Mesh Field Theory – Lecture 07: Modular Phase Fields and Periodicity

From First Principles: Periodicity as Causal Geometry

1. Introduction

In traditional quantum algorithms, periodicity is detected through function evaluation, controlled phase accumulation, and the quantum Fourier transform.

In Mesh, periodicity emerges from the **structure of phase fields** — real spatial functions that lock and repeat.

This lecture defines modular phase behavior, explains how periodicity is revealed through field overlap, and sets the stage for Mesh-native Simon and Shor algorithms.

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2. Modular Structure in Mesh

Let a phase field be defined as:

$$\phi(x) = \frac{2\pi}{r}x \mod 2\pi$$

This defines a spatial repetition of coherence every r units.

Then the coherence field is:

$$\vec{C}(x,t) = \nabla\phi(x) \cdot \chi(x,t)$$

Because $\phi(x+r) = \phi(x)$, this field is **periodic in space**.

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3. Why This Is Periodicity

This is not symbolic periodicity — it is physical:

- The direction and magnitude of coherence repeat. - Collapse regions form at regular spatial intervals. - The Fourier projection (next lecture) reveals this physically, not algebraically.

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4. Detecting Periodicity via Mesh Interference

Prepare N spatial regions $\{x_0, \ldots, x_{N-1}\}$, each with:

$$\vec{C}_i(x) = \nabla\phi(x) \cdot \chi(x,t)$$

Then compute Mesh interference with a frequency-probing field:

$$\phi_k(x) = \frac{2\pi k}{N}x \quad \Rightarrow \quad \vec{C}_k(x) = \nabla\phi_k(x) \cdot \chi(x,t)$$

Measure projection:

$$\mathcal{F}(k) = \int \vec{C}(x,t) \cdot \vec{C}_k(x,t)\, dx$$

Peaks in $\mathcal{F}(k)$ occur where $k \approx \frac{N}{r}$
This is Mesh-native periodicity detection.
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5. Divergence Collapse from Constructive Match

When probing frequency k matches modular structure r, constructive interference builds:

$$\Gamma(x,t) = \nabla \cdot \vec{C}_{\text{total}}(x,t) \quad \Rightarrow \quad \Gamma > \Gamma_{\text{crit}}$$

Collapse occurs only at harmonics of the correct period.
No measurement operators — only physical resonance.
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6. Worked Example: Period-4 Field on 8 Points

Let:

$$\phi(x) = \frac{2\pi}{4} x \pmod{2\pi}$$

Then coherence field has peaks at $x = 0, 4$
Probe with $k = 2$ field:

$$\phi_k(x) = \frac{2\pi \cdot 2}{8} x = \frac{\pi}{2} x$$

Then:

$$\mathcal{F}(2) = \int \vec{C}(x,t) \cdot \vec{C}_k(x,t)\, dx \quad \text{maximized}$$

Divergence collapses at points matching $x = 0, 4$
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7. Comparison to Classical Modulo

— Concept — Classical — Mesh — ———————————————— — Modular arithmetic — $f(x) = f(x+r)$ — $\phi(x) = \phi(x+r) \mod 2\pi$ — — Periodicity check — Evaluate outputs — Physical field repetition — — Frequency detection — FFT / DFT — Field projection via $\vec{C} \cdot \vec{C}_k$ — — Collapse detection — Not applicable — Divergence threshold met at harmonics —
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8. Summary

Modular periodicity in Mesh is:
- **Physical**: built into the structure of the phase field - **Detectable**: via real coherence projections - **Measurable**: collapse occurs when frequency matches the structure - **Reusable**: this forms the foundation for both Simon's and Shor's algorithms

Next: Simon's problem — redundancy as causal symmetry.