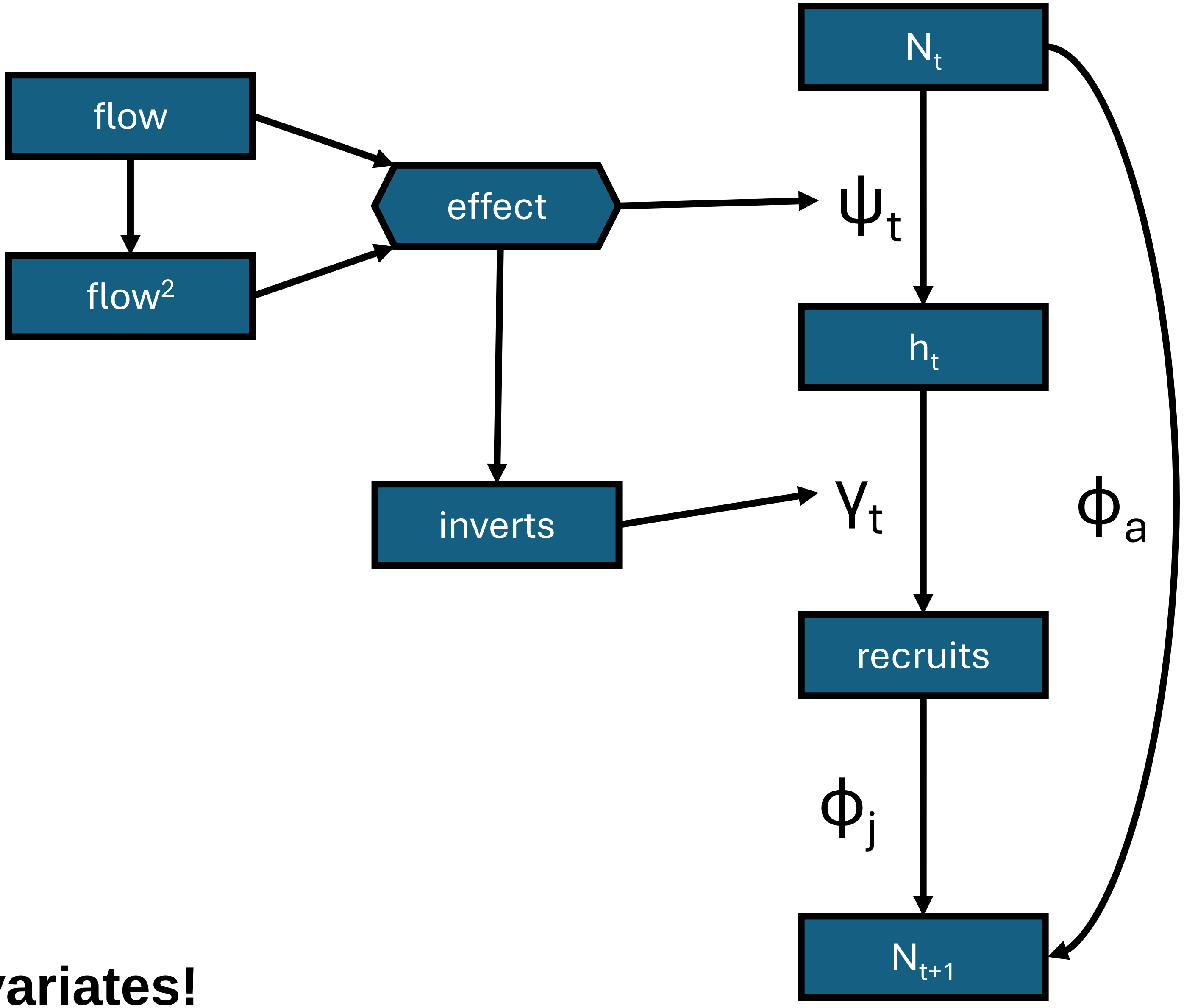
Case study: modeling reproductive success via path analysis

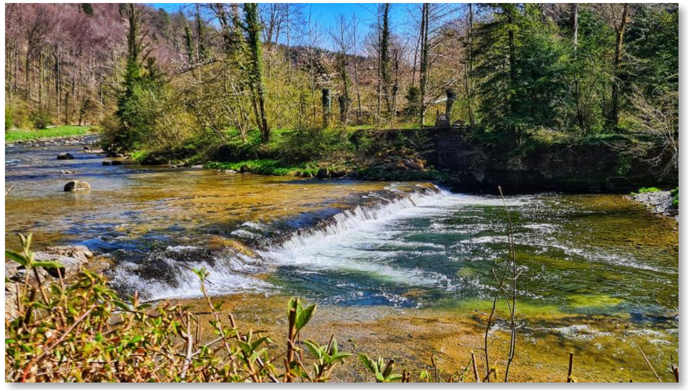


Also: introducing composite covariates!

Today we’re going to use composite covariates

*Do not use ‘blavaan’ in JAGS to develop code for composite covariates (bug alert!)
See blavaan_issue.R

Simulated data (*based on a true story)



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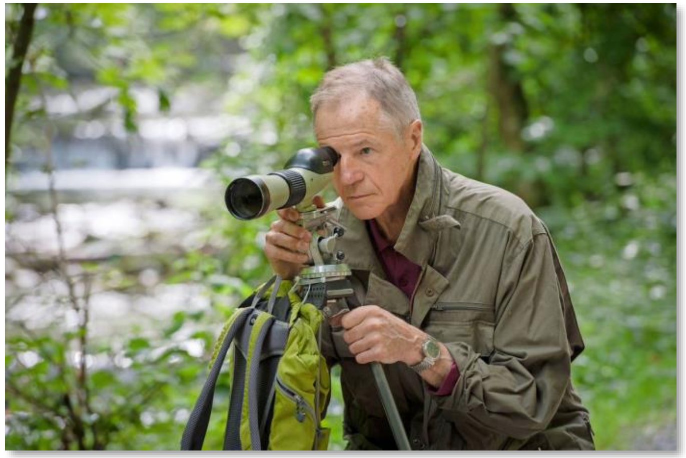
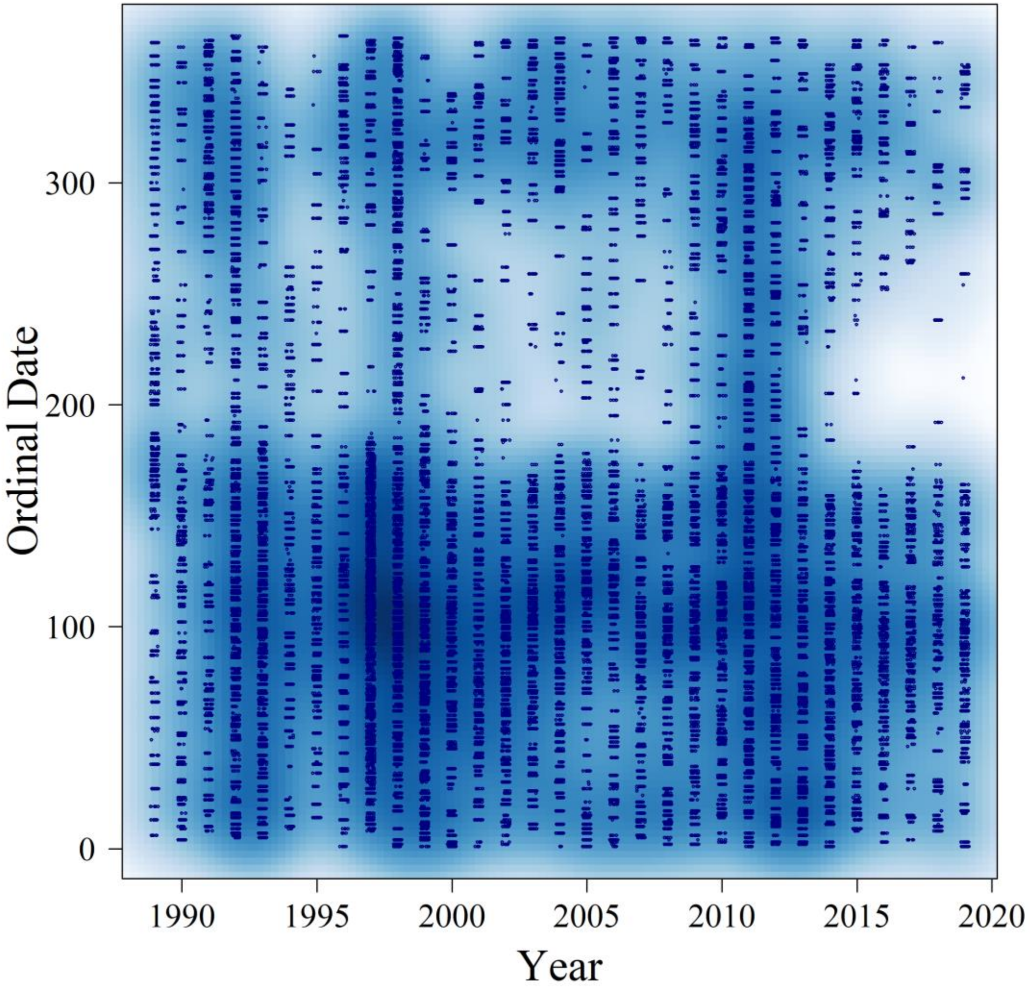
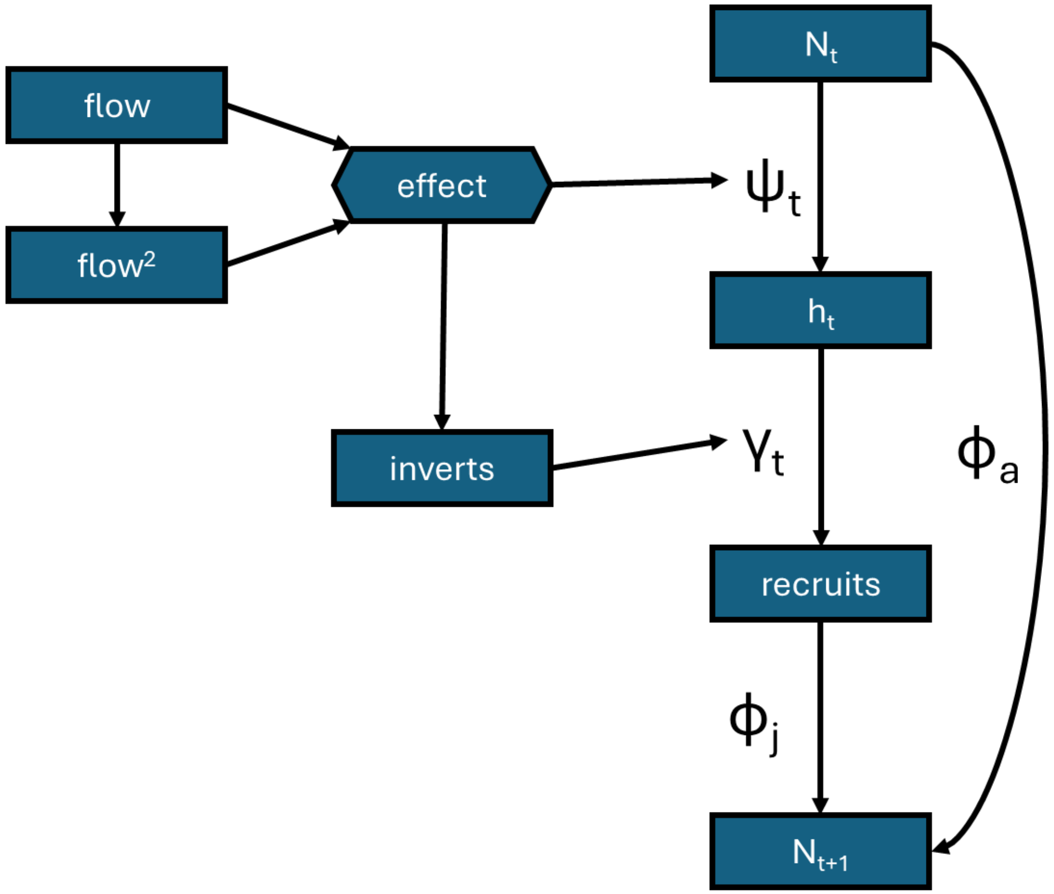




8.5k rings, 3k recaps, 40k resights

We’ll measure several things

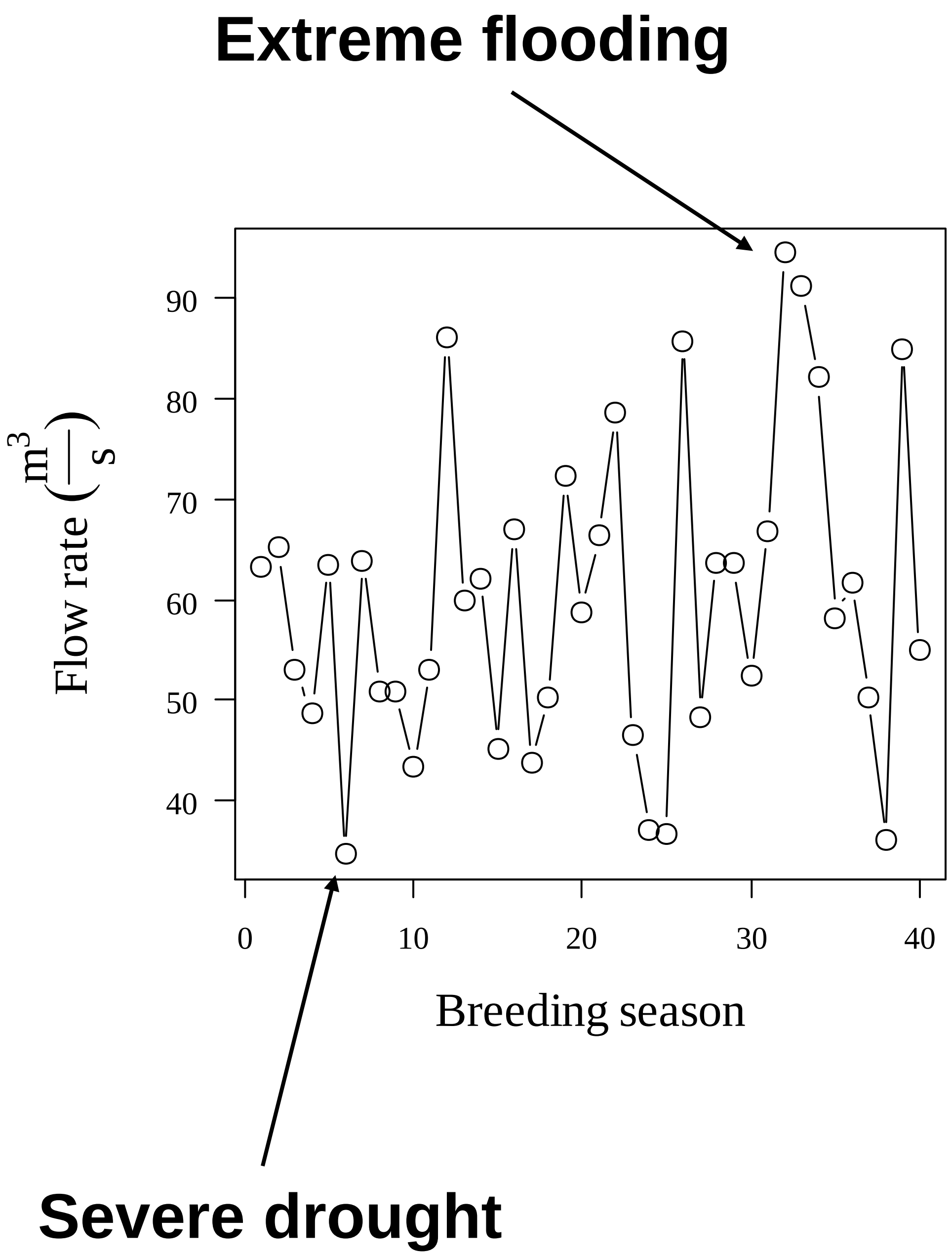
1. Flow rate of our study stream (f)
2. Invertebrate abundance (i)
3. A census (y) of breeding pairs
4. The number of Hatched nests (h)
5. The number of potential Recruits (r)



We could build an IPM + SEM with these data!

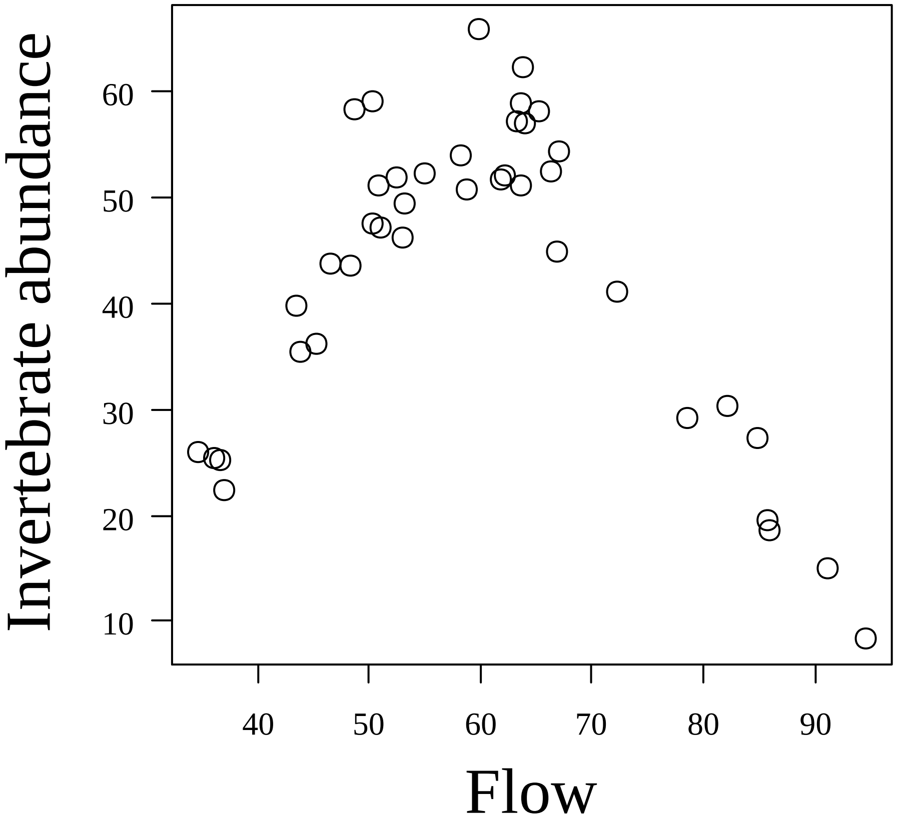
1. Flow rate of our study stream (f)

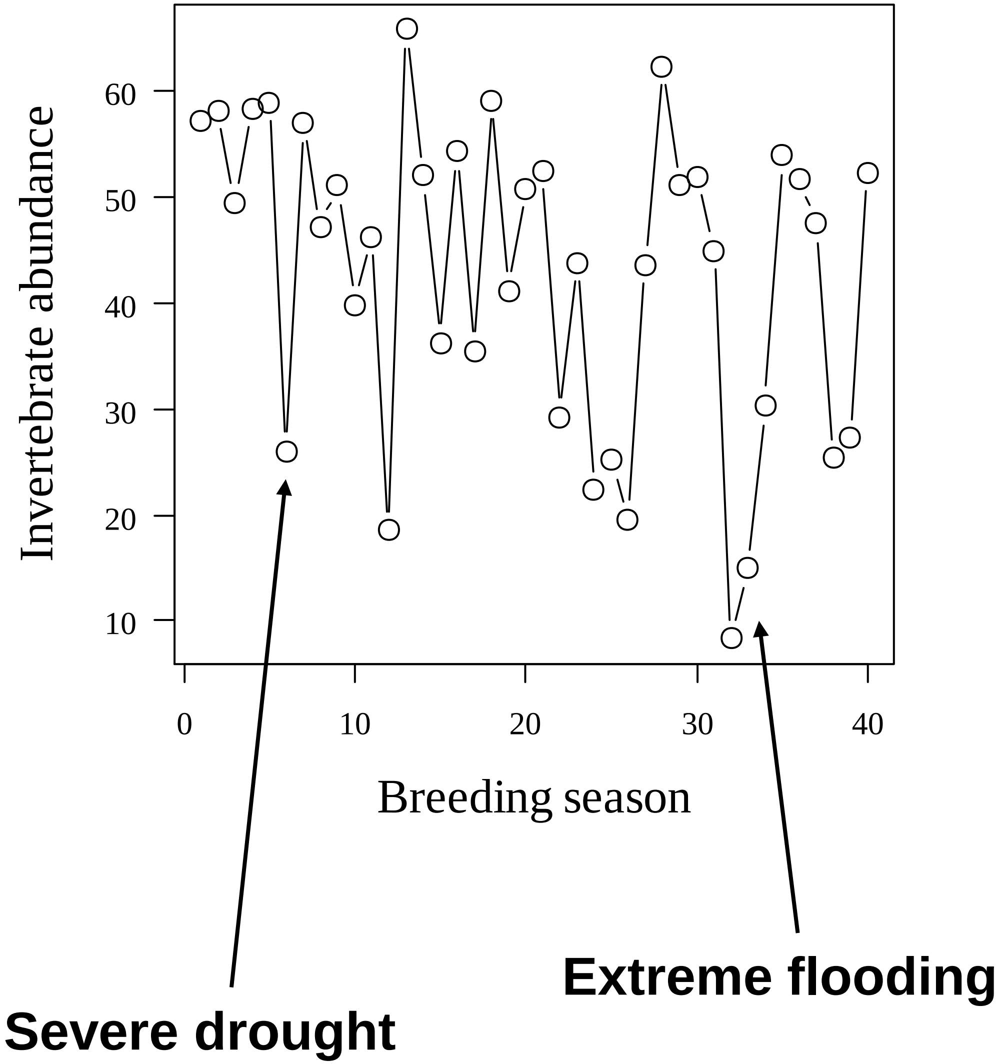
$$\boldsymbol{f} \sim \text{lognormal}(\mu_f = 4, \sigma_f^2 = 0.0625)$$



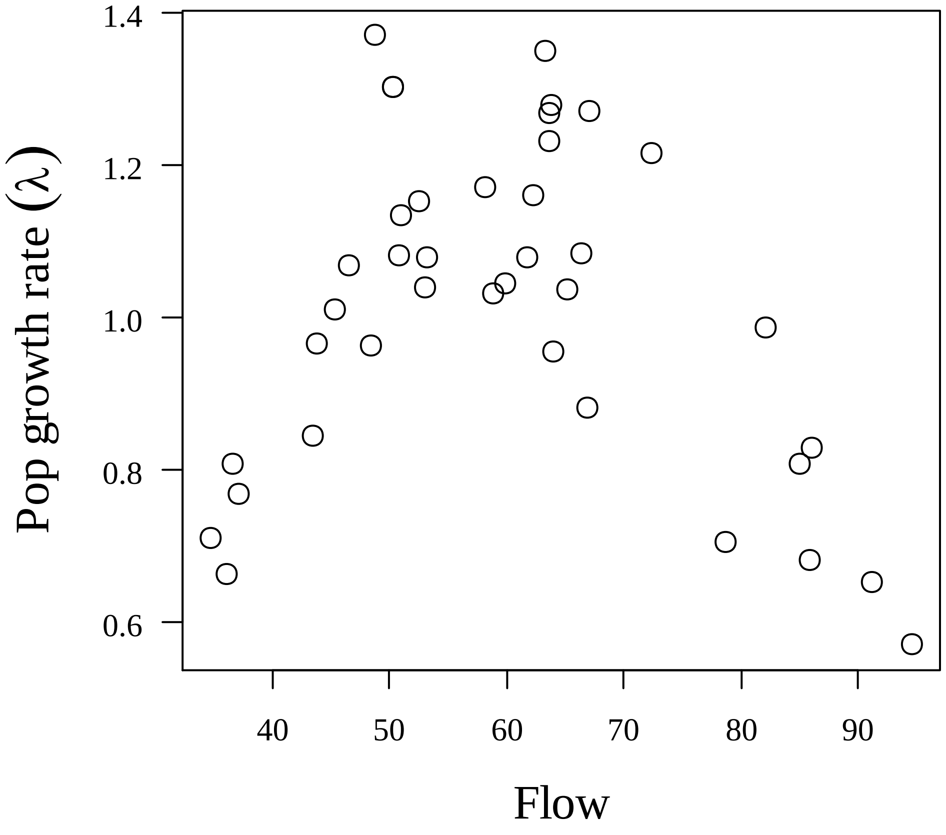
2. Invertebrate abundance (i)

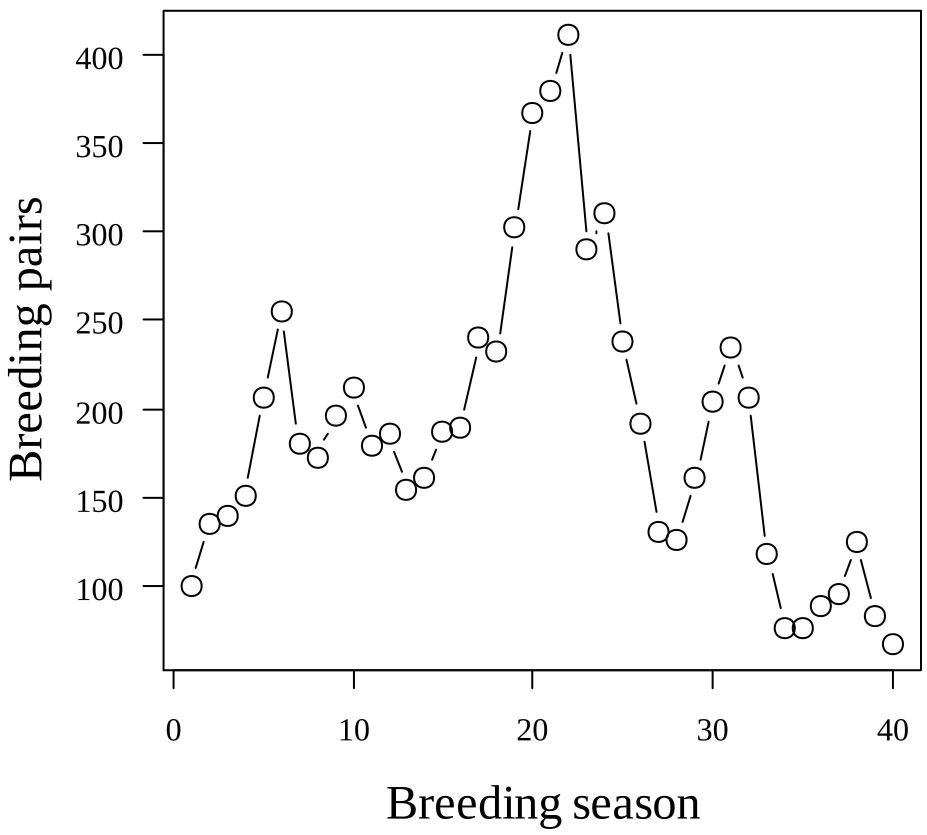
$$\boldsymbol{i} \sim \text{lognormal}(\alpha_1 + \beta_1 \boldsymbol{f} + \beta_2 \boldsymbol{f}^2, \sigma_i^2)$$





3. The number of breeding adults (y)

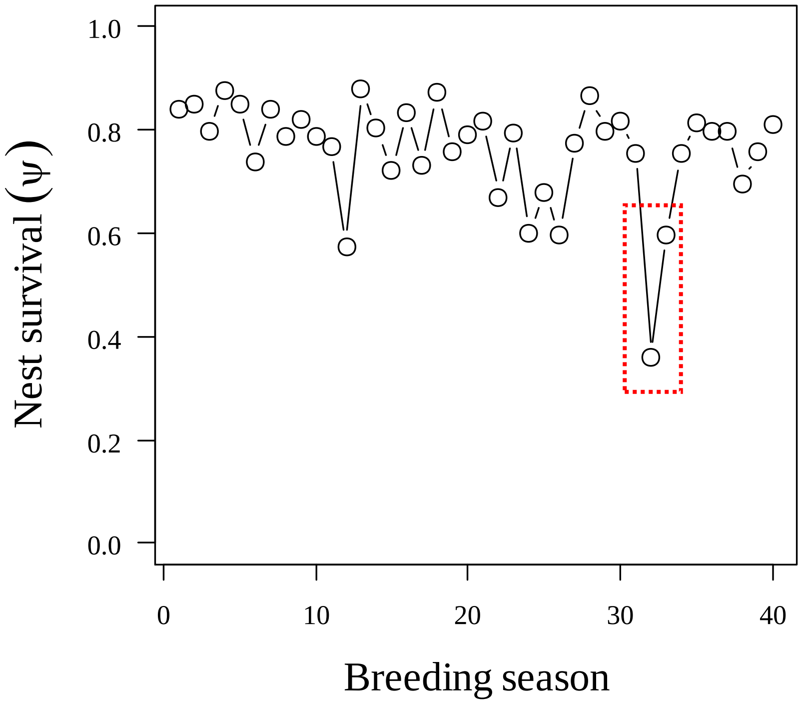


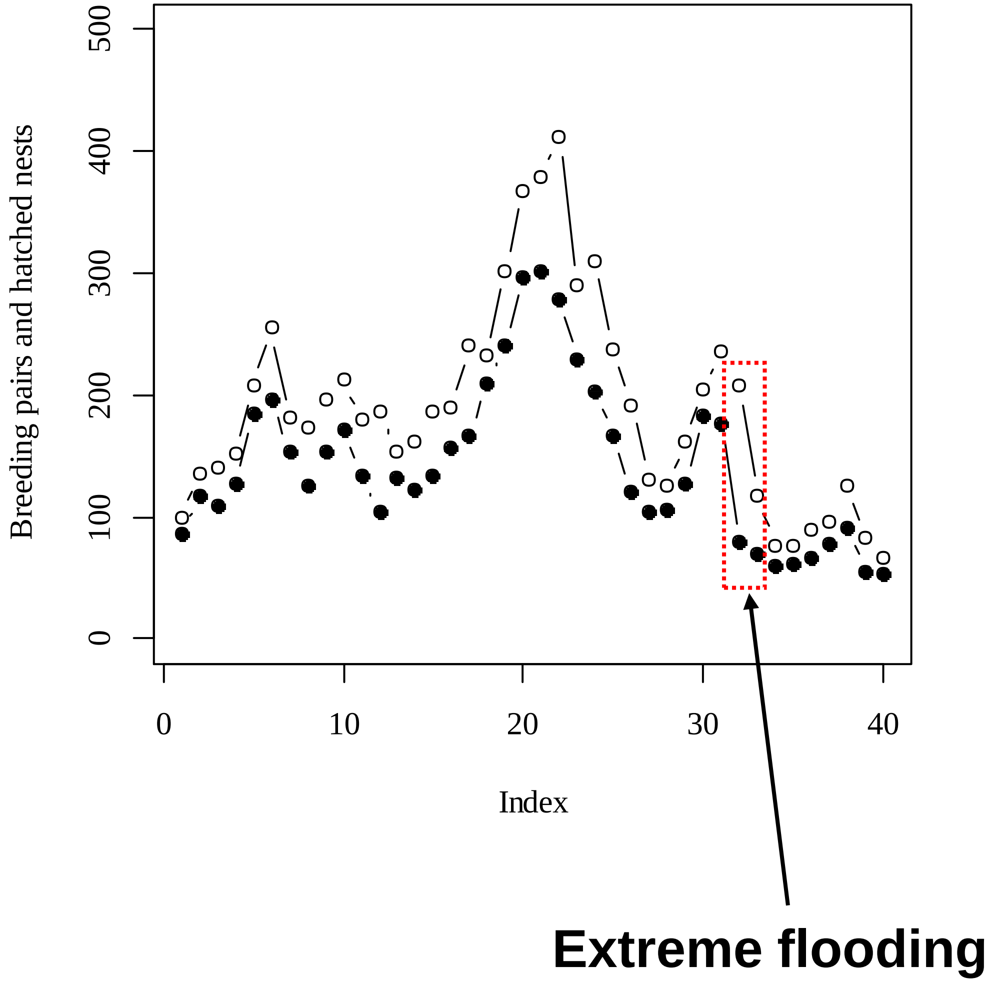


4. The number of hatched nests (h)

$$\boldsymbol{h} \sim \mathrm{binomial}(\boldsymbol{y}, \boldsymbol{\psi})$$

$$\mathrm{logit}(\boldsymbol{\psi}) = \alpha_2 + \beta_3 \boldsymbol{f} + \beta_4 \boldsymbol{f}^2$$

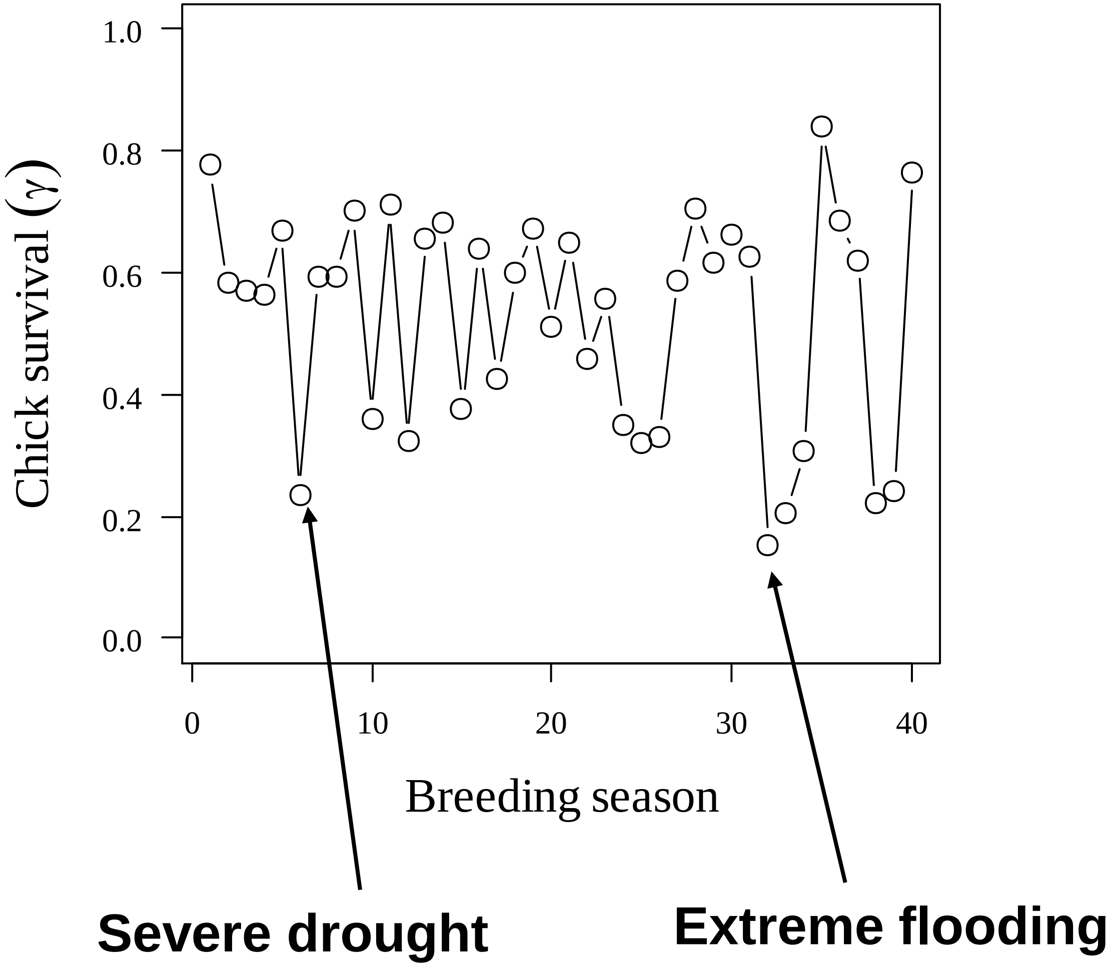




5. The number of surviving potential Recruits (r)

$$\boldsymbol{r} \sim \text{Poisson}(\boldsymbol{h} \times \zeta \times \boldsymbol{\gamma})$$

$$\zeta = 4$$



$$\text{logit}(\boldsymbol{\gamma}) = \alpha_3 + \beta_5 \boldsymbol{i}$$

ζ is clutch size; γ is probability of becoming an independent 'juvenile'

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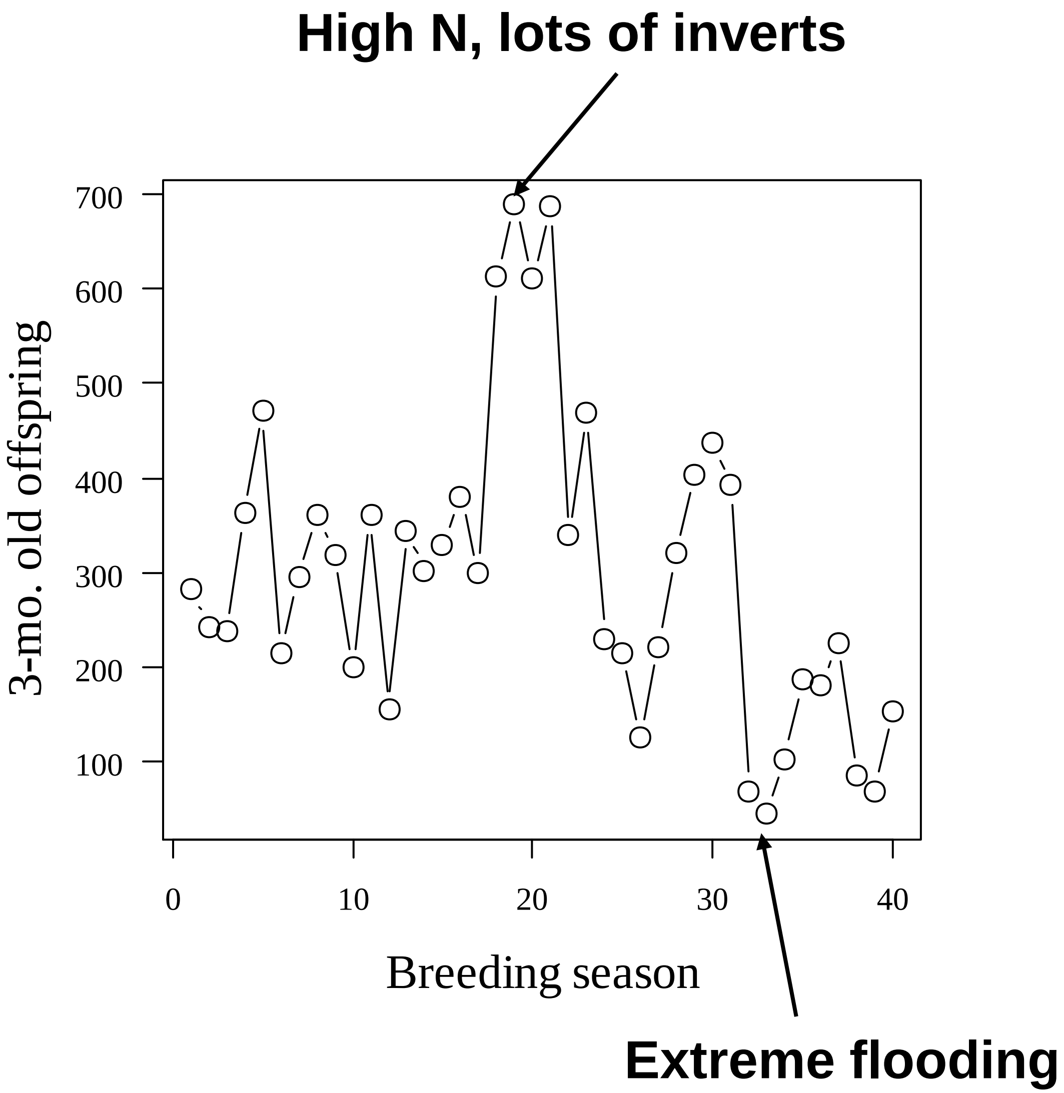

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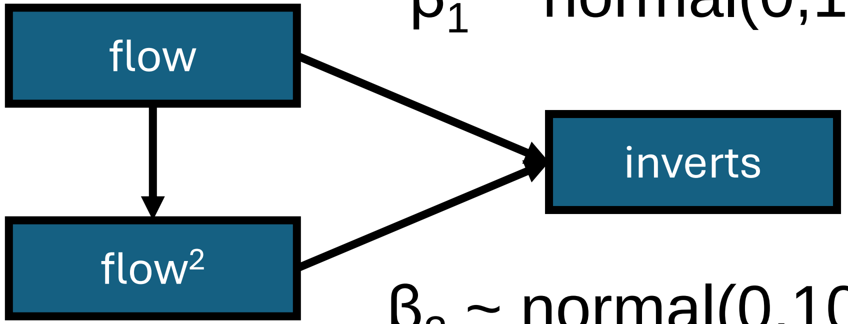
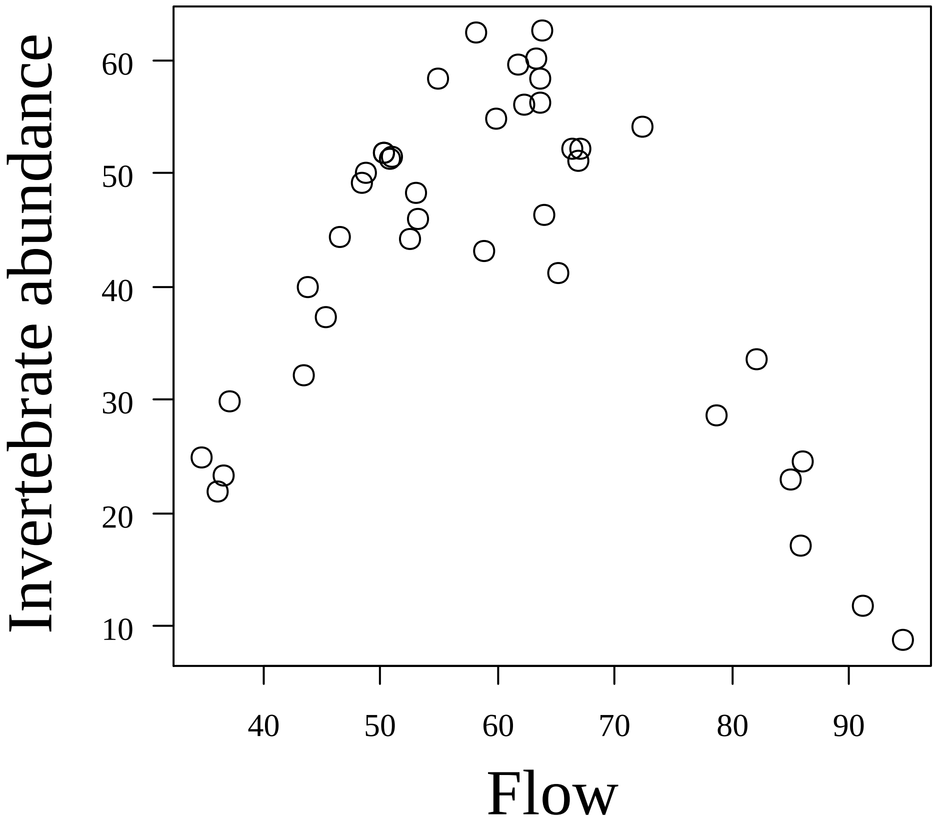
Ok, so what is a composite covariate?!



$\beta_1 \sim \text{normal}(0,10)$

$\beta_2 \sim \text{normal}(0,10)$

$$\boldsymbol{i} \sim \text{lognormal}(\beta_0 + \beta_1 \boldsymbol{f} + \beta_2 \boldsymbol{f}^2, \sigma_i^2)$$



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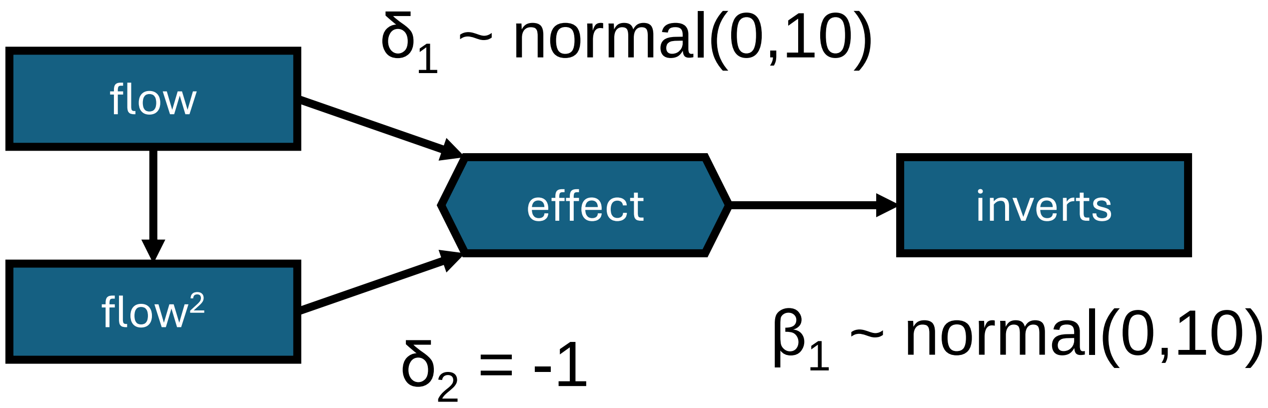
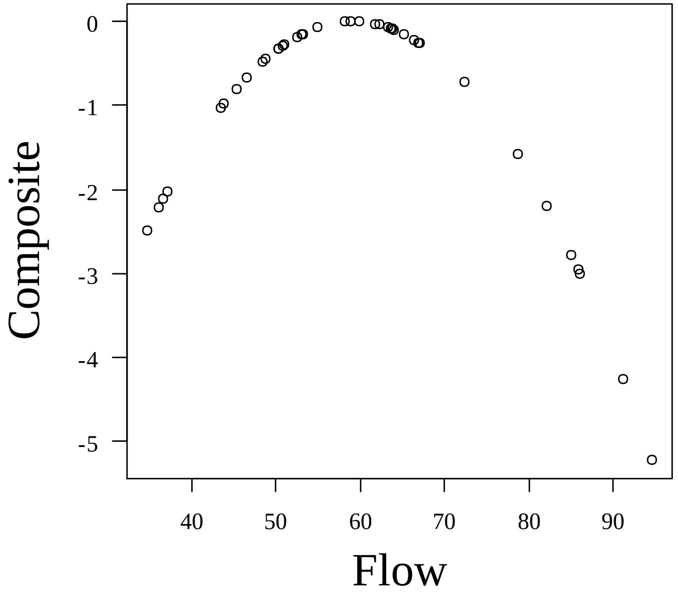
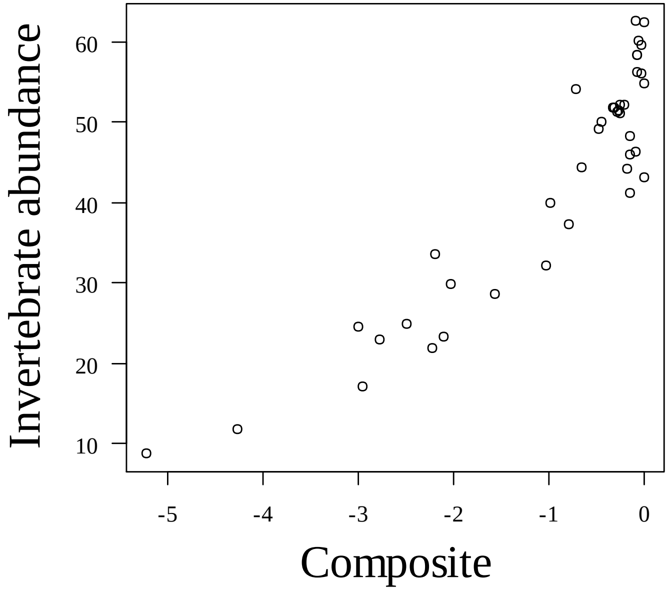


$$\boldsymbol{c} = \delta_1 \boldsymbol{f} + \delta_2 \boldsymbol{f}^2$$

$$\boldsymbol{i} \sim \text{lognormal}(\beta_0 + \beta_1 \boldsymbol{c}, \sigma_i^2)$$





1. What's the difference? 2. Why would we do that?

$$\boldsymbol{i} \sim \mathrm{lognormal}(\beta_0 + \beta_1 \boldsymbol{f} + \beta_2 \boldsymbol{f}^2, \sigma_i^2)$$

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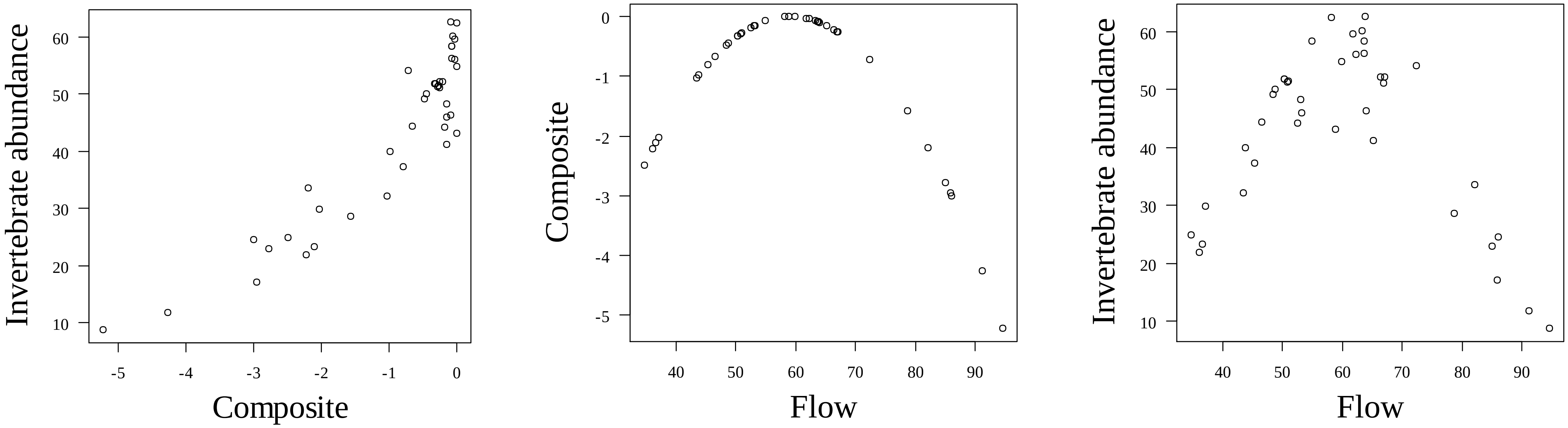

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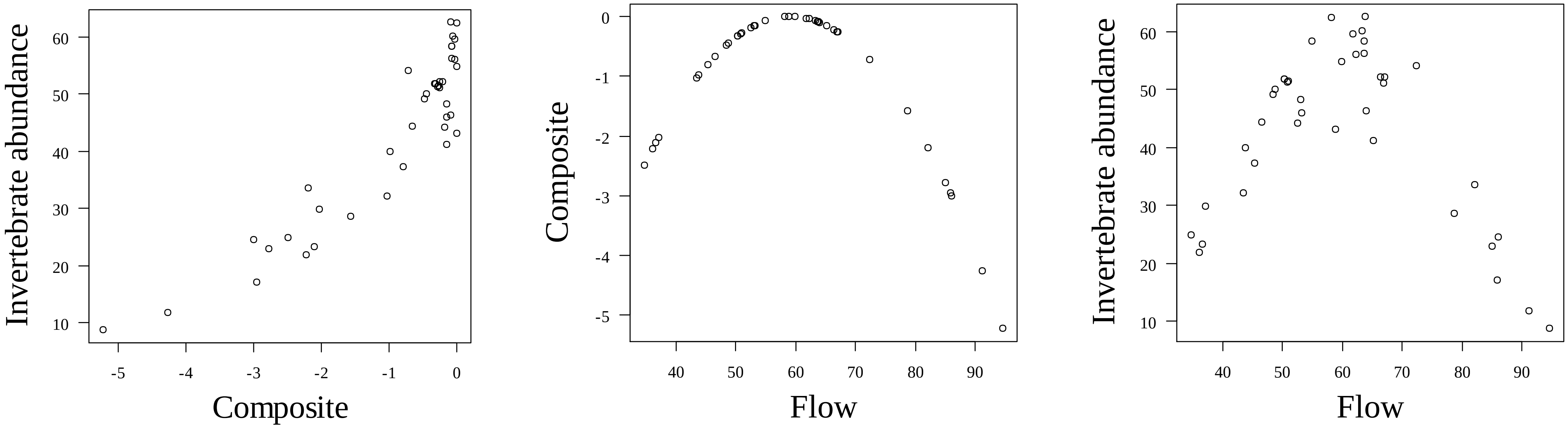


Go to the ‘in_class_question.R’ script

Data simulation

$\mathbf{x_1} \sim \text{normal}(0,1)$

$\mathbf{x_2} = \mathbf{x_1} + \text{normal}(0,1)$

$\mathbf{x_3} = \mathbf{x_1} + \text{normal}(0,1)$

$\boldsymbol{\beta} = [1, 0.5, 2]$

$\mathbf{y} \sim \text{normal}(\boldsymbol{\beta}\mathbf{X}, \sigma^2)$

Linear model

$\boldsymbol{\beta} \sim \text{normal}(0,1)$

$\mathbf{y} \sim \text{normal}(\boldsymbol{\beta}\mathbf{X}, \sigma^2)$

Composite covariate

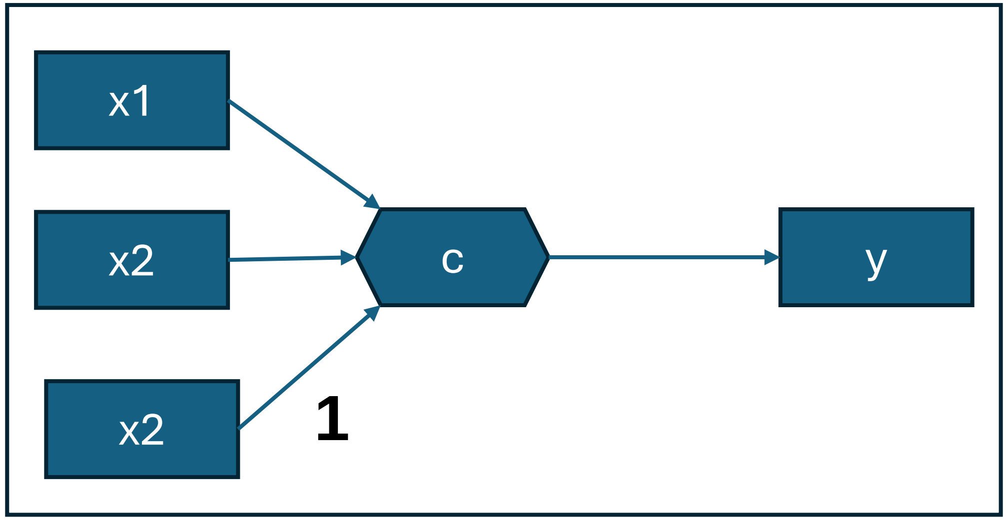
$\boldsymbol{\beta} \sim \text{normal}(0,1)$

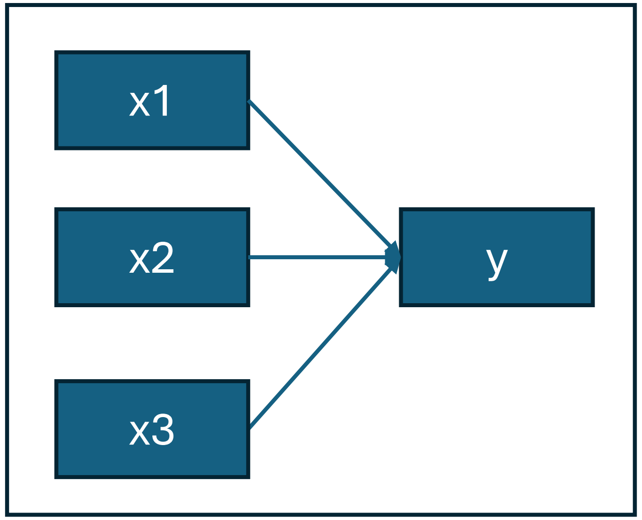
$\mathbf{c} = \boldsymbol{\beta}\mathbf{X}$

$\mathbf{y} \sim \text{normal}(\mathbf{c}, \sigma^2)$

1. What’s the difference? 2. Why would we do that?

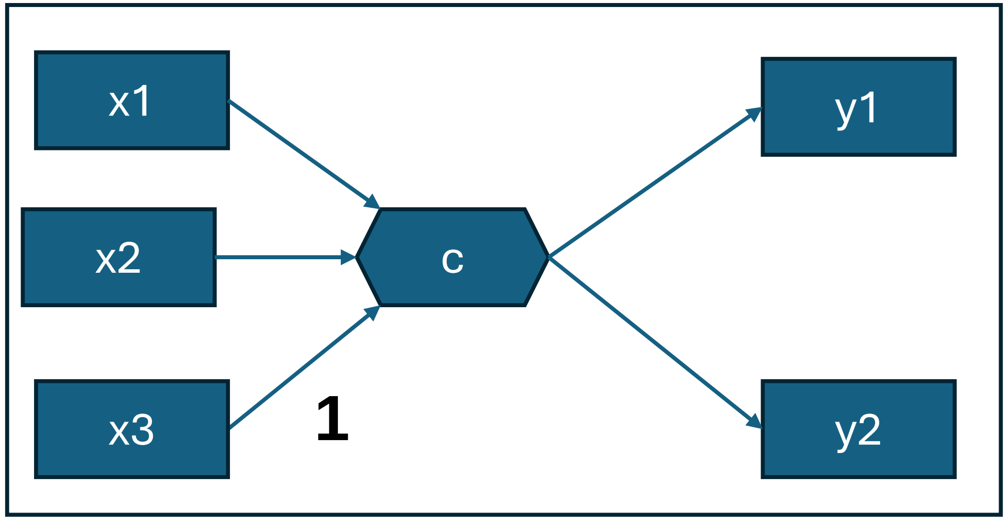
1. What's the difference?

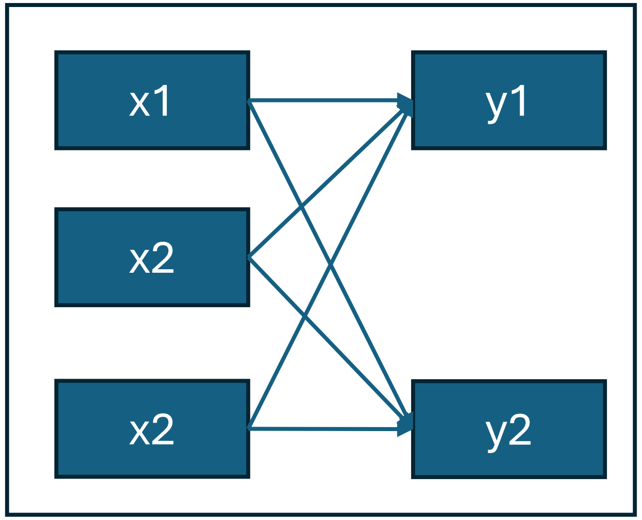




These models are equivalent?!

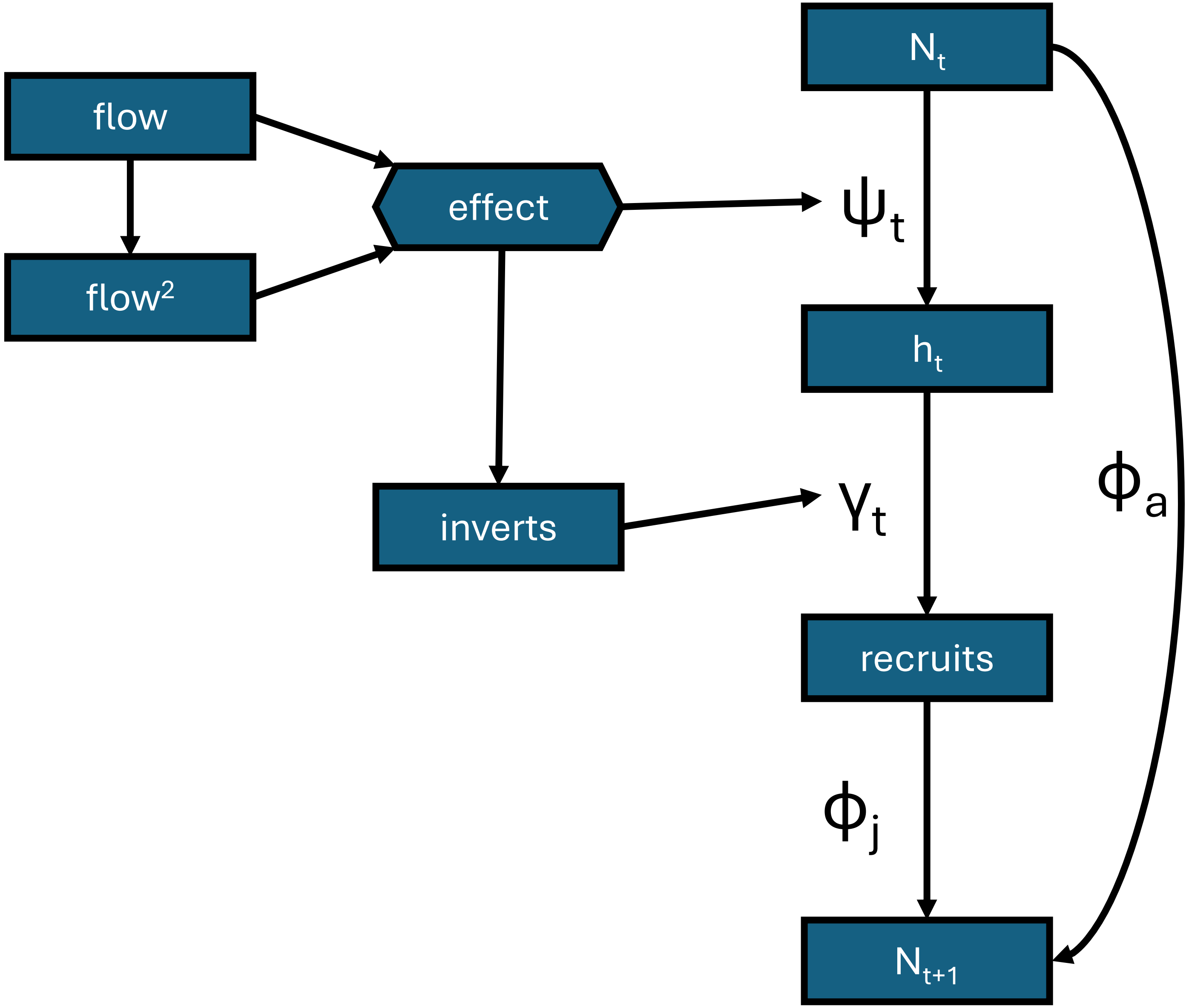
2. Why would we do that?





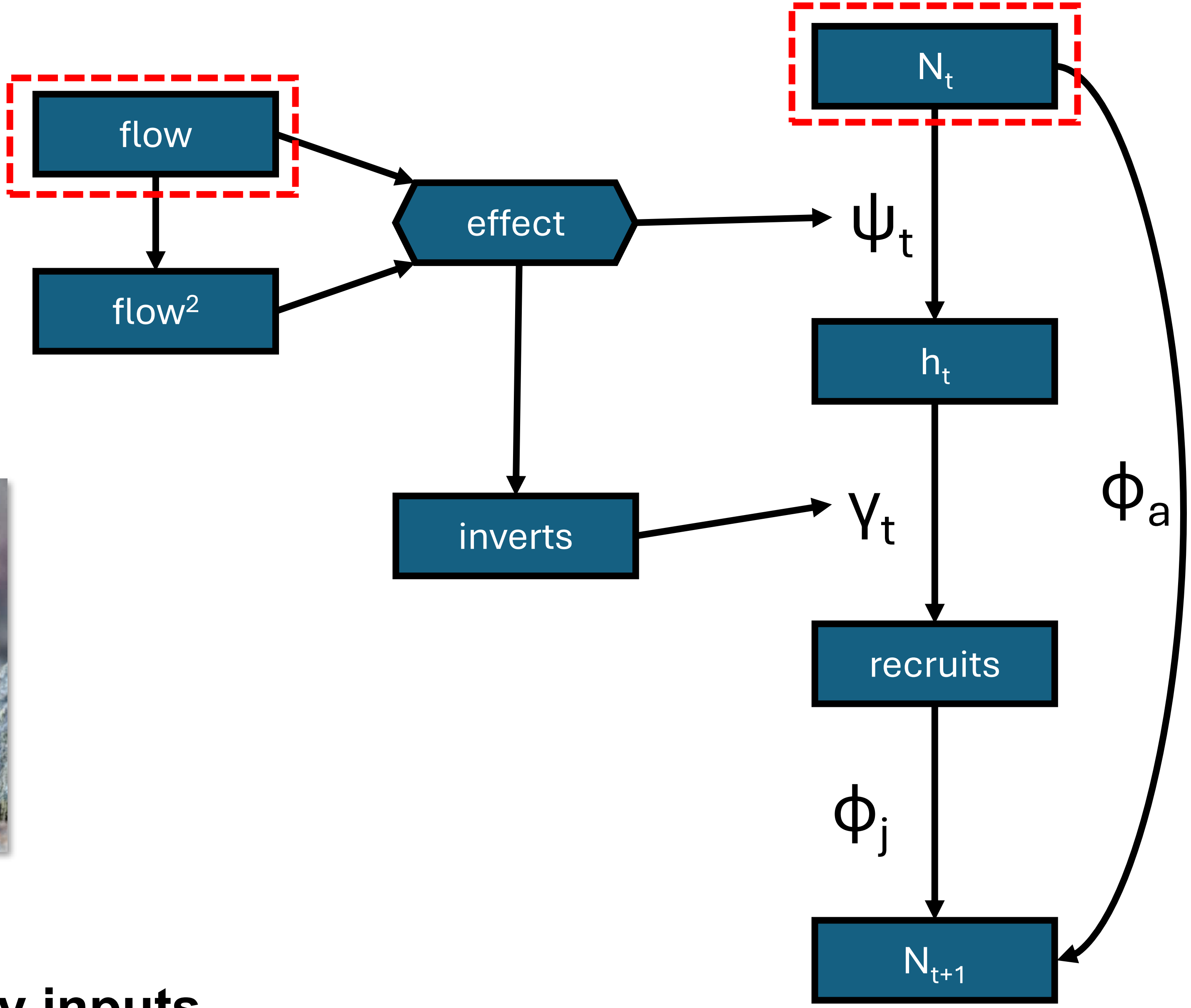
These models don't have to be

Let’s simulate this population



This looks complicated!

Let's simulate this population



These two things don't have any inputs…

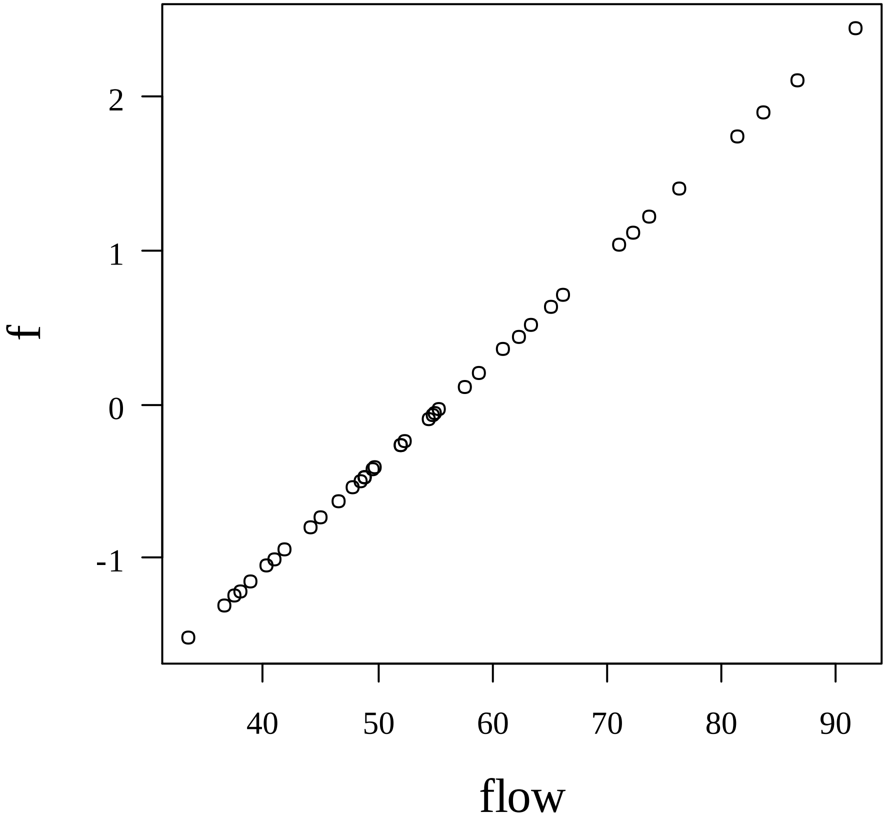
We'll start with some number of breeding pairs (N_1 = 100)…

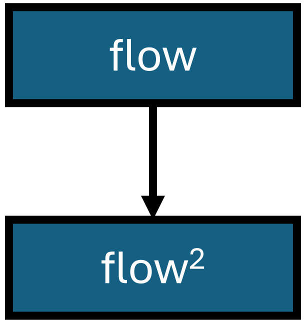
N_t

flow

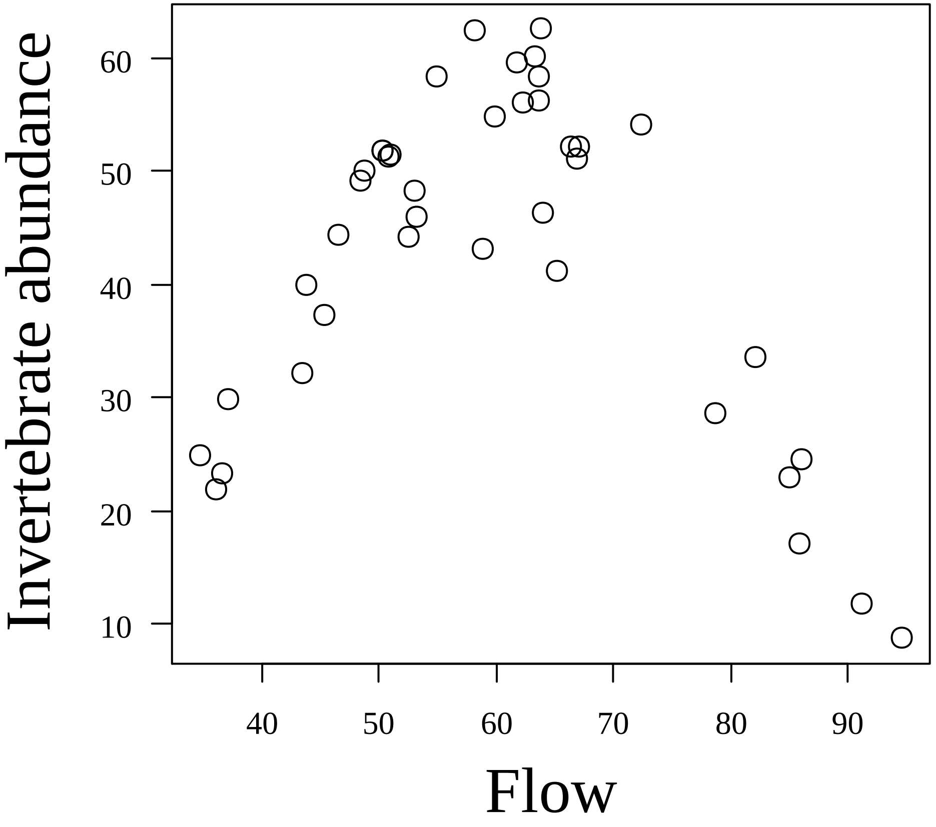
And a flow rate (f_1 = 63.2 m^3/s; about half the size of the Clark Fork in spring)

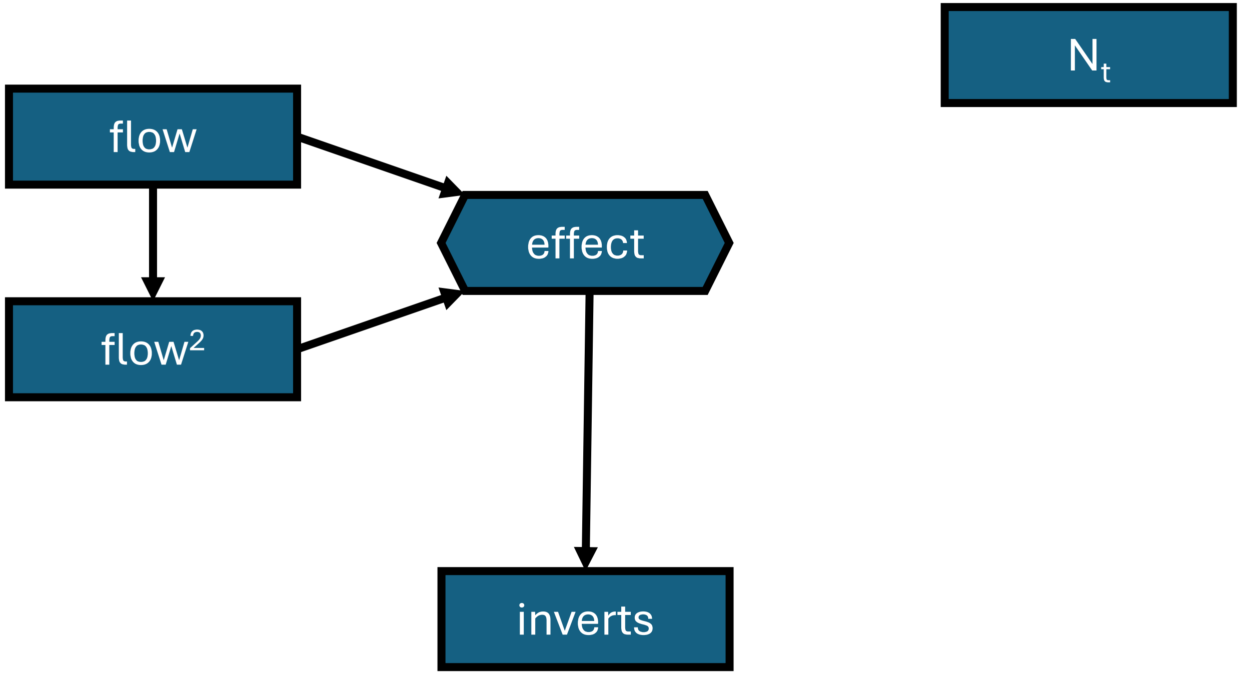
From there we can z-standardize flow (f), and square it (f^2)





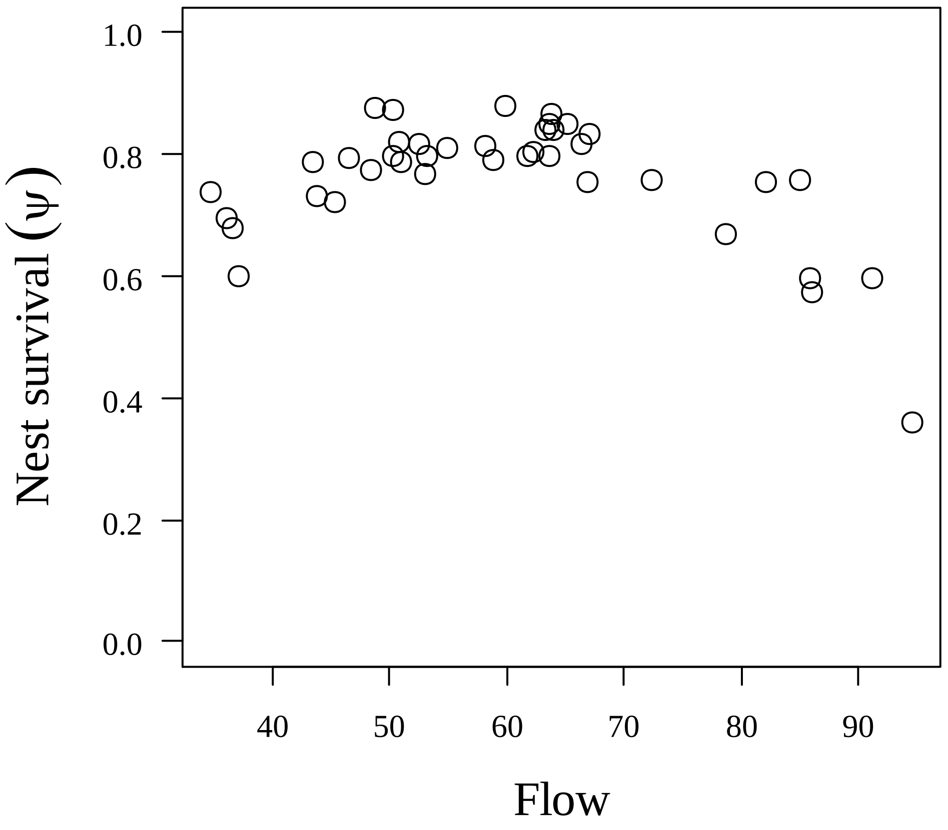
Now, we can simulate invertebrates as a function of flow

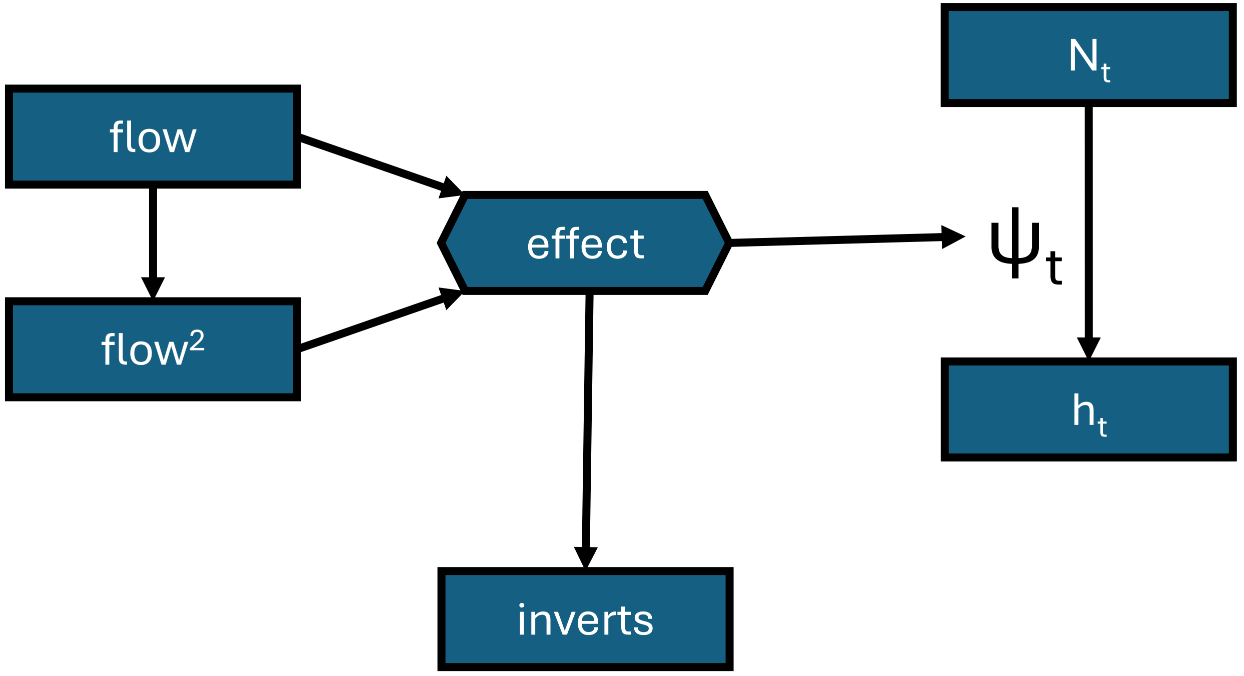




$$\boldsymbol{i} \sim \text{lognormal}(\alpha_1 + \beta_1 \boldsymbol{f} + \beta_2 \boldsymbol{f}^2, \sigma_i^2)$$

We can do the same thing with nest survival to simulate hatched nests (h)

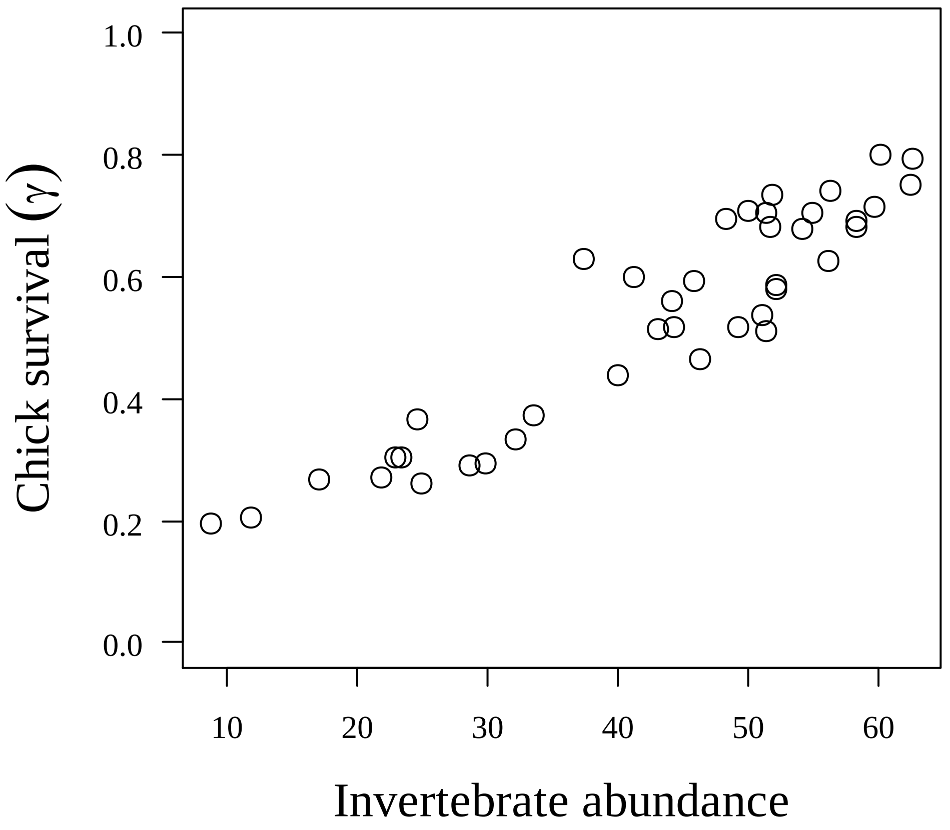


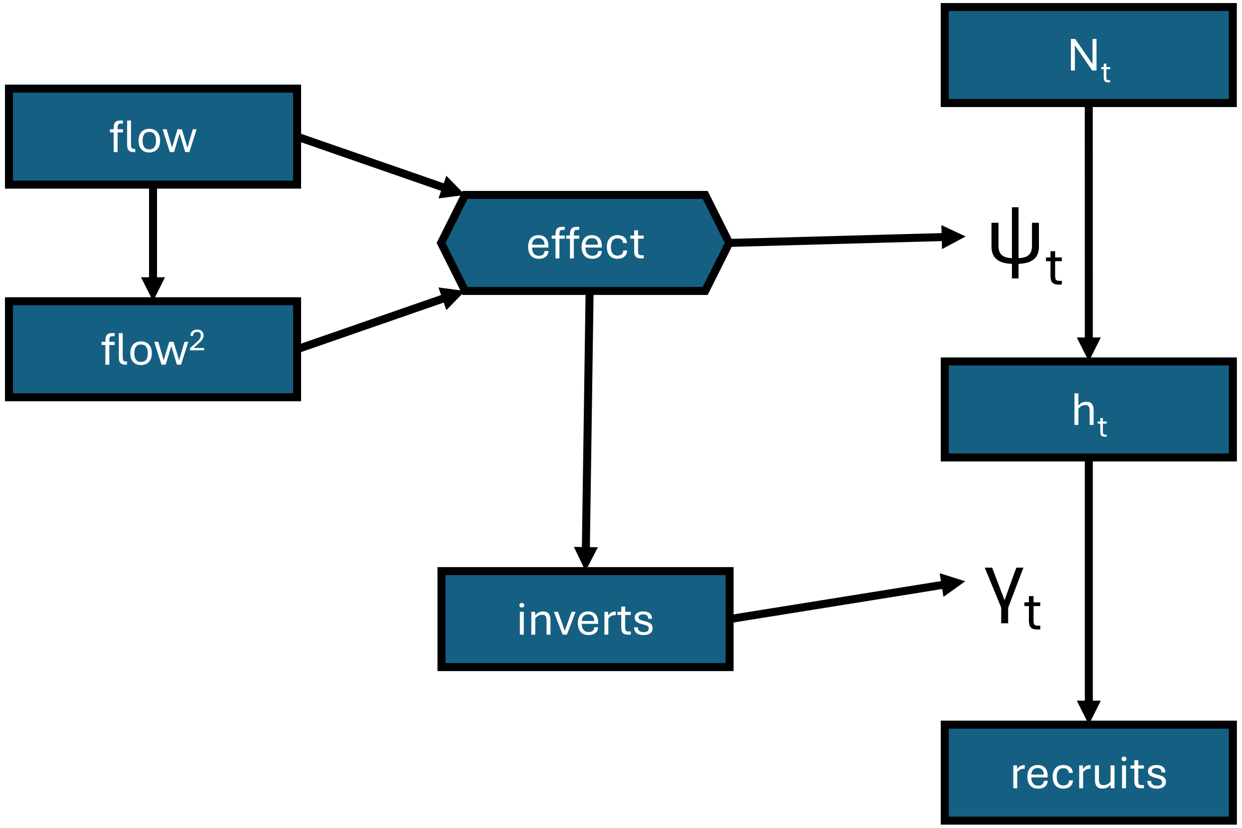


$$\boldsymbol{h} \sim \text{binomial}(\boldsymbol{y}, \boldsymbol{\psi})$$

$$\text{logit}(\boldsymbol{\psi}) = \alpha_2 + \beta_3 \boldsymbol{f} + \beta_4 \boldsymbol{f}^2$$

Then we simulate chick survival as a function of invertebrates to get recruits

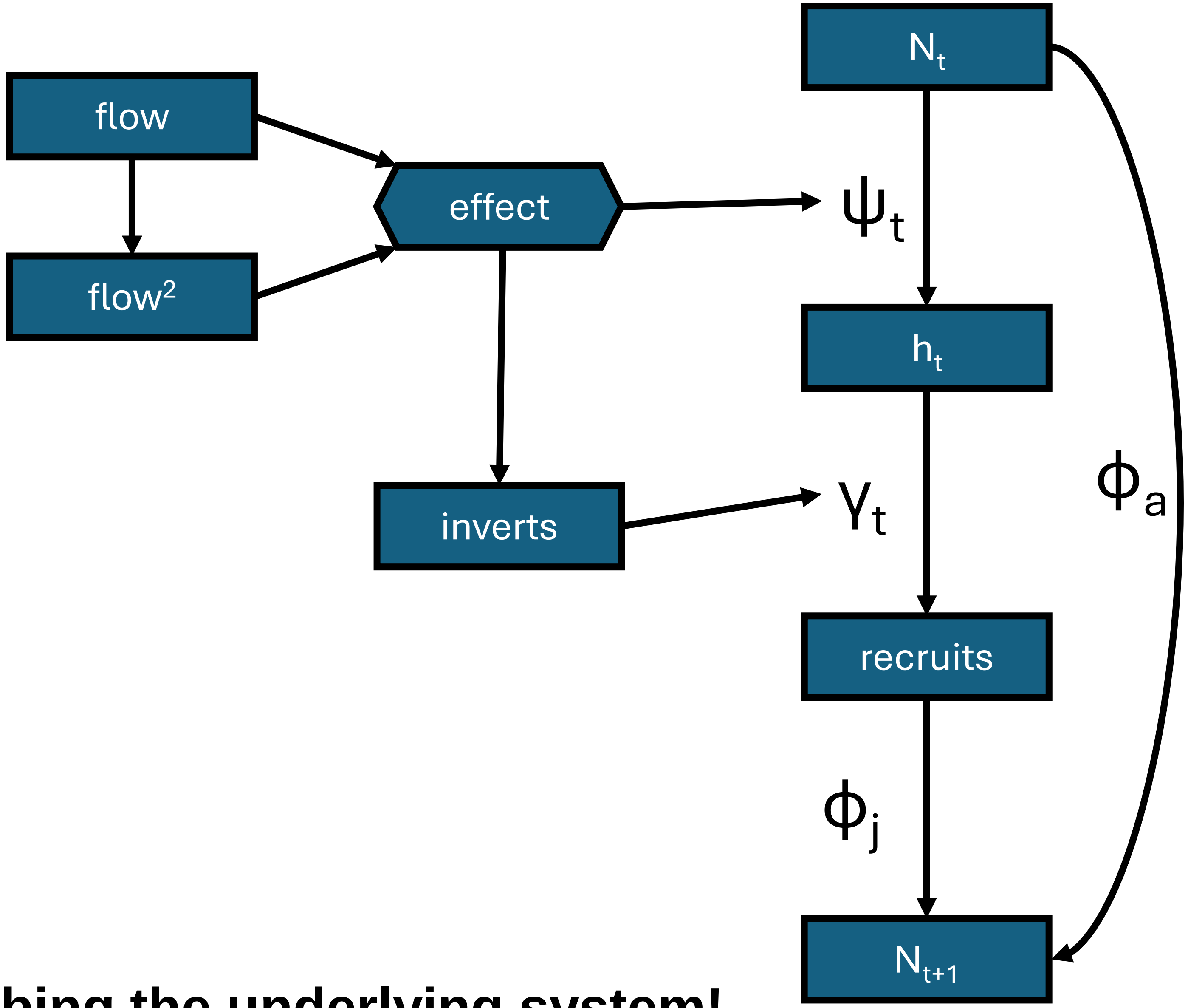




$$\boldsymbol{r} \sim \text{Poisson}(\boldsymbol{h} \times \zeta \times \boldsymbol{\gamma})$$

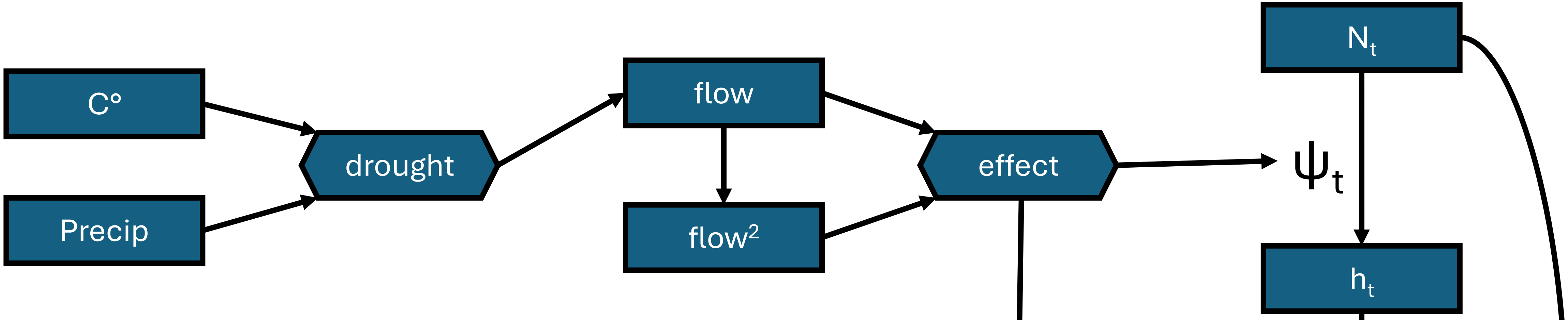
$$\text{logit}(\boldsymbol{\gamma}) = \alpha_3 + \beta_5 \boldsymbol{i}$$

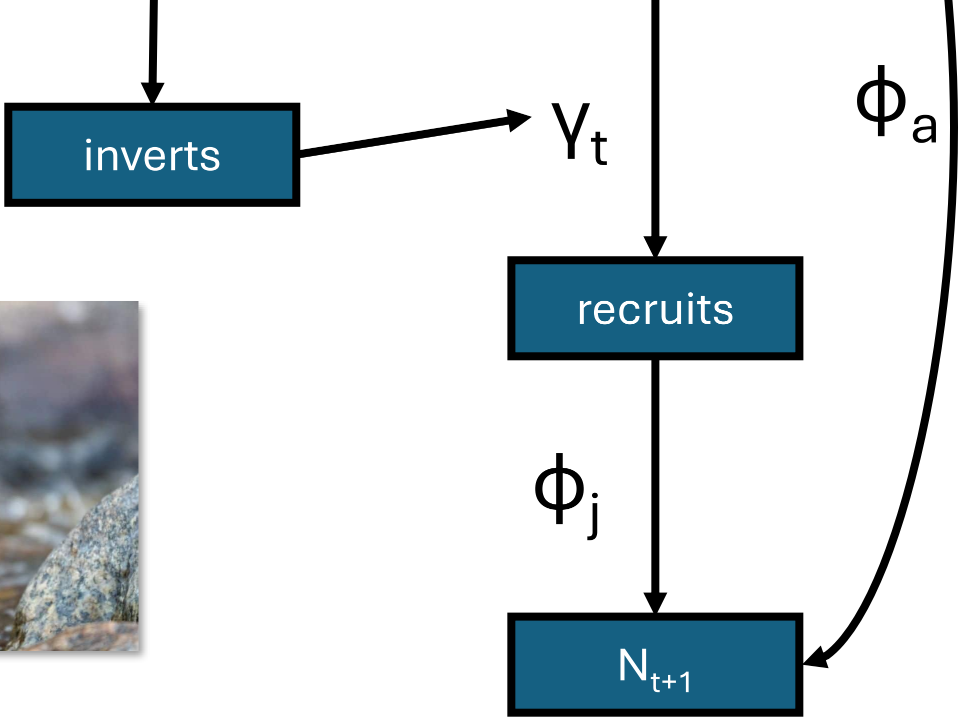
And allow adults and potential recruits to survive into the next year

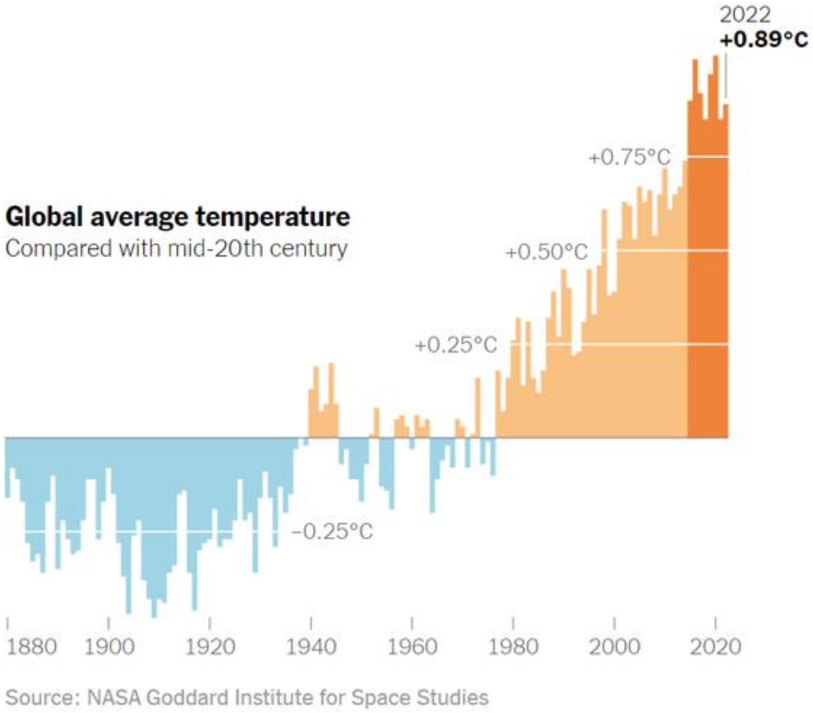


This is an IPM with a SEM describing the underlying system!

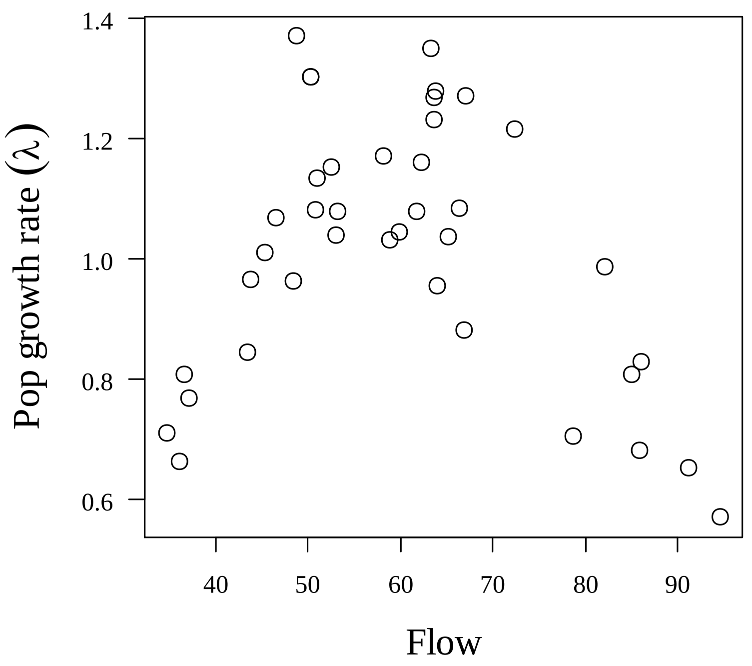
Think about how useful this is!

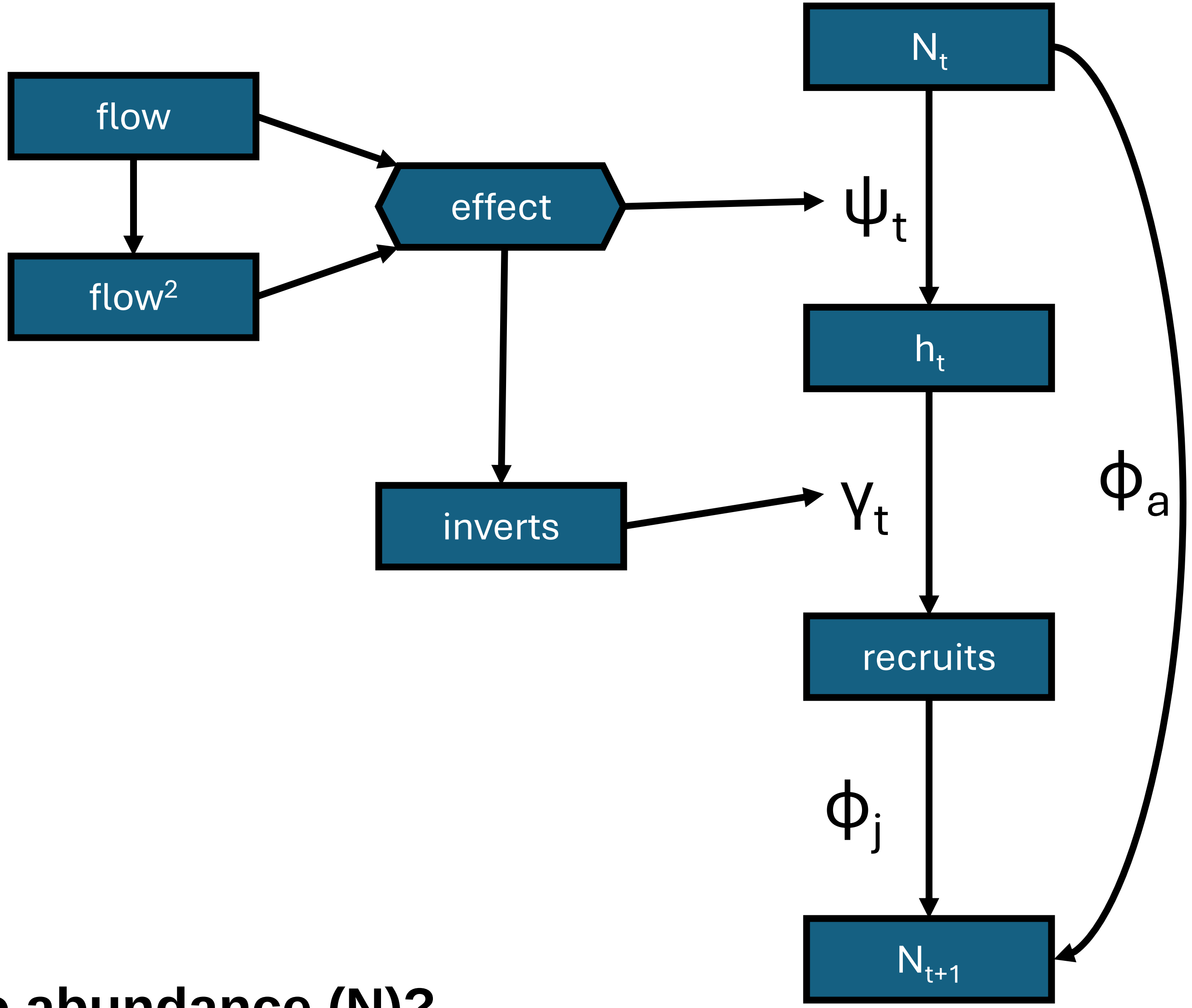






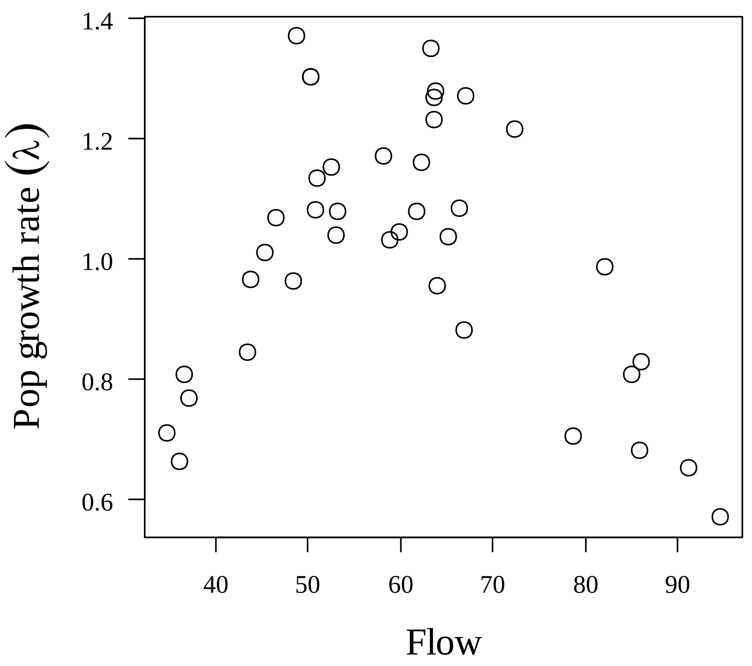
Imagine that the mean of flow declines…

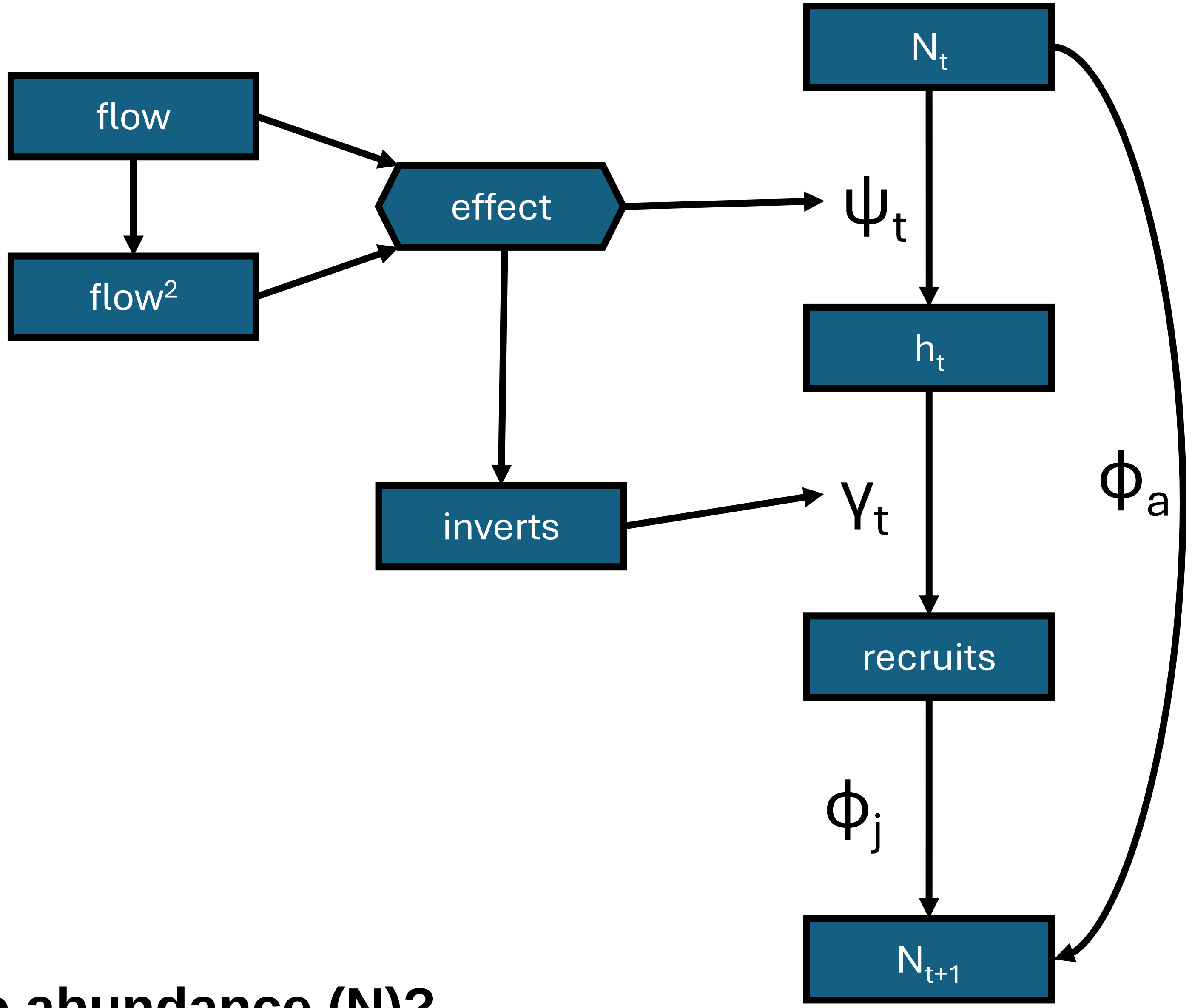




What do you think will happen to abundance (N)?

Now imagine that the variance of flow increases (flood or dry)…





What do you think will happen to abundance (N)?