Mesh Field Theory – Lecture 01: Causal Coherence Fields and Twist Basis (Expanded)

Mirroring CMU Quantum Computation Lecture 01

Introduction

This lecture mirrors the structure of CMU Lecture 01 from the 15-859BB Quantum Computation course at Carnegie Mellon University, but reconstructs all foundations from the viewpoint of **Mesh Field Theory**.

Mesh Field Theory derives quantum computational behavior causally and deterministically, using coherence fields, twist structures, and divergence-triggered collapse, without relying on Hilbert spaces, probabilistic postulates, or unitary matrices.

1 Fundamental Object: Causal Coherence Fields

The basic structure in Mesh is the **coherence vector field**:

$$\vec{C}(x,t) = \nabla\phi(x,t) \cdot \chi(x,t)$$

where:

- $\phi(x,t)$ is a real scalar phase field.
- $\chi(x,t) \in [0,1]$ is the local coherence support function.

Physical Interpretation

- $\phi(x,t)$ represents causal phase readiness for local transmission.
- $\chi(x,t) = 1$ means full causal coherence; $\chi(x,t) = 0$ means no coherence transport possible.
- $\vec{C}(x,t)$ is a real, causal, spatially distributed vector field, not an abstract probability amplitude.

Origin of Coherence Evolution

Causal field evolution is governed by the Mesh causal transport equation:

$$\chi(x)\nabla_\mu \left(g^{\mu\nu}(x)\partial_\nu\phi(x)\right) = 0$$

where $g^{\mu\nu}(x)$ is the local curvature metric.

This structure emerges directly from the Mesh Lagrangian density:

$$\mathcal{L} = \frac{1}{2}\chi(x)g^{\mu\nu}(x)\partial_\mu\phi(x)\partial_\nu\phi(x)$$

Euler-Lagrange variation gives the causal wave equation above.

Thus, field evolution is not postulated — it emerges from first physical principles.

2 Basis States: Quantized Twist Configurations

Instead of an abstract computational basis $\{|0\rangle, |1\rangle\}$, Mesh defines discrete structural basis states by quantized **twist configurations**:

$$T(x) = [T^1(x), T^2(x), T^3(x)] \quad \text{where} \quad T^i \in \{0, 1\}$$

Allowed Twist Basis Set

$$B = \{[0,0,0], [1,0,0], [0,1,0], [0,0,1], [1,1,0], [1,0,1], [0,1,1], [1,1,1]\}$$

Physical Interpretation

- $T^i = 1$ indicates causal coherence locking along spatial axis i.
- $[0,0,0]$ corresponds to a neutrino-like remainder field: no twist locking.
- $[1,1,1]$ corresponds to full soliton coherence: all spatial channels locked.

Why Three Channels?

Mesh Field Theory's causal structure arises from 3+1 spacetime dimensions:

- 3 spatial coherence axes correspond to three orthogonal directions of causal field propagation, - Twist structures encode discrete coherence locking across these directions.

Thus, exactly three coherence channels exist — no more, no less.

Concrete Example

Consider:

$$T = [1, 0, 0]$$

This represents a twist-locked coherence structure along the x-axis only. Causal transport along y and z axes remains free.

3 State Representation in Mesh

A full Mesh causal field "state" is:

$$\text{State} = (\vec{C}(x,t), T(x))$$

where:

- $\vec{C}(x,t)$ determines phase readiness and causal field energy.
- $T(x)$ encodes the topological twist structure of causal coherence.

The system's causal evolution is determined by the PDE:

$$\chi(x)\nabla_\mu(g^{\mu\nu}\partial_\nu\phi) = 0$$

and collapse occurs when divergence exceeds critical threshold.

4 Causal Collapse Mechanism

Collapse (analogous to quantum measurement) occurs when:

$$\Gamma(x) = \nabla \cdot \vec{C}(x,t) \quad \text{with} \quad \Gamma(x) > \Gamma_{\text{crit}}$$

Derivation Sketch

From energy conservation:

- Coherence vector divergence measures net causal flux into/out of a region. - When divergence exceeds critical coherence density, field collapse becomes energetically unavoidable.

Thus, collapse is not postulated — it arises from real field instability.

5 Summary

In this Mesh reconstruction of CMU Lecture 01, we have:

- Introduced causal coherence fields $\vec{C}(x,t)$ as the foundation for computational states.
- Defined discrete twist basis structures $T(x)$ replacing abstract computational basis states.
- Derived field evolution and collapse dynamics from causal PDEs, not probabilistic postulates.
- Provided concrete examples and short derivations to build reader confidence without assumption.

This completes the Mesh mirror of CMU Lecture 01.