Mesh Model Equations Cheat Sheet

Overview

This cheat sheet summarizes key equations from the Mesh Model framework, covering discrete mesh mechanics, quantum field emergence, and geometric structure. It reflects the core pipeline developed in the Mesh-Field Transformer and its alignment with both classical and quantum field theory.

Legend

q_i	Generalized coordinate
$\dot{q}_i$	Time derivative of q_i
T	Kinetic energy
V	Potential energy
m	Mass
R	Radial or rotational coordinate
θ	Angular position
G	Gravitational constant
M	Mass of a black hole or object
r	Radial coordinate
c	Speed of light
$\rho(x)$	Energy density at position x
$\hbar$	Reduced Planck's constant
ω	Angular frequency
$\hat{H}$	Hamiltonian operator
$\hat{p}$	Momentum operator
$\hat{x}$	Position operator
E_n	Energy of the n-th quantum state
ϕ_i	Field value at node i
$\phi(x)$	Continuous field approximation
$\pi(x)$	Canonical momentum density
a	Lattice spacing
$\mathcal{L}$	Lagrangian density
ds	Line element in spacetime
$f(x)$	Emergent metric factor
S	Action
ΔS	Change in action across a curvature gradient

1 Lagrangian Mechanics

1.1 Generalized Coordinates

$$L(q_i, \dot{q}_i, t) = T - V$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0$$

1.2 Particle on a Circle

$$L = \frac{1}{2}mR^2\dot{\theta}^2$$

2 General Relativity Integration

2.1 Schwarzschild Metric

$$ds^2 = -\left(1 - \frac{2GM}{r}\right)c^2dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1}dr^2 + r^2d\Omega^2$$

2.2 Mesh-Emergent Metric

$$ds^2 = -f(x)^2dt^2 + dx^2, \quad f(x) = \sqrt{1 - \frac{2G\rho(x)}{c^2}}$$

3 Quantum Mechanics

3.1 Quantum Harmonic Oscillator

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}k\hat{x}^2$$

$$E_n = \hbar\omega\left(n + \frac{1}{2}\right)$$

4 Field Theory on a Mesh

4.1 Discrete Scalar Field (Lattice)

$$L = \sum_i \left[\frac{1}{2}\dot{\phi}_i^2 - \frac{1}{2a^2}(\phi_{i+1} - \phi_i)^2\right]$$

4.2 Continuum Limit (Wave Equation)

$$m\frac{\partial^2 \phi}{\partial t^2} = k\frac{\partial^2 \phi}{\partial x^2}$$

4.3 Lagrangian Density (Continuum)

$$\mathcal{L} = \frac{1}{2}m\left(\frac{\partial \phi}{\partial t}\right)^2 - \frac{1}{2}k\left(\frac{\partial \phi}{\partial x}\right)^2$$

4.4 Canonical Quantization

$$[\hat{\phi}(x), \hat{\pi}(y)] = i\hbar\delta(x-y)$$

5 Mesh Drive and Coherence Gradient

5.1 Curvature Propulsion

$$\Delta S = S_{\text{front}} - S_{\text{rear}} < 0$$

Motion arises from asymmetry in the mesh curvature gradient.

6 Core Equations of the Mesh Model

Coupled Lagrangian (Structure + Curvature + Coherence)

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + \hbar\, t_{\mu\nu}(x)$$
$$t_{\mu\nu}(x) = \frac{1}{T_0}\nabla_\mu\phi\,\nabla_\nu\phi$$
$$\mathcal{L} = -T_0\sqrt{1 - \frac{1}{T_0}\tilde{g}^{\mu\nu}\nabla_\mu\phi\,\nabla_\nu\phi} - V(\phi)$$
$$+ \frac{1}{2}K^{\mu\nu\alpha\beta}\nabla_\mu h_{\alpha\beta}\nabla_\nu h^{\alpha\beta}$$
$$- \frac{1}{2\kappa}t^{\mu\nu}(x)h_{\mu\nu}$$

Tagline: Spacetime emerges when coherence meets resistance. The form of $t_{\mu\nu} = \nabla_\mu\phi\nabla_\nu\phi/T_0$ is introduced as a structural Ansatz—selected for symmetry, covariant consistency, and minimal-energy coupling to geometry.

6.1 Mesh–Field Transformer (Curved-Compatible)

$$\phi(x) = \sum_i \phi_i\,\psi_i(x), \quad \text{with}$$

$$\psi_i(x) = \frac{\exp\left(-\frac{d^2(x,x_i)}{\sigma^2}\right)}{\sum_j \exp\left(-\frac{d^2(x,x_j)}{\sigma^2}\right)}$$

$$V_{\text{field}} = \int \frac{1}{2}k\,g^{\mu\nu}(x)\,\partial_\mu\phi(x)\,\partial_\nu\phi(x)\,\sqrt{-g(x)}\,d^4x$$

Tagline: "Structure becomes field — the mesh becomes physics."

6.2 Inversion Equation (Structure Defines Geometry)

$$g^{\mu\nu}(x) \propto \frac{1}{\mathcal{E}(x)}\sum_{i,j}\phi_i\phi_j\,\partial^\mu\psi_i(x)\,\partial^\nu\psi_j(x)$$

Tagline: "Curvature is structure. Geometry is earned."

Quantum-Corrected Metric and Tension Tensor

$$\tilde{g}_{\mu\nu}(x) = g_{\mu\nu}(x) + \hbar\, t_{\mu\nu}(x)$$

$$t_{\mu\nu}(x) = \frac{1}{T_0}\,\partial_\mu\phi(x)\,\partial_\nu\phi(x)$$

$$\mathcal{H}(x) = \frac{1}{2}\pi^2(x) + \frac{1}{2}\tilde{g}^{\mu\nu}(x)\,\partial_\mu\phi(x)\,\partial_\nu\phi(x)$$

Interpretation: The tension field contributes to geometry via a rank-2 tensor $t_{\mu\nu}$ derived from its gradients. This defines a quantum-corrected metric $\tilde{g}_{\mu\nu}$, ensuring tensor consistency. Classical general relativity is recovered in the limit $\hbar\to 0$.

7 Causal Structure and Collapse Dynamics

7.1 Coherence Vector Field

$$\vec{C}(x,t) = \nabla\phi(x,t)\cdot\chi(x,t)$$

Interpretation: Directional causal availability from local phase gradient and coherence support.

7.2 Tension-Dependent Propagation Velocity

$$v^2(x) = \frac{T(x)}{\mu(x)} \quad \Rightarrow \quad \vec{v}(x) = \sqrt{\frac{T_{ij}(x)}{\mu}} \cdot \hat{n}$$

Interpretation: Anisotropic signal velocity determined by mesh stiffness and density.

7.3 Curvature Resistance (Accumulated Coherence Loss)

$$\mathcal{R}(x) = \int_\gamma (1 - \chi(x(s)))\, ds$$

Interpretation: Pathwise delay and bending from coherence failure — analog of gravitational curvature.

7.4 Effective Causal Cone

$$\text{Cone}_{\text{eff}}(x) = f\left(\vec{C}(x), \vec{v}(x), \mathcal{R}(x)\right)$$

Interpretation: Emergent light cone from composite geometry of field structure.

7.5 Causal Transport Equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) + \rho \Gamma(x) = 0$$

$$\Gamma(x) = \nabla \cdot \vec{C}(x)$$

Interpretation: General transport law governing ripple propagation, coherence collapse, and decoherence.

7.6 Effective Mass from Collapse and Resistance

$$m_{\text{eff}}^2(x) \propto \Gamma(x) + \mathcal{R}(x)$$

Interpretation: Mass emerges structurally from coherence divergence and accumulated resistance.

7.7 Entropy Bound from Coherence Divergence

$$S_{\text{max}} \leq \frac{1}{4} \int_\Sigma \left| \nabla \cdot \vec{C}(x) \right| dA$$

Interpretation: A structural analog of the Bousso bound based on ripple support loss.

7.8 Decay Probability from Divergence

$$P(t) = 1 - \exp\left(-\int \Gamma(x(t))\, dt \right)$$

Interpretation: Structural analog of exponential decay from coherence divergence.

7.9 Horizonless Tunneling Rate

$$\Gamma \sim \exp\left(-\frac{\Delta \mathcal{R}}{\hbar} \right)$$

Interpretation: Black hole radiation as ripple tunneling through steep coherence gradient.

7.10 Interference Region (Double Slit Geometry)

$$\mathcal{I}(x) = \left\{ x \mid \vec{C}_L(x) \cdot \vec{C}_R(x) > 0, \quad \mathcal{R}(x) < \infty \right\}$$

Interpretation: Interference only forms in regions with overlapping coherence and finite resistance.

7.11 Commutator from Coherence Misalignment

$$[\vec{C}^a, \vec{C}^b] := f^{abc}\vec{C}^c, \quad \text{where} \quad f^{abc}(x) \propto \epsilon^{\mu\nu} \partial_\mu \chi^a \, \partial_\nu \chi^b$$

Interpretation: Effective SU(N)-like structure from interference of coherence gradients.

Particle Structure and Coherence Algebra

Spin-$\frac{1}{2}$ Phase Winding

$$\Psi(x) = e^{i\theta(x)/2} \quad \Rightarrow \quad \Psi \to -\Psi \text{ under } \oint \nabla\theta \cdot d\ell = 2\pi$$

Interpretation: Spin-$\frac{1}{2}$ behavior from double-valued coherence phase geometry.

Chiral Collapse Asymmetry

$$\Delta\Gamma^a(x) = \Gamma_L^a(x) - \Gamma_R^a(x)$$

Interpretation: Structural origin of chirality from directional coherence collapse.

Quark Fractional Charge

$$Q^a = \frac{1}{2\pi} \oint \nabla \theta^a(x) \cdot d\ell = \frac{n_a}{k_a}$$

Interpretation: Fractional charge from coherence winding density.

Gluon Field Strength Tensor

$$\mathcal{F}^{ab}_{\mu\nu} = \partial_\mu C^a_\nu - \partial_\nu C^a_\mu + f^{abc} C^b_\mu C^c_\nu$$

Interpretation: SU(3)-like curvature from coherence cone interactions.

Gluon Field Evolution

$$\nabla^\mu \mathcal{F}^{ab}_{\mu\nu} = J^b_\nu(x) \quad \text{with} \quad J^a_\nu = \phi^b \partial_\nu \phi^c f^{abc}$$

Interpretation: Gluon transport sourced by quark coherence mode transitions.

Mesh Field Lagrangian (Structural Form)

$$\mathcal{L}_{\text{Mesh}} = \underbrace{\mathcal{L}_T}_{\text{Twist}} + \underbrace{\mathcal{L}_C}_{\text{Curvature}} + \underbrace{\mathcal{L}_K}_{\text{Kinetic}} + \underbrace{\mathcal{L}_R}_{\text{Remainder}} + \underbrace{\mathcal{L}_\tau}_{\text{Tension Field (EM-like)}}$$

$$\mathcal{L}_T = \chi(x) \sum_{i=1}^{3} T^i |\nabla \phi_i|^2 - V_T(\vec{T})$$

$$\mathcal{L}_C = -\frac{1}{2\kappa} R + \beta_C \sum T^i f(\partial_\mu \phi_i)^2$$

$$\mathcal{L}_K = \frac{1}{2} \chi g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

$$\mathcal{L}_R = \rho_R \left[\frac{1}{2} \partial^\mu \psi_\nu \partial_\mu \psi_\nu - \frac{1}{2} m_\nu^2 \psi_\nu^2 \right]$$

$$\mathcal{L}_\tau = -\frac{1}{4} \tau_{\mu\nu} \tau^{\mu\nu} + j^\mu \tau_\mu, \quad \tau_{\mu\nu} = \partial_\mu \tau_\nu - \partial_\nu \tau_\mu$$

Dual Euler–Lagrange Systems

Tracking Form: (Cone-propagated coherence wave)

$$\chi g \frac{\partial^2 \phi}{\partial x^2} + \chi \frac{\partial g}{\partial x} \frac{\partial \phi}{\partial x} + g \frac{\partial \chi}{\partial x} \frac{\partial \phi}{\partial x} = 0$$

Structural Form: (Soliton phase locking and twist evolution)

$$\frac{\delta \mathcal{L}}{\delta \phi} = (\text{coherence field collapse})$$

$$\frac{\delta \mathcal{L}}{\delta \tau} = -\partial_x^2 \tau(x) - j(x)$$

Hamiltonian Systems

Tracking Hamiltonian:

$$\mathcal{H}_{\text{tracking}} = \frac{1}{2} \chi g \left(\frac{\partial \phi}{\partial x} \right)^2$$

Structural Hamiltonian (Full Mesh):

$$\mathcal{H}_{\text{Mesh}} = \frac{1}{2} \chi g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - j^\mu \tau_\mu + \frac{1}{2} (\partial_\mu \tau^\nu)^2$$

Twist Structure and Soliton Reaction Logic

Twist Quantization:

$$T = [T^1, T^2, T^3], \quad T^i \in \{0, 1\}, \quad \sum T^i \leq 3$$

Valid Configurations:

$$[0,0,0], \ [1,0,0], \ [1,1,0], \ [1,1,1]$$

Soliton Build Sequence:

Tension $\Rightarrow$ Coherence $\Rightarrow$ Curvature $\Rightarrow$ Twist $\Rightarrow$ Momentum

Soliton Reaction Sequence:

Twist $\Rightarrow$ Curvature $\Rightarrow$ Momentum

Twist Failures:

$T = [0,0,0] \Rightarrow$ Neutrino (remainder) or $\gamma + \gamma$ (if pure kinetic)