Mesh Field Theory – Lecture 08: Mesh Simon's Problem Reconstruction

Mirroring CMU Quantum Computation Lecture 08

Introduction

This lecture mirrors the structure of CMU Lecture 08 from the 15-859BB Quantum Computation course at Carnegie Mellon University, but reconstructs Simon's problem causally through Mesh Field Theory.

In Mesh, hidden symmetry detection arises from causal coherence field alignment and interference, rather than amplitude-based probability and Hilbert space projections.

1 Problem Statement in Mesh

Let $f : \{0,1\}^n \to \{0,1\}^n$ be a function such that:

$$f(x) = f(x \oplus s) \quad \text{for a hidden } s \in \{0,1\}^n \setminus \{0^n\}$$

In Mesh Field Theory:

- Inputs x and $x \oplus s$ map to spatial causal coherence fields $\vec{C}_x(x,t)$, $\vec{C}_{x\oplus s}(x,t)$.
- Oracle is realized as a causal field operation enforcing:

$$\vec{C}_f(x) = \vec{C}_f(x \oplus s)$$

2 Initialization: Mesh Superposition Over Input Space

Instead of amplitude superposition, Mesh prepares real causal coherence fields:

$$\vec{C}_{\text{total}}(x) = \sum_{x\in\{0,1\}^n} \vec{C}_x(x,t)$$

- Each coherence region $\vec{C}_x$ is initialized identically.
- Real fields propagate causally.

3 Oracle Action: Hidden Coherence Linking

Oracle enforces real causal linking:

$$\vec{C}_x(x) \equiv \vec{C}_{x\oplus s}(x) \quad \text{for all } x$$

- Input regions with hidden symmetry are physically linked.
- Phase structures are enforced identically.

4 Causal Interference

Interference occurs causally:

$$\mathcal{I}_{xy} = \vec{C}_x(x,t) \cdot \vec{C}_y(x,t)$$

- Constructive interference when $x \oplus y = s$.
- Destructive elsewhere.

Mesh replaces amplitude probabilities with causal overlap energy.

5 Collapse and Measurement

Divergence measurement:

$$\Gamma(x) = \nabla \cdot \vec{C}(x,t) \quad \Rightarrow \quad \text{collapse when } \Gamma(x) > \Gamma_{\text{crit}}$$

Each measurement produces a bit-string $z \in \{0,1\}^n$ satisfying:

$$z \cdot s = 0 \mod 2$$

Each collapse yields a linear constraint on s.

6 Recovery of Hidden Shift

After collecting $n-1$ independent bit-strings $\{z_i\}$, solve:

$$z_i \cdot s = 0 \mod 2$$

The solution yields the hidden symmetry shift s.
This step is classical and unchanged — Mesh simply changes how z_i is obtained.

7 Worked Example: Mesh Simon with $n = 3$, $s = 101$

Oracle links:

$$\vec{C}_{000} = \vec{C}_{101}, \quad \vec{C}_{001} = \vec{C}_{100}, \quad \vec{C}_{010} = \vec{C}_{111}, \quad \vec{C}_{011} = \vec{C}_{110}$$

Mesh coherence fields are prepared and interfered.
Collapse regions produce constraints:

$$z_1 = 011, \quad z_2 = 110$$

Solve:

$$\begin{cases} z_1 \cdot s = 0 \Rightarrow 011 \cdot 101 = 0 \\ z_2 \cdot s = 0 \Rightarrow 110 \cdot 101 = 0 \end{cases} \Rightarrow s = 101$$

8 Summary

In this Mesh mirror of CMU Lecture 08, we established:

- Inputs are encoded as real causal coherence regions.
- Oracle links symmetry regions via real field structure.
- Interference emerges from real causal vector overlaps.
- Measurement arises from divergence collapse, not probability.
- Final solution recovered deterministically from causal constraints.

Thus, Mesh Field Theory causally reconstructs Simon's Algorithm without Hilbert states, probabilistic amplitudes, or tensor product measurements.