Quantitative Portfolio Management: Theory and Practice

Chapter 8: Portfolios Exploiting Systematic and Unsystematic Risk

Raman Uppal
EDHEC Business School

2025-2026

The big picture: Plan for the entire course

Part A: Preliminaries

Chapter 1: Managing financial data

Chapter 2: Performance measurement (especially out of sample)

Part B: Modern portfolio management

Chapter 3: Mean-variance portfolios that *ignore* estimation error

Chapter 4: Mean-variance portfolios that *adjust for* estimation error

Part C: Post-Modern Portfolio Management

Chapter 5: CAPM-based portfolios: Black-Litterman model

Chapter 6: Factor-based portfolios: Parametric portfolio policies

Chapter 7: Volatility-timed factor portfolios

Chapter 8: Portfolios exploiting systematic and unsystematic risk

Table of contents

Road map

What do we want to do in Chapter 8

In this chapter, first we study the concept of a stochastic discount factor (SDF).

Then we show that an SDF that is good at pricing assets much include compensation for bearing unsystematic risk.

Finally, we study how to build portfolios that exploit both systematic and unsystematic risk.

Timeline: Quantitative portfolio management ideas . . . I

- We can see how ideas about investment have progressed over time.

.... *The thinking in ancient times* ....

- 4th century: $1/N$
  - "One should always divide his wealth into three parts: a third in land, a third in merchandise, and a third ready to hand [cash].
    Rabbi Issac bar Aha, Babylonian Talmud: Tractate Baba Mezi'a, folio 42a
  - "My ventures are not in one bottom trusted"
    "Merchant of Venice, "Shakespeare ((1564–1616) on the importance of diversification in investing
  - Do not put all your eggs in one basket

Timeline: Quantitative portfolio management ideas . . . II

. . . . *Below are the topics we covered in Chapters 3–5* . . . .

- 1950s: Mean-variance optimization
  (Markowitz 1952, 1959)

- 1964: CAPM
  (Sharpe 1964)

- 1970–2000s: Bayesian shrinkage
  (Klein and Bawa 1976; Bawa, Brown, and Klein 1979; Jorion 1985; Jorion 1988; Jorion 1992; Pástor and Stambaugh 2000)

- 1990s: Black-Litterman model
  (Black and Litterman 1990, 1991a, 1991b, 1992; He and Litterman 1999; Litterman 2003)

Timeline: Quantitative portfolio management ideas . . . III

. . . . *Chapter 6* . . . .

- 1970s: Factor models
  (Ross 1976, 1977)

- 1980s Macro factor models
  (Chen, Roll, and Ross 1986)

- 1990–2020s: Fundamental (firm-characteristic-based) factor models
  (Fama and French 1992, 1993, 2012, 2015, 2018).

- 2009-2023: Parametric portfolio policies
  (Brandt, Santa-Clara, and Valkanov 2009; DeMiguel, Martín-Utrera, Nogales, and Uppal 2020).

Timeline: Quantitative portfolio management ideas ... IV

.... *Last time's class: Chapter 7* ....

- 2017-2024: Volatility-timing of factors
  - Moreira and Muir (2017, 2019)
  - Cederburg, O'Doherty, Wang, and Yan (2020)
  - Barroso and Detzel (2021)
  - DeMiguel, Martín-Utrera, and Uppal (2024).

.... *Today's class: Chapter 8* ....

- 2023-2024: Portfolio construction: Beyond systematic risk factors
  - Raponi, Uppal, and Zaffaroni (2023)
  - Dello-Preite, Uppal, Zaffaroni, and Zviadadze (2024).

Timeline: Quantitative portfolio management ideas . . . V

- For a more detailed history of the development of ideas about investment, see the book by Rubinstein (2006).

Road map

Motivation for the material in this chapter

- Existing factor models of asset pricing perform poorly in explaining the cross-section of stock returns;
- This means that portfolios constructed relying on these models are missing an important part of the driver of stock returns.
- In this chapter, we want to find out
  - what is missing in standard models of asset pricing;
  - how to exploit the missing component for portfolio construction.

This chapter is divided into three parts:

1. Understanding the theory of stochastic discount factors (SDFs);
2. Using SDFs to identify what is missing in asset-pricing factor models.
3. Exploiting the missing component for portfolio construction.

Part 1 of this chapter

Understanding the theory of stochastic discount factors (SDFs).

These notes are based on material from the book:

Back, K. E. 2017. *Asset pricing and portfolio choice theory.* Oxford University Press.

Road map

Road map

The stochastic discount factor (SDF) and beta-pricing models

- A key objective of finance is to show how to adjust cash flows
  - for risk and
  - for time.
- We will now study how all the traditional results that you are familiar with can be expressed in terms of the SDF.
- Every asset-pricing model/theory implies a particular SDF:
  - So, all we need to know about any asset-pricing model is the particular SDF implied by it;
  - Conversely, knowing the SDF fully characterizes the asset-pricing model it comes from.

SDF and gross returns

- The price at date $t = 0$ of a non-dividend paying asset, $P_{0,n}$, in terms of the SDF at $t = 1$, m_1, can be expressed as:

$$P_{0,n} = \mathbb{E}\left[m_1 \, P_{1,n}\right].$$

  - The above expression tells us that m_1 has adjusted $P_{1,n}$ for risk and time appropriately, which, therefore, allows us to get $P_{0,n}$.

- If $P_{0,n} > 0$, then we can divide the above equation by $P_{0,n}$ to get:

$$1 = \mathbb{E}\left[m_1 \frac{P_{1,n}}{P_{0,n}}\right] = \mathbb{E}\left[m_1 \, R_{1,n}\right], \quad \ldots \text{gross return } R_{1,n} = P_{1,n}/P_{0,n}.$$

  - The above equation says that after we adjust for time and risk via m_1, the expected gross return R_n on every asset is 1.

SDF and expected excess returns . . . I

- In finance, we often study excess returns. So, next we identify the relation between the SDF and excess returns.

- We start by reproducing the last equation on the previous slide:

$$1 = \mathbb{E}\left[m_1 \frac{P_{1,n}}{P_{0,n}}\right] = \mathbb{E}\left[m_1 R_{1,n}\right], \quad \ldots \text{gross return } R_{1,n} = P_{1,n}/P_{0,n}.$$

- Taking the difference of the two expressions above, we get an expression in terms of excess returns:

$$0 = \mathbb{E}\left[m_1 \left(R_{1,n} - R_{1,m}\right)\right], \quad \ldots \text{where } R_m \text{ is the gross return on asset } m.$$

- The expression above is true for any asset or portfolio.

- It says that once we adjust for risk and time, the excess return on every asset is 0.

Road map

Expected returns in terms of covariance with the SDF . . . I

- Recall from basic statistics that:

$$\text{Cov}[x, y] = \mathbb{E}[x\, y] - \mathbb{E}[x]\,\mathbb{E}[y],$$

which implies that:

$$\mathbb{E}[x\, y] = \text{Cov}[x, y] + \mathbb{E}[x]\,\mathbb{E}[y].$$

- Starting with our basic result and writing m_1 as m and $R_{1,n}$ as R_n:

$$\begin{aligned}
1 &= \mathbb{E}\left[m\, R_n\right], && \text{. . . our basic result} \\
1 &= \text{Cov}[m, R_n] + \mathbb{E}[m]\,\mathbb{E}[R_n] && \text{. . . because } \mathbb{E}[x\, y] = \text{Cov}[x, y] + \mathbb{E}[x] \\
\mathbb{E}[R_n]\,\mathbb{E}[m] &= 1 - \text{Cov}[m, R_n] && \text{. . . rearranging terms} \\
\mathbb{E}[R_n] &= \frac{1}{\mathbb{E}[m]} - \frac{1}{\mathbb{E}[m]}\,\text{Cov}[m, R_n] && \text{. . . dividing by } \mathbb{E}[m].
\end{aligned}$$

Road map

SDF and the risk-free rate . . . I

- We start with our result on the previous slide:

$$\mathbb{E}[R_n] = \frac{1}{\mathbb{E}[m]} - \frac{1}{\mathbb{E}[m]}\,\mathrm{Cov}[m, R_n].$$

- Now consider the special case of the risk-free asset, so that
  - $R_n = R_f$, and $\mathrm{Cov}[m, R_f] = 0$,
  - which, using the equation above, then implies that

$$\mathbb{E}[R_f] = R_f = \frac{1}{\mathbb{E}[m]}$$ . . . a useful result that will appear often.

Road map

SDF, risk premia, and covariance . . . I

- Substituting $R_f = \frac{1}{\mathbb{E}[m]}$ in the result

$$\mathbb{E}[R_i] = \frac{1}{\mathbb{E}[m]} - \frac{1}{\mathbb{E}[m]} \operatorname{Cov}[m, R_n]$$

leads to

$$\mathbb{E}[R_n] = R_f - R_f \operatorname{Cov}[m, R_n]$$

which, upon rearranging terms, gives

$$\mathbb{E}[R_n] - R_f = -R_f \operatorname{Cov}[m, R_n] \quad \text{. . . useful result.} \tag{1}$$

SDF, risk premia, and covariance . . . II

- The equation

$$\mathbb{E}[R_n] - R_f = -R_f \, \text{Cov}[m, R_n]$$

says that the risk-premium on asset n over R_f, is negatively related to the covariance of R_n with the SDF, m.

- The reason for the negative relation between the risk-premium and the covariance of R_n with the SDF m is that
  - when wealth decreases, SDF increases (SDF is like marginal utility);
  - Thus, assets that pay off in states of the world where wealth is low (and SDF is high),
  - need to pay only a small risk-premium (because they have a higher payoff in states where we value this payoff much more).

Road map

Single-factor beta pricing model

- A single-factor beta-pricing model with factor f is said to exist if there are constants R_z and λ such that for each return R_n

$$\mathbb{E}[R_n] = R_z + \lambda \underbrace{\frac{\mathrm{Cov}[f, R_n]}{\mathbb{V}[f]}}_{beta} .$$

- Note that if a risk-free asset exists, then $\mathrm{Cov}[f, R_f] = 0$ and so setting $R_n = R_f$ in the equation above, gives us $R_z = R_f$.

- More generally, R_z (where the "z" stands for "*zero* beta") is the expected value of the return satisfying $\mathrm{Cov}[f, R_z] = 0$.

- In the absence of a risk-free asset, different beta-pricing models can have different expected zero-beta returns (that is, each model may imply a different return for the asset with a beta of zero).

Multifactor beta pricing model

- It is straightforward to extend the single-factor result to a multi-factor setting.
- A multi-factor beta-pricing model with factors $F = (f_1, f_2, \ldots, f_K)^\top$ is said to exist if there are constants R_z and $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_K)^\top$ such that for each return R_n

$$\underbrace{\mathbb{E}[R_n]}_{1\times 1} = \underbrace{R_z}_{1\times 1} + \underbrace{\lambda^\top}_{1\times K} \overbrace{\underbrace{V_F^{-1}}_{K\times K} \underbrace{\mathrm{Cov}[F, R_n]}_{K\times 1}}^{\text{betas}},$$

where

- V_F is the $K \times K$ invertible covariance matrix of the vector F;
- $\mathrm{Cov}[F, R_n]$ is a $K \times 1$ column vector with elements $\mathrm{Cov}[f, R_n]$;
- $V_F^{-1}\,\mathrm{Cov}[F, R_n]$ is the vector of multiple regression betas of R_n on F;
- λ are the $K \times 1$ factor risk premia.

Beta-Pricing Models in Terms of Covariances . . . I

- One can always express beta-pricing models in terms of covariances instead of betas:
- For instance, we can write the single-factor beta model

$$\mathbb{E}[R_n] = R_z + \lambda \underbrace{\frac{\text{Cov}[f, R_n]}{\mathbb{V}[f]}}_{beta}$$

in terms of a covariance

$$\mathbb{E}[R_n] = R_z + \psi \, \text{Cov}[f, R_n], \quad \text{where } \psi = \frac{\lambda}{\mathbb{V}[f]}.$$

Beta-Pricing Models in Terms of Covariances . . . II

- Similarly, we can write the multifactor beta model

$$\mathbb{E}[R_n] = R_z + \lambda^\top V_F^{-1} \operatorname{Cov}[F, R_n],$$

in terms of covariances instead of betas

$$\mathbb{E}[R_n] = R_z + \psi^\top \operatorname{Cov}[F, R_n], \quad \text{where } \psi = V_F^{-1}\lambda.$$

Number of factors is not uniquely determined

- It is important to realize that the number of factors in a beta-pricing model is not uniquely determined.
- For example, given a K-factor model, one can always use $\lambda^\top V_F^{-1} F$ as a single factor.
- One can also use the SDF m as the single systematic factor, provided $\mathbb{E}[m] \neq 0$, as we showed earlier, and show again below:

$$\mathbb{E}[R_n] = \frac{1}{\mathbb{E}[m]} - \frac{1}{\mathbb{E}[m]} \operatorname{Cov}[m, R_n]$$

with $1/\mathbb{E}[m] = R_f$ if there exists a risk-free asset.

Single-factor Models with *returns* as factors . . . I

- In general, a factor need not be a return.
  - For example, macro-factors, such as inflation, are not returns.
- But some factors may be returns; for example, the return on the market factor.
- If a factor is a return, then its factor risk premium is its ordinary risk premium, treating R_z as a proxy for the risk-free return.

Single-factor Models with *returns* as factors . . . II

- To see this, suppose that there is a single-factor beta-pricing model with the factor being the return R_*. Then,

$$\mathbb{E}[R_n] = R_z + \lambda \frac{\text{Cov}[R_*, R_n]}{\mathbb{V}[R_*]}.$$

- But, the above equation must be true for $R_n = R_*$

$$\begin{aligned}
\mathbb{E}[R_*] &= R_z + \lambda \frac{\text{Cov}[R_*, R_*]}{\mathbb{V}[R_*]} && \text{. . . the general result from earlier} \\
&= R_z + \lambda \frac{\mathbb{V}[R_*]}{\mathbb{V}[R_*]} && \text{. . . because } \text{Cov}[R_*, R_*] = \mathbb{V}[R_*] \\
&= R_z + \lambda, && \text{. . . } \mathbb{V}[R_*]/\mathbb{V}[R_*] = 1
\end{aligned}$$

and so, rearranging terms, we get that

$$\lambda = \mathbb{E}[R_*] - R_z.$$

Beta-pricing model with a return as a single factor

- Substituting $\lambda = \mathbb{E}[R_*] - R_z$ into our earlier result

$$\mathbb{E}[R_n] = R_z + \lambda \frac{\text{Cov}[R_*, R_n]}{\mathbb{V}[R_*]},$$

leads to the following new result:

$$\mathbb{E}[R_n] = R_z + (\mathbb{E}[R_*] - R_z) \frac{\text{Cov}[R_*, R_n]}{\mathbb{V}[R_*]}.$$

- Thus, there is a beta-pricing model with a return as a single factor if and only if
  - the return is on mean-variance frontier and
  - is not equal to the risk-free rate, if one exists, and
  - otherwise not equal to the global minimum-variance portfolio (GMV).

The CAPM

- The Capital Asset Pricing Model (CAPM) states that the result on the previous slide, reproduced below

$$\mathbb{E}[R_n] = R_z + \lambda \frac{\text{Cov}[R_*, R_n]}{\mathbb{V}[R_*]},$$

is true if the factor is the return on the market portfolio, $R_* = R_{mkt}$:

$$\mathbb{E}[R_n] = R_z + (\mathbb{E}[R_{mkt}] - R_z) \frac{\text{Cov}[R_{mkt}, R_n]}{\mathbb{V}[R_{mkt}]},$$

which is equivalent to

  - the market portfolio being on the mean-variance frontier, and
  - not equal to the risk-free rate, if one exists, and
  - otherwise not equal to global minimum-variance portfolio.

Road map

SDF and beta pricing

- There is a beta-pricing model with respect to some factors with the expected zero-beta return being non-zero
- if and only if there is an SDF m such that:
  1. $\mathbb{E}[m] \neq 0$; and,
  2. m is an affine function of the factors: $m = a + b\,f$.
- This is true for both single- and multi-factor models.
- In a single-factor model such as

$$\mathbb{E}[R_n] = R_z + \psi \text{ Cov}[f, R_n], \quad \text{where: } \psi = \frac{\lambda}{\mathbb{V}[f]} \text{ and } R_z \neq 0.$$

- the SDF is given by:

$$m = \frac{1}{R_z} - \frac{1}{R_z}\psi\,(f - \mathbb{E}[f]).$$

How to prove that m is a valid SDF? ... I

- Suppose that there is a one-factor beta-pricing model

$$\mathbb{E}[R_n] = R_z + \psi \ \mathrm{Cov}[f, R_n].$$

- We now wish to show that the following is a valid SDF:

$$m = \frac{1}{R_z} - \frac{1}{R_z}\psi \left(f - \mathbb{E}[f]\right).$$

Proof of being a valid SDF . . . I

- To show that $m = \frac{1}{R_z} - \frac{1}{R_z}\psi\left(f - \mathbb{E}[f]\right)$ is a valid SDF,
  - we need to show that $\mathbb{E}[m\,R_n] = 1$.
- To prove this, start by multiplying m by R_n:

$$m\,R_n = \frac{R_n}{R_z} - \frac{1}{R_z}\left(\psi\,f\,R_n - \mathbb{E}[\psi\,f]R_n\right).$$

- Now take expectations;

$$\mathbb{E}[m\,R_n] = \frac{\mathbb{E}[R_n]}{R_z} - \frac{1}{R_z}\Big(\mathbb{E}[\psi\,f\,R_n] - \mathbb{E}[\psi\,f]\mathbb{E}[R_n]\Big).$$

- On the next page, we simplify this expression.

Proof of being a valid SDF ... II

- We write the expression from the previous page and then simplify it:

$$\mathbb{E}[m\,R_n] = \frac{\mathbb{E}[R_n]}{R_z} - \frac{1}{R_z}\Big(\mathbb{E}[\underbrace{\psi\, f}_{x}\ \underbrace{R_n}_{y}] - \mathbb{E}[\underbrace{\psi\, f}_{x}]\mathbb{E}[\underbrace{R_n}_{y}]\Big)$$

$$= \frac{\mathbb{E}[R_n]}{R_z} - \frac{1}{R_z}\,\mathrm{Cov}[\psi\, f, R_n] \qquad \ldots \text{because } \mathrm{Cov}[x,y] = \mathbb{E}[x\,y] - \mathbb{E}[x]\,\mathbb{E}[y]$$

$$= \frac{1}{R_z}\left(\mathbb{E}[R_n] - \psi\ \mathrm{Cov}[f, R_n]\right) \quad \ldots \text{because } \psi \text{ is a constant}$$

$$= \frac{1}{R_z}\left(R_z\right) \qquad \ldots \text{from } \mathbb{E}[R_n] = R_z + \psi\ \mathrm{Cov}[f, R_n]$$

$$= 1.$$

- So, we have proved that indeed $\mathbb{E}[m\,R_n] = 1$.

Road map

What is the SDF in the CAPM?

- If the CAPM is true, then

$$f = R_{mkt} \qquad \text{and} \qquad \psi = \frac{\mathbb{E}[R_{mkt}] - R_z}{\mathbb{V}[R_{mkt}]}.$$

- Substituting the above expression into our general result, which is reproduced below

$$m = \frac{1}{R_z} - \frac{1}{R_z}\psi\left(f - \mathbb{E}[f]\right),$$

- gives us the following result for the SDF in the CAPM:

$$m = \frac{1}{R_z} - \frac{1}{R_z}\underbrace{\left(\frac{\mathbb{E}[R_{mkt}] - R_z}{\mathbb{V}[R_{mkt}]}\right)}_{\psi}\left(\underbrace{R_{mkt}}_{f} - \mathbb{E}[\underbrace{R_{mkt}}_{f}]\right).$$

Road map

Mean-variance portfolio weights in terms of the SDF

- We now show how the mean-variance portfolio weights are related to the SDF.
- Denote by R^e the vector of excess returns on N assets (i.e., in excess of the risk-free rate)
- Recall our condition for mean-variance optimal portfolio weights:

$$w = \frac{1}{\gamma} V_{R^e}^{-1} \mathbb{E}[R^e].$$

- Recall our result in Equation (1) on page 26:

$$\mathbb{E}[R^e] = -R_f \operatorname{Cov}[m, R^e] \approx \operatorname{Cov}[m, R^e].$$

- Substituting $\mathbb{E}[R^e]$ into the expression for the optimal weights gives

$$w = (-)\frac{1}{\gamma} V_{R^e}^{-1} \operatorname{Cov}[m, R^e] \quad \ldots \text{mean-variance weights in terms of } m.$$

SDF in terms of mean-variance portfolio returns

- Conversely, the SDF can be expressed in terms of the return on the mean-variance optimal portfolio, R^e_{MV}; i.e.,

$$m = 1 - \gamma R^e_{MV} = 1 - \gamma(w^\top R^e), \quad \text{where} \quad w = \frac{1}{\gamma} V_{R^e}^{-1} \mathbb{E}[R^e]$$

and $R^e_{MV} = (w^\top R^e)$ is the excess return on the MV optimal portfolio.

- **Proof:**

$$\begin{aligned}
\text{Cov}[m, R^e] &= \text{Cov}[1 - \gamma(w^\top R^e), R^e] && \text{. . . substituting the definition of } m \\
&= -\gamma \,\text{Cov}[(w^\top R^e), R^e] && \text{. . . moving } \gamma \text{ out of cov operator} \\
&= -\gamma w \,\text{Cov}[R^e, R^e] && \text{. . . moving } w \text{ out of cov operator} \\
&= -\gamma w \mathbb{V}[R^e] && \text{. . . } \text{Cov}[x, x] = \mathbb{V}[x],
\end{aligned}$$

which, upon rearranging (i.e., multiplying both sides by $-\gamma^{-1}(\mathbb{V}[R^e])^{-1}$), gives

$$w = (-)\frac{1}{\gamma} V_{R^e}^{-1} \,\text{Cov}[m, R^e].$$

This brings us to an end to Part 1 of this chapter on understanding the SDF.

We now apply this knowledge to characterize the SDF empirically.

Part 2 of this chapter

Use SDF to show that expected returns include compensation for unsystematic risk.

This part of the chapter is based on the paper:

Dello-Preite, M., R. Uppal, P. Zaffaroni, and I. Zviadadze. 2024. Cross-sectional asset pricing with unsystematic risk. Available at SSRN 4135146 and can be downloaded using this link.

Start of focus

Road map

The SDF with systematic and unsystematic risk

- In this part of the chapter, we study the drivers of the SDF.
- In particular,
  - we study which systematic risk factors drive the SDF;
  - whether unsystematic risk is an important driver of the SDF.

Road map

Objective and Motivation

- Major challenge in finance is to price the cross-section of stock returns.
  - That is, to explain why stocks differ in their expected returns?
- The answer to this question is essential for
  - Portfolio-investment: optimal portfolio design;
  - Risk-management: how to adjust returns for risk.
  - Capital-budgeting: cost of capital;

The world of Sharpe (1964) . . . perfect diversification

- In the world of **frictionless markets** assumed by the CAPM of Sharpe (1964),
  - investors hold a perfectly diversified portfolio of risky assets,
  - which, in equilibrium, is the market portfolio.
  - Thus, the only source of risk is the systematic risk of the market.

In the real world . . . less than perfect diversification

- In the data, however,
  - investors do not hold diversified portfolios, and
  - risk compensation for asset exposures to the market factor, performs poorly in explaining the cross-section of expected stock returns.

Post-CAPM . . . factor zoo

- When the CAPM performed poorly in the cross-section, researchers
  - continued to assume that investors hold perfectly diversified portfolios,
  - empirically examined a large number of alternative proxies for systematic risk,
  - leading to the **factor zoo** (Cochrane 2011).

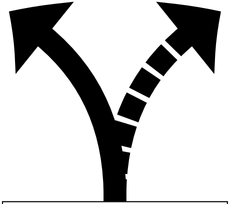
Status of models with systematic risk factors

- Of the more than four hundred systematic risk factors from the factor zoo, none explains the cross-section of stock returns.
- Virtually all models—more than 2000 trillion—featuring factors from this zoo have sizable pricing errors (Bryzgalova, Huang, and Julliard 2023).
- Bottom line: Existing factor models cannot explain differences in expected stock returns.

Which road to take?

- **Assume: fully diversified portfolios**
- **Search for: systematic risk factors?**

- **Assume: under-diversified portfolios**
- **Allow for: priced unsystematic risk?**



Failure of CAPM

What is unsystematic risk?

- Our research focuses on a single question:

Q. Is unsystematic risk priced (i.e., rewarded)?

Unsystematic risk = **Total risk** − **Systematic risk**

- What is unsystematic risk?
- Unsystematic risk can represent
  - Asset-specific risk (i.e., risk specific to a particular stock).
  - Weak factors (i.e., factors that explain returns of only a small number of assets).

The world of Merton (1987) . . . imperfect diversification

- Merton (1987) relaxes the assumption of frictionless markets and derives an equilibrium in which investors hold underdiversified portfolios.
  - consistent with a large body of empirical evidence.
- If investors hold underdiversified portfolios, they will demand compensation for bearing unsystematic risk.
- Building on Merton (1987), we study how compensation for unsystematic risk
  - provides an avenue for explaining the cross-section of expected asset returns and
  - resolving the factor zoo.

Our work is founded on the Arbitrage Pricing Theory (APT)

- Instead of specifying a particular equilibrium model like Merton (1987),
- we use instead the Arbitrage Pricing Theory (APT) of Ross (1976, 1977)
- to explore the possibility that unsystematic risk is compensated (priced),
- So we call our model "Priced Unsystematic Risk (PUR) model."

Benefits of using the APT

- There are several benefits to using the APT:
  - It is agnostic about the relevant systematic risk factors, which are latent.
  - It allows risk premium to deviate from reward for bearing only systematic risk.

Total expected excess returns	=	Expected return for systematic risk	+	Expected return for unsystematic risk

Arbitrage Pricing Theory (APT) ... our starting point

- APT makes only two assumptions:
  1. Gross unexpected returns are given by a K latent linear factor model

     $$R_{t+1} - \mathbb{E}(R_{t+1}) = \beta\big(f_{t+1} - \mathbb{E}(f_{t+1})\big) + e_{t+1},$$

     - f_{t+1} is $K \times 1$ vector of common (latent) risk factors with risk premia λ and covariance matrix V_f;
     - $\beta = (\beta_1, \beta_2, \dots, \beta_N)'$ is the $N \times K$ full-rank matrix of loadings;
     - e_{t+1} is vector of unsystematic shocks with zero mean and $N \times N$ covariance matrix $V_e > 0$.
  2. Asymptotic no-arbitrage: as N increases, it is not possible to have a portfolio whose risk goes to zero, but its return is strictly positive.

From the two assumptions of the APT, we get . . .

- Expected excess returns are:

Expected excess returns	=	Expected return for systematic risk	+	Expected return for unsystematic risk
$\mathbb{E}(R^e_{t+1})$	$=$	$\beta\lambda$	$+$	α

  with the vector α satisfying the asymptotic no-arbitrage restriction: as $N \to \infty$:

$$\underbrace{\alpha' V_e^{-1} \alpha}_{(\mathsf{SR}^\alpha)^2} \le \delta_{\mathsf{apt}}{}^2 < \infty$$

- By construction, unsystematic risk is *orthogonal* (uncorrelated) to systematic risk.

Road map

Our estimation approach

- Our estimation approach is . . . simple and exhaustive.

Model selection and parameter estimation

- To estimate the data-generating process of asset returns with K latent factors

$$R_{t+1} - \mathbb{E}(R_{t+1}) = \beta(f_{t+1} - \mathbb{E}(f_{t+1})) + e_{t+1} \tag{2}$$

$$\text{subject to: } \alpha' V_e^{-1} \alpha \leq \delta_{\text{apt}}^2, \tag{3}$$

- we need to determine only two parameters:
  - K = number of systematic (latent) risk factors;
  - δ_{apt} = magnitude of the Sharpe ratio of the alpha portfolio.

Our estimation approach

- Estimate K using
  - Statistical methods of factor analysis:
    1. Ahn and Horenstein (2013),
    2. Onatski (2012), and
    3. Scree plots
  - Cross-validation (CV), to avoid overfitting
- Estimate δ_{apt} by five-fold cross-validation with four performance metrics
  1. Hansen-Jagannathan distance (HJ) ... Financial economist
  2. Root-mean-square error (RMSE) ... Statistician
  3. Sharpe ratio (SR) ... Asset manager
  4. Generalized-least-square (R2GLS) ... Econometrician

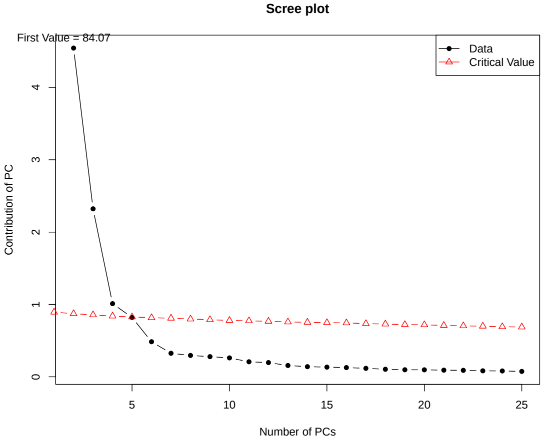
Road map

Data

- Basis assets: 303 characteristic-based double-sorted portfolios with monthly returns from 1963:07 to 2024:12.
  - 25 portfolios each sorted by Size and: Book-to-Market, Operating profitability, Investment, Momentum, Short-term reversal, Long-term reversal, Accruals, Market beta, Variance or Residual variance, (for a total of 25 x 10)
  - 35 portfolios sorted by Size and: Net Share Issues
  - 6 portfolios sorted by Size and: Earnings/Price, Cashflow/Price, Dividend yield
- We also consider several other datasets (including, in a companion paper, data on individual stock returns from the S&P 500).

Road map

Number of systematic (latent) risk factors: $K = \{1, 3, 4\}$



- Ahn and Horenstein: 1 factor
- Onatski: 3 factors
- Scree plot: 4 factors

How to find if unsystematic risk is compensated?

- Q. Do expected excess returns include compensation for unsystematic risk?
- Compare performance of two models across four metrics
  1. PCA model that includes only common (latent) risk factors (i.e., $\delta_{\text{apt}} = 0$)
  2. Our model (DUZZ) that includes
     - the same risk factors as the PCA model
     - and compensation for unsystematic risk: $\delta^2_{\text{apt}} = \underbrace{\alpha' V_e^{-1} \alpha}_{(\text{SR}^\alpha)^2} > 0$

Is $\delta_{\text{apt}} = (\text{SR}^{\alpha}) > 0$ when K fixed by statistical criteria? Yes

- Fix number of factors K as per statistical criteria: $K = \{1, 3, 4\}$
- Look for optimal δ_{apt} in cross-validation using the four metrics

		Metric for estimating δ_{apt} and evaluating performance							
		HJ_{reg}		$\text{RMSE}\times\sqrt{N}\times 100$		SR		R^2_{gls}	
Model (1)	K (2)	δ_{apt} (3)	Value (4)	δ_{apt} (5)	Value (6)	δ_{apt} (7)	Value (8)	δ_{apt} (9)	Value (10)
Panel A: When K is estimated by ER									
PUR-ER	1	0.20	**0.67**	0.80	**7.11**	0.63	**0.57**	0.72	**0.21**
PCA-ER	1	0.00	0.78	0.00	7.95	0.00	0.14	0.00	−0.16
Panel B: When K is estimated by GR									
PUR-GR	1	0.20	**0.67**	0.80	**7.11**	0.63	**0.57**	0.72	**0.21**
PCA-GR	1	0.00	0.78	0.00	7.95	0.00	0.14	0.00	−0.16
Panel C: When K is estimated by Onatski									
PUR-Onatski	3	0.16	**0.71**	0.89	**7.15**	0.88	**0.55**	0.73	**0.21**
PCA-Onatski	3	0.00	0.78	0.00	7.57	0.00	0.23	0.00	0.00
Panel D: When K is estimated by scree plots									
PUR-scree plot	4	0.15	**0.70**	0.85	**7.15**	0.54	**0.55**	0.71	**0.19**
PCA-scree plot	4	0.00	0.77	0.00	7.46	0.00	0.27	0.00	0.04

Is $\delta_{\text{apt}} > 0$ if we choose both K and δ_{apt} in cross-validation? "Yes"

	Metric for estimating K, δ_{apt}, and K^{pc} and for evaluating performance											
	HJ_{reg}			RMSE$\times$ $\sqrt{N}\times 100$			SR			R^2_{gls}		
Model (1)	K, K^{pc} (2)	δ_{apt} (3)	Value (4)	K, K^{pc} (5)	δ_{apt} (6)	Value (7)	K, K^{pc} (8)	δ_{apt} (9)	Value (10)	K, K^{pc} (11)	δ_{apt} (12)	Value (13)
PUR	1	0.20	**0.67**	1	0.80	**7.11**	1	0.63	0.57	1	0.72	**0.21**
PCA	4	0.00	0.77	153	0.00	7.21	123	0.00	**0.58**	51	0.00	0.13

- Performance of PCA model improves with larger K;
- Recall, however, that number of latent risk factors is at most $K = 4$;
- So, remaining risk factors represent weak factors (unsystematic risk).

Is $\delta_{\text{apt}} > 0$ if we define "systematic factor" more generally?

1. Lettau and Pelger (2020) define a factor to be systematic if
   - large contribution to the covariance of returns
   - or large contribution to cross-sectional variation of *expected returns*
2. Kozak, Nagel, and Santosh (2020) define a factor to be systematic if
   - large contribution to the covariance of returns
   - and high price of risk (i.e., high expected returns)

Is $\delta_{\text{apt}} > 0$ if we define "systematic factor" differently? Yes

- Each model was estimated as recommended in the original paper.

		Metric for estimating $\delta_{\text{apt}}/\kappa/\gamma$ and evaluating performance							
		HJ_{reg}		RMSE$\times\sqrt{N}\times 100$		SR		R^2_{gls}	
Model (1)	K (2)	$\delta_{\text{apt}}/\kappa/\gamma$ (3)	Value (4)	$\delta_{\text{apt}}/\kappa/\gamma$ (5)	Value (6)	$\delta_{\text{apt}}/\kappa/\gamma$ (7)	Value (8)	$\delta_{\text{apt}}/\kappa/\gamma$ (9)	Value (10)
Panel A: PUR models with K estimated by statistical criteria									
PUR-ER	1	0.20	**0.67**	0.80	**7.11**	0.63	**0.57**	0.72	**0.21**
PUR-GR	1	0.20	**0.67**	0.80	**7.11**	0.63	**0.57**	0.72	**0.21**
PUR-Onatski	3	0.16	0.71	0.89	7.15	0.88	0.55	0.73	0.21
PUR-scree plot	4	0.15	0.70	0.85	7.15	0.54	0.55	0.71	0.19
Panel B: Traditional models with observable risk factors									
CAPM	1	—	0.79	—	8.58	—	0.14	—	−0.12
FF3	3	—	0.79	—	8.01	—	0.19	—	−0.11
FFC	4	—	0.78	—	7.78	—	0.29	—	0.00
FF5	5	—	0.80	—	7.54	—	0.30	—	0.03
FF6	6	—	0.85	—	7.45	—	0.27	—	0.06
Panel C: State-of-the-art models with a more general definition of systematic risk									
KNS	6	1.18	0.76	16.00	7.36	16.00	0.28	5.23	0.09
LP	5	10.00	0.95	10.00	7.19	10.00	0.33	10.00	0.15
LP	5	20.00	0.97	20.00	7.19	20.00	0.35	20.00	0.14

Road map

Out-of-sample analysis using six subperiods

- Divide the entire sample into six periods, approximately ten years each (with 123 monthly observations).
- Then, estimate the models on one ten-year window.
- Evaluate it on the next ten-year window.

Is $\delta_{apt} > 0$ even out-of-sample? Yes

Model	Metric for estimating and evaluating models			
	HJ_{reg}	RMSE$\times \sqrt{N} \times 100$	SR	R^2_{gls}
(1)	(2)	(3)	(4)	(5)
Panel A: When K is estimated by ER, and δ_{apt} by CV with each metric				
PUR	**0.40**	**2.76**	**0.51**	**0.73**
PCA	0.55	5.02	0.14	-0.11
Panel B: When K is estimated by GR, and δ_{apt} by CV with each metric				
PUR	**0.42**	**2.23**	**0.51**	**0.76**
PCA	0.55	3.95	0.15	0.02
Panel C: When K is estimated by Onatski, and δ_{apt} by CV with each metric				
PUR	**0.46**	**2.28**	**0.50**	**0.73**
PCA	0.54	3.81	0.18	0.03
Panel D: When K is estimated by scree plot, and δ_{apt} by CV with each metric				
PUR	**0.45**	**2.29**	**0.52**	**0.74**
PCA	0.57	4.15	−0.04	-0.03
Panel E: When K and δ_{apt} for PUR model and K^{pc} for PCA model are estimated by CV				
PUR	**0.40**	**2.70**	**0.46**	**0.69**
PCA	0.55	3.11	−0.24	0.45

Results of other out-of-sample are even stronger.

Is unsystematic risk persistent over time? Yes

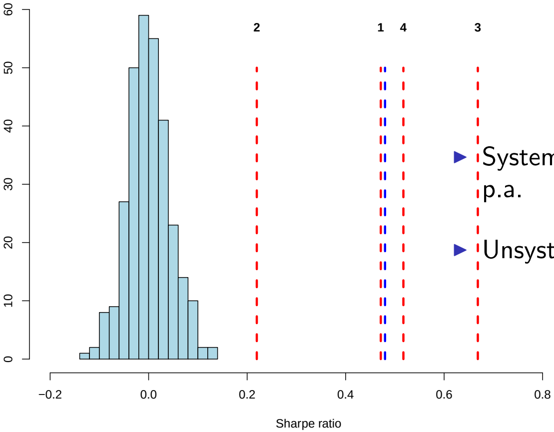
Correlations of unsystematic risk across subperiods

	1973–1984	1984–1994	1994–2004	2004–2014	2014–2024
Panel: K is chosen by Ahn-Horenstein					
1973–1984	1.00	0.62	0.56	0.34	0.31
1984–1994		1.00	0.24	0.26	0.07
1994–2004			1.00	0.30	0.68
2004–2014				1.00	0.09
2014–2024					1.00

Road map

Is priced unsystematic risk important?

- To be conservative, look at the case of $K = 4$ and use the HJ metric.



- Systematic SR ($K = 4$) is 0.99 p.a.
- Unsystematic SR is 0.48 p.a.

Properties of unsystematic component of present-value operator (SDF)

- Small stocks contribute most to the unsystematic SDF component.
  - The most prominent portfolios are those sorted by Momentum, Short-term reversal, Variance, Residual FF3 variance, and Investment.
- Variation of the unsystematic SDF component,
  - Short-Term Reversal Factor explains less than 5%,
  - Investment factor explains less than 6%,
  - Idiosyncratic-volatility factor of Ang et. al (2006) explains less than 8%, and
  - Momentum factor explains less than 12%.

Properties of systematic component of present-value operator (SDF)

- Market factor explains most of the variation (88%) of the first PC, PC_1.
- Size factor of FF explains most of the variation in PC_2;
- Value and Profitability factors of FF explain most of the variation in PC_3; and,
- Momentum factor explains most of the variation in PC_4.

Road map

Conclusion

- Developed a method to examine potential significance of unsystematic risk.
- Key result: establish quantitative importance of priced unsystematic risk
- Our finding has important implications for any question involving the risk-return tradeoff; particularly, portfolio selection, asset allocation, and risk management.

This brings us to the end to Part 2 of this chapter on the importance in the cross-section of stock returns of not just compensation for systematic risk but also unsystematic risk.

Part 3 of this chapter

Exploiting unsystematic risk for portfolio construction.

This part of the chapter is based on the paper:

Raponi, V., R. Uppal, and P. Zaffaroni. 2023. Robust portfolio choice. Working Paper, SSRN eLibrary which can be downloaded using this link.

Road map

Portfolio construction with systematic & unsystematic risk

- The optimal mean-variance portfolio with unsystematic risk is

$$w^{\mathrm{mv}} = \phi^\alpha\, w^\alpha + \phi^\beta\, w^\beta, \quad \text{where}$$

$\phi^\alpha = \frac{\gamma^\alpha}{\gamma}$, $\phi^\beta = \frac{\gamma^\beta}{\gamma}$, w^α and w^β are orthogonal to each other.

- The beta portfolio has the usual mean-variance form, but where $\beta\lambda$ is only the systematic component of returns:

$$w^\beta = \frac{1}{\gamma^\beta} V^{-1}(\beta\lambda)$$

- The alpha portfolio also has the usual mean-variance form, but where α is the unsystematic component of returns:

$$w^\alpha = \frac{1}{\gamma^\alpha}\Sigma^+\alpha + \mathcal{O}(1), \quad \text{where: } \Sigma^+ = \left[\Sigma^{-1} - \Sigma^{-1}\beta(\beta^\top\Sigma^{-1}\beta)^{-1}\beta^\top\Sigma^{-1}\right].$$

1. Two inefficient funds span the efficient frontier

- Under the APT, the entire set of mean-variance efficient portfolios is generated by two inefficient portfolios:
  - "beta" portfolio: depends on observed factor risk premia;
  - "alpha" portfolio: depends on unsystematic risk.

  $$(\mathrm{SR}^{\beta})^2 + (\mathrm{SR}^{\alpha})^2 \to (\mathrm{SR}^{mv})^2.$$
- If one ignores alpha portfolio,s resulting portfolio will not be efficient, regardless of how many risk factors are included in the model.

2. Alpha portfolio weights dominate beta portfolio weights

- As N increases, elements of the alpha portfolio dominate, in terms of magnitude, the corresponding elements of the beta portfolio.

$$\frac{w_i^\beta}{w_i^\alpha} \to 0 \quad \text{if} \quad a \neq 0_N.$$

- This is a consequence of: $w_i^\beta = O\left(\frac{1}{N}\right)$ but $w_i^\alpha = O\left(\frac{1}{\sqrt{N}}\right)$.

3. No need to estimate factor moments

- For beta portfolio: identify conditions under which, as N increases,
  - beta portfolio can be replaced, without any loss of performance,
  - by a benchmark portfolio (e.g., equal- or value-weighted portfolio),
  - which is functionally independent, and hence immune to misspecification, in
    - mean vector (risk premia) of observed factors and
    - covariance matrix of these factors' returns.

$$\boxed{(\mathrm{SR}^{\text{bench}})^2} + (\mathrm{SR}^{\alpha})^2 \to (\mathrm{SR}^{mv})^2 \tag{4}$$

instead of:

$$(\mathrm{SR}^{\beta})^2 + (\mathrm{SR}^{\alpha})^2 \to (\mathrm{SR}^{mv})^2. \tag{5}$$

Empirical results
Evaluate out-of-sample performance

Data: Four datasets

- Datasets of monthly stock returns: two emprical, two simulated.
- For empirical datasets, follow closely Ao, Li, and Zheng (2019).
  - Monthly returns from 1977 to 2016.
    1. $N = 30$ stocks comprising the Dow Jones that month.
    2. $N = 100$ randomly selected stocks from S&P 500 that month.
- For simulated returns, match the empirical datasets:
  - Monthly returns
    1. $N = 30$ stocks;
    2. $N = 100$ stocks.
- In all cases, returns augmented by Fama-French $K = 3$ factors.

Design of experiment

- Compare out-of-sample performance of various portfolios that have a target volatility of 5% per month, as in Ao, Li, and Zheng (2019).
  - Estimate APT model at date t
    - Employ pseudo-MLE (maximum-likelihood-estimation) with the no-arbitrage constraint.
    - Use cross-validation to identify when no-arbitrage constraint binding.
    - Use a rolling window of 120 months
  - Compute portfolio weight at date t.
  - Evaluate out-of-sample portfolio return at date $t+1$.
  - And, so on ...

Use five reference portfolios from existing literature

- Compare performance of our strategies, with five reference portfolios:

1. MV: Mean-variance efficient portfolio, using sample return moments.
2. GMV-LW: Global minimum-variance portfolio, with covariance matrix estimated using shrinkage (Ledoit and Wolf 2003) .
3. PCA_n, with $n = \{1, 2, \ldots, 10\}$: Treat PCs as observed factors.
4. EW: Equally weighted portfolio (DeMiguel, Garlappi, and Uppal 2009).
5. MAXSER (Ao, Li, and Zheng 2019)

⋆ DeMiguel, Garlappi, and Uppal (2009) and Ao, Li, and Zheng (2019) show, respectively, that the EW and MAXSER portfolios outperform fourteen other strategies proposed in the literature.

Our portfolio strategies: Robust-Mean-Variance (RMV)

1. RMV using V (covariance matrix of N stock returns)
2. RMV using Ω (covariance matrix of K factor returns), because beta portfolio return from investing in N assets is equal to that from investing in only the $K = 3$ Fama-French factors.
3. RMV using V: OptComb combines optimally alpha and beta portfolios, recognizing that with finite N the two are not necessarily orthogonal.
4. RMV using Ω: OptComb combines optimally alpha and beta-equivalent portfolios using only $K = 3$ Fama-French factors.

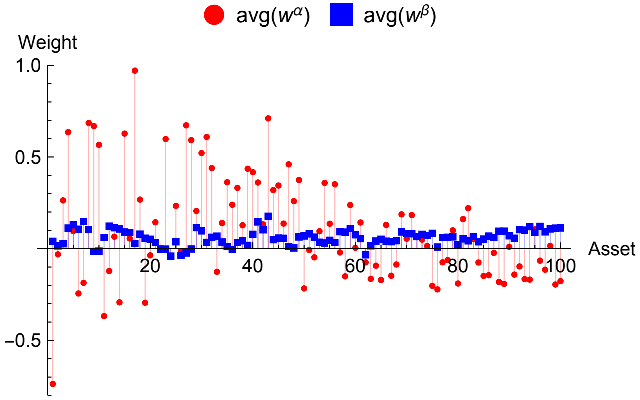
⋆ Because returns may not be normal, compute t-statistic for difference in Sharpe ratios using heteroskedasticity and autocorrelation robust (HAC) kernel estimation approach (Ledoit and Wolf 2008).

- Display results only for S&P stocks ($N = 100$)
- Results for other three datasets are similar

Out-of-sample performance: For S&P stock returns

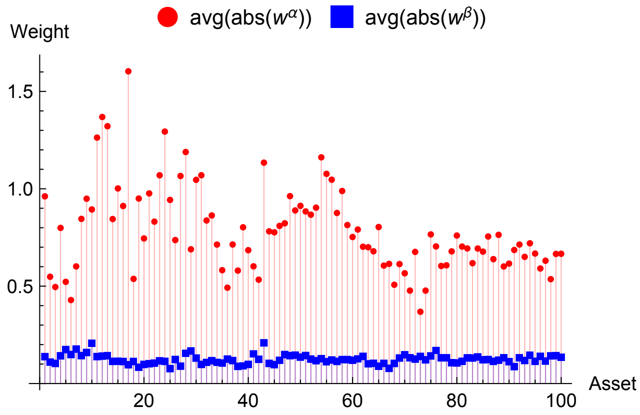
	Mean p.a.	SR p.a.	$\frac{(SR_k - SR_{EW})}{SR_{EW}}$	$\frac{(SR_k - SR_{MAXS})}{SR_{MAXS}}$	t-stat for diff in SR wrt EW	t-stat for diff in SR wrt MAXSER
MV	−0.024	− 0.190	−1.385	−1.283	−2.052	−3.112
GMV-LW	0.019	0.146	−0.704	−0.782	−1.176	−1.816
PCA2	−0.001	−0.001	−1.003	−1.002	−2.206	−2.985
PCA3	0.056	0.334	−0.324	−0.502	−0.712	−1.497
PCA4	0.075	0.460	−0.068	−0.314	−0.150	−0.937
PCA10	0.033	0.203	−0.587	−0.696	−1.292	−2.075
EW	0.070	0.494	0.000	−0.265	—	−0.495
MAXSER	0.094	0.672	0.360	0.000	0.495	—
RMV using V	0.116	0.763	0.546	0.137	0.703	0.406
RMV using Ω	0.137	1.016	1.055	0.511	1.959	1.623
RMV using V: OptComb	0.114	0.642	0.298	−0.046	0.952	0.875
RMV using Ω: OptComb	0.206	1.222	1.472	0.818	2.169	2.203

Properties of alpha and beta portfolio weights



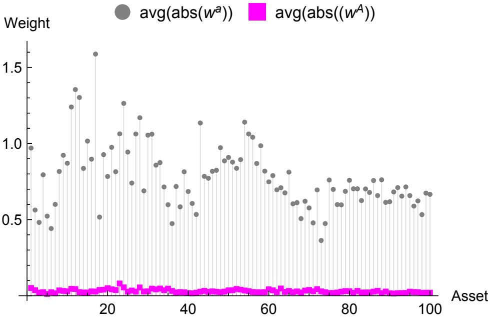
- Weights of beta portfolio are small and mostly positive
- Weights of alpha portfolio are large and have long-short positions

Importance of alpha portfolio relative to beta portfolio



- Weights of alpha portfolio dominate weights of beta portfolio.

Relative size of missing systematic & unsystematic components



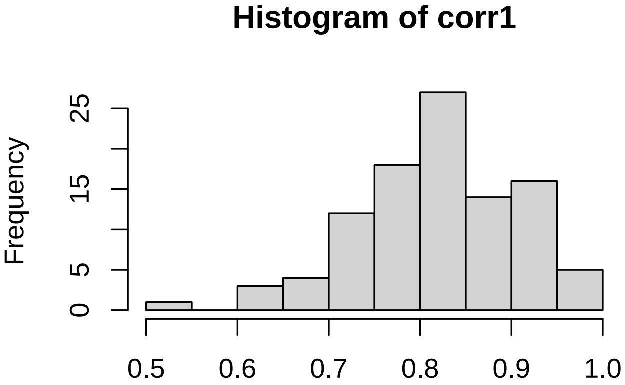
- When beta portfolio is based on Fama-French three-factor model,
- Then, the alpha portfolio weights show that:
  - The effect of omitted systematic risk factors is negligible;
  - The effect of omitted unsystematic risk dominates.

Source of performance of our portfolio (RMV using V)

Portfolio	w^{rmv}	w^{β}	w^{α}	$w^{\beta_{\mathbf{mis}}}$
Mean	0.1160	0.0088	0.1070	0.0001
Standard deviation	0.1520	0.0302	0.1520	0.0136
Sharpe ratio	0.7630	0.2920	0.7020	0.0090
$\frac{(\mathrm{SR}^k)^2}{(\mathrm{SR}^{rmv})^2}$	100%	14.65%	84.65%	0.01%

- When beta portfolio is based on Fama-French three-factor model, then to the squared SR of the mean-variance optimal portfolio:
  - Fama-French factors contribute only: 14.65%
  - Omitted unsystematic risk contributes: 84.65%
  - Omitted systematic risk factors contribute: 0.01%

How persistent is w^{α} (or α)?



- Average serial correlation is greater than 0.80.

Key takeaway:
Unsystematic risk is priced

Key implication:
Optimal portfolio should exploit unsystematic risk

Conclusion

- Our theoretical and empirical results
  - highlight the importance of priced unsystematic risk.
- What has typically been viewed as a pricing error,
  - should instead be viewed as an integral part of asset-pricing models.
- An optimal portfolio should exploit unsystematic risk.

End of focus

This brings us to an end to Part 3 of this chapter
on how to construct portfolios that exploit
both systematic and unsystematic risk.

- The rest of the chapter contains
  1. first, a summary of all the chapters in the book,
  2. then, some advice about the final information, and,
  3. finally, some concluding remarks.

Road map

Road map

One-page summary of the entire book

Part A: Preliminaries

Chapter 1: Properties of asset returns (when managing financial data)

Chapter 2: Out-of-sample Sharpe ratio (for performance measurement)

Part B: Modern portfolio management

Chapter 3: Markowitz model (which ignores estimation error)

Chapter 4: Shrinkage in the Markowitz model (to adjust for estimation error)

Part C: Post-Modern Portfolio Management

Chapter 5: Black-Litterman model (portfolio choice using the CAPM)

Chapter 6: Parametric portfolio policies (portfolio choice with FF characteristics)

Chapter 7: Volatility-timed factor portfolios (*timing* systematic risk factors)

Chapter 8: Exploiting unsystematic risk (portfolio choice beyond systematic factors)

Road map

Summary of Chapter 1: Managing financial data

- The Tidy Finance website tells us how we can obtain
  1. Stock-price data
  2. Stock-characteristics data
  3. Macroeconomic data.
- From daily stock-price data, we can construct
  - daily returns, monthly, returns, and annual returns;
  - we can then estimate return means, volatilities, and covariances.
- Estimate of expected returns based on sample mean is imprecise.
  - Additional data does not lead to improvement in precision.
- Estimate of return variance based on sample data is precise.
  - Additional data does lead to an improvement in precision.

Summary of Chapter 2: Performance measurement

- Performance of myopic portfolios can be evaluated by
  - estimating their out-of-sample returns
  - based on a rolling or expanding estimation window
  - performance metrics include Sharpe ratio, Sortino ratio, Treynor ratio, alpha, maximum drawdown, VaR, and CVaR.
- When evaluating performance, we
  - test the difference in Sharpe ratios,
  - for which the test-statistic must be computed correctly,
  - given the properties of the distribution of the underlying returns.

Summary of Chapter 3: Markowitz model . . . I

- It is important to look at an asset as part of a portfolio instead of in isolation; i.e., it is important to take into account asset correlations.
- The variance of the return on a portfolio with many assets equals the average covariance of returns in the portfolio.
- The entire efficient frontier can be generated from two efficient portfolios:
  - if a risk-free asset is not available, then we need two portfolios on the efficiency frontier;
  - if a risk-free asset is available, then we need the tangency portfolio and the risk-free asset.

Summary of Chapter 3: Markowitz model . . . II

- In the absence of a risk-free asset, every portfolio on the frontier can be written as

$$w_p = w_0 + (w_1 - w_0)\mu_{\text{targ}} = w_0(1 - \mu_{\text{targ}}) + w_1\mu_{\text{targ}}$$

where w_0 is a portfolio with $\mu_{\text{targ}} = 0$ and w_1 with $\mu_{\text{targ}} = 1$.

- In the presence of a risk-free asset, the optimal portfolio weights are:

$$\begin{aligned} w &= \left[\frac{(\mu_{\text{targ}} - R_f)}{(\mu - R_f 1_N)^\top V^{-1} (\mu - R_f 1_N)}\right] V^{-1} (\mu - R_f 1_N) \\ &= \frac{1}{\gamma} V^{-1} (\mu - R_f 1_N). \end{aligned}$$

Summary of Chapter 3: Markowitz model . . . III

- The marginal contribution of an asset to the portfolio's expected return and to the portfolio's volatility are, respectively

$$\frac{\partial \mathbb{E}[R_p]}{\partial w_n} = \mathbb{E}[R_n] - R_f \quad \text{and} \quad \frac{\partial \sigma_p}{\partial w_n} = \frac{\sigma_{np}}{\sigma_p}.$$

- For any frontier portfolio p, the return-to-risk ratio of all the risky assets in it must be the same:

$$\frac{\mathbb{E}[R_n] - R_f}{(\sigma_{np}/\sigma_p)} = \frac{\mathbb{E}[R_p] - R_f}{(\sigma_{pp}/\sigma_p)} = \frac{\mathbb{E}[R_p] - R_f}{\sigma_p} \qquad \text{. . . recall that } \sigma_{pp} = \sigma_p^2,$$

which implies that

$$\mathbb{E}[R_n] - R_f = \beta_{np}\big(\mathbb{E}[R_p] - R_f\big),$$

which links optimal portfolio choice to beta-pricing models!

Summary of Chapter 3: Markowitz model . . . IV

- The Markowitz analysis extends in a straightforward manner
  - to mean-variance optimization with respect to a benchmark portfolio,
  - with the Information ratio replacing the Sharpe ratio, and
  - the Appraisal ratio replacing the Treynor ratio.

Summary of Chapter 4: Shrinking Markowitz model . . . I

- The mean-variance portfolio weights

  $$w = \frac{1}{\gamma} V^{-1} (\mu - R_f 1_N)$$

  perform very poorly out-of-sample because

  - expected returns are estimated imprecisely;
  - the covariance matrix is ill-conditioned.

- Bayesian shrinkage of sample mean returns is not very effective at reducing the effect of estimation error on out-of-sample performance of mean-variance portfolios.

- Shortsale constraints on mean-variance weights reduce turnover but are less effective at improving its out-of-sample Sharpe ratio.

Summary of Chapter 4: Shrinking Markowitz model . . . II

- The global-minimum-variance (GMV) portfolio, which ignores expected returns entirely, achieves a higher Sharpe ratio than mean-variance portfolios;
- Performance of the GMV portfolio can be improved further by
  - imposing short-sale constraints or
  - Ledoit-Wolf shrinkage of the covariance matrix.

Summary of Chapter 5: Black-Litterman model . . . I

- Instead of starting with sample moments and shrinking them,
- Black and Litterman's approach has two steps:
  1. start with the weights of the market portfolio
  2. tilt these weights to reflect the views of the investor.

Summary of Chapter 5: Black-Litterman model . . . II

- The Black-Litterman model has two steps:
  1. Back out expected returns from the CAPM.

$$w_{\text{mkt}} = \frac{1}{\gamma}\mathbb{V}[R_{\text{sample}}]^{-1}(\mathbb{E}[R_{\text{CAPM}}] - R_f 1_N).$$

$$\underbrace{\mathbb{E}[R_{\text{CAPM}}]}_{N\times 1} - R_f 1_N = w_{\text{mkt}}\gamma\,\mathbb{V}[R_{\text{sample}}] \quad \text{. . . CAPM-implied expected returns}$$

  2. Blend investor's views with CAPM-implied expected returns to get $\mathbb{E}[R_{\text{CAPM}}|\text{views}]$ and $\mathbb{V}[R_{\text{sample}}|\text{views}]$, to compute

$$w = \frac{1}{\gamma}\mathbb{V}[R_{\text{sample}}|\text{views}]^{-1}(\mathbb{E}[R_{\text{CAPM}}|\text{views}] - R_f 1_N).$$

Summary of Chapter 5: Black-Litterman model . . . III

1. You can get the CAPM-implied expected excess returns from

$$\mathbb{E}[R_{\text{CAPM}}] - R_f 1_N = \gamma V_{\epsilon_R} w_{\text{mkt}}, \quad \text{where } \gamma = \frac{\mathbb{E}[R_{\text{mkt}} - R_f]}{\sigma^2_{\text{mkt}}}.$$

2. Conditional on an investor's views, the Black-Litterman formula
   2.1 for posterior mean returns is:

$$\begin{aligned}\mu_{\text{BL}} &= \mathbb{E}[R_{\text{CAPM}}|\text{views}] \\ &= \left[(\tau\Sigma)^{-1} + P^\top \Omega^{-1} P\right]^{-1} \left[(\tau\Sigma)^{-1} \underbrace{\mathbb{E}[R_{\text{CAPM}}]}_{\text{model}} + P^\top \Omega^{-1} \underbrace{q}_{\text{views}}\right].\end{aligned}$$

   2.2 for posterior covariance matrix of returns is:

$$\Sigma_{\text{BL}} = \mathbb{V}[R_{\text{sample}}|\text{views}] = \Sigma + \left[(\tau\Sigma)^{-1} + P^\top \Omega^{-1} P\right]^{-1}.$$

Summary of Chapter 6: Parametric portfolios . . . I

- In the absence of a single-factor model, we needed to estimate:
  - N mean returns
  - N variances
  - $(N^2 - N)/2$ covariances
  - for a total of $\frac{N(N+3)}{2} \approx \frac{N^2}{2}$ parameters.
- In the presence of a single-factor model, we needed to estimate:
  - N alphas
  - N betas
  - N asset-specific volatilities, $\sigma^2_{e_n}$
  - mean and volatility of the market excess return, i.e., λ_m and σ_m
  - for a total of $(3N + 2) \approx N$ parameters.

Summary of Chapter 6: Parametric portfolios . . . II

- But the market model is bad at explaining the cross-section of stock returns.
- Thus, we need a model with $K > 1$ factors, which could be
  1. macroeconomic factors,
  2. fundamental factors (firm characteristics), or
  3. statistical factors (from principal-component analysis).
- For a K-factor model for N assets, we need to estimate:
  - α_n for each of the N stocks . . . N;
  - $\sigma^2_{e_n}$ for each of the N stocks . . . N;
  - $\beta_{n,k}$ for each of the N stocks for each of the K factors; . . . $N \times K$;
  - λ_k and $\sigma^2_{F_k}$ for the K factors. . . . $2K$;
- which is a total of: $2N + 2K + (N \times K)$.

Summary of Chapter 6: Parametric portfolios . . . III

- Instead of a factor model for asset returns,
- Brandt, Santa-Clara, and Valkanov (2009) propose, a K-factor model for portfolio weights:

$$w_t(\theta) = w_{b,t} + (F_{1,t}\theta_1 + F_{2,t}\theta_2 + \ldots + F_{K,t}\theta_K)/N_t.$$

  - Note that θ does not depend on time.
- Then, the investor choses the weights θ by maximizing mean-variance utility:

$$\max_{\theta} \quad \mathbb{E}_t[r_{p,t+1}(\theta)] - \frac{\gamma}{2}\mathbb{V}_t[r_{p,t+1}(\theta)].$$

- If you want, you can impose nonnegativity or other constraints on θ and apply shrinkage to the covariance matrix.

Summary of Chapter 7: Volatility-timed factors . . . I

- Moreira and Muir (2017) show that the mean return of a factor does not change proportionately with its variance.
- Thus, conditioning the weight on return volatility will allow you to earn an "alpha" relative to the return of the unconditional factor.
- The return on the volatility-managed factor is

$$f^{\sigma}_{t+1} = \frac{c}{\sigma^2_t(f)} \times f_{t+1}, \quad \text{where}$$

  - $\sigma_t(f)$ is the previous month's realized volatility, estimated using daily data
  - choose c so f^{σ} has the same unconditional volatility as f.

Summary of Chapter 7: Volatility-timed factors . . . II

- The findings of Moreira and Muir (2017) have been criticized:
  - Cederburg, O'Doherty, Wang, and Yan (2020) show gains from volatility timing cannot be realized out of sample.
  - Barroso and Detzel (2021) show that transaction costs erode the gains entirely.
  - Barroso and Detzel (2021) show that gains from volatility timing the market are achieved only during periods of "high sentiment."

Summary of Chapter 7: Volatility-timed factors . . . III

- DeMiguel, Martín-Utrera, and Uppal (2024) address these criticisms by proposing a volatility-timing strategy, with four distinct features:
  1. Multifactor, instead of individual-factor portfolios.
  2. Relative factor weights can vary (as a function of market volatility), instead of having a fixed-weight multifactor portfolio.

     $$\theta_{k,t} = a_k + \frac{b_k}{\sigma_t}.$$

  3. Account for trading diversification (netting of trades across factors) when computing transaction costs;
  4. Optimize factor weights accounting for transaction costs.

Summary of Chapter 8: Harvesting unsystematic risk . . . I

- Dello-Preite, Uppal, Zaffaroni, and Zviadadze (2024) show that:
  - Expected stock returns include compensation for unsystematic risk.
  - A large part of the SDF's variation is explained by unsystematic risk.
  - Contrary to the factor-zoo literature, a single systematic risk factor, the market return, is sufficient to explain most of the variation in the SDF's systematic component.
- Raponi, Uppal, and Zaffaroni (2023) show how the optimal mean-variance efficient portfolio must have two components:
  - beta portfolio that depends only on systematic risk factors;
  - alpha portfolio that depends only on unsystematic risk.
- They also show as N increases, the weight of each asset in the alpha portfolio dominates the weight of that asset in the beta portfolio.

Road map

What we plan to do in the next chapter

What's next?

Today's class is the last class for this course.

I hope you continue to enjoy learning about quantitative portfolio management (and Python) in your working life.

I wish you the very best in your journey of learning.

Road map

Bibliography . . . I

Ahn, S. C., and A. R. Horenstein. 2013. Eigenvalue ratio test for the number of factors. *Econometrica* 81 (3): 1203–1227. (Cited on page 69).

Ao, M., Y. Li, and X. Zheng. 2019. Approaching mean-variance efficiency for large portfolios. *Review of Financial Studies* 32 (7): 2890–2919. (Cited on pages 98–100).

Back, K. E. 2017. *Asset pricing and portfolio choice theory.* Oxford University Press. (Cited on page 15).

Barroso, P., and A. L. Detzel. 2021. Do limits to arbitrage explain the benefits of volatility-managed portfolios? *Journal of Financial Economics* 140 (3): 744–767. (Cited on pages 10, 133).

Bawa, V. S., S. Brown, and R. Klein. 1979. *Estimation risk and optimal portfolio choice.* North Holland, Amsterdam. (Cited on page 8).

Black, F., and R. Litterman. 1990. Asset allocation: Combining investor views with market equilibrium. Goldman, Sachs & Co. (Cited on page 8).

———. 1991a. Combining investor views with market equilibrium. *Journal of Fixed Income* 1 (2): 7–18. (Cited on page 8).

———. 1991b. Global asset allocation with equities, bonds, and currencies. *Fixed Income Research* 2 (15-28): 1–44. (Cited on page 8).

Bibliography . . . II

Black, F., and R. Litterman. 1992. Global portfolio optimization. *Financial Analysts Journal* 48:28–43. (Cited on page 8).

Brandt, M. W., P. Santa-Clara, and R. Valkanov. 2009. Parametric portfolio policies: Exploiting characteristics in the cross-section of equity returns. *Review of Financial Studies* 22 (9): 3411–3447. (Cited on pages 9, 131).

Bryzgalova, S., J. Huang, and C. Julliard. 2023. Bayesian solutions for the factor zoo: we just ran two quadrillion models. *The Journal of Finance* 78 (1): 487–557. (Cited on page 58).

Cederburg, S., M. S. O'Doherty, F. Wang, and X. Yan. 2020. On the performance of volatility-managed portfolios. *Journal of Financial Economics* 138 (1): 95–117. (Cited on pages 10, 133).

Chen, N.-F., R. Roll, and S. Ross. 1986. Economic forces and the stock market. *Journal of Business* 59:383–403. (Cited on page 9).

Cochrane, J. 2011. Discount rates. *Journal of Finance* 56:1047–1108. (Cited on page 57).

Dello-Preite, M., R. Uppal, P. Zaffaroni, and I. Zviadadze. 2024. Cross-sectional asset pricing with unsystematic risk. Available at SSRN 4135146. (Cited on pages 10, 49, 135).

Bibliography . . . III

DeMiguel, V., L. Garlappi, and R. Uppal. 2009. Optimal versus naive diversification: How inefficient is the $1/N$ portfolio strategy? *Review of Financial Studies* 22 (5): 1915–1953. (Cited on page 100).

DeMiguel, V., A. Martín-Utrera, F. J. Nogales, and R. Uppal. 2020. A Transaction-Cost Perspective on the Multitude of Firm Characteristics. *Review of Financial Studies* 33, no. 5 (April): 2180–2222. (Cited on page 9).

DeMiguel, V., A. Martín-Utrera, and R. Uppal. 2024. A multifactor perspective on volatility-managed portfolios. *Journal of Finance* 79 (6): 3859–3891. (Cited on pages 10, 134).

Fama, E. F., and K. R. French. 1992. The cross-section of expected stock returns. *Journal of Finance* 47, no. 2 (June): 427–465. (Cited on page 9).

———. 1993. Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics* 33 (1): 3–56. (Cited on page 9).

———. 2012. Size, value, and momentum in international stock returns. *Journal of Financial Economics* 105 (3): 457–472. (Cited on page 9).

———. 2015. A five-factor asset pricing model. *Journal of Financial Economics* 116 (1): 1–22. (Cited on page 9).

———. 2018. Choosing factors. *Journal of Financial Economics* 128 (2): 234–252. (Cited on page 9).

Bibliography . . . IV

He, G., and R. Litterman. 1999. The intuition behind Black-Litterman model portfolios. *Investment Management Research (Goldman, Sachs & Company).* (Cited on page 8).

Jorion, P. 1988. Bayes-stein estimation for portfolio analysis. *Journal of Financial and Quantitative Analysis* 21 (3): 279–292. (Cited on page 8).

Jorion, P. 1985. International portfolio diversification with estimation risk [in English]. *Journal of Business* 58 (3): pp. 259–278. (Cited on page 8).

———. 1992. Portfolio optimization in practice. *Financial Analysts Journal* 48 (1): 68–74. (Cited on page 8).

Klein, R. W., and V. S. Bawa. 1976. The effect of estimation risk on optimal portfolio choice. *Journal of Financial Economics* 3 (3): 215–31. (Cited on page 8).

Ledoit, O., and M. Wolf. 2003. Improved estimation of the covariance matrix of stock returns with an application to portfolio selection. *Journal of Empirical Finance* 10 (5): 603–621. (Cited on page 100).

———. 2008. Robust performance hypothesis resting with the Sharpe ratio. *Journal of Empirical Finance* 15:850–859. (Cited on page 101).

Litterman, R. 2003. *Modern investment management: An equilibrium approach.* New York: Wiley. (Cited on page 8).

Bibliography . . . V

Markowitz, H. M. 1952. Portfolio selection. *Journal of Finance* 7 (1): 77–91. (Cited on page 8).

———. 1959. *Portfolio selection: Efficient diversification of investments.* New York: Wiley. (Cited on page 8).

Merton, R. C. 1987. A simple model of capital market equilibrium with incomplete information [in English]. *Journal of Finance* 42 (3): 483–510. (Cited on pages 61, 62).

Moreira, A., and T. Muir. 2017. Volatility-managed portfolios. *Journal of Finance* 72 (4): 1611–1644. (Cited on pages 10, 132, 133).

———. 2019. Should long-term investors time volatility? *Journal of Financial Economics* 131 (3): 507–527. (Cited on page 10).

Onatski, A. 2012. Asymptotics of the principal components estimator of large factor models with weakly influential factors. *Journal of Econometrics* 168 (2): 244–258. (Cited on page 69).

Pástor, Ľ., and R. F. Stambaugh. 2000. Comparing asset pricing models: an investment perspective. *Journal of Financial Economics* 56 (3): 335–381. (Cited on page 8).

Raponi, V., R. Uppal, and P. Zaffaroni. 2023. Robust portfolio choice. Working Paper, SSRN eLibrary. (Cited on pages 10, 91, 135).

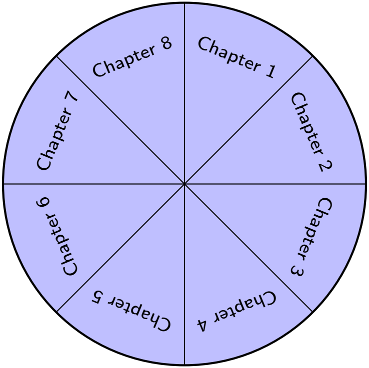
Bibliography . . . VI

Ross, S. A. 1976. The arbitrage theory of capital asset pricing. *Journal of Economic Theory* 13:341–360. (Cited on pages 9, 62).

———. 1977. Return, risk, and arbitrage. In *Risk and return in finance,* edited by I. Friend and J. Bicksler. Cambridge, MA: Ballinger. (Cited on pages 9, 62).

Rubinstein, M. 2006. *A history of the theory of investments: My annotated bibliography.* Hoboken, NJ: John Wiley & Sons. (Cited on page 11).

Sharpe, W. 1964. Capital asset prices: A theory of market equilibrium under conditions of risk. *Journal of Finance* 1919 (3): 425–442. (Cited on pages 8, 55).



End of Chapter 8