Mesh Field Theory – Lecture 09: Mesh Shor's Algorithm — Periodicity Through Modular Phase Collapse

From First Principles: Causal Modularity and Collapse

1. Introduction

Shor's algorithm factors a composite number N by reducing the problem to finding the period r of the modular function $f(x) = a^x \mod N$.

In Mesh, this modular function becomes a **modular phase field**. Periodicity is detected not symbolically, but via real coherence overlap and divergence collapse.

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2. The Modular Phase Field

Given a base $a < N$, and assuming $\gcd(a, N) = 1$, construct a phase field:

$$\phi(x) = \frac{2\pi}{r}x \pmod{2\pi}$$

This defines spatial periodicity in coherence:

$$\vec{C}(x,t) = \nabla\phi(x) \cdot \chi(x,t) \quad \text{with} \quad \vec{C}(x+r,t) = \vec{C}(x,t)$$

This coherence field encodes the modular exponentiation behavior — causally.

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3. Input Register and Oracle Exposure

Prepare an input register of N Mesh coherence regions:

$$\vec{C}_x(x_i,t) = \vec{C}_0$$

Then expose each region to the modular phase field $\phi(x)$ corresponding to $f(x) = a^x \mod N$. This is not an evaluation — it is a **phase pattern injection** that encodes periodicity into real coherence.

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4. Mesh Fourier Projection

To detect the period, prepare Mesh frequency probes:

$$\phi_k(x) = \frac{2\pi k}{N}x \quad \Rightarrow \quad \vec{C}_k(x,t) = \nabla\phi_k(x) \cdot \chi(x,t)$$

Compute projection:

$$\mathcal{F}(k) = \int \vec{C}(x,t) \cdot \vec{C}_k(x,t)\, dx$$

If $\vec{C}(x,t)$ has period r, then $\mathcal{F}(k)$ peaks near $k/N \approx m/r$ for integer m.
This is the Mesh analog of observing peaks in the QFT.
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5. Collapse and Readout

Where coherence alignment is maximal:

$$\Gamma(x,t) = \nabla \cdot \vec{C}(x,t) > \Gamma_{\text{crit}}$$

Collapse occurs at spatial locations corresponding to that frequency k.
Outcome:
- Value k is extracted (spatial label) - Approximation $k/N \approx m/r$ is recovered - Use continued fractions to deduce r
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6. Worked Example: $N = 15$, $a = 2$

Let:

$$f(x) = 2^x \mod 15$$

Construct Mesh phase field with true period $r = 4$:

$$\phi(x) = \frac{2\pi}{4}x$$

Prepare $N = 8$ coherence regions.
Apply Mesh Fourier probe with $k = 2$:

$$\phi_k(x) = \frac{2\pi \cdot 2}{8}x = \frac{\pi}{2}x$$

Strong projection at $k = 2 \Rightarrow k/N = 1/4$
Collapse occurs at harmonic location. Recovered:

$$\frac{1}{4} \Rightarrow r = 4 \quad \Rightarrow \quad \gcd(2^{r/2} \pm 1,\ 15) = \gcd(3, 15) = 3,\ \gcd(5, 15) = 5$$

Factors recovered: 3 and 5
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7. Ensemble and Statistical Stability

As in quantum Shor's algorithm:
- A single k may not yield a useful fraction - But repeated Mesh runs (with different initial phase conditions) produce a distribution of values k - Multiple collapse points yield converging estimates of r

Thus Mesh phase estimation is statistical — but from causal structure, not random measurement.
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8. Comparison to Quantum Shor Algorithm

— Feature — Quantum — Mesh — ———————————— — Modular function — $a^x \mod N$ — Modular phase field $\phi(x) = \frac{2\pi}{r}x$ — — Fourier step — QFT — Mesh projection via $\vec{C}(x) \cdot \vec{C}_k(x)$ — — Collapse — Measurement of frequency register — Divergence-triggered collapse — — Output — Probabilistic frequency detection — Deterministic collapse + statistical ensemble — — Recovery — Continued fractions — Continued fractions (same classical step) —

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9. Summary

Shor's algorithm in Mesh is not symbolic — it is structural.

- Modular periodicity is encoded physically as phase repetition - Detection is through real projection and collapse - Recovery uses the same classical method

The entire quantum algorithm is preserved — but built from geometry, not abstraction.

Next: Error correction — can Mesh fields protect information?