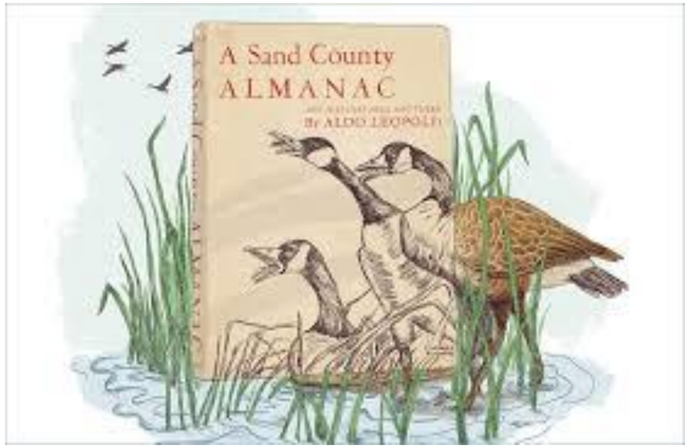
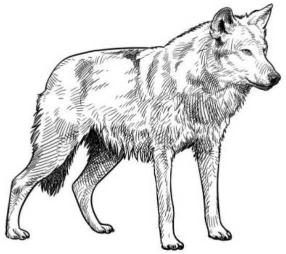
Path analysis: just more than one linear model!

$$u_i \sim \text{lognormal}(\alpha_0 + \alpha_1 w_i, \sigma_u^2)$$

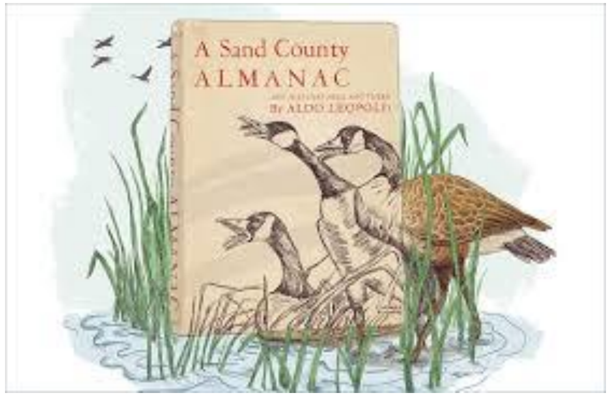
$$v_i \sim \text{lognormal}(\beta_0 + \beta_1 u_i, \sigma_v^2)$$



$$\text{wolves (w)} \xrightarrow{\alpha_1} \text{ungulates (u)} \xrightarrow{\beta_1} \text{vegetation (v)}$$

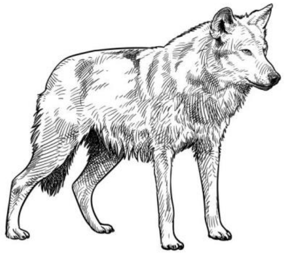


Today’s dataset (the same as example 2 from Thursday!)



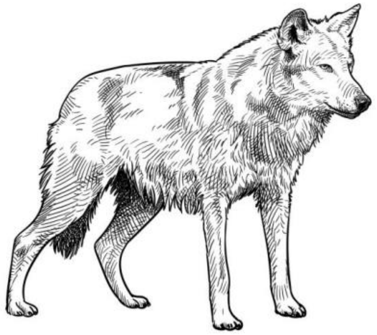
Available browse (***v***; vegetation) as a function of ungulate (***u***) and predator (***w***; wolf) abundance.

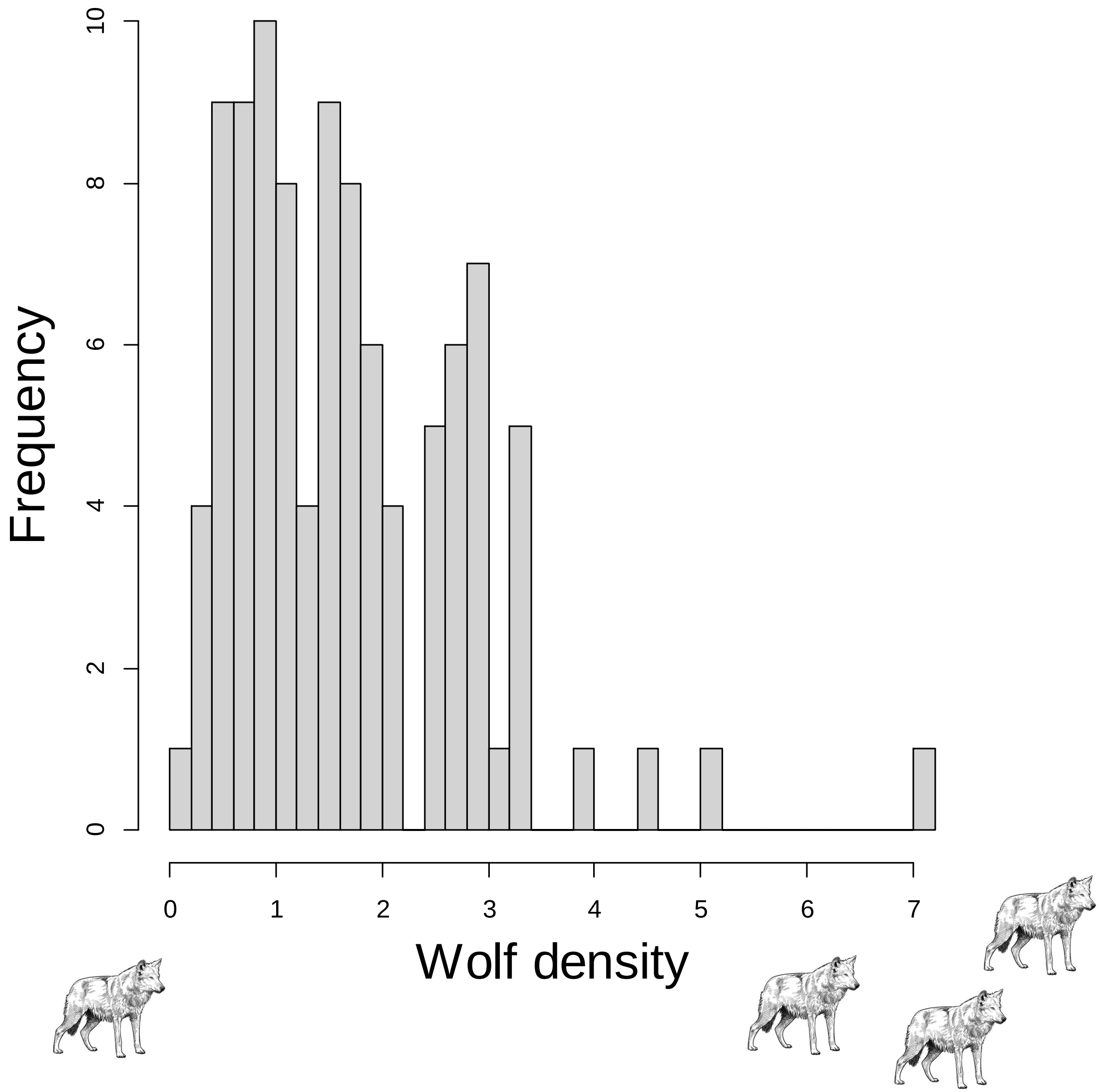
wolves (w) $\xrightarrow{\alpha_1}$ ungulates (u) $\xrightarrow{\beta_1}$ vegetation (v)



First, we'll simulate random variation in wolf density (*w*)

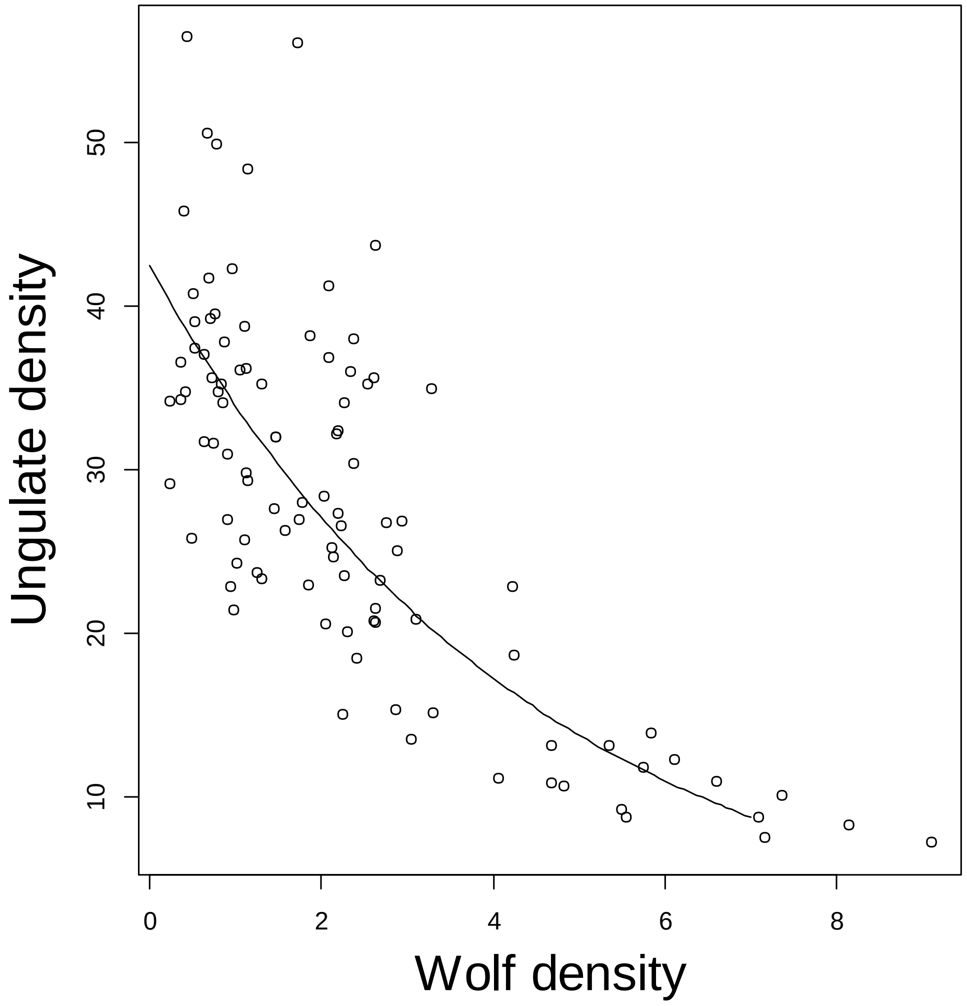
$$w_i \sim \mathrm{lognormal}(0.35, 0.75)$$





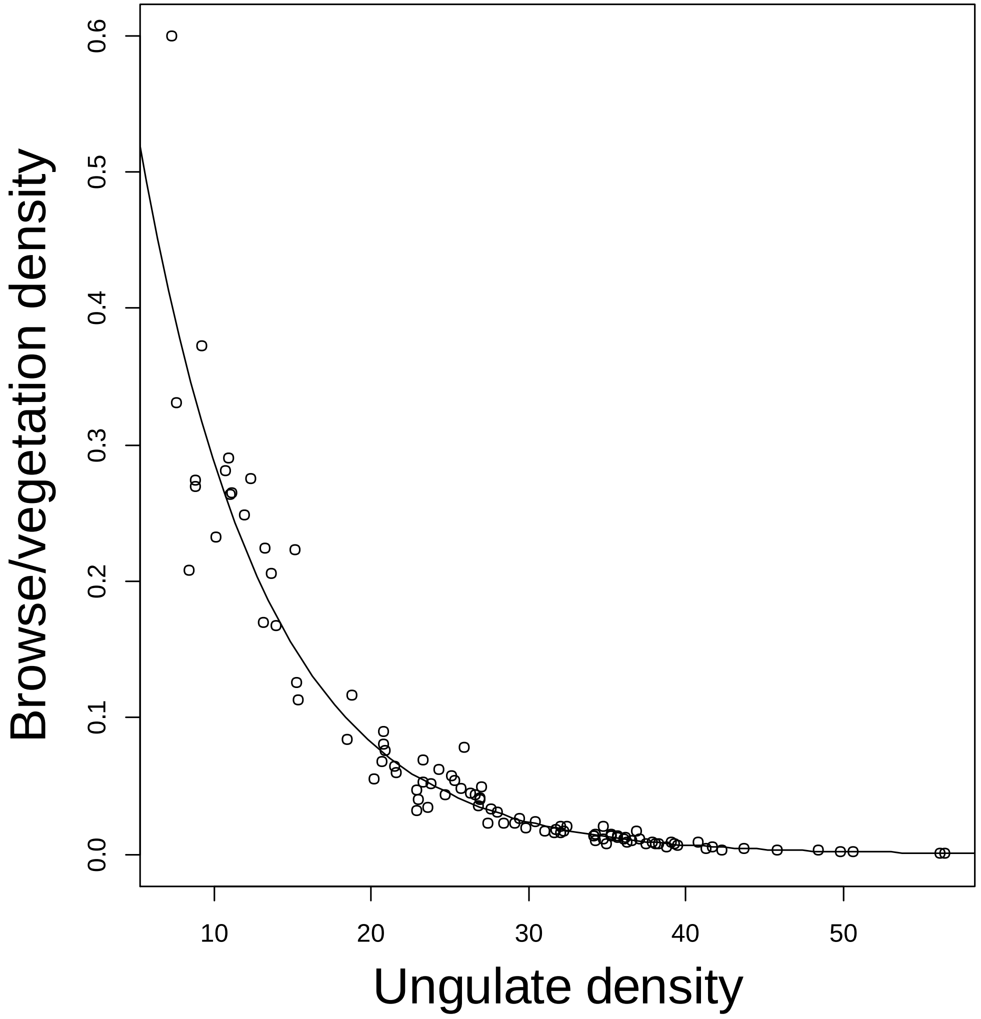
Second, we'll simulate ungulate density (*u*) as a function of *w*

$$u_i \sim \text{lognormal}(\alpha_0 + \alpha_1 w_i, \sigma_v^2)$$



Third, we'll simulate browse density (*v*) as a function of *u*

$$v_i \sim \text{lognormal}(\beta_0 + \beta_1 u_i, \sigma_v^2)$$

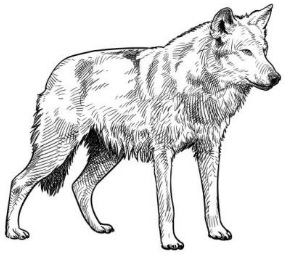


We'll use two (very similar) models to analyze the data

1. Identity link functions (i.e., normal distributions) in R

2. Log-normal distributions (to constrain to 'possible' values) in JAGS

$$\text{wolves (w)} \xrightarrow{\alpha_1} \text{ungulates (u)} \xrightarrow{\beta_1} \text{vegetation (v)}$$

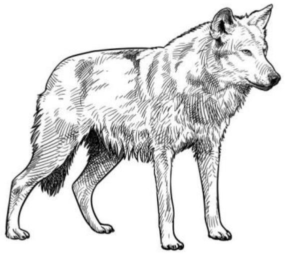


First, we'll use 'identity' link functions in R [lm()]

$$u_i \sim \text{normal}(\alpha_0 + \alpha_1 w_i, \sigma_u^2)$$

$$v_i \sim \text{normal}(\beta_0 + \beta_1 u_i, \sigma_v^2)$$

$$\text{wolves (w)} \xrightarrow{\alpha_1} \text{ungulates (u)} \xrightarrow{\beta_1} \text{vegetation (v)}$$

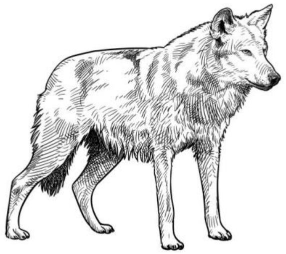


Something to think about

$$u_i \sim \text{normal}(\alpha_0 + \alpha_1 w_i, \sigma_u^2)$$

$$v_i \sim \text{normal}(\beta_0 + \beta_1 u_i, \sigma_v^2)$$

$$\boxed{\text{wolves (w)}} \xrightarrow{\alpha_1} \boxed{\text{ungulates (u)}}$$



```
lm(u ~ w)
```

It's easy to calculate the effect of wolves on ungulate (α_1)!

Something to think about

$$u_i \sim \text{normal}(\alpha_0 + \alpha_1 w_i, \sigma_u^2)$$

$$v_i \sim \text{normal}(\beta_0 + \beta_1 u_i, \sigma_v^2)$$

$$\text{ungulates (u)} \xrightarrow{\beta_1} \text{vegetation (v)}$$

```
lm(v ~ u)
```

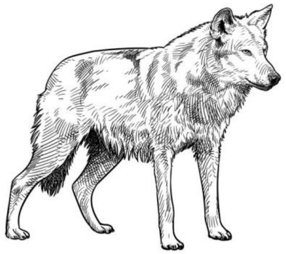
It's also easy to calculate the effect of ungulates on vegetation (β_1)

Something to think about (we'll discuss this shortly!)

$$u_i \sim \text{normal}(\alpha_0 + \alpha_1 w_i, \sigma_u^2)$$

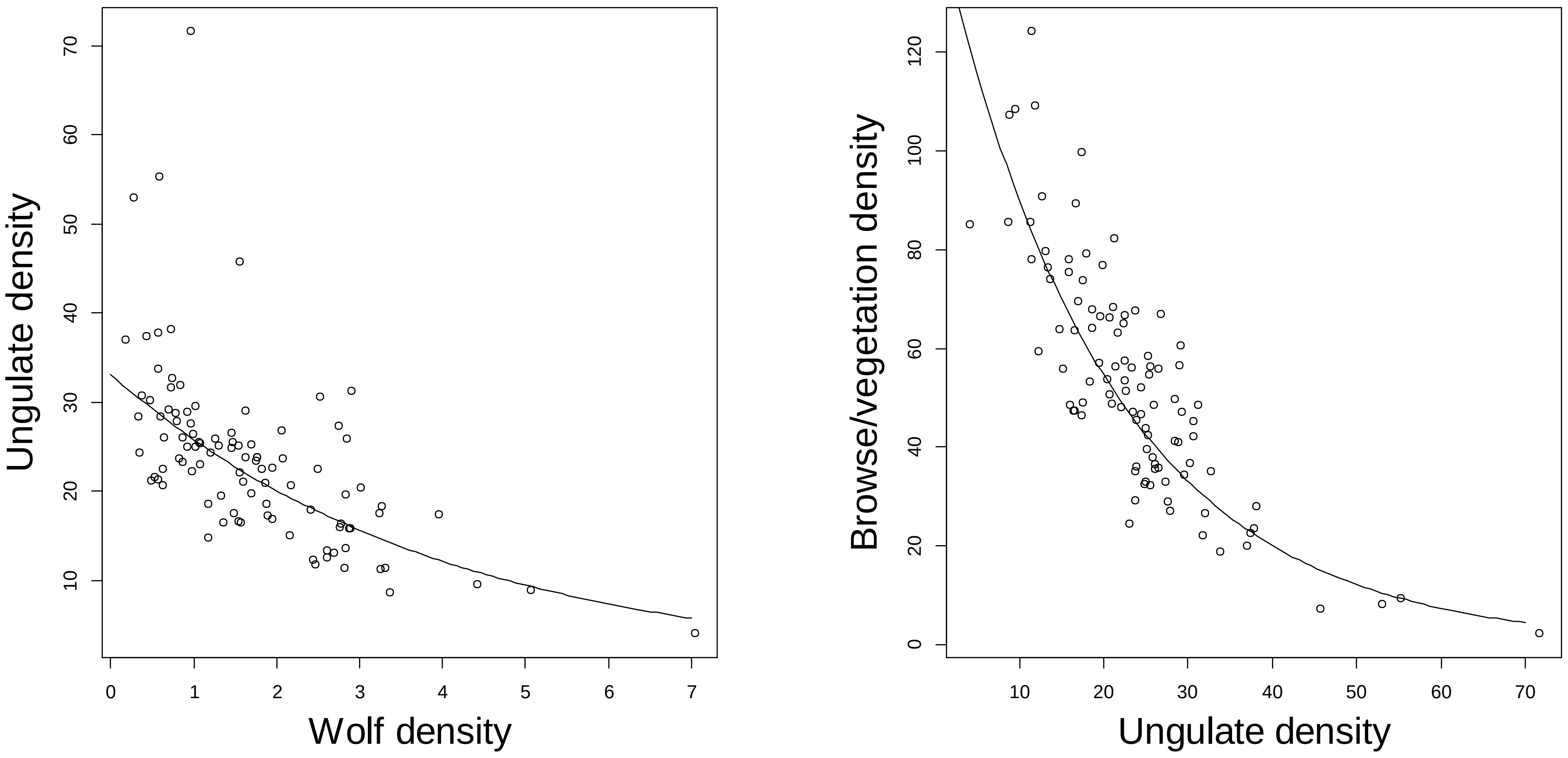
$$v_i \sim \text{normal}(\beta_0 + \beta_1 u_i, \sigma_v^2)$$

wolves (w) $\xrightarrow{\alpha_1}$ ungulates (u) $\xrightarrow{\beta_1}$ vegetation (v)



How would we calculate the effect of wolves on browse?!

How would we calculate the effect of wolves on browse?!



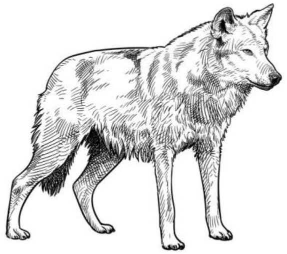
Let’s imagine there is a hypothetical 101st study site with a current density of 2 wolves per ‘unit’

1. What is the current ML estimate of ungulates and browse?

2. What would happen to ungulate density if wolves were extirpated?

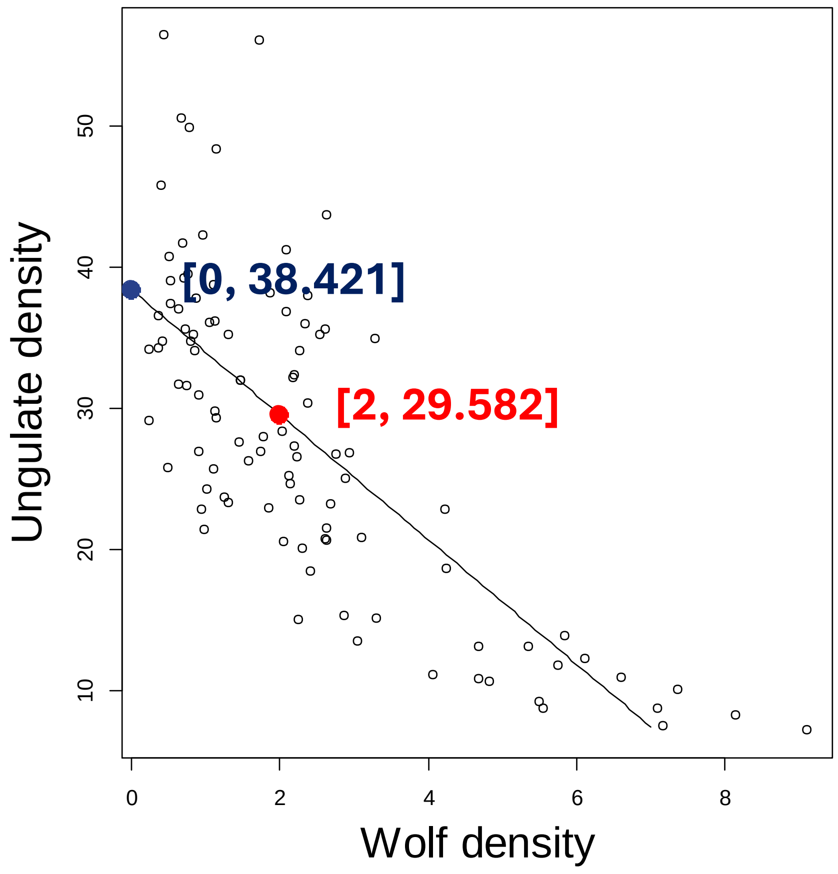
3. What would happen to browse if wolves were extirpated?

$$\text{wolves (w)} \xrightarrow{\alpha_1} \text{ungulates (u)} \xrightarrow{\beta_1} \text{vegetation (v)}$$



Let's visualize an indirect effect!

$$u_i \sim \text{normal}(\alpha_0 + \alpha_1 w_i, \sigma_u^2) \qquad v_i \sim \text{normal}(\beta_0 + \beta_1 u_i, \sigma_v^2)$$



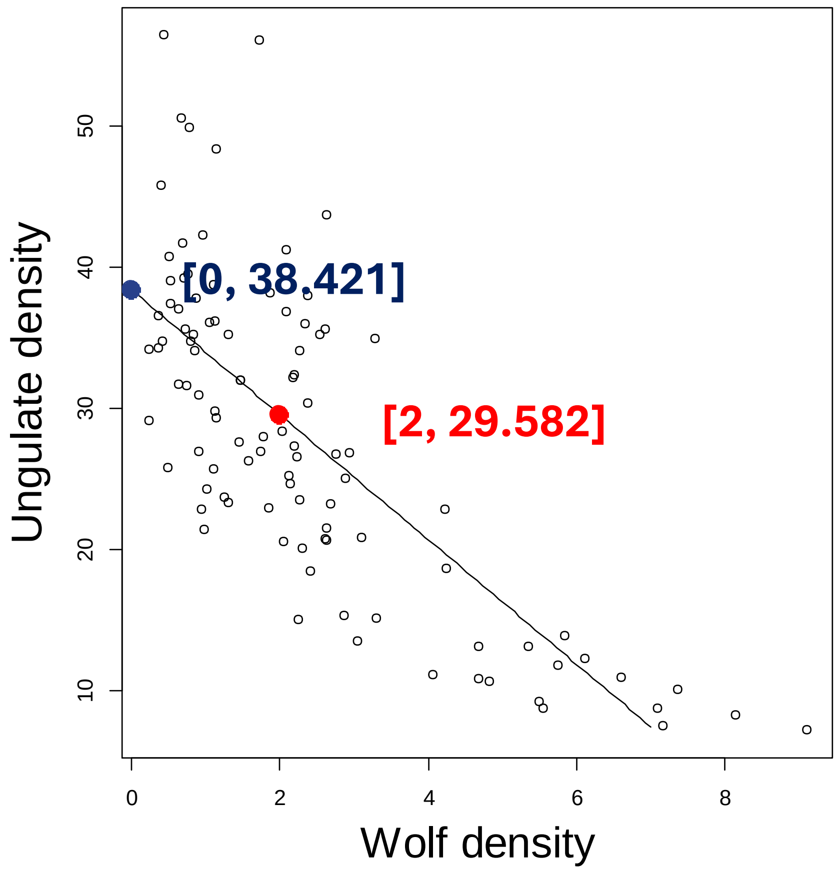
We would gain about 8.83 ungulates, going from 29.582 to 38.421.

$$\alpha_0 = 38.4211$$
$$\alpha_1 = -4.4194$$

What would happen if we went from 2 wolves to 0?

Let's visualize an indirect effect!

$u_i \sim \text{normal}(\alpha_0 + \alpha_1 w_i, \sigma_u^2)$ $v_i \sim \text{normal}(\beta_0 + \beta_1 u_i, \sigma_v^2)$



We would gain about 8.83 ungulates, going from 29.582 to 38.421.

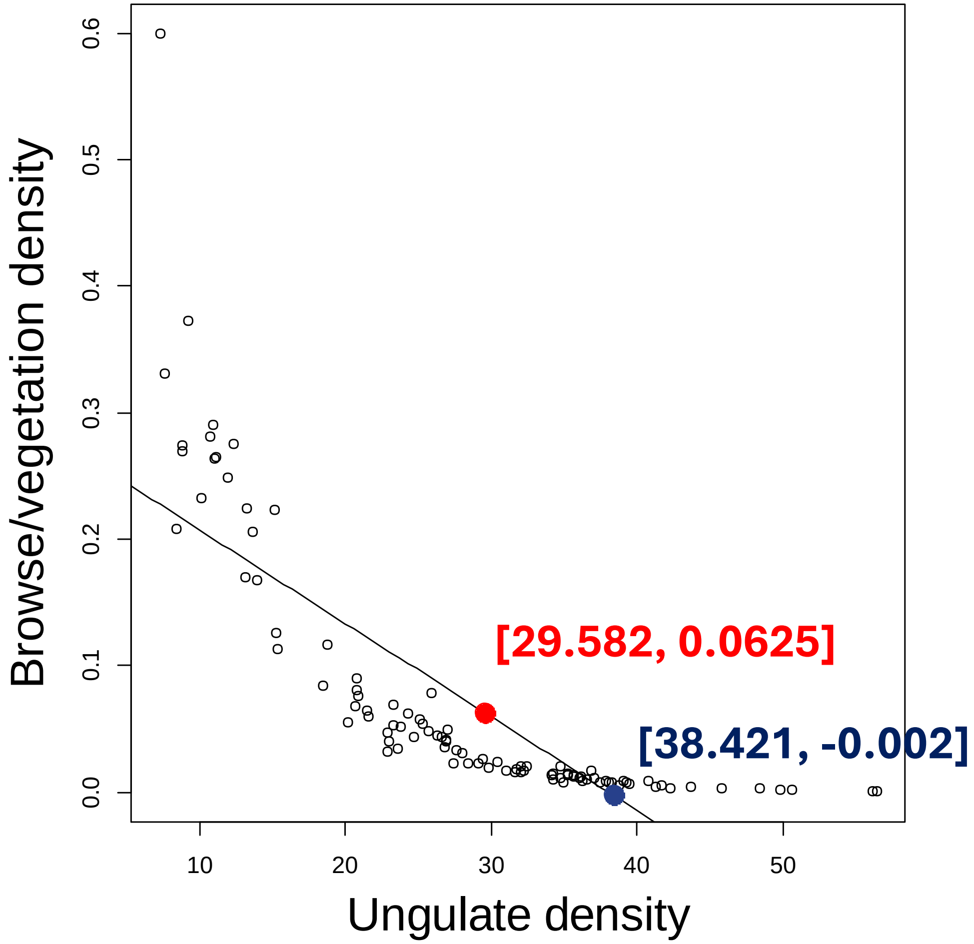
$$\alpha_0 = 38.4211$$
$$\alpha_1 = -4.4194$$

How would that 8.83 gain in ungulates affect browse?

Let's visualize an indirect effect!

$$u_i \sim \text{normal}(\alpha_0 + \alpha_1 w_i, \sigma_u^2)$$

$$v_i \sim \text{normal}(\beta_0 + \beta_1 u_i, \sigma_v^2)$$



If we gain about 8.83 ungulates, we lose 'browse'.

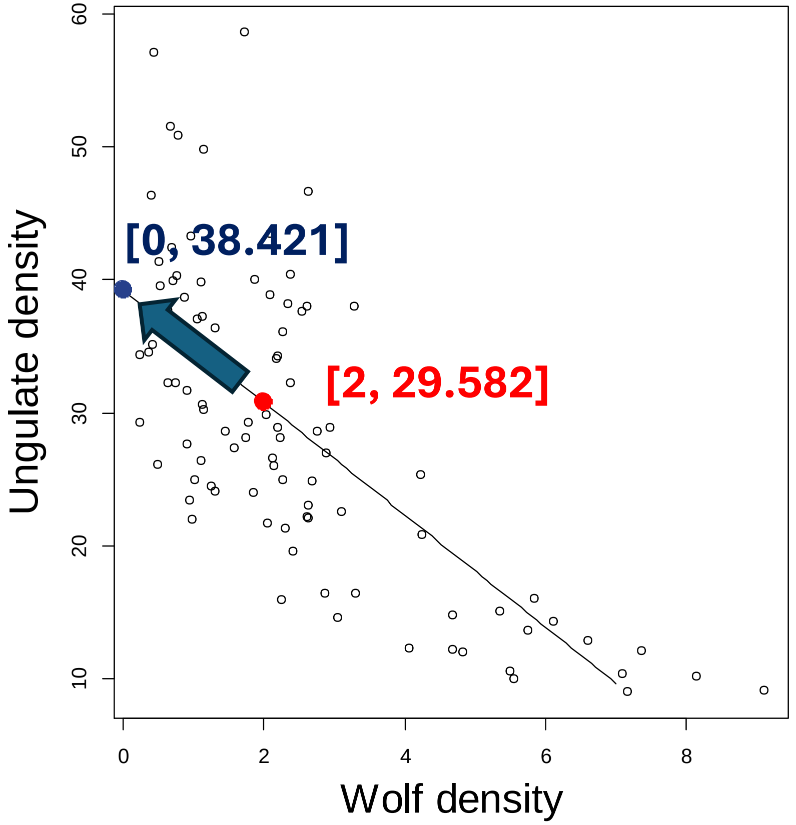
$$\beta_0 = 0.2809$$

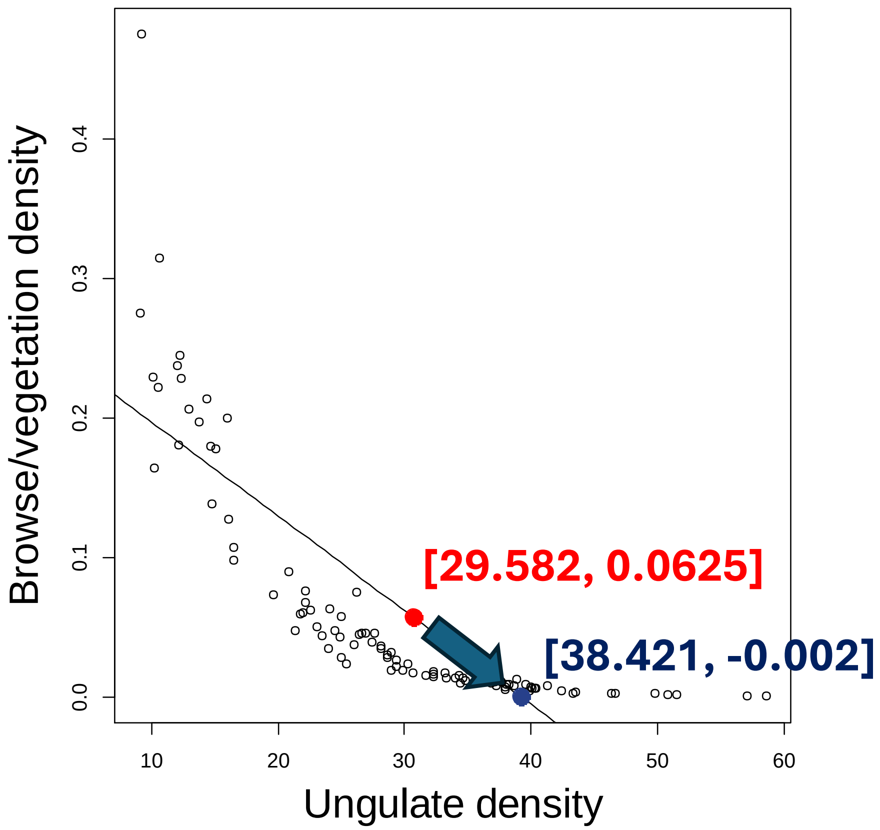
$$\beta_1 = -0.00738$$

Let's visualize an indirect effect!

$$u_i \sim \text{normal}(\alpha_0 + \alpha_1 w_i, \sigma_u^2)$$

$$v_i \sim \text{normal}(\beta_0 + \beta_1 u_i, \sigma_v^2)$$



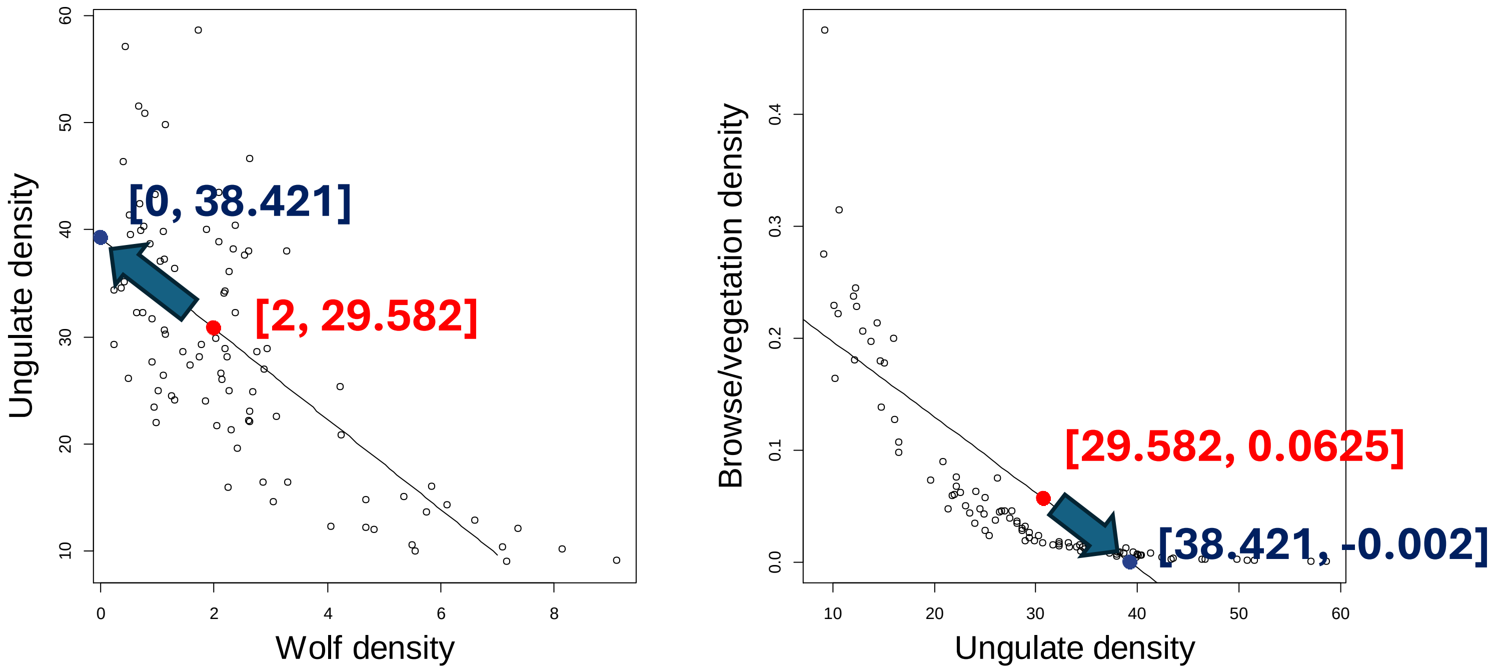


Losing wolves -> more deer; more deer -> less browse

Let's visualize an indirect effect!

$$u_i \sim \text{normal}(\alpha_0 + \alpha_1 w_i, \sigma_u^2)$$

$$v_i \sim \text{normal}(\beta_0 + \beta_1 u_i, \sigma_v^2)$$



$$\beta_1 = -0.00738$$

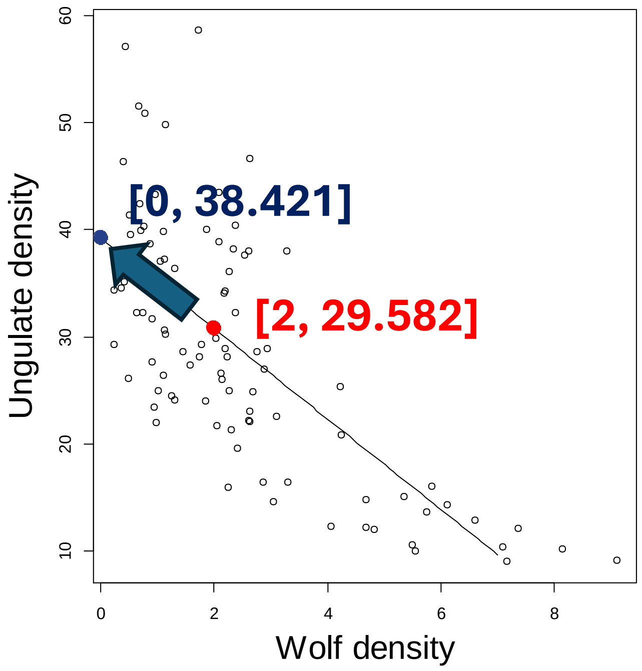
$$\alpha_1 = -4.4194$$

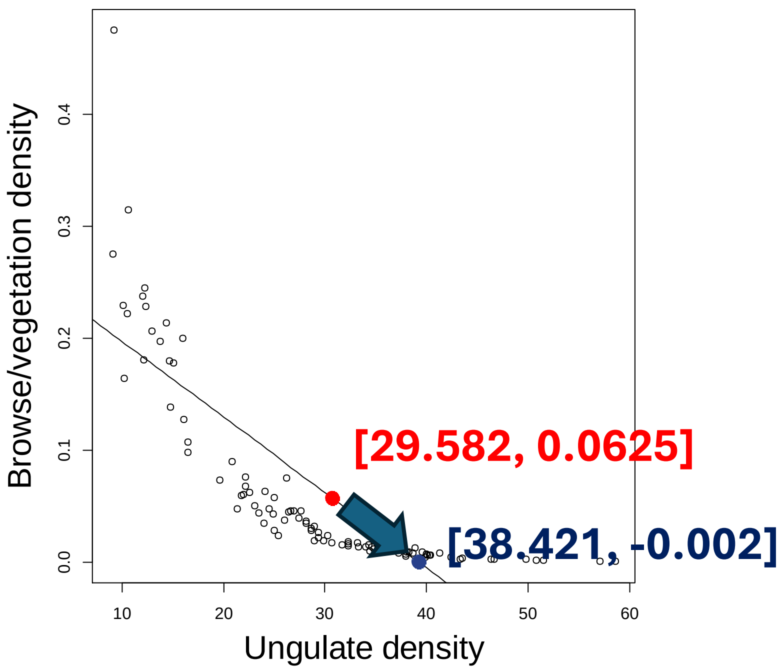
$$-0.06524 = \boxed{-0.002 - 0.0625}$$

Change in browse

Let’s visualize an indirect effect!

$$u_i \sim \text{normal}(\alpha_0 + \alpha_1 w_i, \sigma_u^2) \qquad v_i \sim \text{normal}(\beta_0 + \beta_1 u_i, \sigma_v^2)$$





$$\beta_1 = -0.00738$$

$$\alpha_1 = -4.4194$$

$$-0.06524 = \boxed{-0.002 \; - \; 0.0625} = \alpha_1(\beta_1)(-2)$$

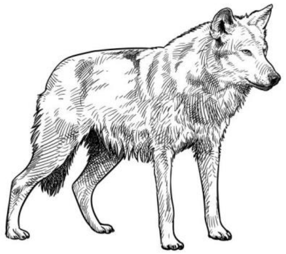
Change in browse

Take-home 1: indirect effects aren't as 'horrifying' as they seem

$$u_i \sim \mathrm{normal}(\alpha_0 + \alpha_1 w_i, \sigma_u^2)$$

$$v_i \sim \mathrm{normal}(\beta_0 + \beta_1 u_i, \sigma_v^2)$$

wolves (w) $\xrightarrow{\alpha_1}$ ungulates (u) $\xrightarrow{\beta_1}$ vegetation (v)

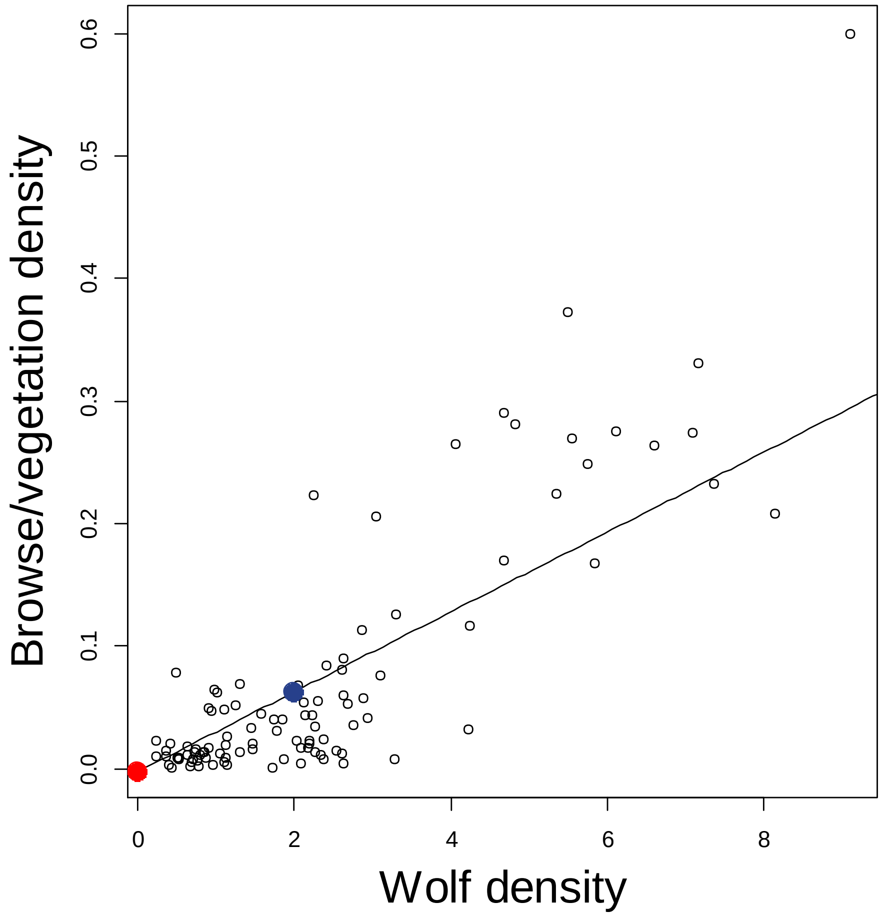


i.e., if identity link, then effect of wolves on veg = $\alpha_1\beta_1$

$$E(v_i|w_i) = [\beta_0 + \beta_1(\alpha_0)] + \alpha_1\beta_1 w_i$$

slope $= \alpha_1\beta_1$

intercept =
what value of veg would be given number of deer if there were no wolves



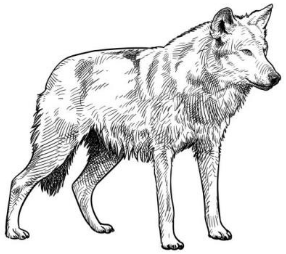
Let’s go to the code and prove it!

Second, we'll use the data-generating model to analyze the data

$$u_i \sim \text{lognormal}(\alpha_0 + \alpha_1 w_i, \sigma_u^2)$$

$$v_i \sim \text{lognormal}(\beta_0 + \beta_1 u_i, \sigma_v^2)$$

$$\boxed{\text{wolves (w)}} \xrightarrow{\alpha_1} \boxed{\text{ungulates (u)}} \xrightarrow{\beta_1} \boxed{\text{vegetation (v)}}$$

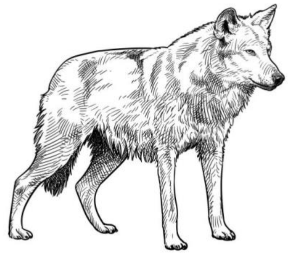


Take-home: this takes some careful thinking!

$$u_i \sim \text{lognormal}(\alpha_0 + \alpha_1 w_i, \sigma_u^2)$$

$$v_i \sim \text{lognormal}(\beta_0 + \beta_1 u_i, \sigma_v^2)$$

$$\text{wolves (w)} \xrightarrow{\alpha_1} \text{ungulates (u)} \xrightarrow{\beta_1} \text{vegetation (v)}$$



i.e., what link functions are we using, and how do we incorporate those?

We may be transforming the y-axis (log- & logit-links)

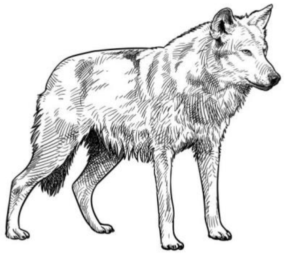
i.e., what link functions are we using, and how do we incorporate those?

Take-home: this takes some careful thinking!

$$u_i \sim \text{lognormal}(\alpha_0 + \alpha_1 w_i, \sigma_u^2)$$

$$v_i \sim \text{lognormal}(\beta_0 + \beta_1 u_i, \sigma_v^2)$$

$$\text{wolves (w)} \xrightarrow{\alpha_1} \text{ungulates (u)} \xrightarrow{\beta_1} \text{vegetation (v)}$$



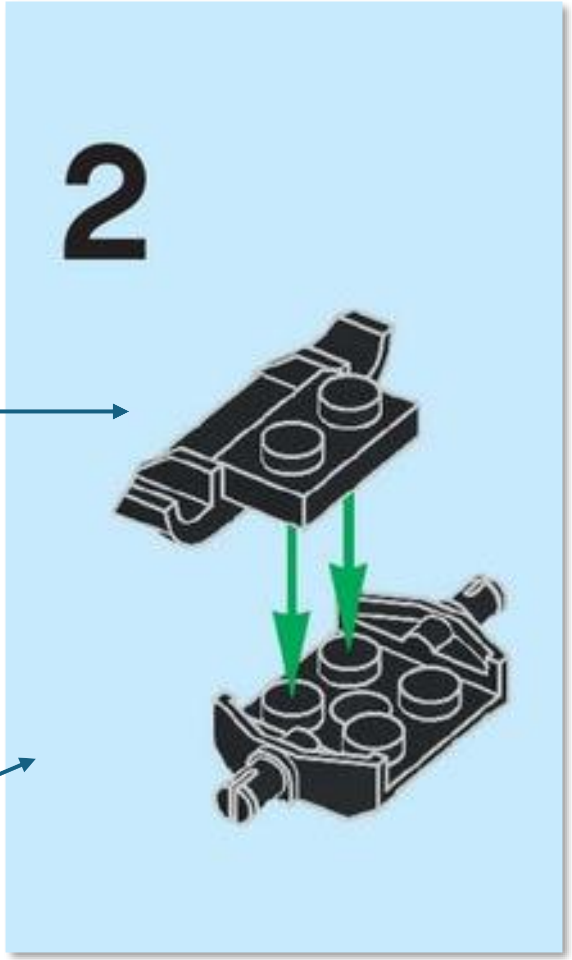
i.e., are these z-standardized covariates? How do we back-transform axes?

We may be transforming the x-axis (z-standardizing)

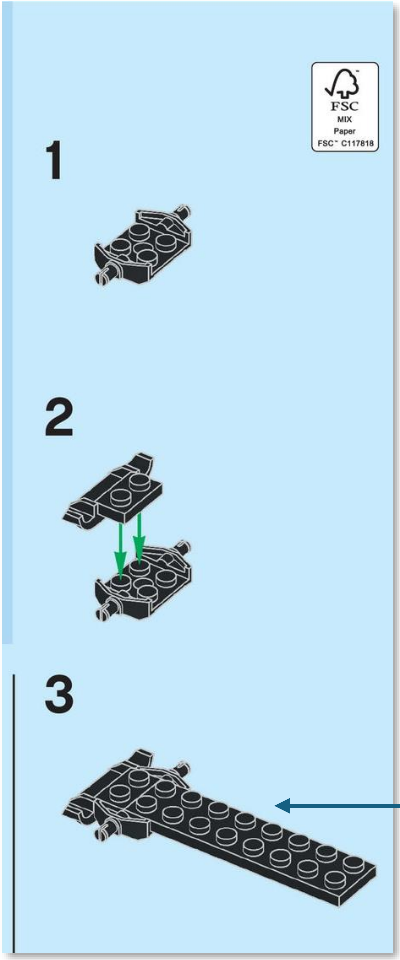
i.e., how are we scaling our covariates?

We have to keep track of a bunch of stuff

This piece is z-standardized



This piece is on the log-link



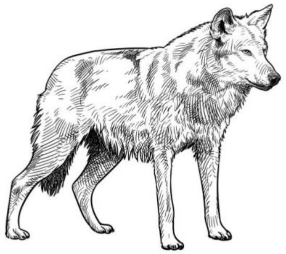
This piece is on the logit-link

Take-home

$$u_i \sim \mathrm{lognormal}(\alpha_0 + \alpha_1 w_i, \sigma_u^2)$$

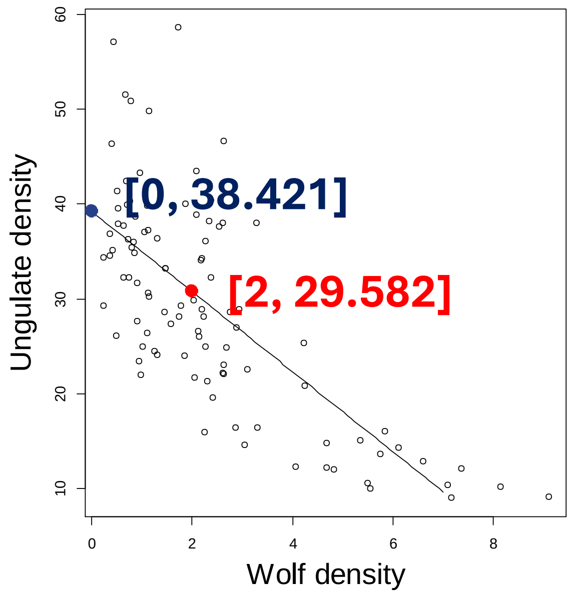
$$v_i \sim \mathrm{lognormal}(\beta_0 + \beta_1 u_i, \sigma_v^2)$$

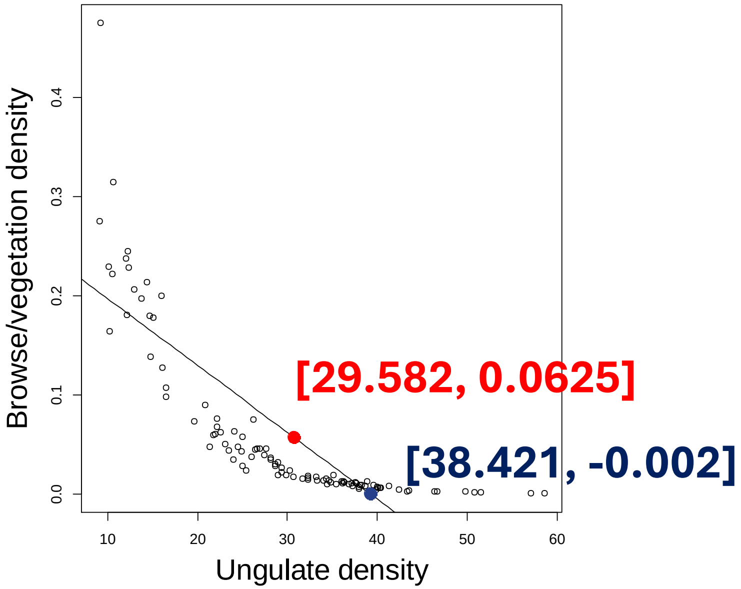
$$\boxed{\text{wolves (w)}} \xrightarrow{\alpha_1} \boxed{\text{ungulates (u)}} \xrightarrow{\beta_1} \boxed{\text{vegetation (v)}}$$



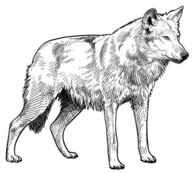
JAGS/Bayes will take some time, but it comes with advantages too!

Take-home: the math is <u>tedious and requires careful thought</u>, but the concept is simple!





wolves (w) $\xrightarrow{\alpha_1}$ ungulates (u) $\xrightarrow{\beta_1}$ vegetation (v)



Take-home: on Thursday, we'll review and expand!