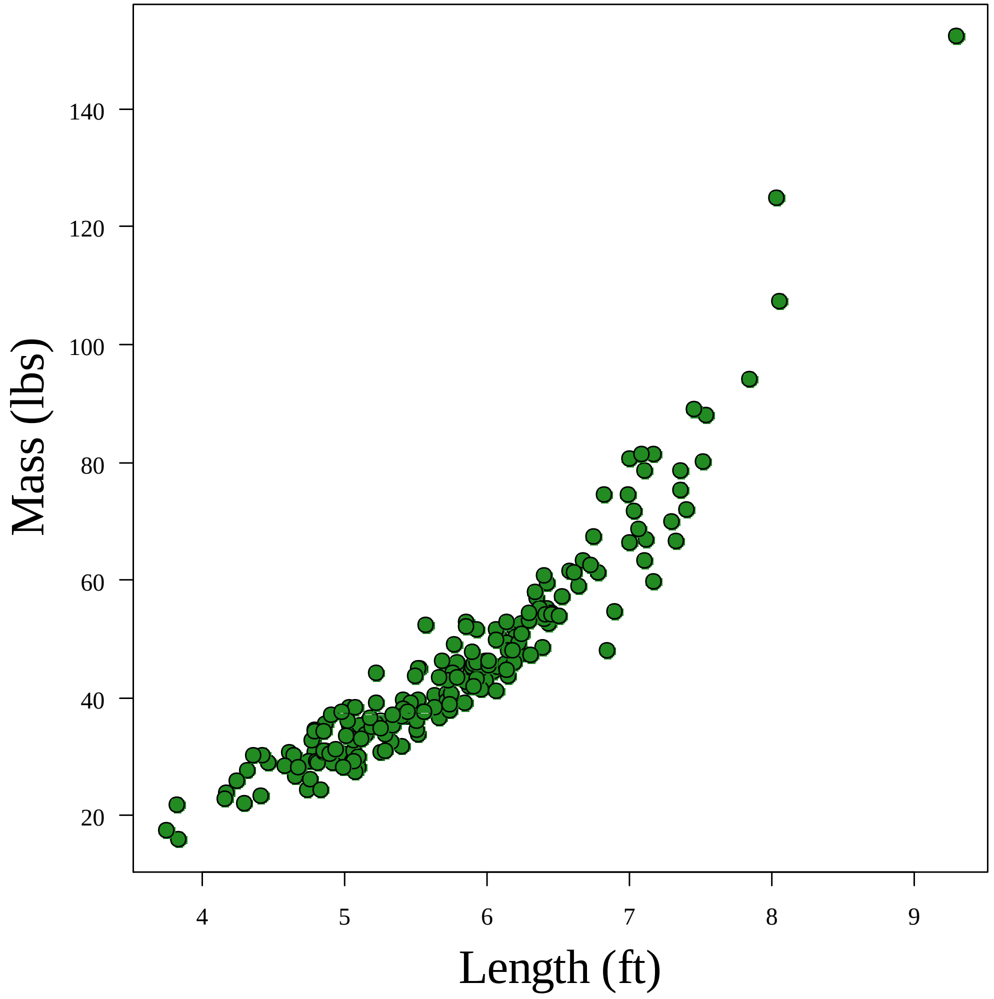
Introducing linear models*: our first JAGS excursion



*With a non-linear example!

First, let's wrap-up August 29th!

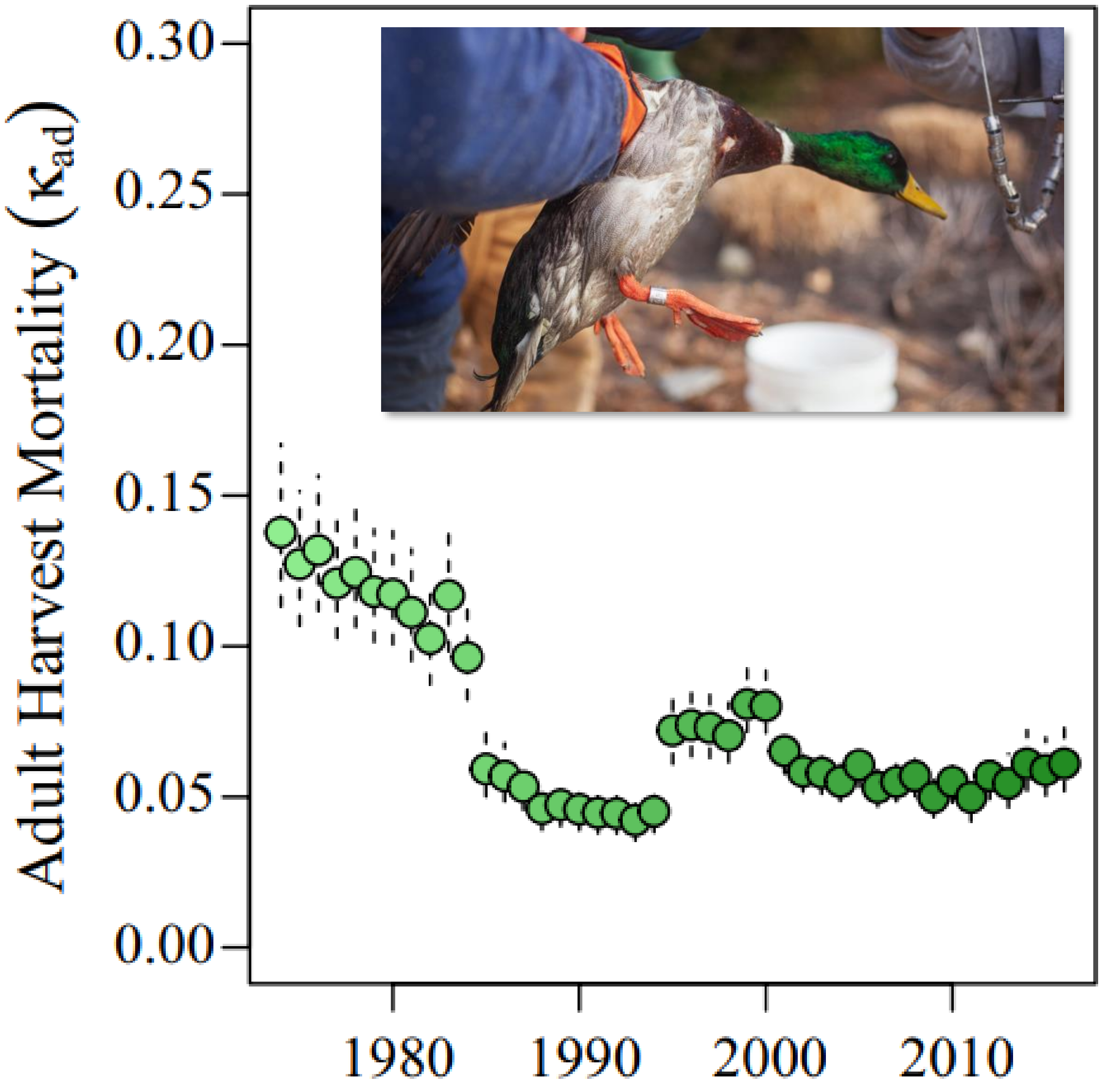
$$y \sim \text{binomial}(n, f)$$

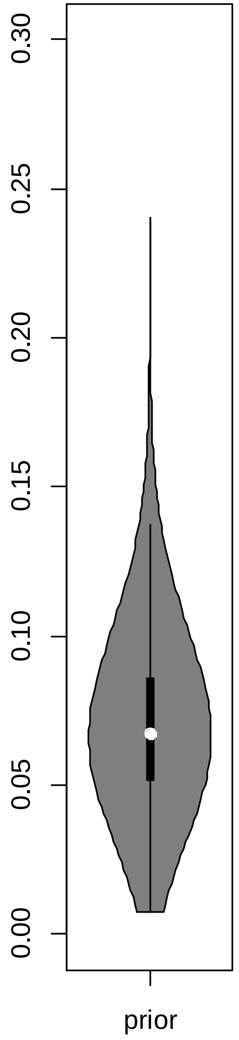
$$f \sim \text{beta}(\alpha, \beta)$$

Just look at a figure and come up with a prior?!

$$y \sim \text{binomial}(n, f)$$

$$f \sim \text{beta}(7?, 93?)$$





That can't be right…?

Let's be more objective!

$$y \sim \text{binomial}(n, f)$$

$$f \sim \text{beta}(\alpha, \beta)$$

Imagine that we have an estimate of average teal band-recovery probability. It's reported in a manuscript as 0.0487 (sd = 0.01).

How would we use that information for a prior?

We could do this? Problem?

$$y \sim \text{binomial}(n, f)$$

$$f \sim \text{normal}(0.0487, \sigma = 0.01)$$

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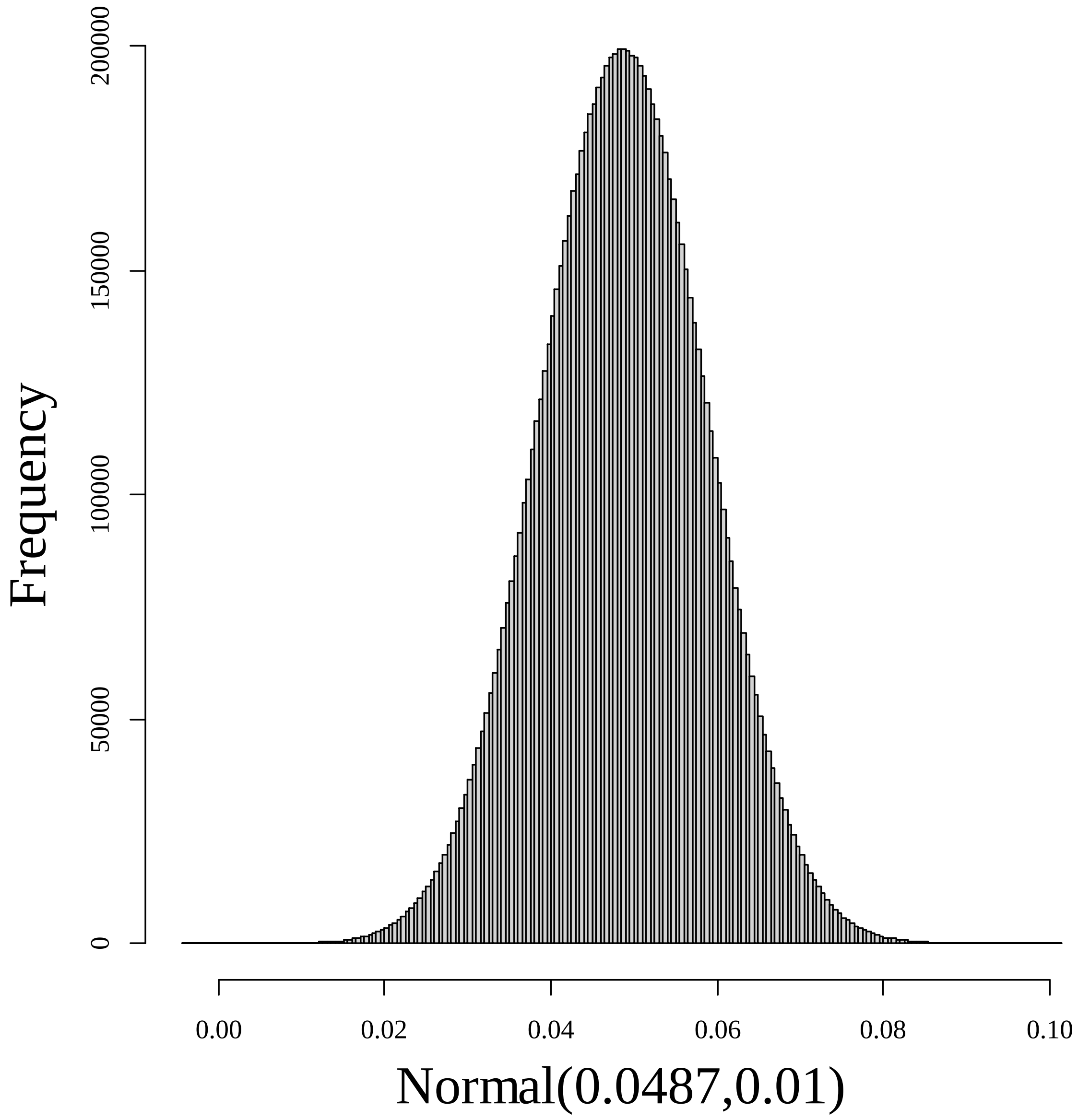


We can swap distributions via moment matching

If we have an estimate with <u>$\mu = 0.0487$</u> and <u>$\sigma = 0.01$</u>, we can derive the alpha and beta parameters of a beta distribution given the mean and sd of a normal

$$\alpha = \left(\frac{1-\mu}{\sigma^2} - \frac{1}{\mu}\right)\mu^2$$

$$f \sim \mathrm{beta}(\alpha, \beta)$$

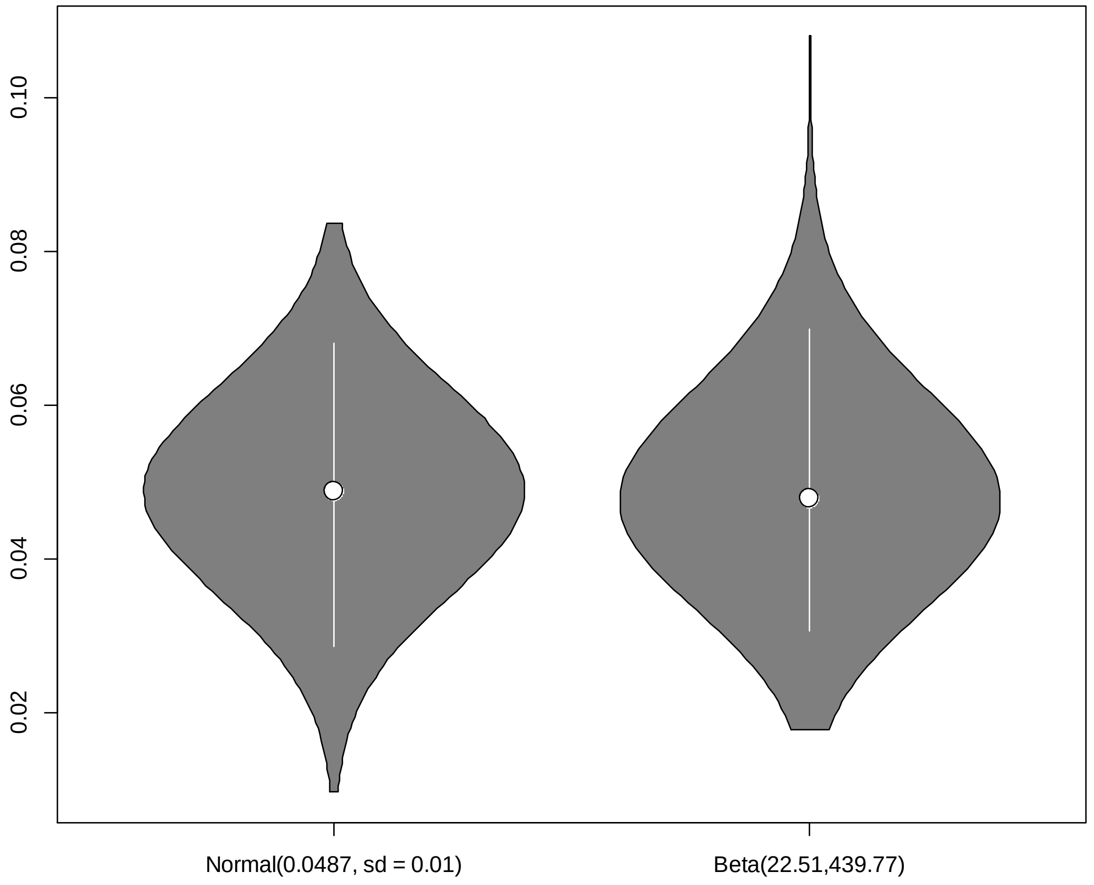
$$\beta = \alpha\left(\frac{1}{\mu} - 1\right)$$

Moment-matching!

If we have an estimate with μ=0.0487 and σ=0.01

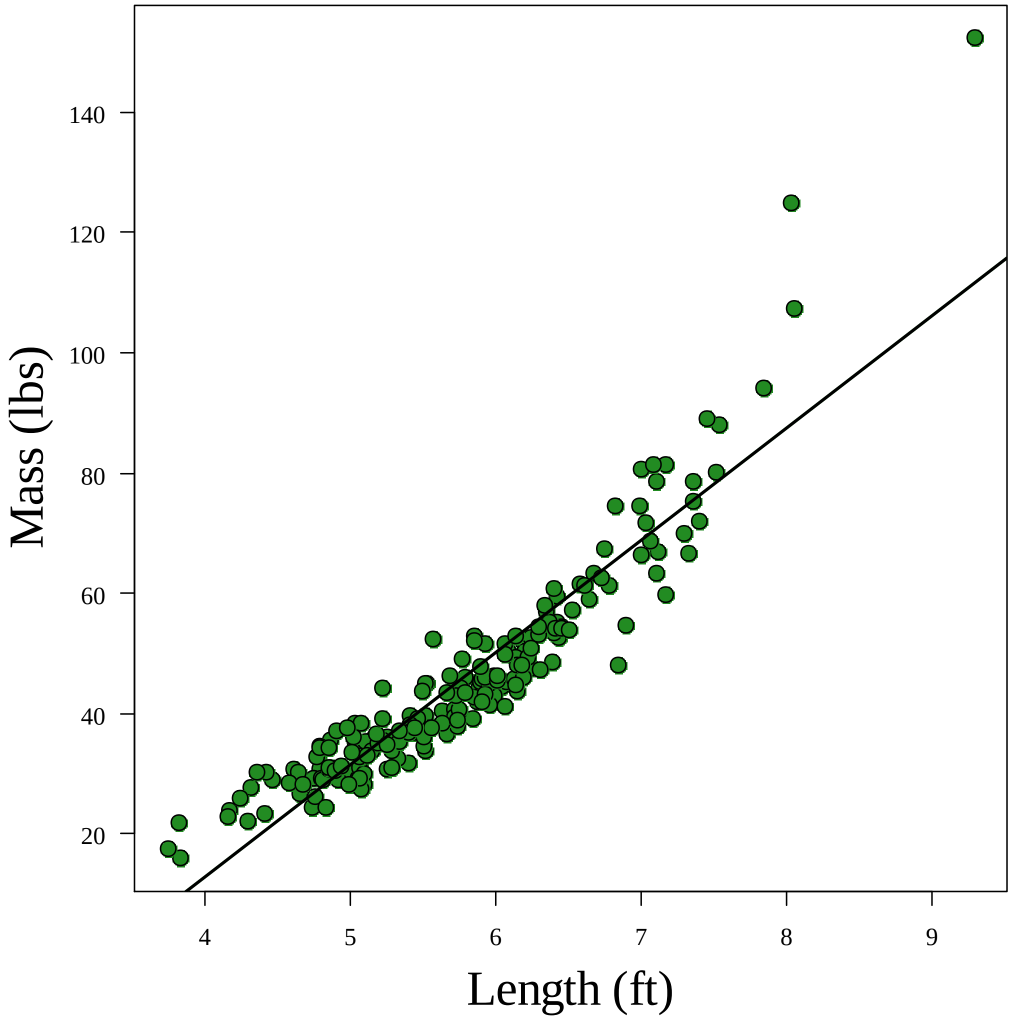
$$\alpha = \left(\frac{1-\mu}{\sigma^2} - \frac{1}{\mu}\right)\mu^2$$

$$\beta = \alpha\left(\frac{1}{\mu} - 1\right)$$



Beta distributions are constrained to [0, … , 1]

Introducing linear models: our first JAGS example



Notation

$$y = mx + b$$

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$$E(y_i) = \beta_0 + \beta_1 \times x_i$$
$$y_i \sim \mathrm{normal}(E(y_i), \sigma^2)$$

This is the same thing.

Notation

$$\mathrm{y = mx + b}$$

$$\mu_i = \beta_0 + \beta_1 \times x_i$$
$$y_i \sim \mathrm{normal}(\mu_i, \sigma^2)$$

This is also the same thing

Notation

$$\mathrm{y} = \mathrm{mx} + \mathrm{b}$$

$$\boldsymbol{\mu} = \boldsymbol{\beta}\boldsymbol{X}$$
$$\boldsymbol{y} \sim \mathrm{normal}(\boldsymbol{\mu}, \sigma^2)$$

This is also the same thing

Let’s go one piece at a time…

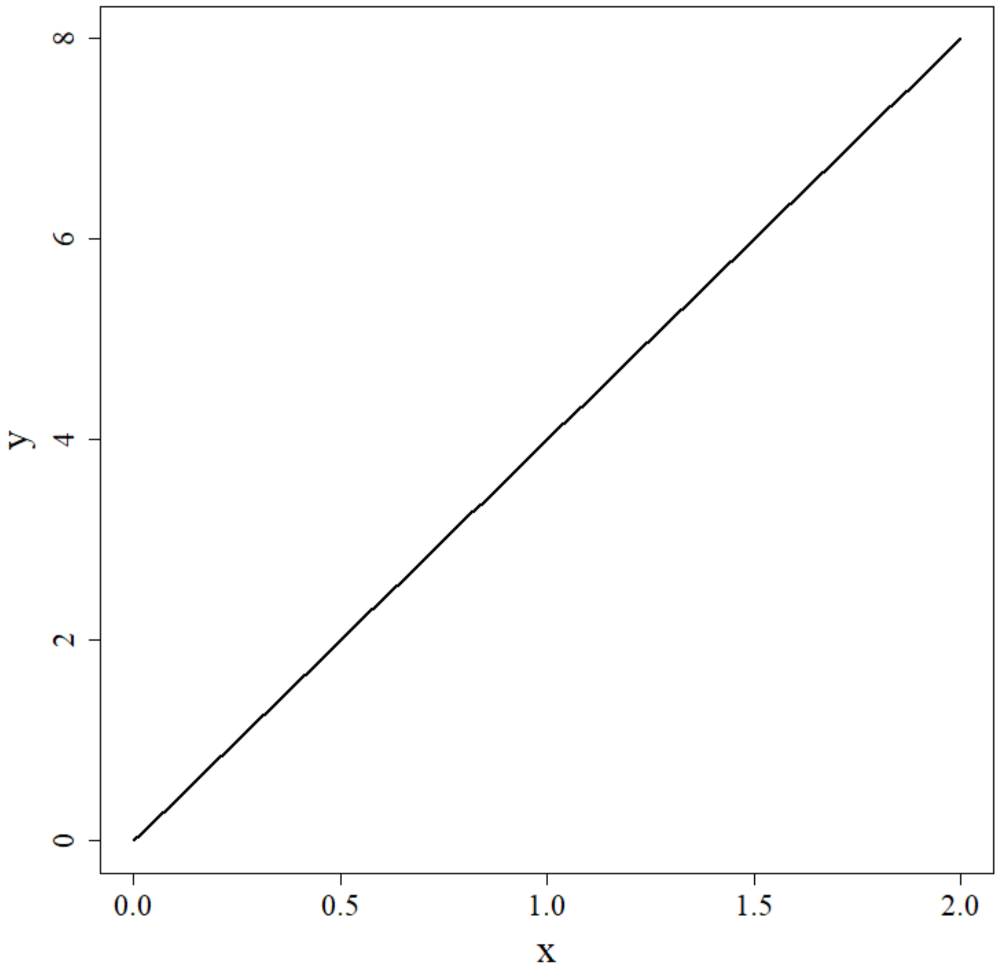
Slopes (m or β_1)

The key idea is to understand the relationship between a **covariate** (e.g., predator abundance, NDVI, distance to road) and a **response** variable (e.g., survival, fecundity, behavior, stress)

Slopes

y = mx + 0

$$E(y_i) = 0 + \beta_1 \times x_i$$
$$y_i \sim \text{normal}(E(y_i), \sigma^2)$$



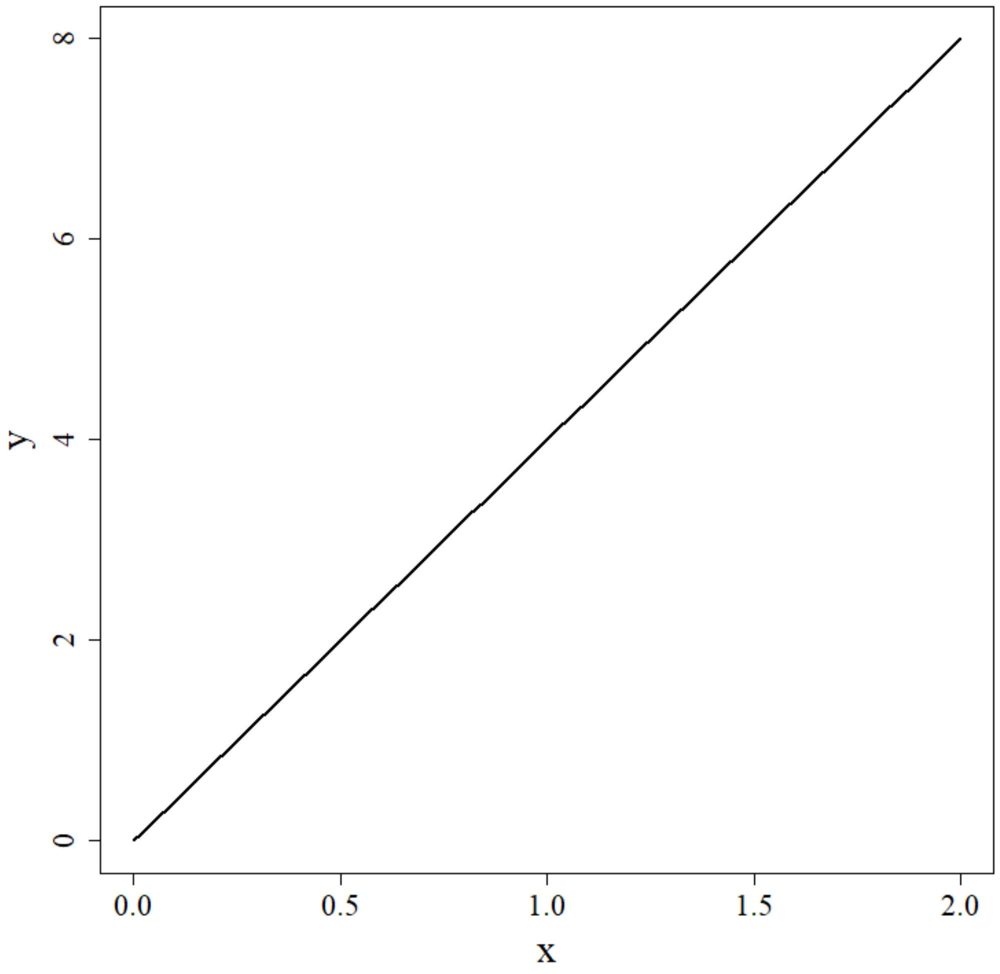
What’s the slope of this line?

Slopes

$$y = mx + 0$$

$$E(y_i) = 0 + \beta_1 \times x_i$$
$$y_i \sim \text{normal}(E(y_i), \sigma^2)$$

$$\beta_1 = \text{m} = 4$$



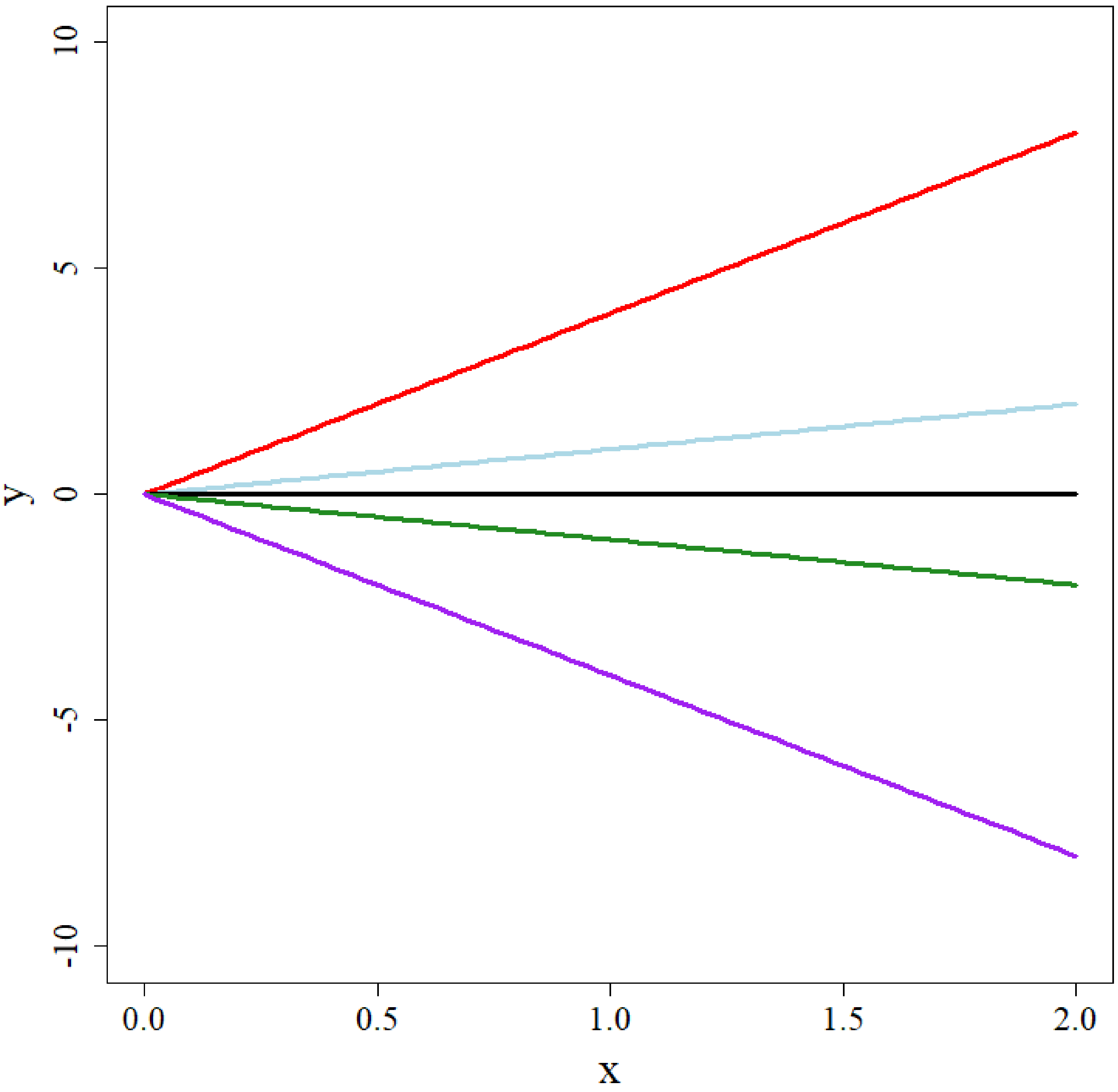
As x increases from 0 to 1, y increases from 0 to 4

Slopes

$$y = mx + 0$$

$$E(y_i) = 0 + \beta_1 \times x_i$$
$$y_i \sim \text{normal}(E(y_i), \sigma^2)$$

$$\beta_1 = \text{m} = [\mathbf{4}, \mathbf{1}, \mathbf{0}, -\mathbf{1}, -\mathbf{4}]$$

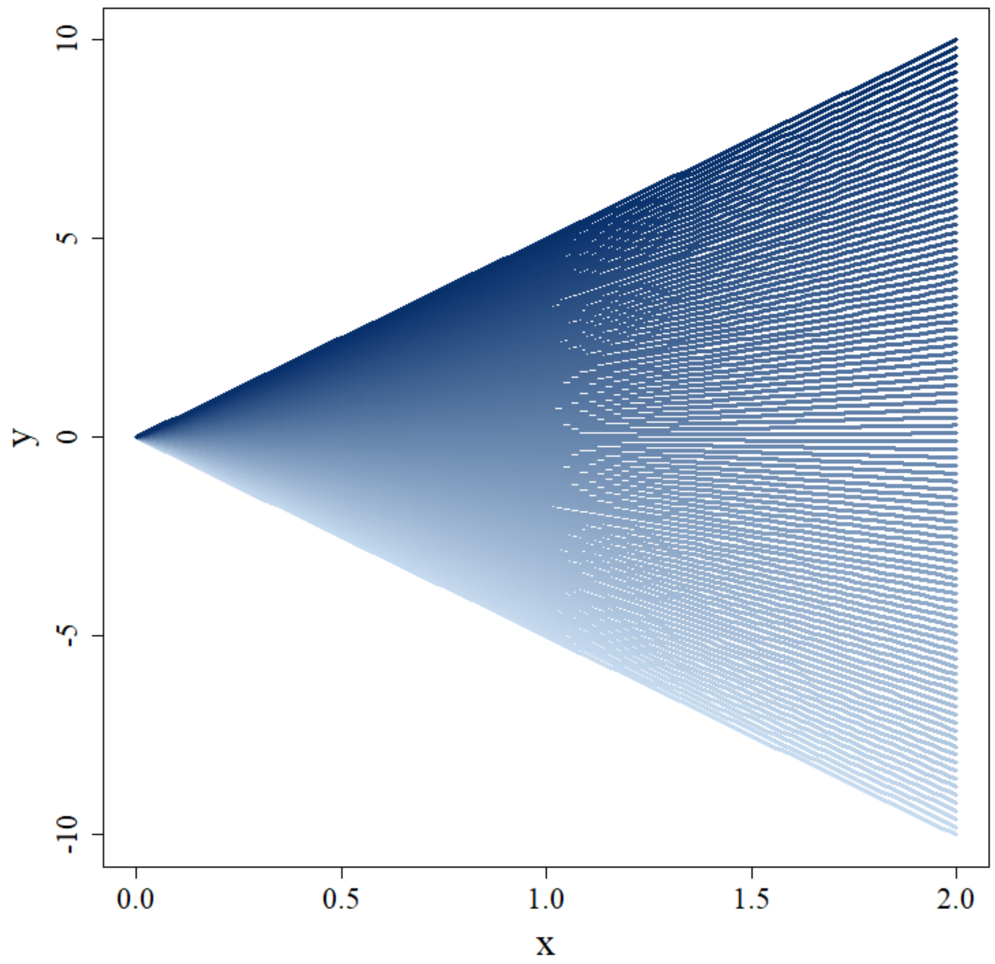


Slopes

$$y = mx + 0$$

$$E(y_i) = 0 + \beta_1 \times x_i$$
$$y_i \sim \text{normal}(E(y_i), \sigma^2)$$

$$\beta_1 = \text{m} = [-5, -4.9, \ldots, 4.9, 5]$$



Slopes

Positive values means the response variable will increase as the covariate increases.

Negative values means the response variable will decrease as the covariate increases

Slopes

Larger positive values means the response variable will increase faster as the covariate increases.

Lower negative values means the response variable will decrease faster as the covariate increases

We need to be cognizant of the scale of our covariates!

Intercepts

y = mx + b

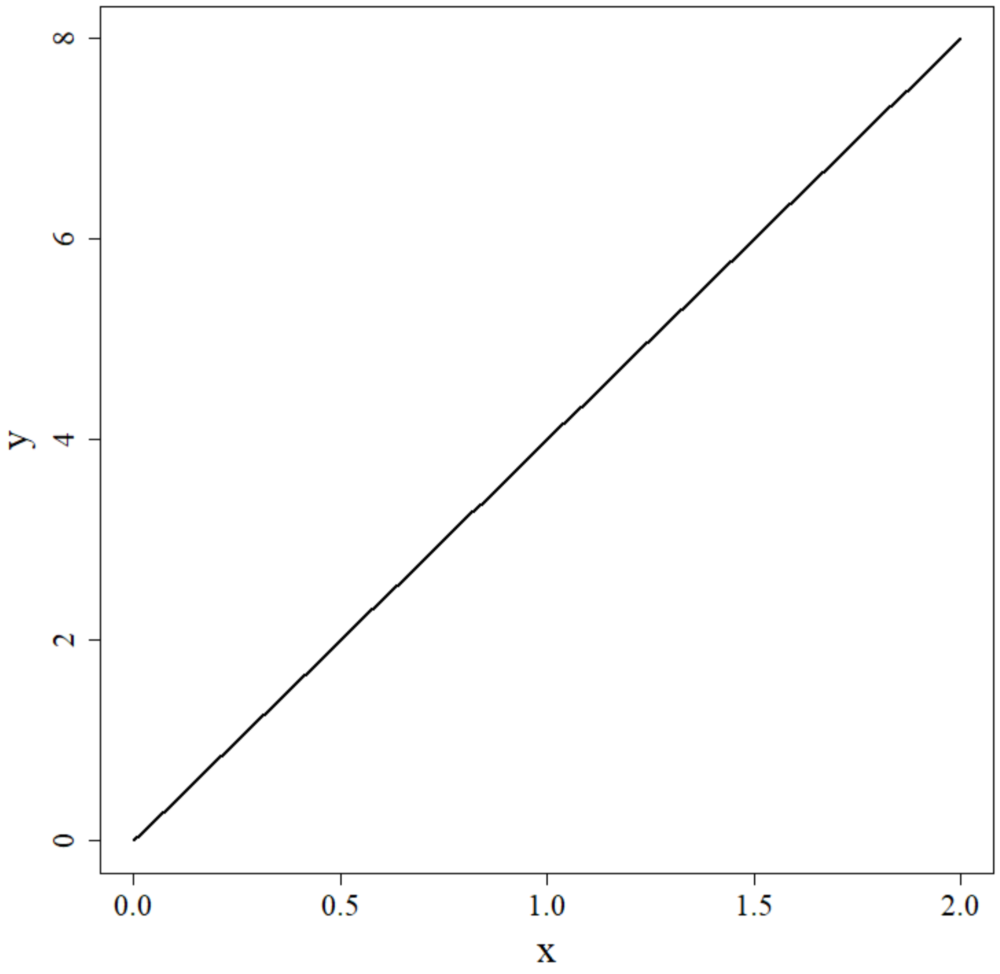
$$E(y_i) = \beta_0 + \beta_1 \times 0$$
$$y_i \sim \mathrm{normal}(E(y_i), \sigma^2)$$

What does y equal when x = 0?

Intercepts

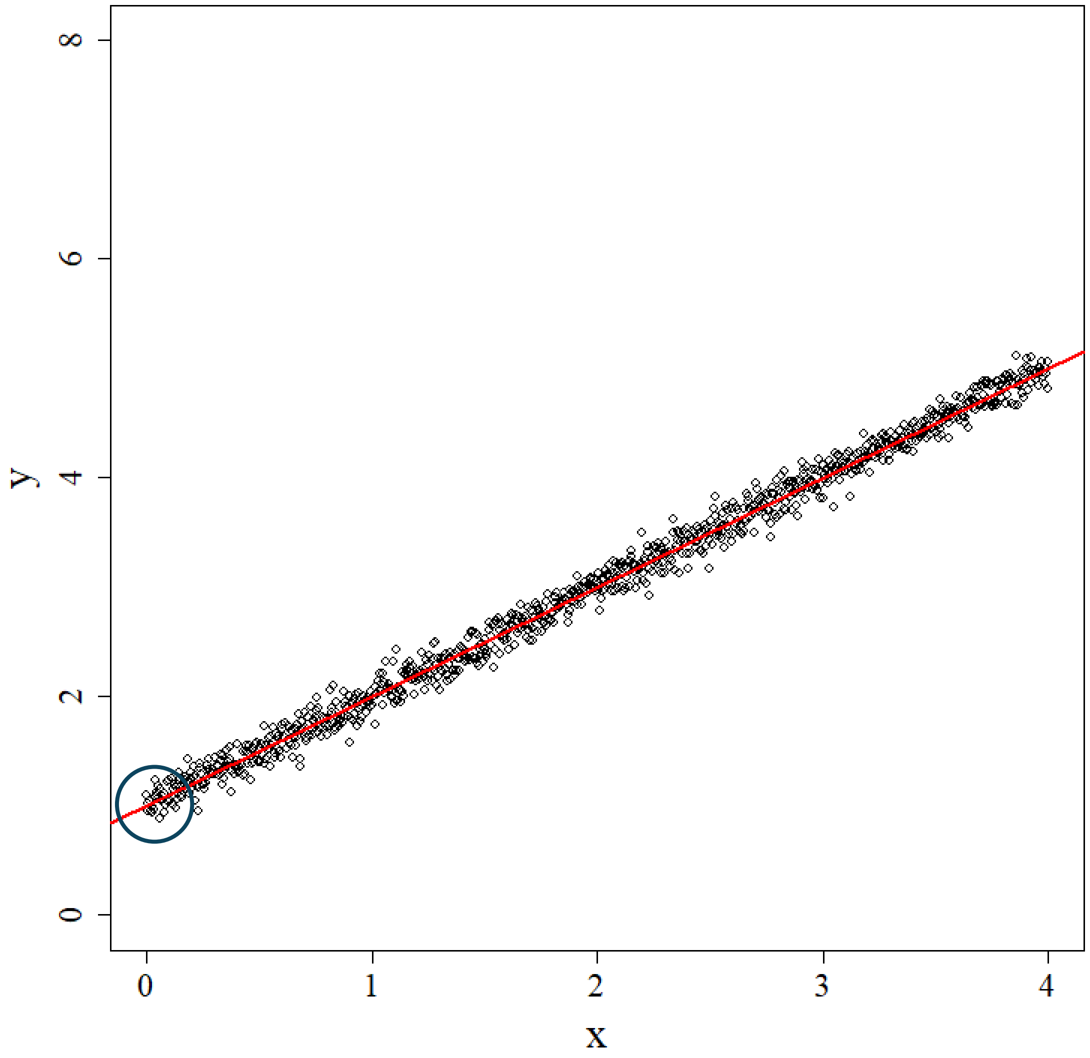
y = mx + b

$$E(y_i) = \beta_0 + \beta_1 \times 0$$
$$y_i \sim \mathrm{normal}(E(y_i), \sigma^2)$$



What is the intercept of this line?

You can think of an intercept as an anchor…



Or the expected response when all covariates = 0

Variance

y = mx + b

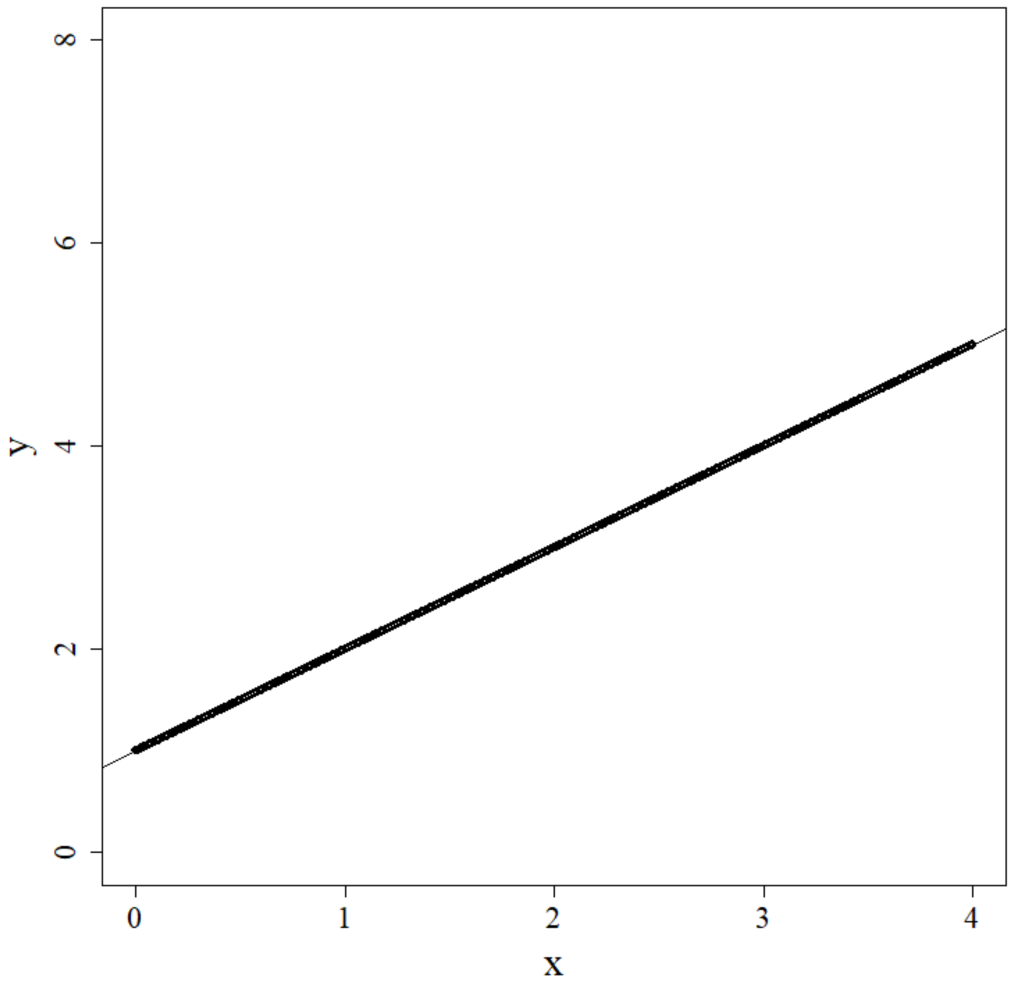
$$\mu_i = \beta_0 + \beta_1 \times x_i$$
$$y_i \sim \text{normal}(\mu_i, \sigma^2)$$

No variance

y = mx + b

$$\mu_i = \beta_0 + \beta_1 \times x_i$$
$$y_i \sim \text{normal}(\mu_i, \sigma^2)$$

$$\sigma = 0$$

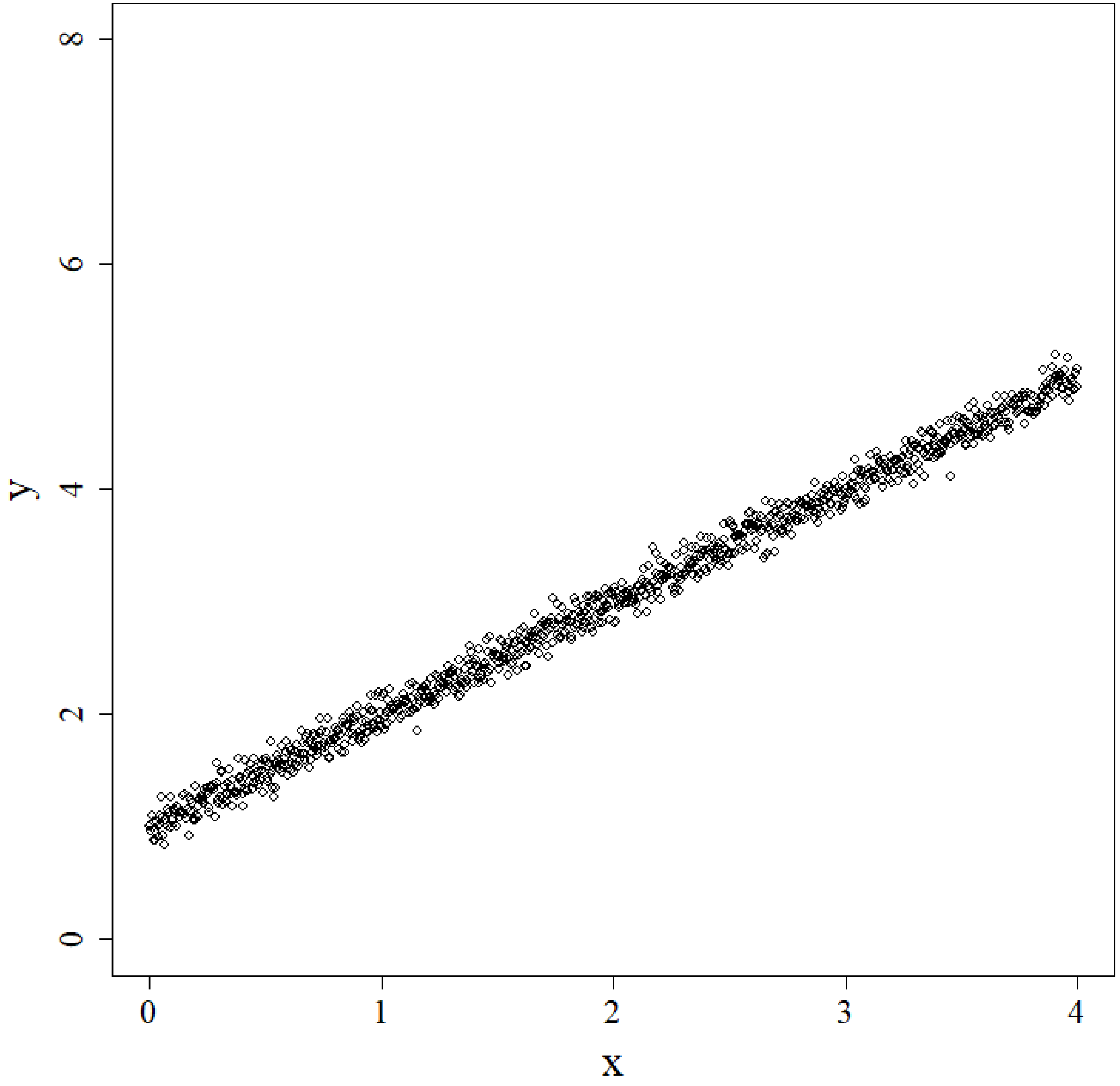


Small variance

y = mx + b

$$\mu_i = \beta_0 + \beta_1 \times x_i$$
$$y_i \sim \text{normal}(\mu_i, \sigma^2)$$

$$\sigma = 0.1$$

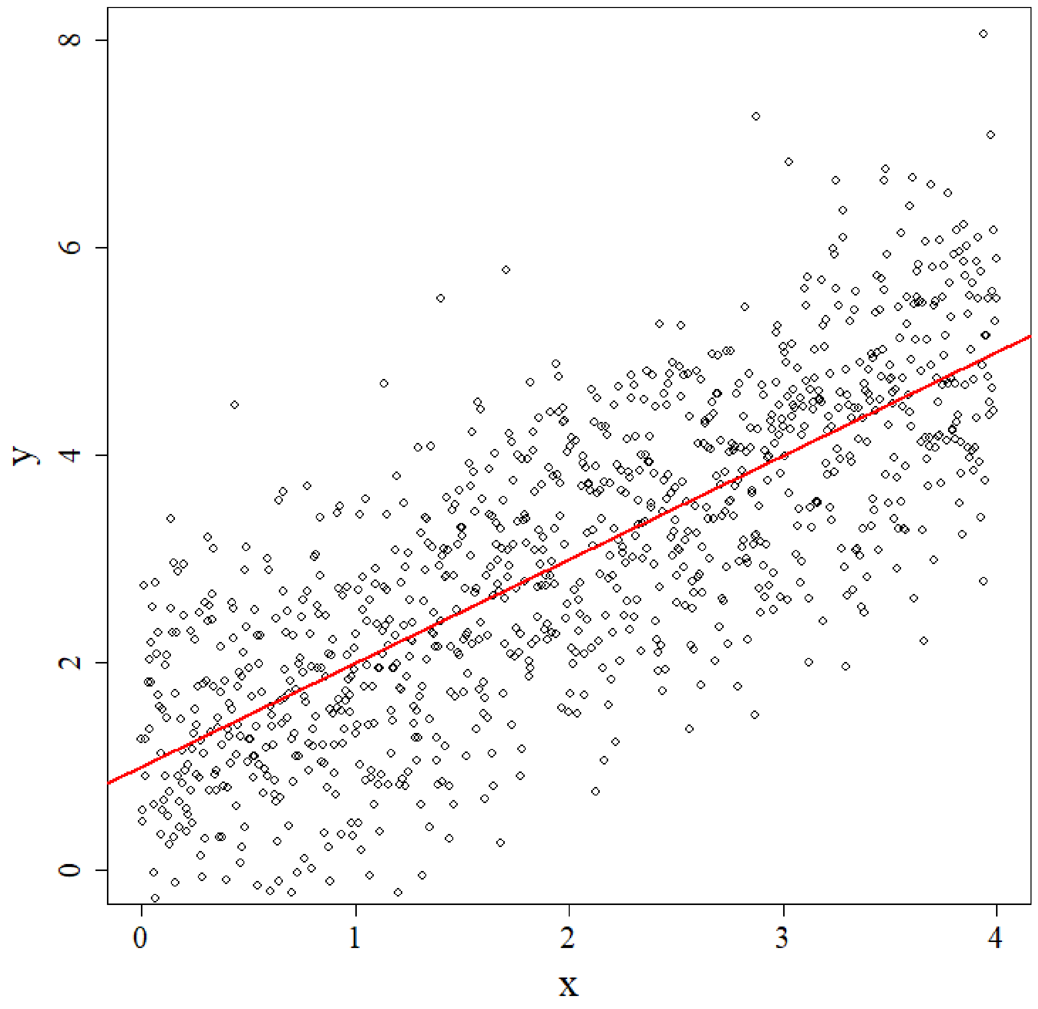


More variance

y = mx + b

$$\mu_i = \beta_0 + \beta_1 \times x_i$$
$$y_i \sim \text{normal}(\mu_i, \sigma^2)$$

$$\sigma = 1$$

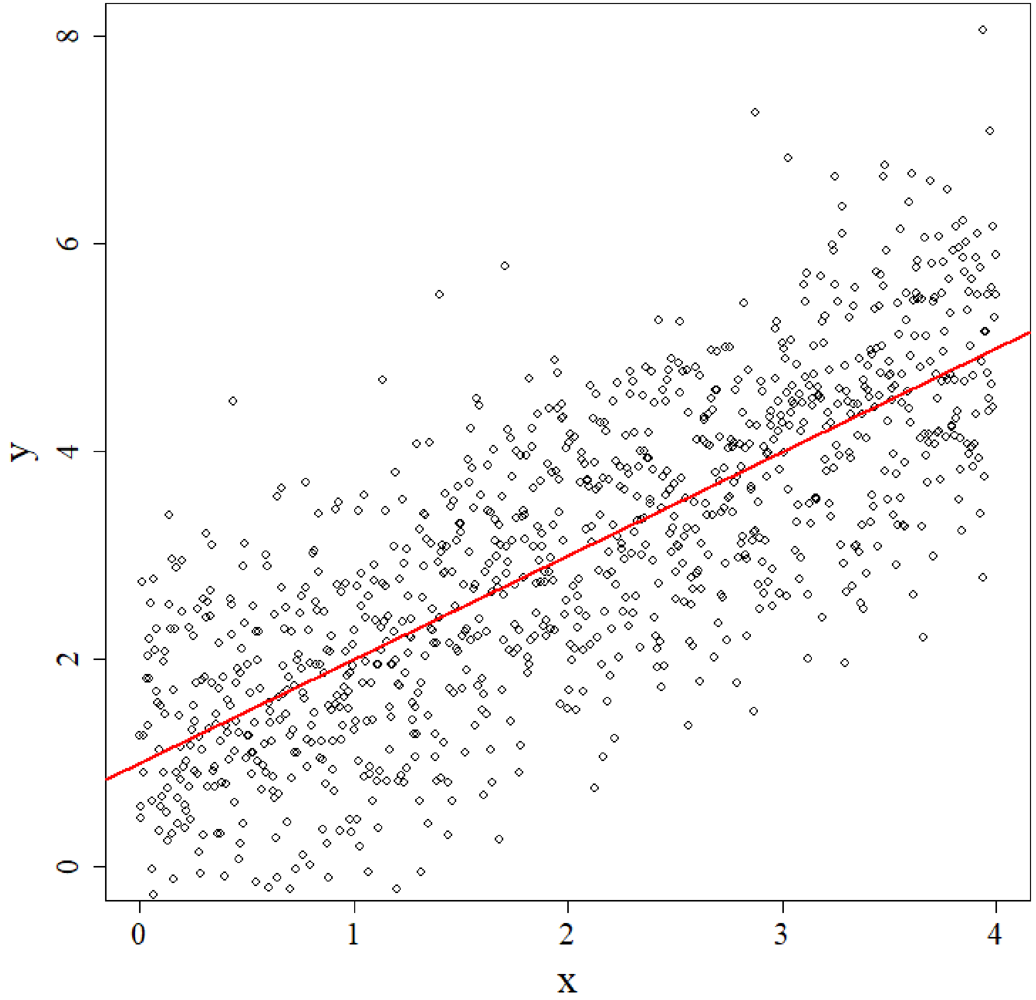


More variance means a broader distribution of data around the best fitting linear relationship

y = mx + b

$$\mu_i = \beta_0 + \beta_1 \times x_i$$
$$y_i \sim \text{normal}(\mu_i, \sigma^2)$$

$$\sigma = 1$$



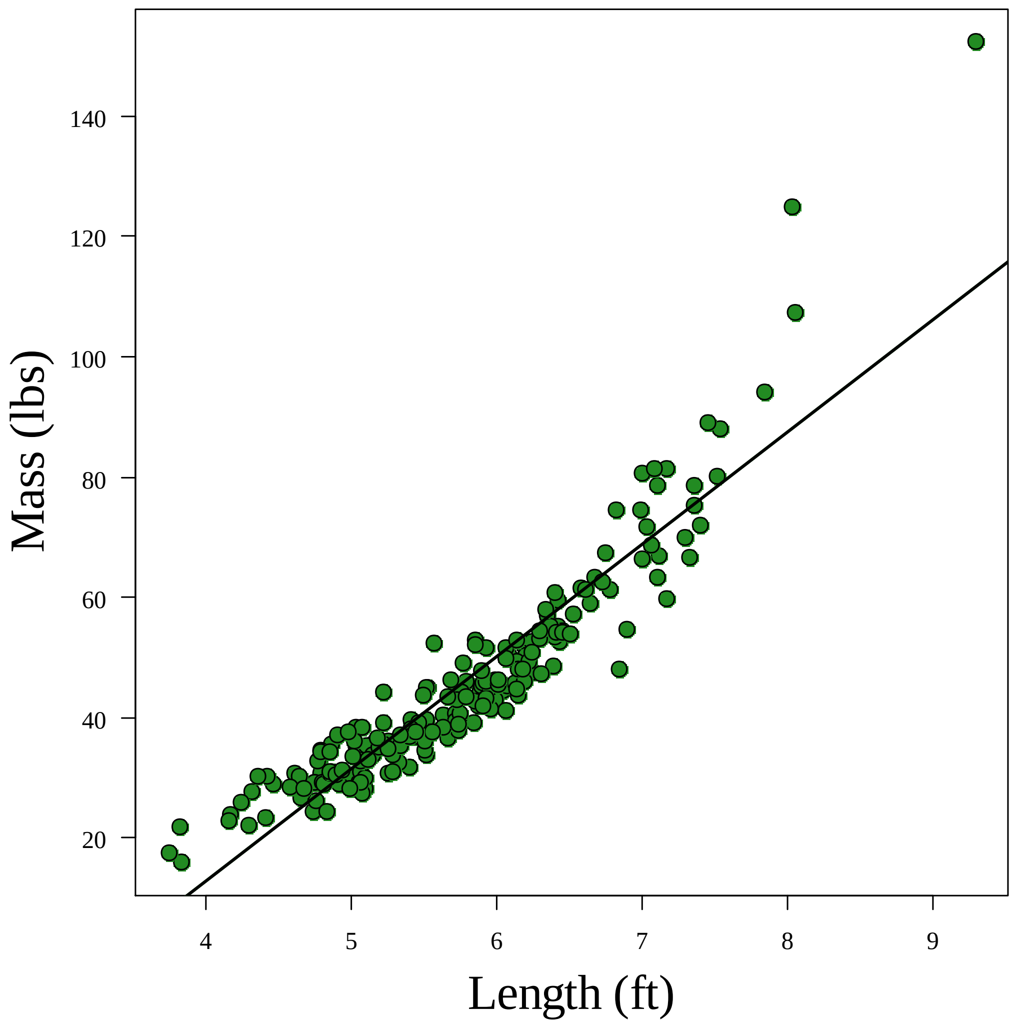
Simple linear model take-homes

1. *Slopes (or effect sizes) are the relationships between covariates and response variables (rise/run)*

2. *Intercepts are the values when the covariate = 0*

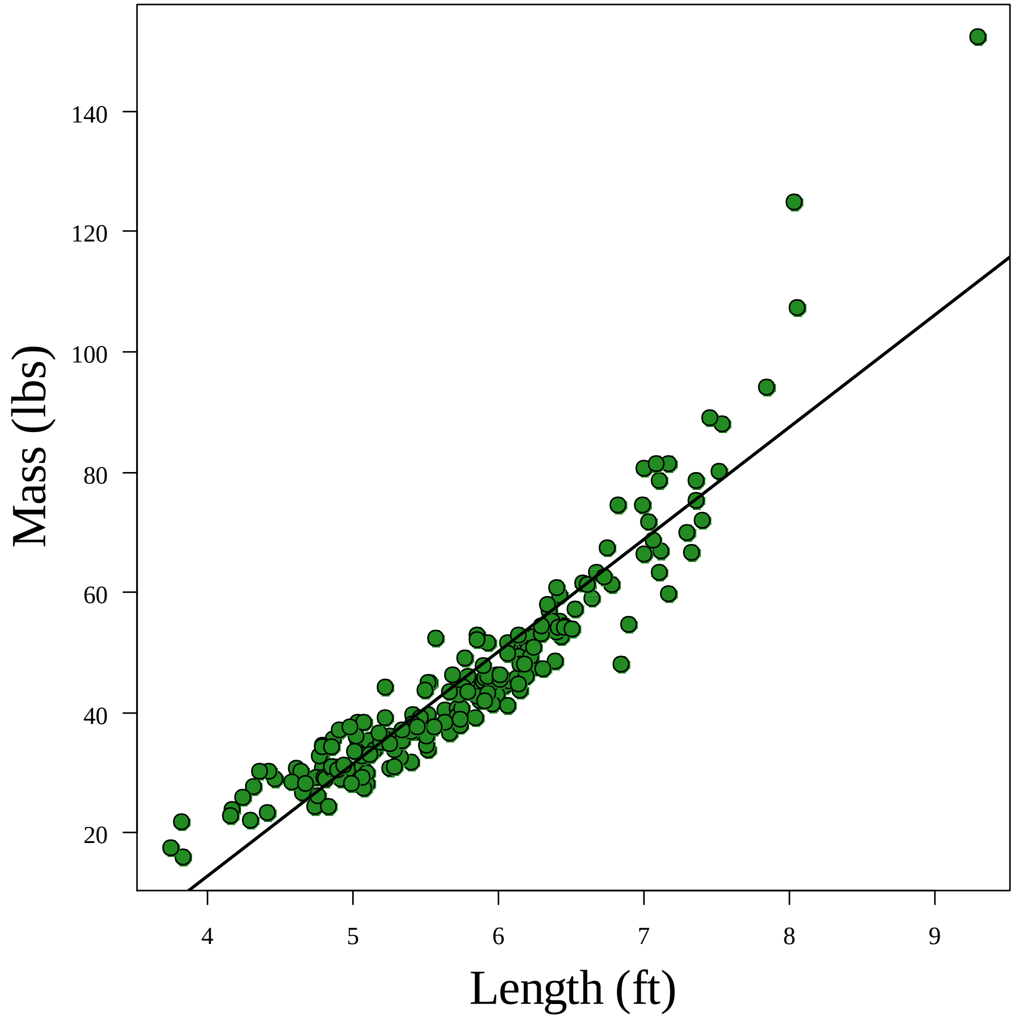
3. *Variance is the amount of variation around the best-fitting regression relationship*

The estimate of the slope (β_1) is 18.71 lbs/ft



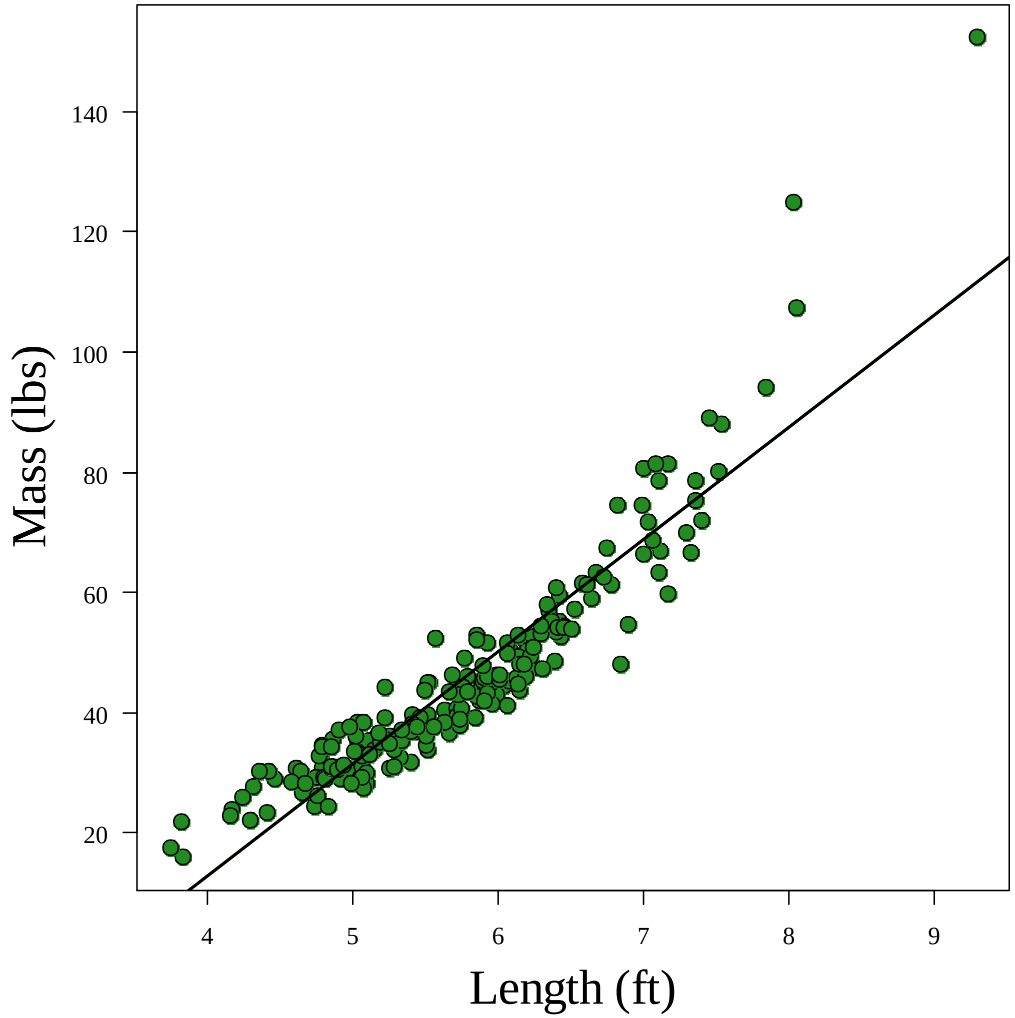
Those are linear models (easy peasy)

$$\mu_i = \beta_0 + \beta_1 \times x_i$$
$$y_i \sim \text{normal}(\mu_i, \sigma^2)$$



There's all kinds of other cool stuff!

$$\mu_i = \beta_0 + \beta_1 \times x_i$$
$$y_i \sim \text{normal}(\mu_i, \sigma^2)$$

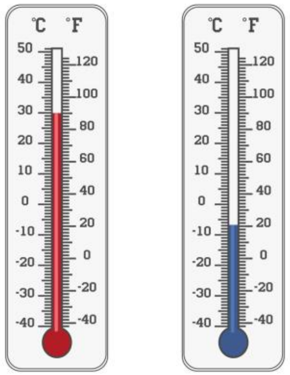


Let's think about 'scaling' our covariate values

A quick note about ‘scaling’

- Miles vs. kilometers per hour

- Inches vs cm



- Fahrenheit vs. Celsius

We measure things with different ‘scales’ all the time.

Linear model take-homes

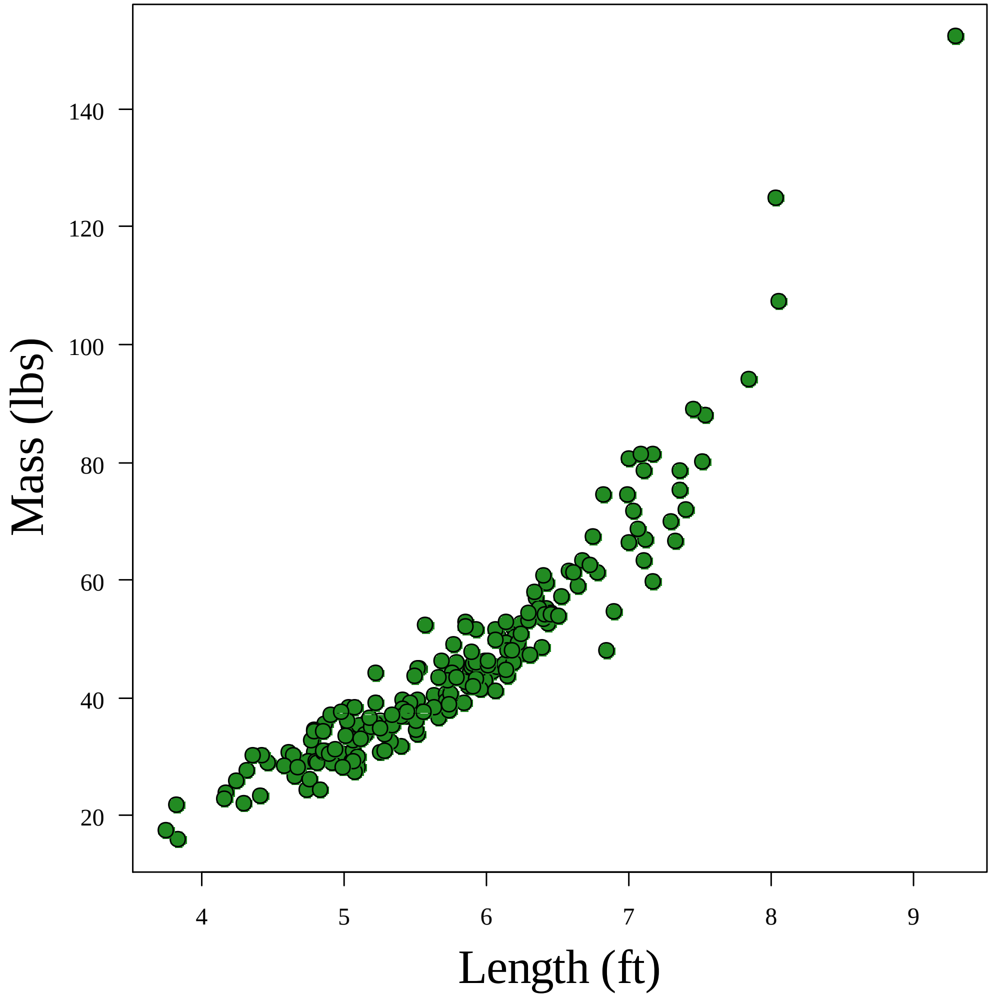
Slopes are the relationships between covariates and response variables (rise/run)

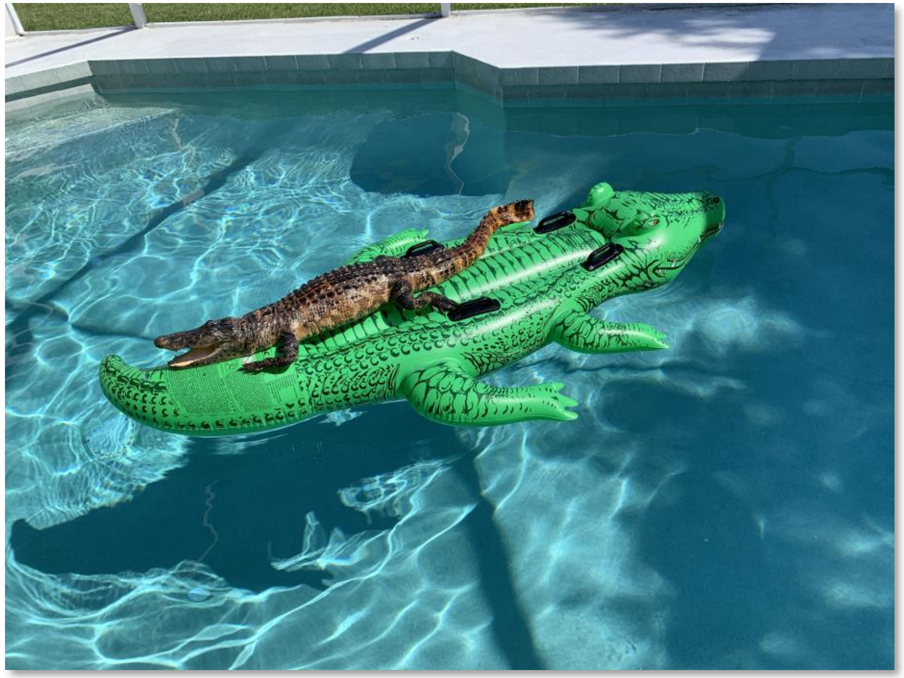
Intercepts are the values when the covariate = 0

Variance is the amount of variation around the best-fitting regression relationship

Think about the relationship between sample size and uncertainty

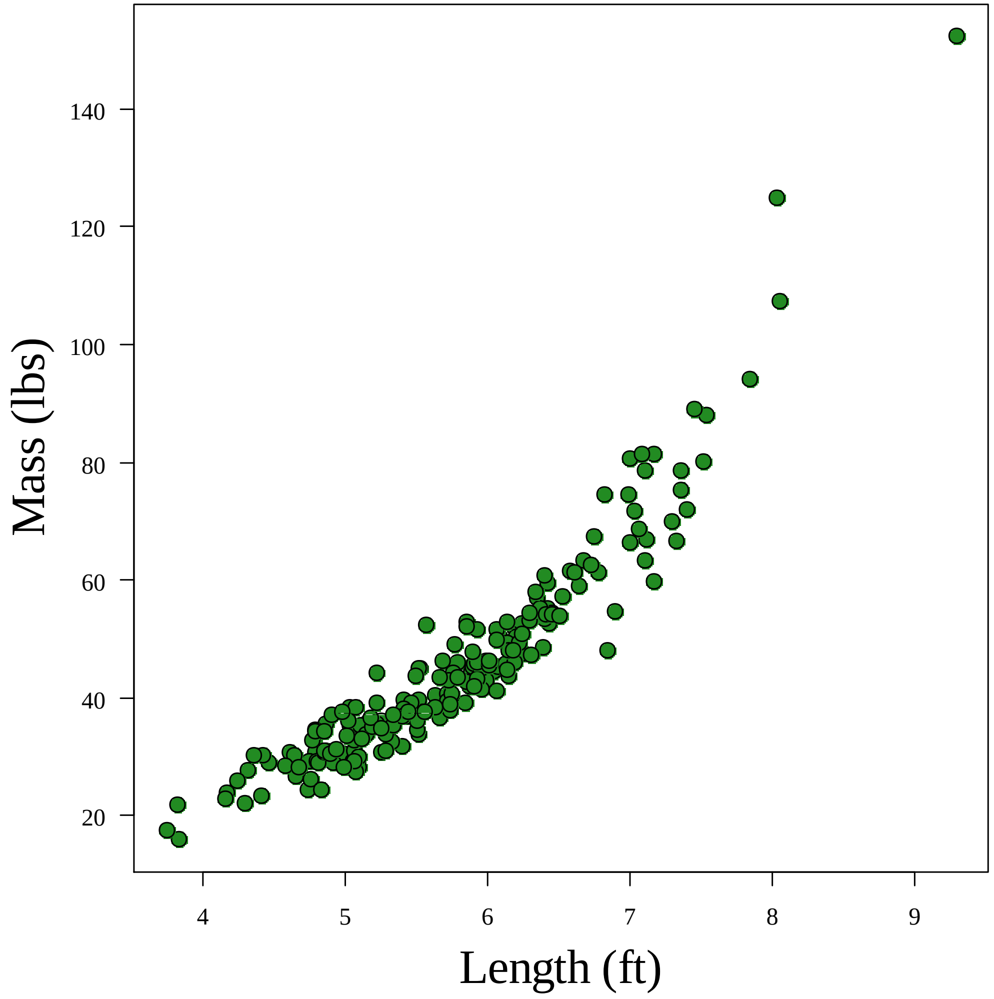
Let's relate this back to a Bayesian framework…

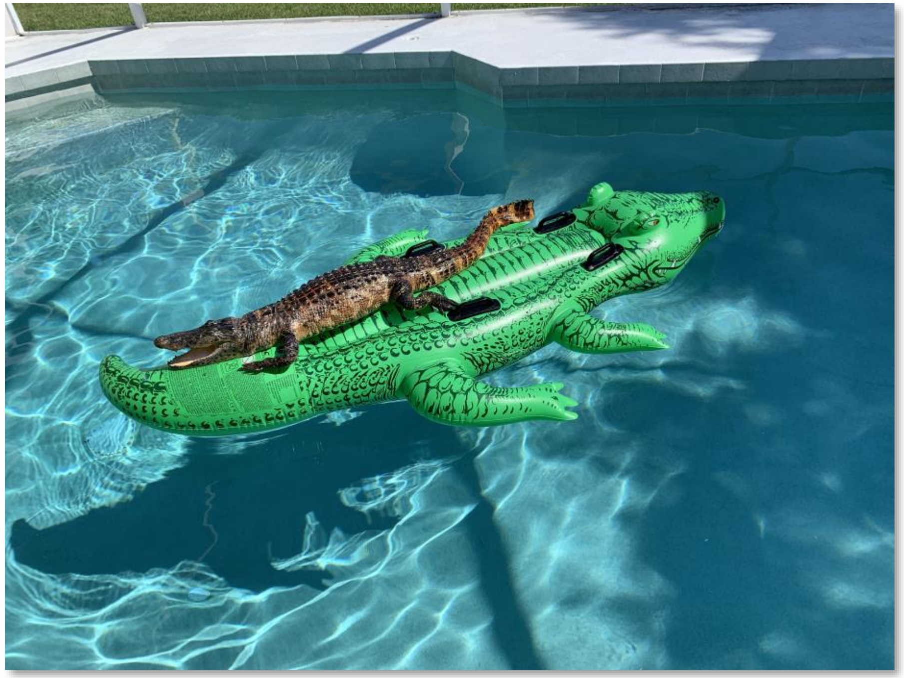




What would our intercept represent here?

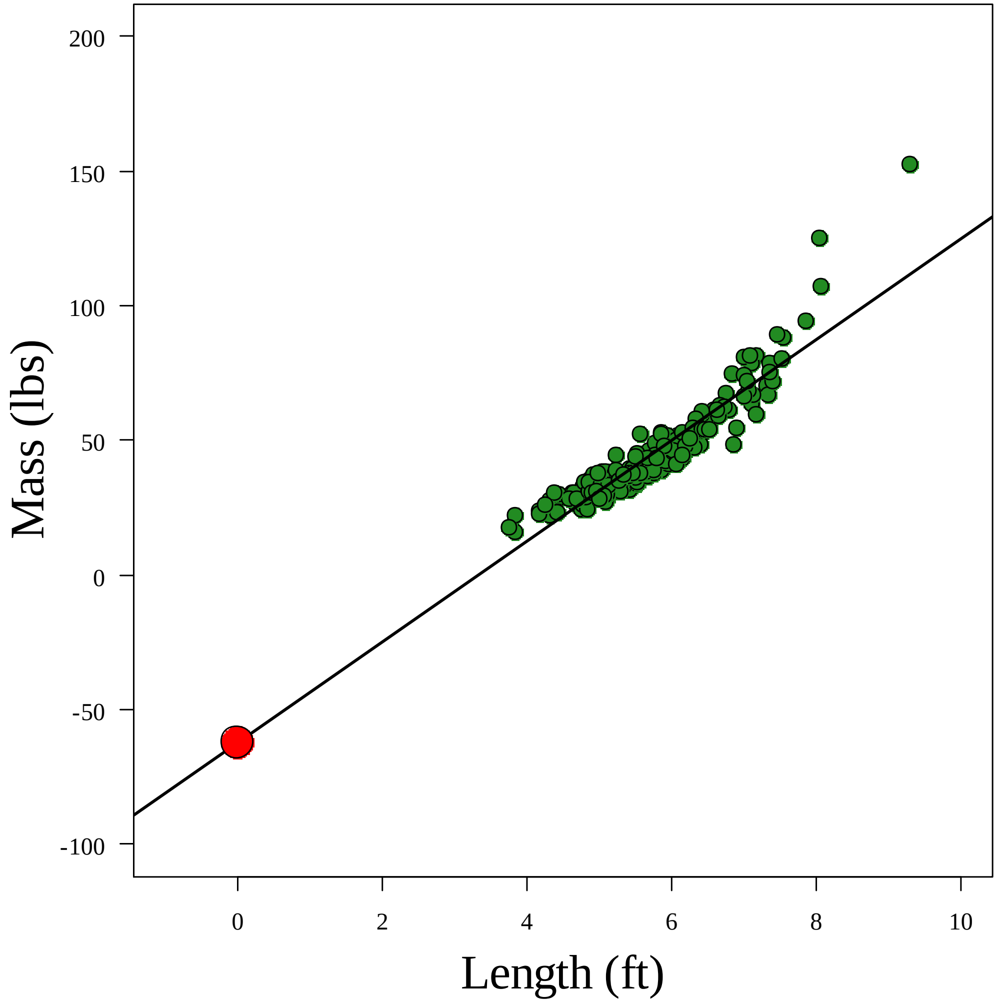
The mass of a 0 ft (or inch, or mm) long alligator

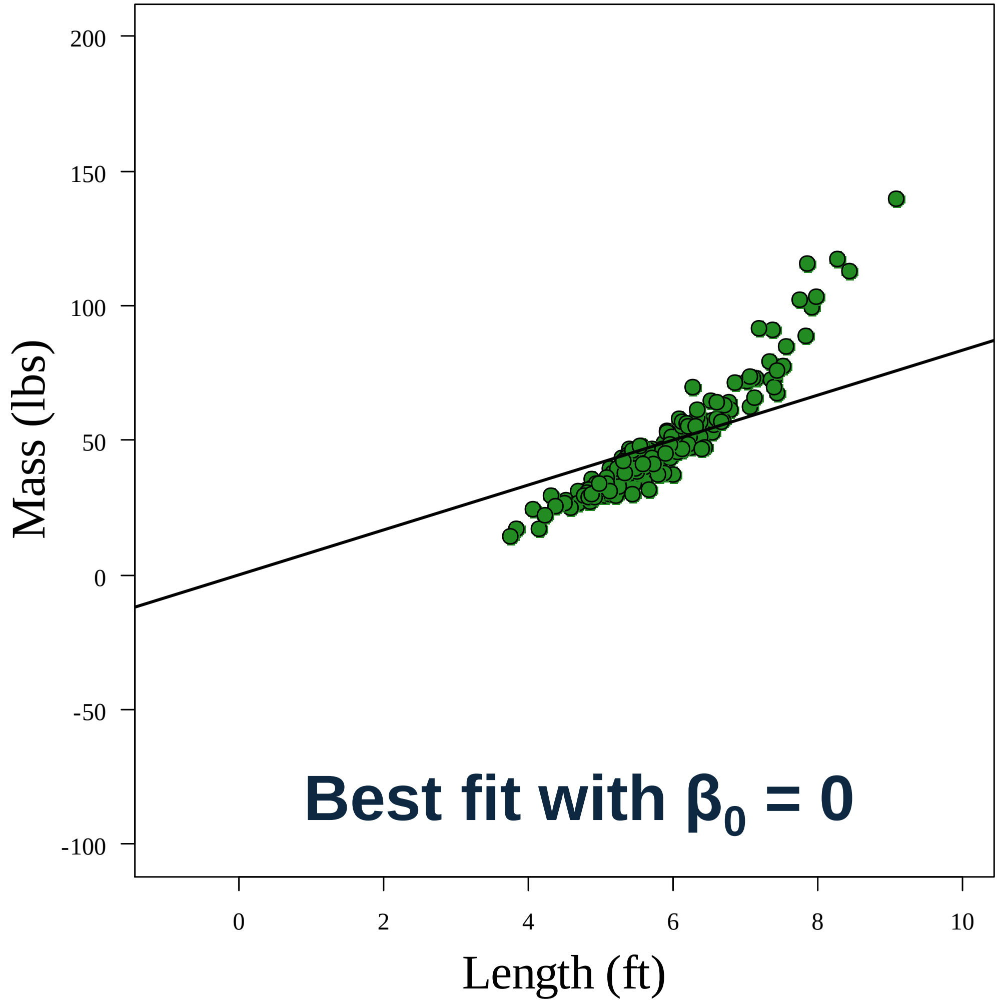




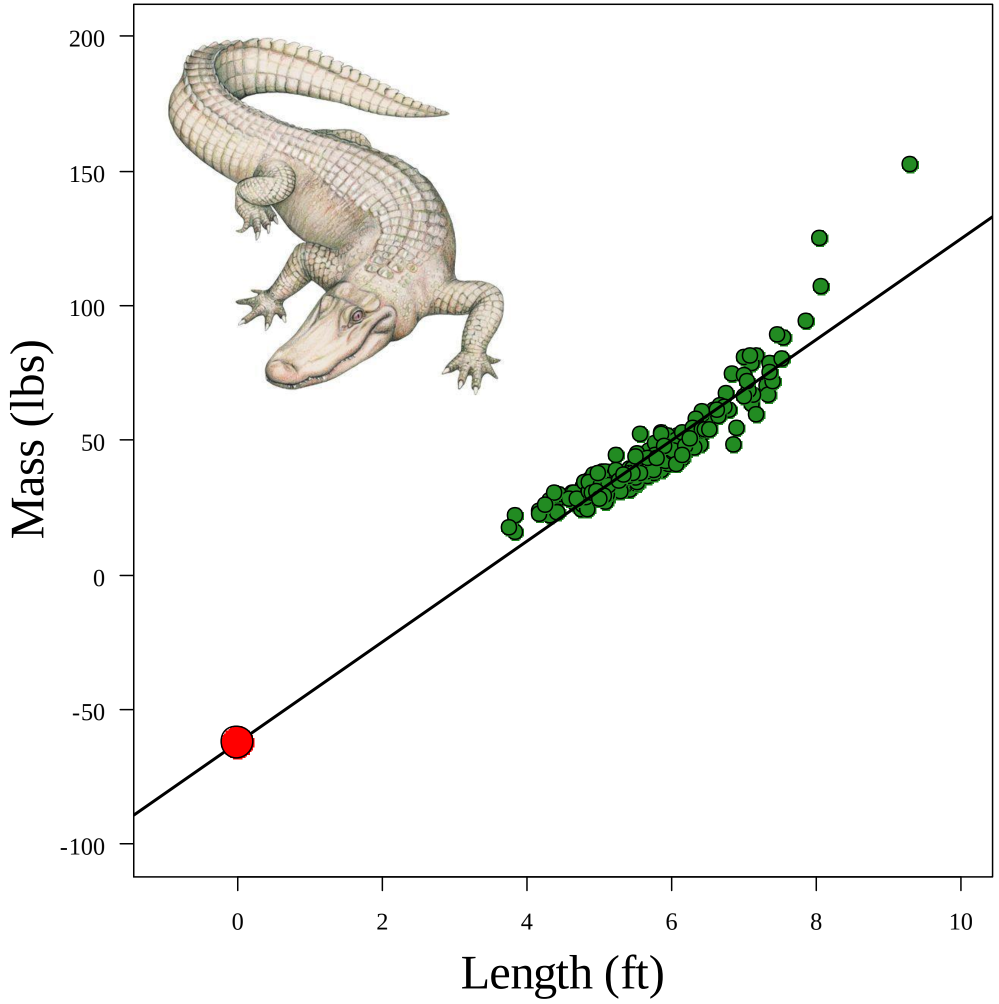
What would our prior be for that?

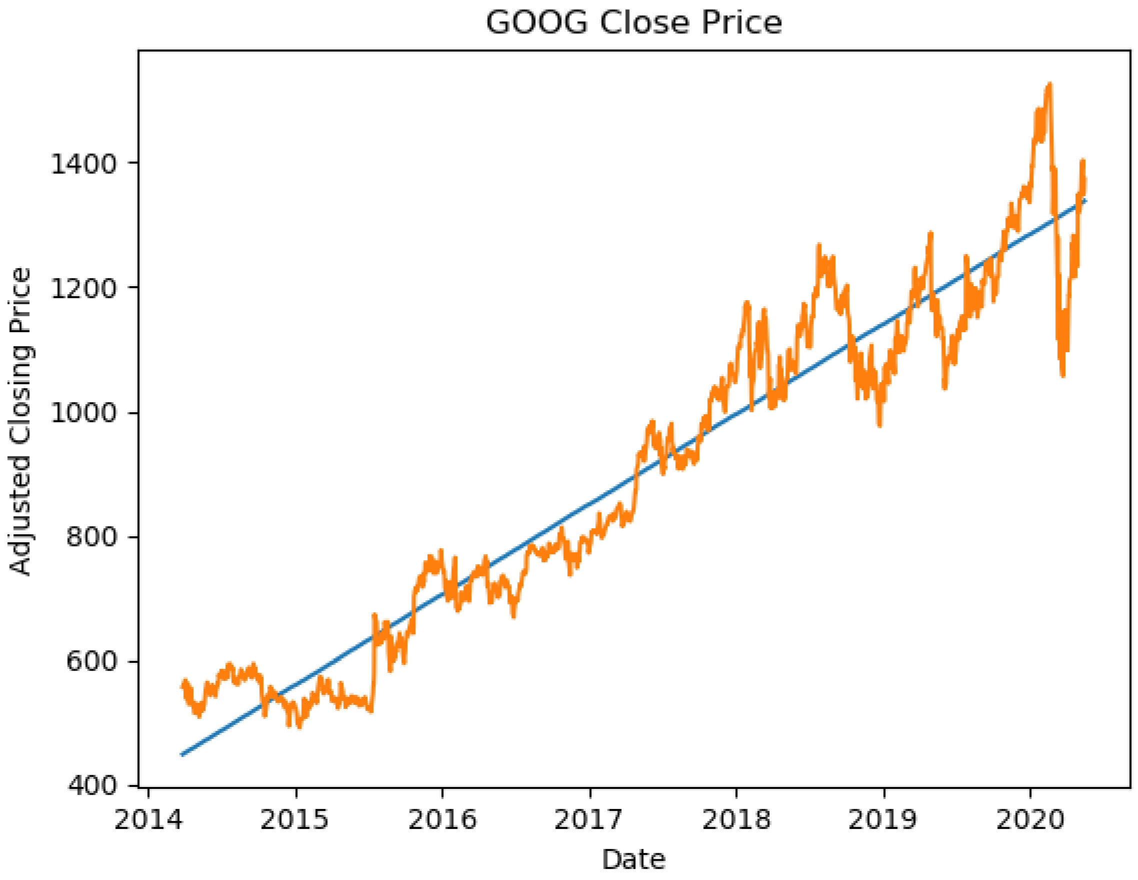
The actual ML estimate of the intercept (β_0) is -62.2 lbs





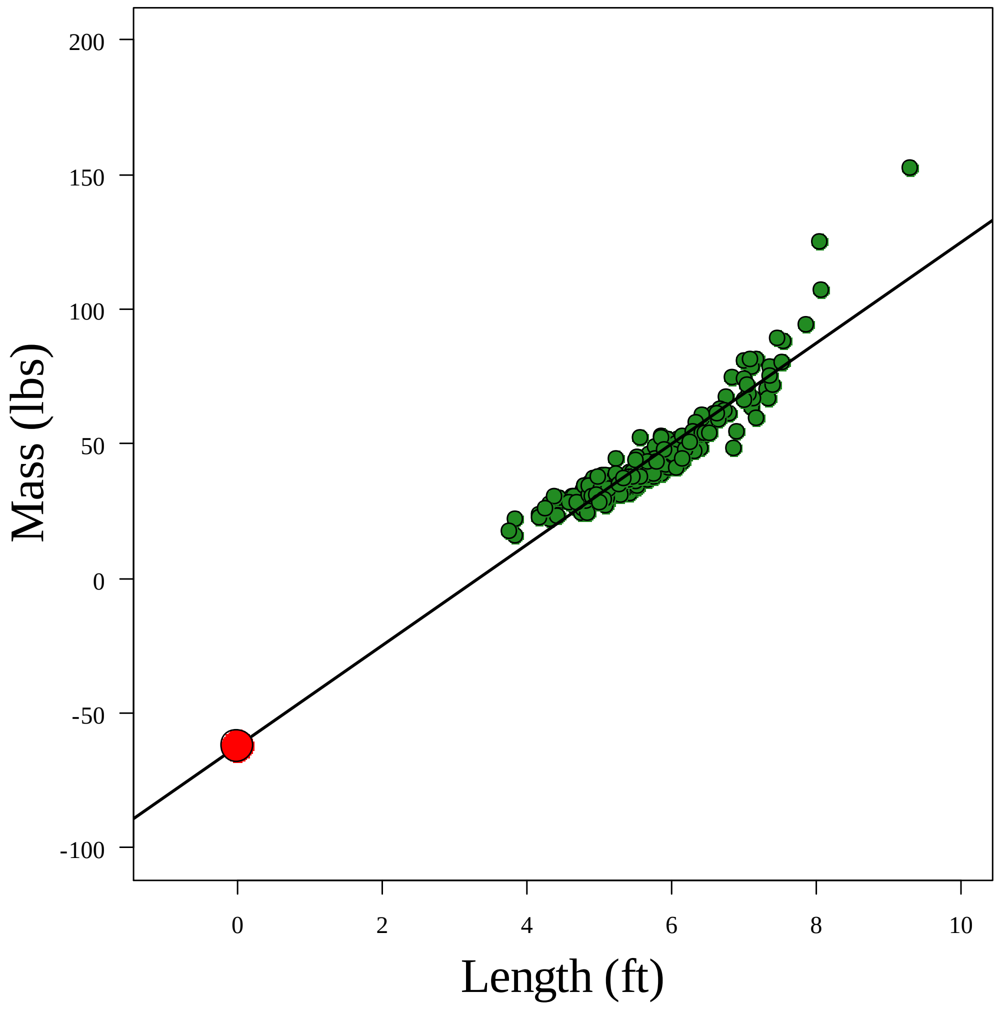
Sometimes intercepts at x = 0 are absurd





What if we wanted our intercept to mean something?

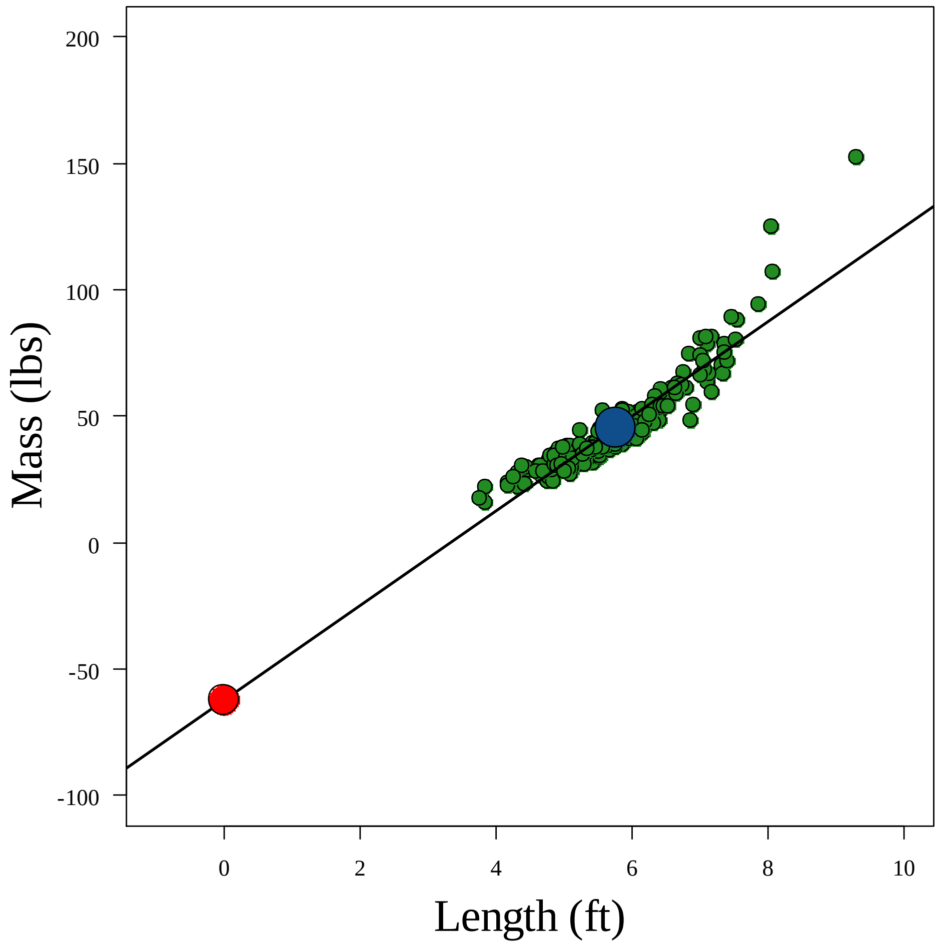
$$\mu_i = \beta_0 + \beta_1 \times x_i$$
$$y_i \sim \mathrm{normal}(\mu_i, \sigma^2)$$



What should it mean?

Perhaps the average weight of an average length gator?

$$\mu_i = \beta_0 + \beta_1 \times x_i$$
$$y_i \sim \text{normal}(\mu_i, \sigma^2)$$



How would we make the mean value of x = 0?

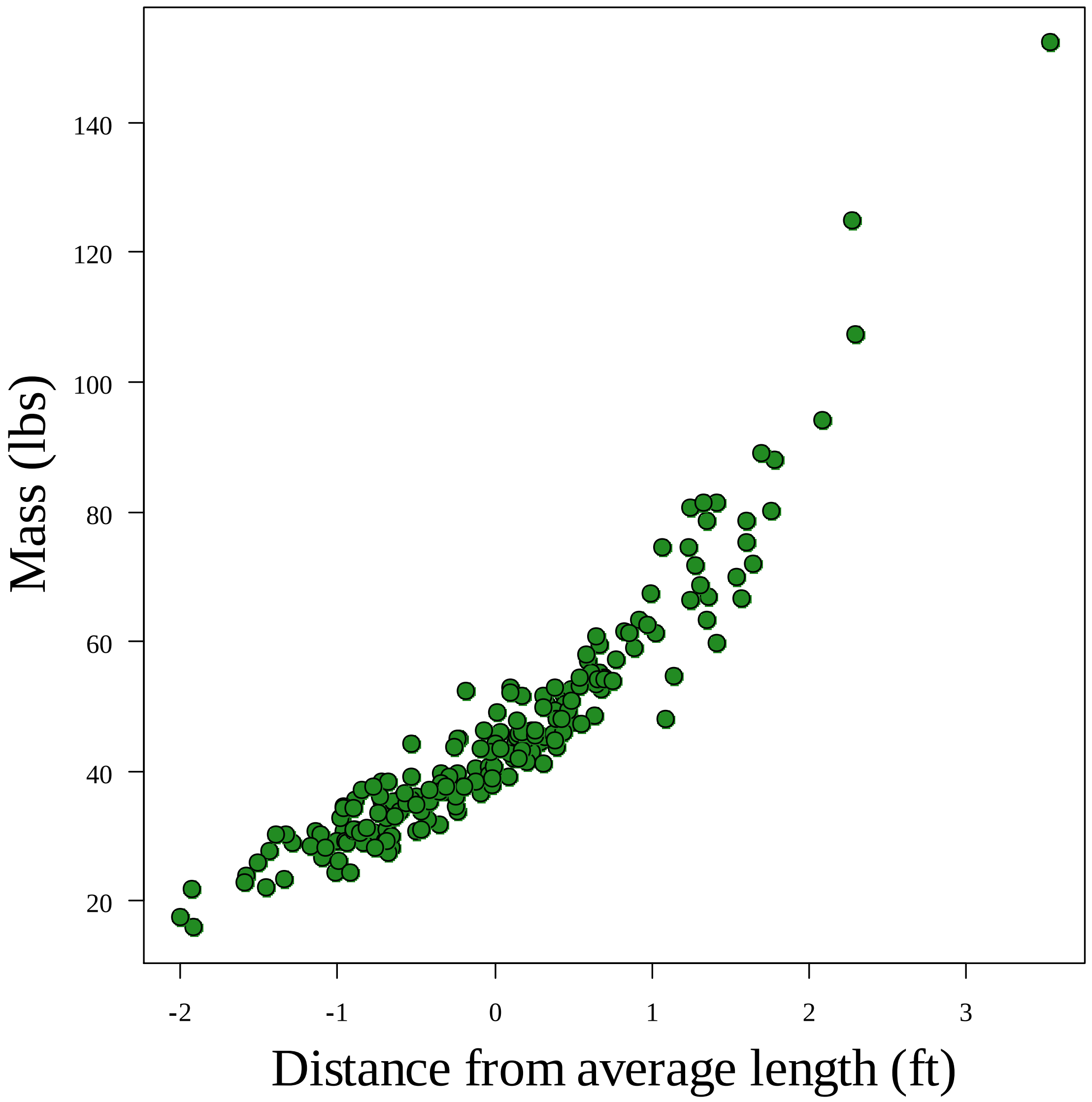
Two key tools

1. **Centering (making the intercept the mean of x)**

2. **Z-standardizing (centering and <u>scaling</u>)**

Centering!

$$\mu_i = \beta_0 + \beta_1 \times x_i'$$
$$y_i \sim \mathrm{normal}(\mu_i, \sigma^2)$$

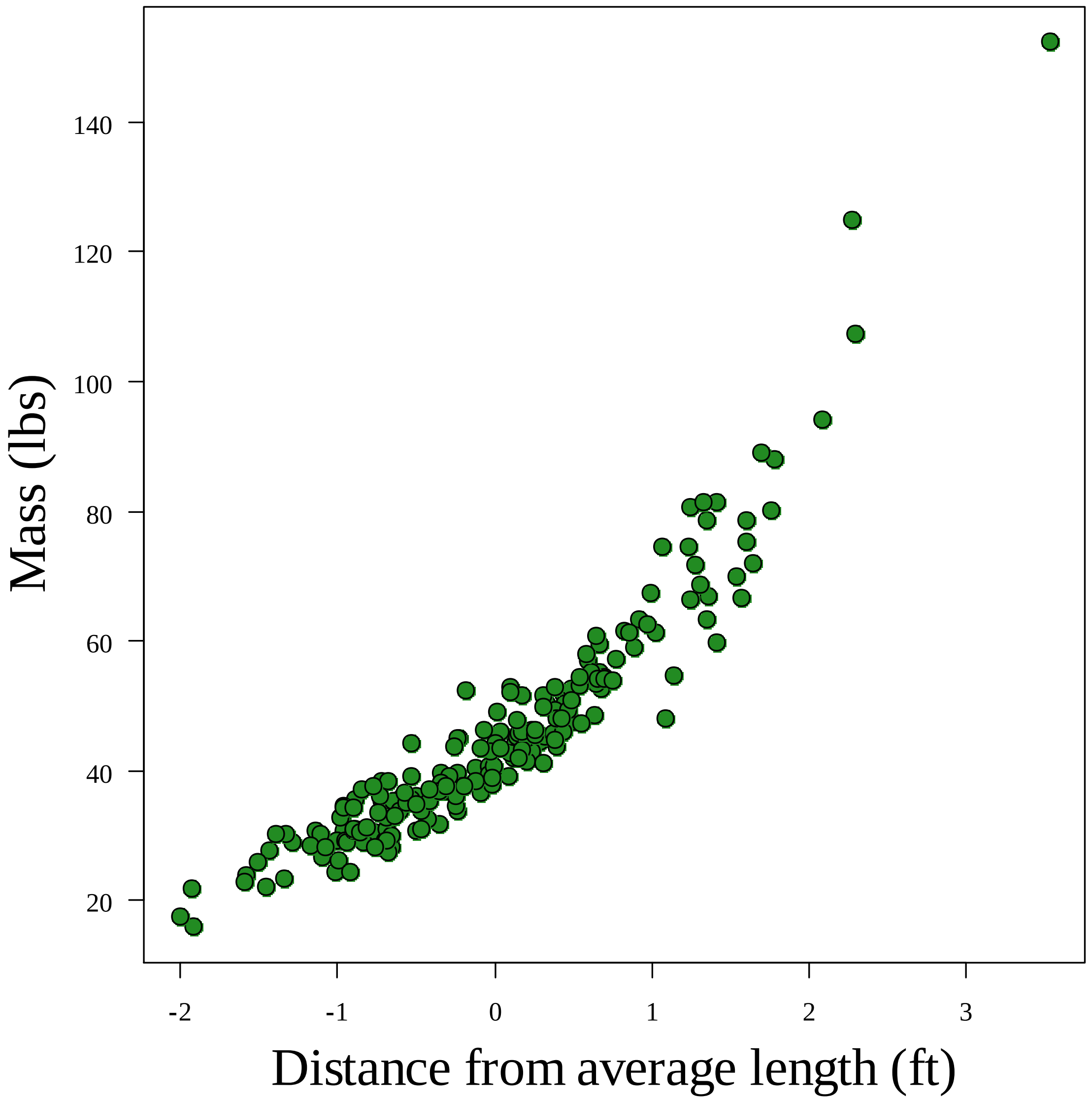


$$x_i' = x_i - \bar{x}$$

Centering!

$$\mu_i = \beta_0 + \beta_1 \times x_i$$
$$y_i \sim \text{normal}(\mu_i, \sigma^2)$$

$$\beta_0 = 45.46$$
$$\beta_1 = 18.71$$

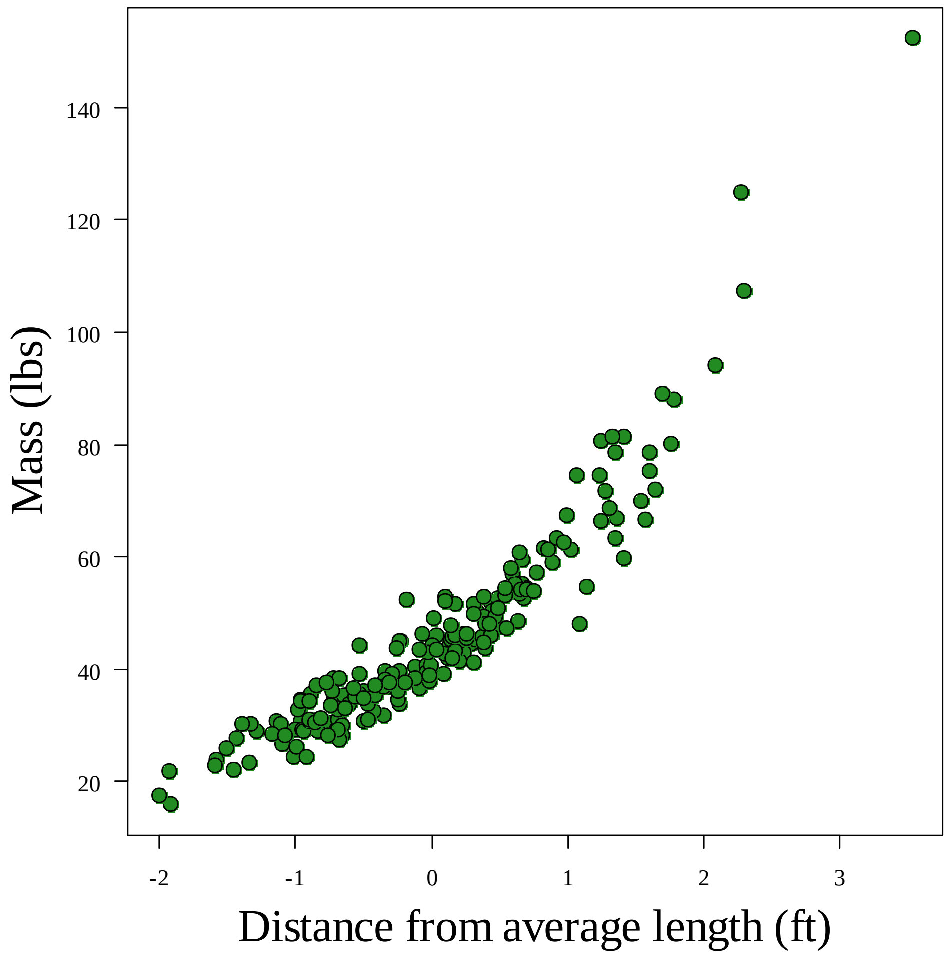


Our slope is the same as before… why?

What if we wanted to 'scale' our covariate (i.e., not in ft)?

$$\mu_i = \beta_0 + \beta_1 \times x_{1,i}$$
$$y_i \sim \text{normal}(\mu_i, \sigma^2)$$

$$\beta_0 = 45.46$$
$$\beta_1 = 18.71$$

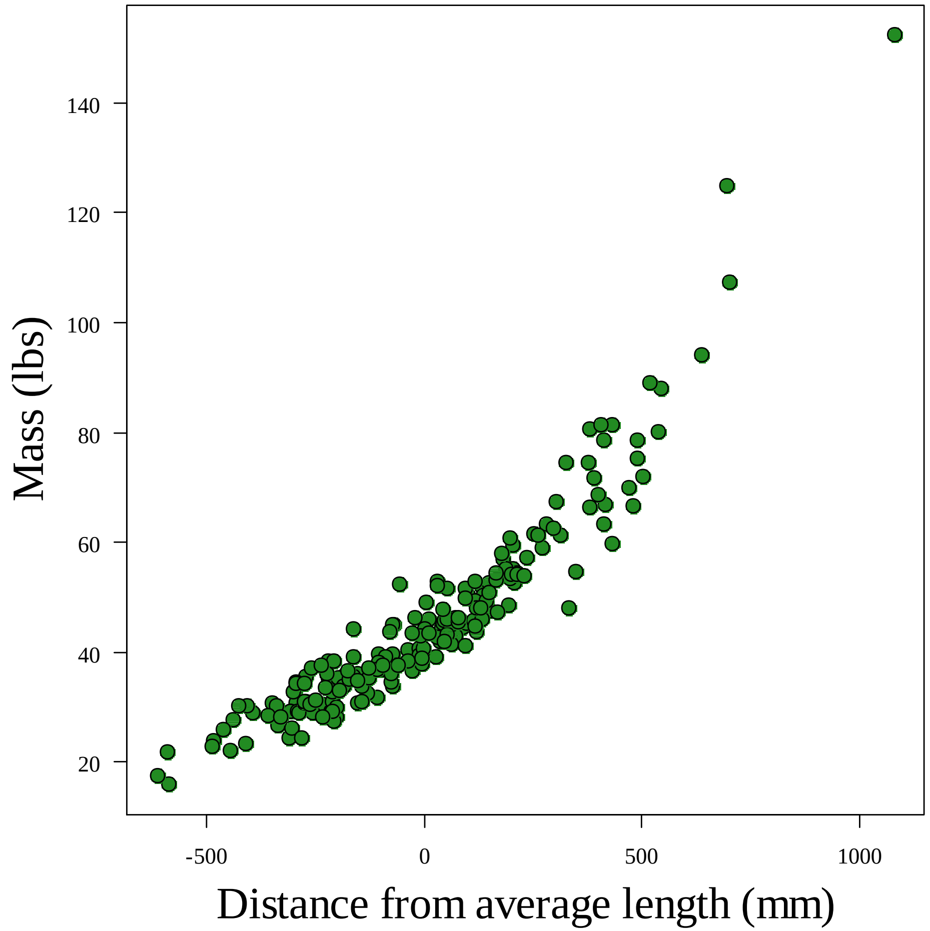


1st let's define the problem better?!

e.g., here is the same relationship in mm

$$\mu_i = \beta_0 + \beta_1 \times x_{1,i}$$
$$y_i \sim \text{normal}(\mu_i, \sigma^2)$$

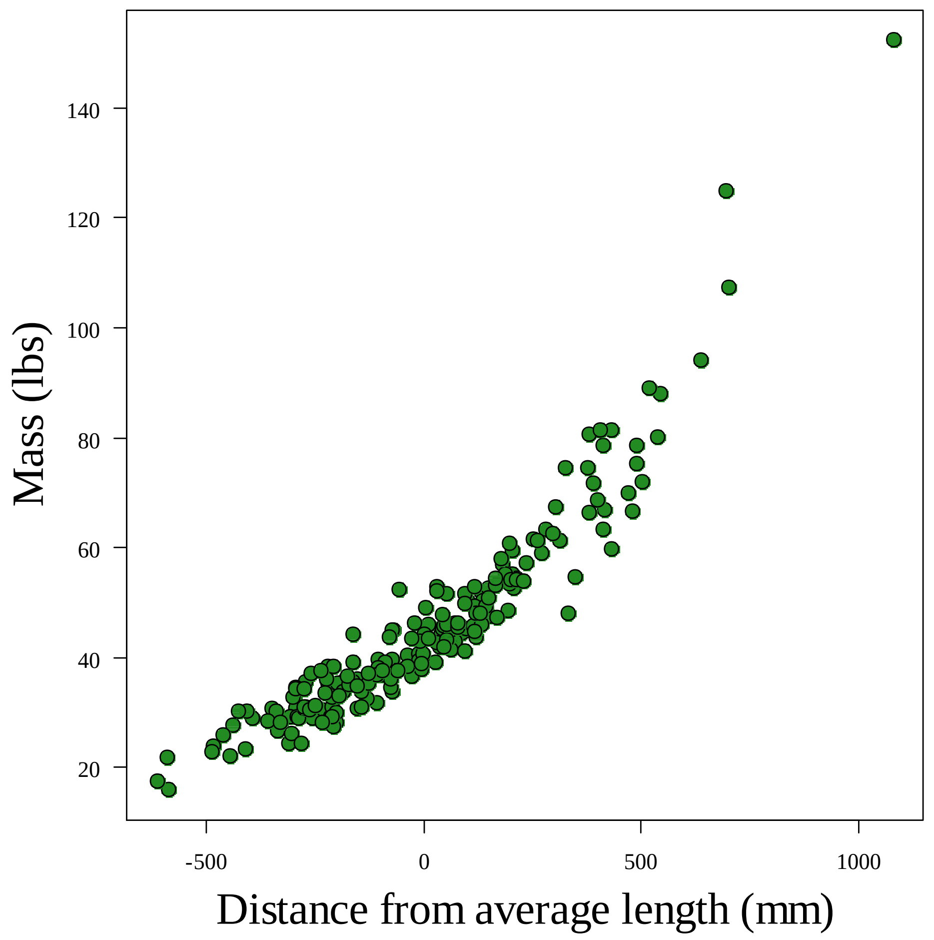
$$\beta_0 = 45.46$$
$$\beta_1 = 0.061$$



Imagine I’m trying to compare the importance of two covariates

$$\mu_i = \beta_0 + \beta_1 \times x_i + \beta_2 \times \text{sex}_i$$
$$y_i \sim \text{normal}(\mu_i, \sigma^2)$$

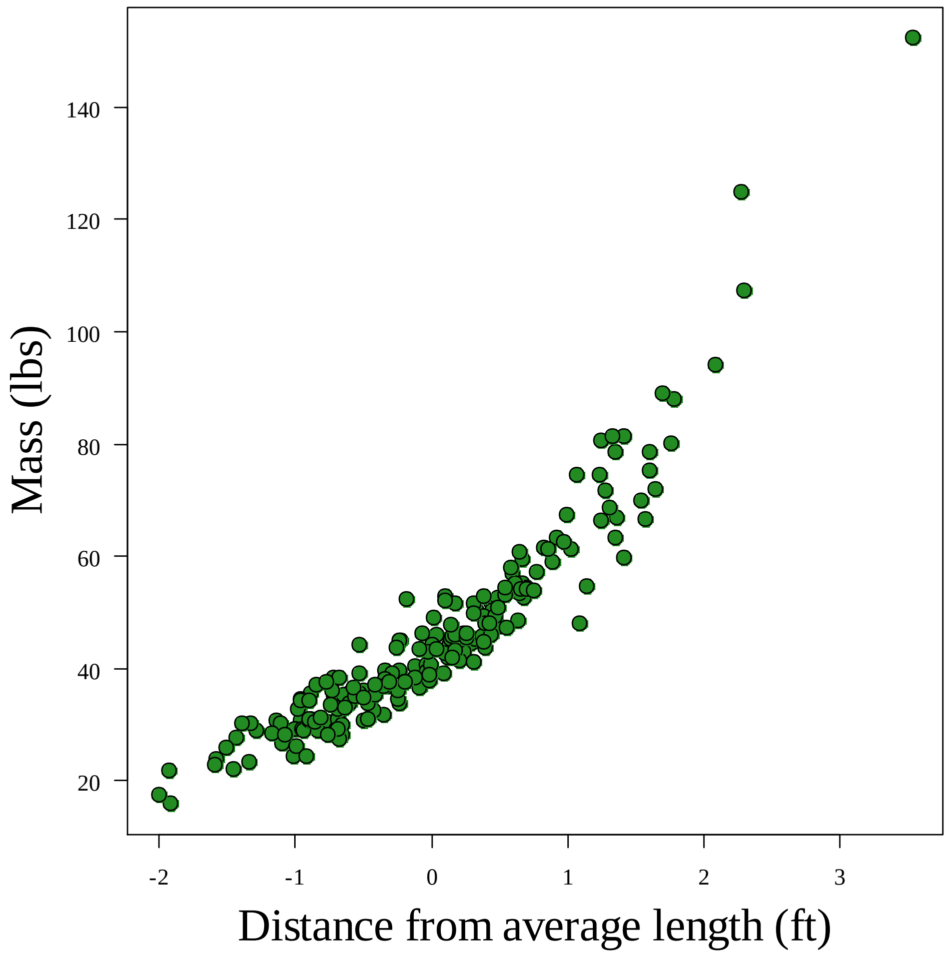
$$\beta_0 = 45.46$$
$$\beta_1 = 0.061$$
$$\beta_2 = 5$$



Imagine I'm trying to compare the importance of two covariates

$$\mu_i = \beta_0 + \beta_1 \times x_i + \beta_2 \times \mathrm{sex}_i$$
$$y_i \sim \mathrm{normal}(\mu_i, \sigma^2)$$

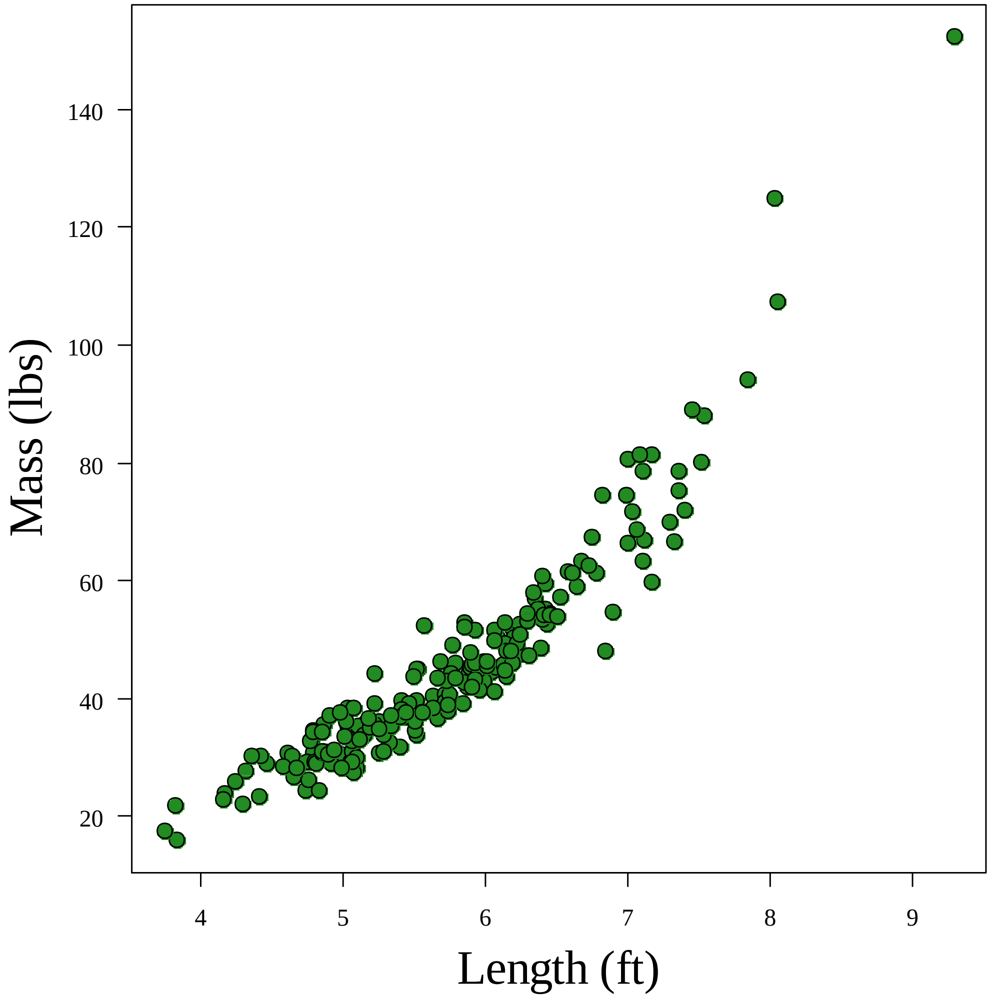
$$\beta_0 = 45.46$$
$$\beta_1 = 18.71$$
$$\beta_2 = 5$$



Z-standardizing

$$\mu_i = \beta_0 + \beta_1 \times z_i$$
$$y_i \sim \text{normal}(\mu_i, \sigma^2)$$

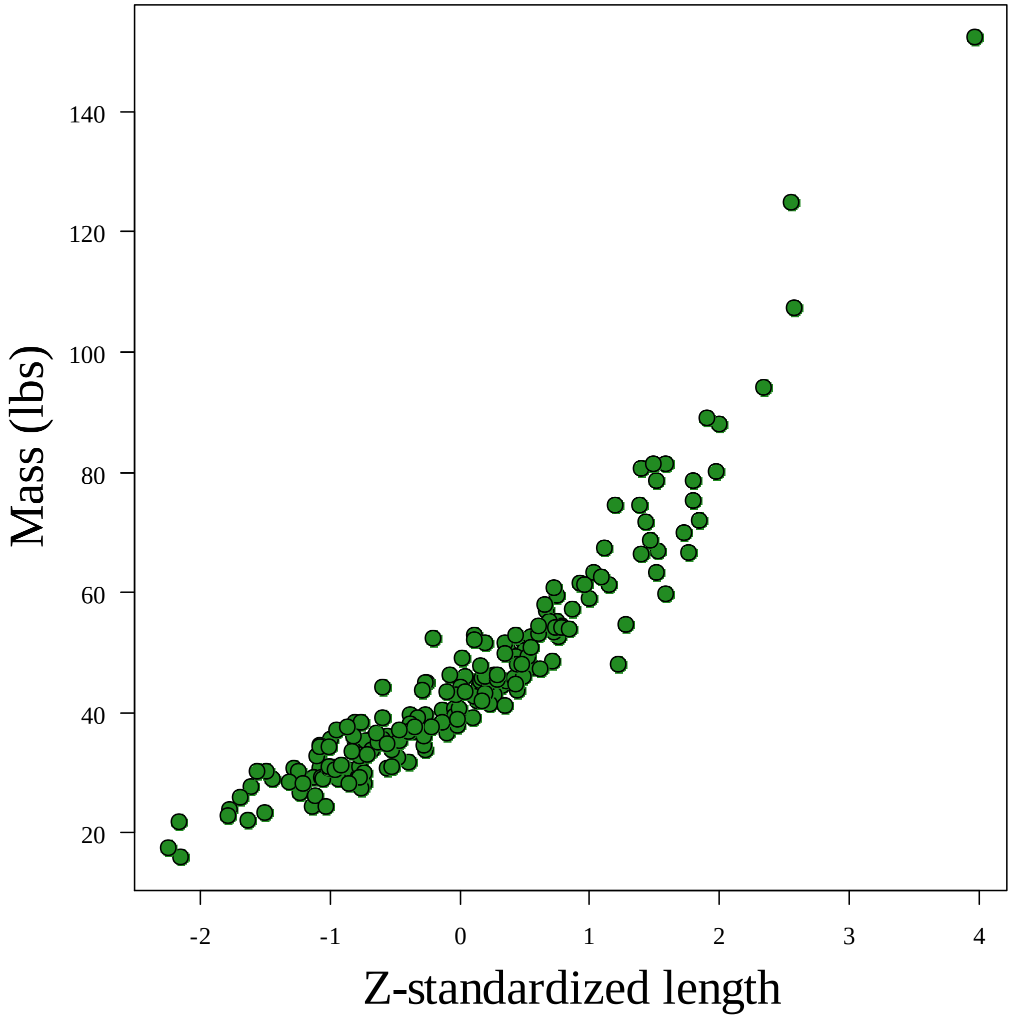
$$z_i = \frac{x_i - \bar{x}}{\sigma_x}$$



Z-standardized

$$\mu_i = \beta_0 + \beta_1 \times z_i$$
$$y_i \sim \text{normal}(\mu_i, \sigma^2)$$

$$\beta_0 = 45.46$$
$$\beta_1 = 16.711$$

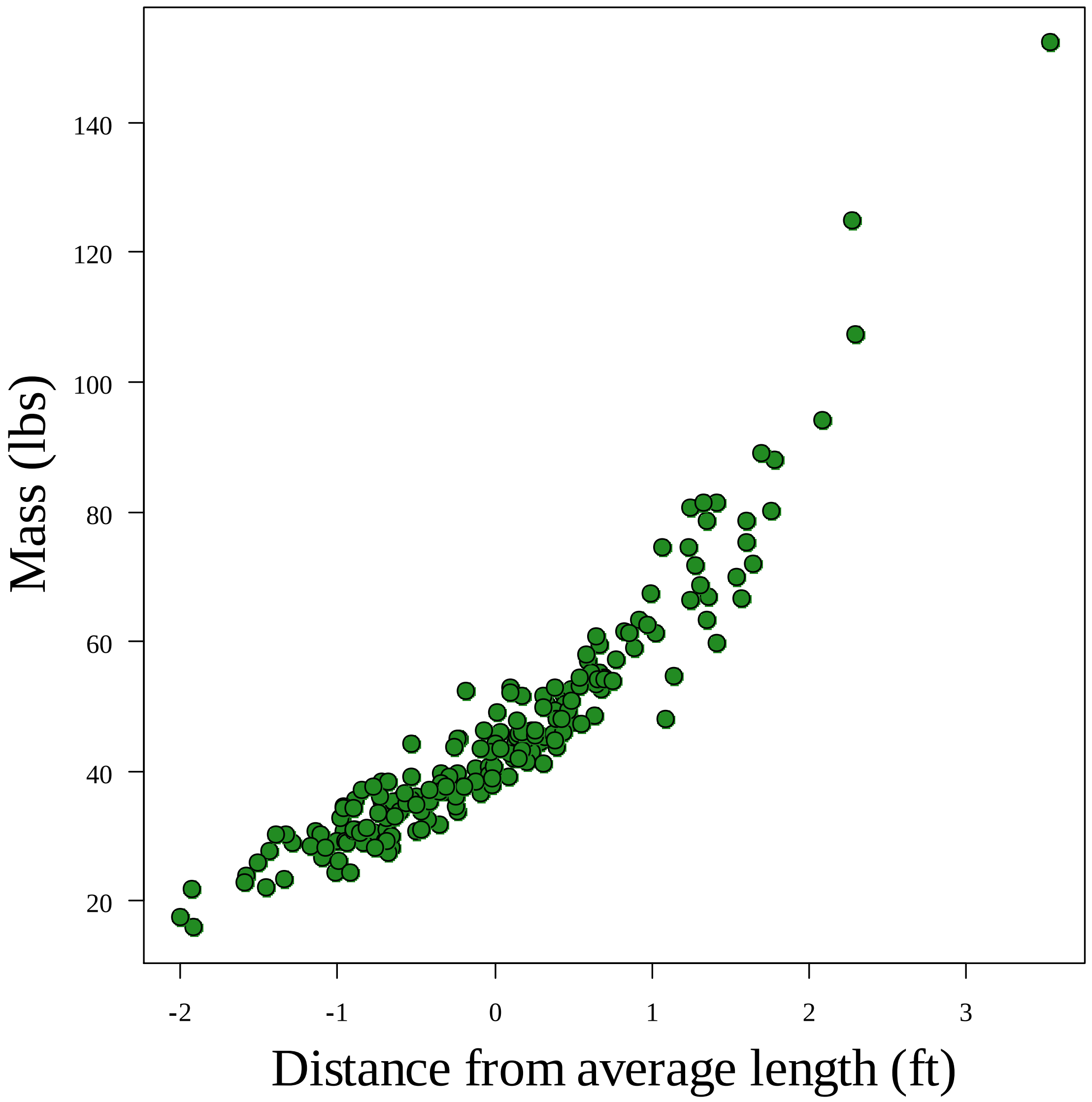


Note that the x-axis would be the same with ft or mm!

Centered

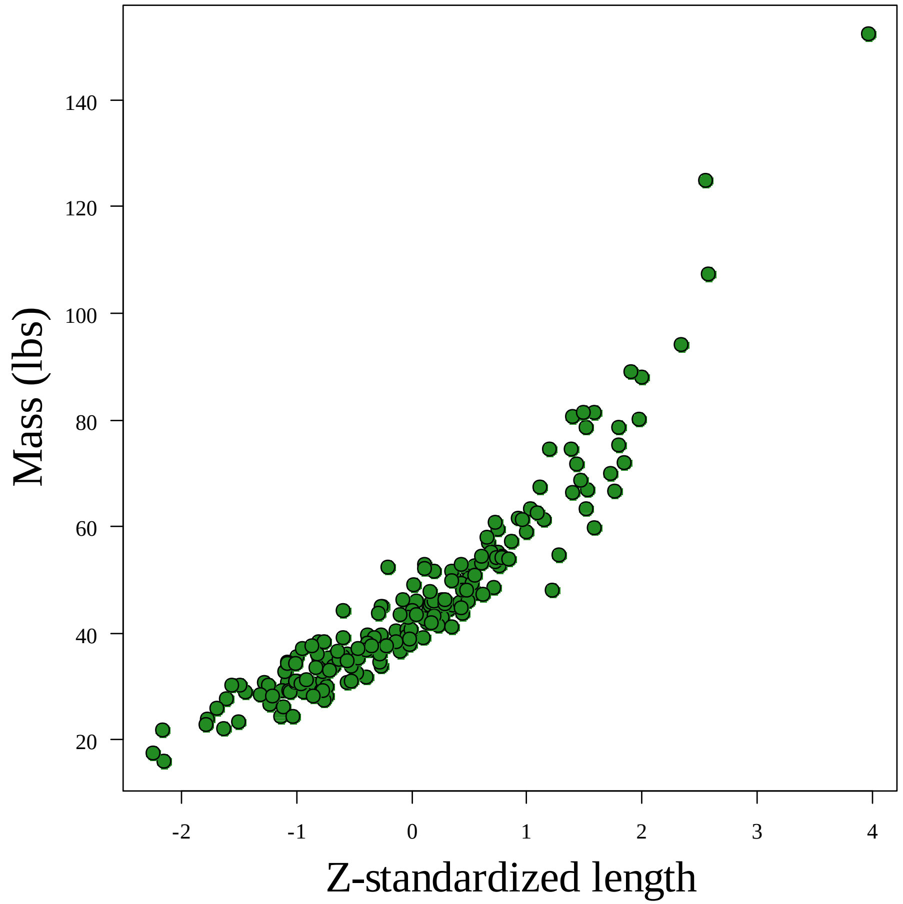
$$\mu_i = \beta_0 + \beta_1 \times x_i'$$
$$y_i \sim \mathrm{normal}(\mu_i, \sigma^2)$$

$$\beta_0 = 45.46$$
$$\beta_1 = 18.71$$

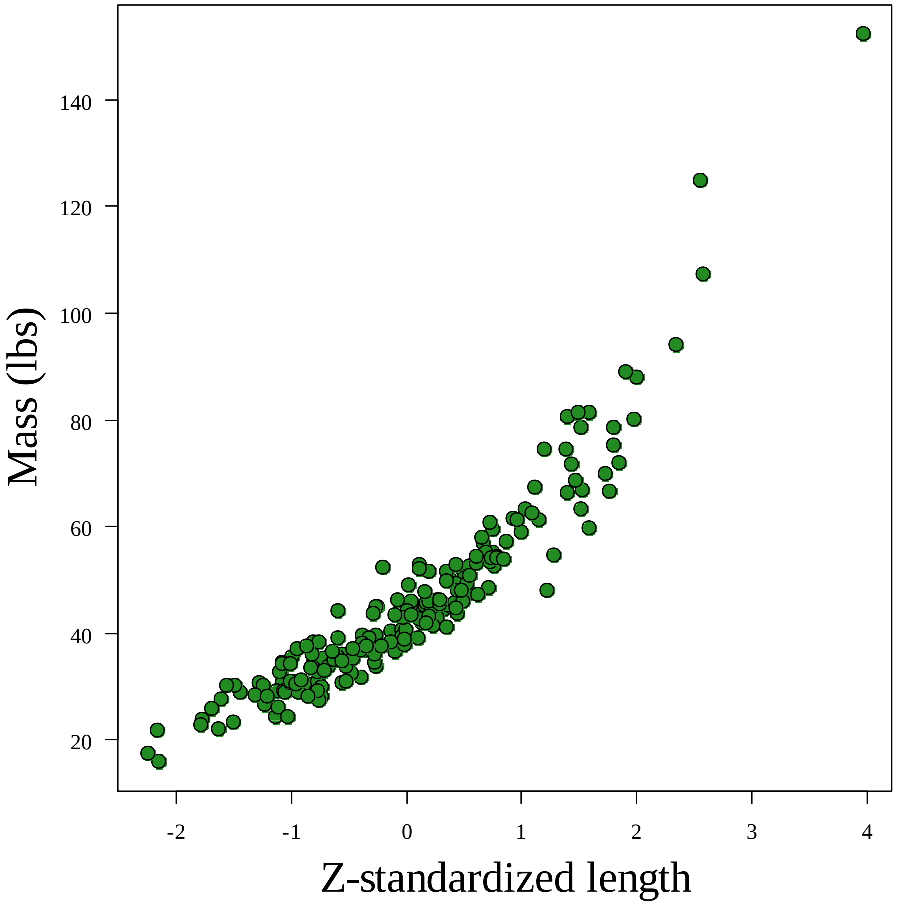


$$x_i' = x_i - \bar{x}$$

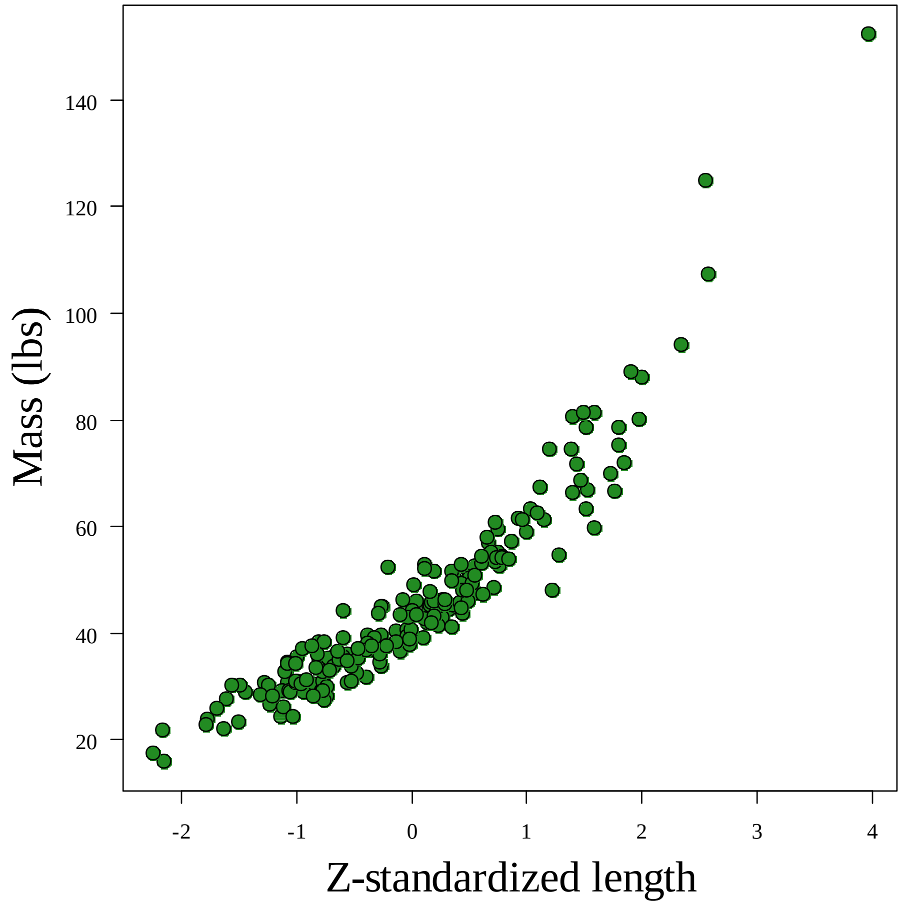
Take-home: slopes are simply the change in y per 1 unit change in x



Take-home: intercepts are the expected value when the covariate(s) = 0

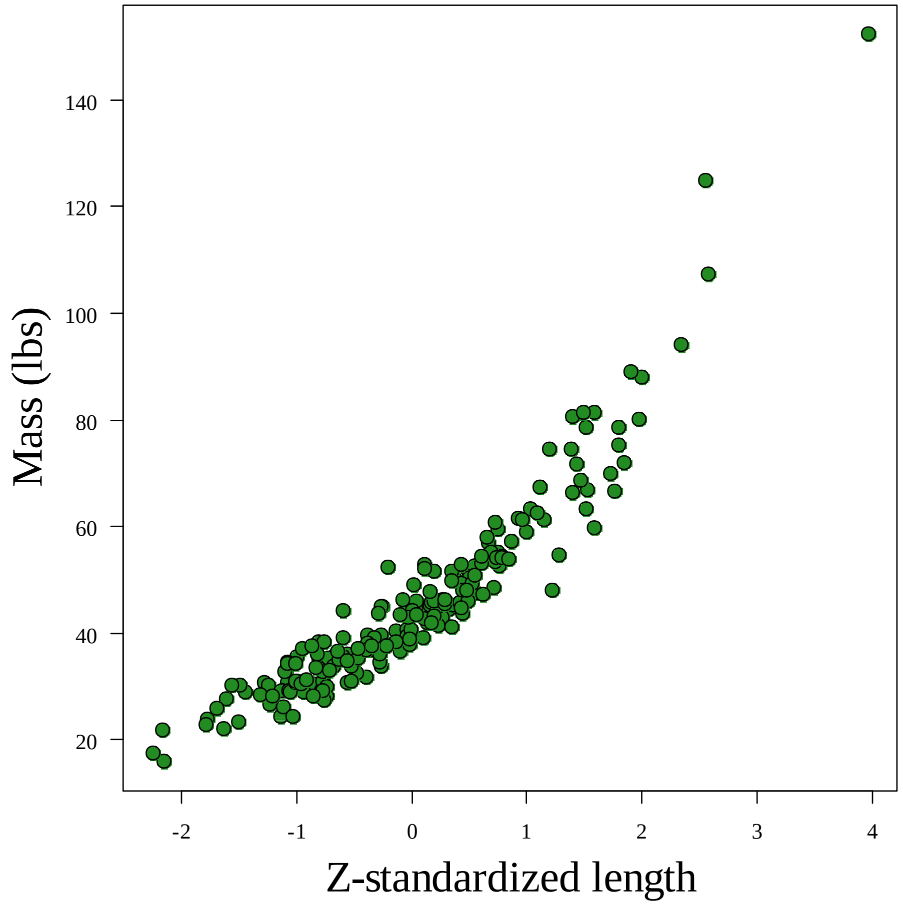


Take-home: scaling lets us control what the intercept <u>means</u>



It doesn’t change predictions (i.e., you get the same line), it’s just more meaningful.

Take-home: scaling lets us control what the intercept <u>means</u>



This will become very important as we start to think about multiple causal pathways

Let's run a JAGS model and talk about it…

$$\mu_i = \beta_0 + \beta_1 \times x_i$$
$$y_i \sim \text{normal}(\mu_i, \sigma^2)$$