Some of the theoretical contents are taken from the instructor manual of the book „Digital Image Processing (4th Edition)“ by Rafael C. Gonzalez and Richard E. Woods. Therefore, the confidentiality class of this document is „internally extended“. The data can be used by OST members as of September 1, 2020, but must not be passed on to third parties.

Image Processing and Computer Vision 1

Chapter 3 – Intensity Transformation – week 4

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1 Book

1.1 Book by Gonzalez and Woods, 3.10

Assuming continuous values, show by example that it is possible to have a case in which the transformation function given in Eq. (3.11) satisfies condition (a) and (b) in Section 3.3, but its inverse may fail condition (a').

We are interested in just one example in order to satisfy the statement of the problem. Consider the probability density function in Fig. P3.10(a). A plot of the transformation $T(r)$ in Eq. (3-11) using this particular density function is shown in Fig. P3.10(b), because $p_r(r)$ is a probabilty density function we know from the discussion in Section 3.3 that the transformation $T(r)$ satisfies conditions (a) and (b) stated in that section. However, we see from Fig. P3.10(b) that the inverse transformation from s back to r is not single valued, as there are an infinite number of possible mappings from, for example, $s = (L-1)/2$ back to r. It is important to note that the reason the inverse transformation function turned out not to be single valued is the gap in $p_r(r)$ in the interval $[L/4, 3L/4]$.

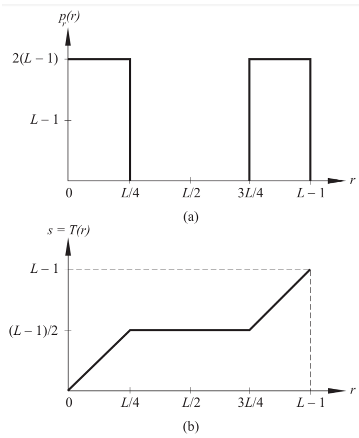

Figure P3.10

1.2 Inversion of cumulative distributions

Specify a procedure that inverts discrete cumulative distributions as shown in Fig. P3.22(c). The inversion is ambiguous in general. Define a meaningful and unique inversion. Apply this to create a lookup table that can be used to invert the function in the Fig. P3.22(c).

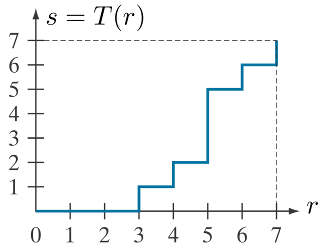

Figure P3.22(c)

The discrete cumulative distribution defines a mapping $T : r \mapsto s$ with $r \in \{0, 1, 2, \ldots, L-1\}$ and $s \in [0, L-1]$, with $s \in \mathbb{R}$. The function $T()$ is not strictly monotonously increasing and thus cannot be inverted uniquely. Despite of this, we can define an inversion rule $T^{-1} : s \mapsto r$ with $r, s \in \{0, 1, 2, \ldots, L-1\}$ that gives a unique result. We achieve this by a decision: If multiple values of r are mapped to the same s then their median should be used as the output of the inversion. Remember the convention $r_k = k$, also note that $s_k \neq k$. The rule can be defined as follows:

(a) round $s \in 0, 1, \ldots, L-1$ to the closest possible value $s_k = T(r_k)$ for $k = r_k \in \{0, 1, 2 \ldots, -1\}$:

$$k := \arg \min_{i \in 0,1,2,\ldots,L-1} \{|s - s_i|\}$$

If this result is ambiguous i.e. if multiple values of k solve the minimization problem, then use the median of these values to define a unique k.

(b) Now $s_k := T(r_k)$. consequently, as an output of the inversion define the value $r_k = k$.

(c) The lookup table maps indices to intensities according to $T^{-1} : (0, 1, 2, 3, 4, 5, 6, 7) \mapsto (1, 3, 4, 4, 5, 5, 6, 7)$

1.3 Book by Gonzalez and Woods, 3.12

Two in general different images, $f(x,y)$ and $g(x,y)$ have unnormalized histograms h_f and h_g. Give the conditions (on the values of the pixels in g) under which you can determine the histograms of images formed as follows:

(a) $f(x,y) + g(x,y)$

(b) $f(x,y) - g(x,y)$

(c) $f(x,y) \times g(x,y)$

(d) $f(x,y) \div g(x,y)$

Show how the histograms would be formed in each case. The arithmetic operations are elementwise operations, as defined in Section 2.6.

The purpose of this simple problem is to make the student think of the meaning of histograms and arrive at the conclusion that histograms carry no information about spatial properties of images. Thus, the only time that the histogram of the images formed by the operations shown in the problem statement can be determined in terms of the original histogram is when one (both) of the images is (are) constant. In (d) we have the additional requirement that none of the pixels of $g(x,y)$ can be 0. It is given that the histograms are not normalized, so, for example, $h_f(r_k)$ is the number of pixels in $f(x,y)$ having intensity level r_k. Assume that all the pixels in $g(x,y)$ have constant value c. The pixels of both images are assumed to be positive. Finally, let u_k denote the intensity levels of the pixels of the images formed by any of the arithmetic operations given the problem statement. Under the preceeding set of conditions, the histograms are determined as follows:

(a) We obtain the histogram $h^+(u_k)$ of the sum of f and g by letting $u_k = r_k + c$, and also $h^+(u_k) = h_f(r_k)$ for all k. In other words, the values (height) of the components of h^+ are the same as the components of h_f, but their locations on the intensity axis are shifted right by an amount c.

(b) Similarly, the histogram $h^-(u_k)$ of the difference of f and g has the same components as h_f but their locations are moved left by an amount c as a result of the subtraction operation. The subtraction cannot result in negative numbers, so this placees a limitation on s which depends on the values of the pixels of corresponding pairs of pixels in the images.

Note by the lecturer: In general the result can be negative. With the same right the author forbids negative values for histogram bins, he could forbid values larger than $L-1$ in the above answer.

(c) Following the same reasoning, the values (heights) of the components of the histogram $h^\times(u_k)$ of the product of f and g are the same as h_f, but their locations are at $u_k = c \times r_k$. Note that while the spacing between components of the resulting histograms in (a) and (b) was not affected, the spacing between components of $h^\times(u_k)$ will be spread out by an amount c.

(d) Finally, assuming that the values of g are $c > 0$, the components of $h^\div(u_k)$ are the same as those of h_f, but their location will be at $u_k = r_k \div c$. Thus, the spacing between components of $h^\div(u_k)$ will be compressed by an amount equal to $1/c$. The preceeding solutions are applicable also when both images are constant. In this case the four histograms just discussed would each have only one component. The locations of those components would be affected as described (a) through (d).

1.4 Book by Gonzalez and Woods, 3.21

The local histogram processing method discussed in Section 3.3 requires that a histogram be computed at each neighborhood location. Propose a method for updating the histogram from one neighborhood to the next, rather than computing a new histogram each time.

The value of the histogram component corresponding to the kth intensity level in a neighborhood is

$$p_r(r_k) = \frac{n_k}{n}$$

for $k = 0, 1, 2, \ldots, K-1$*, where* n_k *is the number of pixels having intensity level* r_k*,* n *is the total number of pixels in the neighborhood, and* K *is the total number of possible intensity levels. Suppose that the neighborhood is moved one pixel to the right (we are assuming rectangular neighborhoods). This deletes the leftmost column and introduces a new column on the right. The updated histogram then becomes*

$$p'_r(r_k) = \frac{1}{n}\left[n_k - n_{L_k} + n_{R_k}\right]$$

for $k = 0, 1, 2, \ldots, K-1$*, where* n_{L_k} *is the number of occurences of level* r_k *on the left column and* n_{R_k} *is the similar quantity on the right column. The preceding equation can ve written also as*

$$p'_r(r_k) = p_r(r_k) + \frac{1}{n}\left[n_{R_k} - n_{L_k}\right]$$

for $k = 0, 1, 2, \ldots, K-1$*. The same concept applies to other modes of neighborhoods motion:*

$$p'_r(r_k) = p_r(r_k) + \frac{1}{n}\left[b_k - a_k\right]$$

$k = 0, 1, 2, \ldots, K-1$*, where* a_k *is the number of pixels with value* r_k *in the neighborhood area deleted by the move, and* b_k *is the corresponding number introduced by the move.*

Note by the lecturer: A formulation closer to an implementation is the following:

```
for each coordinate (x, y) on the image do
    if this is the first iteration then
        compute the non-normalized histogram n_k for the region of interest S_xy
    else
        for all pixel coordinates (v, w) that fall out of S_xy just since the current iteration do
            k = f(v, w)
            decrement n_k
        end
        for all pixel coordinates (s, t) that fall into S_xy just since the current interation: do
            k = f(s, t)
            increment n_k
        end
    end
    compute the normalized histogram p_r(r_k) form n_k
end
```

1.5 Book by Gonzalez and Woods, 3.99

Given is the picture f and the kernel h:

$$f = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \qquad h = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

(a) Calculate the convolution of f with h.

(b) Describe in your own words, what the filter h does.

(a)

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(b) The filter h smears/averages the image along the y-axis