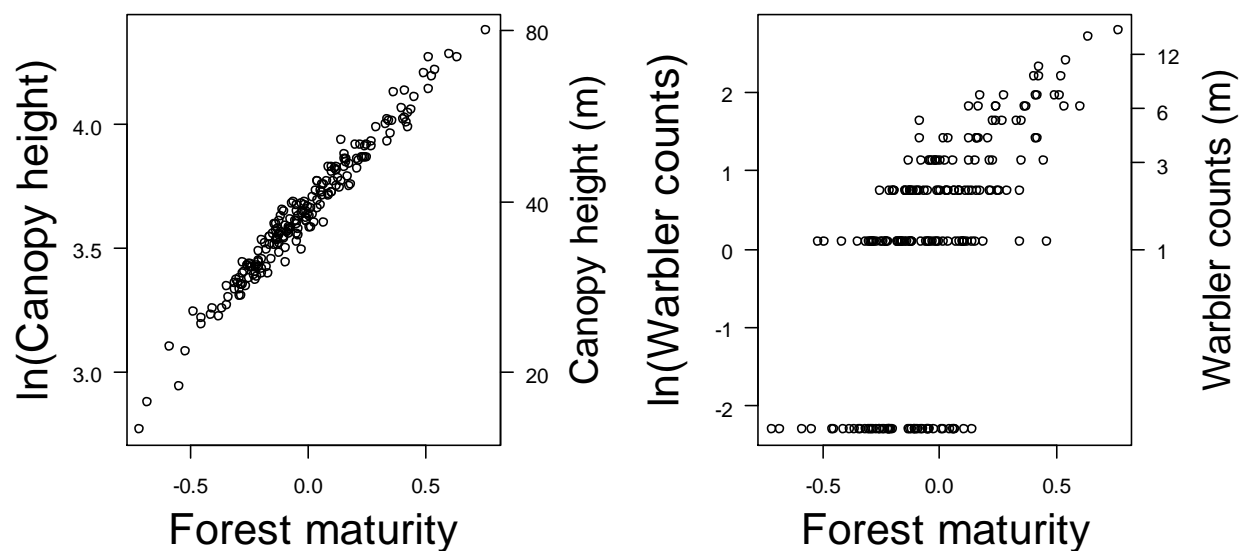
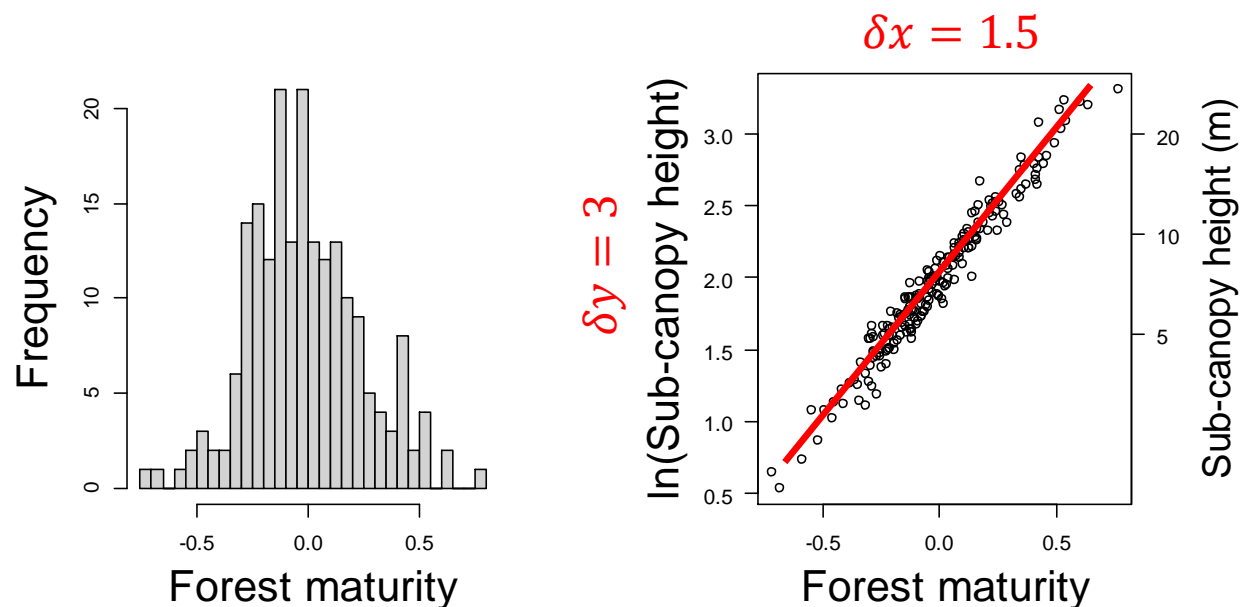
So what's the effect of m on s ?



$$\delta y = \delta x \beta$$

$$3 = 1.5 \times ?$$

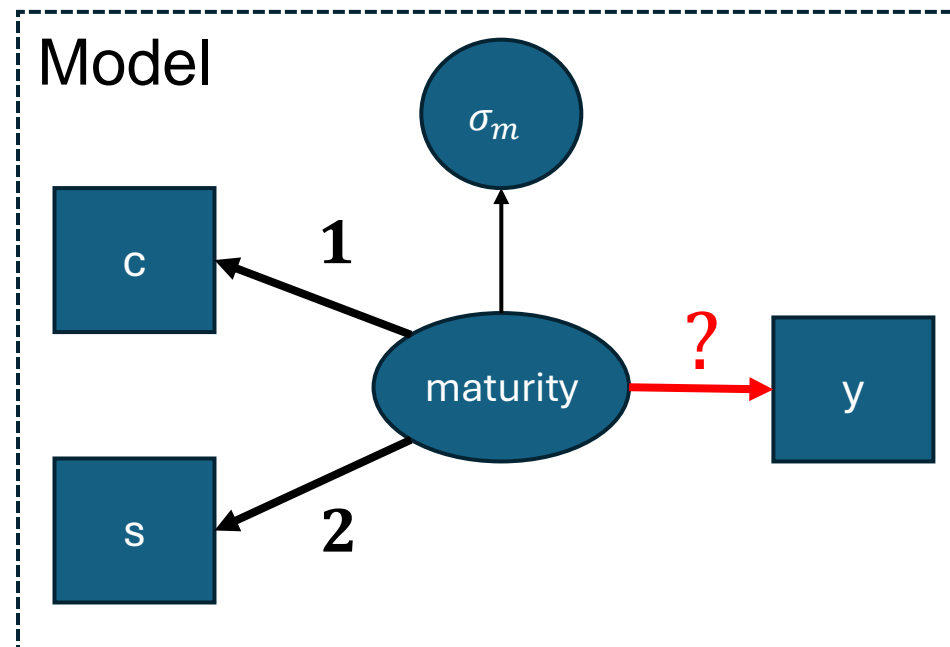
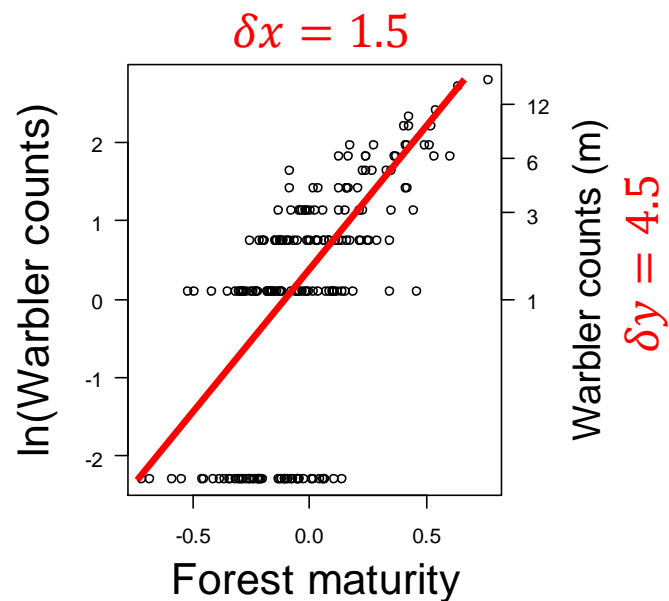
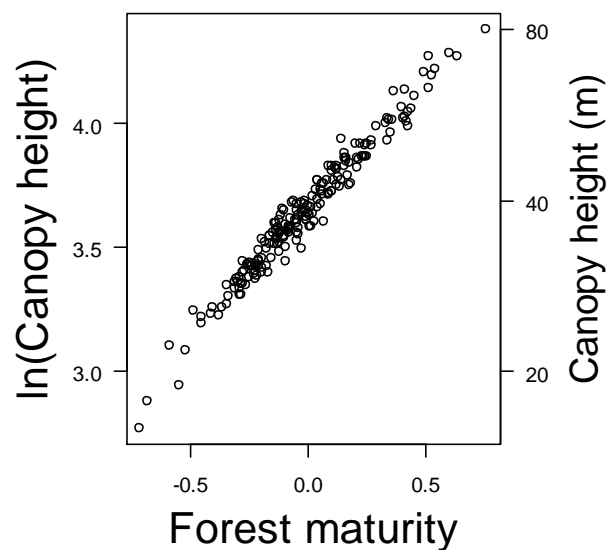
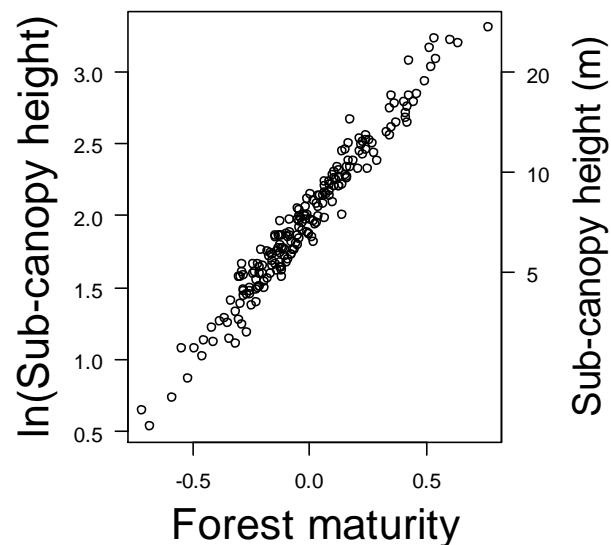
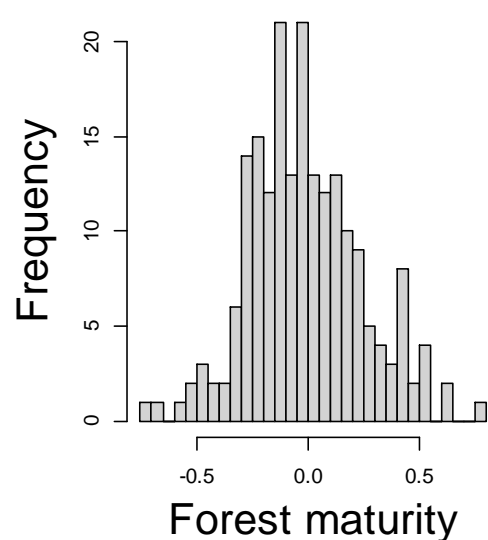
So what's the effect of m on s



$$\delta y = \delta x \beta$$

$$3 = 1.5 \times 2$$

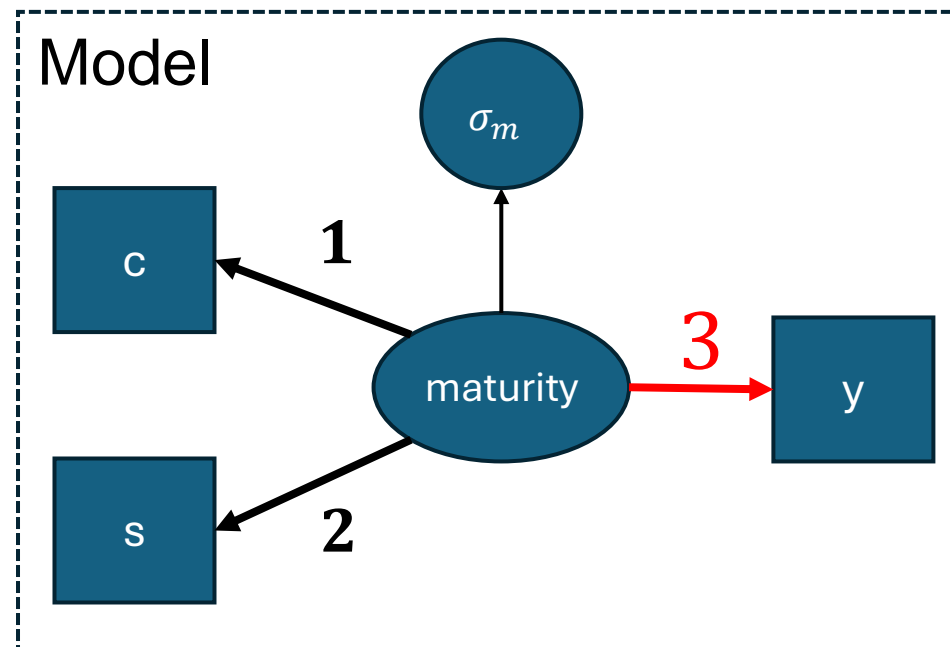
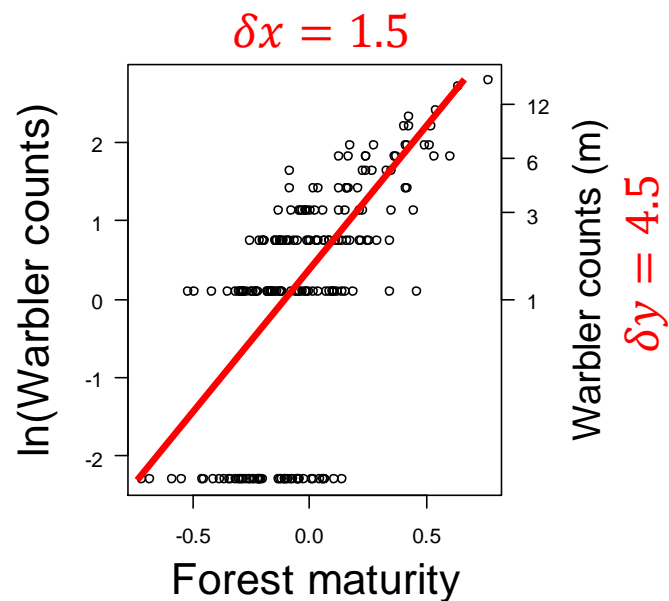
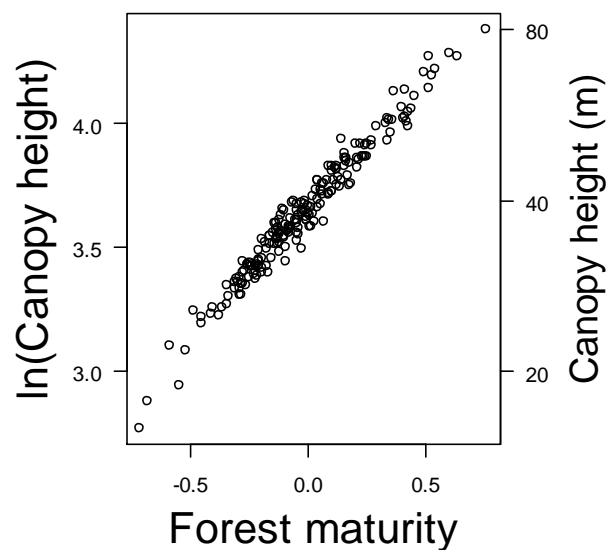
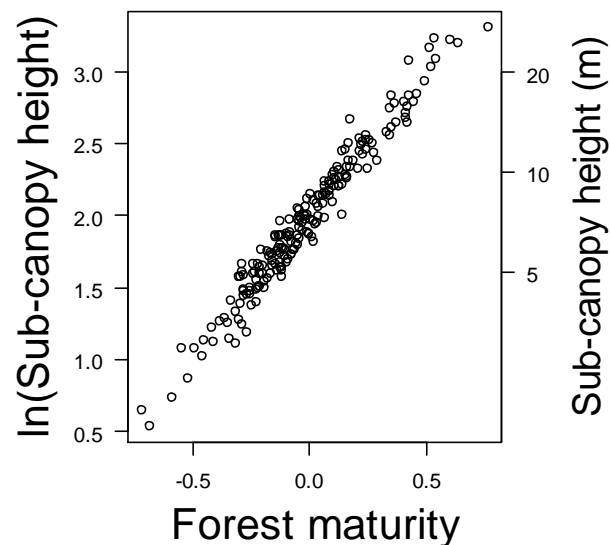
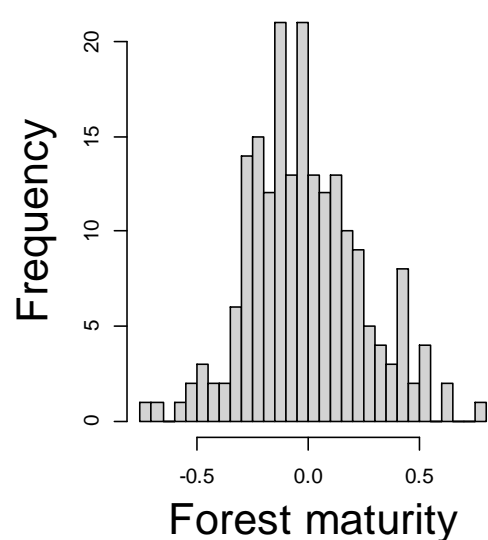
What about effect of m on y ?



$$\delta y = \delta x \beta$$

$$4.5 = 1.5 \times ?$$

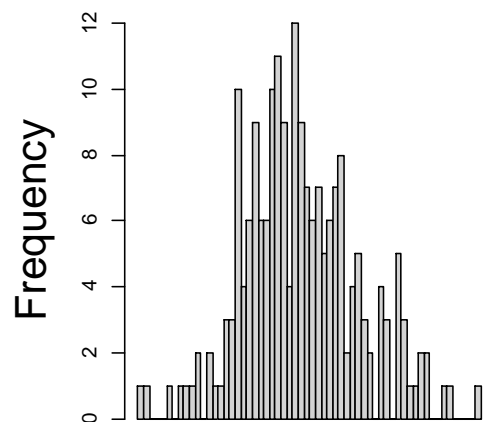
What about effect of m on y ?



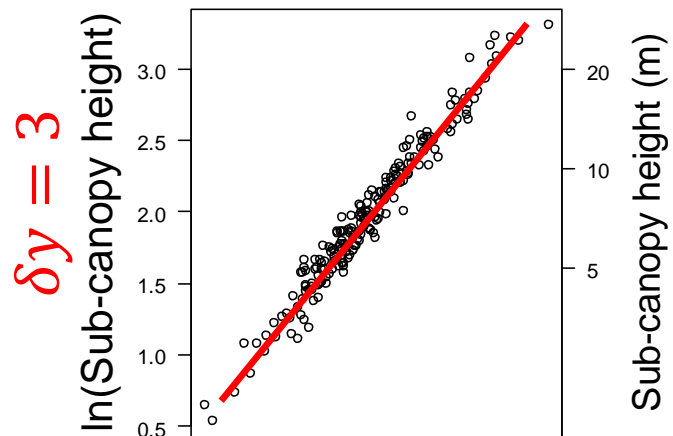
$$\delta y = \delta x \beta$$

$$4.5 = 1.5 \times 3$$

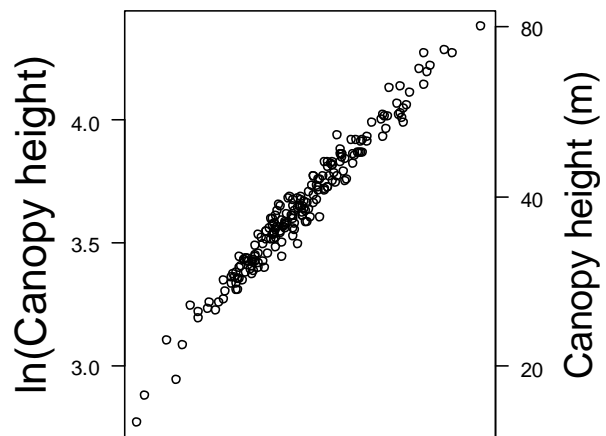
Let's do it again!!



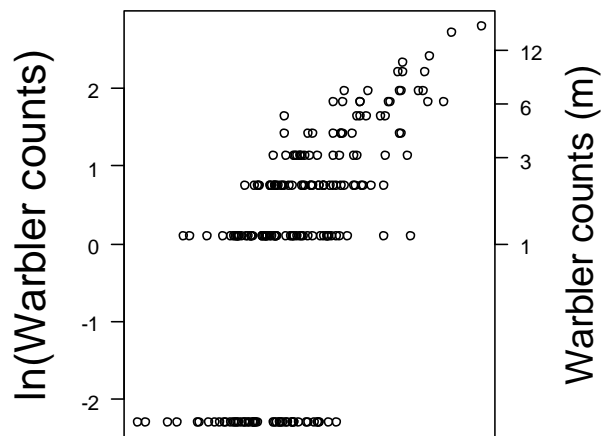
Forest maturity



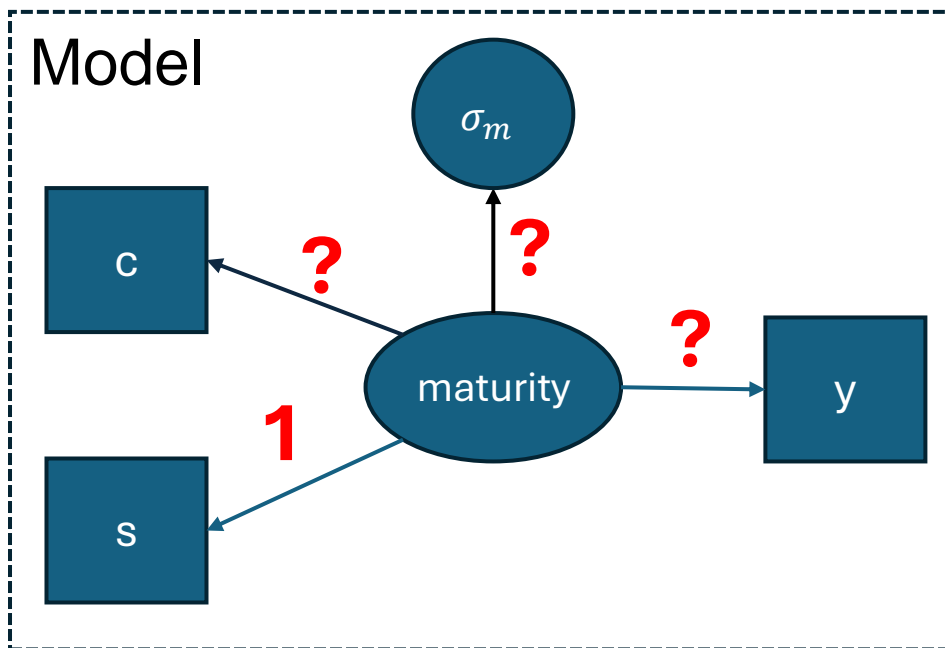
Forest maturity



Forest maturity

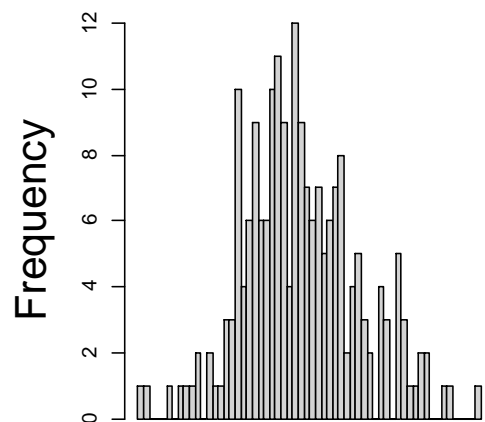


Forest maturity

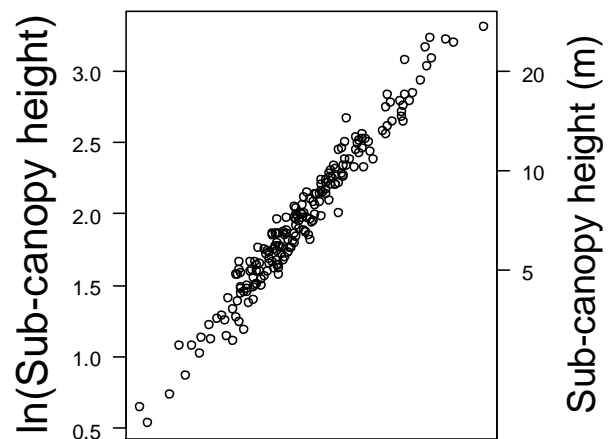


$$\delta y = \delta x \beta$$

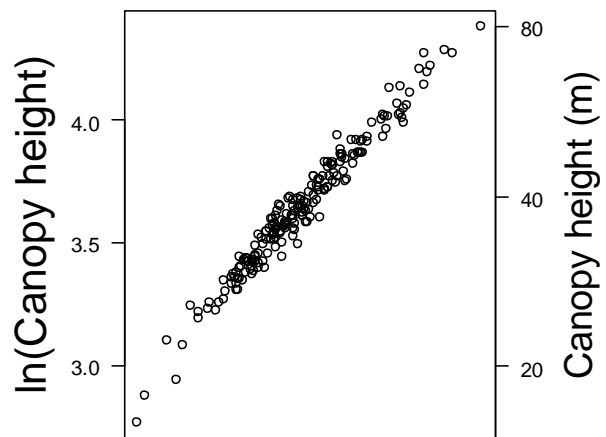
Play with the script some later!



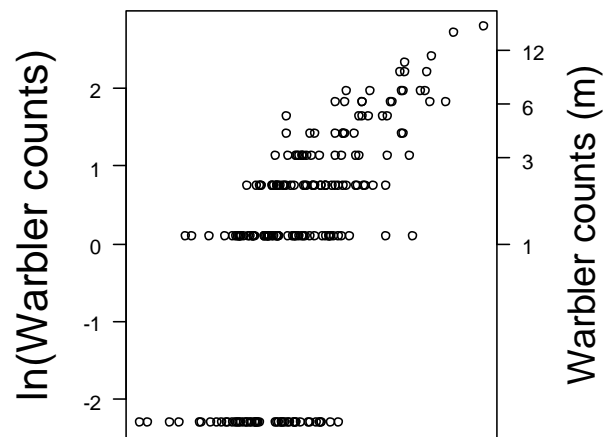
Forest maturity



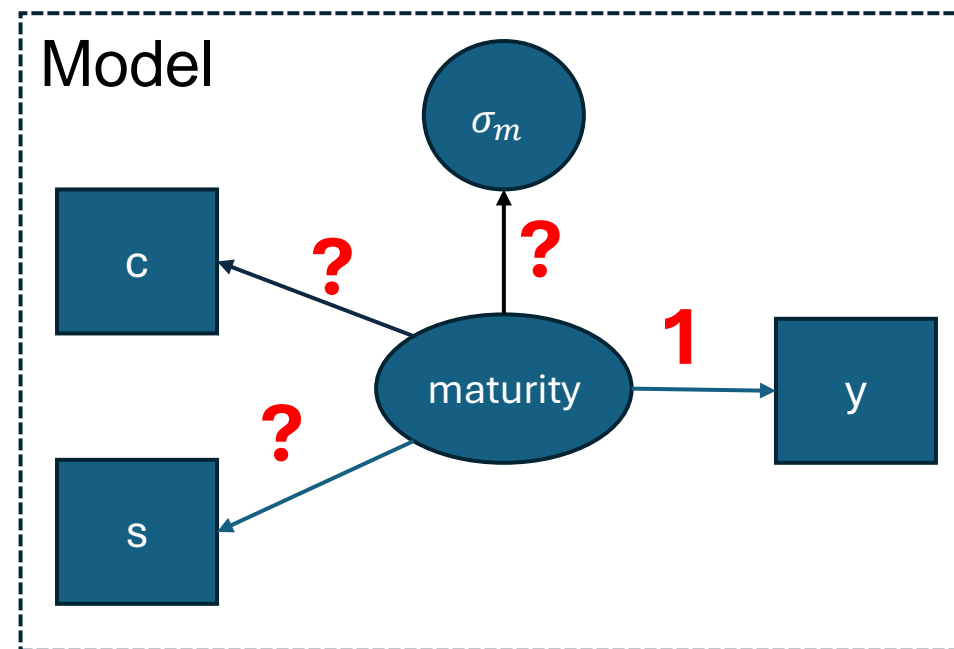
Forest maturity



Forest maturity



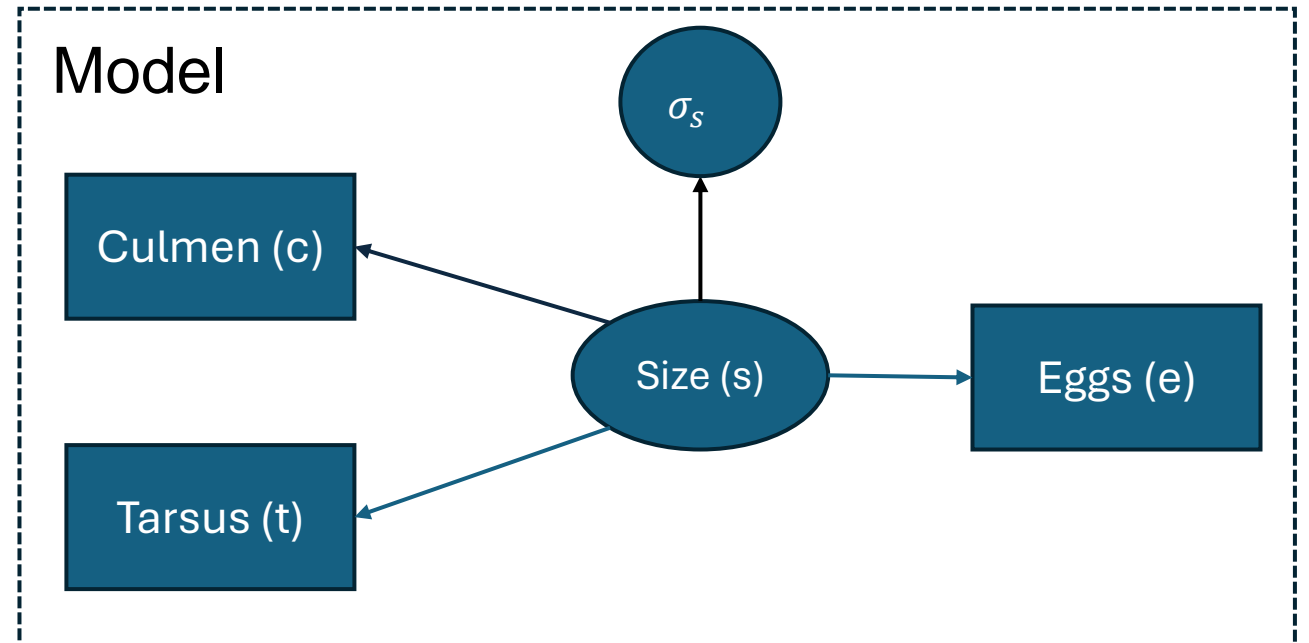
Forest maturity



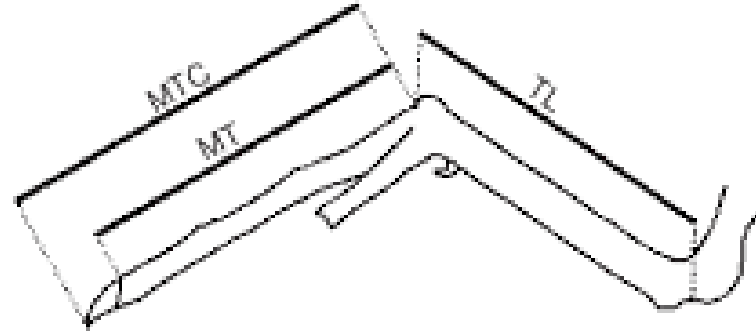
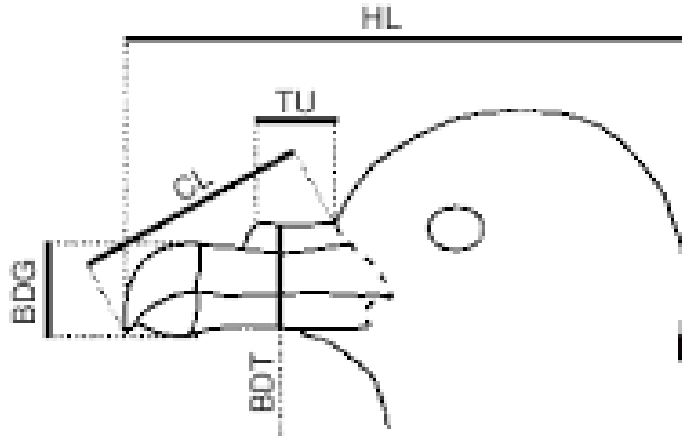
Today's coding exercise... build a SEM to estimate 'size'



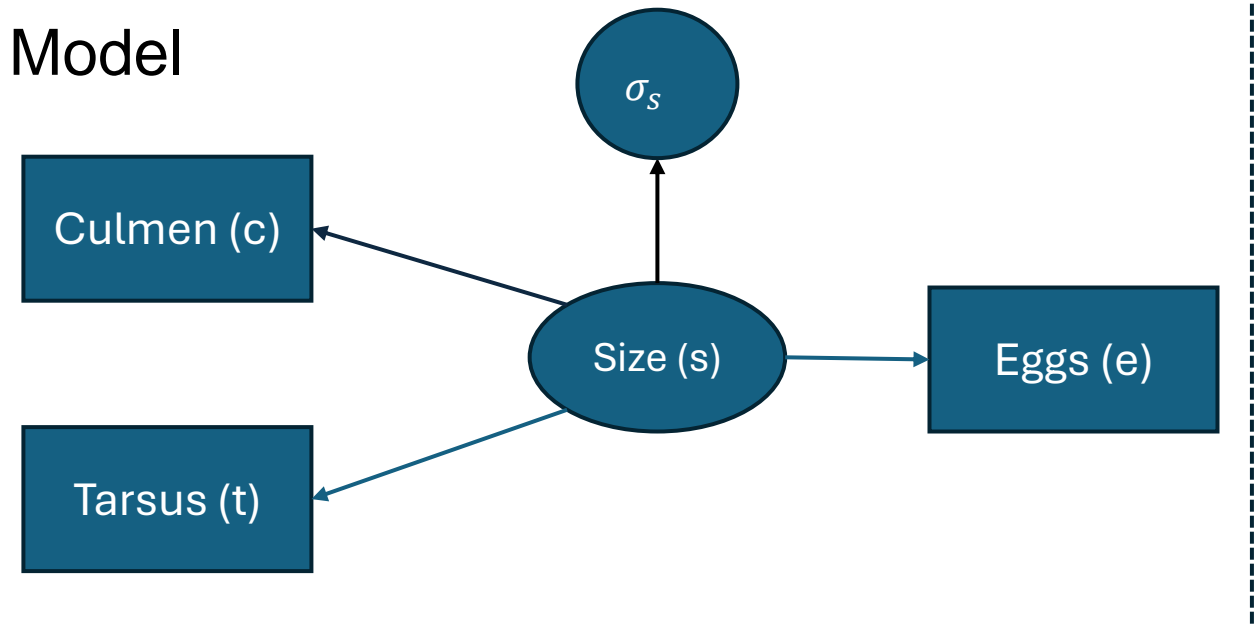
- Adult body size varies by ~25%
- Egg size can range from 50 cm³ to >100 cm³



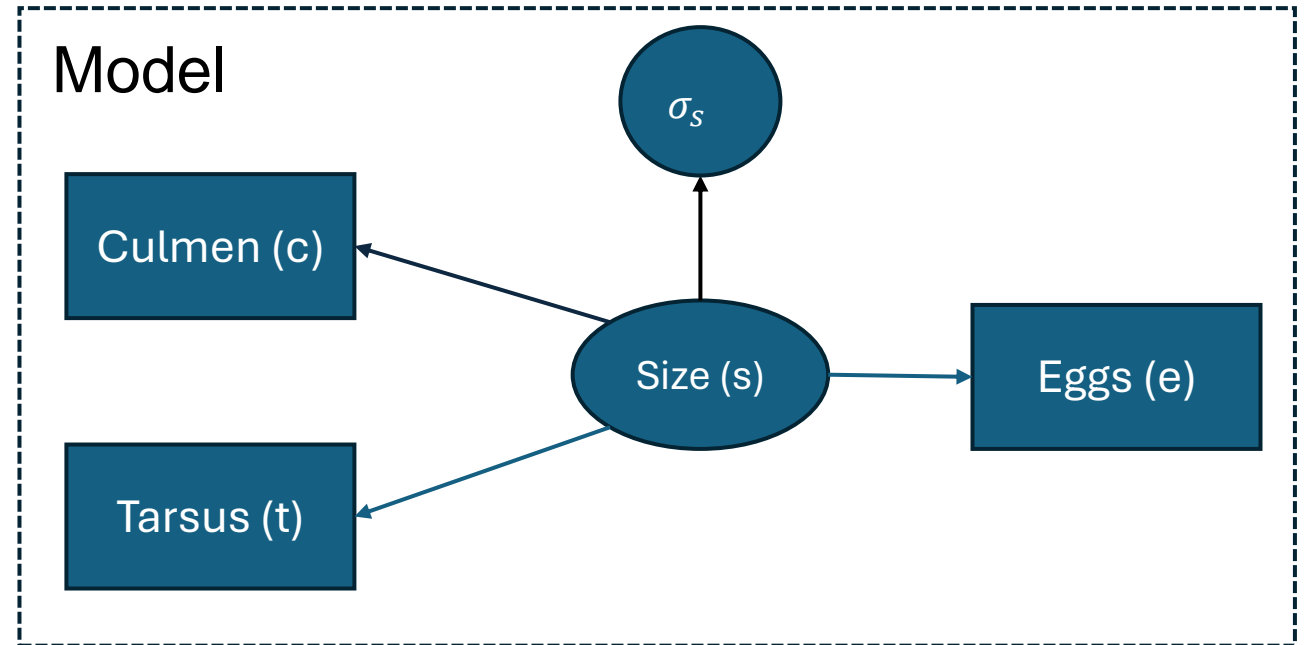
We take multiple morphometric measurements



Model



And we'll attempt to estimate the effect of size on clutch size



I've left a couple of things 'blank' in the model for you to supply