Deep Learning

Theoretical Exercises – Week 2 – Chapter 3

Exercises on the book "Deep Learning" written by Ian Goodfellow,
Yoshua Bengio, and Aaron Courville.
Exercises and solutions by T. Méndez and G. Schuster

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1 Exercises on Probability and Information Theory

1. Given a Bernoulli distributed random variable x with $P(\mathrm{x} = 1) = \frac{3}{4}$ and $P(\mathrm{x} = 0) = \frac{1}{4}$.
   (a) Draw the probability mass function $P_{\mathrm{x}}(x)$.
   (b) Find the expected value of x, i.e. $\mathbb{E}_{\mathrm{x}}[x]$.
   (c) Find the variance of x, i.e. $\mathrm{Var}_{\mathrm{x}}(x)$.
2. Given a uniformly distributed random variable x between -0.5 and 1.
   (a) Draw the probability density function $f_{\mathrm{x}}(x)$.
   (b) Find the expected value of x, i.e. $\mathbb{E}_{\mathrm{x}}[x]$.
   (c) Find the variance of x, i.e. $\mathrm{Var}_{\mathrm{x}}(x)$.
3. There are 5000 yellow and 100 red cabs in a city. In a hit-and-run accident, a witness saw a red cab. What is the probability that it was actually a red cab if witnesses in 95% of the cases state the car's colour correctly?
4. Given are the following sets of samples of two different classes c_1 and c_2:

$$S_1 = \{5.3,\ 5.7,\ 6.1,\ 6.3,\ 6.6\},$$

$$S_2 = \{6.2,\ 6.5,\ 6.9,\ 7.7,\ 8.0,\ 8.3,\ 8.9\}.$$

   (a) Estimate the mean (expected value) and the variance of both classes and sketch their probability density function assuming that the samples of both classes are normally distributed.
   (b) Estimate the prior probabilities $P(y = c_i)$ of both classes.
   (c) Assign the new samples $x_1 = 5$ and $x_2 = 7$ to the class with the highest posterior probability.
   (d) Calculate the entropy $H(x \mid y)$ of x with $y = c_1$ and $y = c_2$ respectively.