Mesh Field Theory – Lecture 09: Mesh Shor's Algorithm Reconstruction

Mirroring CMU Quantum Computation Lecture 09

Introduction

This lecture mirrors the structure of CMU Lecture 09 from the 15-859BB Quantum Computation course at Carnegie Mellon University, but reconstructs Shor's Algorithm causally through Mesh Field Theory.

In Mesh, modular exponentiation is represented by modular phase fields, and period-finding is achieved via real causal interference and Mesh Fourier transform, with no reliance on Hilbert spaces, unitaries, or probability amplitudes.

1 Problem Statement

Given a composite number N, choose an integer $a < N$ with $\gcd(a, N) = 1$. The goal is to find the period r such that:

$$a^r \equiv 1 \mod N$$

Recovering r allows us to compute a nontrivial factor of N with high probability using classical post-processing.

2 Mesh Initialization: Modular Phase Field Construction

Define a scalar modular phase field:

$$\phi_a(x) = \frac{2\pi}{r}x \mod 2\pi$$

Coherence field:

$$\vec{C}_a(x,t) = \nabla\phi_a(x,t) \cdot \chi(x,t)$$

This field satisfies:

$$\vec{C}_a(x+r,t) = \vec{C}_a(x,t)$$

The modular structure of $a^x \mod N$ is encoded causally as periodic field behavior with period r.

3 Superposition of Input Coherence Fields

Mesh constructs an array of causal coherence fields $\{\vec{C}_x(x,t)\}$ corresponding to each value $x \in \{0, \ldots, N-1\}$.

Each region:

$$\vec{C}_x(x,t) = \vec{C}_0$$

This replaces quantum uniform superposition with real uniform causal field initialization.

4 Oracle Action: Causal Modular Exposure

Expose each input coherence region to the modular phase field:

$$\phi_x(x,t) \mapsto \phi_x(x,t) + \phi_a(f(x))$$

where $f(x) = a^x \mod N$.

The causal interaction imprints modular periodicity onto each input coherence region.

5 Mesh Fourier Transform: Detecting Periodicity

Apply the Mesh Fourier Transform to the input register field:

$$\mathcal{F}(k) = \int_\Sigma \vec{C}_{\text{input}}(x,t) \cdot \vec{C}_k(x,t)\, d^3x$$

with basis:

$$\phi_k(x) = \frac{2\pi k}{N} x \quad \Rightarrow \quad \vec{C}_k(x,t) = \nabla\phi_k(x,t) \cdot \chi(x,t)$$

Properties:

- Peaks in $\mathcal{F}(k)$ reveal hidden periodicity r.
- Constructive interference arises when $k/N \approx m/r$ for integer m.

6 Collapse and Readout

Causal divergence at frequency-mode-aligned locations triggers collapse:

$$\Gamma(x,t) = \nabla \cdot \vec{C}(x,t) \quad \text{with} \quad \Gamma(x,t) > \Gamma_{\text{crit}}$$

Readout gives a rational approximation $k/N \approx m/r$. Classically extract r using continued fractions.

7 Worked Example: Mesh Shor with $N = 15$, $a = 2$

- Construct modular phase field for $f(x) = 2^x \mod 15$
- Build input Mesh register with $N = 8$ coherence regions
- Apply oracle phase exposure: $\phi_x \mapsto \phi_x + \phi_a(f(x))$
- Apply Mesh Fourier Transform
- Detect peak at $k = 2 \Rightarrow \frac{k}{N} = \frac{2}{8} = \frac{1}{4} \Rightarrow r = 4$

Collapse occurs at causal coherence maxima aligned to $k = 2$, yielding $r = 4$. Then, compute $\gcd(a^{r/2} \pm 1, N) \Rightarrow \gcd(2^2 \pm 1, 15) = 3, 5$

8 Summary

In this Mesh mirror of CMU Lecture 09, we established:

- Modular periodicity is encoded as causal phase fields with real coherence flow.
- Oracle operations are physical phase interactions, not matrix exponentiation.
- Period detection uses Mesh Fourier projection, not amplitude interference.
- Measurement is deterministic via divergence-triggered collapse.
- Final result is extracted classically from Mesh causal dynamics.

Thus, Mesh Field Theory reconstructs Shor's Algorithm fully causally, without Hilbert vectors, probability amplitudes, or unitarity.