Image Processing and Computer Vision 1

Chapter 4 – Filter Design in the Frequency Domain – week 10

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1 Book

1.1 Book by Gonzalez and Woods, 4.99

Without MATLAB, Python or calculator (exam example) calculate the 2-D Discrete Fourier Transform of the images:

$$\text{(a)} \quad f_a = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \text{(b)} \quad f_b = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \text{(c)} \quad f_c = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

1.2 Book by Gonzalez and Woods, 4.53

Given an image of size $M \times N$, you are asked to perform an experiment that consists of repeatedly lowpass filtering the image using a Gaussian lowpass filter with a given cutoff frequency, D_0. You may ignore computational round-off errors.

(a) Let K denote the number of applications of the filter. Can you predict (without doing the experiment) what the result (image) will be for a sufficiently large value of K? If so, what is that result?

1.3 Book by Gonzalez and Woods, 4.59

Each spatial highpass kernel in Fig. 4.52 has a strong spike in it the center. Explain the source of these spikes.

2 Practical Exercise

Implement a homomorphic filter (4.9-29) and play with the different parameters. What do you observe?

Homomorphic Filtering
An image can be expressed as the product of its illumination $i(x,y)$, and reflectance, $r(x,y)$.

$$f(x,y) = i(x,y)r(x,y) \tag{4-134}$$

This equation cannot be used directly to operate on the frequency components of illumination and reflectance, because the Fourier transform of a product is not the product of the transforms:

$$\mathcal{F}\left[f(x,y)\right] \neq \mathcal{F}\left[i(x,y)\right]\mathcal{F}\left[r(x,y)\right] \tag{4-135}$$

However, suppose that we define

$$\begin{aligned} z(x,y) &= \ln f(x,y) \\ &= \ln i(x,y) + \ln r(x,y) \end{aligned} \tag{4-136}$$

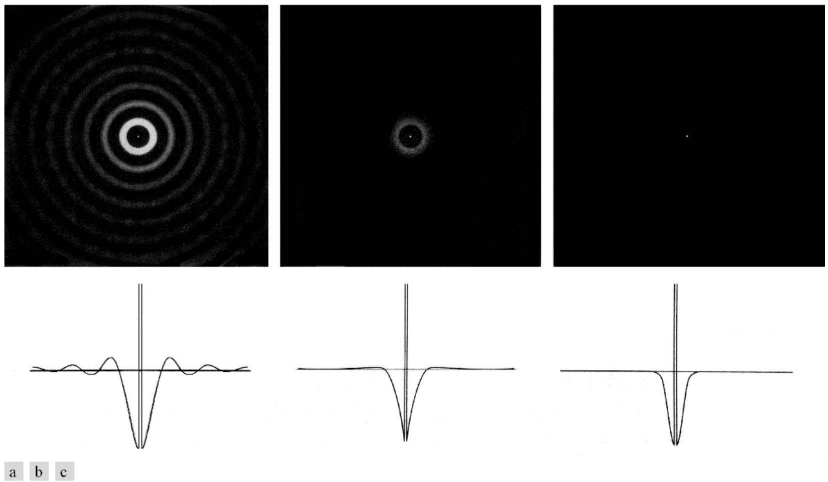
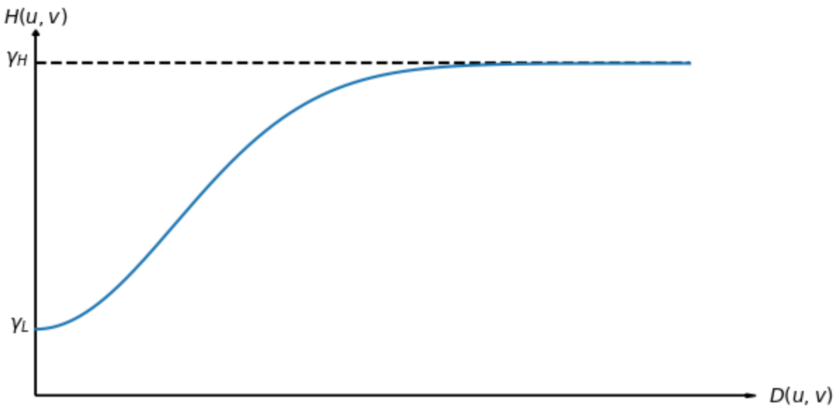


FIGURE 4.52 Spatial representation of typical (a) ideal, (b) Butterworth, and (c) Gaussian frequency domain highpass filters, and corresponding intensity profiles through their centers.

Book figure 4.52 *Spatial representation of typical highpass filters*



Plot of function in Eq. (4-147)

Then,

$$\mathcal{F}[z(x,y)] = \mathcal{F}[\ln f(x,y)]$$
$$= \mathcal{F}[\ln i(x,y)] + \mathcal{F}[\ln r(x,y)] \tag{4-137}$$
$$Z(u,v) = F_i(u,v) + F_r(u,v) \tag{4-138}$$

where $F_i(u,v)$ and $F_r(u,v)$ are the Fourier transforms of $\ln i(x,y)$ and $\ln r(x,y)$, respectively. We can filter $Z(u,v)$ using filter transfer function $H(u,v)$ so that

$$S(u,v) = H(u,v)Z(u,v)$$
$$= H(u,v)F_i(u,v) + H(u,v)F_r(u,v) \tag{4-139}$$

The filtered image in the spatial domain is then

$$s(x,y) = \mathcal{F}^{-1}[S(u,v)]$$
$$= \mathcal{F}^{-1}[H(u,v)F_i(u,v)] + \mathcal{F}^{-1}[H(u,v)F_r(u,v)] \tag{4-140}$$

By defining

$$i'(x,y) = \mathcal{F}^{-1}[H(u,v)F_i(u,v)] \tag{4-141}$$
$$r'(x,y) = \mathcal{F}^{-1}[H(u,v)F_r(u,v)] \tag{4-142}$$

we can express Eq. (4-140) in the form $s(x,y) = i'(x,y) + r'(x,y)$.
Finally, because $z(x,y)$ was formed by taking the natural logarithm of the input image, we reverse the process by taking the exponential of the filtered result to form the output image:

$$\begin{aligned} g(x,y) &= e^{s(x,y)} \\ &= e^{i'(x,y)} e^{r'(x,y)} \\ &= i_0(x,y) r_0(x,y) \end{aligned} \tag{4-144}$$

The illumination component of an image generally is characterized by slow spatial variations, while the reflectance component tends to vary abruptly. Therefore a highpass filter is needed to remove the illumination component. In practice a modified form of the GHPF function is often used

$$H(u,v) = (\gamma_H - \gamma_L)\left[1 - e^{-cD^2(u,v)/D_0^2)}\right] + \gamma_L \tag{4-147}$$