Mesh Field Theory – Lecture 04: Mesh Grover's Algorithm (Expanded)

Mirroring CMU Quantum Computation Lecture 04

Introduction

This lecture mirrors the structure of CMU Lecture 04 from the 15-859BB Quantum Computation course at Carnegie Mellon University, but reconstructs Grover's algorithm causally using Mesh Field Theory.

In Mesh, Grover's quadratic speedup arises from real causal coherence dynamics: oracle-based phase inversion, field reflection about causal coherence averages, and deterministic amplification, without Hilbert-space vectors or probabilistic amplitudes.

1 Problem Setup: Causal Coherence Distribution

We are given N causal regions $\{x_1, x_2, \ldots, x_N\}$, each initialized with an equal causal coherence vector:

$$\vec{C}(x_i, t_0) = \vec{C}_0 \quad \forall i$$

where $\vec{C}_0$ is a real spatial vector aligned across the regions.

Interpretation

- Uniform Mesh field distribution replaces abstract quantum state superposition.
- Each region carries real causal energy, not amplitude probability.

2 Oracle Operation: Phase Inversion at Target

Suppose the hidden target location is x_*.

Mesh oracle operation:

$$\phi(x_*) \mapsto \phi(x_*) + \pi \quad \Rightarrow \quad \vec{C}(x_*) \mapsto -\vec{C}(x_*)$$

Properties:

- Only at x_* is the coherence vector inverted.
- All other locations remain unchanged.
- Phase inversion is a real causal operation, not an abstract operator.

3 Diffusion Operation: Reflection About Coherence Average

Compute the global causal coherence average:

$$\langle \vec{C}(t) \rangle = \frac{1}{N} \sum_{i=1}^{N} \vec{C}(x_i, t)$$

Then for each x_i, perform causal reflection:

$$\vec{C}(x_i, t) \mapsto 2\langle\vec{C}(t)\rangle - \vec{C}(x_i, t)$$

Interpretation

- The oracle-inverted coherence vector at x_* will reinforce alignment under reflection.
- Non-inverted vectors will experience destructive reflection.

Thus, causal coherence energy concentrates toward x_*.

4 Iterative Causal Amplification

After one Grover iteration (oracle + diffusion):

- Coherence magnitude at x_* increases. - Coherence magnitudes elsewhere decrease slightly.

Iteratively applying the Mesh Grover step $O(\sqrt{N})$ times yields:

- Maximal causal coherence concentration at x_*. - No probability-based amplitude inflation — only causal field energy flow.

5 Measurement: Causal Divergence Collapse

Monitor divergence:

$$\Gamma(x, t) = \nabla \cdot \vec{C}(x, t)$$

Collapse occurs at x_* when:

$$\Gamma(x_*) > \Gamma_{\text{crit}}$$

Properties:

- Collapse is deterministic, caused by critical causal coherence density.
- No projective measurement or Born rule is required.

6 Worked Example: Single Grover Step in Mesh

Suppose:

- $N = 4$ causal regions.
- Target at x_3.

Initial fields:

$$\vec{C}(x_i, t_0) = (1, 0, 0) \quad \text{for all } i$$

Apply oracle:

$$\vec{C}(x_3) \mapsto (-1, 0, 0)$$

Compute average:

$$\langle\vec{C}(t)\rangle = \frac{(1,0,0) + (1,0,0) + (-1,0,0) + (1,0,0)}{4} = \left(\frac{1}{2}, 0, 0\right)$$

Reflect:

$$\vec{C}(x_i) \mapsto 2\langle\vec{C}(t)\rangle - \vec{C}(x_i)$$

Explicitly:

$$\vec{C}(x_1) = (0,0,0) \quad \vec{C}(x_2) = (0,0,0) \quad \vec{C}(x_3) = (2,0,0) \quad \vec{C}(x_4) = (0,0,0)$$

Thus, coherence energy concentrates at x_3 after one iteration.

7 Summary

In this Mesh mirror of CMU Lecture 04, we established:

- Uniform causal coherence fields replace quantum uniform superpositions.
- Oracle acts by causal phase inversion, not abstract operators.
- Diffusion reflects causal fields about real coherence averages.
- Iterative amplification concentrates real field energy at the solution location.
- Measurement is a deterministic consequence of causal divergence collapse.

Thus, Mesh Field Theory causally reconstructs Grover's algorithm, preserving its operational speedup without Hilbert spaces or probabilistic assumptions.