Image Processing and Computer Vision 1

Chapter 5 – Image restauration – week 12

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1 Book

1.1 Book by Gonzalez and Woods, 5.34

Using the transfer function in Problem 5.33, give the expression $W(u,v)$ for a Wiener filter transfer function, assuming that the ratio of power spectra of the noise and undegraded signal is a constant.

Transfer function of Problem 5.33:

Spatial domain	$h(x,y) = \frac{x^2+y^2-2\sigma^2}{\sigma^4} \cdot e^{-\frac{x^2+y^2}{2\sigma^2}}$
Frequency domain	$H(u,v) = -8\pi^4\sigma^2 \cdot (u^2+v^2) \cdot e^{-2\pi^2\sigma^2(u^2+v^2)}$

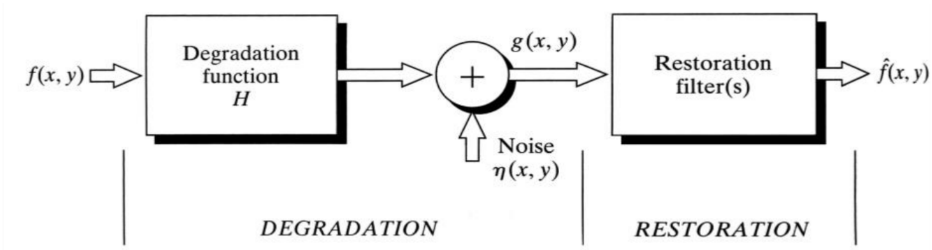
1.2 Book by Gonzalez and Woods, 5.38

Assume that the model in Fig. 5.1 is linear and position invariant and that the noise and image are uncorrelated. Show that the power spectrum of the output is

$$|G(u,v)|^2 = |H(u,v)|^2 \cdot |F(u,v)|^2 + |N(u,v)|^2$$

Refer to Eq. (5-65) and Eq. (4-89).

FIGURE 5.1 A model of the image degradation/restoration process



Book figure 5.1 *A model of the image degradation / restoration process*

$$G(u,v) = H(u,v)F(u,v) + N(u,v) \tag{5-65}$$
$$P(u,v) = |F(u,v)|^2 = R^2(u,v) + I^2(u,v) \tag{4-89}$$

$P(u,v)$ is the power spectrum, $R(u,v)$ and $I(u,v)$ are the real an imaginary parts of the Digital Fourier Transform $F(u,v)$.

1.3 Book by Gonzalez and Woods, 5.39

Cannon [1974] suggested a restoration filter $R(u,v)$ satisfying the condition

$$|\hat{F}(u,v)|^2 = |R(u,v)|^2 |G(u,v)|^2$$

The restoration filter is based on the premise of forcing the power spectrum of the restored image, $|\hat{F}(u,v)|^2$, to equal the spectrum of the original image, $|F(u,v)|^2$. Assume that the image and noise are uncorrelated.

(a) Find $R(u,v)$ in terms of $|F(u,v)|^2$ and $|N(u,v)|^2$.
Hint: Refer to Fig. 5.1, Eq.(5.65), and Problem 5.38.

(b) Use your result in (a) to state a result in the form of Eq. (5.81).

$$\hat{F} = \left[\frac{H^* \cdot S_f}{S_f \cdot |H|^2 + S_\eta} \right] \cdot G = \left[\frac{H^*}{|H|^2 + \dfrac{S_\eta}{S_f}} \right] \cdot G = \left[\frac{1}{H} \cdot \frac{|H|^2}{|H|^2 + \dfrac{S_\eta}{S_f}} \right] \cdot G \tag{5.81}$$

We show $\hat{F}$, H, G, S_f and S_η instead of $\hat{F}(u,v)$, $H(u,v)$, $G(u,v)$, $S_f(u,v)$ and $S_\eta(u,v)$ to simplify the notation.

2 Practical Exercise

An image contains a motion blur, which occurred during acquisition and can be modeled as follows

$$g(r,c) = f(r,c) + f(r-1,c-1) + f(r-2,c-2) + \ldots f(r-D,c-D).$$

Write a program which will create such a motion blur and adds zero mean white gaussian noise to the image. Write a program which restores the blurred and noisy image (play with the noise power) using a Wiener filter (play with the factor K).