Quantitative Portfolio Management: Theory and Practice

Chapter 7:
Volatility-Timed Factor Portfolios

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The big picture: Plan for the entire book

Part A: **Preliminaries**

Chapter 1: Managing financial data

Chapter 2: Performance measurement (especially out of sample)

Part B: **Modern portfolio management**

Chapter 3: Mean-variance portfolios that *ignore* estimation error

Chapter 4: Mean-variance portfolios that *adjust for* estimation error

Part C: **Post-Modern Portfolio Management**

Chapter 5: CAPM-based portfolios: Black-Litterman model

Chapter 6: Factor-based portfolios: Parametric portfolio policies

Chapter 7: Volatility-timed factor portfolios

Chapter 8: Portfolios exploiting systematic risk factors *and* unsystematic risk

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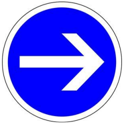
Road map

What do we want to do in Chapter 7



In this chapter, we study if it is possible to *time* our investment in risk factors.

Specifically, first we study if we can use the volatility of a factor to time the investment in that factor.

Then, we study the performance of a portfolio of *many* factors where the weight in each factor depends on its own volatility or on market volatility.

Timeline: Quantitative portfolio management ideas . . . I

- We can see how ideas about investment have progressed over time.

. . . . *The thinking in ancient times* . . . .

- 4th century: $1/N$
  - "One should always divide his wealth into three parts: a third in land, a third in merchandise, and a third ready to hand [cash].
    Rabbi Issac bar Aha, Babylonian Talmud: Tractate Baba Mezi'a, folio 42a
  - "My ventures are not in one bottom trusted"
    "Merchant of Venice, "Shakespeare ((1564–1616) on the importance of diversification in investing
  - Do not put all your eggs in one basket

Timeline: Quantitative portfolio management ideas . . . II

. . . . *Below are the topics we covered in Chapters 3–5* . . . .

- 1950s: Mean-variance optimization
  (Markowitz 1952, 1959)

- 1964: CAPM
  (Sharpe 1964)

- 1970–2000s: Bayesian shrinkage
  (Klein and Bawa 1976; Bawa, Brown, and Klein 1979; Jorion 1985; Jorion 1988; Jorion 1992; Pástor and Stambaugh 2000)

- 1990s: Black-Litterman model
  (Black and Litterman 1990, 1991a, 1991b, 1992; He and Litterman 1999; Litterman 2003)

Timeline: Quantitative portfolio management ideas . . . III

. . . . *Last time: Chapter 6* . . . .

- 1970s: Factor models
  (Ross 1976, 1977)

- 1980s Macro factor models
  (Chen, Roll, and Ross 1986)

- 1990–2020s: Fundamental (firm-characteristic-based) factor models
  (Fama and French 1992, 1993, 2012, 2015, 2018).

- 2009-2023: Parametric portfolio policies
  (Brandt, Santa-Clara, and Valkanov 2009; DeMiguel, Martín-Utrera, Nogales, and Uppal 2020).

Timeline: Quantitative portfolio management ideas . . . IV

. . . . *Today: Chapter 7* . . . .

- 2017-2024: Volatility-timing of factors
  - Moreira and Muir (2017, 2019)
  - Cederburg, O'Doherty, Wang, and Yan (2020)
  - Barroso and Detzel (2021)
  - DeMiguel, Martín-Utrera, and Uppal (2024).

. . . . *Next: Chapter 8* . . . .

- 2023-2024: Portfolio construction: Beyond systematic risk factors
  - Raponi, Uppal, and Zaffaroni (2023)
  - Dello-Preite, Uppal, Zaffaroni, and Zviadadze (2024).

Timeline: Quantitative portfolio management ideas . . . V

- For a more detailed history of the development of ideas about investment, see the book by Rubinstein (2006).

Road map

Road map

Motivation for the material in this chapter

- Factor investing: Quantitative-investment approach that exploits firm characteristics that predict expected stock returns.
- Smart beta: *low-cost* approach that exploits *commonly known* characteristics with the aim to outperform a capitalization-weighted benchmark in a rules-based, transparent manner.
- Smart beta strategies are designed to provide investors with exposure to specific factors that have historically been associated with
  - higher returns,
  - lower risk, or
  - other desirable investment properties (e.g., lower drawdown or higher ESG score).

Some common firm characteristics

Characteristic	Acronym	**Buy**	**Sell**
Size	SMB	Small firms	Large firms
Value	HML	High book-to-market	Low book-to-market
Momentum	UMD	Winners (Up)	Losers (Down)
Investment	CMA	Conservative	Aggressive
Profitability	RMW	Robust	Weak

- In the assignment based on this chapter, we will look at 9 factors.

Key components of smart beta strategies . . . I

- Factor exposure
  - Smart beta strategies are often based on exposure to specific factors, which are characteristics or risk premiums that have been shown to drive returns over the long term.
  - Common factors include value, momentum, size, and low volatility.
  - E.g., a value-based smart beta strategy might involve selecting undervalued stocks based on price-to-earnings ratios or book value.
- Rules-based methodology
  - Smart beta strategies use a predetermined set of rules for portfolio construction and rebalancing,
  - as opposed to relying on market-capitalization-weighted portfolios.

Key components of smart beta strategies . . . II

- Systematic and transparent
  - Smart beta strategies are systematic, meaning they follow a consistent and predefined methodology.
  - They are also transparent, allowing investors to understand the underlying factors and rules guiding the portfolio.
  - E.g., a low-volatility smart beta strategy might select stocks with historically lower price volatility.
- Diversification
  - Smart beta strategies often maintain a level of diversification to reduce specific stock or sector risk.
  - E.g., a multifactor smart beta strategy might combine exposure to value, momentum, and quality factors to create a diversified portfolio that aims to outperform traditional market-cap-weighted indices.

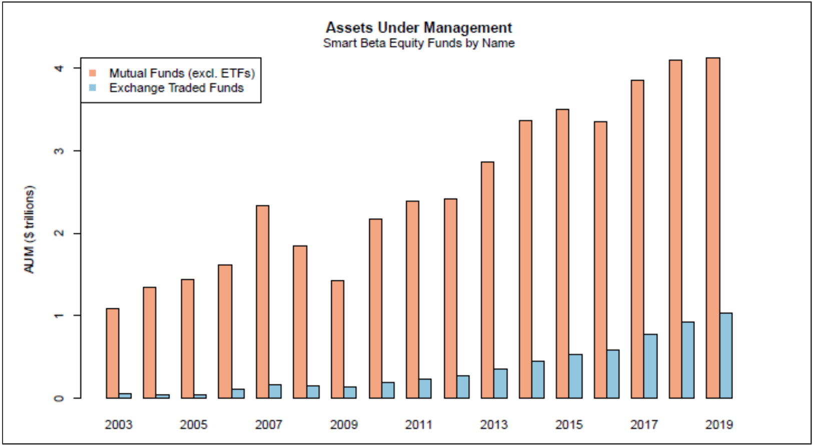
Key components of smart beta strategies . . . III

- Performance evaluation and risk considerations
  - Smart beta strategies are evaluated based on their historical performance and risk characteristics relative to traditional benchmarks.
  - E.g., a smart beta strategy focusing on dividend yield might attract investors seeking income, with the performance evaluated against a dividend-focused benchmark.
- Huang, Song, and Xiang (2020) find that smart beta strategies have underperformed by 1% on average since launch.

Factor investing growth

- Assets under management growing fast:

  Johansson, Sabbatucci, Tamoni (2020) *since 2010, the total assets under management for the U.S. Smart Beta ETF market have grown 30% per year."*



Factor investing and parametric portfolio policies

- In the last class, we saw how to construct portfolios using factor models.
- In particular, we studied parametric portfolio policies, which showed how to reduce the dimensionality of the portfolio problem when investing in factors.
  - Instead of modeling asset returns in terms of factors, and then constructing the optimal portfolio,
  - parametric portfolio policies directly specify portfolio weights in terms of factors.

$$w_t(\theta) = w_{b,t} + (\theta_1 F_{1,t} + \theta_2 F_{2,t} + \ldots + \theta_K F_{K,t})/N_t.$$

Start of focus

Road map

Parametric portfolios whose weights depend on volatility

- In this class, we build on parametric portfolio policies,
  - by modeling the portfolio weight on each factor, θ_k,
  - to depend on factor or market volatility.

$$\theta_{k,t} = a_k + \frac{b_k}{\sigma_{k,t}} \qquad \text{or} \qquad \theta_{k,t} = a_k + \frac{b_k}{\sigma_{\mathbf{mkt},t}}.$$

Motivation for studying volatility-timing of factors

A fundamental premise in finance:

strong risk-return tradeoff

$$\mathbb{E}\left[\text{Return}\right] \propto \text{Risk}$$

If volatility timing was successful,
it would imply breakdown of risk-return tradeoff

Importance of volatility-timing strategies

- Volatility-timing strategies have important implications for:
  - investors,
  - asset managers, and, therefore,
  - they received considerable attention in the financial press.

Articles in the financial press about volatility timing

- "Reassessing the classic risk-return tradeoff," The Financial Times, March 9, 2016. Link to article.
- "When markets get scary, panicking is smart," CNBC, March 23, 2016. Link to article and video.
- "Authers' Note: Healthy correction?," The Financial Times, February 7, 2018. Link to article.
- Video interview of Alan Moreira by CEPR & VideoVox Economics in 2017. Link to video.

Volatility-timing strategies in practice

- BlackRock offers the following description of the investment strategy for its Managed Volatility V.I. Fund:
  - "In periods of heightened volatility, the portfolio will de-risk into less volatile assets like fixed income and cash and re-risk when market turbulence subsides."
  - Performance plots available from this link to FT.

Can you time factors?
Cliff Asness (AQR) believes you cannot . . .

The Journal of Portfolio Management

INVITED EDITORIAL COMMENT

The Siren Song of Factor Timing

aka "Smart Beta Timing" aka "Style Timing"

CLIFFORD S. ASNESS

Link to Cliff Asness (2016) article

(look for the "View" button on the right side of the page)

Can you time factors?
Moreira and Muir (2017) believe you can ...

- Moreira and Muir (2017) show investors can increase Sharpe ratios by reducing exposure to risk factors when their volatility is high.
- The success of the volatility-timing strategy implies that:
  - changes in factor volatility are not offset
  - by proportional changes in expected returns.
- This is a challenge to key insight about the risk-return tradeoff.

How this class is structured

- In today's class, we will study four papers:
    1. Moreira, A., and T. Muir. 2017. Volatility-managed portfolios. *Journal of Finance* 72 (4): 1611–1644. Available from this link.
    2. Cederburg, S., M. S. O'Doherty, F. Wang, and X. Yan. 2020. On the performance of volatility-managed portfolios. *Journal of Financial Economics* 138 (1): 95–117. Available from this link.
    3. Barroso, P., and A. L. Detzel. 2021. Do limits to arbitrage explain the benefits of volatility-managed portfolios? *Journal of Financial Economics* 140 (3): 744–767. Available from this link.
    4. DeMiguel, V., A. Martín-Utrera, and R. Uppal. 2024. A multifactor perspective on volatility-managed portfolios. *Journal of Finance* 79 (6): 3859–3891. Available from this link.
- The first and last papers in detail, the other two papers briefly.

Start of our discussion of Moreira and Muir (2017).

Road map

Outline for our discussion of Moreira and Muir (2017)

- Moreira and Muir (2017) is described in the following steps:
  1. The empirical evidence on volatility-timing strategies
  2. Data used by Moreira and Muir (2017)
  3. Analysis of gains from timing individual factors
  4. Analysis of gains from timing portfolio of multiple factors
  5. The implications of the performance of volatility-timing strategies
- Python code to implement volatility-timing strategies is available from this link, which is written by Alan Moreira.
  - The code includes an excellent explanation of volatility timing.
  - The code relies on access to WRDS, but you can use it instead with the data I have provided for the next assignment.

Road map

What Moreira and Muir (2017) do? ...I

- Volatility managed portfolio: scale aggregate priced factor by $1/\sigma_t^2$
- Motivation: we know from Markowitz that the demand for a risky asset is:

$$w_t = \frac{1}{\text{[risk aversion]}} \frac{\text{[excess mean returns]}}{\text{[return variance]}} = \frac{1}{\gamma} \frac{\mu_t}{\sigma_t^2}.$$

- If volatility doesn't forecast returns, then a volatility-timing strategy will outperform an unconditional strategy with constant weight, w_t.
  - That is, if μ_t does not change fully when σ_t^2 changes,
  - then, this strategy will make returns in excess of the returns of an unconditional strategy (i.e., it will earn an alpha).

What Moreira and Muir (2017) find?

- Volatility-timing
  - increases Sharpe ratios and
  - generates large alphas relative to the original factors.
- The strategy takes less risk in recessions when σ is high.
  - Strategy sells after the market crashes (1929, 1987, 2008).
  - That is, when volatility increases, returns do not increase proportionately.
  - So, reducing your position when volatility is high reduces risk more than it reduces returns.

Road map

Data used by Moreira and Muir (2017)

- Daily and monthly return data for several factors
- Factors:
  - Market, SMB, HML, UMD, Profitability, ROE, Investment, Carry (FX), BAB.
- Sample periods
  - 1926–2015 (Mkt, SMB, HML, Momentum),
  - Post-1960 for the remaining factors.
- All numbers are annualized.

Road map

How to construct volatility-managed factors

- Define f_{t+1} to be an excess return
  - If you invest one unit of the factor, your return is f_{t+1}
- Construct a new volatility-managed factor, whose return is

$$f^{\sigma}_{t+1} = \frac{c}{\sigma^2_t(f)} \times f_{t+1}, \quad \text{where}$$

  - $\sigma_t(f)$ is the previous month's realized volatility, estimated using daily data
  - choose c so f^{σ} has the same unconditional volatility as f;
  - If you invest $\frac{c}{\sigma^2_t(f)}$ units, your return is $f^{\sigma}_{t+1} = \frac{c}{\sigma^2_t(f)} \times f_{t+1}$.
  - Note that quantity invested decreases as σ^2_t increases.

Calculation of mean-variance weights

- You can then do mean-variance optimization to combine
  1. The original (without timing) factor, f_{t+1};
  2. The volatility-managed version of this factor, f^{σ}_{t+1}.
- The mean-variance optimization will give you the weight to put on
  - the original factor and
  - the volatility-timed factor.
- Comparing the Sharpe ratios of
  - the portfolio with just the original factor and
  - the portfolio that includes also the volatility-timed factor.
- will tell you whether volatility-timing improves performance.
- *This is what the next assignment asks you to do.*

Calculation of alphas

- An "equivalent" way to assess the performance gains from volatility timing is to use regression analysis (Moreira and Muir 2017).
  - Later on we will see that the two are not "equivalent." Why?
- Run the following regression:

$$f_{t+1}^{\sigma} = \alpha + \beta f_{t+1} + \epsilon_{t+1}.$$

- Compute α, which Moreira and Muir (2017) show is theoretically equal to

$$\alpha = -\operatorname{Cov}\left(\frac{\mu_t}{\sigma_t^2}, \sigma_t^2\right) \frac{c}{\mathbb{E}\left[\sigma_t^2\right]};$$

which we interpret on the next slide.

Market return mean and variance

- Consider the market factor
- When studying Black-Litterman model, we showed for the market

$$\gamma_{\mathsf{mkt}} = \frac{\text{expected excess return of market}}{\text{variance of market return}} = \frac{\mu_{t,\mathsf{mkt}}}{\sigma^2_{t,\mathsf{mkt}}}.$$

- If the above result is true, then

$$\begin{aligned}
\alpha &= -\operatorname{Cov}\left(\frac{\mu_{t,\mathsf{mkt}}}{\sigma^2_{t,\mathsf{mkt}}}, \sigma^2_{t,\mathsf{mkt}}\right)\frac{c}{\mathbb{E}\left[\sigma^2_{t,\mathsf{mkt}}\right]} && \ldots \text{from previous page} \\
&= -\operatorname{Cov}\left(\gamma_{\mathsf{mkt}}, \sigma^2_{t,\mathsf{mkt}}\right)\frac{c}{\mathbb{E}\left[\sigma^2_{t,\mathsf{mkt}}\right]} && \ldots \text{setting } \gamma_{\mathsf{mkt}} = \frac{\mu_{t,\mathsf{mkt}}}{\sigma^2_{t,\mathsf{mkt}}} \\
&= 0 && \ldots \text{if } \gamma_{\mathsf{mkt}} \text{ is constant, cov} = 0.
\end{aligned}$$

- So, if $\mu_{t,\mathsf{mkt}} = \gamma_{\mathsf{mkt}}\sigma^2_{t,\mathsf{mkt}}$, we should find $\alpha = 0$.
  - But, if $\alpha \neq 0$, then it implies risk is not proportional to return.

Road map

Univariate α and β for volatility-managed portfolios . . . I

- Moreira and Muir (2017, Panel A of Table 1 – see next page) run a
  - time-series regressions of each volatility-managed factor
  - on the unmanaged factor:

  $$f_t^\sigma = \alpha + \beta f_t + \epsilon_t,$$

  where f_t plays the role of the benchmark portfolio return.
- The table shows that the betas are significant, as one would expect.
- The surprising result is that the estimated alphas are significant: they are more than two standard errors away from zero for MKT, UMD, RMW, ROE, IA, BAB (but not for SMB, HML, CMA).

Univariate α and β for volatility-managed portfolios . . . II

	(1) Mkt^{σ}	(2) SMB^{σ}	(3) HML^{σ}	(4) UMD^{σ}	(5) RMW^{σ}	(6) CMA^{σ}	(7) FX^{σ}	(8) ROE^{σ}	(9) IA^{σ}	(10) BAB^{σ}
					Regression betas and standard errors (in parenthesis)					
MktRF	0.61 (0.05)									
SMB		0.62 (0.08)								
HML			0.57 (0.07)							
UMD				0.47 (0.07)						
RMW					0.62 (0.08)					
CMA						0.68 (0.05)				
Carry							0.71 (0.08)			
ROE								0.63 (0.07)		
IA									0.68 (0.05)	
BAB										0.57 (0.05
					Regression alphas and standard errors (in parenthesis)					
α	4.86	-0.58	1.97	12.51	2.44	0.38	2.78	5.48	1.55	5.67
std. error	(1.56)	(0.91)	(1.02)	(1.71)	(0.83)	(0.67)	(1.49)	(0.97)	(0.67)	(0.98)
N	1,065	1,065	1,065	1,060	621	621	360	575	575	996
R^2	0.37	0.38	0.32	0.22	0.38	0.46	0.33	0.40	0.47	0.33
RMSE	51.39	30.44	34.92	50.37	20.16	17.55	25.34	23.69	16.58	29.73

Multivariate alphas controlling for Fama-French 3-factors

- Moreira and Muir (2017, Panel B of Table 1) show that the alphas remain significant
  - for MKT, HML, UMD, RMW, ROE, BAB
  - even after controlling for the Fama-French three factors.

	(1) Mkt^σ	(2) SMB^σ	(3) HML^σ	(4) UMD^σ	(5) RMW^σ	(6) CMA^σ	(7) FX^σ	(8) ROE^σ	(9) IA^σ	(10) BAB^σ
	Panel B of Table 1: Regression alphas and standard errors (in parenthesis)									
α	5.45	-0.33	2.66	10.52	3.18	-0.01	2.54	5.76	1.14	5.63
std. error	(1.56)	(0.89)	(1.02)	(1.60)	(0.83)	(0.68)	(1.65)	(0.97)	(0.69)	(0.97)

Performance gains from volatility-managed factors . . . I

- How much do we increase Sharpe ratio/expand the MVE frontier?
- Note that the improvement in Sharpe ratio from using a volatility-managed factor is given by the appraisal ratio;

$$\begin{aligned}\text{Appraisal ratio} &= \text{Additional Sharpe ratio relative to benchmark} \\ &= \frac{\text{additional mean return relative to the benchmark}}{\text{additional risk relative to the benchmark}} \\ &= \frac{\alpha}{\sigma_\epsilon} \\ &= \frac{\alpha}{\text{RMSE}},\end{aligned}$$

where RMSE is the root mean squared error;
i.e., the volatility of the regression residual.

Performance gains from volatility-managed factors . . . II

Additional Sharpe ratio from volatility timing

Factor	α	RMSE	$\frac{\alpha}{\text{RMSE}} \times \sqrt{12}$
MktRf	4.86	51.39	0.32
SMB	−0.58	30.44	−0.06
HML	1.97	34.92	0.19
UMD	12.51	50.37	0.86
RMW	2.44	20.16	0.41
CMA	0.38	17.55	0.07
Carry (FX)	2.78	25.34	0.38
ROE	5.48	23.69	0.80
IA	1.55	16.58	0.32
BAB	5.67	29.73	0.66

- The additional Sharpe ratio generated from volatility-timing is large:
  - For MktRf factor it is 32%;
  - For UMD factor it is 86%;
  - For ROE factor it is 80%;
  - For BAB factor it is 66%.

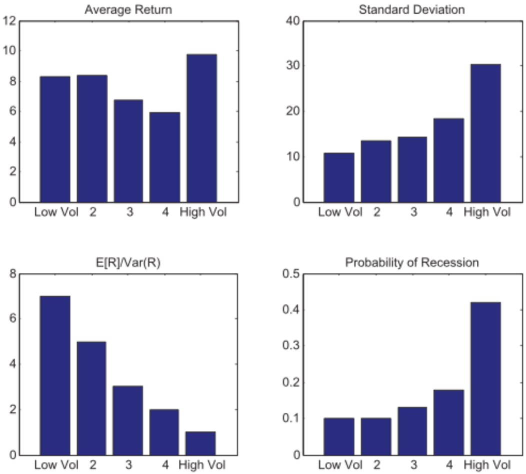
Why does volatility-timing work? . . . I

- To explain why volatility timing performs well, Moreira and Muir provide a nice explanation using pictures for the market factor.
- They use the monthly time series of realized volatility to sort the *following* month's returns into five buckets.
  - The lowest, "low vol," looks at the properties of returns over the month following the lowest 20% of realized volatility months.
- Note that for the market, the average (excess) return per unit of variance represents the optimal weight of a mean-variance investor;
  - it also represents "market's risk-aversion," as we saw when studying the Black-Litterman model:

$$\underbrace{\mathbb{E}_t \left[R_{\mathsf{mkt},t+1}\right]}_{\text{excess return}} = \gamma_{\mathsf{mkt}}\, \sigma^2_{\mathsf{mkt},t}.$$

Why does volatility-timing work? Explained in pictures

- Top, right figure shows:
  Market volatility persists
- Top, left figure shows:
  Lagged market volatility uncorrelated with future returns
- Bottom, left figure shows:
  Market risk-return tradeoff deteriorates with lagged market volatility
- Conclusion: Volatility timing (for market) works due to weak risk-return trade-off



From: Moreira and Muir (2017, Figure 1)

Road map

Empirical results: Gains from timing multiple factors . . . I

- Some investors invest in multiple factors beyond the market
- So, Moreira-Muir extend their analysis to a (static) mean-variance efficient (MVE) portfolio of multiple risk factors.
- Moreira-Muir construct the MVE portfolio to match exactly the procedure for individual factors.
  - Recall that for individual factors, the volatility-managed factor's return is

$$f^{\sigma}_{t+1} = \frac{c}{\sigma^2_t(f)} \times f_{t+1}.$$

Empirical results: Gains from timing multiple factors . . . II

- Let F_{t+1} be a vector of factor returns and b the static weights that produce the maximum in-sample Sharpe ratio.

- Then the MVE portfolio is

$$f_{t+1}^{MVE} = b^{\top} F_{t+1} \qquad \ldots \text{optimal combination of } K \text{ factors}$$

- Then, the volatility-timed MVE portfolio is

$$\begin{aligned} f_{t+1}^{MVE,\sigma} &= \frac{c}{[\text{variance of MVE portfolio}]}[\text{return of MVE portfolio}] \\ &= \frac{c}{\hat{\sigma}_t^2 \left(f_{t+1}^{MVE} \right)} f_{t+1}^{MVE}, \end{aligned}$$

where again c is a constant that normalizes the variance of the volatility-managed portfolio so it is equal to the MVE portfolio.

Empirical results: Gains from timing multiple factors . . . III

- We start by rewriting the volatility-time MVE portfolio:

$$
\begin{aligned}
f_{t+1}^{MVE,\sigma} &= \frac{c}{[\text{variance of MVE portfolio}]}[\text{return of MVE portfolio}] \\
&= \frac{c}{\hat{\sigma}_t^2\left(f_{t+1}^{MVE}\right)} f_{t+1}^{MVE},
\end{aligned}
$$

- From the above, we see that the volatility-managed portfolio
  - shifts the conditional weight on the entire MVE portfolio
  - but does not change the relative weights across the individual factors.

Volatility-timed mean-variance efficient portfolios

- Moreira-Muir consider seven sets of risk factors.
  - FF means Fama and French
  - HXZ means Hou, Xue, and Zhang
- Find substantial performance gains from volatility timing.

Statistic	(1) MKT	(2) FF3	(3) FF3&UMD	(4) FF5	(5) FF5&UMD	(6) HXZ	(7) HXZ&UMD
α	4.86	4.99	4.04	1.34	2.01	2.32	2.51
Standard error	(1.56)	(1.00)	(0.57)	(0.32)	(0.39)	(0.38)	(0.44)
Original Sharpe	0.42	0.52	0.98	1.19	1.34	1.57	1.57
Vol-Managed Sharpe	0.51	0.69	1.09	1.20	1.42	1.69	1.73
Appraisal Ratio	0.33	0.50	0.69	0.56	0.77	0.91	0.91
Observations	1,065	1,065	1,060	621	621	575	575
R^2	0.37	0.22	0.25	0.42	0.40	0.46	0.43
RMSE	51.39	34.50	20.27	8.28	9.11	8.80	9.55

Road map

Implications of profitability of volatility-timing strategies

- Portfolio choice for long term investors
  - Large wealth gains for both short and long-term oriented investors.
- Reduced-form pricing
  - Risk-adjust mutual fund/ hedge fund strategies.
- General-equilibrium asset pricing models
  - Puzzle: the price of risk is low when volatility is high.

Road map

Python code for volatility-timing strategies

- Alan Moreira has written Python code for volatility timing.
  - The code is available from this link.
- Alan Moreira has also written an excellent online book on Quantitative Investing that is available from this link.
  - This book parallels closely our course.
- The Moreira and Muir (2017) paper is an excellent example of very well-executed research.
  - Please read it if you are interested in advanced material.

End of our discussion of Moreira and Muir (2017)

Now we look at two papers criticizing the result in Moreira and Muir (2017)

The finding of Moreira and Muir (2017) has been criticized

1. Out of sample
   - Cederburg, O'Doherty, Wang, and Yan (2020) show gains from volatility timing cannot be realized out of sample.
2. Transaction costs
   - Barroso and Detzel (2021) show that transaction costs entirely erode the gains.
3. Sentiment
   - Barroso and Detzel (2021) also show that gains from volatility timing the market portfolio are achieved only during periods of "high sentiment."

We first look at the critique in the paper by Cederburg, O'Doherty, Wang, and Yan (2020).

Road map

Cederburg, O'Doherty, Wang, and Yan (2020) . . . I

- Using a comprehensive set of 103 equity strategies, they analyze the value of volatility-managed portfolios for real-time investors.
- Volatility-managed portfolios do not systematically outperform their corresponding unmanaged portfolios in direct comparisons.
- Consistent with Moreira and Muir (2017), volatility-managed portfolios tend to exhibit significantly positive alphas in spanning regressions.

Cederburg, O'Doherty, Wang, and Yan (2020) . . . II

- However, the trading strategies implied by these regressions are not implementable in real time.
  - The regression alpha is based on the entire sample,
  - but an investor would not have access to future data.
- Out-of-sample versions generally earn lower Sharpe ratios than the simple investments in the original, unmanaged portfolios.
- This poor out-of-sample performance for volatility-managed portfolios stems primarily from structural instability in the underlying spanning regressions.

Understanding why the alpha estimated from regression analysis may not be achievable

- Consider the regression to measure the alpha from volatility timing:

$$f_t^\sigma = \alpha + \beta f_t + \epsilon_t.$$

- Gibbons, Ross, and Shanken (1989) explain that to actually earn this alpha, you need to choose the optimal mean-variance portfolio that is a combination of
  1. the factor, f_t, and
  2. the volatility-timing strategy, f_t^σ.
- But the weights of this portfolio have to be chosen in real time
  - using only past (historical) data;
  - without knowing what will happen in the future (no look-ahead bias).
- The regression alpha, because it is estimated from the entire data, suffers from look-ahead bias.

Data used by Cederburg, O'Doherty, Wang, and Yan

- 9 factors used by Moreira and Muir (2017),
- plus 94 anomaly returns from
  - Hou, Xue, and Zhang (2015) and
  - McLean and Pontiff (2016).

Results of Cederburg, O'Doherty, Wang, and Yan

- They directly compare the Sharpe ratios earned by
  - volatility-scaled strategies with those of
  - unscaled strategies.
- Find no systematic evidence that volatility-managed portfolios outperform their unmanaged versions.
  - Volatility scaling generates a higher Sharpe ratio for 5 of the 9 equity factors examined by Moreira and Muir (2017).
  - Volatility-managed versions outperform in 53 cases out of 103.
  - Find that only eight strategies in the broad sample yield statistically significant Sharpe ratio differences in favor of volatility management, which are concentrated among momentum-related strategies.

End of our discussion of
Cederburg, O'Doherty, Wang, and Yan (2020)

Next, we look at the second paper
criticizing Moreira and Muir (2017)

Road map

Barroso and Detzel (2021)

- Barroso and Detzel (2021) study whether "limits to arbitrage" can explain the benefits of volatility-timing strategies.
- In particular, they investigate if
  - transaction costs,
  - arbitrage risk, and
  - short-sale impediments
- explain the abnormal returns of volatility-managed equity portfolios.

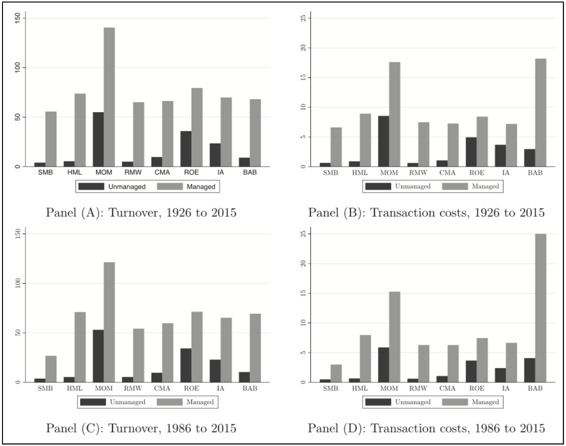
Findings of Barroso and Detzel (2021)

- After transaction costs, volatility management of factors (other than the market) generally significantly reduces Sharpe ratios.
- In contrast, the volatility-managed market portfolio is
  - robust to transaction costs (which are very small for market factor),
  - concentrated in the most easily arbitraged stocks
    (those with low arbitrage risk and few impediments to short selling).
- But, the volatility-timed market strategy has superior performance
  - only when sentiment is high,
  - consistent with theory that sentiment traders underreact to volatility.

Findings of Barroso and Detzel (2021)

- Factors other than the market have very high transaction costs
  - because they take relatively large positions in small-cap stocks
  - that are expensive to trade.
- Time-varying leverage inherent to volatility-managed portfolios further increases these trading costs by
  - increasing the maximum possible trade size and
  - forcing trades when none would otherwise exist in corresponding unmanaged portfolios.
- Turnover of volatility-managed factors is up to 15 times higher.

Turnover and transaction costs of unmanaged and managed strategies



From: Barroso and Detzel (2021, Figure 2)

End of our discussion of
Barroso and Detzel (2021)

Next, we look at the fourth paper
that resurrects the gains from volatility timing,
DeMiguel, Martín-Utrera, and Uppal (2024)

Road map

DMU address these criticisms of Moreira and Muir (2017)

- DMU = DeMiguel, V., A. Martín-Utrera, and R. Uppal. 2024. A multifactor perspective on volatility-managed portfolios. *Journal of Finance* 79 (6): 3859–3891. Available from this link.

- DMU show that volatility-timing can improve performance even
  1. Out of sample
  2. Net of transaction costs
  3. Independent of sentiment

Outline of discussion of DMU

1. Introduction . . . motivation and objective
2. Methodology . . . data and modeling approach
3. Results—performance gains . . . what DMU find
4. Source of performance gains . . . what drives DMU's results

Contribution of DMU

- DMU propose a new volatility-timing strategy, with four distinguishing features:

  1. Multifactor, instead of individual-factor portfolios.
  2. Relative factor weights can vary (as a function of market volatility), instead of having a fixed-weight multifactor portfolio.
  3. Account for trading diversification (netting of trades across factors) when computing transaction costs, as in DeMiguel, Martín-Utrera, Nogales, and Uppal (2020).
  4. Optimize factor weights accounting for transaction costs.

DMU: Key finding

- DMU's proposed volatility-managed multifactor strategy outperforms its unconditional counterpart (and Moreira and Muir strategies) even
  1. out-of-sample,
  2. net of transaction costs, and
  3. during both high- and low-sentiment periods.
- Implication: breakdown of risk-return tradeoff even more puzzling.

Road map

Related papers: Timing individual factors

- Factor-momentum strategies, which rely on the positive autocorrelation of factor returns (Ehsani and Linnainmaa 2019; Gupta and Kelly 2019).
- Timing the market using a business-cycle predictor derived from macroeconomic data (Gómez-Cram 2021).
- In contrast, DMU's strategy times multiple factors in combination.

Related papers: Timing combination of factors . . . I

- Dynamic portfolio approach using classification-tree analysis (Miller, Li, Zhou, and Giamouridis 2015).
- Factor portfolios conditional on macroeconomic state variables (Bass, Gladstone, and Ang 2017; Hodges, Hogan, Peterson, and Ang 2017; Amenc, Esakia, Goltz, and Luyten 2019; Bender, Sun, and Thomas 2018).
- Conditioning on macroeconomic regimes identified using Nowcasting (Blin, Ielpo, Lee, and Teiletche 2018).

Related papers: Timing combination of factors . . . II

- Multivariate Markov regime-switching model for the three traditional Fama-French factors (De Franco, Guidolin, and Monnier 2017).
- Timing the market and first five principal components of a large set of equity factors using value spread as timing variable (Haddad, Kozak, and Santosh 2020).
- In contrast, DMU study multifactor portfolios whose
  - weights change with market volatility,
  - which allows us to study breakdown of the risk-return tradeoff.

Related papers: Factor portfolios with transaction costs

- Gupta and Kelly (2019) study performance of factor momentum net of transaction costs.
- Barroso and Detzel (2021) study performance of volatility-managed portfolios net of transaction costs.
- In contrast, DMU's strategy
  - combines factors optimally and
  - accounts for trading diversification (netting out trades across factors before computing the transaction costs).

Road map

Data . . . I

- DMU's data contains all firms traded on
  - NYSE,
  - AMEX, and
  - NASDAQ.
- Use CRSP & Compustat data from Jan. 1967–Dec. 2020.
  - For out-of-sample analysis, DMU use an expanding-window approach, with the first window = 120 months, starting 1967.
  - To ensure fair comparison with out-of-sample results, the in-sample results are reported for the same period, Jan. 1977 to Dec. 2020.

Data . . . II

- Nine factors, as in Moreira and Muir & Barroso and Detzel:
  - Fama and French (2015):
    1. market (MKT),
    2. small-minus-big (SMB),
    3. high-minus-low (HML),
    4. robust-minus-weak (RMW),
    5. conservative-minus-aggressive (CMA)
  - Carhart (1997):
    6. momentum (UMD, up-minus-down)
  - Hou, Xue, and Zhang (2015):
    7. profitability (ROE),
    8. investment (IA).
  - Frazzini and Pedersen (2014)
    9. betting-against-beta (BAB)
- Robustness check: consider larger set of 60 risk factors.

Road map

Methodology in existing literature: (1) Individual factors

- Unmanaged factor return: f_{t+1}
- Volatility-managed factor return: f^{σ}_{t+1}

$$\overbrace{f^{\sigma}_{t+1}}^{\text{managed}} = \text{constant} \times \frac{\overbrace{f_{t+1}}^{\text{unmanaged}}}{\underbrace{(\sigma^f_t)^2}_{\text{scaling}}}$$

  - $(\sigma^f_t)^2$: variance of factor in month t (estimated using realized daily volatility over previous month).
  - "constant" equates volatility of unmanaged & managed factors.

- Combine managed and unmanaged factors to maximize mean-variance utility.

Methodology in existing literature: (2) Fixed-weight multifactor portfolios

- Optimal combination of
  - unconditional mean-variance multifactor portfolio and its
  - volatility-managed counterpart,
    - scaled using unconditional portfolio's past-month return variance.
- This portfolio assigns same relative weight to each factor as the unconditional mean-variance multifactor portfolio
  - thus, DMU call this "conditional fixed-weight multifactor portfolio."

DMU's methodology: General idea

- DMU consider a conditional mean-variance multifactor portfolio that
  - allows relative weights $\theta_{k,t}$ of different factors to vary over time, and
  - uses as the conditioning variable inverse market volatility, σ_t.

$$\theta_{k,t} = a_k + \frac{b_k}{\sigma_t}.$$

Conditional multifactor portfolio return

- Return of conditional multifactor portfolio is

$$r_{p,t+1}(\theta_t) = \sum_{k=1}^{K} r_{k,t+1}\theta_{k,t} = \sum_{k=1}^{K} r_{k,t+1}\left(a_k + \frac{b_k}{\sigma_t}\right).$$

- DMU use this return to do mean-variance optimization.

Conditional mean-variance multifactor portfolio

- Conditional mean-variance multifactor portfolio (CMV) solves

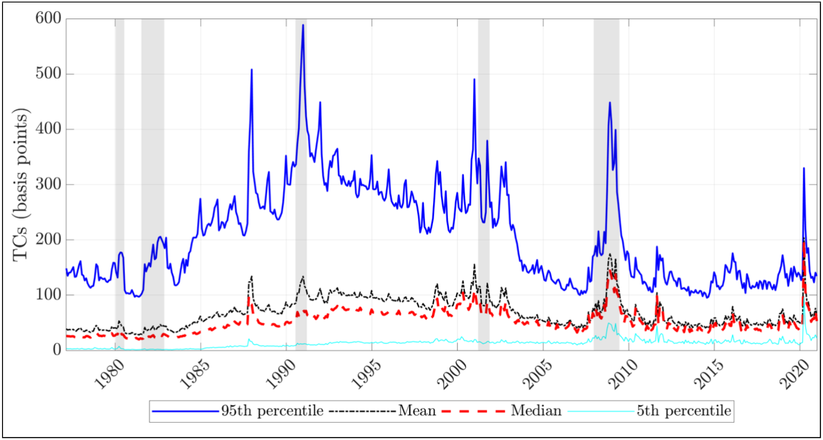
$$\max_{\{a_k \geq 0, b_k \geq 0\}} \quad \mathbb{E}[r_{p,t+1}(\theta_{k,t})] - \mathrm{TC}(\theta_{k,t}) - \frac{\gamma}{2} \mathrm{Var}[r_{p,t+1}(\theta_{k,t})]$$

  - $\mathbb{E}[r_{p,t+1}(\theta_{k,t})]$ is mean of multifactor portfolio return.
  - $\mathrm{TC}(\theta_{k,t})$ is average proportional transaction cost
  - $\mathrm{Var}[r_{p,t+1}(\theta_{k,t})]$ is the variance the multifactor portfolio return
- That is, to find the optimal portfolio, we need to maximize the above objective function over $a_k \geq 0$ and $b_k \geq 0$:
  - a_k: the portfolio component that is unconditional
  - b_k: the portfolio component that is conditional on volatility

Big picture understanding of transaction costs

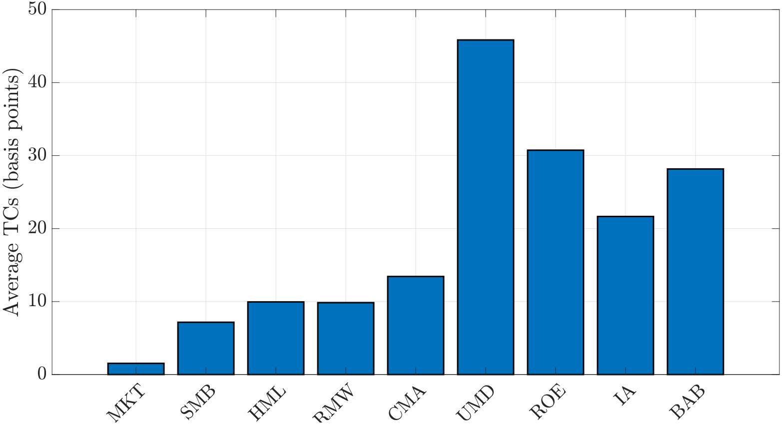
1. Estimate stock-level transaction-cost, as in Abdi and Ranaldo (2017) – figure on next slide shows why it is important to do this.
2. For each factor, determine trading in all the stocks required to rebalance the factor.
3. Net out trades in each stock across the nine factors in the portfolio.
4. Compute aggregate transaction cost for trading all stocks to rebalance all nine factors – figure on next-to-next slide shows that the cost for trading various factors is very different.

Transaction costs at level of individual stocks



- Transaction costs of individual stocks are highly time varying.
- The time variation is particularly strong for less liquid stocks.

Transaction costs for various factors



- As one would expect,
  - the cost for trading the market factor is the lowest;
  - the cost for trading the momentum factor is the highest.

Details: Transaction costs of rebalancing trades

- With trading diversification, transaction cost are estimated as

$$\mathrm{TC} = \frac{1}{T-1} \sum_{t=1}^{T-1} \|\Lambda_t (w_{t+1} - w_t^+)\|_1,$$

  - Λ_t, diagonal transaction-cost matrix whose ith diagonal element contains transaction cost parameter $\kappa_{i,t}$ of stock i,
  - $\|a\|_1 = \sum_{i=1}^{N} |a_i|$ is the 1-norm, and
  - w_t^+ is the conditional mean-variance multifactor portfolio before rebalancing at time $t+1$.

Details: Transaction costs of rebalancing trade

- With trading diversification, transaction cost are estimated as

$$\text{TC} = \frac{1}{T-1}\sum_{t=1}^{T-1} \|\Lambda_t(w_{t+1} - w_t^{+})\|_1,$$

  - That is, first net out the trades in each stock across the nine factors.
  - Then, add up the transaction costs to execute the net trades.

- If one ignores trading diversification, transaction costs are given by

$$\text{TC}(\eta) = \frac{1}{T-1}\sum_{t=1}^{T-1}\sum_{k=1}^{K} \|\Lambda_t\left(x_{k,t+1}\theta_{k,t+1} - x_{k,t}^{+}\theta_{k,t}\right)\|_1,$$

where $x_{k,t}^{+} = x_{k,t} \circ (e_t + r_{t+1})$ is the portfolio before rebalancing.

  - That is, first compute transaction cost for rebalancing each factor;
  - Then, add up the transaction costs across factors;
  - This is suboptimal.

Road map

Start by showing results in the existing literature
but using the DMU dataset.

Performance of individual-factor portfolios

- Performance measured using Sharpe ratio (SR)

	MKT	SMB	HML	RMW	CMA	UMD	ROE	IA	BAB
Panel I: In-sample without transaction costs									
SR(f)	0.530	0.208	0.170	0.506	0.399	0.474	0.722	0.508	0.880
SR(f, f^σ)	**0.585**	**0.246**	**0.215**	**0.739**	**0.419**	**1.088**	**1.153**	**0.621**	**1.397**
p-value($SR(f, f^\sigma) - SR(f)$)	0.244	0.376	0.338	0.038	0.308	0.000	0.001	0.099	0.000
Panel II: In-sample net of transaction costs (but without trading diversification)									
SR(f)	0.519	0.126	0.054	0.357	0.162	0.117	0.313	0.109	0.627
SR(f, f^σ)	**0.521**	0.126	0.054	0.357	0.162	**0.261**	**0.335**	0.109	**0.740**
p-value($SR(f, f^\sigma) - SR(f)$)	0.464	0.500	0.500	0.500	0.500	0.223	0.389	0.500	0.127
Panel III: Out-of-sample and net of transaction costs (but without trading diversification)									
SR(f)	0.519	0.126	0.054	0.357	0.162	0.117	0.313	0.109	0.627
SR(f, f^σ)	0.325	−0.292	−0.038	−0.442	−0.043	**0.204**	0.274	-0.122	**0.727**
p-value($SR(f, f^\sigma) - SR(f)$)	0.979	1.000	0.879	0.999	1.000	0.324	0.672	1.000	0.249

Road map

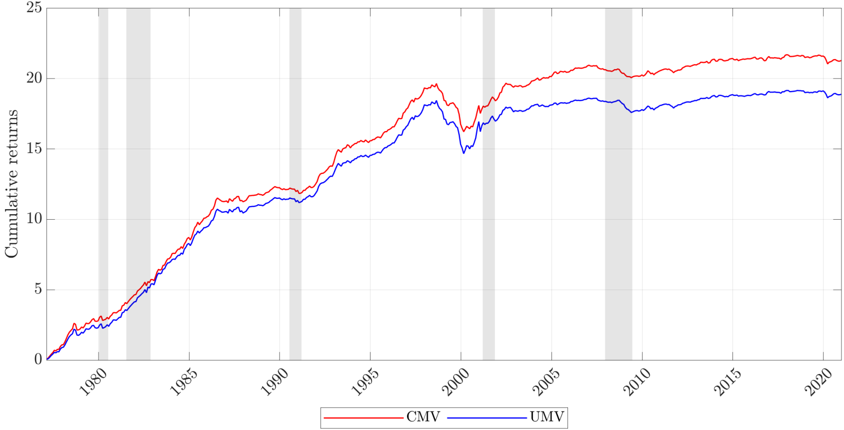
DMU's main results

- DMU's main result: for the new multifactor mean-variance portfolio, volatility timing improves performance even
  - OOS
  - net of TC
  - in both high- and low-sentiment periods

Main result: Performance of conditional multifactor portfolio (out of sample & net of transaction costs)

	unconditional UMV	conditional CMV
Mean	0.446	0.507
Standard deviation	0.459	0.450
Sharpe ratio	0.971	1.126
p-value($SR_{\text{CMV}} - SR_{\text{UMV}}$)	—	0.000
α (%)	—	8.195
$t(\alpha)$	—	4.416
TC	0.149	0.188

Cumulative wealth (OOS & net of TC)



Strategy	Return p.a.	Amt: 1977–2020
Unconditional multifactor portfolio	14.87%	$445
Conditional multifactor portfolio	17.60%	$1,253

Road map

Another key finding:

Volatility timing improves portfolio performance
in both high- and low-sentiment periods

Performance (Sharpe ratios) in different sentiment regimes

- Definition of high-sentiment years: sentiment in December of prior year is above its median value for entire sample.

	Entire sample			High sentiment			Low sentiment		
	Uncond.	Cond.	p-val.	Uncond.	Cond.	p-val.	Uncond.	Cond.	p-val.
Panel A: In sample									
Market	0.519	**0.532**	0.295	0.178	**0.217**	0.093	0.954	0.940	0.656
Multifactor	1.130	**1.339**	0.000	1.250	**1.594**	0.000	1.102	**1.150**	0.282
Panel B: Out of sample									
Market	0.519	0.449	0.889	0.178	0.082	0.887	0.954	0.952	0.496
Multifactor	0.971	**1.126**	0.001	1.403	**1.600**	0.003	0.412	**0.569**	0.016

- Out-of-sample, conditional multifactor portfolio significantly outperforms unconditional portfolio during both high and low sentiment.
- Thus, sentiment does not explain the out-of-sample and net-of-costs performance of DMU's conditional multifactor portfolio.

Road map

To identify sources of good performance
decompose the main result
into several small steps

Disentangling the source of the gains

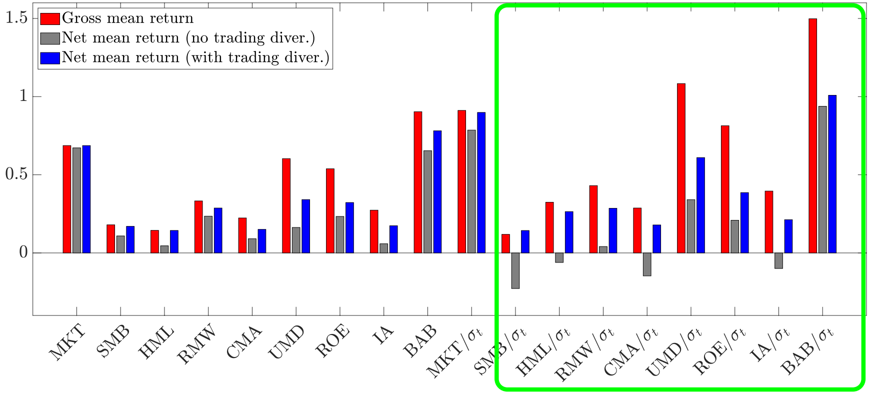
- In the paper,
  - DMU decompose performance into **16 steps**;
  - here, will show only 4 steps.
- Each of the columns reports the performance of multifactor portfolios evaluated in a different way:
  1. in-sample without TC,
  2. out-of-sample (OOS) without TC,
  3. out-of-sample with TC but ignoring trading diversification,
  4. out-of-sample with TC and with trading diversification.

Disentangling source of gains of CMV relative to UMV

	(1) In-sample without TC		(2) Out-of-sample without TC		(3) Out-of-sample with TC no trading diver.		(4) Out-of-sample with TC with trading diver.	
	UMV	CMV	UMV	CMV	UMV	CMV	UMV	CMV
Sharpe ratio	1.379	1.729	1.296	1.543	0.761	0.748	0.971	1.126
p-value($SR_{\text{CMV}} - SR_{\text{UMV}}$)		0.000		0.000		0.682		0.001
α		10.915		12.885		0.357		8.193
$t(\alpha)$		7.625		6.433		0.193		4.416

1. In-sample, CMV significantly outperforms UMV.
2. OOS, UMV & CMV perform less well, but CMV still outperforms UMV.
3. OOS & net of costs, without trading divers., CMV underperforms UMV.
4. OOS & net of costs with trading diversification, CMV outperforms UMV.

Gross and net-of-costs mean factor returns



- TCs reduce returns substantially, especially for managed factors.
- Net mean returns are negative for four factors (ignoring trading diver.)
- But, they have positive net mean returns with trading diversification.

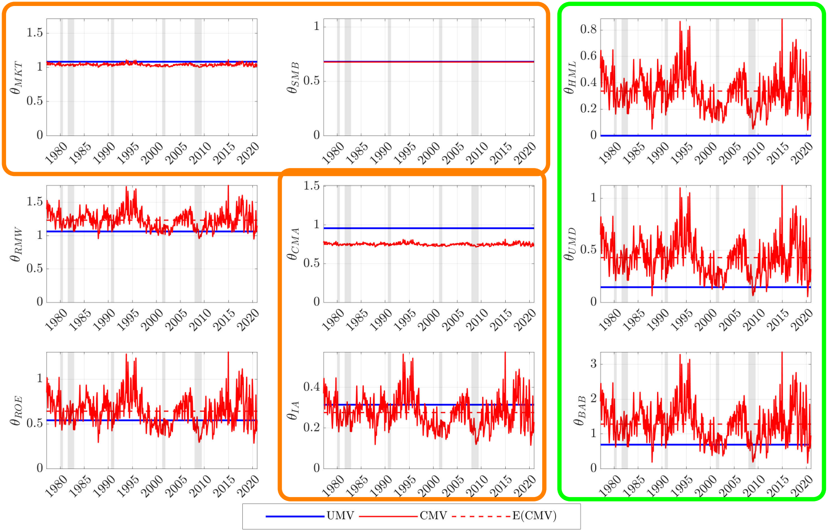
Trading diversification across and within factors

	(1) with full trading div. within & across factors		(2) with trading div. only within factors	(3) without any trading div.
	UMV	CMV	CMV	CMV
Sharpe ratio	0.971	1.126	0.790	0.748
p-value wrt SR_{UMV}		0.000	1.000	1.000
α		8.193	−7.028	−8.879
$t(\alpha)$		4.416	−3.664	−4.589
TC	0.149	0.188	0.339	0.358

- Without trading diversification, $SR_{CMV} < SR_{UMV}$.
- Trading diversification only within factors does not reduce TC much.
- Thus, trading diversification across factors is the main driver of TCs reduction, which can only be achieved with multifactor portfolios.

Time-variation of portfolio weights:

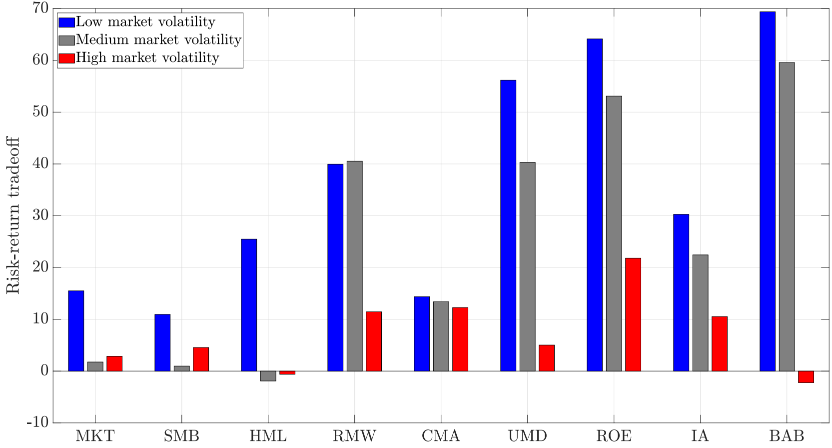
Unconditional (UMV), Conditional mean-variance (CMV)



Market volatility and returns

- DMU wish to study how the risk-return tradeoff for the nine factors in their dataset varies with realized market volatility.
- DMU use the monthly time series of realized market volatility to sort the months in their sample into terciles.
- For each factor, DMU then estimate the risk-return tradeoff for month t as the
  - realized factor return for month $t+1$
  - divided by the monthly realized factor variance estimated as the sample variance of daily returns for month t.
- Finally, they report the risk-return tradeoff averaged across the months in each tercile.

Key takeaway from experiment . . . I



Key takeaway from experiment . . . II

- For all nine individual factors the risk-return tradeoff weakens with market volatility.
- Moreover, the weakening of the risk-return tradeoff is
  - substantial for some of the factors (UMD, ROE, and BAB) but
  - less striking for others (SMB and CMA).
- This motivates using
  - a conditional multifactor portfolio that
  - allows the relative weights of the different factors to vary with market volatility.

Market volatility and factor returns: Regressions . . . I

- Study more formally the tradeoff between realized market volatility and returns of
  - nine individual factors
  - unconditional multifactor portfolio (UMV) using regression analysis.

$$r_{k,t+1} = \alpha + \beta\, \sigma_t + \epsilon_{t+1}$$

- If strong risk-return tradeoff, β should be positive.

Market volatility and factor returns: Regressions . . . II

Factor	MKT	SMB	HML	RMW	CMA	UMD	ROE	IA	BAB	UMV
Intercept	0.709	−0.221	0.567	0.127	0.108	1.248	0.622	0.188	1.471	2.670
t-stat	[1.311]	[−1.046]	[2.200]	[0.735]	[0.741]	[2.472]	[2.547]	[1.434]	[4.965]	[4.578]
Slope (β)	−0.041	0.361	− 0.569	0.118	−0.018	− 1.202	−0.426	−0.141	− 0.904	− 0.742
t-stat	[−0.064]	[1.593]	[−1.953]	[0.554]	[−0.121]	[−1.861]	[−1.389]	[−1.045]	[−2.503]	[−1.145]

- Of the slopes for individual factors, none are significantly positive.
- Slope for UMV portfolio is negative, with a t-stat of −1.145.
- Slopes for HML, UMD, and BAB are negative, with large t-statistics.
- CMV exploits heterogeneity in risk-return tradeoff across factors: times HML, UMD, and BAB more aggressively.

Road map

Robustness checks

1. Consider a large set of 60 factors, instead of just 9.
2. Performance during periods of high volatility.
3. Exclude the market or BAB from the multifactor portfolio.
4. Relax non-negativity constraints.
5. Constrain leverage.
6. Condition on each factor's own volatility, instead of market volatility.
7. Condition on the value spreads in addition to market volatility.
8. Condition on business-cycle variables, besides market volatility.
9. Alternative multifactor portfolio with a general covariance matrix.
10. Evaluate performance using alternative risk measures.
11. Consider investment horizons of up to 18 months.

Road map

Takeaways

- What DMU do: New strategy to time investment in multiple factors
  - Allow relative weight on each factor to vary with market volatility.
  - Optimize for transaction costs with trading divers. across factors.
- What DMU find: Volatility timing leads to large performance gains
  - out-of-sample,
  - net of transaction costs, and
  - for both low- and high-sentiment periods.
- What this means: Breakdown of risk-return tradeoff
  - is even more puzzling than previously thought.

End of discussion of
DeMiguel, Martín-Utrera, and Uppal (2024)

End of focus

Road map

What we plan to do in the next chapter

What's next?

In the next chapter, we will test if it is actually optimal to diversify away unsystematic risk, or, instead, is it better to construct a portfolio to earn the compensation for bearing unsystematic risk.

To do for next class

- Readings
  - The best reference for volatility-timed portfolios is Moreira and Muir (2017); you can download the original article from this link.
  - Python code to implement volatility-timing strategies is available from this link, which is written by Alan Moreira.

Road map

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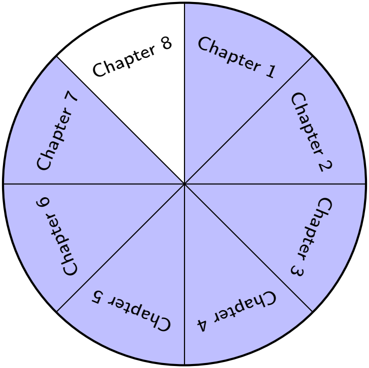

End of Chapter 7