Mesh Field Theory – Lecture 02: Superposition and Interference (Expanded)

Mirroring CMU Quantum Computation Lecture 02

Introduction

This lecture mirrors the structure of CMU Lecture 02 from the 15-859BB Quantum Computation course at Carnegie Mellon University, but reconstructs superposition and interference from the perspective of **Mesh Field Theory**.

In Mesh, superposition and interference emerge causally and deterministically from real coherence fields and causal vector overlaps, without requiring Hilbert space linearity or probabilistic amplitudes.

1 Causal Superposition of Coherence Fields

In Mesh, superposition of states corresponds to real causal addition of coherence vectors:

$$\vec{C}_{\text{total}}(x,t) = \vec{C}_a(x,t) + \vec{C}_b(x,t)$$

where:

- $\vec{C}_a(x,t)$ and $\vec{C}_b(x,t)$ are independent coherence vector fields.
- Superposition occurs physically when causal cones and coherence regions overlap.

There is no abstract vector space — only real causal flows merging or interfering in spacetime.

2 Mesh Coherence Overlap: Causal Inner Product Analog

The Mesh analog of the quantum inner product is the real spatial scalar product between coherence fields:

$$\mathcal{I}_{ab}(x,t) = \vec{C}_a(x,t) \cdot \vec{C}_b(x,t)$$

Interpretation

- $\mathcal{I}_{ab}(x,t) > 0$ indicates constructive causal alignment.
- $\mathcal{I}_{ab}(x,t) < 0$ indicates destructive causal opposition.
- $\mathcal{I}_{ab}(x,t) = 0$ indicates incoherent or orthogonal causal structure.

3 Mesh Norm and Coherence Conservation

The causal norm of a coherence field is defined as:

$$\|\vec{C}\|^2 = \int_\Sigma \vec{C}(x,t) \cdot \vec{C}(x,t)\, d^3x$$

where:

- Σ is the causal region under consideration.

Properties

- Norm measures total causal coherence energy.
- Norm is conserved under free Mesh field evolution.
- Collapse occurs only when local divergence exceeds critical threshold.

Thus, causal coherence replaces amplitude norms naturally.

4 Interference Conditions

Mesh defines constructive and destructive interference causally:

Constructive Interference

Occurs when coherence vectors align:

$$\vec{C}_a(x,t) \cdot \vec{C}_b(x,t) > 0$$

Destructive Interference

Occurs when coherence vectors anti-align:

$$\vec{C}_a(x,t) \cdot \vec{C}_b(x,t) < 0$$

No-Interference

Occurs when coherence vectors are orthogonal:

$$\vec{C}_a(x,t) \cdot \vec{C}_b(x,t) = 0$$

No complex amplitudes are needed — only real spatial causal structure.

5 Measurement and Divergence Collapse

As interference patterns develop, causal divergence $\Gamma(x,t)$ indicates local coherence instability:

$$\Gamma(x,t) = \nabla \cdot \vec{C}(x,t)$$

Collapse occurs deterministically when:

$$\Gamma(x,t) > \Gamma_{\text{crit}}$$

Outcomes depend entirely on local causal structure, not abstract probabilistic rules.

6 Worked Example: Causal Superposition

Suppose two Mesh fields:

- $\vec{C}_a(x,t)$ points along the x-axis.
- $\vec{C}_b(x,t)$ points along the y-axis.

Superposition:

$$\vec{C}_{\text{total}}(x,t) = \vec{C}_a(x,t) + \vec{C}_b(x,t)$$

Interference:

$$\mathcal{I}_{ab}(x,t) = \vec{C}_a(x,t) \cdot \vec{C}_b(x,t) = 0$$

(no interference if orthogonal).
If phases were shifted to align vectors partially, $\mathcal{I}_{ab}(x,t) > 0$ and constructive interference would occur.

7 Summary

In this Mesh mirror of CMU Lecture 02, we established:

- Superposition corresponds to real causal addition of coherence fields.
- Interference arises from real scalar product of causal coherence vectors.
- Norm conservation follows naturally from coherence energy conservation.
- Measurement outcomes emerge from deterministic divergence-triggered collapse.

Thus, Mesh Field Theory replaces quantum superposition and interference with causal geometric structures, maintaining full operational equivalence without probabilistic assumptions.