1 Book

1.1 Book by Gonzalez and Woods, 3.1

Give a single intensity transformation function for spreading the intensities of an image so that the lowest intensity becomes 0 and the highest becomes $L-1$. Let f denote the original image with its lowest intensity $f_{\min}$ and highest intensity $f_{\max}$.

First subtract the minimum value to yield a function whose minimum value is 0*:*

$$g_1 = f - f_{\min}$$

Next, divide g_1 *by its maximum value to yield a function in the range* $[0, 1]$ *and multiply the result by* $L-1$ *to yield a function with values in the range* $[0, L-1]$*:*

$$g = \frac{L-1}{\max(g_1)} g_1 = \frac{L-1}{(f_{\max} - f_{\min})}(f - f_{\min})$$

Keep in mind that $f_{\min}$ *is a scalar and* f *is an image.*

1.2 Gamma Transform

Assume you want to apply the gamma transform $s = T(r) = c \cdot r^{\gamma}$. Find the proper value for the constant c such that $T(0) = 0$ and $T(L-1) = L-1$ for $L = 256$

We have the two constraints $T(0) = c \cdot r^{\gamma} = 0$ *and* $T(L-1) = c \cdot (L-1)^{\gamma} = L-1$*. The first constraint is always satisfied. The second constraint is satisfied when*

$$c = \frac{L-1}{(L-1)^{\gamma}} = (L-1)^{1-\gamma} = 255^{-0.2} = 0.3301352998405312.$$

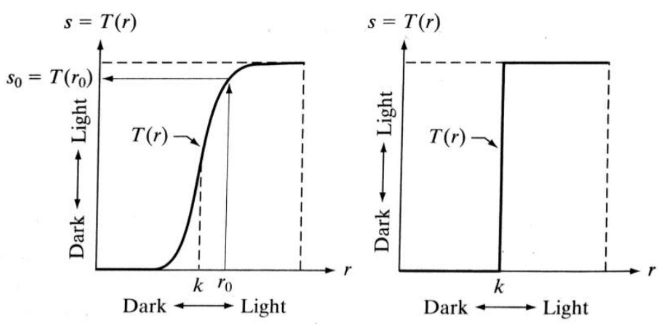
1.3 Book by Gonzalez and Woods, 3.2a

Give a continuous function $s = T(r)$ for implementing the contrast stretching transformation shown in Fig. 3.2(a). In addition to k, your function must include a parameter, E, for controlling the slope of the function as it transitions from low to high. Your function should be normalized so that its minimum and maximum values are 0 and 1, respectively.

Hint: $\frac{1}{2} = T(k), 0 = T(0), 1 = T(\infty)$

a b

FIGURE 3.2 Intensity transformation functions. (a) Contrast-stretching function. (b) Thresholding function.



$$s = T(r) = \frac{1}{1 + (k/r)^E}$$

1.4 Book by Gonzalez and Woods, 3.5a

What effect would setting to zero the lower-order bit planes have on the histogram of an image in general?

The number of pixels having different intensity level values would decrease, thus causing the count in the lower-order bins in the histogram to decrease. Because the number of pixels would not change, this would cause the number of counts of the higher order bins to increase, resulting in taller histogram peaks in the higher values. This will brighten the image but its tonality would decrease.

Note by the lecturer: The text of the solution must in part to be corrected and in part to be sharpened. For the latter consider the definitions:

(a) A grayvalue x falls into a lower-order bin if its binary representation has some of its lower-order bits set to one. It falls into a higher-order bin if the opposite is the case.

(b) Tonality is the overall appearance of an image regarding to the range and distribution of tones (gray values) and the smoothness of gradation between them. Tonality plays an important role in photography.

In contrast to the statment in the solution, the histogram peaks becoming taller are those that account for lower, not for higher gray values. The reason is that setting any bit of a binary number to zero will decrease its value. As a consequence, the image will darken, not brighten. Setting lower-order bits to zero will reduce the number of grayvalues in the image, hence, the gray value quantization becomes more coarse. A photographer would say, "the image's tonality decreases".

1.5 Book by Gonzalez and Woods, 3.6

Explain why the discrete histogram equalization technique does not, in general, yield a flat histogram.

All that histogram equalization does is remap histogram components on the intensity scale. To obtain a uniform (flat) histogram would require in general that pixel intensities actually be redistributed so that there are L groups of n/L pixels with the same intensity, where L is the number of allowed discrete intensity levels and $n = MN$ is the total number of pixels int the input image. The histogram equalization method has no provisions for this type of intensity redistribution process.