Mesh Field Theory – Lecture 07: Mesh Fourier Transform Foundations

Mirroring CMU Quantum Computation Lecture 07

Introduction

This lecture mirrors the structure of CMU Lecture 07 from the 15-859BB Quantum Computation course at Carnegie Mellon University, but reconstructs the quantum Fourier transform using Mesh Field Theory.

In Mesh, Fourier transforms are realized causally through phase-modulated coherence fields and real interference measurements, without Hilbert vectors, complex amplitudes, or unitary gates.

1 Fourier Basis in Mesh Field Theory

The Fourier basis in Mesh is constructed from causal phase ramp fields:

$$\phi_k(x) = \frac{2\pi k}{N} x \quad \text{for} \quad k = 0, 1, \ldots, N-1$$

Corresponding causal coherence fields:

$$\vec{C}_k(x,t) = \nabla \phi_k(x,t) \cdot \chi(x,t)$$

Properties:

- Each k encodes a real phase gradient across the causal region.
- No imaginary numbers are needed — only real phase slopes.

2 Mesh Fourier Transform: Causal Projection

Given a causal coherence field $\vec{C}(x,t)$, its Mesh Fourier Transform is defined as:

$$\mathcal{F}(k) = \int_\Sigma \vec{C}(x,t) \cdot \vec{C}_k(x,t)\, d^3x$$

where:

- $\vec{C}_k(x,t)$ is the Fourier basis coherence field for frequency k.
- Σ is the spatial region of interest.

Interpretation:

- $\mathcal{F}(k)$ measures the causal coherence alignment with frequency k.
- Peaks in $\mathcal{F}(k)$ reveal periodicities.

3 Physical Interpretation of Fourier Peaks

If $\vec{C}(x,t)$ has hidden modular periodicity r, then $\mathcal{F}(k)$ peaks near:

$$k \approx \frac{N}{r}$$

Thus:

- Real causal field structure reveals modular periods.
- No need for abstract complex Fourier coefficients.

4 Worked Example: Simple Modular Field

Suppose a coherence field:

$$\phi(x) = \frac{2\pi}{4}x \pmod{2\pi}$$

on $N = 8$ sites.
Thus, period $r = 4$.
Projection:

$$\mathcal{F}(k) = \int_\Sigma \vec{C}(x,t) \cdot \vec{C}_k(x,t)\, d^3x$$

Peaks at $k = 2$, since $8/4 = 2$.
Collapse will be triggered at causal regions aligned with $k = 2$ Fourier mode.

5 Collapse and Measurement

As usual, Mesh measurement is driven by divergence:

$$\Gamma(x,t) = \nabla \cdot \vec{C}(x,t) \quad \text{with collapse if} \quad \Gamma(x,t) > \Gamma_{\text{crit}}$$

Collapse preferentially occurs at coherence maxima aligned with k-basis fields.
Thus, Mesh Fourier Transform structure guides deterministic causal readout.

6 Comparison to Quantum Fourier Transform (QFT)

— Feature — Quantum Fourier Transform — Mesh Fourier Transform — —:——:——:—— — Basis — Complex exponential vectors — Real causal phase ramp fields — — State — Complex amplitude vectors — Real causal coherence vectors — — Operation — Unitary transformation — Causal interference projection — — Readout — Probabilistic measurement — Deterministic divergence collapse —

Thus, Mesh reconstructs the operational behavior of QFT causally.

7 Summary

In this Mesh mirror of CMU Lecture 07, we established:

- Fourier basis fields are constructed from real phase gradients.
- Fourier transforms correspond to real coherence projections, not abstract amplitude transformations.
- Modular periodicities are revealed through causal interference peaks.

- Measurement is deterministic, caused by causal divergence instability.

Thus, Mesh Field Theory reconstructs the quantum Fourier transform causally, maintaining its operational behavior without requiring Hilbert space vectors or complex amplitudes.