Image Processing and Computer Vision 1

Chapter 4 – Two Dimensional Fourier Transform – week 7

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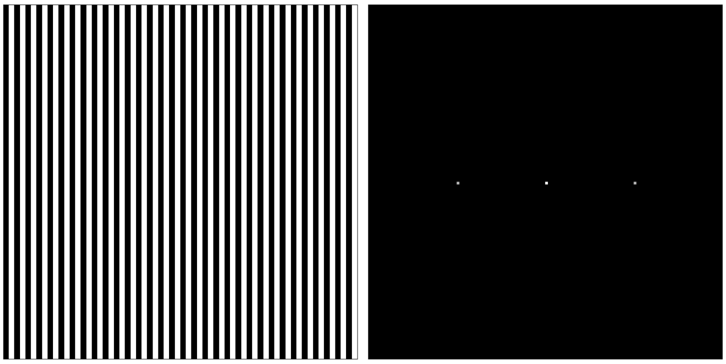
1 Book

1.1 Book by Gonzalez and Woods, 4.25

The image on the left in Fig. 4.25 consists of alternating stripes of black/white, each stripe being two pixels wide. The image on the right is the Fourier spectrum of the image on the left, showing the DC term and the frequency terms corresponding to the stripes. (Remember, the spectrum is symmetric so all components, other than the DC term, appear in two symmetric locations.)

(a) Suppose that the stripes of an image of the same size are four pixels wide. Sketch what the spectrum of the image would look like, including only the DC term and the two highest-value frequency terms, which correspond to the two spikes in the spectrum of Fig. 4.25.



Exercise Figure 4.25

1.2 Book by Gonzalez and Woods, 4.28

With reference to the discussion on linearity in Section 2.5, demonstrate that

(a) The 2-D continuous Fourier transform is a linear operator.

(b) The 2-D DFT is a linear operator also.

Definition of linearity

$$H\left[af_1(x,y)+bf_2(x,y)\right]=aH\left[f_1(x,y)\right]+bH\left[f_2(x,y)\right], \tag{2-23}$$

where $a,\, b \in \mathbb{R}$ and $f_1(x,\, y),\ f_2(x,\, y)$ are arbitrary real valued images.

1.3 Book by Gonzalez and Woods, 4.29

With reference to Eqs. (4-71) and (4-72), demonstrate the validity of the following translation (shifting) properties of 2-D, discrete Fourier transform pairs from Table 4.4. (Hint: Study the solutions to Problem 4.17.) (*Hint:* Study the solutions to Problem 4.11)

(a) $f(x, y) \cdot e^{j2\pi(u_0 x/M + v_0 y/N)} \Leftrightarrow F(u - u_0, v - v_0)$

Book section 4.5 *Extensions to Functions of Two Variables*

$$F(\mu, \nu) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(t, z) e^{-j2\pi(\mu t + \nu z)} dt dz \tag{4-59}$$

$$f(t, z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(\mu, \nu) e^{j2\pi(\mu t + \nu z)} d\mu d\nu \tag{4-60}$$

1.4 Book by Gonzalez and Woods, 4.39

The following problems are related to the properties in Table 4.1.

(a) Demonstrate the validity of property 2.

(b) Demonstrate the validity of property 4.

(c) Demonstrate the validity of property 5.

(d) Demonstrate the validity of property 7.

Table 4.1 Some symmetry properties of the 2-D DFT and its inverse. $R(u,v)$ and $I(u,v)$ are the real and imaginary parts of $F(u,v)$, respectively. The term *complex* indicates that a function has nonzero real and imaginary parts.

	Spatial Domain		Frequency Domain
1)	$f(x,y)$ *real*	$\Leftrightarrow$	$F^*(u,v) = F(-u,-v)$
2)	$f(x,y)$ *imaginary*	$\Leftrightarrow$	$F^*(-u,-v) = -F(u,v)$
3)	$f(x,y)$ *real*	$\Leftrightarrow$	$R(u,v)$ *even*; $I(u,v)$ *odd*
4)	$f(x,y)$ *imaginary*	$\Leftrightarrow$	$R(u,v)$ *odd*; $I(u,v)$ *even*
5)	$f(-x,-y)$ *real*	$\Leftrightarrow$	$F^*(u,v)$ *complex*
6)	$f(-x,-y)$ *complex*	$\Leftrightarrow$	$F(-u,-v)$ *complex*
7)	$f^*(x,y)$ *complex*	$\Leftrightarrow$	$F^*(-u,-v)$ *complex*

Book Table 4.1 Symmetry Properties of the 2-D DFT / 2-D IDFT

2 Practical Exercise – Spectrum of an Image

Run the script `template_spectrum.py` that computes and visualizes the specturm of an image $f(x, y)$. Observe the high frequency components in the specturm. Now consider the window $w(x, y)$ used to compute a smoothly faded out version $f'(x, y) = w(x, y) \cdot f(x, y)$ of the image $f(x, y)$ towards boundaries. Explain the effects of the window on the magnitude of the spectrum. Also observe, how the rolling operation of the spectrum can be achieved by rolling in the frequency-domain and by multiplying with a specific function in the time-domain.

3 Practical Exercise – Phase Correlation Method

Write a program which implements the phase correlation method for image registration:
http://en.wikipedia.org/wiki/Phase_correlation

Additional Task

Try to track an object in the field of view of the camera using the phase correlation method.