Mesh Field Theory – Lecture 06: Mesh Modular Structures and Simon's Algorithm Foundations

Mirroring CMU Quantum Computation Lecture 06

Introduction

This lecture mirrors the structure of CMU Lecture 06 from the 15-859BB Quantum Computation course at Carnegie Mellon University, but reconstructs modular periodic structures and the foundations of Simon's Algorithm causally through Mesh Field Theory.

In Mesh, modular periodicity emerges from real causal coherence structures, not abstract function properties. Hidden symmetry detection uses real causal interference patterns without Hilbert-space operations.

1 Modular Structures in Mesh Field Theory

In Mesh, modular periodicity is implemented via scalar phase fields:

$$\phi(x) = \frac{2\pi}{r}x + \phi_0 \mod 2\pi$$

where:

- r is the hidden period to be discovered,
- ϕ_0 is an initial phase offset.

The coherence field:

$$\vec{C}(x,t) = \nabla\phi(x,t) \cdot \chi(x,t)$$

thus satisfies:

$$\vec{C}(x+r,t) = \vec{C}(x,t)$$

Causal modularity is encoded physically, not algebraically.

2 Hidden Symmetry and Mesh Oracle Structure

Suppose we are given a causal coherence field $\vec{C}_f(x)$ constructed such that:

$$\vec{C}_f(x) = \vec{C}_f(x \oplus s)$$

where $s \in \{0,1\}^n$ is the hidden shift vector.

This models the Mesh equivalent of the oracle structure in Simon's Problem.

Properties:

- Inputs x and $x \oplus s$ map to identical causal coherence configurations.
- Symmetry is real and spatially embedded, not abstractly function-defined.

3 Superposition of Coherence Fields

Prepare a uniform causal superposition:

$$\vec{C}_{\text{total}}(x) = \sum_{x \in \{0,1\}^n} \vec{C}_x(x,t)$$

where each $\vec{C}_x$ is initialized uniformly across spatial causal regions.

Thus, causal fields from all possible inputs coexist and interfere.

4 Oracle Application: Hidden Symmetry Imprinting

The Mesh oracle links coherence fields:

- Causally merge fields corresponding to x and $x \oplus s$. - Establish hidden redundancy in causal field structure.

No unitary gate needed — only causal structure adjustment.

5 Causal Interference and Symmetry Extraction

Causal interference between coherence fields creates standing wave patterns.

Measurement structure:

$$\mathcal{I}_{xy}(x,t) = \vec{C}_x(x,t) \cdot \vec{C}_y(x,t)$$

Constructive interference occurs for causal field pairs satisfying the hidden symmetry.

Collapse will be concentrated along the hidden shift s.

6 Collapse and Recovery of Hidden Shift

Divergence triggers collapse:

$$\Gamma(x,t) = \nabla \cdot \vec{C}(x,t) \quad \text{with} \quad \Gamma(x,t) > \Gamma_{\text{crit}}$$

Each collapse event provides a linear constraint:

$$x \cdot s = 0 \mod 2$$

After collecting $n-1$ independent constraints, solve for s.

All collapse and constraint acquisition is deterministic and causally enforced.

7 Worked Example: Simple Mesh Simon Oracle

Suppose $n = 2$ and $s = (1,0)$.

Oracle causal structure:

$$\vec{C}_f(00) = \vec{C}_f(10) \quad \vec{C}_f(01) = \vec{C}_f(11)$$

Prepare causal coherence regions for all 4 inputs.

Apply Mesh oracle by causally linking 00 and 10, 01 and 11.

Coherence interference reveals symmetry.

Collapse measurements provide linear equations:

$$\begin{cases} x_1 \oplus 0 = 0 \\ x_2 \quad \text{unconstrained} \end{cases}$$

Solution yields $s = (1,0)$.

8 Summary

In this Mesh mirror of CMU Lecture 06, we established:

- Modular periodic structures are encoded causally in phase fields.
- Hidden symmetries arise from real spatial coherence redundancies.
- Oracle action is causal coherence linking, not abstract functional evaluation.
- Interference and divergence collapse reveal hidden shift deterministically.

Thus, Mesh Field Theory reconstructs the foundations of Simon's Algorithm causally, without Hilbert vectors, Fourier amplitudes, or probabilistic operations.