4^{nd} Homework

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Exercise 1

a) $E_D[(f(x;D) - E[y|x])^2] = 0 \Rightarrow \sigma_n^2 = 0 \Rightarrow n = 0$

b) In practice, this is impossible since we always have nose.

Exercise 2

a) Large error on training set indicates that the model complexity is too low to explain the data.
b) Large error on test set indicates that the model is more complex than the model that generate the data.
c) The predictions of that model are biased.
d) The dataset has no variable.

Exercise 3

a) $\int_0^1 \left(\int_{x^2}^1 \frac{3}{2} dy \right) dx \; = \; \int_0^1 \frac{3}{2} [y]_{x^2}^1 dx \; = \; \int_0^1 \frac{3}{2}(1 - x^2) dx = \frac{3}{2}[x]_0^1 - \frac{3}{2}\left[\frac{x^3}{3}\right]_0^1 = \frac{3}{2} - \frac{3}{2}\frac{1}{3} = 1$

b) $p_x(x) = \int_{x^2}^1 \frac{3}{2} dy = \frac{3}{2}[y]_{x^2}^1 = \frac{3}{2}(1 - x^2)$

c) $P_{y|x}(y|x) = \frac{p(x,y)}{p_x(x)} = \frac{\frac{3}{2}}{\frac{3}{2}(1-x^2)} = \frac{6}{6(1-x^2)} = \frac{1}{1-x^2}$

d) $E[y|x] = \int_{x^2}^1 yp(y|x)\, dy = \int_{x^2}^1 y \frac{1}{1-x^2} dy = \frac{1}{1-x^2}\left[\frac{y^2}{2}\right]_{x^2}^1 = \frac{1-x^4}{2(1-x^2)}$

Exercise 4

a) $g(x) = E[y|x] = \mu_y + \frac{a\sigma_y}{\sigma_x}(x - \mu_x) = 2 + \frac{1}{2}(x - 2) = 0.5x + 1$

f) From our experiment, we conclude that when the size of dataset is 5000, we get estimates closer to the optimum mean square error and variance of LS estimates is lower than that of 50 data points. The variance term has decreased but the bias term has increased in the following equation.

$$E_D[(f(x;D) - E[y|x])^2] = E_D[(f(x;D) - E_D[f(x;D)])^2] + (E_D[f(x;D] - E[y|x])^2$$