Mesh Field Theory – Lecture 11: Mesh Error Correction via Twist Redundancy

Mirroring CMU Quantum Computation Lecture 11

Introduction

This lecture mirrors the structure of CMU Lecture 11 from the 15-859BB Quantum Computation course at Carnegie Mellon University, but reconstructs quantum error correction using Mesh Field Theory.

In Mesh, coherence preservation is achieved through spatial twist redundancy, divergence monitoring, and causal realignment, without logical qubits, stabilizers, or syndrome measurements.

1 Error Correction in Mesh Field Theory

In Mesh, a coherence field $\vec{C}(x,t)$ represents a real causal state. To protect information, the coherence structure is distributed across multiple independent causal regions.

Let:

$$\{R_1, R_2, \ldots, R_M\}$$

be M coherence-supporting regions, each encoding:

$$\vec{C}_i(x,t) = \nabla\phi_i(x,t) \cdot \chi_i(x,t) \quad \text{with} \quad T_i(x) = T_L(x)$$

2 Twist Redundancy Encoding

Each region carries:

- Identical twist structure T_L
- Causal phase coherence $\phi(x,t)$

The logical state is defined by the majority alignment of the distributed twist structure.

3 Error Detection via Divergence Monitoring

Errors are detected by observing local divergence:

$$\Gamma_i(x,t) = \nabla \cdot \vec{C}_i(x,t)$$

If:

$$\Gamma_i(x,t) > \Gamma_{\text{crit}}$$

then coherence in region R_i is unstable, indicating a causal error.

Key Property

Mesh does not require measurement to detect errors — divergence is a real observable quantity.

4 Error Correction via Causal Realignment

Suppose one region R_i experiences error:

- Remaining regions $\{R_j\}_{j \neq i}$ maintain coherence
- Damaged region is realigned causally:

$$\phi_i(x,t) \mapsto \phi_j(x,t) \quad \text{for majority-consistent } j$$

No syndrome extraction is needed — correction uses causal field consensus.

5 Worked Example: Mesh Repetition Code

Logical twist: $T_L = [1, 0, 0]$

Distribute across 3 regions R_1, R_2, R_3 with:

$$\vec{C}_1 = \vec{C}_2 = \vec{C}_3 = \vec{C}_L$$

Suppose noise corrupts R_2:

$$\Gamma_2 > \Gamma_{\text{crit}} \Rightarrow \text{Error detected}$$

Correction:

$$\phi_2(x,t) \mapsto \phi_1(x,t) \quad \text{(majority realignment)}$$

Logical coherence is preserved.

6 Comparison to Quantum Error Correction

— Feature — Standard QEC — Mesh Field Theory — —:——:——:—— — Encoding — Logical qubits into multiple physical qubits — Logical twist field across multiple coherence regions — — Error detection — Syndrome measurement — Divergence detection — — Error correction — Logical gates based on syndrome — Causal phase realignment — — Collapse — Needed to readout syndrome — Not needed — errors handled causally —

Mesh offers a deterministic, fully physical model of error correction, without projection or probabilistic steps.

7 Summary

In this Mesh mirror of CMU Lecture 11, we established:

- Logical states are preserved via causal twist redundancy.
- Error detection arises from real-time divergence monitoring.
- Errors are corrected by realigning coherence fields with majority phase consensus.
- No syndrome extraction or projective collapse is required.

Thus, Mesh Field Theory reconstructs quantum error correction causally, without logical qubits, stabilizers, or measurement-based control.