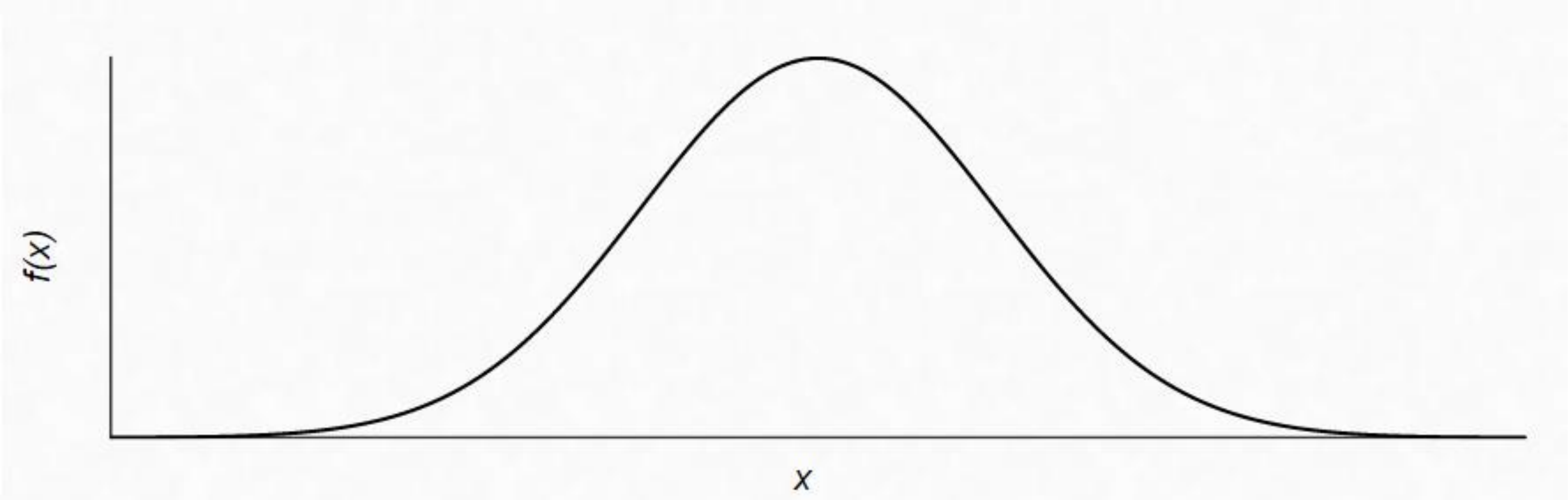
Categorical covariates: ‘random’ and ‘fixed’ effects



Outline

1. **Basic introduction**
   - *Key concepts*
     - *Defining ‘random’ and ‘fixed’ effects*
     - *Normal distributions*
     - *Hierachical models and hyper-parameters (hey-O!)*
     - *Shrinkage*

2. **Case study: adult size variation among populations**

3. **Perils and pitfalls, review take home ideas**

Definitions and distinctions

The terms ‘fixed’ and ‘random’ effect are used fairly widely (also, wildly) across a number of ‘fields’...

Gelman **(2005)** *The Annals of Statistics*

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3. When a sample exhausts the population, use fixed effects. When the sample is small (i.e., negligible), the variable is random (Green and Tukey 1960)

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5. Fixed effects are estimated using least squares, and random effects are estimated with shrinkage (Snijders and Bosker 1999)

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What is a random effect?!

```
lm(y ~ as.factor(x))
```

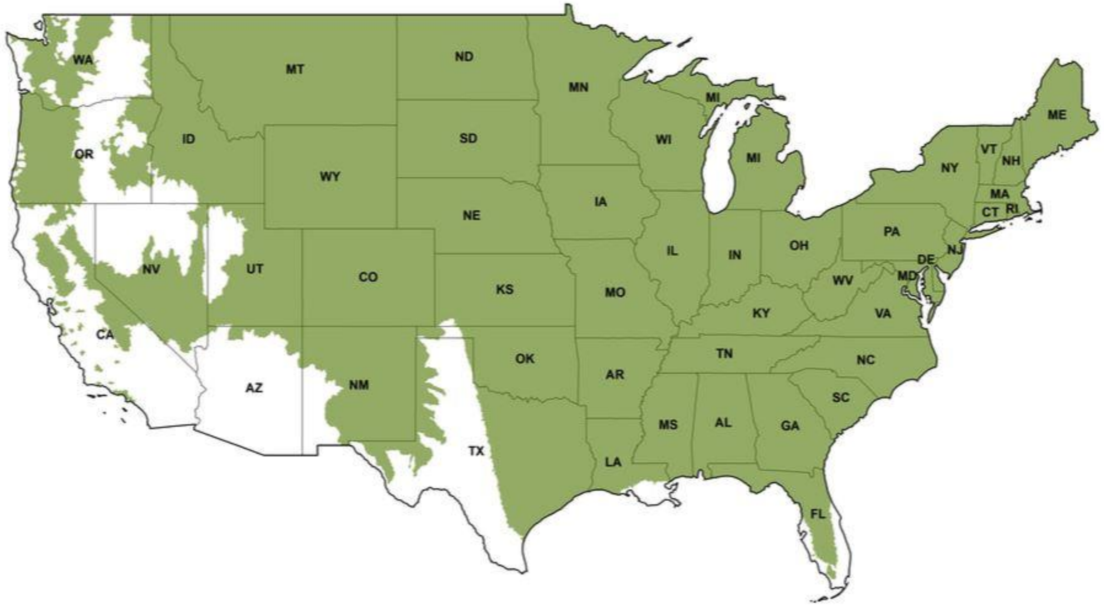
Fixed effect of x

```
lmer(y ~ (1 | as.factor(x)))
```

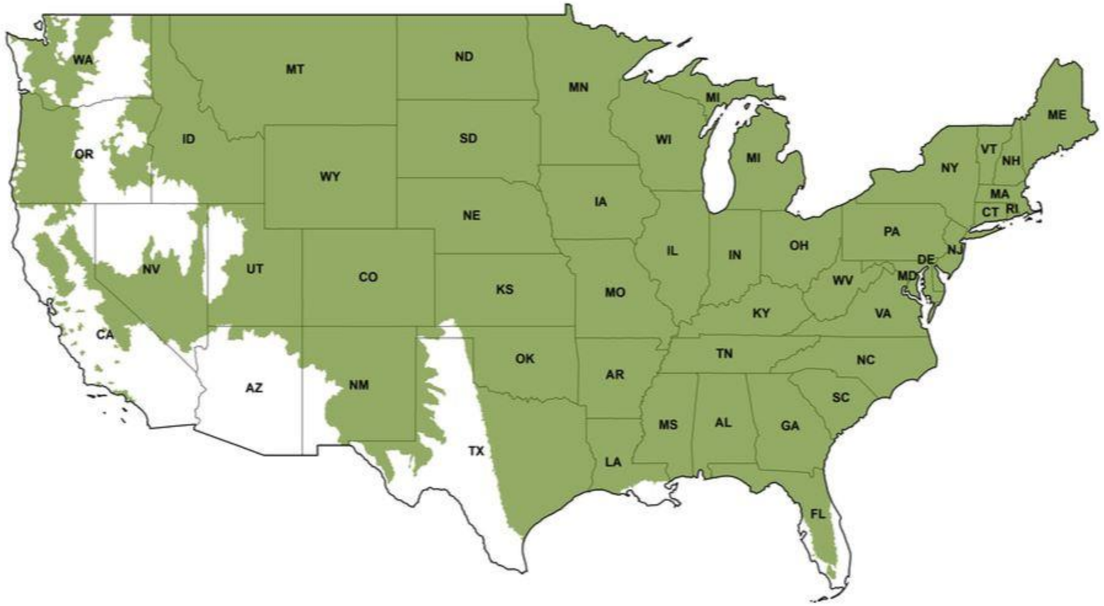
Random effect of x

Using them in a Bayesian context can help clarify what they are!

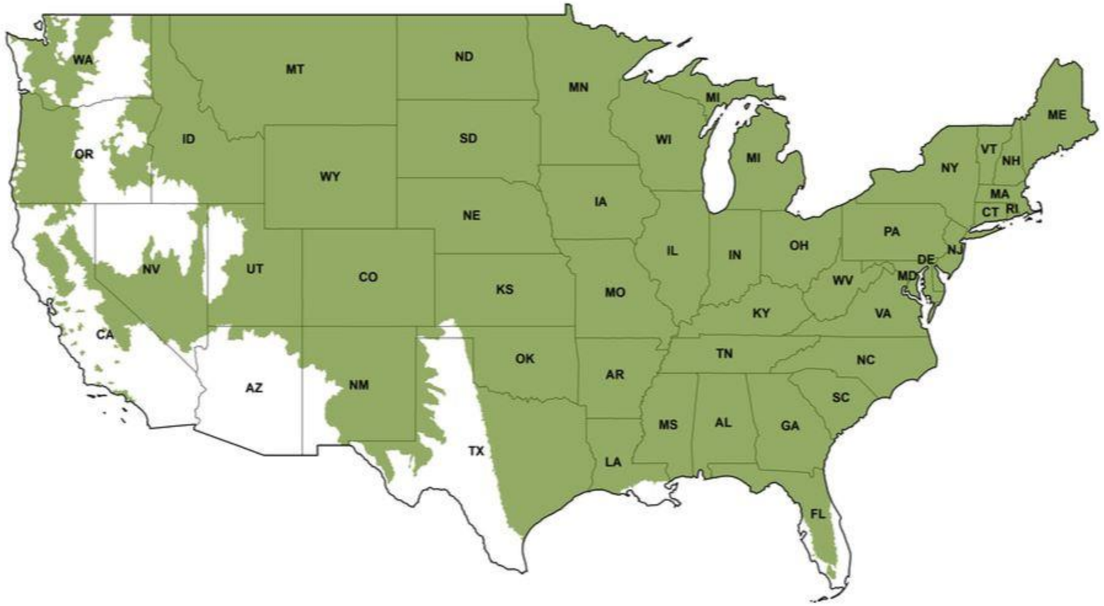
First, let’s describe our data



We'll sample 30 populations of foxes, and weigh a certain number of adult females from each population



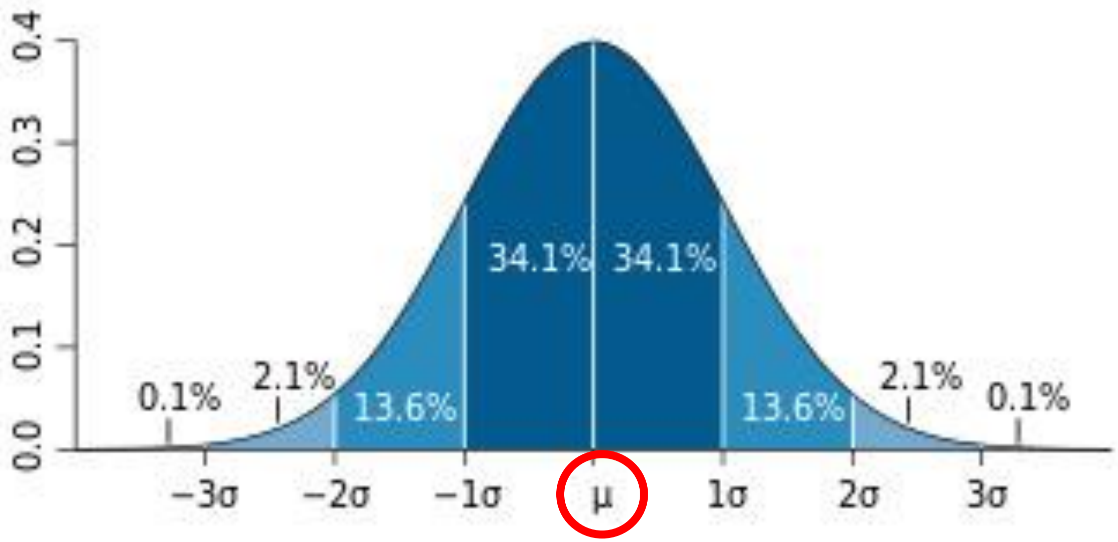
The data (y) will be female mass. We'll have two associated covariates; latitude of the population (continuous, x) and a population identifier (categorical, p).



So what is a random effect?

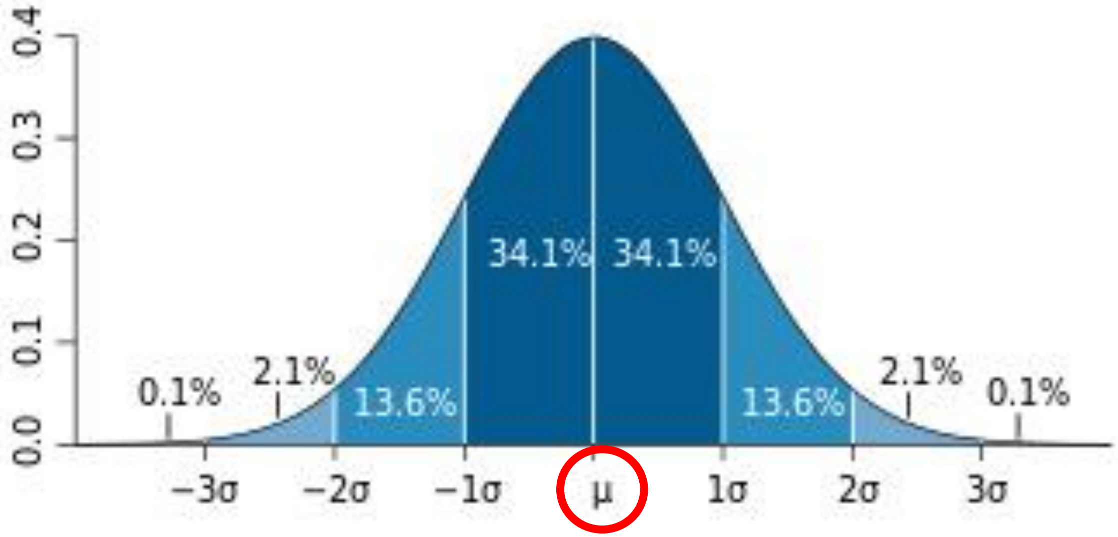
Let's remember a beautiful and simple idea…

$$y_i \sim \text{normal}(\mu, \sigma^2)$$



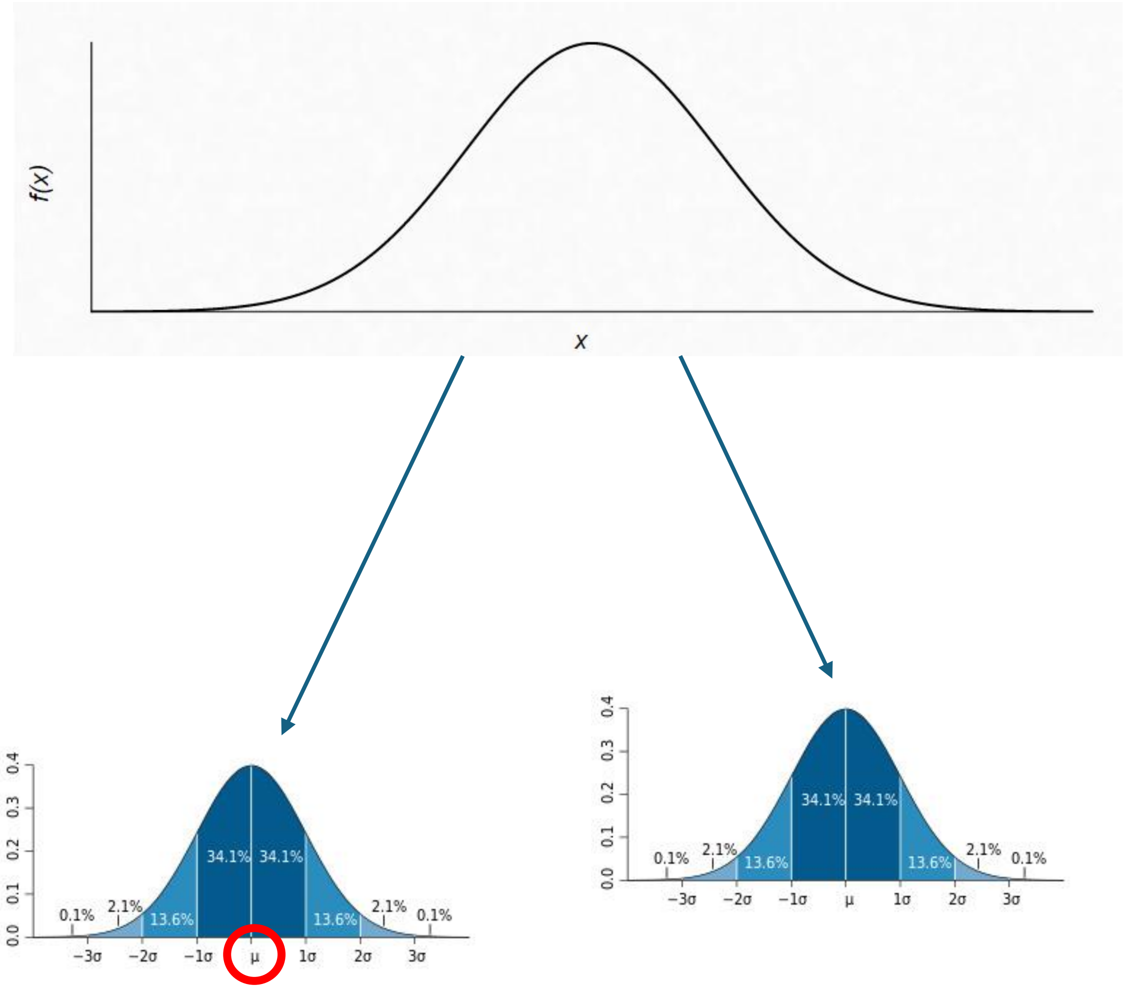
Now let's imagine we have 30 populations, each with a mean…

$$y_i \sim \mathrm{normal}(\mu, \sigma^2)$$



Each of those populations is a sample from the total population!

$$\mu_j \sim \text{normal}(\mu^*, \sigma^2)$$



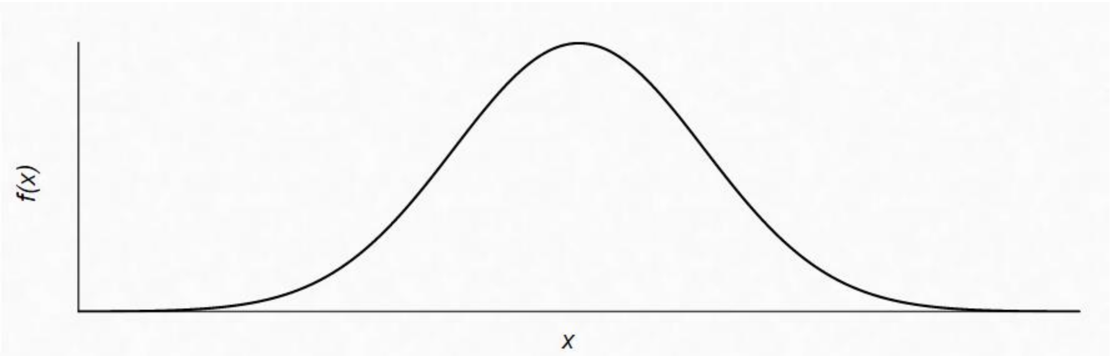
μ_j: each populations mean

μ^*: an average female fox

σ^2: variance among pops

Now let's imagine we have 30 populations, each with a mean…

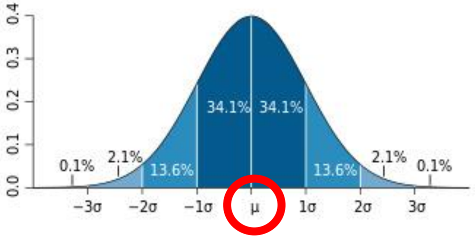
$$\mu_j \sim \text{normal}(\mu^*, \sigma^2)$$

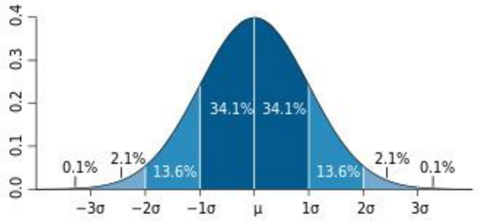


y_i: **the weight of each fox**

ς^2: **variance within pops**

$$y_i \sim \text{normal}(\mu_j, \varsigma^2)$$

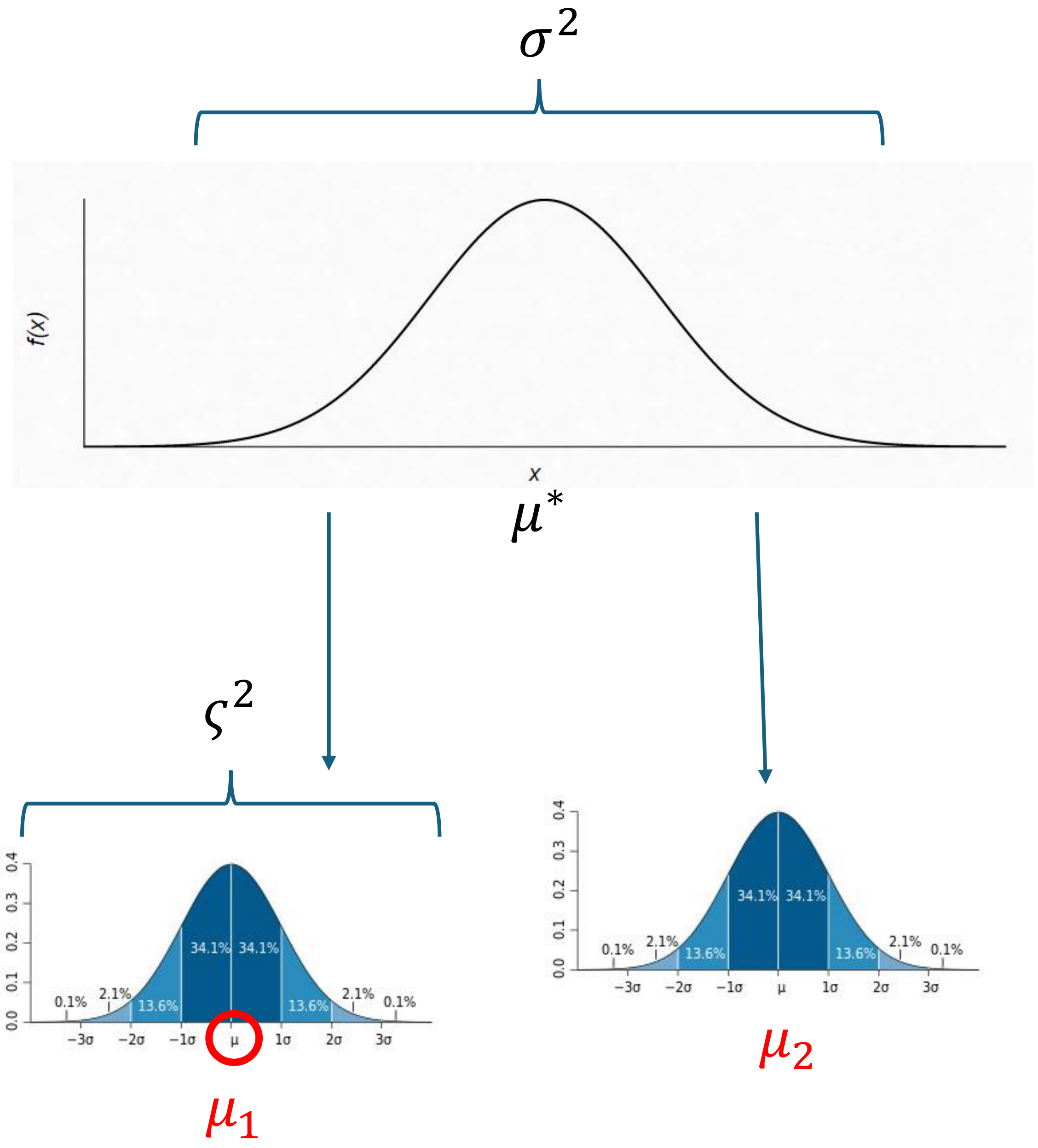




Now let's imagine we have 30 populations, each with a mean…

$$\mu_j \sim \text{normal}(\mu^*, \sigma^2)$$

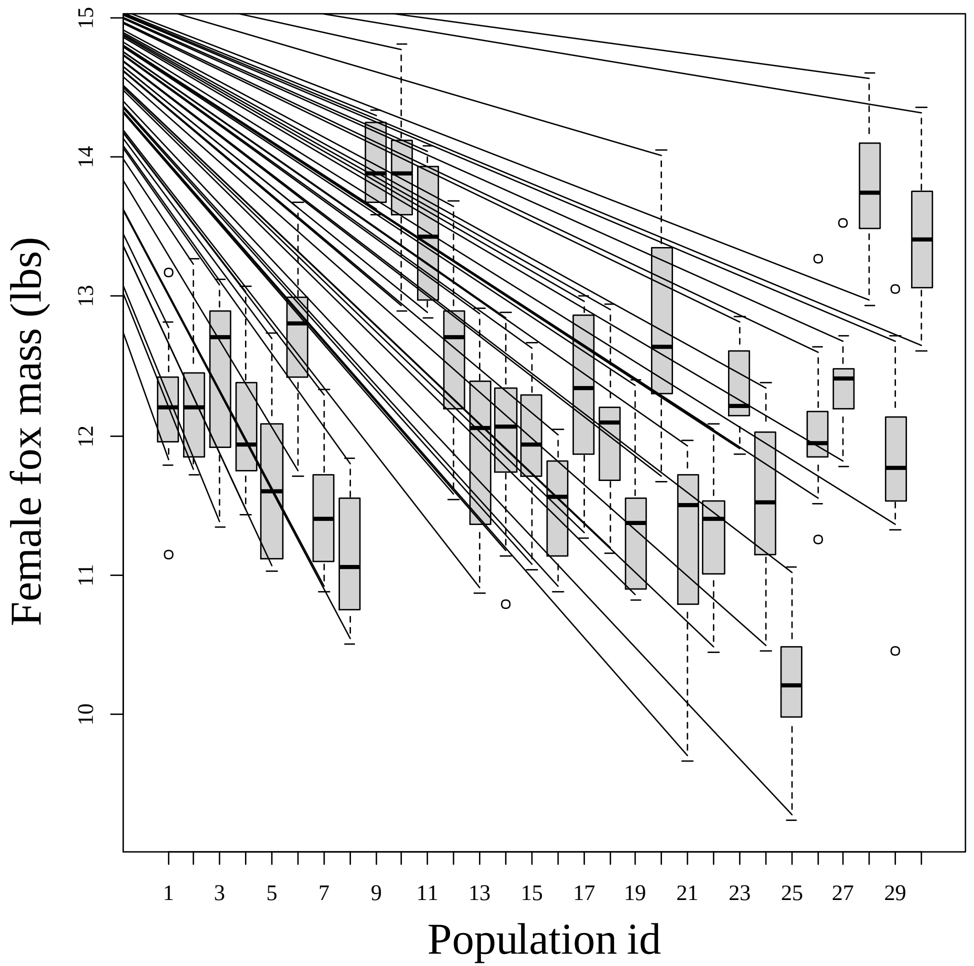
$$y_i \sim \text{normal}(\mu_j, \varsigma^2)$$



The foxes in each population are normally distributed

$$\mu_j \sim \text{normal}(\mu^*, \sigma^2)$$

$$y_i \sim \text{normal}(\mu_j, \varsigma^2)$$

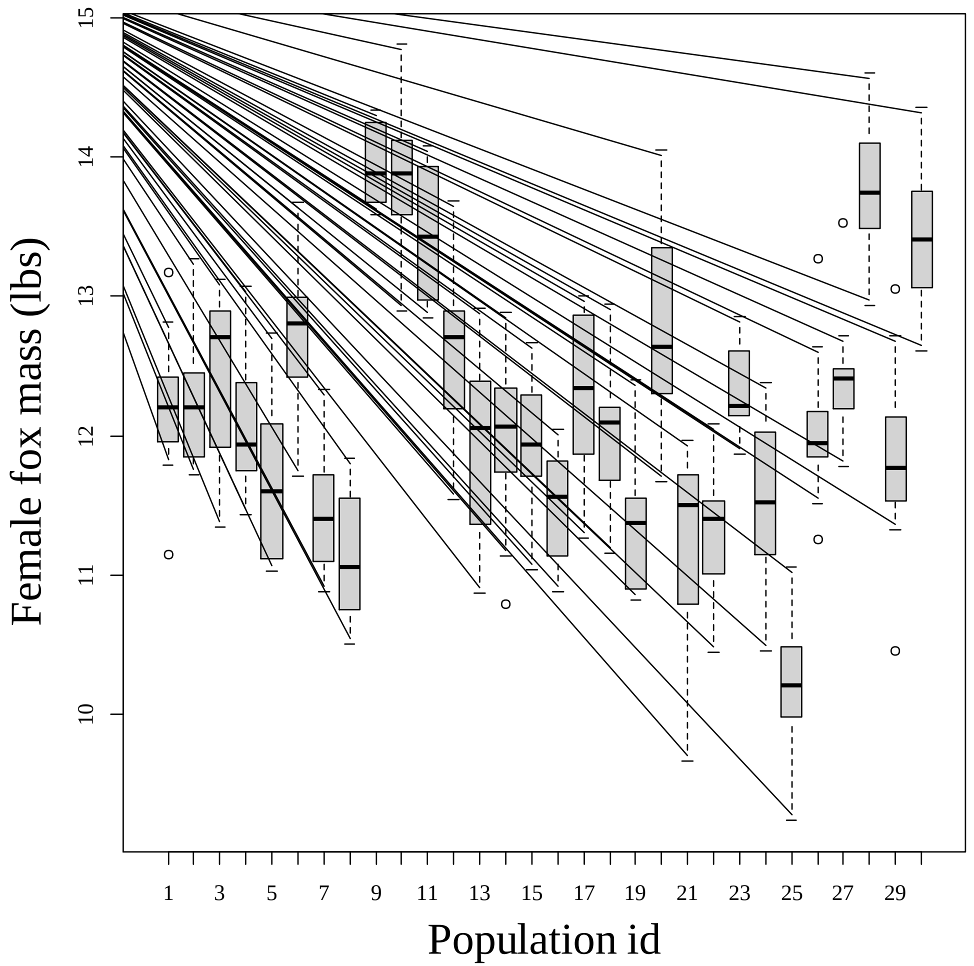


The populations are normally distributed (random effects)

Why do we need a certain number (7?, 10?, more?) of groups…

$$\color{red}{\mu_j} \sim \mathrm{normal}(\mu^*, \sigma^2)$$

$$y_i \sim \mathrm{normal}(\color{red}{\mu_j}, \varsigma^2)$$

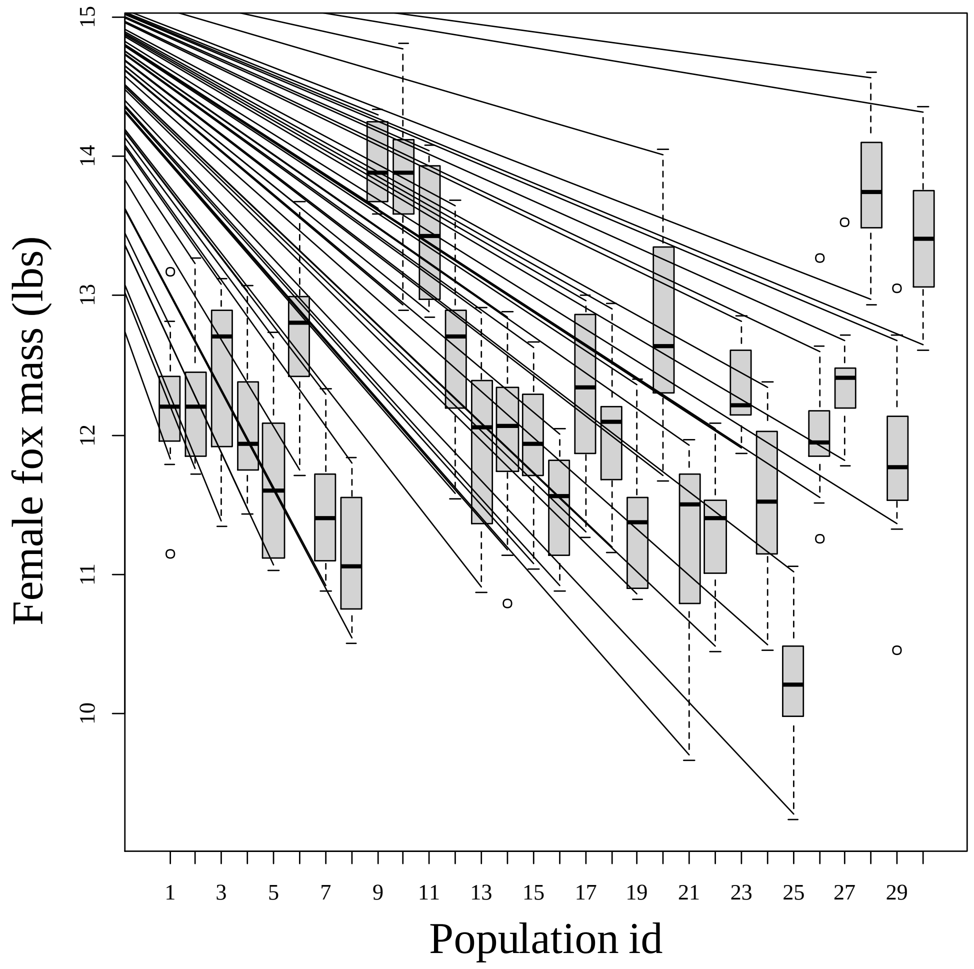


to estimate random effects?

So, what is a fixed effect?!

$$\mu_j \sim \text{normal}(12, 10)$$

$$y_i \sim \text{normal}(\mu_j, \varsigma^2)$$

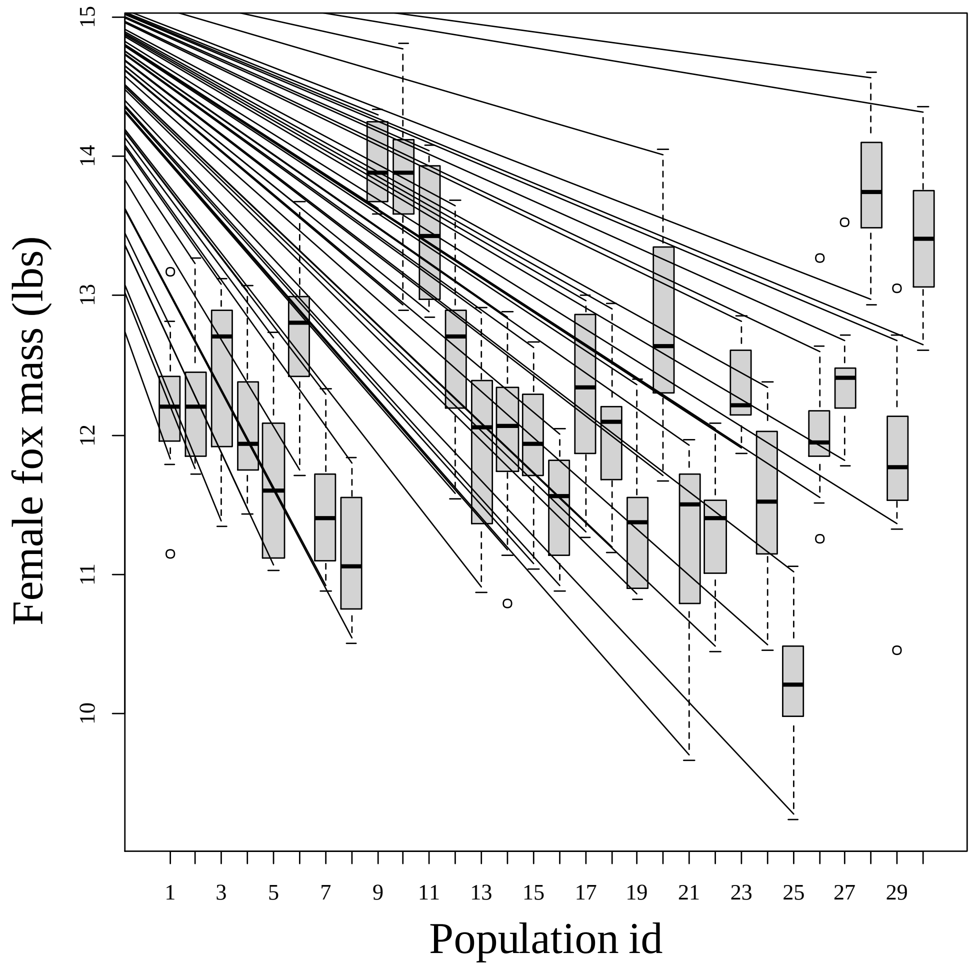


Fixed effects assume that each group is independent.

Random effects are nearly ~equivalent to fixed effects if…

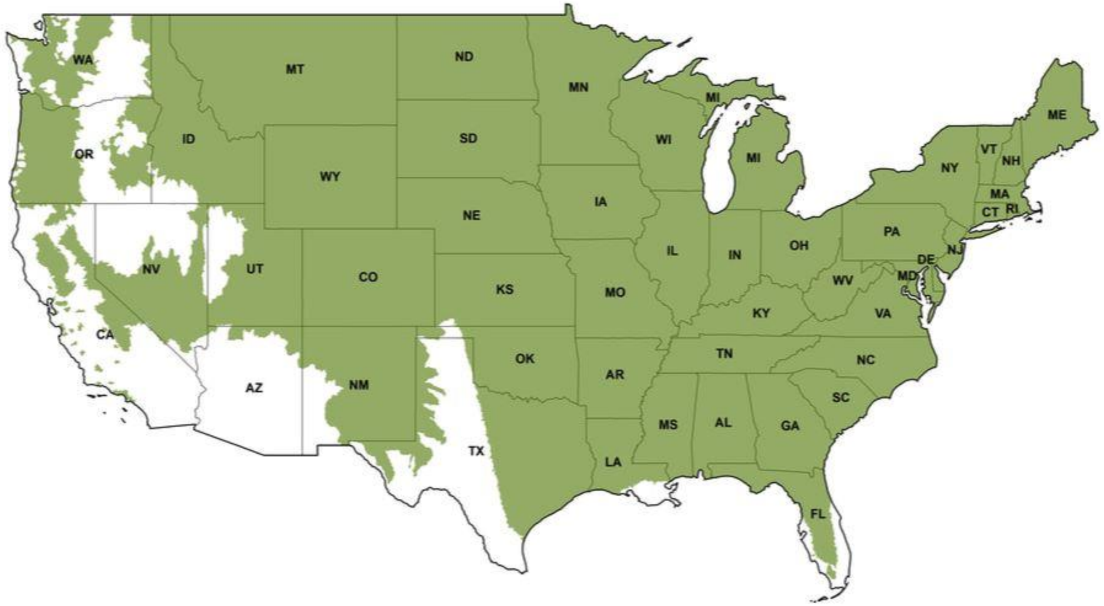
$$\mu_j \sim \text{normal}(\mu^*, \infty)$$

$$y_i \sim \text{normal}(\mu_j, \varsigma^2)$$



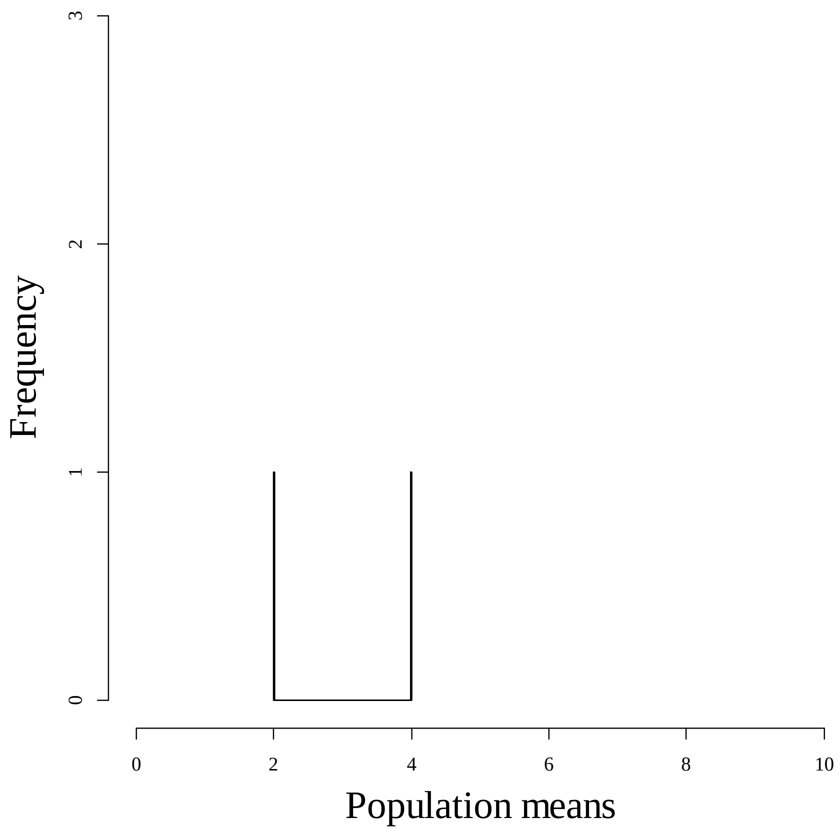
Or as the number of samples increases…

Case study I: adult size variation among populations



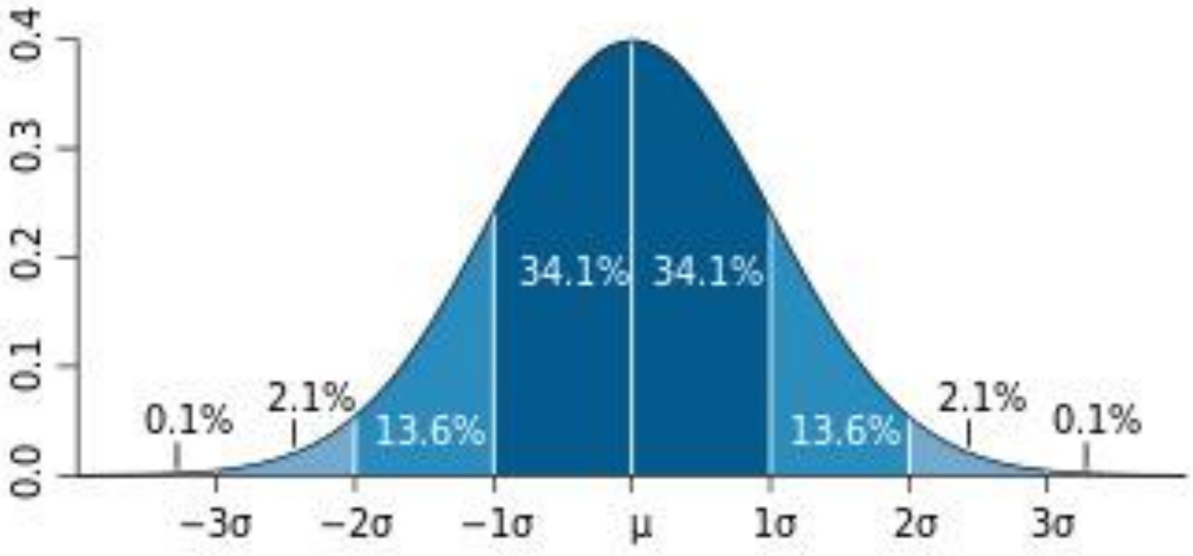
To JAGS!

Take-home idea: <u>there's a reason you need *n* groups!</u>



Imagine trying to estimate the mean (3?) and variance (?) of two groups?

Take-home idea: <u>REs are just another normal distribution ☺!</u>



Take-home idea: <u>estimates 'shrink' more when *n* is small</u>

Take-home idea: as *n* increases, random fx ~= fixed fx

Take-home idea: <u>there are LOTS of bad explanations of random fx</u>

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Seeing the actual model helps us understand what they are.