Mesh Field Theory – Lecture 03: Entanglement via Causal Coherence Overlap

Mirroring CMU Quantum Computation Lecture 03

Introduction

This lecture mirrors the structure of CMU Lecture 03 from the 15-859BB Quantum Computation course at Carnegie Mellon University, but reconstructs the concept of entanglement causally and deterministically through Mesh Field Theory.

In Mesh, entanglement emerges naturally from causal coherence cone overlaps and field alignments, without invoking tensor products, nonlocality, or probabilistic postulates.

1 Coherence Fields and Causal Cone Structure

Recall the fundamental object in Mesh:

$$\vec{C}(x,t) = \nabla\phi(x,t) \cdot \chi(x,t)$$

where:

- $\phi(x,t)$ is the scalar causal phase field,
- $\chi(x,t)$ is the coherence support.

Causal coherence propagates through light-cone-like structures:

$$\mathrm{Cone}(x,t) = \{(y,s) \mid (y-x)^2 = c^2(s-t)^2\}$$

Entanglement requires causal cone interaction.

2 Definition: Entanglement via Causal Overlap

Two coherence fields, $\vec{C}_A(x,t)$ and $\vec{C}_B(x,t)$, are **entangled** if:

1. Their causal cones overlap:

$$O_{AB}(t) = \mathrm{Cone}_A(x,t) \cap \mathrm{Cone}_B(x,t) \neq \emptyset$$

2. Their coherence vectors are locally aligned:

$$\vec{C}_A(x,t) \cdot \vec{C}_B(x,t) > 0 \quad \text{for some} \quad x \in O_{AB}(t)$$

Entanglement in Mesh is thus a **real geometric linkage** through shared causal structure and coherence alignment.

3 Entanglement Energy and Causal Structure

Define the **entanglement overlap energy**:

$$\mathcal{E}_{\text{entangle}} = \int_{O_{AB}(t)} \vec{C}_A(x,t) \cdot \vec{C}_B(x,t)\, d^3x$$

Properties:

- If $\mathcal{E}_{\text{entangle}} > 0$, the regions are causally coherently linked.
- If $\mathcal{E}_{\text{entangle}} = 0$, no causal coherence linkage exists.

Thus, entanglement strength corresponds to real causal field overlap.

4 Measurement and Collapse of Entangled Fields

Collapse still governed by divergence:

$$\Gamma(x) = \nabla \cdot \vec{C}(x,t)$$

Collapse can occur independently at either A or B, triggering causal reconfiguration of the linked partner field.

Collapse sequence:

1. Local divergence at A exceeds threshold: $\Gamma_A(x) > \Gamma_{\text{crit}}$.
2. Causal signal propagates through O_{AB} to B.
3. Coherence structure at B deterministically realigns.

No probabilistic nonlocal collapse is needed.

5 Worked Example: Entangled Coherence Fields

Suppose two coherence fields:

- $\vec{C}_A(x,t)$ localized around x_A,
- $\vec{C}_B(x,t)$ localized around x_B,

such that:

$$|x_A - x_B| < c\Delta t$$

(i.e., causal cones overlap.)

Initial phases set:

$$\phi_A(x,t) = \phi_B(x,t) + \delta$$

where δ is a small phase shift.

Thus:

$$\vec{C}_A(x,t) \cdot \vec{C}_B(x,t) > 0 \quad \text{in } O_{AB}(t)$$

Entanglement established.

Measurement at A via divergence collapse causes deterministic structural adjustment at B.

6 Summary

In this Mesh mirror of CMU Lecture 03, we established:

- Entanglement arises from real causal cone overlaps and coherence vector alignment.
- Entanglement strength is measured by real field overlap energy, not abstract tensor products.
- Collapse propagates causally and deterministically through field structures.
- No nonlocality or probabilistic postulates are necessary.

Mesh Field Theory thus reconstructs quantum entanglement causally, geometrically, and deterministically, while preserving all operational behavior required for quantum information processing.