# Deep Learning

## Theoretical Exercises – Week 1 – Chapter 2

Exercises on the book ”Deep Learning” written by Ian Goodfellow, Yoshua Bengio, and Aaron Courville.
Exercises and solutions by T. Méndez and G. Schuster

FS 2024

## 1 Exercises on Linear Algebra

1. Calculate the following matrix products.

   (a) $\boldsymbol{C} = \boldsymbol{A}\boldsymbol{B}$ and $\boldsymbol{D} = \boldsymbol{B}\boldsymbol{A}$
   with $\boldsymbol{A} = \begin{pmatrix} 7 & 7 & 0 \\ 8 & 4 & 2 \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} 7 & 0 \\ 0 & 1 \\ 3 & 8 \end{pmatrix}$

   (b) $\boldsymbol{b} = \boldsymbol{A}\boldsymbol{x}$
   with $\boldsymbol{A} = \begin{pmatrix} 7 & 7 & 0 \\ 8 & 4 & 2 \end{pmatrix}$ and $\boldsymbol{x} = \begin{pmatrix} 7 \\ 3 \\ 10 \end{pmatrix}$

   (c) $\boldsymbol{c} = \boldsymbol{h}^T\boldsymbol{A}$
   with $\boldsymbol{A} = \begin{pmatrix} 7 & 7 & 0 \\ 8 & 4 & 2 \end{pmatrix}$ and $\boldsymbol{h} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$

2. Calculate the inverse of the orthogonal matrix

$$\boldsymbol{A} = \frac{1}{3}\begin{pmatrix} 2 & -2 & 1 \\ 1 & 2 & 2 \\ 2 & 1 & -2 \end{pmatrix}.$$

   and check whether your result is correct. Also find the solution of $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}$ with

$$\boldsymbol{b} = \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$$

   using the inverse matrix.

3. Calculate the $L^p$ norm of the vector $\boldsymbol{y} = \begin{pmatrix} 4 & -3 \end{pmatrix}^T$ for $p = 1, 2, 3$.

4. The following overdetermined, linear system of equations $\boldsymbol{Ax} = \boldsymbol{b}$ is given, where

$$\boldsymbol{A} = \begin{pmatrix} 1 & 2 \\ 1 & 4 \\ 1 & 6 \end{pmatrix}, \quad \text{and} \quad \boldsymbol{b} = \begin{pmatrix} 1.8 \\ 3.3 \\ 4.1 \end{pmatrix}.$$

Since there are more linear independent equations than unknowns, there is no solution. For this reason, the most reasonable solution should be found, by using the Moore-Penrose pseudoinverse

$$\boldsymbol{A}^+ = \boldsymbol{V}\boldsymbol{D}^+\boldsymbol{U}^T,$$

for which a singular value decomposition must be carried out. To perform a singular value decomposition, follow the steps below.

(a) Determine the eigenvalues $\lambda_1$ and $\lambda_2$ of $\boldsymbol{B} = \boldsymbol{A}^T\boldsymbol{A}$, where $\lambda_1 > \lambda_2$.

(b) Determine the Matrix $D$ as $D = \begin{pmatrix} \sqrt{\lambda_1} & 0 \\ 0 & \sqrt{\lambda_2} \\ 0 & 0 \end{pmatrix}$

(c) Determine matrix $\boldsymbol{V} = \begin{pmatrix} \boldsymbol{v}^{(1)} & \boldsymbol{v}^{(2)} \end{pmatrix}$, where $\boldsymbol{v}^{(i)}$ are the eigenvectors of $\boldsymbol{B} = \boldsymbol{A}^T\boldsymbol{A}$. Note that $\boldsymbol{v}^{(1)}$ is the eigenvector to the larger eigenvalue $\lambda_1$.

(d) Determine matrix $\boldsymbol{U} = \begin{pmatrix} \boldsymbol{u}^{(1)} & \boldsymbol{u}^{(2)} & \boldsymbol{u}^{(3)} \end{pmatrix}$, where $\boldsymbol{u}^{(i)}$ are the eigenvectors of $\boldsymbol{C} = \boldsymbol{A}\boldsymbol{A}^T$. Note that $\boldsymbol{u}^{(1)}$ is the eigenvector to the largest eigenvalue and $\boldsymbol{u}^{(3)}$ is the eigenvector to the smallest eigenvalue.

(e) Check weather $\boldsymbol{U}$ and $\boldsymbol{V}$ are orthogonal matrices and weather $\boldsymbol{A} = \boldsymbol{U}\boldsymbol{D}\boldsymbol{V}^T$

(f) Determine the Matrix $D^+$.

(g) Determine the Moore-Penrose pseudoinverse $A^+$.

(h) Determine the most reasonable solution $\boldsymbol{x}$.

**Hint:**
Use the calculator or python to determine the eigenvalues and eigenvectors.