Quantitative Portfolio Management

Sample Final Exam

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1 Managing financial data and performance evaluation

Assume that you are an investment manager who wishes to estimate the mean and volatility (standard deviation) of stock market returns over the last 100 years.

Suppose you have data for **daily prices** for the last 100 years; assume that there are 250 trading days each year, so you have a total of 25 000 prices. The data is in an Excel file called "data.xlsx". The first column has the dates in the format yyyy mm dd and the second column has the prices.

Q1.1 (**1 point**) Please write the Python code to calculate the *annualized mean and volatility* (standard deviation) of **daily** log returns over the 100 years. Include code to import any libraries needed for the calculation. Include comments to make it easier to understand the Python code.

Q1.2 (**1 point**) Please write the Python code to calculate, using the same data, the annualized mean of **monthly** log returns and volatility of log returns over the 100 years. There is no need to import the libraries or data again; i.e., there is no need to repeat any code you have written for the previous question.

Q1.3 (**1 point**) Please **compare** the precision of the **mean** of log returns computed from **daily** data to that computed from **monthly** data. That is, will one be more precise than the other?

Q1.4 (**1 point**) Please **compare** the precision of the **volatility** of log returns computed from **daily** data to that computed from **monthly** data. That is, will one be more precise than the other?

Q1.5 (**1 point**) This part is unrelated to the earlier ones.
Consider the setting where Greta works for an asset management company. Greta has a client, Celine Dijon, who has 100 million euros invested in a market index. She now wishes to invest an additional 1 million euros, and she is considering investing in a biotech company that is doing innovative research on the frontier of medical research. The mean returns in excess of the risk-free rate, volatility of excess returns, and Sharpe ratios are given below.

Item	Market	Biotech
Mean of excess returns	0.10	0.15
Volatility of excess returns	0.20	0.50
Sharpe ratio	0.50	0.30

If Greta says that investing in the biotech company is not a good idea because its Sharpe ratio is lower than the market's, what would be your response to her?

END of Q1 ♠

2 Mean-variance portfolio choice (without and with shrinkage)

Suppose that you are a portfolio manager. You have a client who wishes to invest in three risky assets but wants to minimize her risk as much as possible. So, to satisfy the client's desire to minimize risk, you recommend that she invest in the global minimum variance (GMV) portfolio. The client tells you that the three risky assets are **uncorrelated** and have annual return means of μ_1, μ_2, μ_3, and annual return volatilities of σ_1, σ_2, σ_3.

Q2.1 (**1 point**) Write down the **objective function** of the investor and any **constraints** that the optimization problem must satisfy.

Q2.2 (**1 point**) Write the **Python code** to solve the constrained optimization problem above. Include code to import any libraries needed for the optimization.

Q2.3 (**2 points**) **Solve this optimization problem** (by hand; i.e., analytically) to obtain the expression for the optimal weights in the three assets. Your answer should be an expression depending on the return moments, i.e., the annual return means, μ_1, μ_2, μ_3, and the annual return volatilities, σ_1, σ_2, σ_3.

Q2.4 (**1 point**) Would **Ledoit-Wolf shrinkage** applied to the returns covariance matrix improve the out-of-sample performance of the optimal portfolio weights you estimate? Explain why or why not.

END of Q2 ♠

3 Black-Litterman model

Imagine that you work for an investment bank that uses the Black-Litterman model to construct portfolios for its clients.

Q3.1 (**0.5 points**) Explain to your new client the underlying **logic and motivation** of the two major steps of the Black-Litterman model, which are to:
(i) back-out expected excess returns from the CAPM; and
(ii) blend expected excess returns from the CAPM with the investor's views.

Q3.2 (**0.5 points**) Explain to the client why the Black-Litterman approach, which is based on the two steps described above, is better than simply **blending the market-portfolio weights** with the views of the investor.

Q3.3 (**2 points**) Find the market risk aversion, γ_{mkt}, if

$$\gamma_{\text{mkt}} = \frac{\text{SR}_{\text{mkt}}}{\sigma_{\text{mkt}}},$$

and the market consists of only three risky assets, $n = \{1, 2, 3\}$, whose excess returns are **uncorrelated** and their relative stock-market capitalizations (i.e., market weights), mean excess returns, and volatilities (standard deviations) are given below:

Item	Asset 1	Asset 2	Asset 3
Weight in market portfolio	w_1	w_2	w_3
Mean of sample excess returns	μ_1	μ_2	μ_3
Volatility of sample excess returns	σ_1	σ_2	σ_3

Q3.4 (**1 point**) Now, find the **CAPM-implied expected excess returns**, $\mathbb{E}[R_{\text{CAPM}}]$, if

$$w_{\text{mkt}} = \frac{1}{\gamma_{\text{mkt}}} \mathbb{V}[R_{\text{sample}}]^{-1} \mathbb{E}[R^{\text{capm}}], \quad \text{where:} \quad \gamma_{\text{mkt}} = \frac{\text{SR}_{\text{mkt}}}{\sigma_{\text{mkt}}},$$

which, in expanded form, can be written as

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \frac{1}{\gamma_{\text{mkt}}} \left(\begin{bmatrix} \sigma_1^2 & 0 & 0 \\ 0 & \sigma_2^2 & 0 \\ 0 & 0 & \sigma_3^2 \end{bmatrix} \right)^{-1} \begin{bmatrix} E[R_1^{\text{capm}}] \\ E[R_2^{\text{capm}}] \\ E[R_3^{\text{capm}}] \end{bmatrix}$$

and, **as before**, the market consists of only three risky assets: $n = \{1, 2, 3\}$ whose excess returns are **uncorrelated** and their relative market capitalizations (i.e., market weights), mean excess returns, and volatilities (standard deviations) are:

Item	Asset 1	Asset 2	Asset 3
Weight in market portfolio	w_1	w_2	w_3
Mean of sample excess returns	μ_1	μ_2	μ_3
Volatility of sample excess returns	σ_1	σ_2	σ_3

Q3.5 (**1 point**) Please write the **pick matrix** P and the **view vector** Q if the investor's beliefs are the following:

- The first and second assets will have equal returns on average;
- The third asset is expected to have a return of 8%.

END of Q3 ♠

4 Factor-based portfolios

Suppose that you work for an asset-management company that believes that the N stock returns are driven by the three-factor ($K = 3$) Fama-French model, where the three factors are the returns on the market (mkt) minus the risk-free asset, size (smb), and value (hml) long-short portfolios. That is, excess returns on assets i and j can be written as:

$$R_i - R_f = \alpha_i + \beta_{i,\text{mkt}}(R_{\text{mkt}} - R_f) + \beta_{i,\text{smb}}R_{\text{smb}} + \beta_{i,\text{hml}}R_{\text{hml}} + e_i, \quad \text{where } \mathbb{E}[e_i] = 0,\ \text{Cov}[e_i, e_j] = 0,$$
$$R_j - R_f = \alpha_j + \beta_{j,\text{mkt}}(R_{\text{mkt}} - R_f) + \beta_{j,\text{smb}}R_{\text{smb}} + \beta_{j,\text{hml}}R_{\text{hml}} + e_j, \quad \text{where } \mathbb{E}[e_j] = 0,\ \text{Cov}[e_i, e_j] = 0,$$

and σ^2_{mkt}, σ^2_{smb}, and σ^2_{hml} are the variances of the returns on the three factors, which are orthogonal to one another; i.e., the covariance between each pair of factors is zero.

Q4.1 (**1 point**) What is the covariance between the returns on Asset i and Asset j?

Q4.2 (**1 point**) Write down the steps for using this asset-returns factor model for constructing a mean-variance optimal portfolio of N assets; that is, a factor-based portfolio policy (**not** parametric portfolio policy, which is the next question).

Q4.3 (**1 point**) Now we consider **parametric portfolio policies**. What is the logic underlying parametric portfolio policies? What are the strengths of parametric portfolio policies as opposed to using a factor model for asset returns and then choosing portfolio weights based on the factor model?

Q4.4 (**1 point**) Explain how one can implement **parametric portfolio policies** for the three-factor Fama-French model. In particular, write down the optimization problem of an investor who wishes to maximize **mean-variance utility** to obtain the optimal parametric portfolio. Make sure to write down the **choice variables** over which the investor will be optimizing and also the expressions for the **mean** and **variance** of the returns of the parametric portfolio.

Q4.5 (**1 point**) Suppose you find empirical evidence that the risk-return relation is not strong; i.e., changes in factor variance do not lead to proportional changes in factor mean returns. Therefore, you would like to extend the parametric portfolios you have described above so that they are **conditional on the volatility of each factor**.

Explain how you will construct a parametric portfolio policy whose weights on each factor are conditional on the volatility of that factor. Make sure that the portfolio you recommend depends only on historical data and does not suffer from look-ahead bias.

END of Q4 ♠

END of exam ♠