6th Homework

Thomas Saltos

Exercise 1

The Lagrangian function is:

$$L(P_1, P_2, \dots, P_m) = \sum_{i=1}^{N} \sum_{j=1}^{m} P(j|x_i) \ln P_j + \lambda \left(\sum\nolimits_{j=1}^{m} P_j - 1 \right)$$

$$\frac{\partial L(P_1, P_2, \dots, P_m)}{\partial P_j} = 0 \Rightarrow \sum_{i=1}^{N} \sum_{j=1}^{m} P(j|x_i) \frac{1}{P_j} + \lambda = 0$$

For P_1 the last equation is $P_1 = -\frac{\sum_{i=1}^{N} P(1|x_i)}{\lambda}$

For P_2 the last equation is $P_2 = -\frac{\sum_{i=1}^{N} P(2|x_i)}{\lambda}$

And P_m the last equation is $P_m = -\frac{\sum_{i=1}^{N} P(m|x_i)}{\lambda}$

$$\sum\nolimits_{j=1}^{m} P_j = 1 \Rightarrow -\frac{\sum_{i=1}^{N} P(1|x_i)}{\lambda} - \frac{\sum_{i=1}^{N} P(2|x_i)}{\lambda} - \dots - \frac{\sum_{i=1}^{N} P(m|x_i)}{\lambda} = 1 \Rightarrow$$

$$\lambda = -\sum\nolimits_{i=1}^{N} \sum\nolimits_{j=1}^{m} P(j|x_i)$$

We have that: $\sum_{i=1}^{N} \sum_{j=1}^{m} P(j|x_i) \frac{1}{P_j} - \sum_{i=1}^{N} \sum_{j=1}^{m} P(j|x_i) = 0 \Rightarrow P_j = \frac{\sum_{i=1}^{N} P(j|x_i)}{N}$

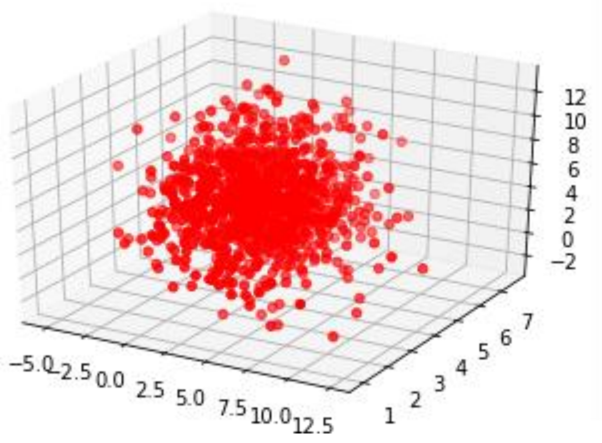
Exercise 2

a)

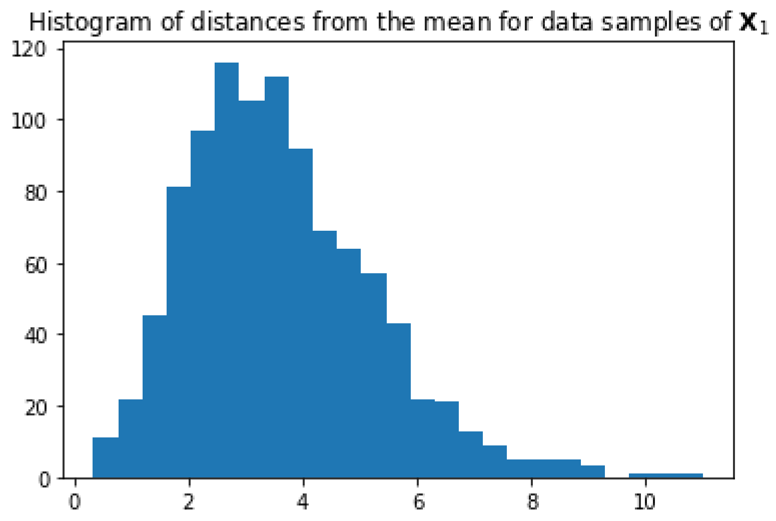
```
import scipy.io as sio
import numpy as np
import matplotlib.pyplot as plt

import numpy.matlib
from scipy.spatial import distance
from mpl_toolkits.mplot3d import Axes3D
from scipy.stats import multivariate_normal, norm

Dataset = sio.loadmat('Dataset.mat')
X1 = Dataset['X1']
```

```
X2 = Dataset['X2']
```

```
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
ax.scatter(X1[:,1],X1[:,2],X1[:,3],c='r', marker='o')
plt.show()
```



```
x1_mean = np.mean(X1,axis=0)
Dista = np.zeros(len(X1))
for i in range(len(X1)):
   Dista[i] = np.linalg.norm(X1[i,:] - x1_mean)

plt.hist(Dista,bins='auto')
plt.title('Histogram of distances from the mean for data samples of $\mathbf{X}_1$')
plt.show()
```



```
mean_X1 = np.mean(X1,axis=0)
N1 = len(X1)
Cov_X1 = (1/N1)*(((X1 - np.matlib.repmat(mean_X1,N1,1)).T).dot((X1 -
np.matlib.repmat(mean_X1,N1,1))))
mean_X1
```

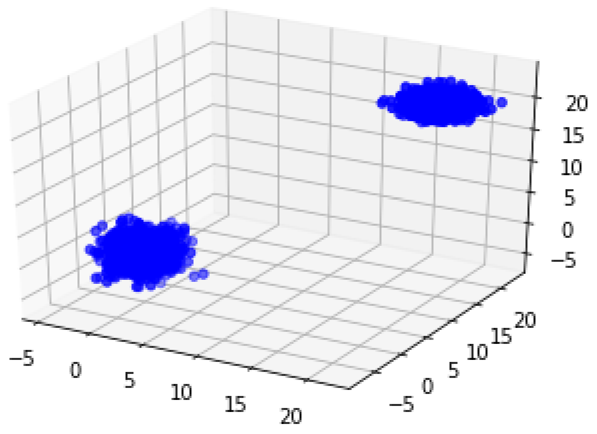
```
array([1.88364427, 2.94930523, 3.94277602, 4.91643708])
```

```
X1_test = np.array([[2.01,2.99,3.98,5.02],[20.78,-15.76,19.38,-25.02],[3.08,3.88,4.15,6.02]])
P1 = multivariate_normal.pdf(X1_test,mean_X1,Cov_X1)
P1
```

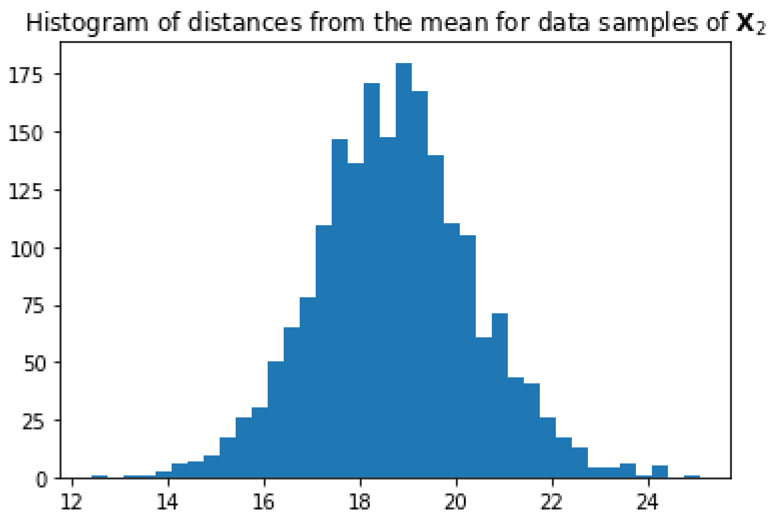
```
array([3.37904075e-003, 1.65657893e-208, 1.95207559e-003])
```

b)

```
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
ax.scatter(X2[:,1],X2[:,2],X2[:,3],c='b', marker='o')
plt.show()
```



```
x2_mean = np.mean(X2,axis=0)
Dista2 = np.zeros(len(X2))
for i in range(len(X2)):
    Dista2[i] = np.linalg.norm(X2[i,:] - x2_mean)

plt.hist(Dista2,bins='auto')
plt.title('Histogram of distances from the mean for data samples of $\mathbf{X}_2$')
plt.show()
```



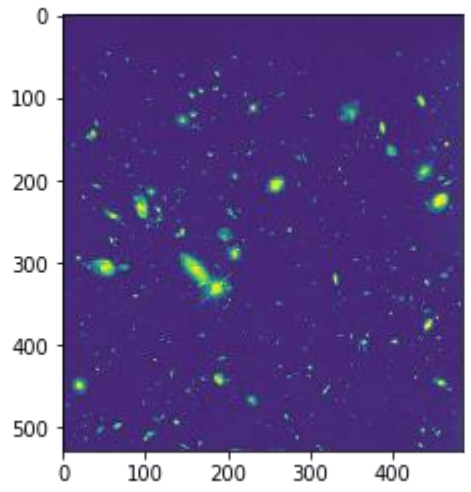
```
from sklearn import mixture
clf = mixture.GaussianMixture(n_components=2, covariance_type='full')
clf.fit(X2)
X2_test = np.array([[0.05, 0.15,-0.12,-0.08],[7.18, 7.98,9.12,9.94],[3.48,4.01,4.55,4.96],[20.78,-15.76,19.38,-25.02]])
P2 = np.exp(clf.score_samples(X2_test))
P2
```

```
array([1.57831095e-003, 1.80488930e-024, 8.69261803e-009,
7.60251661e-105])
```

Exercise 3

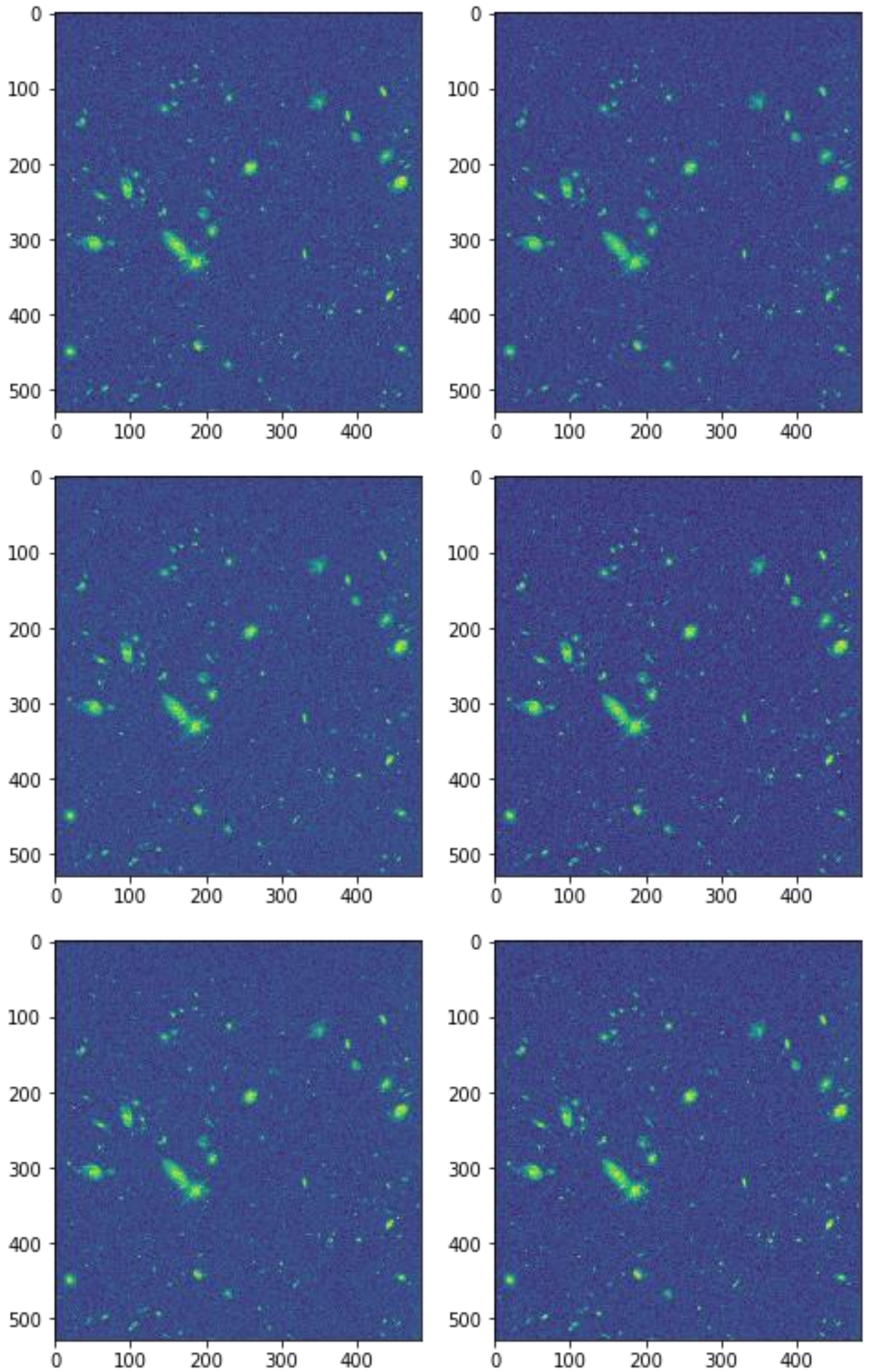
a)

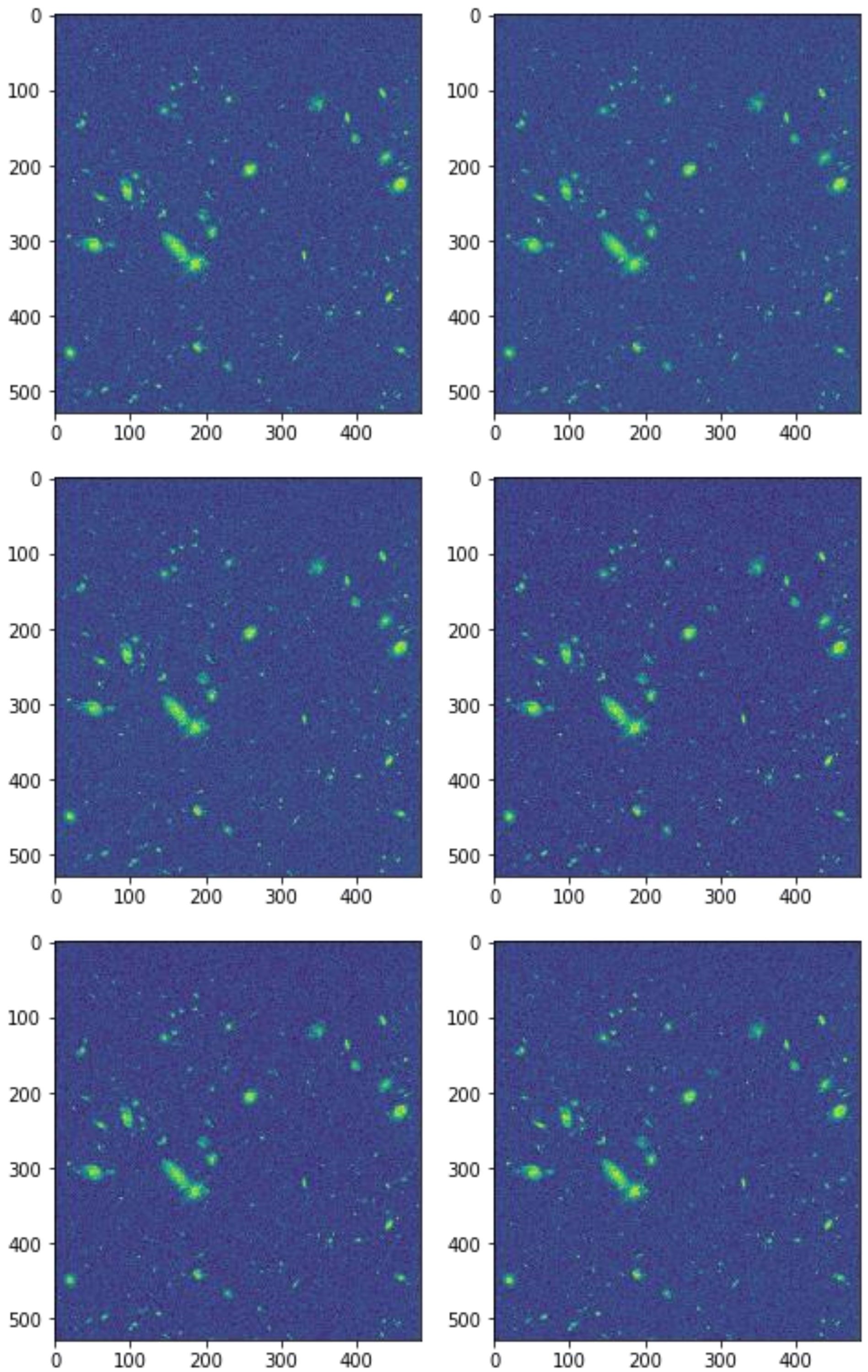
```
import matplotlib.image as mpimg
A = mpimg.imread('huble_tel.jpg')
plt.imshow(A)
plt.show()
```

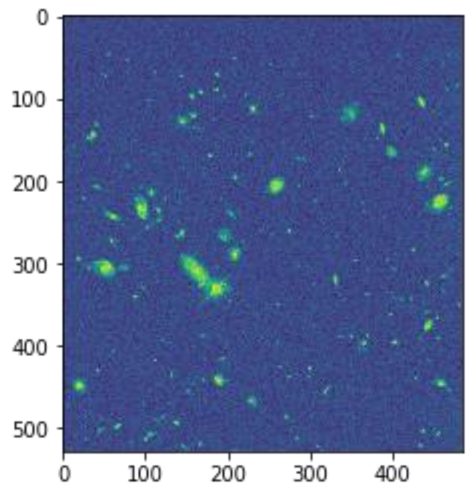


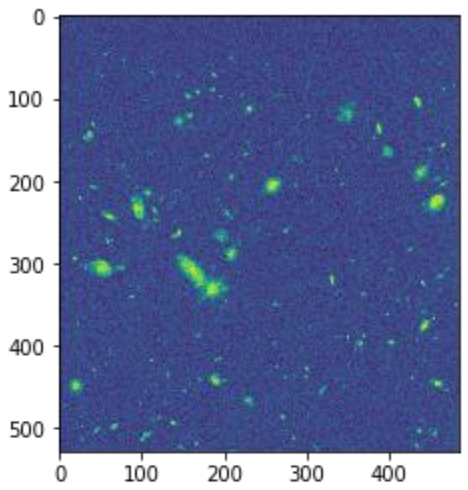
b) Production of 15 noisy versions of the image adding Gaussian noise with zero mean and variance 256.

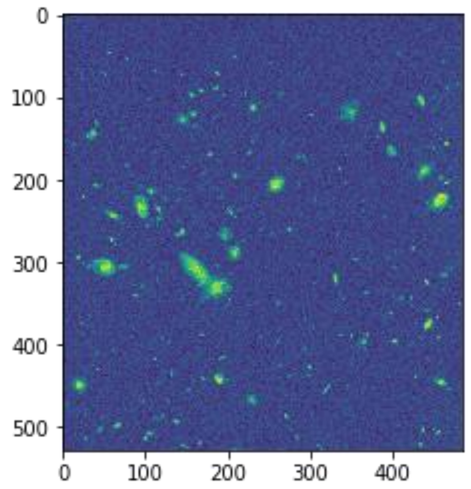
```
from numpy import empty

C_sum = empty(A.shape)

for i in range(15):
   B_i=np.sqrt(256)*np.random.normal(0,1,A.shape) #gaussian noise matrix
   C_i=A+B_i #noisy version of A
   plt.imshow(C_i)
   plt.show()
   C_sum += C_i  # the sum will be used below to compute the average of the noisy versions
```


0
100
200
300
400
500


0
100
200
300
400
500

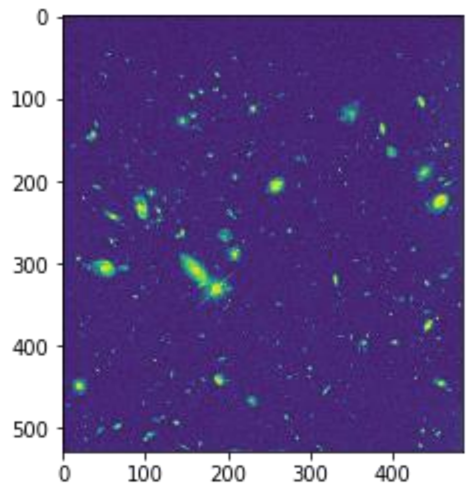






c) Average of the 15 noisy versions of A.

```
C_avg = C_sum/15
plt.imshow(C_avg)
plt.show()
```



The averaged image over the 15 noisy versions of the original image is clearly less noisy than the noisy versions. Compared with the original image, noise is still present in the averaged image but has been efficiently suppressed considering from where we started. The above method is in fact the ML method applied for 15 different data sets assuming the data were generated by a normal distribution of unknown mean. Improved noise suppression/removal can be achieved if we increase the number of available noisy images.