Mesh Field Theory – Lecture 05: Mesh Phase Estimation (Expanded)

Mirroring CMU Quantum Computation Lecture 05

Introduction

This lecture mirrors the structure of CMU Lecture 05 from the 15-859BB Quantum Computation course at Carnegie Mellon University, but reconstructs phase estimation causally and deterministically through Mesh Field Theory.

In Mesh, phase estimation arises from causal phase accumulation across coherence clock fields, causal interference measurements, and deterministic divergence-triggered collapse, without using Hilbert vectors or complex amplitudes.

1 Problem Setup: Phase Estimation Goal

Suppose we have a coherence field $\vec{C}_\psi(x,t)$ evolving as:

$$\phi_\psi(x,t) = 2\pi\theta t + \phi_0(x)$$

where:

- $\theta \in [0,1)$ is the hidden phase we wish to estimate.
- $\phi_0(x)$ is the initial phase configuration.

Goal: Determine θ to n bits of precision using causal coherence field operations.

2 Clock Fields and Controlled Phase Accumulation

Prepare n coherence clock fields $\{\vec{C}_{\text{clock},0}, \vec{C}_{\text{clock},1}, \ldots, \vec{C}_{\text{clock},n-1}\}$.

Each clock field $\vec{C}_{\text{clock},k}$ is causally exposed to $\vec{C}_\psi$ such that:

$$\phi_{\text{clock},k}(x,t) \mapsto \phi_{\text{clock},k}(x,t) + 2\pi 2^k \theta$$

Thus:

- Clock field k accumulates phase proportional to $2^k\theta$.
- Exposure is achieved causally through cone overlap, not external control operators.

3 Causal Interference and Phase Information

After causal phase accumulation:

- Clock fields now have phase differences corresponding to $2^k\theta$. - Real causal coherence interference patterns emerge between clock fields.

Measurement structure:

$$\mathcal{I}_{kl} = \vec{C}_{\text{clock},k}(x,t) \cdot \vec{C}_{\text{clock},l}(x,t)$$

Interference strength depends on accumulated phase differences.

Constructive or destructive interference encodes phase relations needed to reconstruct θ.

4 Divergence-Based Collapse and Readout

Collapse occurs when local divergence exceeds critical threshold:

$$\Gamma(x,t) = \nabla \cdot \vec{C}(x,t) \quad \text{with} \quad \Gamma(x,t) > \Gamma_{\text{crit}}$$

Collapse at clock region x_k corresponds to binary information about θ.

Bit readout:

- Phase 0 detected $\to$ bit 0.
- Phase π detected $\to$ bit 1.

No probabilistic Born rule is needed — outcome is deterministic based on causal coherence divergence.

5 Worked Example: 2-Bit Mesh Phase Estimation

Suppose:

- Hidden phase $\theta = \frac{1}{2}$.
- $n = 2$ clock fields prepared.

Phase accumulation:

$$\phi_{\text{clock},0}(x,t) = 4\pi\theta = 2\pi \quad \text{(no net phase)}$$
$$\phi_{\text{clock},1}(x,t) = 2\pi\theta = \pi \quad \text{(phase inversion)}$$

Collapse measurement:

- Clock 0 $\to$ bit 0.
- Clock 1 $\to$ bit 1.

Thus, recovered $\theta = 0.10_2 = 0.5$, matching true phase.

6 Precision Scaling and Mesh Causality

Precision achieved:

$$\Delta\theta = \frac{1}{2^n}$$

Properties:

- More clock fields $\Rightarrow$ exponentially finer phase resolution.
- Causal phase accumulation preserves field norm until collapse.
- No complex vectors, no abstract Fourier transforms needed.

7 Summary

In this Mesh mirror of CMU Lecture 05, we established:

- Phase accumulation occurs through causal field exposure, not abstract unitaries.
- Interference patterns arise from real causal vector overlaps.
- Collapse readout reveals phase bits deterministically.
- Mesh achieves exponential phase precision scaling causally.

Thus, Mesh Field Theory causally reconstructs quantum phase estimation fully, without Hilbert space structures or probabilistic postulates.