# Image Processing and Computer Vision 1

Chapter 4 – 2-D Convolution using the DFT – week 9

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# 1 Book

## 1.1 Book by Gonzalez and Woods, 4.40

You know from Table 4.3 that the DC-term, $F(0,0)$, of a DFT is proportional to the average value of its corresponding spatial image. Assume that the image is of size $M \times N$. Suppose that you pad the image with zeros to size $P \times Q$, where $P$ and $Q$ are given in Eqs. (4.102) and (4.103). Let $F_p(0,0)$ denote the DC-term of the DFT of the padded function $F_p(u,v)$.

(a) What is the ratio of the average values of the original and padded images?

(b) Is $F_p(0,0) = F(0,0)$? Support your answer mathematically.

Book section 4.6.6 *The 2-D Convolution Theorem*

$$P = 2M \tag{4.102}$$

$$Q = 2N \tag{4.103}$$

## 1.2 Filter mask interpration in the frequency domain

Consider a $3 \times 3$ spatial Mask that averages the four closest neighbors of a point $(x,y)$, but excludes the point itself from the average.

(a) Find the equivalent filter, $H(u,v)$, in the frequency domain.

(b) Show that the $H(u,v)$ is a lowpass filter transfer function for intensity oscillations along the $x$ axis and along the $y$ axis.

Book table 4.4 *Summary of DFT pairs*

$$f(x - x_0, y - y_0) \Leftrightarrow F(u,v) \cdot e^{-j2\pi(ux_0/M + vy_0/N)} \qquad \text{(3) Translation (general)}$$

## 1.3 Book by Gonzalez and Woods, 4.56

Do the following:

(a) Show that the Laplacian of a continuous function $f(t,z)$ of two continuous variables $t$ and $z$ satisfies the following Fourier transform pair:

$$\nabla^2 f(t,z) = \frac{\partial^2 f(t,z)}{\partial t^2} + \frac{\partial^2 f(t,z)}{\partial z^2} \Leftrightarrow -4\pi^2 \cdot \left(\mu^2 + \nu^2\right) \cdot F(\mu,\nu)$$

*Hint:* See Eq.(3-59) and study entry 12 in Table 4.4.

(b) The result in (a) is valid only for continuous variables. How would you implement the continuous frequency domain transfer function $H(\mu, \nu) = -4\pi^2 \cdot \left(\mu^2 + \nu^2\right)$ for discrete frequency variables $u$ and $v$?

(c) As you saw in Example 4.21, the Laplacian result in the frequency domain was similar to the result of using a spatial mask with a center coefficient equal to -8. Explain the reason why the frequency domain result was not similar instead to the result of using a spatial mask with a center coefficient of -4.

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \tag{3-59}$$

Book from Table 4.4 entry 12: *Differentiation Property*

$$\left(\frac{\partial}{\partial t}\right)^m \left(\frac{\partial}{\partial z}\right)^n f(t,z) \Leftrightarrow (j2\pi\mu)^m \cdot (j2\pi\nu)^n \cdot F(\mu,\nu)$$
$$\frac{\partial^m f(t,z)}{\partial t^m} \Leftrightarrow (j2\pi\mu)^m \cdot F(\mu,\nu)$$
$$\frac{\partial^n f(t,z)}{\partial z^n} \Leftrightarrow (j2\pi\nu)^n \cdot F(\mu,\nu)$$

### 1.4 Book by Gonzalez and Woods, 4.57

Can you think of a way to use the Fourier transform to compute (or partially compute) the magnitude of the gradient [Eq. (3-58)] for use in image differentiation? If your answer is yes, give a method to do it. If your answer is no, explain why.

Book section 3.6.4

$$\nabla f \equiv \text{grad}(f) \equiv \begin{bmatrix} g_x \\ g_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix} \tag{3-67}$$

$$M(x,y) = \text{mag}(\nabla f) = \sqrt{g_x^2 + g_y^2} \tag{3.6-11}$$

## 2 Practical Exercise

Write a program, which implements the filter in problem 1.2 in the frequency and in the spatial domain. Filter a digital image using these two filters and compare the resulting images.

## Additional Task

Apply the frequency domain filter from the exercise to the images of the camera. Try to manipulate and show the camera images in real time. Try to implement the Laplacian in the frequency domain.