Image Processing and Computer Vision 1

2 – Introduction to Image Processing – week 2

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1 Exercises from the Book

1.1 Book by Gonzalez and Woods, 2.22

In Chapter 3 we will deal with operators whose function is to compute the sum of pixel values in a small sub image area, S_{xy}, as in Eq. (2-43), where the denominator mn equals the number of pixels in the area S_{xy}:

$$g(x,\, y) = \frac{1}{mn} \sum_{(r,c) \in S_{xy}} f(r,\, c) \tag{2-43}$$

Show that these are linear operators.

1.2 Book by Gonzalez and Woods, 2.26

Averaging of noisy images for noise reduction. Prove the validity of

(a)

$$\mathbb{E}\left\{\bar{g}(x,y)\right\} = f(x,y) \tag{2-27}$$

(b)

$$\sigma^2_{\bar{g}(x,y} = \frac{1}{K}\sigma^2_{\eta(x,y)} \tag{2-28}$$

For part (b) you will need the following facts from probability: (1) the variance of a constant times a random variable is equal to the constant squared times the variance of the random variable. (2) The variance of the sum of uncorrelated random variables is equal to the sum of the variances of the individual random variables.

1.3 Book by Gonzalez and Woods, 2.32

Give expressions (in terms of sets A, B, and C) for the sets shown shaded in the following figures. The shaded areas in each figure constitute one set, so give only one expression for each of the four figures.

Write the expressions using the set operators $\cup$, $\cap$ and ^{c} for union, intersection and the compliment, respectively.

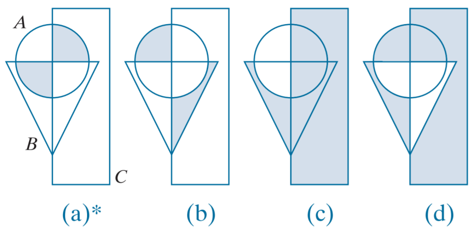

Figure belonging to exercise 2.34

1.4 Book by Gonzalez and Woods, 2.42

Show that 2-D transforms with separable, symmetric kernels can be computed by

(a) computing 1-D transforms along the individual rows (columns) of the input, followed by

(b) computing 1-D transforms along the columns (rows) of the result from step (a)

2 Practical Exercise

Let $f(x, y)$ be a noiseless image. We add to $f(x, y)$ a furterh image $\eta(x, y)$ consisting of pure noise and thus yield a noise image version

$$g(x, y) = f(x, y) + \eta(x, y). \tag{2.25}$$

The noise realizations $\eta(x, y)$ of each pixel with coordinates (x, y) are Gaussian distributed with expected value $\mu = 0$ and standard deviation $\sigma = 1$. For this the short hand notation $\eta(x, y) \sim \mathcal{N}(\mu, \sigma^2)$ is common. Using `numpy` we can generate a matrix whose components are uncorrelated samples of a gaussian random variable with zero mean and unit variance using the command `np.random.randn()`, see the documentation of `randn` for details. Scale the noise image $\eta(x, y)$ so that its variance becomes equivalent to the variance of the image $f(x, y)$, this choice makes the noise clearly visible.

Create 100 uncorrelated versions of noise matrices $\eta_i(x, y)$ to form 100 noisy images $g_i(x, y) = f(x, y) + \eta_i(x, y)$. Now average these images i.e. compute

$$\bar{g}(x, y) = \frac{1}{K} \sum_{i=1}^{K} g_i(x, y) \tag{2.26}$$

what variance dos the average image have?

Additional Exercise

Use the script `webcam.*` to read images from the webcam. Set the exposure time to a low value or close the iris of the camera to get a very dark image. Then use the gain to increase the image brightness. You will get a very noisy image. Use the code from the practical exercise above to remove the noise.