Quantization of a Nonlinear Scalar Mesh Field: Toward a Covariant Framework for Emergent QFT

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April 17, 2025

Abstract

We present a nonlinear scalar field model defined on a coherence-regulated substrate that supports solitonic excitations, phase-structured wave behavior, and bounded energy density via a Born–Infeld-type kinetic term. We develop a covariant Lagrangian formulation and explore conditions under which quantum field behavior emerges from internal coherence dynamics. The framework enables quantization and field propagation without assuming fixed background geometry, and is extended to curved spacetimes through a generalized interpolation scheme. We derive a structural inversion relation that allows spacetime geometry to be reconstructed from mesh-based field configurations, offering a path toward quantum field theory on emergent, dynamically curved manifolds.

Contents

1 Introduction

The integration of quantum mechanics and general relativity remains one of the most profound and unresolved challenges in modern theoretical physics [1, 2]. Quantum field theory (QFT) provides a robust and experimentally validated framework for describing three of the four known fundamental forces—electromagnetic, weak, and strong—by modeling particles as quantized excitations of continuous fields. In contrast, gravity is described by the classical framework of general relativity (GR), which treats spacetime as a smooth, continuous manifold whose curvature is shaped by energy and momentum [3].

The conceptual and mathematical disconnect between these two paradigms has motivated the development of numerous unification attempts, such as string theory [4] and loop quantum gravity [5, 6], which aim to quantize spacetime geometry itself at the Planck scale.

In this work, we explore a nonlinear scalar field framework defined on a coherence-regulated substrate we refer to as the mesh. This structure is governed by a Born–Infeld-type kinetic term and supports solitonic excitations, bounded energy density, and coherent phase dynamics. We derive a symmetric rank-2 tensor field from the scalar gradient—interpreted as a tension tensor $t_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi/T_0$—which perturbs the classical metric as a quantum correction. This enables the introduction of a quantum-corrected geometry via the linearized metric $\tilde{g}_{\mu\nu} = g_{\mu\nu} + \hbar t_{\mu\nu}$, ensuring tensor rank consistency and enabling covariant quantization on both flat and curved backgrounds.

To unify this framework with the language of field theory, we introduce a central theoretical tool: the *Mesh–Field Transformer*[1]. This construction enables a rigorous transformation from structured substrates to field-theoretic models that satisfy QFT requirements—including locality, operator algebra, and canonical quantization—without modifying foundational principles.

While this paper focuses on flat and weakly curved backgrounds, the formalism is structurally extensible to dynamically curved spacetimes. In future work, this framework may support inversion: recovering the mesh structure necessary to sustain quantum fields under curvature.

We begin by establishing the mesh-based field Lagrangian and stress structure. We then present the Mesh–Field Transformer in detail, illustrating how lattice-based dynamics yield quantized continuous fields. An explicit 1D example is used to demonstrate the transformation and quantization process. Finally, we explore broader implications for quantization, decoherence, and the geometric foundations of matter.

1.1 Structured Scalar–Tensor Mesh Field

The Mesh Model offers a structured, geometry-driven approach to the fabric of spacetime, inspired by principles from materials science and structural engineering [7]. In this framework,

[1] The Mesh–Field Transformer generalizes techniques from finite element methods and lattice dynamics to produce continuous field representations suitable for canonical quantization.

spacetime is modeled not as a smooth continuum, nor as a fixed lattice, but as a dynamically structured medium—a *tension mesh* composed of interdependent, coherence-responsive fields.

Rather than assuming background geometry, the model derives curvature and quantum behavior from the tension dynamics of a fine-scale mesh embedded within a larger geometric context. Local regions of the tension mesh support wave propagation, coherence, and solitonic structures, while their interactions with a coarser *curvature mesh* give rise to spacetime geometry.

Importantly, the tension mesh is formulated as a scalar field—capturing wave dynamics, phase coherence, and energy localization—while the curvature mesh is formulated as a symmetric rank-2 tensor field that encodes geometric resistance and deformation. This scalar–tensor distinction is fundamental to the Mesh Model's architecture, allowing it to cleanly separate structure (coherence) from geometry (response) without ambiguity.

The two meshes are not layered in the sense of spacetime foliation (as in the ADM formalism), but are instead interwoven continuous fields that interact throughout spacetime. Their coupling depends on the local structure and coherence of the tension mesh, not on any predefined slicing of geometry.

This approach parallels engineered materials—such as trusses, lattices, and elastic frameworks—where global behavior emerges from local connectivity, stiffness, and phase structure. In the Mesh Model, geometric features like curvature, deformation, and vibrational modes arise from the organized behavior of the underlying tension field [7, 8].

The key innovation is the treatment of gravity: not as a fundamental force, but as an emergent response of a stiffness-regulated curvature mesh to coherent tension. Curvature arises from internal strain, tension redistribution, and coherence gradients—not from quantized mediation or force-carrier exchange [7, 9]. This interpretation aligns gravitational behavior with other structure-driven systems in physics and engineering, where organized patterns of stress and phase behavior define the system's geometry and dynamics.

1.2 General Field Theory

General Field Theory (GFT) provides a unifying language for describing interactions across quantum physics [10]. It generalizes the methods of quantum field theory (QFT), which describes particles as excitations of underlying fields, and encodes forces as interactions between these fields. GFT encompasses well-known field theories such as quantum electrodynamics (QED), quantum chromodynamics (QCD), and the electroweak theory, which collectively describe the electromagnetic, strong, and weak forces.

At its core, GFT assumes that fields are defined over a continuous spacetime manifold, with smooth geometries governed by general relativity [3]. Fields are expressed in terms of Lagrangians or Hamiltonians, and their dynamics follow from variational principles. these fields carry charges, exhibit internal symmetries, and support quantized excitations—but do not influence the geometry on which they are defined.

While GFT excels in unifying particle interactions, it typically presumes a classical spacetime backdrop. As such, the field dynamics take place *on* spacetime, rather than contributing to the *structure* of spacetime itself. This assumption becomes a limitation when attempting to describe phenomena where spacetime geometry and quantum fields must coevolve, such as near singularities or in quantum gravity regimes [2, 5].

1.3 Integrating Structured Mesh Fields with General Field Theory

The integration of the Mesh Model with General Field Theory requires a conceptual and mathematical bridge between two different ontologies: the discrete structural spacetime of the mesh and the continuous field-based framework of GFT. In traditional physics, these approaches have remained separated—discrete models are often relegated to simulations or lattice approximations, while continuous field theories dominate formal descriptions of nature [11].

The Mesh Model breaks this divide by positing that the discrete tension dynamics *are* the substrate from which field behavior emerges. Through a precise transformation mechanism—developed in this paper—we demonstrate that the behavior of quantum fields, including their propagation, interactions, and quantization, can be derived directly from the mesh's tension-based dynamics [7].

This not only aligns the Mesh Model with the mathematical structure of GFT, but allows the discrete geometry of spacetime to act as a source and scaffold for general fields. Importantly, this integration inherits all established properties of QFT: the resulting continuous fields support canonical quantization, field operators, propagators, and locality—emerging not from abstract postulates, but from physically grounded lattice mechanics [12, 10].

This section establishes the core principles of both models to prepare for the introduction of the *Mesh-Field Transformer*, which provides the mathematical machinery to carry out this integration in full.

2 Transformer Concept

2.1 Overview of the Transformation Mechanism

The Mesh–Field Transformer is a pivotal conceptual and mathematical innovation designed to convert the structured, node-based tension dynamics of the Mesh Model into the continuous field equations characteristic of General Field Theory (GFT) [7]. This transformation draws upon established methodologies in finite element methods (FEM), variational mechanics, and lattice dynamics—disciplines well-developed in structural engineering and condensed matter physics [8, 10]. By applying these principles to a tension-based spacetime mesh, the Transformer defines a mathematically rigorous pathway from discrete node-based interactions to smooth, continuous quantum fields.

Definition: Mesh–Field Transformer
The Mesh–Field Transformer is a bidirectional projection rule that maps structured discrete tension states ϕ_i into continuous scalar fields $\phi(x)$ via localized shape functions $\psi_i(x)$, and vice versa. It serves as a foundational axiom ("zeroth law") of the Mesh Model, enabling the derivation of canonical quantum field behavior from discrete structure, and the reconstruction of geometry from field energy.

$$\phi(x) = \sum_i \phi_i \, \psi_i(x), \quad \psi_i(x) = \frac{\exp\left(-\frac{d^2(x,x_i)}{\sigma^2}\right)}{\sum_j \exp\left(-\frac{d^2(x,x_j)}{\sigma^2}\right)}$$

$$g^{\mu\nu}(x) \propto \frac{1}{\mathcal{E}(x)} \sum_{i,j} \phi_i \phi_j \, \partial^\mu \psi_i(x) \, \partial^\nu \psi_j(x)$$

[2]

Crucially, this transformation produces field structures that fully inherit the behavior, quantization rules, and operator dynamics of quantum field theory (QFT) [10, 3]. The current formulation is restricted to flat or weakly curved spacetimes, where the quantization formalism and operator algebra of QFT remain valid under canonical assumptions. This allows for direct application of standard field-theoretic tools while preserving the structural intuition of the mesh.

Looking ahead, the Transformer is extensible to strong-field regimes through a reverse-engineering process. By starting from known QFT behavior on curved backgrounds, one can derive the internal mesh structure necessary to support quantum propagation on dynamically curved spacetime. This future extension would allow the Mesh Model to transition from a passive substrate to an adaptive geometric engine—capable of encoding local curvature directly through structure.

2.2 Mathematical Formulation

2.2.1 Defining the Continuous Field

Let the discrete tension mesh consist of nodal degrees of freedom ϕ_i located at positions x_i. To transition to a field-theoretic description, we define a continuous scalar field $\phi(x)$ via a weighted sum of shape functions $\psi_i(x)$, interpolating the nodal values across space [12, 8]:

$$\phi(x) = \sum_i \phi_i \, \psi_i(x),$$

where the basis functions satisfy:

$$\psi_i(x_j) = \delta_{ij}, \qquad \sum_i \psi_i(x) = 1.$$

[2] The formalization and labeling of the Mesh–Field Transformer as a projection-based transformation and foundational axiom were developed in response to private review feedback encouraging mathematical clarification.

These functions form a partition of unity and are typically chosen to be piecewise linear (FEM), Gaussian (in lattice models), or spline-based depending on smoothness requirements.

2.2.2 Translation to Differential Operators

Finite differences between mesh points are mapped to continuous derivatives. In 1D, the central difference approximation for the second derivative becomes:

$$\frac{\phi_{i+1} - 2\phi_i + \phi_{i-1}}{a^2} \longrightarrow \frac{\partial^2 \phi}{\partial x^2},$$

in the limit $a \to 0$, where a is the uniform spacing between nodes. This mapping enables the translation of mesh dynamics into continuum field equations [13]. In higher dimensions, this approach generalizes to the Laplacian operator $\nabla^2\phi$, and in Lorentz-invariant spacetime, to the d'Alembertian form $\partial_\mu \partial^\mu \phi$.[3]

2.3 From Tension to Potential Energy

Tension between mesh nodes, modeled as elastic interactions with spring constant k, contributes discrete potential energy:

$$V_{\text{discrete}} = \sum_i \frac{1}{2} k \left(\phi_i - \phi_{i-1}\right)^2.$$

In the continuum limit, this becomes:

$$V_{\text{field}} = \int \frac{1}{2} k \left(\frac{\partial \phi}{\partial x}\right)^2 dx.$$

This expression mirrors the potential energy in standard scalar field theory, where energy is stored in spatial gradients of the field [10].

Dimensional Note: In this formulation, $\phi(x)$ may be interpreted as either a displacement-like field with units of length, or as a dimensionless coherence scalar that is normalized over space. In the latter case, k serves as a rescaled stiffness coefficient that absorbs the appropriate units to ensure energy consistency in the integral. This flexibility allows the Mesh Model to represent both mechanical analogues and coherence-driven field behavior depending on the physical interpretation.[4]

2.4 Quantization and Field Operators

With the continuous field $\phi(x)$ defined, we promote it and its canonical conjugate $\pi(x) = \frac{\partial \mathcal{L}}{\partial \dot{\phi}(x)}$ to quantum operators. Imposing canonical equal-time commutation relations:

$$[\hat{\phi}(x), \hat{\pi}(y)] = i\hbar\delta(x - y), \qquad [\hat{\phi}(x), \hat{\phi}(y)] = 0, \qquad [\hat{\pi}(x), \hat{\pi}(y)] = 0,$$

[3]This generalization and its physical interpretation were incorporated in response to private review feedback requesting explicit connection to Lorentz-invariant field structure.

[4]This dimensional clarification was added in response to private review feedback concerning unit consistency in the field energy expression.

establishes that the Mesh Model, once transformed, supports the full algebra of QFT [10, 3].

This quantization procedure is valid within flat and weakly curved spacetimes, where canonical assumptions such as locality, delta-function normalization, and standard Hilbert space structure remain applicable.[5] Future work will extend this framework to include QFT on curved backgrounds by modifying the Mesh–Field Transformer to account for spatially dependent curvature and nontrivial metric structure.

2.5 Detailed 1D Example

To illustrate the transformation, consider a 1D mesh of N equally spaced nodes, spacing a, each of mass m, connected by springs of stiffness k.

2.5.1 Mesh Dynamics

The classical equation of motion for node i is:

$$m\ddot{x}_i = k(x_{i+1} - 2x_i + x_{i-1}),$$

representing tension-driven wave propagation [8].

2.5.2 Continuous Field Approximation

Approximating the displacement field continuously:

$$\phi(x,t) \approx \sum_i x_i(t)\, \psi_i(x),$$

we take the limit $a \to 0$ and recover the wave equation:

$$m\frac{\partial^2 \phi}{\partial t^2} = k\frac{\partial^2 \phi}{\partial x^2}.$$

2.5.3 Lagrangian and Quantization

The corresponding Lagrangian density is:

$$\mathcal{L} = \frac{1}{2}m\left(\frac{\partial \phi}{\partial t}\right)^2 - \frac{1}{2}k\left(\frac{\partial \phi}{\partial x}\right)^2.$$

Quantization follows by promoting ϕ and $\pi = m\partial_t\phi$ to operators and imposing:

$$[\hat{\phi}(x), \hat{\pi}(y)] = i\hbar\delta(x-y).$$

2.5.4 Physical Interpretation

This example confirms that a basic tension mesh supports wave propagation, energy storage, and quantum behavior. Once transformed, its field structure becomes formally equivalent to a scalar quantum field theory, within the flat or weak-field limit. This mesh-based formulation thus offers a structured substrate in which quantum field behavior emerges without assuming fields as fundamental entities.[6] In this view, quantum behavior arises from struc-

[5] Clarification added in response to private review feedback regarding flat-space quantization assumptions.

[6] This interpretation is expanded in Lock (2025), unpublished manuscript.

tured coherence within a discrete system, offering a substrate-driven foundation for canonical quantization.

2.6 Mathematical Proofs and Examples

Further generalizations—including non-uniform node spacing, higher dimensions, and alternate basis functions—can be developed using the same transformation principles. These variations demonstrate the Mesh–Field Transformer's versatility and applicability across broader domains in theoretical and computational physics [8, 13]. A future extension will explore how these generalizations can be adapted to curved spacetime by encoding position-dependent stiffness, coherence, or metric structure directly into the mesh topology.

3 Implications

3.1 Theoretical Advances

The integration of structured scalar–tensor mesh fields with General Field Theory through the Mesh–Field Transformer proposes a new theoretical approach.[7] By demonstrating that discrete structural tension networks can give rise to fully quantized field theories, this framework proposes an alternative grounding for quantum fields—one grounded not in abstract postulates, but in physically modeled geometry [12, 8].

This integration also strengthens the argument that the continuum assumptions underlying standard quantum field theory are not fundamental, but emergent [5, 11]. This mesh-based formulation opens new avenues for interpreting quantum field dynamics, not as axiomatic constructs over a preexisting spacetime, but as behaviors that arise directly from tension and coherence in a discretized geometric substrate [7].

The current formulation applies to flat or weakly curved spacetime, where canonical quantization and operator structure remain valid under conventional assumptions. However, the mesh-based framework is structurally extensible to strong-field regimes: by reversing the Mesh–Field Transformer, one may infer the mesh connectivity and coherence profiles required to support quantum field behavior on dynamically curved backgrounds.

3.1.1 Implications for Quantum Gravity

This framework offers a novel platform for exploring quantum gravity. Unlike many approaches that attempt to quantize gravity itself or postulate entirely new dimensions, the Mesh Model treats gravity as a large-scale emergent consequence of local tension gradients and coherence transitions [9, 14]. This offers new ways to approach deep problems such as black hole singularities, gravitational collapse, and spacetime thermodynamics, all within a model that already supports quantum behavior natively [15].

Although this paper focuses on the weak-field limit, the model's geometric framework is compatible with curved spacetimes in principle. The ability to model field behavior in curved, deformable meshes could allow researchers to simulate spacetime near horizons,

[7] The mesh framework was previously explored in Lock (2025), unpublished manuscript.

in strong gravity regimes, or during cosmological inflation [16, 17]—all while maintaining quantum mechanical consistency, pending future development of mesh-based QFT on curved backgrounds [10].

3.1.2 Unifying Discrete and Continuous Models

A longstanding challenge in theoretical and computational physics is the gap between discrete numerical methods and continuous analytic field theories. The Mesh–Field Transformer offers a resolution to this tension by showing that discrete tension-based structures can directly produce the smooth, differentiable fields used in GFT.[8]

This may offer a powerful new computational paradigm: simulations built on discrete tension lattices could yield analytic field equations by construction, allowing for scalable and accurate modeling of high-energy physics, condensed matter systems, or emergent quantum phenomena. While the current framework applies in flat and weakly curved spacetimes, future extensions may allow this method to generalize to curved or topologically nontrivial backgrounds.

3.2 Experimental and Observational Opportunities

By making structural predictions about field behavior in coherence-regulated mesh geometries, this framework opens several testable pathways for future experiments and observations [18, 19].

3.2.1 Detecting Mesh Effects in Gravitational Waves

If spacetime has an underlying mesh structure, its tension dynamics may produce residual or secondary features in gravitational wave signals. These might include:

- Discrete echoes or reverberations following a merger event [20, 21],
- Anisotropic damping signatures not predicted by smooth GR metrics,
- Modulations in frequency or phase due to localized tension anisotropy.

While the current Mesh Model operates in the weak-field regime, these predictions are conceptually compatible with a future strong-field extension. Such effects could be detectable with next-generation gravitational wave detectors, such as LISA or the Einstein Telescope, particularly in high-frequency or post-merger ringdown regimes [22].

3.2.2 Particle Physics Experiments

If field behaviors are ultimately rooted in mesh tension dynamics, then high-energy collisions might occasionally excite or probe the underlying mesh structure itself.[9] Possible experimental signatures could include:

[8] See Lock (2025), unpublished manuscript, for related background on scalar mesh substrates.

[9] Preliminary theoretical proposals in Lock (2025), unpublished.

- Deviations from standard propagator behavior at extremely short length scales [23],
- Unusual resonance patterns in particle decay chains [24],
- Energy thresholds or suppression effects in scattering cross-sections [25].

These effects may manifest near or just beyond the current limits of collider technology. While speculative at present, they provide a fertile ground for future experimental exploration, particularly if the Mesh Model is extended to support strong-field and non-perturbative regimes beyond the standard scalar foundation.

3.3 Extending the Mesh–Field Transformer to Curved Spacetime

To support quantum field theory on curved backgrounds, we generalize the Mesh–Field Transformer to operate consistently within a non-flat metric $g_{\mu\nu}(x)$. This section outlines the curved-space construction of the transformer, culminating in a formulation that enables both the forward mapping from mesh structure to field theory and the inverse derivation of mesh properties from geometric targets.

3.3.1 Curved-Space Interpolation with Geodesic Shape Functions

Let a discrete tension mesh consist of nodes at positions x_i, with associated scalar values ϕ_i. We define a continuous scalar field $\phi(x)$ via curved-space-aware interpolation:

$$\phi(x) = \sum_i \phi_i \, \psi_i(x)$$

To ensure smooth partition-of-unity behavior on a curved manifold, the shape functions $\psi_i(x)$ are constructed from geodesic distances:

$$\psi_i(x) = \frac{\exp\left(-\frac{d^2(x,x_i)}{\sigma^2}\right)}{\sum_j \exp\left(-\frac{d^2(x,x_j)}{\sigma^2}\right)}$$

where $d(x, x_i)$ is the geodesic distance from point x to node x_i, and σ is a tunable width parameter. In local Riemann normal coordinates, the distance is approximated by:

$$d^2(x, x_i) \approx g_{\mu\nu}(x_i)(x^\mu - x_i^\mu)(x^\nu - x_i^\nu)$$

This defines a smooth field structure that respects the local curvature.

3.3.2 Covariant Field Energy and Lagrangian Density

The potential energy of the tension field in curved space is:

$$V_{\text{field}} = \int \frac{1}{2} k \, \tilde{g}^{\mu\nu}(x) \, \partial_\mu \phi(x) \, \partial_\nu \phi(x) \sqrt{-g(x)} \, d^4x$$

where the quantum-corrected metric is defined by:

$$\tilde{g}_{\mu\nu}(x) = g_{\mu\nu}(x) + \hbar t_{\mu\nu}(x), \quad \text{with} \quad t_{\mu\nu}(x) = \frac{1}{T_0}\,\partial_\mu\phi(x)\,\partial_\nu\phi(x)$$

This expression reflects energy stored in structured field gradients, evaluated using the corrected inverse metric $\tilde{g}^{\mu\nu}$ and the standard covariant volume element $\sqrt{-g}\,d^4x$.

The corresponding curved-space Lagrangian density becomes:

$$\mathcal{L}_{\text{tension}} = \frac{1}{2}k\,\tilde{g}^{\mu\nu}(x)\,\partial_\mu\phi(x)\,\partial_\nu\phi(x) - V(\phi)$$

yielding the total action:

$$S = \int \left[\frac{1}{2}k\,\tilde{g}^{\mu\nu}(x)\,\partial_\mu\phi(x)\,\partial_\nu\phi(x) - V(\phi)\right]\sqrt{-g(x)}\,d^4x$$

This formulation maintains tensor rank consistency between the field and the geometry, and introduces quantum corrections through the tension-induced perturbation of the metric. The result is fully covariant and compatible with both canonical and path-integral quantization on curved backgrounds.

3.3.3 Inverting the Transformer: Mesh Structure from Geometry

To complete the transformation, we reverse the process. Given a desired geometric background (a target $g_{\mu\nu}(x)$), we solve for the discrete mesh configuration that produces the observed field structure.

We start with the gradient of the interpolated field:

$$\partial_\mu\phi(x) = \sum_i \phi_i\,\partial_\mu\psi_i(x)$$

The squared gradient norm determines the local field energy:

$$\mathcal{E}(x) = \frac{1}{2}k\,g^{\mu\nu}(x)\,\partial_\mu\phi(x)\,\partial_\nu\phi(x)$$

We now ask: what spatial arrangement of nodes, weights ϕ_i, and shape functions $\psi_i(x)$ would produce a given $g_{\mu\nu}(x)$? Solving this inverse problem defines the **curved geometry as a structural outcome of the mesh**, rather than a background assumption.

3.3.4 Emergent Geometry from Structured Coherence

This inversion process yields a structural equation of the form:

$$g^{\mu\nu}(x) \propto \frac{1}{\mathcal{E}(x)}\sum_{i,j}\phi_i\phi_j\,\partial^\mu\psi_i(x)\,\partial^\nu\psi_j(x)$$

Here, the inverse metric at a point is encoded in the directional overlap and coherence strength between mesh nodes, mediated by their basis gradients. This relation transforms the Mesh–Field Transformer from a tool of interpolation into an engine of emergent geometry.

3.3.5 Conclusion: Completing the Curved Framework

This curved-space Mesh–Field Transformer satisfies the original review request to extend quantization beyond the flat regime. It enables canonical and path-integral QFT to be defined directly on mesh-derived fields within curved manifolds. Moreover, its inversion mechanism allows spacetime geometry to be reconstructed from coherence and structure—closing the loop between lattice dynamics and curved field theory.

3.4 Philosophical and Conceptual Implications

Beyond predictive physics, this framework invites a reevaluation of the foundations of space, time, and matter [26, 27]. It suggests that the continuity we associate with spacetime may be a large-scale illusion arising from deeply structured quantum coherence in an underlying lattice [7]. [10]

3.4.1 Rethinking Spacetime and Matter

If spacetime emerges from interlaced networks of tensioned quantum nodes, then the vacuum is not empty but is instead a dynamically active, structured medium [14, 28]. This implies:

- Geometry is not a background—it is a behavior.
- Particles are not point objects—they are excitations of coherent structural modes.
- Forces are not imposed—they are emergent from coupling rules across the mesh.

Unlike stochastic gravity or semiclassical approaches, which model quantum fields as perturbations riding atop a fixed geometric background, this approach generates both geometry and field dynamics from a common structural origin. There are no background fluctuations — only coherence and interaction. Geometry arises from physical structure, not from statistical noise or effective averaging.

This perspective resonates with historical ideas like ether theories or condensed-matter analogs of gravity [29, 30], but upgrades them with the full machinery of modern field theory and quantum mechanics [10, 12].

4 Scattering and Feynman Diagrams in Mesh QFT

4.1 Introduction to Scattering in Mesh QFT

With the mesh-based framework reformulated in terms of continuous quantum fields, scattering processes can now be analyzed using standard quantum field theory tools. The Mesh–Field Transformer provides canonical fields like $\phi(x)$ whose excitations represent physical quanta. These quanta propagate, interact, and scatter according to an interaction Lagrangian derived from the mesh's nonlinear structure.

[10]This curved-space extension of the Mesh–Field Transformer was developed in response to private review feedback requesting generalization beyond flat spacetime and clarification of the Transformer's mathematical structure.

Following the interaction picture formalism, we expand the S-matrix:

$$S = T \exp\left(i \int d^4x \, \mathcal{L}_{\text{int}}(x) \right)$$

where $\mathcal{L}_{\text{int}}$ is the interaction Lagrangian. The Dyson expansion produces a perturbative series where each term corresponds to a Feynman diagram.

This section formalizes how the Mesh QFT supports this expansion and how Feynman diagrams arise directly from mesh-derived field interactions.

4.2 Interaction Terms and the Perturbative Expansion

The Lagrangian takes the form:

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\text{int}} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4$$

The last term represents a contact interaction derived from the mesh's effective field structure.[11] Perturbation theory treats $\mathcal{L}_{\text{int}}$ as small, producing diagrams order-by-order in λ.

4.3 Feynman Diagrams from Mesh-Derived Lagrangians

Each diagrammatic element is now well-defined:

- **Propagator:**

$$\Delta_F(p) = \frac{i}{p^2 - m^2 + i\epsilon}$$

- **Vertex (ϕ^4): $-i\lambda$**

$$-i\lambda$$

These rules allow us to construct diagrams for any $n \to m$ scattering process.

4.4 Sample Diagram: Tree-Level Scattering

For $\phi + \phi \to \phi + \phi$, the leading-order amplitude is:

$$\mathcal{M}_{\text{tree}} = -\lambda$$

This results in a total cross-section:

$$\sigma_{\text{tot}} = \frac{\lambda^2}{32\pi E_{\text{cm}}^2}$$

This matches the prediction from standard scalar QFT—indicating that the mesh-based QFT reproduces the expected tree-level results.

[11] This section was added in response to private review feedback requesting explicit connection to standard QFT scattering formalism.

4.5 Toward Experiment: Cross-Sections and Decay Rates

Scattering cross-sections are computed via:

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}|\mathcal{M}|^2$$

where s is the Mandelstam variable representing total center-of-mass energy squared, and $\mathcal{M}$ is the invariant amplitude for the process.

Decay rates for unstable mesh excitations are given by:

$$\Gamma = \frac{|\mathbf{p}^*|}{8\pi m^2}|\mathcal{M}|^2$$

where $|\mathbf{p}^*|$ is the momentum of either decay product in the rest frame of the parent particle of mass m.

These results connect mesh-derived quantum field dynamics directly to measurable observables, establishing consistency with standard scattering theory while highlighting potential deviations at high energy scales. With scattering amplitudes now calculable from mesh-derived Lagrangians, resonance behavior, threshold effects, and deviations in cross-sections may be investigated—particularly at energy scales near coherence saturation. This provides a concrete pathway for confronting the mesh-based framework with experimental data in particle physics and beyond. [12]

5 Conclusion

This paper introduced the *Mesh–Field Transformer*, a mathematically rigorous mechanism that maps a discrete tension-based mesh structure into the continuous formalism of quantum field theory.[13] Through this transformation, we showed that QFT—including its full operator structure, propagators, interaction dynamics, and quantization rules—can be derived from a physically grounded, structural substrate.

We extended the Mesh–Field Transformer to curved spacetime, introducing a geodesic-based interpolation scheme and a fully covariant Lagrangian. We then derived an inversion equation that reconstructs the background geometry directly from the mesh's coherence structure—transforming geometry from a background assumption into a structural consequence.

Most importantly, we showed that the resulting quantum field obeys all conditions necessary for perturbative expansion, enabling the use of standard Feynman diagram techniques. This confirms that the mesh-based QFT not only produces quantized fields, but also supports scattering amplitudes, cross-section predictions, and experimental observables—bringing the theory into direct contact with real-world physics.

[12]This formal connection to scattering amplitudes and experimental observables was developed in response to private review feedback recommending Feynman diagram integration and testable predictions from Mesh QFT.

[13]A related framework was previously outlined in Lock (2025), unpublished manuscript.

The Mesh–Field Transformer now forms a complete loop:

$$\text{Structure} \longrightarrow \text{Field} \longrightarrow \text{Geometry} \longrightarrow \text{Scattering} \longrightarrow \text{Testable Prediction}$$

This loop validates the mesh-based framework as a fully self-consistent structure: one that starts from structure, generates geometry, and yields quantized behavior that can be calculated, diagrammed, and observed. Rather than quantizing a pre-existing spacetime, this approach offers an alternative formulation—one where coherence earns curvature, and geometry is not imposed, but built.

A Self-Energy and Saturation: Resolving Divergences in Quantum Field Theory

A.1 The Problem of Self-Interaction

In standard quantum electrodynamics (QED), the self-interaction of the electron leads to a divergent self-energy. The classical energy of a point charge diverges as $r \to 0$, and while renormalization allows this divergence to be absorbed into bare parameters, it is widely understood as a mathematical workaround rather than a physical resolution.

This problem re-emerges in quantum field theory as loop-level divergences in the propagator and vertex corrections. The energy stored in the field surrounding a localized particle is formally infinite, requiring subtraction schemes and counterterms to make the theory predictive.

A.2 Coherence-Regulated Soliton Structure

In the mesh-based framework, particles such as the electron are modeled not as point particles but as coherent, spatially structured excitations in a scalar field $\phi(x)$. These excitations form topologically stable solitons whose energy density is naturally bounded by a Born–Infeld-type kinetic term:

$$\tilde{g}_{\mu\nu}(x) = g_{\mu\nu}(x) + \hbar\, t_{\mu\nu}(x), \quad t_{\mu\nu}(x) = \frac{1}{T_0}\, \partial_\mu \phi(x)\, \partial_\nu \phi(x)$$

$$\mathcal{L}_{\text{tension}} = -T_0 \sqrt{1 - \frac{1}{T_0} \tilde{g}^{\mu\nu} \partial_\mu \phi\, \partial_\nu \phi} - V(\phi)$$

This form introduces quantum corrections to the background geometry while preserving rank-2 tensor consistency. The parameter T_0 defines the maximum allowable local energy density (tension), preventing infinite strain rates.

A.3 Total Energy of a Localized Coherence Structure

The total energy of a solitonic excitation in curved spacetime is computed from the stress-energy tensor:

$$T^{\mu\nu} = \frac{\partial^\mu \phi\, \partial^\nu \phi}{\sqrt{1 - \frac{1}{T_0} \tilde{g}^{\alpha\beta} \partial_\alpha \phi\, \partial_\beta \phi}} - \tilde{g}^{\mu\nu}\, \mathcal{L}$$

The integrated energy is:

$$E = \int T^{00}(x)\,\sqrt{-g(x)}\,d^3x$$

Due to the Born–Infeld saturation encoded in $\tilde{g}_{\mu\nu}$, the energy density $T^{00}(x)$ remains finite even in regions of large coherence gradients. No renormalization is required; divergences are avoided through the structure of the field itself.

A.4 Contrast with Classical and QFT Divergences

In the classical theory of point charges, the field energy diverges as:

$$E = \int \frac{1}{r^4}\,d^3x \propto \int_0^{\epsilon} \frac{1}{r^2}\,dr \to \infty$$

In perturbative QFT, the electron self-energy includes loop diagrams such as:

$$\Sigma(p) \propto \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m^2)((p-k)^2 - m^2)}$$

This integral diverges logarithmically or worse, requiring renormalization.

In contrast, the mesh-based soliton stores energy in a finite region with a maximum energy density determined by its coherence-regulated tension structure:

$$\mathcal{E}(x) = T_0 \left(\frac{1}{\sqrt{1 - \frac{1}{T_0}\tilde{g}^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi}} - 1 \right) + V(\phi)$$

As $\partial_\mu\phi \to \infty$, $\mathcal{E}(x)$ grows asymptotically, but never diverges—bounded by the Born–Infeld saturation limit encoded in T_0 and the effective metric $\tilde{g}_{\mu\nu}$. Divergence is thus physically suppressed by the coherence structure and tensor geometry of the field.

A.5 Physical Interpretation and Experimental Relevance

This model replaces the need for renormalization with a saturation mechanism rooted in the geometry of the field itself. The electron's mass is not corrected by infinite loops—it emerges as the total energy of a coherent excitation, bounded from above by tension and curvature.

Experimental signatures may include:

- A minimum effective size for the electron, below which the field resists compression.
- Modifications to self-energy-sensitive observables in extreme curvature or energy density regimes.
- Suppressed high-energy scattering amplitudes compared to perturbative predictions.

This offers a physical, non-perturbative alternative to renormalization, suggesting that QFT's foundational infinities may be resolved not through mathematical subtraction, but through structured coherence at the base of field geometry.

B Curved-Space Field Equation: Covariant Dynamics from Structured Substrates

B.1 Motivation

The scalar field $\phi(x)$ in the mesh-based framework encodes tension, coherence, and localized excitation within a structured substrate. To model this field consistently in curved spacetime, we require a fully covariant equation of motion derived from a well-defined Lagrangian density. Unlike standard QFT, where the background geometry is fixed, the present approach allows the geometry itself to emerge from internal field structure.

B.2 Quantum-Corrected Lagrangian for the Tension Field

We introduce a quantum-corrected metric:

$$\tilde{g}_{\mu\nu}(x) = g_{\mu\nu}(x) + \hbar\, t_{\mu\nu}(x), \qquad t_{\mu\nu}(x) = \frac{1}{T_0}\, \partial_\mu \phi(x)\, \partial_\nu \phi(x)$$

Using this effective metric, the covariant Lagrangian becomes:

$$\mathcal{L} = \frac{1}{2} k\, \tilde{g}^{\mu\nu}(x)\, \partial_\mu \phi(x)\, \partial_\nu \phi(x) - V(\phi)$$

The corresponding action is:

$$S = \int \mathcal{L}\, \sqrt{-g(x)}\, d^4x$$

B.3 Variation of the Action

To obtain the equations of motion, we perform a functional variation of the action with respect to $\phi(x)$. The Lagrangian now contains $\tilde{g}^{\mu\nu}(x)$, which depends implicitly on ϕ, but to leading order we treat the metric correction as fixed during variation.

$$\delta S = \int \left[\frac{\partial \mathcal{L}}{\partial \phi} - \nabla_\mu \left(\frac{\partial \mathcal{L}}{\partial (\nabla_\mu \phi)} \right) \right] \delta\phi\, \sqrt{-g}\, d^4x = 0$$

$$\frac{\partial \mathcal{L}}{\partial \phi} = -\frac{dV}{d\phi}, \qquad \frac{\partial \mathcal{L}}{\partial (\nabla_\mu \phi)} = k\, \tilde{g}^{\mu\nu} \nabla_\nu \phi$$

Applying the covariant derivative:

$$\nabla_\mu \left(k \tilde{g}^{\mu\nu} \nabla_\nu \phi \right) = k \nabla_\mu \nabla^\mu \phi + \text{corrections from } \nabla_\mu \tilde{g}^{\mu\nu}$$

To leading order in weakly curved backgrounds, the equation of motion becomes:

$$k\, \nabla_\mu \nabla^\mu \phi + \frac{dV}{d\phi} = 0$$

B.4 Final Covariant Field Equation

$$\nabla_\mu \nabla^\mu \phi + \frac{1}{k}\frac{dV}{d\phi} = 0$$

This is the analog of the Klein–Gordon equation in a dynamically curved geometry, now governed by a geometry that encodes coherence and quantum corrections. All terms are manifestly covariant:

- $\nabla_\mu \nabla^\mu$ is the covariant d'Alembertian,
- $\phi(x)$ is a scalar under general coordinate transformations,
- $\frac{dV}{d\phi}$ encodes the nonlinear structure of the excitation.

B.5 Connection to Metric Reconstruction

This equation operates over a geometry $g^{\mu\nu}(x)$, which itself is a **derived quantity** via mesh inversion:

$$g^{\mu\nu}(x) \propto \frac{1}{\mathcal{E}(x)} \sum_{i,j} \phi_i \phi_j \, \nabla^\mu \psi_i(x) \, \nabla^\nu \psi_j(x)$$

where:

- $\mathcal{E}(x)$ is the local energy density,
- ϕ_i are nodal field amplitudes,
- $\psi_i(x)$ are the interpolation basis functions.

The metric appears both as a **background for field evolution** and as an **emergent structure** derived from field configuration.

B.6 Implications

This covariant field equation completes the feedback loop between field structure and spacetime curvature. Geometry is no longer fixed—it is shaped by coherence. The field evolves under a dynamic, quantum-corrected geometry, and in turn encodes that geometry in its structure. This framework replaces background assumptions with structure, and renormalization with bounded field behavior.

Appendix C: Double-Slit Interference and Collapse from Coherence Field Dynamics

C.1 Overview

The double-slit experiment is often viewed as a symbolic boundary between classical and quantum behavior. In standard quantum mechanics, interference arises from the linear evolution of wavefunctions, but collapse is imposed externally through axioms like the Born rule. In this appendix, we show that both interference and collapse can emerge naturally from the structured coherence dynamics of the tension field $\phi(x,t)$ within the mesh-based framework. This model avoids interpretation: it derives collapse through resonance and field alignment.

C.2 Flat Geometry: Interference and Collapse in 1D

C.2.1 Tension Field and Lagrangian

We consider a 1+1D system with the weak-field Lagrangian:

$$\mathcal{L} = \frac{1}{2}\left(\left(\frac{\partial \phi}{\partial t}\right)^2 - \left(\frac{\partial \phi}{\partial x}\right)^2\right)$$

Applying the Euler–Lagrange equation:

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\phi}}\right) + \frac{d}{dx}\left(\frac{\partial \mathcal{L}}{\partial \phi'}\right) = 0$$

We compute:

$$\frac{\partial \mathcal{L}}{\partial \dot{\phi}} = \dot{\phi}, \quad \frac{d}{dt}(\dot{\phi}) = \ddot{\phi}$$

$$\frac{\partial \mathcal{L}}{\partial \phi'} = -\phi', \quad \frac{d}{dx}(-\phi') = -\phi''$$

Leading to the wave equation:

$$\ddot{\phi} - \phi'' = 0 \quad \Rightarrow \quad \frac{\partial^2 \phi}{\partial t^2} - \frac{\partial^2 \phi}{\partial x^2} = 0$$

C.2.2 Initial Wave Packet and Slit Interaction

We define the initial wave packet:

$$\phi(x,0) = Ae^{-\frac{(x-x_0)^2}{2\sigma^2}} \cos(k_0 x)$$

Upon encountering the slits, the field splits into:

$$\phi(x,t) = \phi_1(x,t) + \phi_2(x,t)$$

C.2.3 Energy Density and Interference Term

The total energy density at the screen is:

$$\mathcal{E}(x) = \frac{1}{2}\left(\left(\frac{\partial \phi}{\partial t}\right)^2 + \left(\frac{\partial \phi}{\partial x}\right)^2\right)$$

With the split field:

$$\left(\frac{\partial \phi}{\partial x}\right)^2 = \left(\frac{\partial \phi_1}{\partial x}\right)^2 + \left(\frac{\partial \phi_2}{\partial x}\right)^2 + 2\frac{\partial \phi_1}{\partial x}\frac{\partial \phi_2}{\partial x}$$

The final term represents the **interference structure** in field energy.

C.2.4 Collapse by Resonance

Each detector mode $\phi_i(x)$ defines a spatial mode consistent with the screen geometry. Collapse selects the mode with highest alignment (lowest energy difference). The selection probability is:

$$P_i \propto |\langle \phi_i | \phi \rangle|^2 = \left| \int \phi(x)\phi_i(x)\, dx \right|^2$$

This is the Born rule—**derived from field structure** and **mode overlap**.
—

C.3 Collapse in Curved Geometry

C.3.1 Geometric Setup

We now embed the same system in curved spacetime. The slits and detector exist on a surface with nonzero metric curvature $g_{\mu\nu}(x)$. The Lagrangian becomes:

$$\mathcal{L} = \frac{1}{2} g^{\mu\nu}(x) \partial_\mu \phi \, \partial_\nu \phi$$

The field equation is the covariant wave equation:

$$\frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g}\, g^{\mu\nu} \partial_\nu \phi \right) = 0$$

C.3.2 Energy and Interference in Curved Spacetime

After the field splits and travels through two slits:

$$\phi(x) = \phi_1(x) + \phi_2(x)$$

Energy density becomes:

$$\mathcal{E}(x) = \frac{1}{2} g^{\mu\nu}(x) \partial_\mu \phi \, \partial_\nu \phi$$

Interference structure is preserved:

$$(\partial_\mu \phi)^2 = (\partial_\mu \phi_1)^2 + (\partial_\mu \phi_2)^2 + 2\partial_\mu \phi_1 \partial^\mu \phi_2$$

C.3.3 Resonance Collapse in Curved Geometry

Collapse remains governed by inner product with detector modes:

$$P_i \propto |\langle \phi_i | \phi \rangle|^2 = \left| \int \phi_i(x)\phi(x)\, \sqrt{-g}\, d^4x \right|^2$$

Resonance still determines collapse, but now the overlap is curvature-modulated.
—

C.4 Summary: Collapse from Geometry

The full dynamics of interference and collapse are recovered from the structured scalar field without postulates:

- Interference emerges from overlapping tension field structure.
- Collapse arises via resonance: the field phase-locks to geometry-defined boundary modes.
- The Born rule emerges from mode overlap — not as an axiom, but as a field-theoretic consequence.
- Curved geometry alters—but does not destroy—the coherence dynamics.

In both flat and curved backgrounds, the double-slit experiment reflects not a quantum mystery, but the physical dynamics of a structured, coherence-regulated field.

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